

Underdetermined System:

number of equations are less than
number of unknowns.

ex $x_1 + x_2 - x_3 = 0$
 $2x_1 + x_2 + x_3 = 5$

2x3 sys.

note:

- ① underdetermined systems are usually consistent.
- ② It is not possible for underdetermined system to have a unique solution.

not always true

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 2 & 1 & 1 & 5 \end{array} \right]$$

max \rightarrow 2 pivots.

max. \rightarrow 2 lead variable.

min \rightarrow 1 free variable

\Rightarrow infinitely many sol.

③ underdetermined system.

①
inconsistent

②
infinitely many solutions.

ex solve

$$\textcircled{a} \quad x_1 + 2x_2 + x_3 = 1$$

$$2x_1 + 4x_2 + 2x_3 = 3$$

sol:
$$\left[\begin{array}{ccc|c} \textcircled{1} & 2 & 1 & 1 \\ 2 & 4 & 2 & 3 \end{array} \right] \rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{array} \right]$$

$[0 \ 0 \ 0 \ | \ 1] \Rightarrow$ this system is inconsistent.

$$\textcircled{b} \quad x_1 + x_2 + x_3 + x_4 + x_5 = 2$$

$$x_1 + x_2 + x_3 + 2x_4 + 2x_5 = 3$$

$$x_1 + x_2 + x_3 + 2x_4 + 3x_5 = 2$$

sol:
$$\left[\begin{array}{ccccc|c} \textcircled{1} & 1 & 1 & 1 & 1 & 2 \\ 1 & 1 & 1 & 2 & 2 & 3 \\ 1 & 1 & 1 & 2 & 3 & 2 \end{array} \right] \rightarrow \left[\begin{array}{ccccc|c} \textcircled{1} & 1 & 1 & 1 & 1 & 2 \\ 0 & 0 & 0 & \textcircled{1} & 1 & 1 \\ 0 & 0 & 0 & 1 & 2 & 0 \end{array} \right]$$

$$\left[\begin{array}{ccccc|c} \textcircled{1} & x_2 & x_3 & x_4 & x_5 & \\ 1 & 1 & 1 & 1 & 1 & 2 \\ 0 & 0 & 0 & \textcircled{1} & 1 & 1 \\ 0 & 0 & 0 & 0 & \textcircled{1} & -1 \end{array} \right]$$

free variables:
 x_2, x_3

$$\Rightarrow \begin{cases} x_2 = \alpha \\ x_3 = \beta \end{cases} \quad \alpha, \beta \in \mathbb{R}$$

Infinity
many
sol.

$$\boxed{x_5 = -1}$$

$$x_4 + x_5 = 1$$

$$x_4 + -1 = 1 \rightarrow \boxed{x_4 = 2}$$

$$x_1 + x_2 + x_3 + x_4 + x_5 = 2$$

$$x_1 + \alpha + \beta + 2 + -1 = 2$$

$$\boxed{x_1 = 1 - \alpha - \beta}$$

solution: $(1 - \alpha - \beta, \alpha, \beta, 2, -1)$

this system has infinitely
many solutions.
