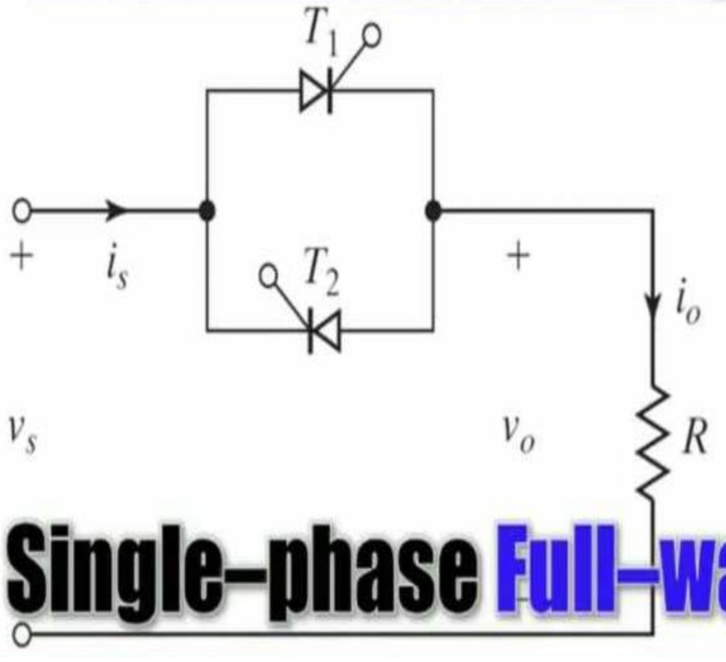
AC TO AC CONVERETERS

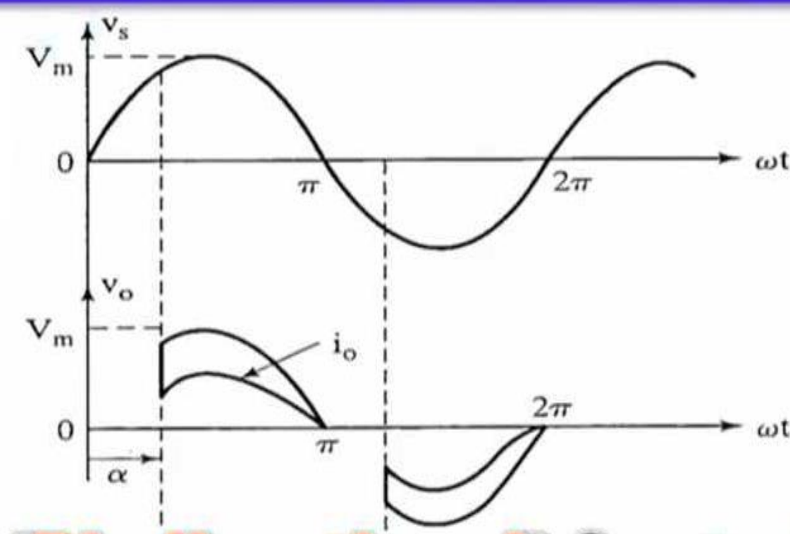
SINGLE PHASE AC VOLTAGE REGULATORS

Eng :Eman Abu Hany

AC – AC Converter

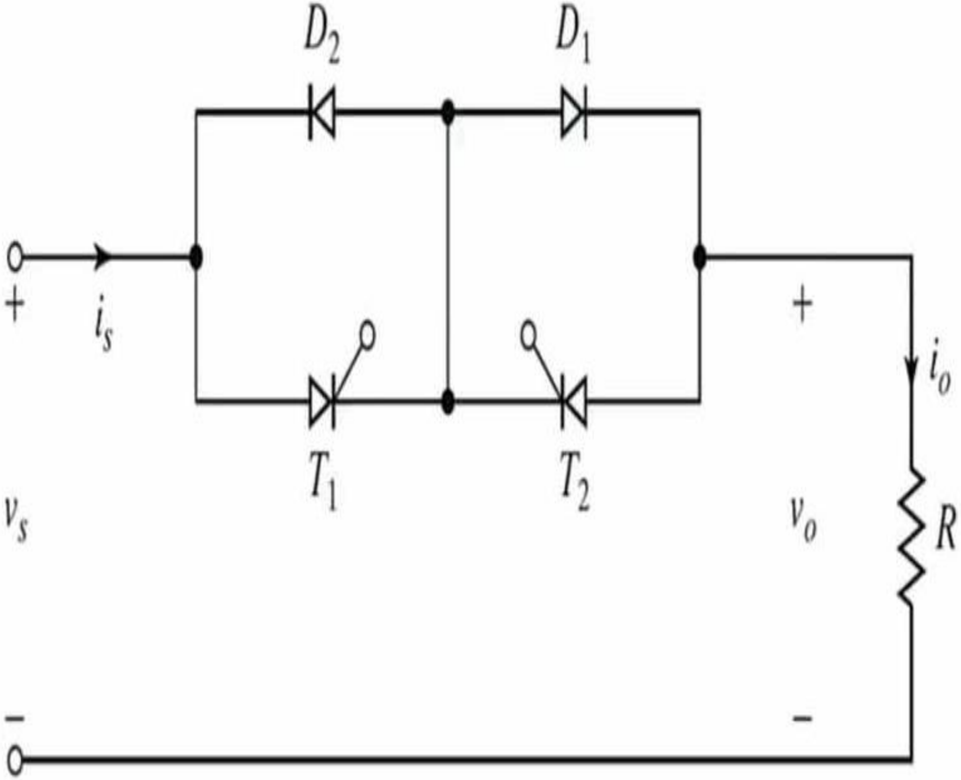
AC Voltage Controllers: Phase Control



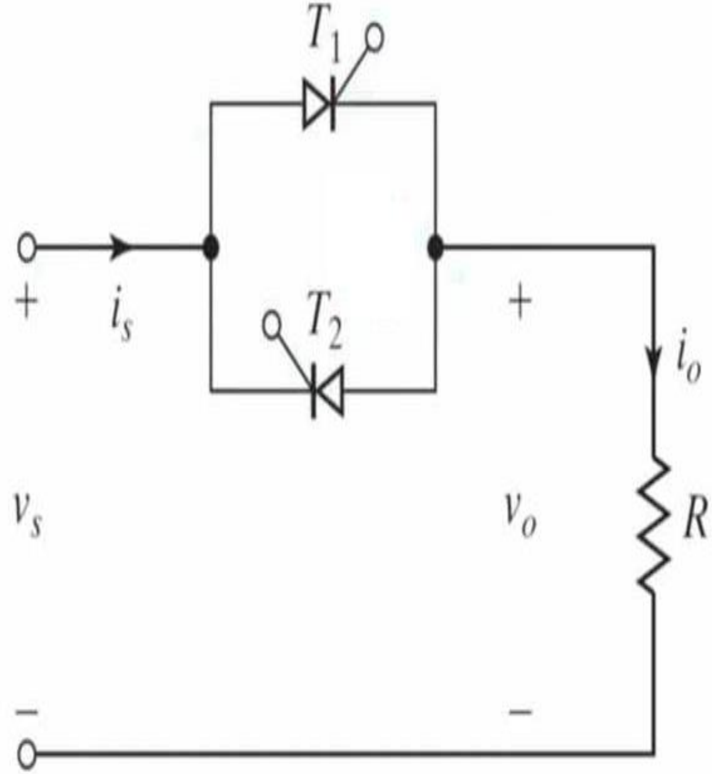


Single–phase Full–wave (Bi–directional) Controller

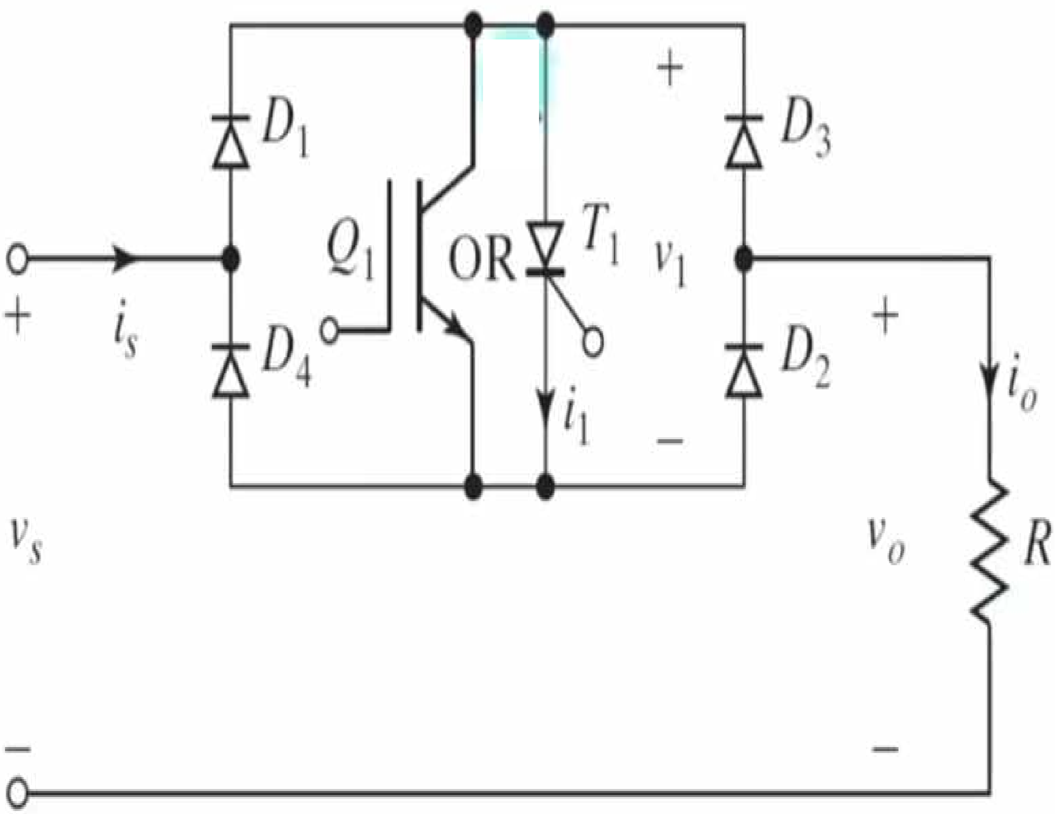
- The principle of phase control can be explained with the following single-phase full-wave (bi-directional) controller
- **For Resistive Load:**

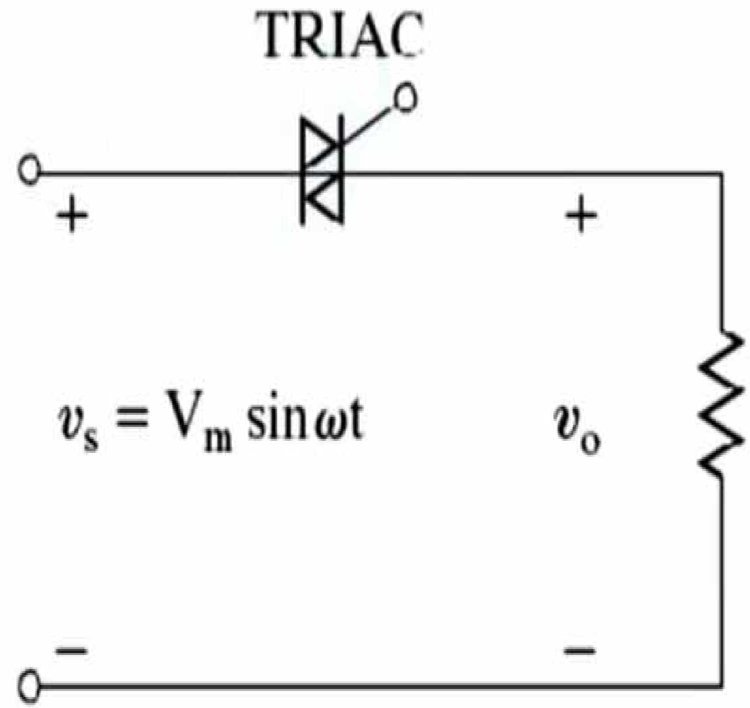


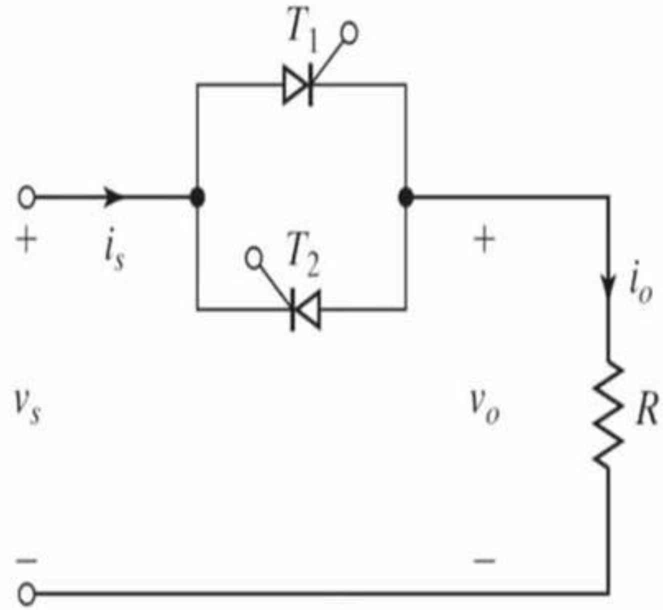
With single common cathode



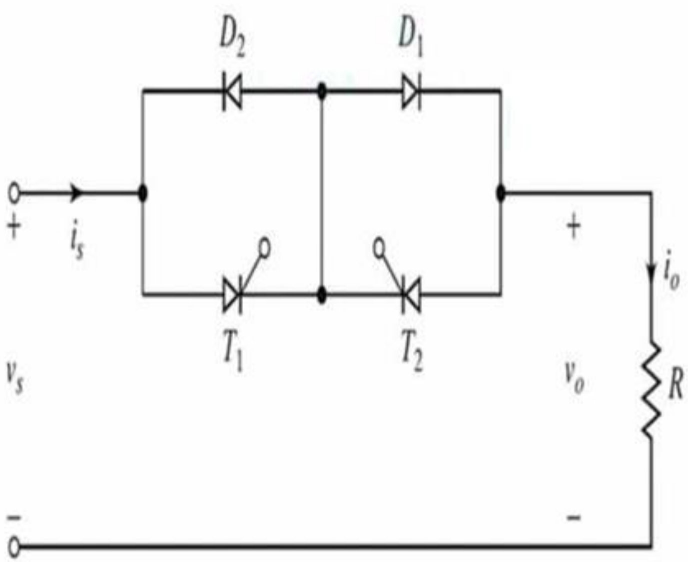
Principle of Phase Control



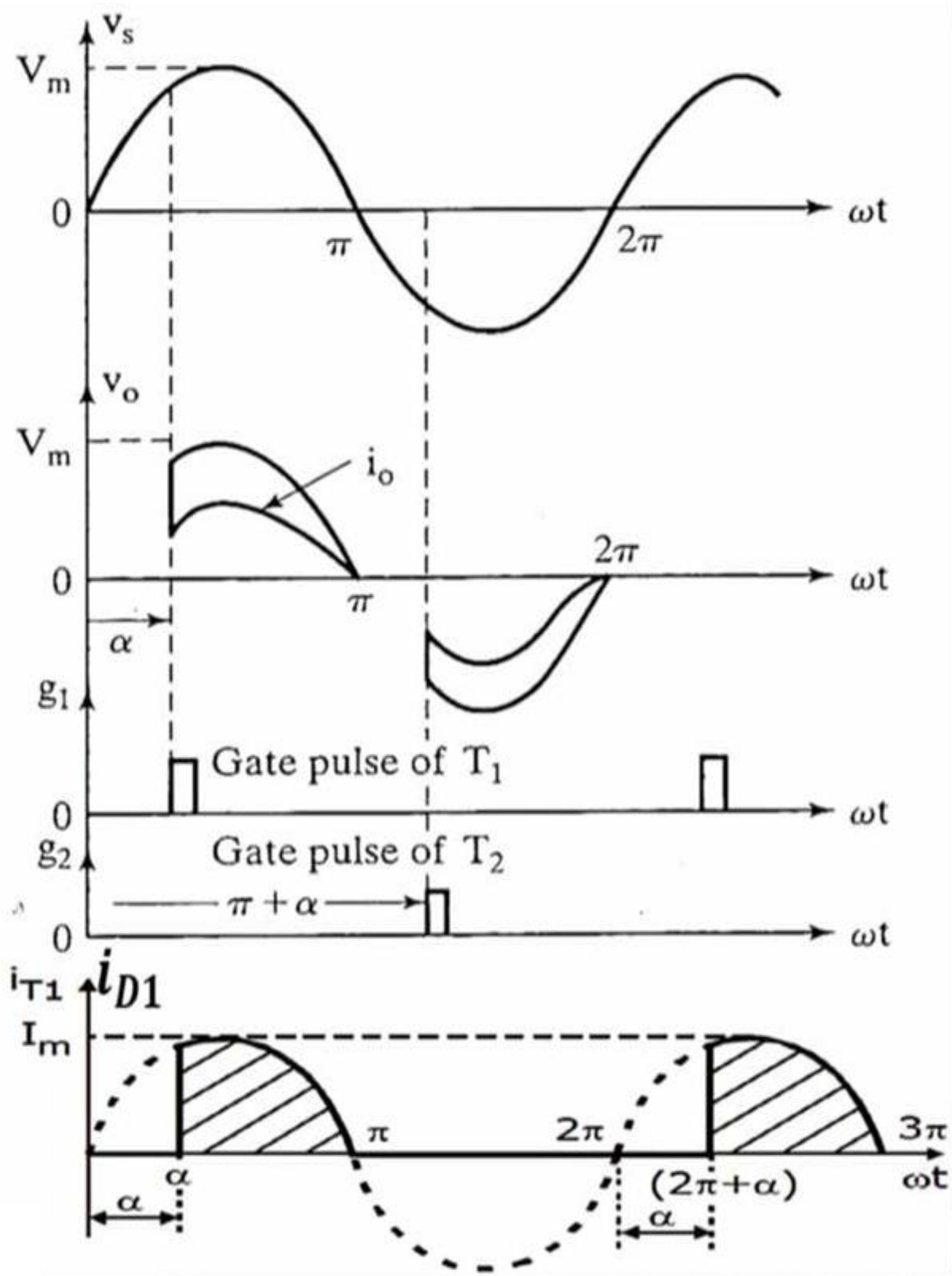




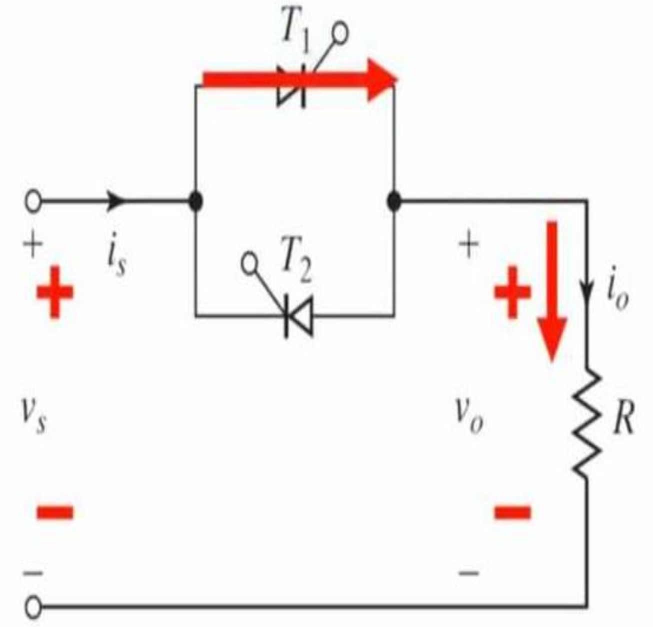
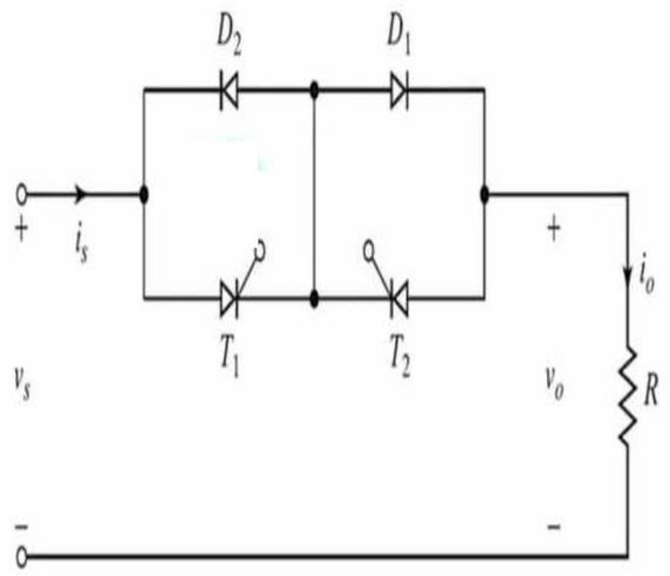
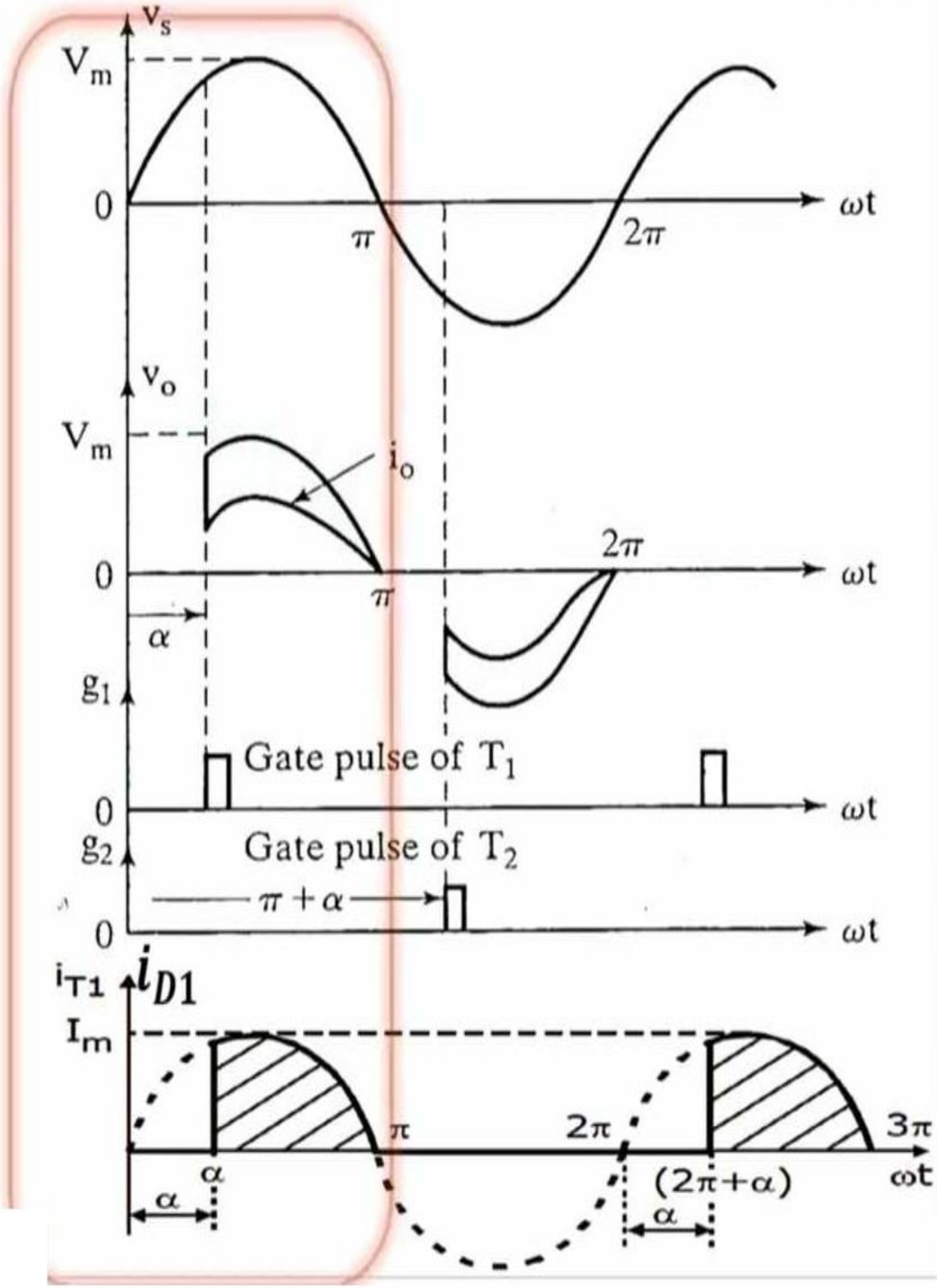
T1
T2
+
is
+
io
vs
vo
R
–
–

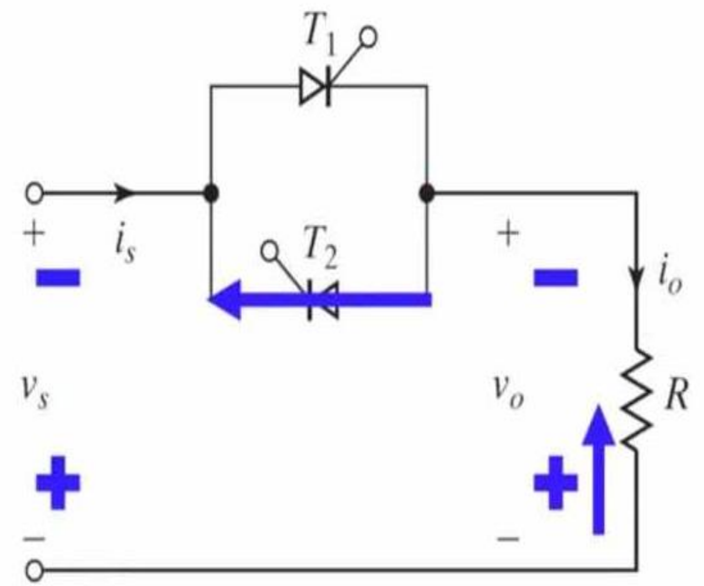


D2
D1
+
is
+
io
T1
T2
vs
vo
R
-
-

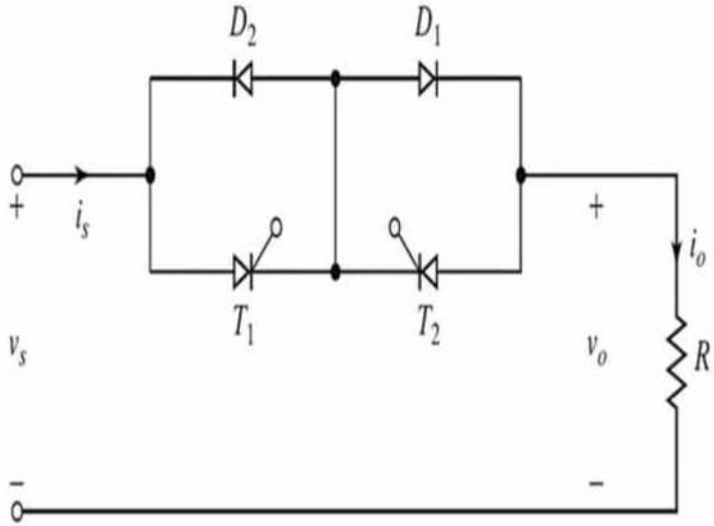


vs
Vm
0
π
2π
ωt
vo
Vm
io
2π
0
π
ωt
α
g1
Gate pulse of T1
0
ωt
g2
Gate pulse of T2
π + α
0
ωt
iT1
iD1
Im
π
2π
3π
α
(2π+α)
ωt
α
α

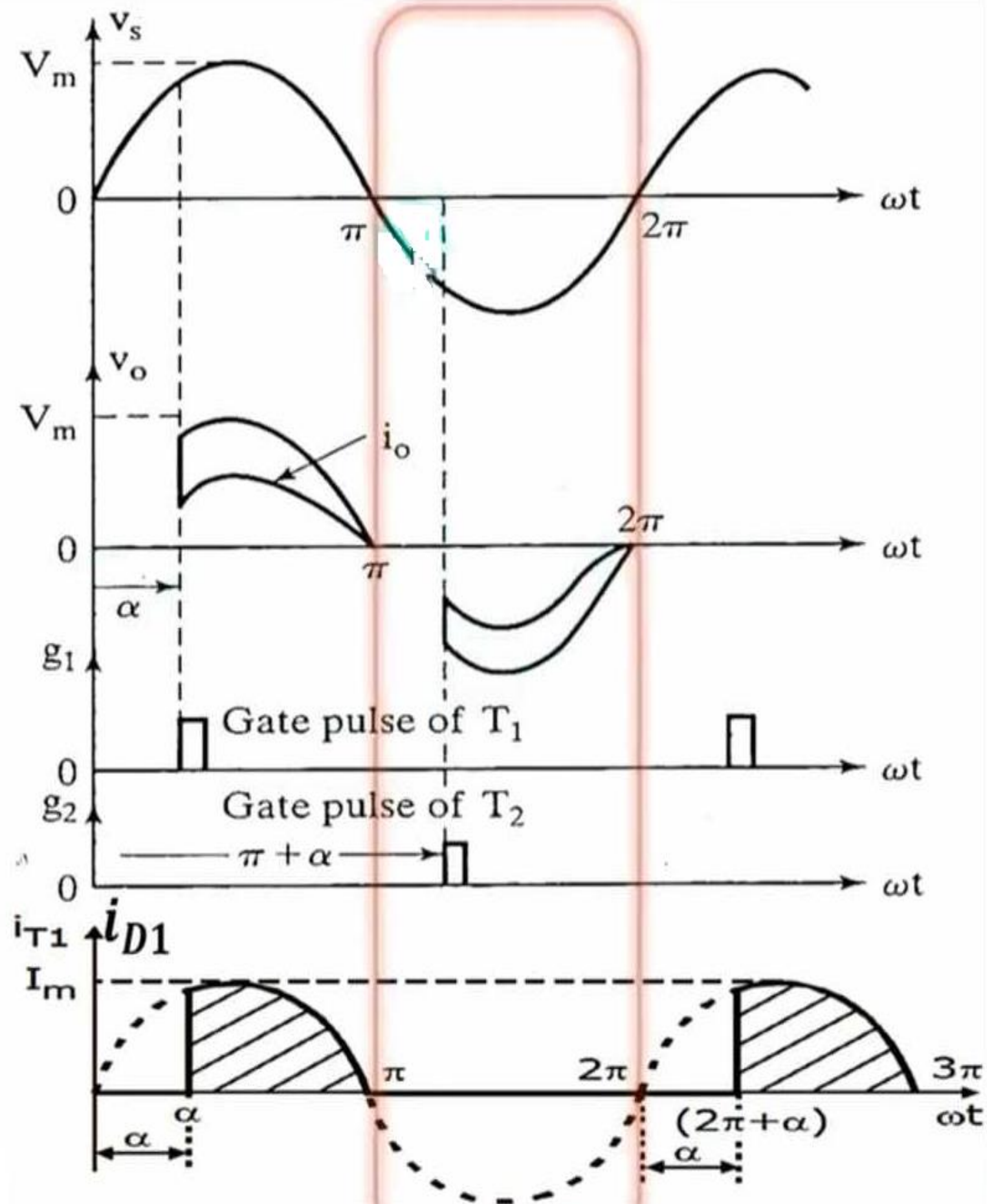

T1
T2
i s
i o
v s
v o
R
+
-

D2
D1
T1
T2
i s
i o
v s
v o
R
+
-

v s
V m
0
π
2π
ωt
v o
i o
α
g 1
Gate pulse of T1
g 2
Gate pulse of T2
π + α
i T1
i D1
I m
3π
(2π+α)



T_1
T_2
i_s
v_s
v_o
i_o
R
+
−



D_2
D_1
T_1
T_2
i_s
v_s
v_o
i_o
R
+
−



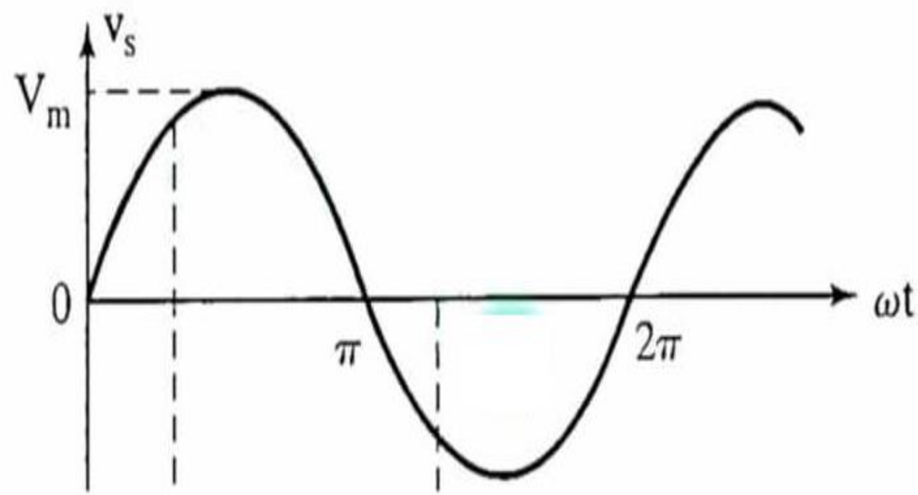
v_s
V_m
0
π
2π
ωt
v_o
V_m
i_o
α
g_1
Gate pulse of T_1
g_2
Gate pulse of T_2
$\pi + \alpha$
i_{T1} i_{D1}
I_m
3π
$(2\pi+\alpha)$

- **For sinusoidal wave input supply Voltage:**

$$v_s = V_m \sin\omega t = \sqrt{2}\, V_{s\,(RMS)} \sin\omega t$$

$$V_{s\,(RMS)} = V_s = V_m/\sqrt{2}$$

$$V_{s\,(Avg)} = V_{s(DC)} = 0$$



▪ The Average Output Load Voltage:

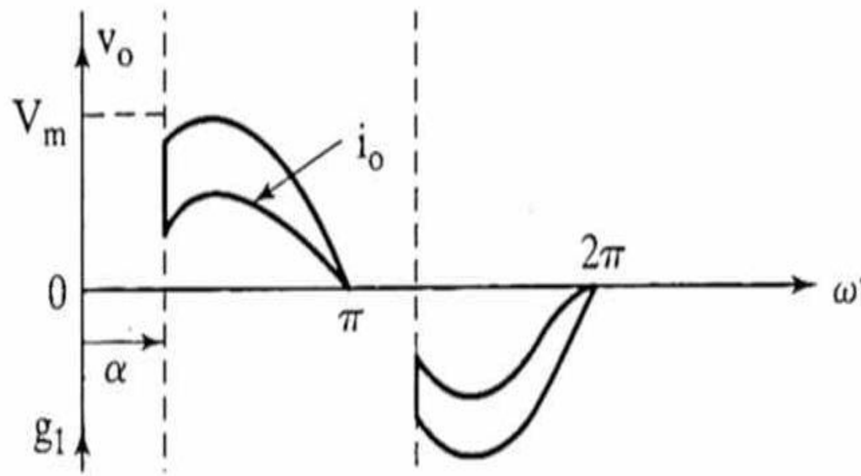
$$V_{o\,(Avg)} = \frac{1}{2\pi}\int_0^{2\pi} v_o\, d\omega t$$

$$= \frac{1}{2\pi}\left(\int_{\alpha}^{\pi} V_m \sin\omega t + \int_{\pi+\alpha}^{2\pi} V_m \sin\omega t\right) d\omega t$$

$$= \frac{-V_m}{2\pi}\left[(\cos\omega t)\Big|_{\alpha}^{\pi} + (\cos\omega t)\Big|_{\pi+\alpha}^{2\pi}\right]$$

$$= \frac{-V_m}{2\pi}\Big(\big(\cos(\pi) - \cos(\alpha)\big) + \big(\cos(2\pi) - \cos(\pi+\alpha)\big)\Big)$$

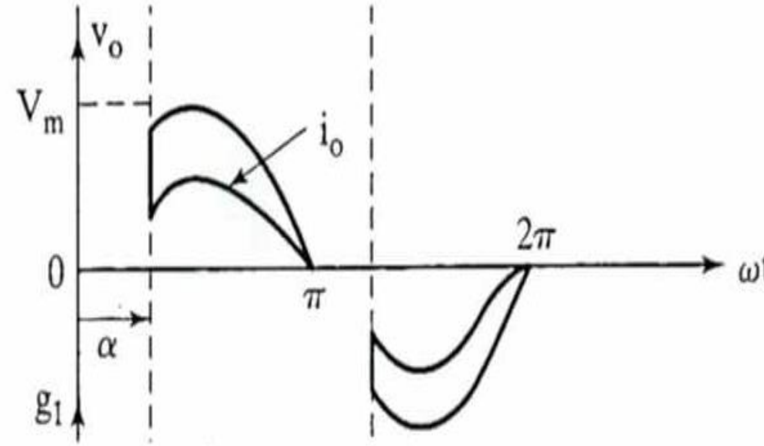
$$= \frac{-V_m}{2\pi}\Big(\big(-1 - \cos(\alpha)\big) + \big(1 + \cos(\alpha)\big)\Big) = 0$$



$$I_{o\,(Avg)} = \frac{1}{2\pi}\int_0^{2\pi} i_o\, d\omega t = \frac{1}{2\pi}\left(\int_{\alpha}^{\pi} I_m \sin\omega t + \int_{\pi+\alpha}^{2\pi} I_m \sin\omega t\right) d\omega t = 0$$

$$I_{o\,(Avg)} = I_{s\,(Avg)} = \frac{V_{o\,(Avg)}}{R} = 0$$

- **The RMS Output Load Voltage:**



$$V_{o\,(RMS)} = \left(\frac{1}{2\pi}\int_0^{2\pi} v_o^2\, d\omega t\right)^{1/2}$$

$$= \left(\frac{1}{2\pi}\left(\int_\alpha^\pi V_m^2 \sin^2\omega t + \int_{\pi+\alpha}^{2\pi} V_m^2 \sin^2\omega t\right) d\omega t\right)^{1/2}$$

$$= \left(\frac{2}{2\pi}\int_\alpha^\pi V_m^2 \sin^2\omega t\, d\omega t\right)^{1/2} = \left(\frac{V_m^2}{\pi}\int_\alpha^\pi \frac{1}{2}(1-\cos 2\omega t)\, d\omega t\right)^{1/2}$$

$$= \left(\frac{V_m^2}{2\pi}\left(\omega t - \frac{1}{2}\sin 2\omega t\right)\Big|_\alpha^\pi\right)^{1/2} = \left(\frac{V_m^2}{2\pi}\left(\left(\pi - \frac{1}{2}\sin(2\pi)\right) - \left(\alpha - \frac{1}{2}\sin(2\alpha)\right)\right)\right)^{1/2}$$

$$= \frac{V_m}{\sqrt{2}}\left(\frac{1}{\pi}\left((\pi-\alpha) + \frac{1}{2}\sin(2\alpha)\right)\right)^{1/2} = V_s\left(\frac{1}{\pi}\left((\pi-\alpha) + \frac{1}{2}\sin(2\alpha)\right)\right)^{1/2}$$

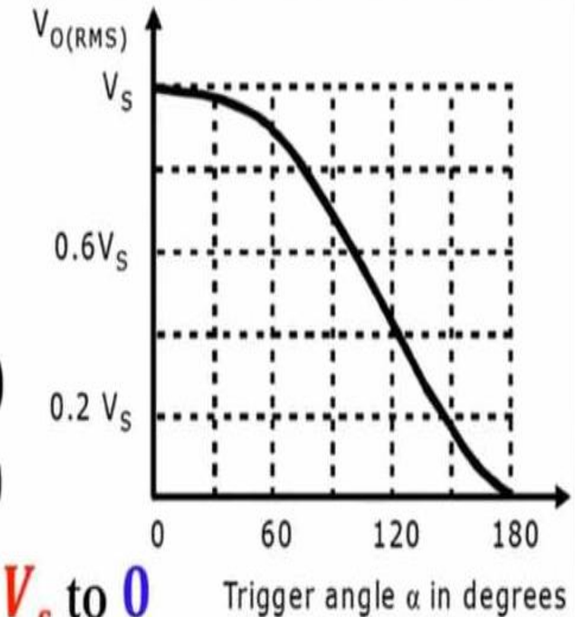
$$I_{o\,(RMS)} = \frac{V_{o\,(RMS)}}{R} = \frac{I_m}{\sqrt{2}}\left(\frac{1}{\pi}\left((\pi-\alpha) + \frac{1}{2}\sin(2\alpha)\right)\right)^{1/2}$$

- **RMS value of output (Load) voltage, $V_{o(RMS)}$:**

$$V_{o(RMS)} = V_s \sqrt{\frac{1}{\pi}\left((\pi - \alpha) + \frac{1}{2}\sin(2\alpha)\right)}$$

$\alpha = 0^\circ \Rightarrow V_{o(RMS)} = 100\% \, V_s$ (Max output)

$\alpha = 180^\circ \Rightarrow V_{o(RMS)} = 0\% \, V_s$ (mini output)



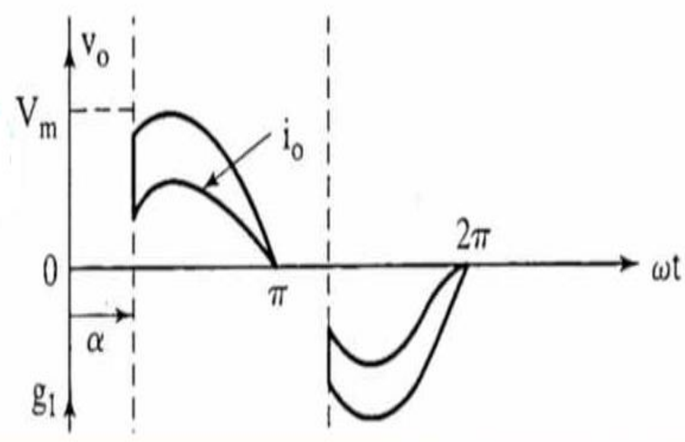
- By varying α from 0 to π, V_o can be varied from V_s to 0

- **Power Factor, PF:**

$$PF = \frac{V_{o(RMS)}}{V_{s(RMS)}} = \sqrt{\frac{1}{\pi}\left((\pi - \alpha) + \frac{1}{2}\sin(2\alpha)\right)}$$

- **RMS Source & Load Current:**

$$I_{o(RMS)} = I_{s(RMS)} = \frac{V_{o(RMS)}}{R}$$

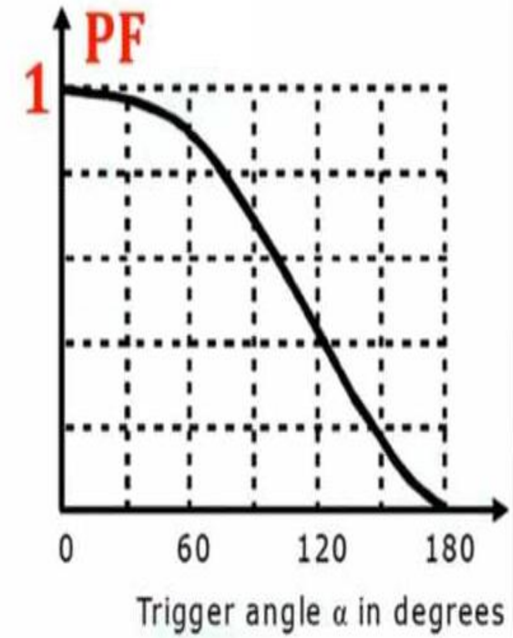


- **Average Output (AC) Load Power:**

$$P_{o\,(Avg)} = I^2_{o\,(RMS)}R = \frac{V^2_{o\,(RMS)}}{R} = I_{o\,(RMS)}\,V_{o\,(RMS)}$$

- **Average Input Supply Volt–Amperes:**

$$VA = I_{s\,(RMS)}\,V_{s\,(RMS)}$$



- **Power Factor, PF, is:**

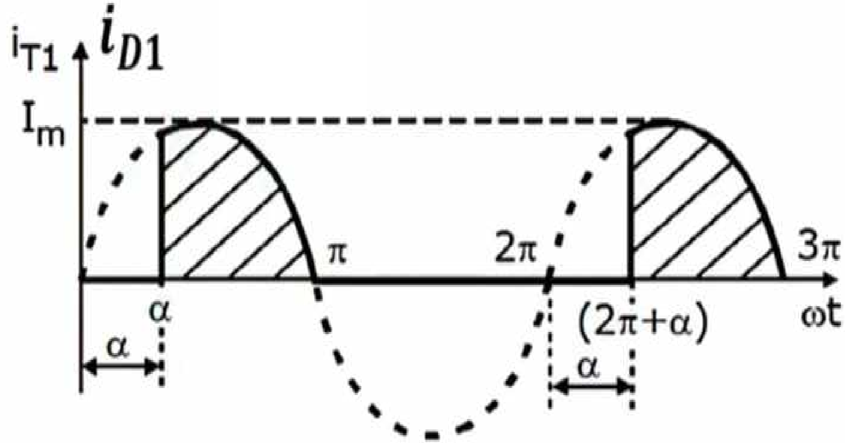
$$PF = \frac{P_{o\,(Avg)}}{VA} = \frac{I_{o\,(RMS)}\,V_{o\,(RMS)}}{I_{s\,(RMS)}\,V_{s\,(RMS)}}$$

$$= \frac{V_{o\,(RMS)}}{V_{s\,(RMS)}} = \sqrt{\frac{1}{\pi}\left((\pi-\alpha)+\frac{1}{2}\sin(2\alpha)\right)}$$

$\alpha = 0^\circ \Rightarrow PF = 1$ → $P_{o\,(Avg)} = 100\%\,P_{s\,(Avg)} = 100\%\,VA$

$\alpha = 180^\circ \Rightarrow PF = 0$ → $P_{o\,(Avg)} = 0\%\,P_{s\,(Avg)} = 0$

- The **Average Thyristor Current**:



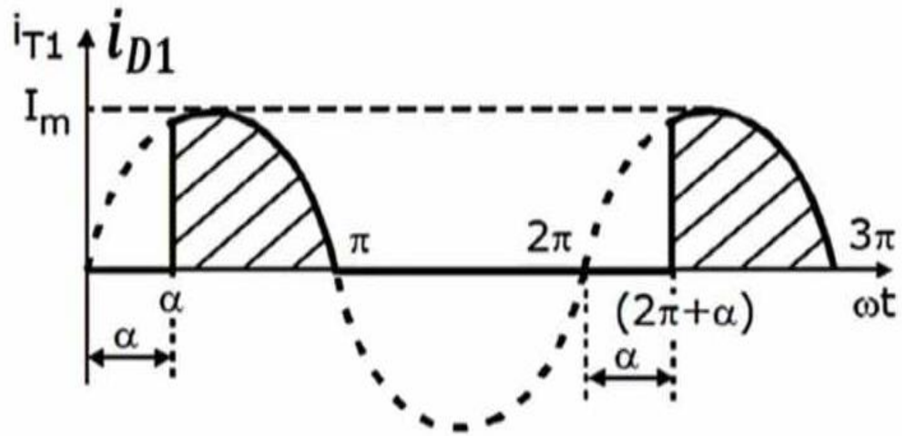
$$I_{T\,(Avg)} = \frac{1}{2\pi}\int_0^{2\pi} i_T \, d\omega t$$

$$= \frac{1}{2\pi}\int_\alpha^{\pi} I_m \sin\omega t \; d\omega t$$

$$= \frac{I_m}{2\pi}(-\cos\omega t)\Big|_\alpha^\pi = \frac{-I_m}{2\pi}\big(\cos(\pi) - \cos(\alpha)\big) = \frac{I_m}{2\pi}\big(1+\cos(\alpha)\big)$$

$$I_m = \frac{V_m}{R} = \frac{V_s\sqrt{2}}{R}$$

- The **RMS Thyristor Current**:



$$I_{T\,(RMS)} = \left(\frac{1}{2\pi}\int_0^{2\pi} i_T^2\, d\omega t\right)^{1/2}$$

$$= \left(\frac{1}{2\pi}\int_\alpha^{\pi} I_m^2 \sin^2\omega t\; d\omega t\right)^{1/2}$$

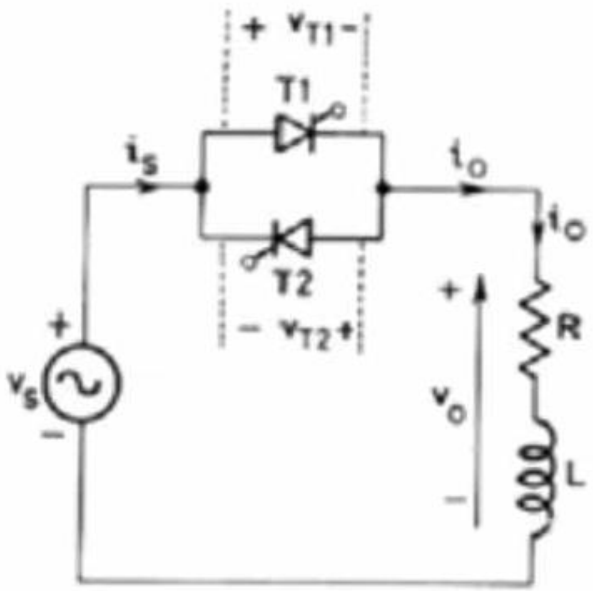
$$= \left(\frac{I_m^2}{2\pi}\int_\alpha^{\pi} \frac{1}{2}(1-\cos 2\omega t)\, d\omega t\right)^{1/2} = \left(\frac{I_m^2}{4\pi}\left(\omega t + \frac{1}{2}\sin 2\omega t\right)\Big|_\alpha^{\pi}\right)^{1/2}$$

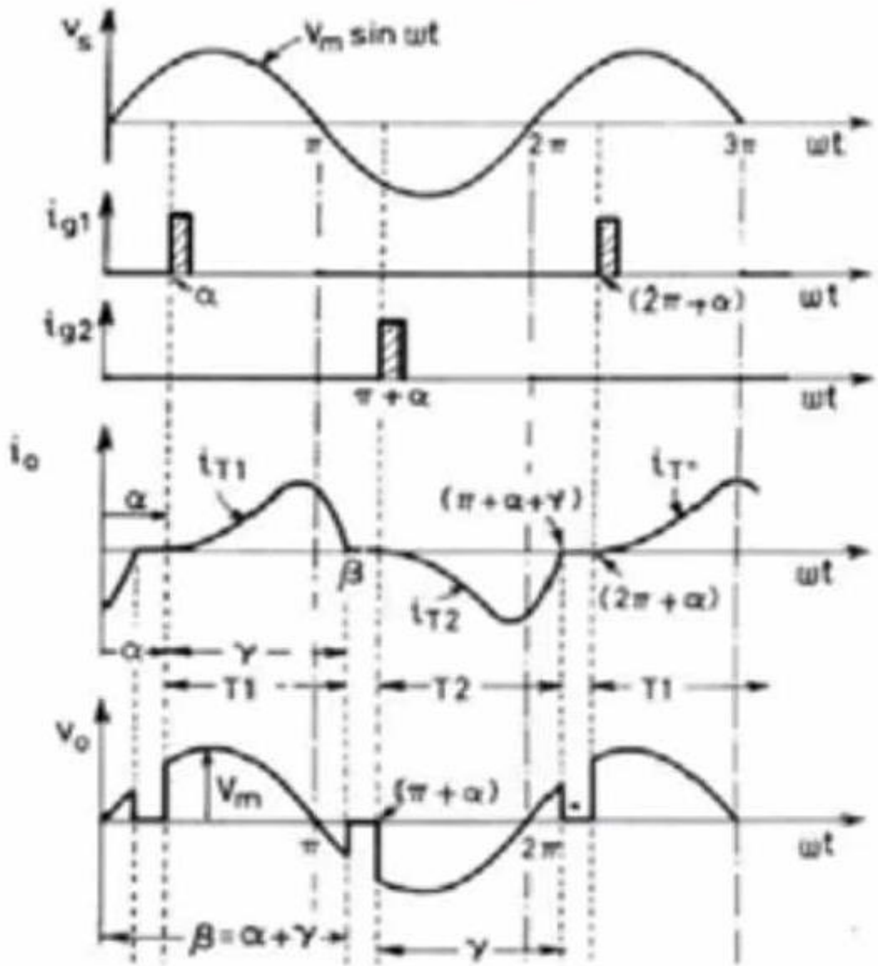
$$= \frac{I_m}{2}\left(\frac{1}{\pi}\left(\left(\pi + \frac{1}{2}\sin(2\pi)\right) - \left(\alpha + \frac{1}{2}\sin(2\alpha)\right)\right)\right)^{1/2}$$

$$= \frac{I_m}{2}\left(\frac{1}{\pi}\left((\pi-\alpha) + \frac{1}{2}\sin(2\alpha)\right)\right)^{1/2}$$

$$I_m = \frac{V_m}{R} = \frac{V_s\sqrt{2}}{R}$$

Single-phase AC voltage controller with inductive (RL) load





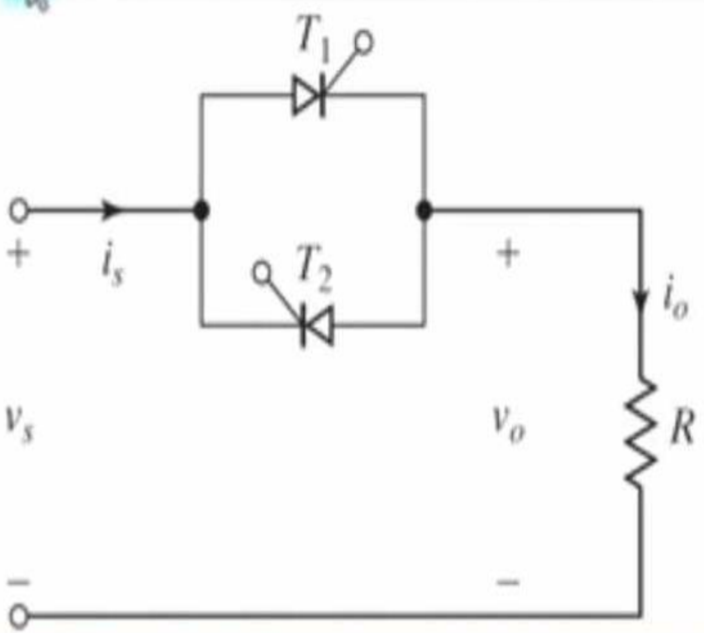
- The problem of dc input current can be prevented by using bidirectional or full-wave controller.
- the AC current pulses are periodic and symmetrical and there is no DC current component. The load voltage also does not have any DC component.
- The rms output voltage can be varied between 0% to 100%
- Suitable for loads with short time constants.
- The firing pulse (instants) of thyristors of T1 and T2 are 180 degrees apart, each occurring in its respective half cycle.

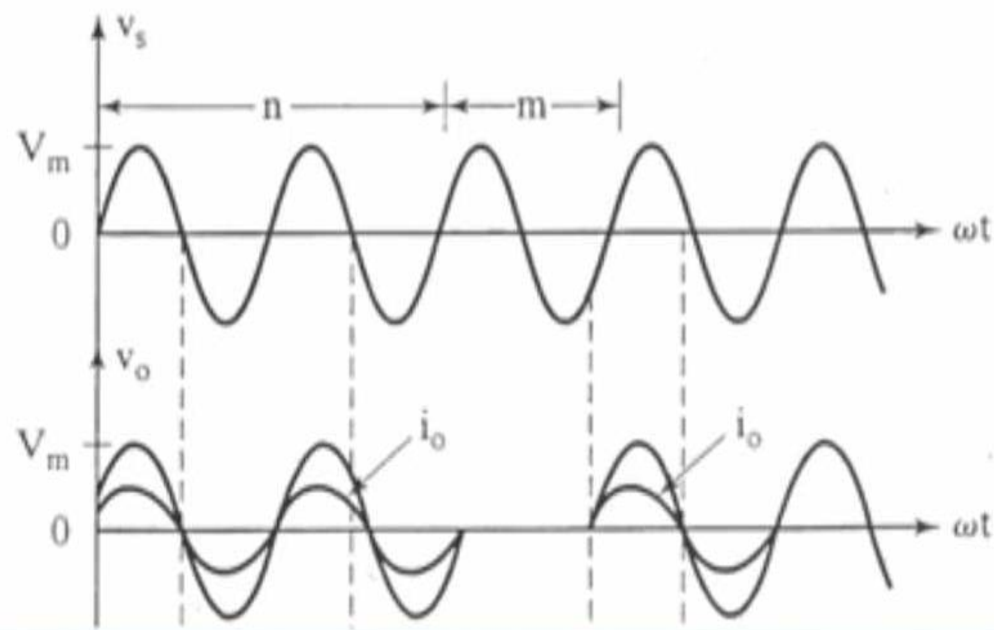
$$0 \leq \alpha \leq 180$$

- By controlling this instant of firing, the effective voltage occurring across the load can be varied.

AC – AC Converter

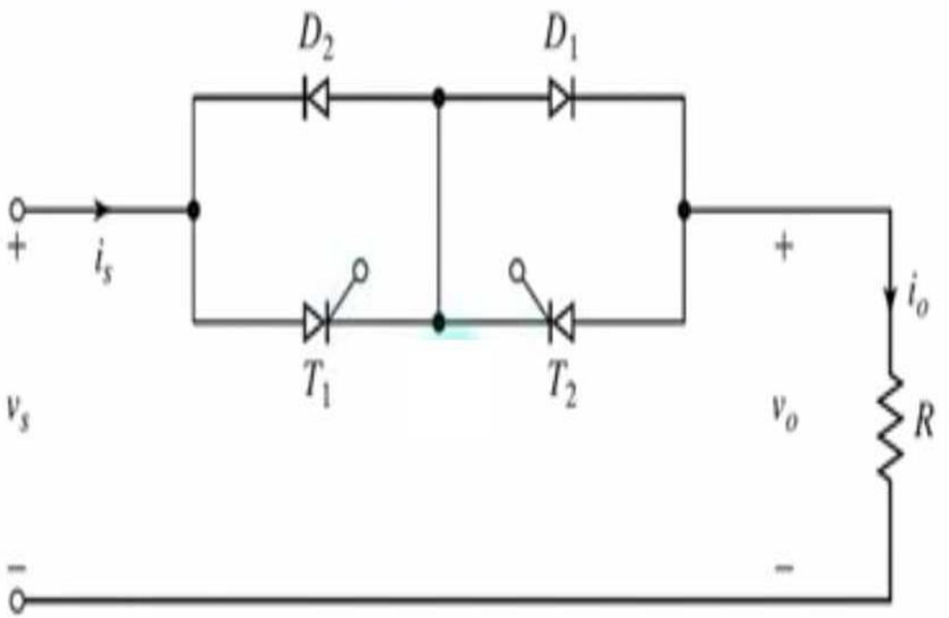
AC Voltage Controllers: ON – OFF Control

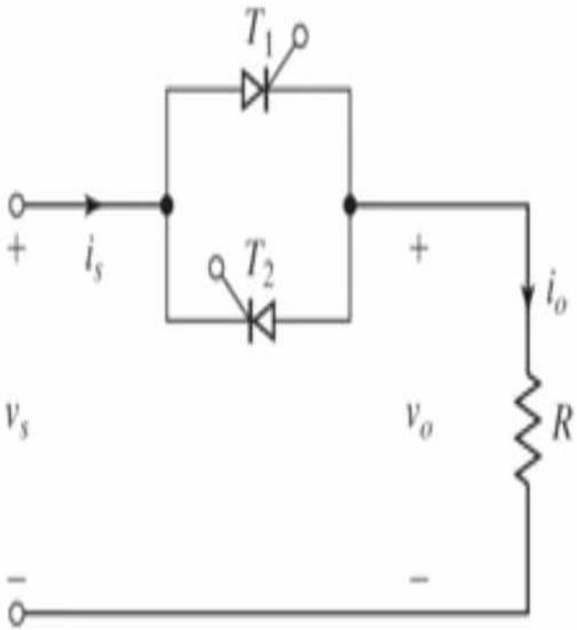




Principle of On – Off Control (Integral Cycle Control)

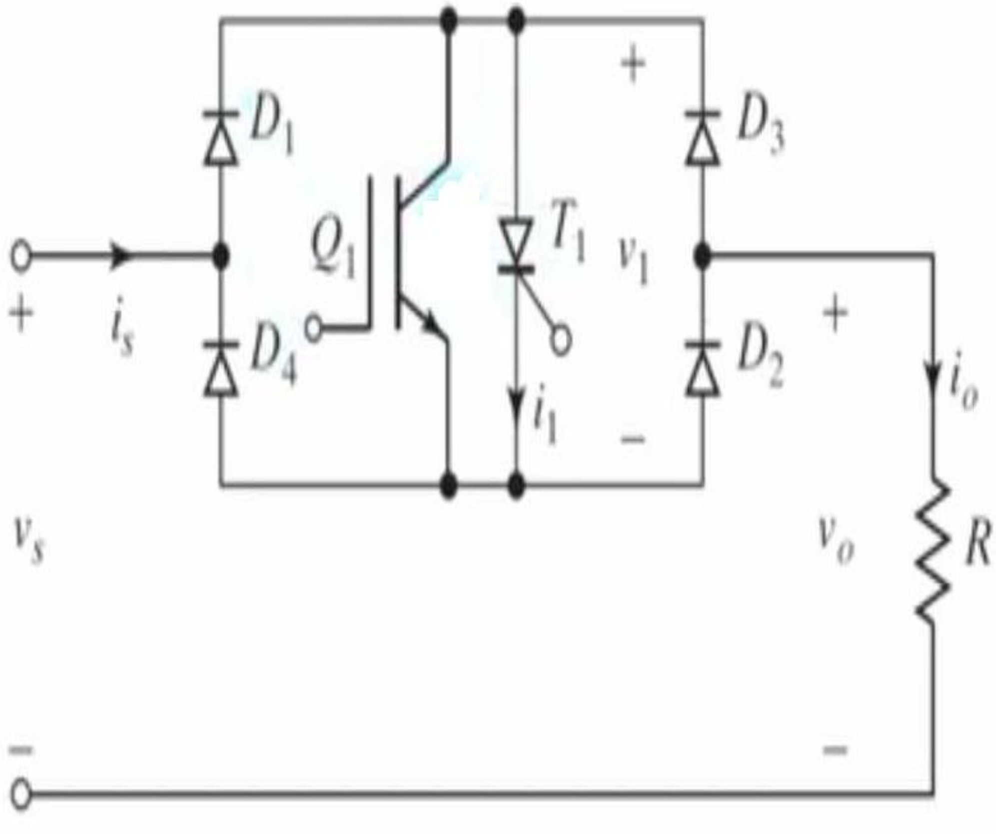
- The principle of on-off control can be explained with the following single-phase full-wave (bi-directional) controller
- <u>For Resistive Load:</u>

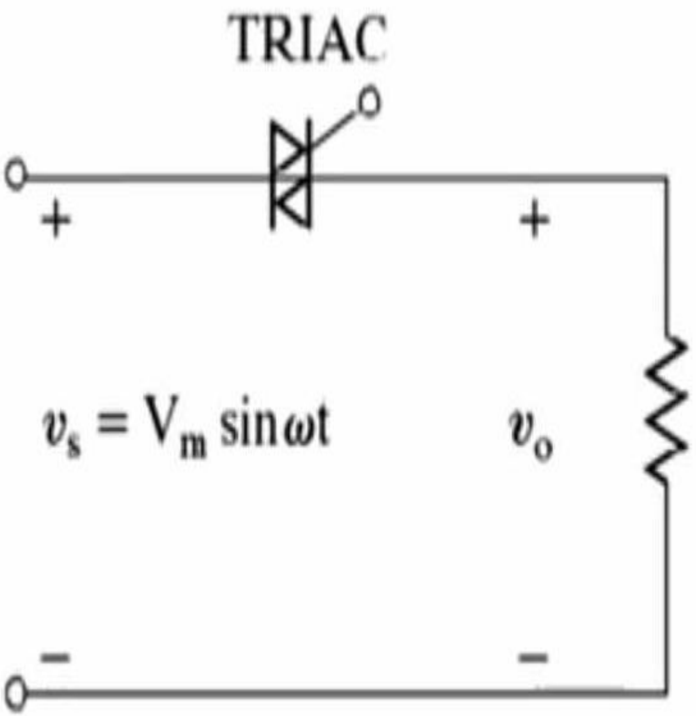


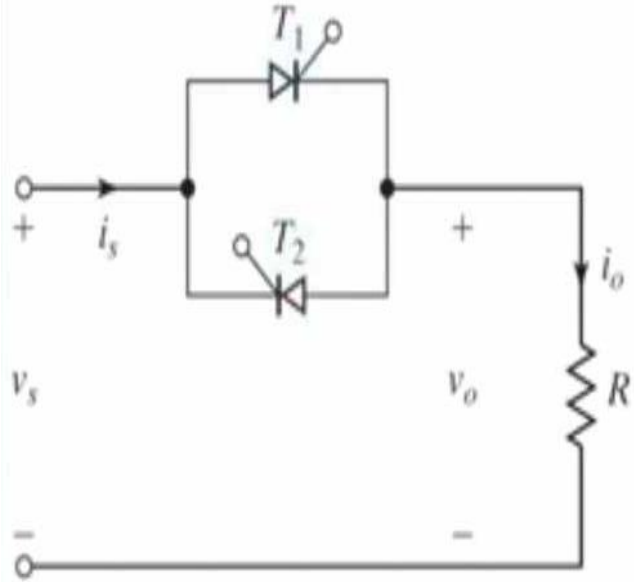


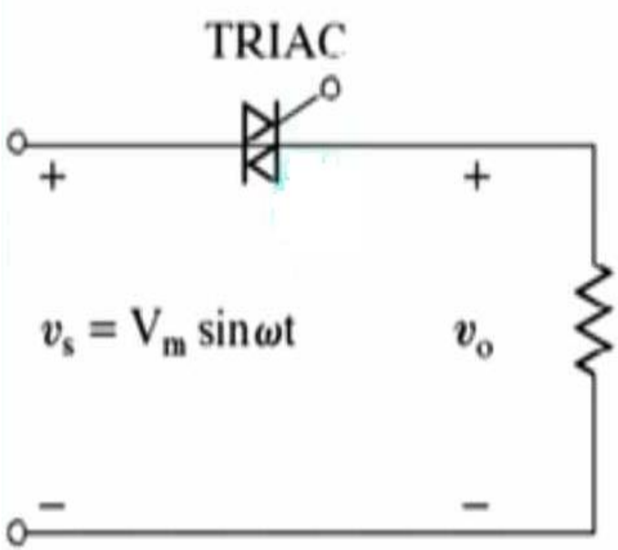
With single common cathode

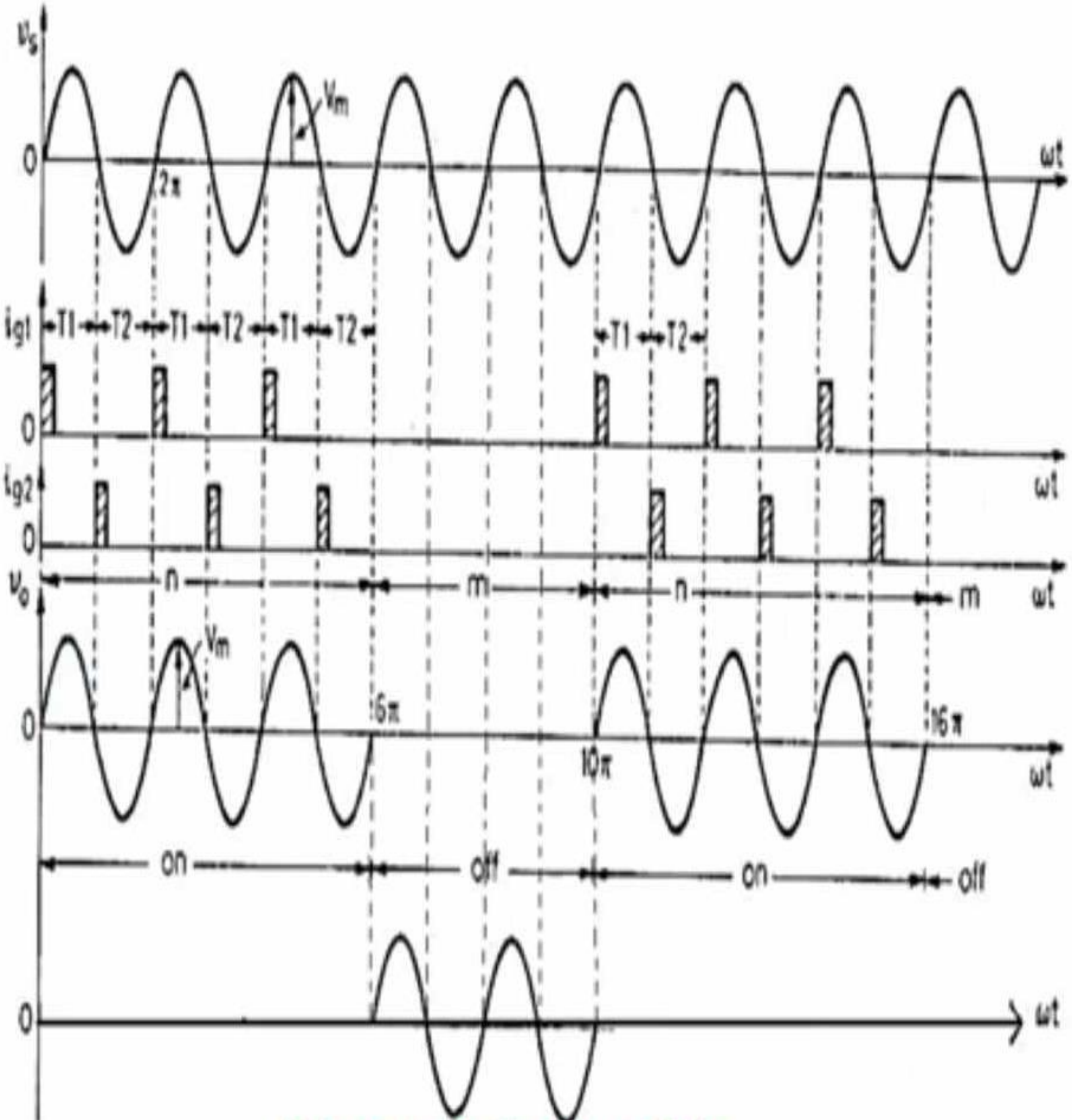
Principle of On – Off Control (Integral Cycle Control)



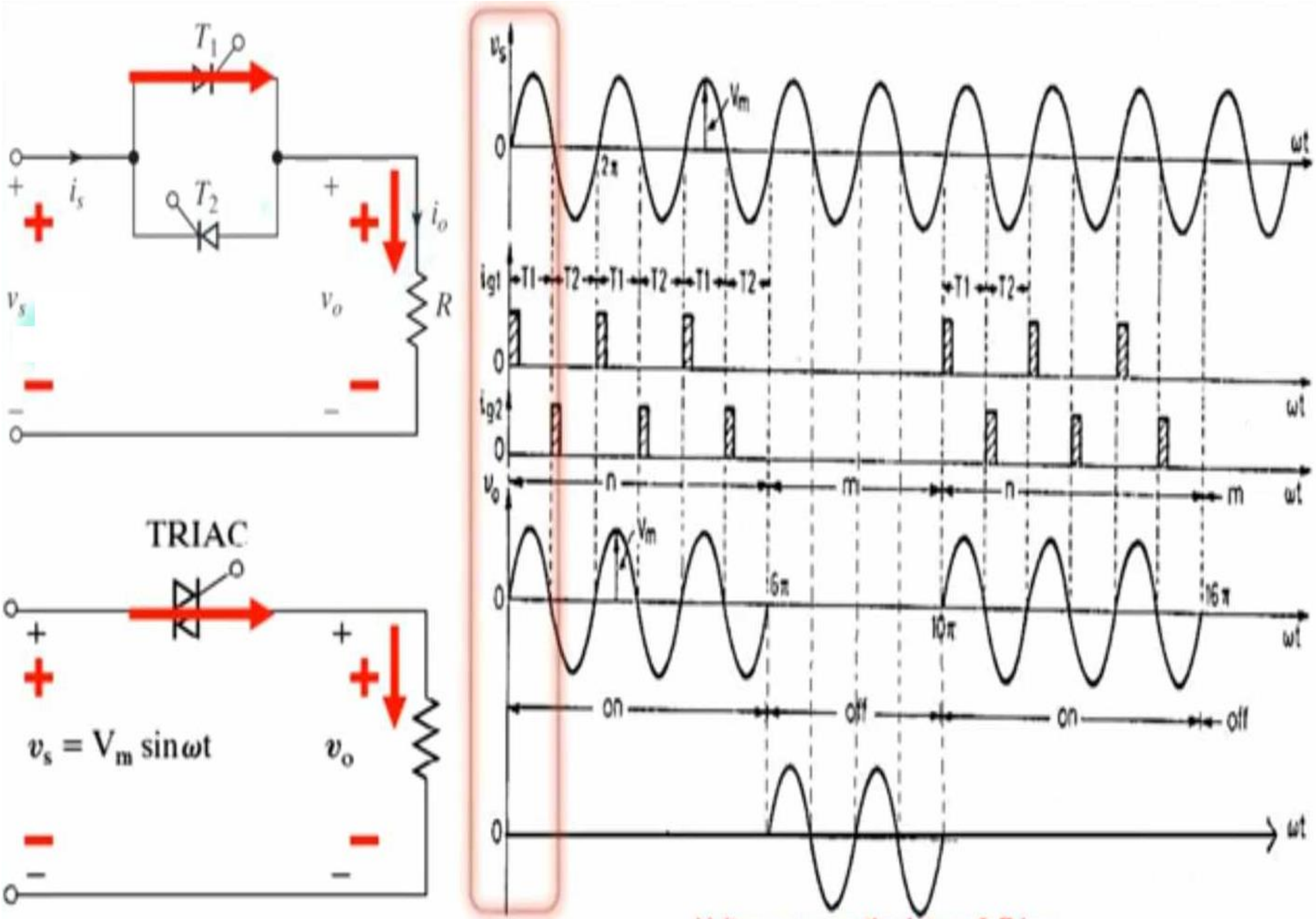




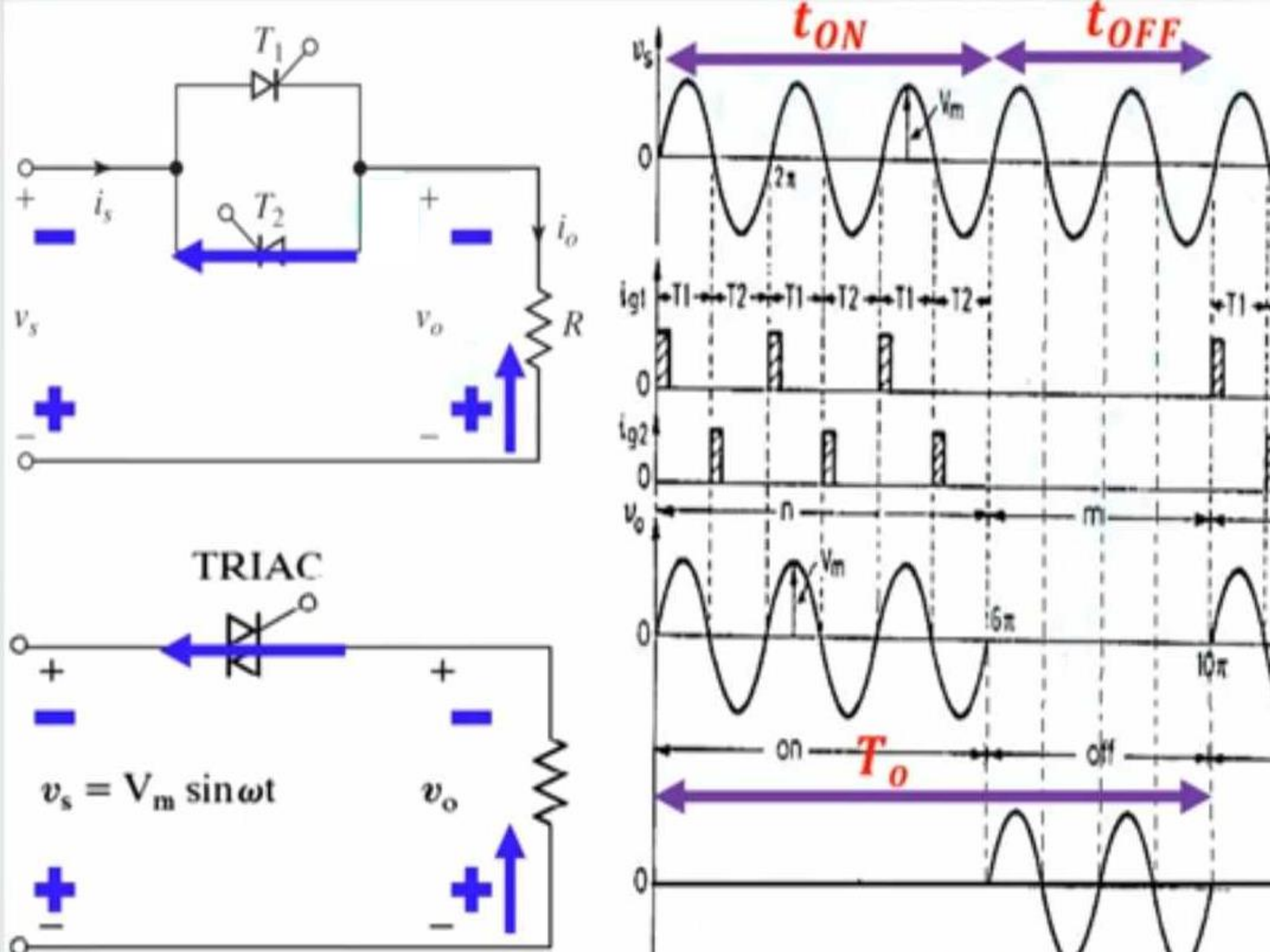




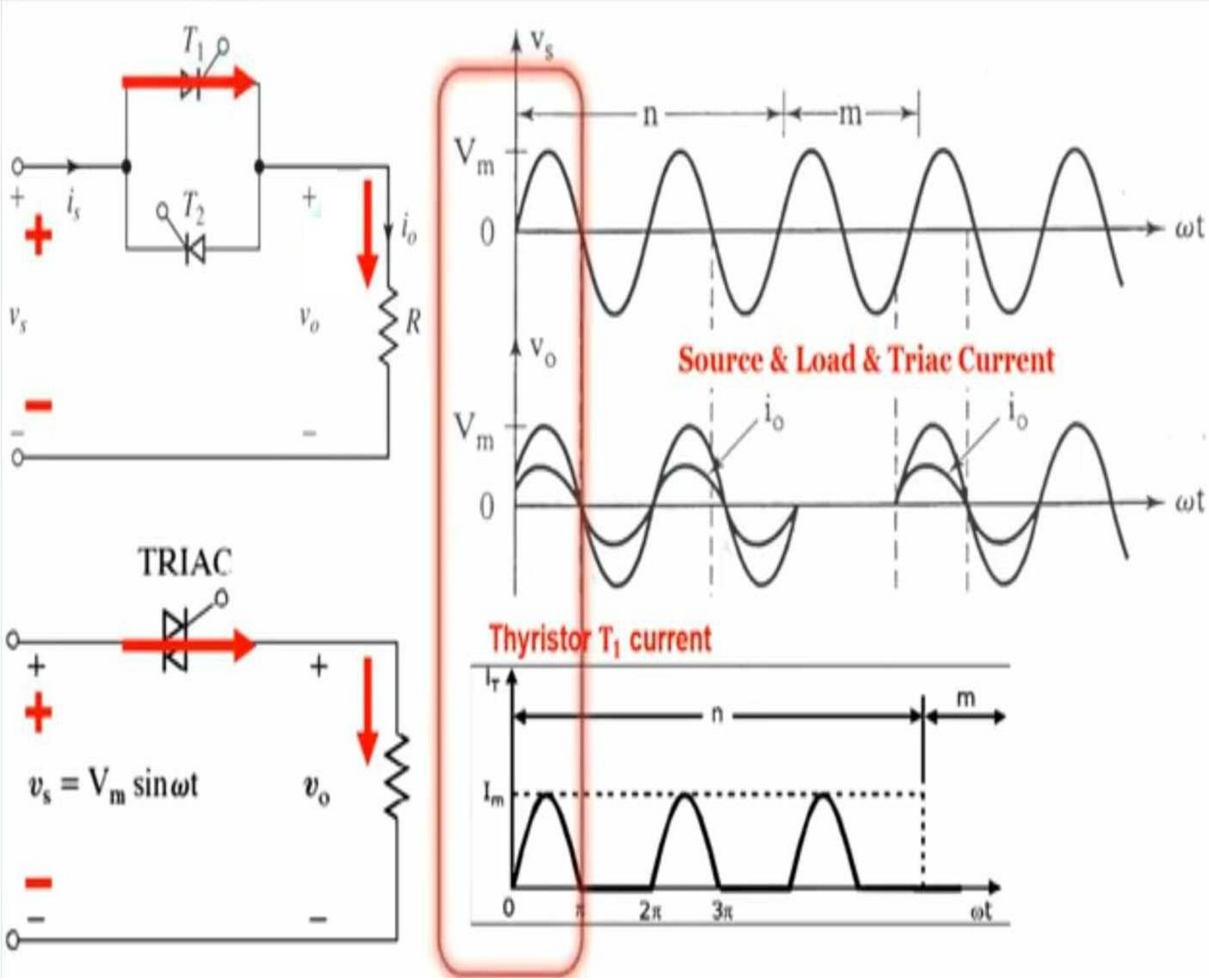
Voltage across thyristors & Triac



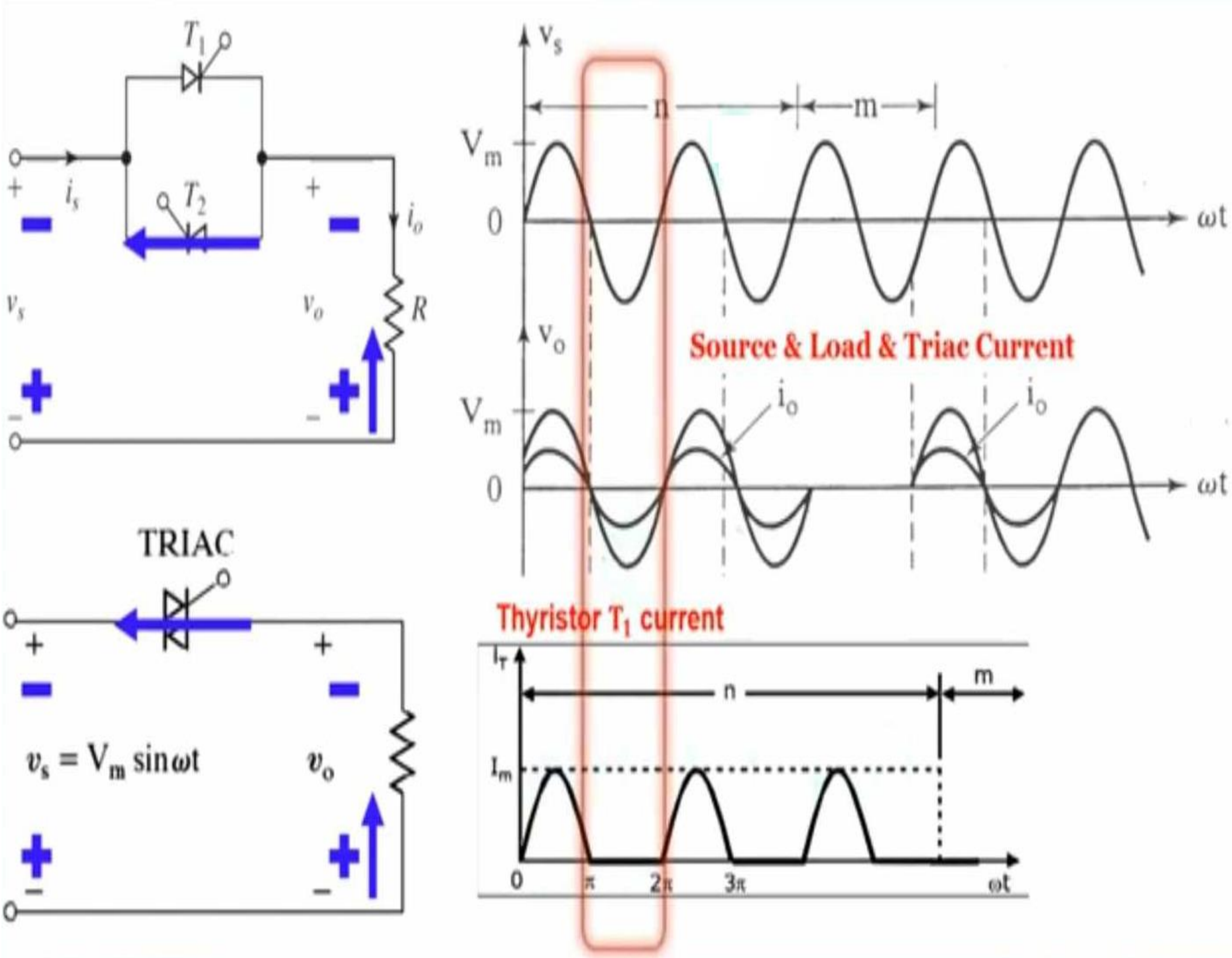
Voltage across thyristors & Triac



T1
T2
i_s
i_o
R
v_s
v_o
TRIAC
v_s = V_m sin ωt
v_o
t_ON
t_OFF
v_s
Vm
2π
ig1
T1
T2
ig2
n
m
Vo
6π
10π
on
off
T_o



T1
T2
is
io
vs
vo
R
+
−
TRIAC
vs = Vm sinωt
vo
vs
n
m
Vm
0
ωt
vo
Source & Load & Triac Current
io
Thyristor T1 current
IT
Im
0
π
2π
3π
ωt



T1
T2
is
io
vs
vo
R
TRIAC
$v_s = V_m \sin\omega t$
v_o
vs
Vm
0
n
m
ωt
vo
io
Source & Load & Triac Current
Thyristor T1 current
IT
Im
0
π
2π
3π

- The input voltage is connected to load (thyristors T_1 & T_2 are turned ON) for 'n' number of input cycles during the time interval t_{ON}

- The input voltage is disconnected from load (thyristors T_1 & T_2 are turned OFF) for 'm' number of input cycles during the time interval t_{OFF}.

- t_{ON} (AC Controller ON time) = nT
- t_{OFF} (AC Controller OFF time) = mT
- $T = 1/f$ = Input cycle time (time period), f = Input supply frquency
- T_o = Output time period = $t_{ON} + t_{OFF} = (n+m)T$

- Thyristors T_1 & T_2 are turned ON precisely at zero voltage crossings of the input supply.

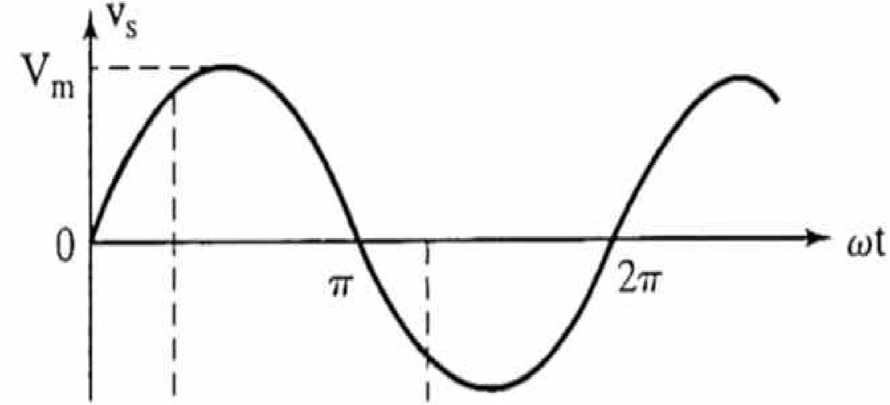
- Due to zero voltage switching of thyristors, the harmonics generated by switching actions are reduced

- **For sinusoidal wave input supply Voltage:**

$$v_s = V_m \sin\omega t = \sqrt{2}\, V_{s\,(RMS)} \sin\omega t$$

$$V_{s\,(RMS)} = V_s = V_m/\sqrt{2}$$

$$V_{s\,(\mathrm{Avg})} = V_{s(DC)} = 0$$



$$V_{s\,(Avg)} = \frac{1}{T}\int_0^T v_s\, \mathrm{d}\omega t = \frac{1}{2\pi}\int_0^{2\pi} V_m \sin\omega t$$

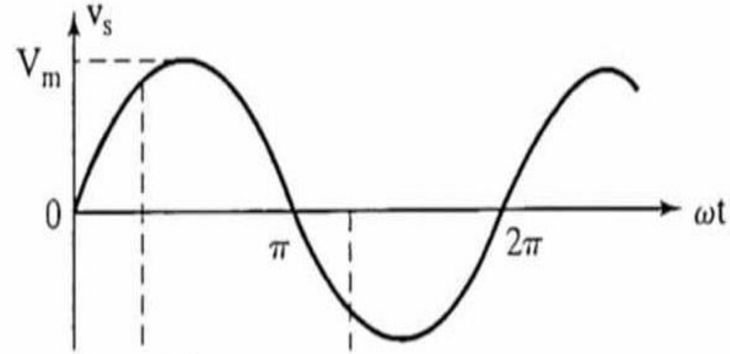
$$= \frac{V_m}{2\pi}(-\cos\omega t)\Big|_0^{2\pi} = \frac{-V_m}{2\pi}\big(\cos(2\pi) - \cos(0)\big) = 0$$

- **The RMS Voltage for input supply :**

$$v_s = V_m \sin\omega t = \sqrt{2}\, V_{s\,(RMS)} \sin\omega t$$

$$V_{s\,(RMS)} = V_s = V_m/\sqrt{2}$$

$$V_{s\,(\mathrm{Avg})} = V_{s(DC)} = 0$$

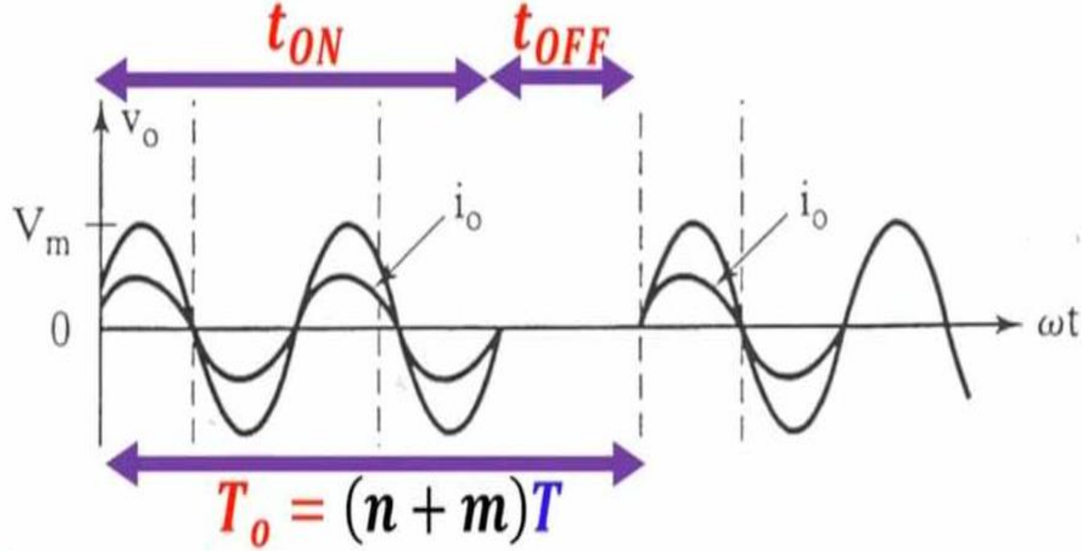


$$V_s = \left(\frac{1}{T}\int_0^T v_s^2\, \mathrm{d}\omega t\right)^{1/2} = \left(\frac{1}{2\pi}\int_0^{2\pi} V_m^2 \sin^2\omega t\ \mathrm{d}\omega t\right)^{1/2}$$

$$= \left(\frac{V_m^2}{2\pi}\int_0^{2\pi} \frac{1}{2}(1-\cos 2\omega t)\, \mathrm{d}\omega t\right)^{1/2} = \left(\frac{V_m^2}{4\pi}\left(\omega t + \frac{1}{2}\sin 2\omega t\right)\Big|_0^{2\pi}\right)^{1/2}$$

$$= \left(\frac{V_m^2}{4\pi}\left(2\pi + \frac{1}{2}\sin(2 * 2\pi) - 0 - \frac{1}{2}\sin(0)\right)\right)^{1/2} = \left(\frac{V_m^2}{4\pi}2\pi\right)^{1/2} = \frac{V_m}{\sqrt{2}}$$

- **The Average Output Load Voltage:**



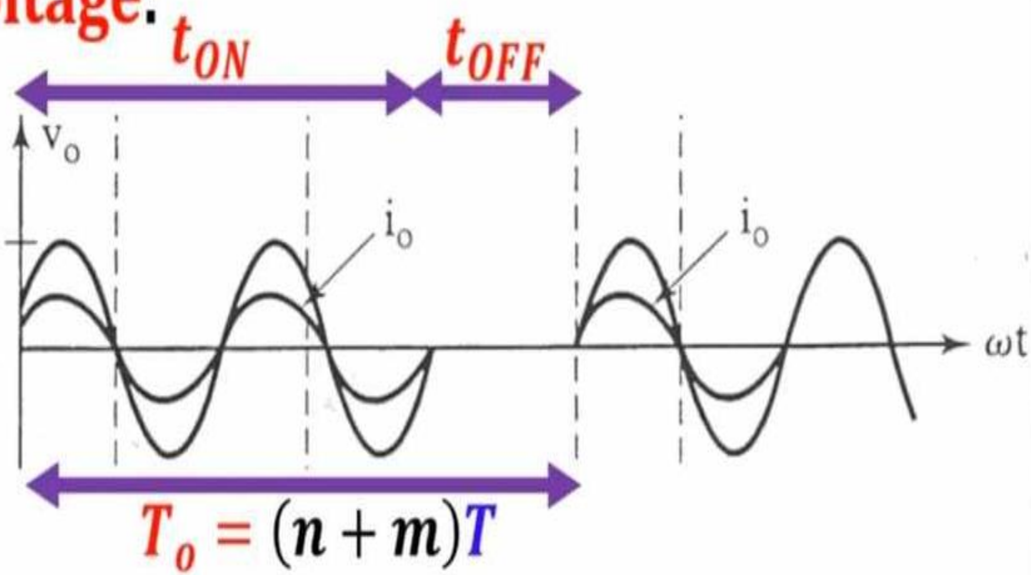
$$V_{o\,(Avg)} = \frac{1}{T_0}\int_0^{T_o} v_o \, d\omega t$$

$$= \frac{1}{(n+m)T} \cdot n \int_0^{2\pi} V_m \sin\omega t \; d\omega t = \frac{V_m}{2\pi} \frac{n}{(n+m)} (-\cos\omega t) \Big|_0^{2\pi}$$

$$= \frac{-V_m}{2\pi} \frac{n}{(n+m)} \big(\cos(2\pi) - \cos(0)\big) = 0$$

$$I_{o\,(Avg)} = I_{Triac\,(Avg)} = I_{s\,(Avg)} = \frac{V_{o\,(Avg)}}{R} = 0$$

- **The RMS Output Load Voltage:**



$$V_{o\,(RMS)} = \left(\frac{1}{T_o}\int_0^{T_o} v_o^2\, d\omega t\right)^{1/2}$$

$$= \left(\frac{1}{(n+m)T}\cdot n\int_0^{2\pi} V_m^2 \sin^2\omega t\ d\omega t\right)^{1/2}$$

$$= \left(\frac{V_m^2}{2\pi}\frac{n}{(n+m)}\int_0^{2\pi}\frac{1}{2}(1-\cos 2\omega t)\, d\omega t\right)^{1/2} = \left(\frac{V_m^2}{4\pi}\frac{n}{(n+m)}\left(\omega t + \frac{1}{2}\sin 2\omega t\right)\Big|_0^{2\pi}\right)^{1/2}$$

$$= \left(\frac{V_m^2}{4\pi}\frac{n}{(n+m)}\left(2\pi + \frac{1}{2}\sin(2*2\pi) - 0 - \frac{1}{2}\sin(0)\right)\right)^{1/2}$$

$$= \left(\frac{V_m^2}{4\pi}2\pi\frac{n}{(n+m)}\right)^{1/2} = \frac{V_m}{\sqrt{2}}\sqrt{\frac{n}{n+m}} = V_s\sqrt{\frac{n}{n+m}} = V_s\sqrt{k}$$

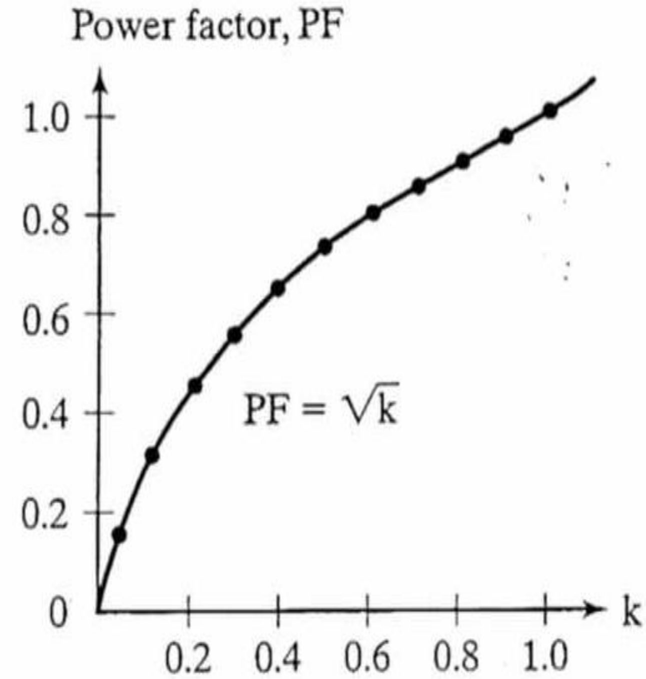
- **RMS value of output (Load) voltage, $V_{o\,(RMS)}$:**

$$V_{o\,(RMS)} = \frac{V_m}{\sqrt{2}}\sqrt{\frac{n}{n+m}} = \frac{V_m}{\sqrt{2}}\sqrt{\frac{t_{ON}}{T_o}} = V_{s\,(RMS)}\sqrt{\frac{n}{n+m}} = V_{s\,(RMS)}\sqrt{k}$$

- **Power Factor, PF, is:**

$$PF = \frac{V_{o\,(RMS)}}{V_{s\,(RMS)}} = \sqrt{k} = \sqrt{\frac{t_{ON}}{T_o}}$$

- **Note that the power factor and RMS output voltage vary with the square root of k**

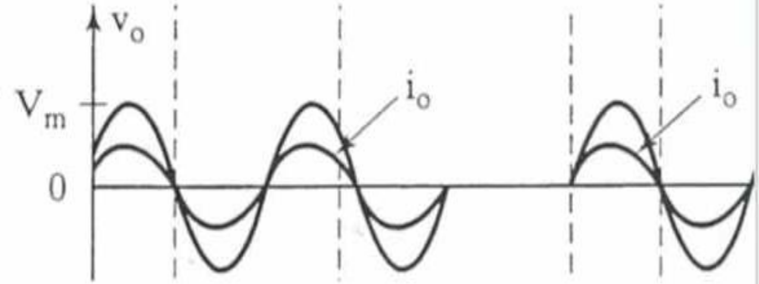


- **Duty Cycle (k):**

$$k = \frac{t_{ON}}{T_o} = \frac{nT}{(n+m)T} = \frac{n}{n+m}$$

▪ RMS Source & Load & Triac Current:

$$I_{o\,(RMS)} = I_{Triac\,(RMS)} = I_{s\,(RMS)} = \frac{V_{o\,(RMS)}}{R}$$



▪ **Average Output (AC) Load Power:**

$$P_{o\,(Avg)} = I^2_{o\,(RMS)} R = \frac{V^2_{o\,(RMS)}}{R} = I_{o\,(RMS)} V_{o\,(RMS)} = \frac{V^2_{s\,(RMS)}}{R} k$$

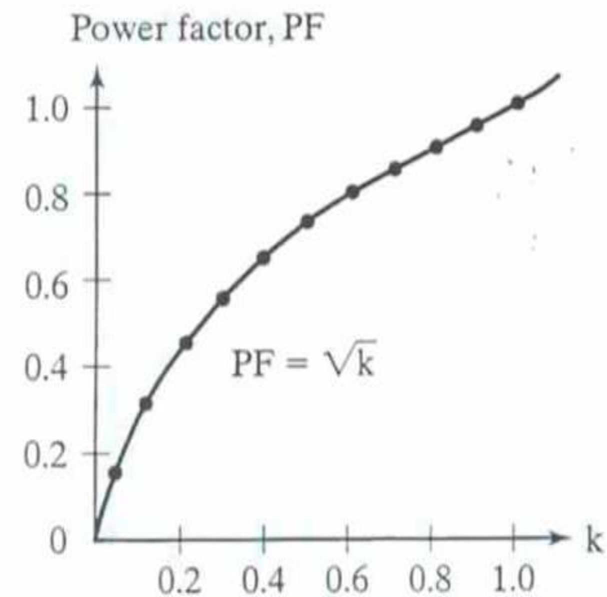
▪ **Average Input Supply Volt–Amperes:**

$$VA = I_{s\,(RMS)} V_{s\,(RMS)}$$

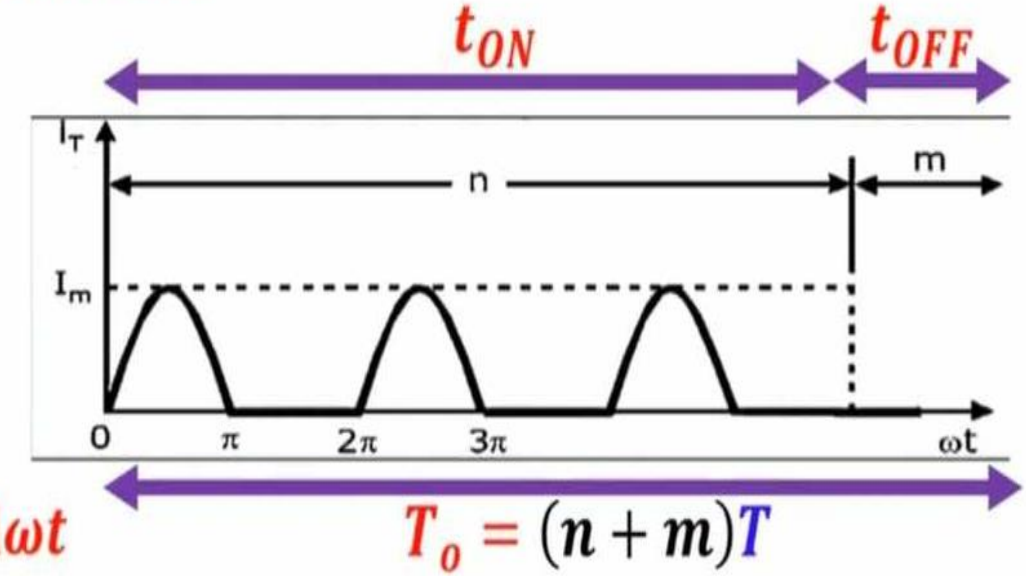
▪ **Power Factor, PF, is:**

$$PF = \frac{P_{o\,(Avg)}}{VA} = \frac{I_{o\,(RMS)} V_{o\,(RMS)}}{I_{s\,(RMS)} V_{s\,(RMS)}} = \frac{V_{o\,(RMS)}}{V_{s\,(RMS)}}$$

$$PF = \sqrt{k}$$



- **The Average Thyristor Current:**



$$I_{Thy\,(Avg)} = \frac{1}{T_0}\int_0^{T_o} i_{Thy}\, d\omega t$$

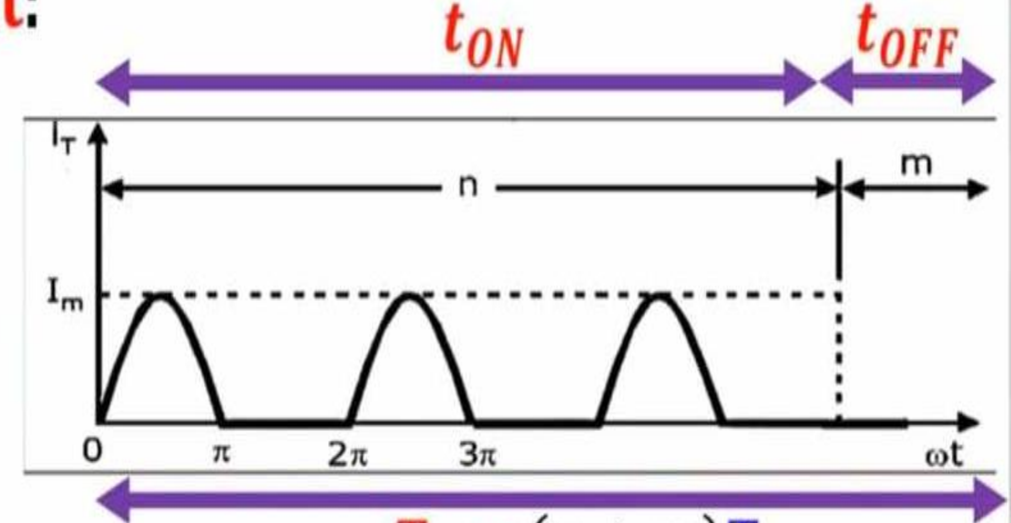
$$= \frac{1}{(n+m)T}\, n\int_0^{2\pi} I_m \sin\omega t\ d\omega t$$

$$= \frac{1}{(n+m)T}\, n\int_0^{\pi} I_m \sin\omega t\ d\omega t = \frac{I_m}{2\pi}\frac{n}{(n+m)}(-\cos\omega t)\Big|_0^{\pi}$$

$$= \frac{-I_m}{2\pi}\frac{n}{(n+m)}\big(\cos(\pi)-\cos(0)\big) = \frac{-I_m}{2\pi}\frac{n}{(n+m)}(-2) = \frac{I_m}{\pi}k$$

$$I_m = \frac{V_m}{R} = \frac{V_s\sqrt{2}}{R}$$

- The RMS Thyristor Current:



$$T_o = (n+m)T$$

$$I_{Thy\,(RMS)} = \left(\frac{1}{T_o}\int_0^{T_o} i_{Thy}^2\, d\omega t\right)^{1/2}$$

$$= \left(\frac{1}{(n+m)T}\cdot n\int_0^{\pi} I_m^2 \sin^2\omega t\ d\omega t\right)^{1/2}$$

$$= \left(\frac{I_m^2}{2\pi}\frac{n}{(n+m)}\int_0^{\pi}\frac{1}{2}(1-\cos 2\omega t)\,d\omega t\right)^{1/2} = \left(\frac{I_m^2}{4\pi}\frac{n}{(n+m)}\left(\omega t+\frac{1}{2}\sin 2\omega t\right)\Big|_0^{\pi}\right)^{1/2}$$

$$= \left(\frac{I_m^2}{4\pi}\frac{n}{(n+m)}\left(\pi+\frac{1}{2}\sin(2\pi)-0-\frac{1}{2}\sin(0)\right)\right)^{1/2}$$

$$= \left(\frac{I_m^2}{4\pi}\pi\frac{n}{(n+m)}\right)^{1/2} = \frac{I_m}{2}\sqrt{\frac{n}{n+m}} = \frac{I_m}{2}\sqrt{\frac{n}{n+m}} = \frac{I_m}{2}\sqrt{k}$$

$$I_m = \frac{V_m}{R} = \frac{V_s\sqrt{2}}{R}$$

- This type of control is applied in applications which have high mechanical inertia and high thermal time constant

- Typical examples are industrial heating and speed control of large motors.

- Duty Cycle (k = Ratio of on time to total output cycle time): controls average load power as well as rms output voltage.
- Average power delivered to the load can be varied from 0% through 100%
- The output voltage and input current are asymmetrical (for resistive load)
- There is no DC component of input supply current and output voltage i.e., the average value for each is Zero.