

Arithmetic Circuits

BCD addition

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BCD Adder Circuit

The digital systems handles the decimal number in the form of binary coded decimal numbers (BCD). A BCD Adder Circuit that adds two BCD digits and produces a sum digit also in BCD. BCD numbers use 10 digits, 0 to 9 which are represented in the binary form 0 0 0 0 to 1 0 0 1, i.e. each BCD digit is represented as a 4-bit binary number. When we write BCD number say 526, it can be represented as

5	2	6
↓	↓	↓
0 1 0 1	0 0 1 0	0 1 1 0

The addition of two BCD numbers can be best understood by considering the three cases that occur when two BCD digits are added.

Sum Equals 9 or less with carry 0

Let us consider additions of 3 and 6 in BCD.

$$\begin{array}{r} 6 \\ + 3 \\ \hline 9 \end{array} \quad \begin{array}{r} 0110 \\ 0011 \\ \hline 1001 \end{array} \quad \begin{array}{l} \leftarrow \text{BCD for 6} \\ \leftarrow \text{BCD for 3} \\ \leftarrow \text{BCD for 9} \end{array}$$

The addition is carried out as in normal binary addition and the sum is 1 0 0 1, which is BCD code for 9.

Sum greater than 9 with carry 0

Let us consider addition of 6 and 8 in BCD

6	0	1	1	0	← BCD for 6
+ 8	1	0	0	0	← BCD for 8
<hr/>	<hr/>				
14	1	1	1	0	← Invalid BCD number

The sum 1 1 1 0 is an invalid BCD number. This has occurred because the sum of the two digits exceeds 9. Whenever this occurs the sum has to be corrected by the addition of six (0110) in the invalid BCD number, as shown below

$\begin{array}{r} 6 \\ + 8 \\ \hline 14 \end{array}$	$\begin{array}{r} 0\ 1\ 1\ 0 \\ 1\ 0\ 0\ 0 \\ \hline 1\ 1\ 1\ 0 \\ 0\ 1\ 1\ 0 \\ \hline 0\ 1\ 0\ 0 \end{array}$	<p>← BCD for 6</p> <p>← BCD for 8</p> <p>← Invalid BCD number</p> <p>← Add 6 for correction</p>
$\underbrace{0\ 0\ 0\ 1}_1$	$\underbrace{0\ 1\ 0\ 0}_4$	<p>← BCD for 14</p>

After addition of 6 carry is produced into the second decimal position.

Sum equals 9 or less with carry 1

Let us consider addition of 8 and 9 in BCD

$$\begin{array}{r}
 8 \\
 + 9 \\
 \hline
 17
 \end{array}
 \qquad
 \begin{array}{r}
 1\ 0\ 0\ 0 \\
 1\ 0\ 0\ 1 \\
 \hline
 0\ 0\ 0\ 1\ 0\ 0\ 0\ 1
 \end{array}
 \begin{array}{l}
 \leftarrow \text{BCD for 8} \\
 \leftarrow \text{BCD for 9} \\
 \leftarrow \text{Incorrect BCD result}
 \end{array}$$

In this, case, result (0001 0001) is valid BCD number, but it is incorrect. To get the correct BCD result correction factor of 6 has to be added to the least significant digit sum, as shown below

$$\begin{array}{r}
 8 \\
 + 9 \\
 \hline
 17
 \end{array}
 \qquad
 \begin{array}{r}
 1\ 0\ 0\ 0 \\
 1\ 0\ 0\ 1 \\
 \hline
 0\ 0\ 0\ 1\ 0\ 0\ 0\ 1 \\
 +\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 0 \\
 \hline
 0\ 0\ 0\ 1\ 0\ 1\ 1\ 1
 \end{array}
 \begin{array}{l}
 \leftarrow \text{BCD for 8} \\
 \leftarrow \text{BCD for 9} \\
 \leftarrow \text{Incorrect BCD result} \\
 \leftarrow \text{Add 6 for correction} \\
 \leftarrow \text{BCD for 17}
 \end{array}$$

Going through these three cases of BCD addition we can summarize the BCD addition procedure as follows :

- 1- Add two BCD numbers using ordinary binary addition.
- 2- If four-bit sum is equal to or less than 9, no correction is needed. The sum is in proper BCD form.
- 3- If the four-bit sum is greater than 9 or if a carry is generated from the four-bit sum, the sum is invalid.
- 4- To correct the invalid sum, add 0110_2 to the four-bit sum. If a carry results from this addition, add it to the next higher-order BCD digit.

Adding Circuits for the 8421-BCD Code

Circuit One: using two 4-bit binary adder and logic gate

4-bit binary adder for initial addition

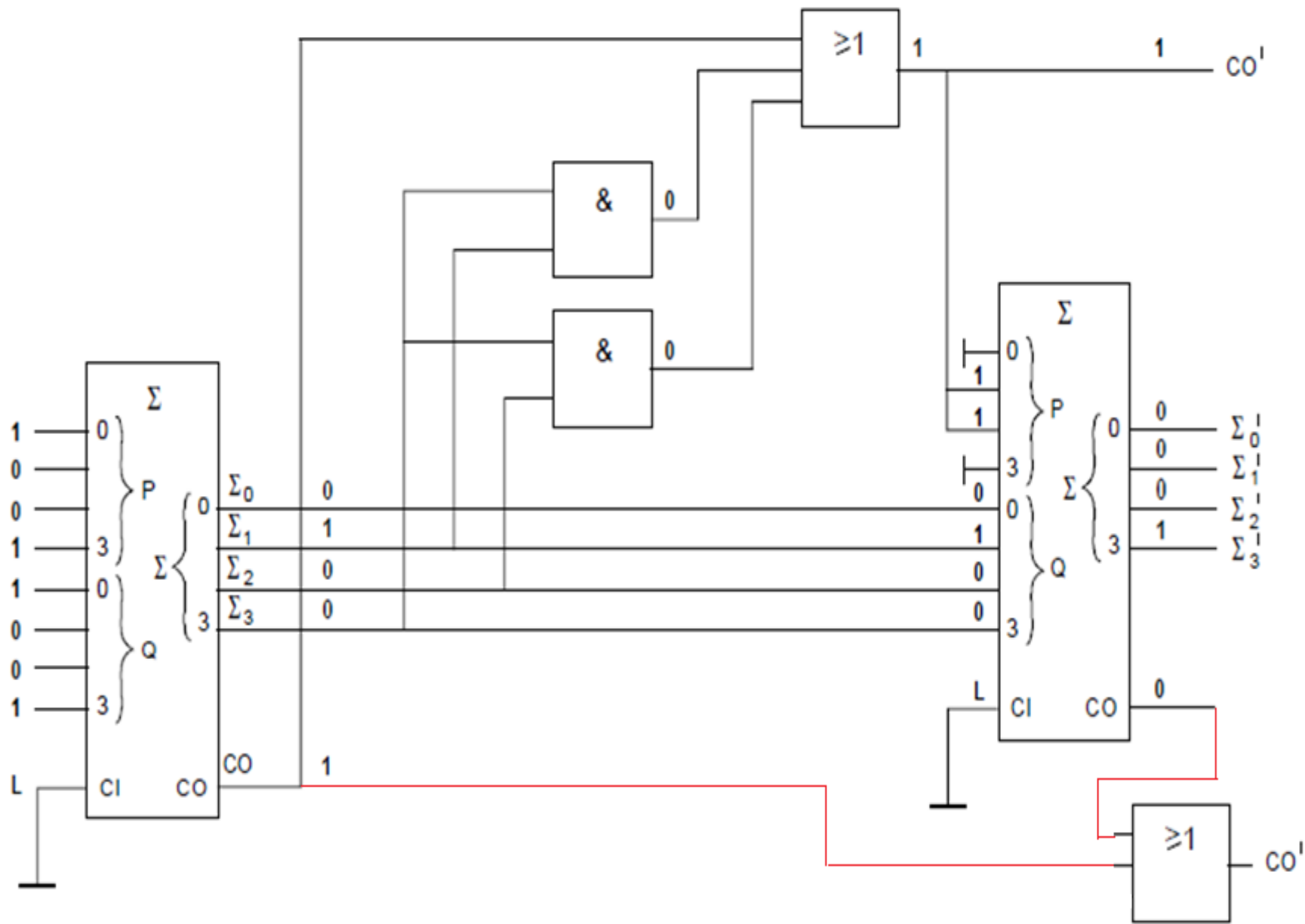
Logic circuit to detect sum greater than 9

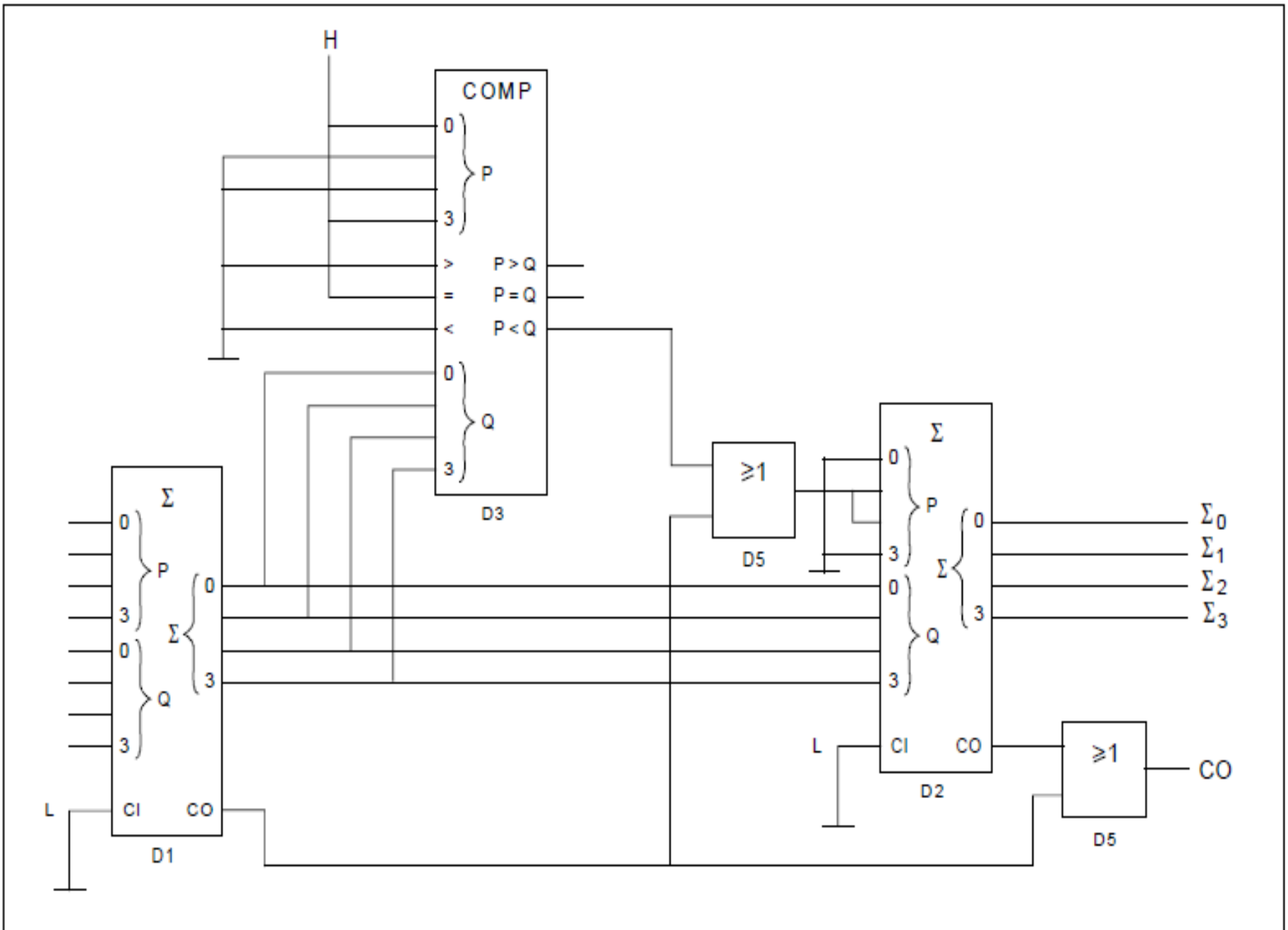
4-bit adder to add 0110_2 in the sum if sum is greater than 9 or carry is 1.

Circuit Two : using two 4-bit binary adder and comparator

The subtotal of the first adder is compared in the comparator with the permanently applied number 9.

if sum is greater than 9 or carry is 1, this must be corrected with a second adder





□ Experiment 3: Addition with the aid of a 4-bit number comparator

Addition task	Full adder D1													Compara- tor D3	Full adder D2									
	P ₃	P ₂	P ₁	P ₀	Q ₃	Q ₂	Q ₁	Q ₀	CO	Σ ₃	Σ ₂	Σ ₁	Σ ₀		P ₃	P ₂	P ₁	P ₀	CO	Σ ₃	Σ ₂	Σ ₁	Σ ₀	
4 + 3	0	1	0	0	0	0	1	1	0	0	1	1	1	0	0	0	0	0	0	0	1	1	1	
8 + 4	1	0	0	0	0	1	0	0	0	1	1	0	0	1	0	1	1	0	1	0	0	1	0	
8 + 8	1	0	0	0	1	0	0	0	1	0	0	0	0	0	0	1	1	0	1	0	1	1	0	
9 + 9	1	0	0	1	1	0	0	1	1	0	0	1	0	0	0	1	1	0	1	1	0	0	0	

Table 6.2.3.2