

2 Overdetermined System:

the number of equations are more than number of unknowns.

- overdetermined system are usually (but not always) inconsistent.

do not usually have solutions

ex

$$\begin{cases} x_1 + x_2 = 1 \\ x_1 - x_2 = 0 \\ 3x_1 + x_2 = 10 \end{cases} \begin{matrix} 2 \times 2 \\ n \times n \end{matrix} \quad \begin{matrix} 3 \times 2 \\ \text{over} \end{matrix}$$

ex solve the linear system.
(use Gaussian elimination):

(a) $x_1 + x_2 = 1$
 $x_1 - x_2 = 3$
 $-x_1 + 2x_2 = -2$

\swarrow pivot = 1

sol:

$$\left[\begin{array}{cc|c} 1 & 1 & 1 \\ 1 & -1 & 3 \\ -1 & 2 & -2 \end{array} \right] \begin{matrix} -R_1 + R_2 \\ R_1 + R_2 \end{matrix} \rightarrow \left[\begin{array}{cc|c} 1 & 1 & 1 \\ 0 & -2 & 2 \\ 0 & 3 & -1 \end{array} \right] \begin{matrix} \frac{-1}{2} R_2 \\ \end{matrix}$$

$$\left[\begin{array}{cc|c} 1 & 1 & 1 \\ 0 & -2 & 2 \\ 0 & 3 & -1 \end{array} \right] \rightarrow \left[\begin{array}{cc|c} 1 & 1 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 2 \end{array} \right] \rightarrow \left[\begin{array}{cc|c} 1 & 1 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{array} \right]$$

اكتب فيه المتغيرات
 $0 \cdot x_1 + 0 \cdot x_2 = 1 \Rightarrow 0 = 1$

$$\Rightarrow \boxed{b=1} \Rightarrow$$

this system has no solution (inconsistent)
لا يوجد حل

$$\left[\begin{array}{cccc|c} 0 & 0 & \dots & 0 & \text{number not equal to zero} \end{array} \right] \Rightarrow \text{inconsistent}$$

ex (6)

$$\begin{aligned} x_1 + 2x_2 + x_3 &= 1 \\ 2x_1 - x_2 + x_3 &= 2 \\ 4x_1 + 3x_2 + 3x_3 &= 4 \\ 2x_1 - x_2 + 3x_3 &= 5 \end{aligned}$$

sol:

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 2 & -1 & 1 & 2 \\ 4 & 3 & 3 & 4 \\ 2 & -1 & 3 & 5 \end{array} \right] \xrightarrow{\text{R.E.F}} \left[\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 5 \\ 0 & 0 & 0 & 3 \end{array} \right]$$

حل النظام
المتساوية

$$\boxed{x_3 = \frac{3}{2}} \rightarrow \frac{15}{10}$$

$$x_2 + \frac{1}{5}x_3 = 0$$

$$x_2 + \frac{1}{5}\left(\frac{3}{2}\right) = 0 \rightarrow x_2 + \frac{3}{10} = 0$$

$$\boxed{x_2 = -\frac{3}{10}}$$

$$x_1 + 2x_2 + x_3 = 1$$

$$x_1 + 2\left(-\frac{3}{10}\right) + \frac{3}{2} = 1$$

$$x_1 - \frac{6}{10} + \frac{15}{10} = \frac{10}{10} \rightarrow \boxed{x_1 = \frac{1}{10}}$$

Sol: $\left(\frac{1}{10}, -\frac{3}{10}, \frac{15}{10}\right) \rightarrow$ unique sol.

$$\text{ex (3)} \quad \begin{aligned} x_1 + 2x_2 + x_3 &= 1 \\ 2x_1 - x_2 + x_3 &= 2 \\ 4x_1 + 3x_2 + 3x_3 &= 4 \\ 3x_1 + x_2 + 2x_3 &= 3 \end{aligned}$$

sol:

$$\left[\begin{array}{ccc|c} \textcircled{1} & 2 & 1 & 1 \\ 2 & -1 & 1 & 2 \\ 4 & 3 & 3 & 4 \\ 3 & 1 & 2 & 3 \end{array} \right] \rightarrow \left[\begin{array}{ccc|c} \textcircled{1} & 2 & 1 & 1 \\ 0 & -5 & -1 & 0 \\ 0 & -5 & -1 & 0 \\ 0 & -5 & -1 & 0 \end{array} \right]$$

$$\xrightarrow{-\frac{1}{5}R_2} \left[\begin{array}{ccc|c} \textcircled{1} & 2 & 1 & 1 \\ 0 & \textcircled{-5} & -1 & 0 \\ 0 & -5 & -1 & 0 \\ 0 & -5 & -1 & 0 \end{array} \right] \rightarrow \begin{array}{ccc|c} x_1 & x_2 & x_3 & \\ \textcircled{1} & 2 & 1 & 1 \\ 0 & \textcircled{-5} & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}$$

← R.EF

variables: x_1, x_2, x_3
lead variables: x_1, x_2

free variables: $x_3 \Rightarrow \boxed{x_3 = \alpha}, \alpha \in \mathbb{R}$
متغيره الامدادية

\Rightarrow this system has infinitely many solutions
بسبب وجود free

$$\boxed{x_3 = \alpha}$$

$$x_2 + \frac{1}{5}x_3 = 0 \Rightarrow x_2 + \frac{1}{5}\alpha = 0 \Rightarrow \boxed{x_2 = -\frac{\alpha}{5}}$$

$$x_1 + 2x_2 + x_3 = 1 \Rightarrow x_1 + 2\left(-\frac{\alpha}{5}\right) + \alpha = 0$$

$$\rightarrow x_1 - \frac{2\alpha}{5} + \frac{5\alpha}{5} = 0 \rightarrow \boxed{x_1 = -\frac{3\alpha}{5}}$$

$$\text{sol: } \left(-\frac{3}{5}\alpha, -\frac{\alpha}{5}, \alpha\right)$$

عدد قيم α من اجل

$$\begin{cases} \alpha = 0 \\ (0, 0, 0) \\ \alpha = 1 \\ \left(-\frac{3}{5}, -\frac{1}{5}, 1\right) \\ \left(-3, -1, 5\right) \dots \end{cases}$$