Engineering Economy

[2-1]

Time Value of Money

Single Cash Flow

The Five Types of Cash Flows

Cash flow transactions can be generally classified into five general categories:

(1) Single cash flow

(2) Uniform series

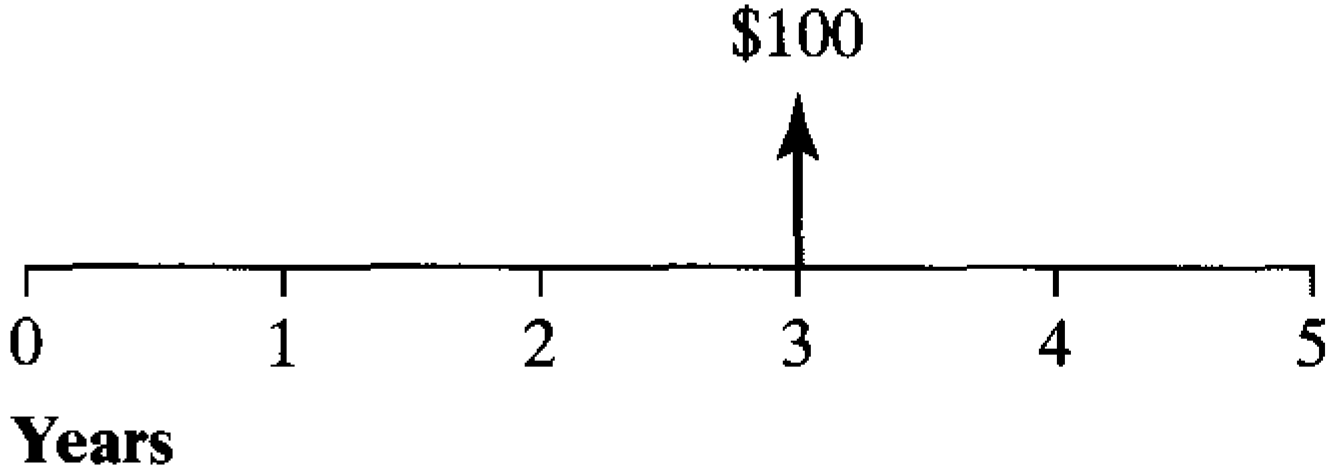
(3) Linear gradient series

(4) Geometric gradient series, and

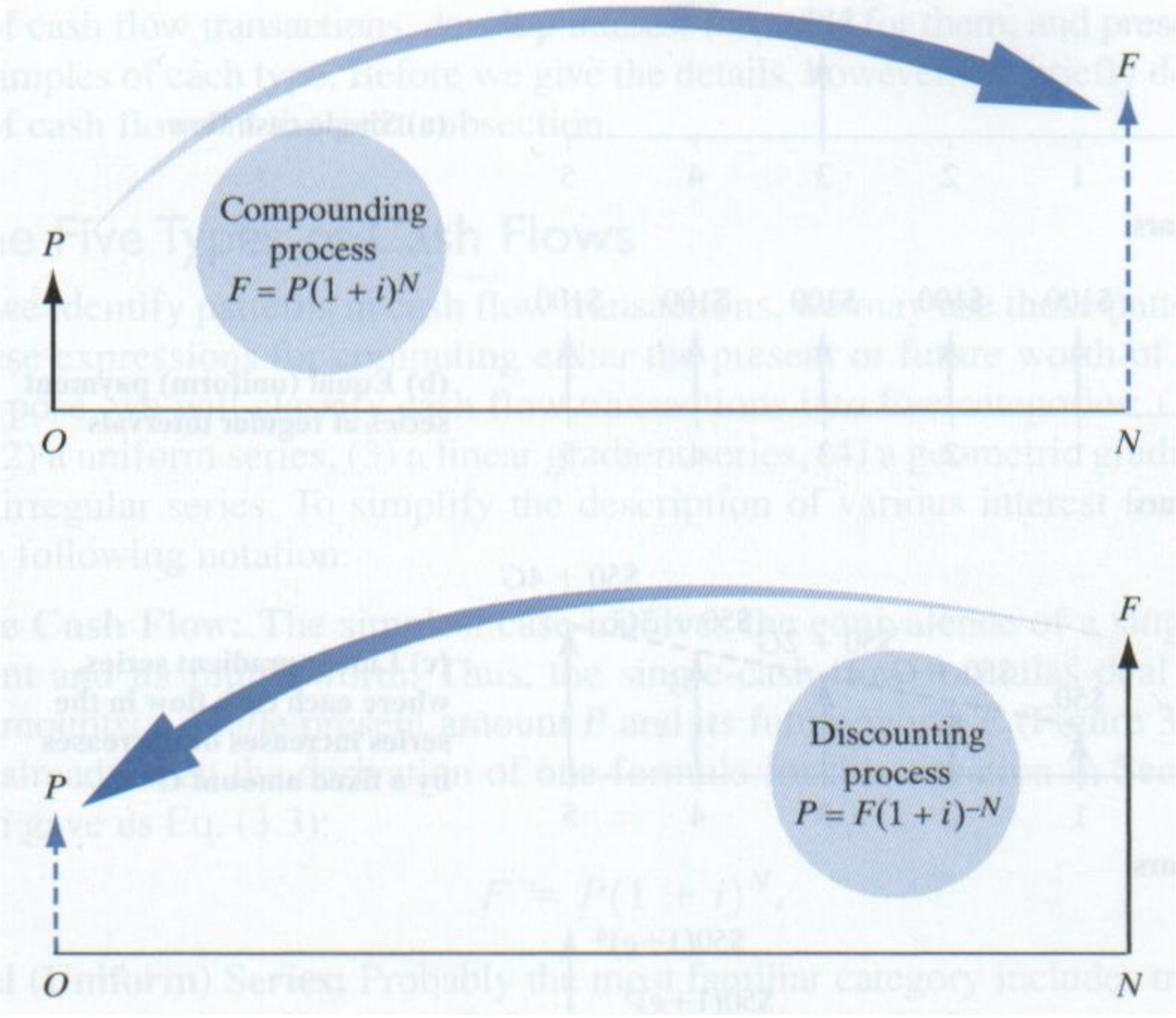
(5) Irregular series

Single Cash Flow

- The simplest case involves the equivalence of a single present amount and its future worth
- Thus, the single-cash-flow formulas deal with only two amounts: a single present amount P and its equivalent future worth F



Single Cash Flow



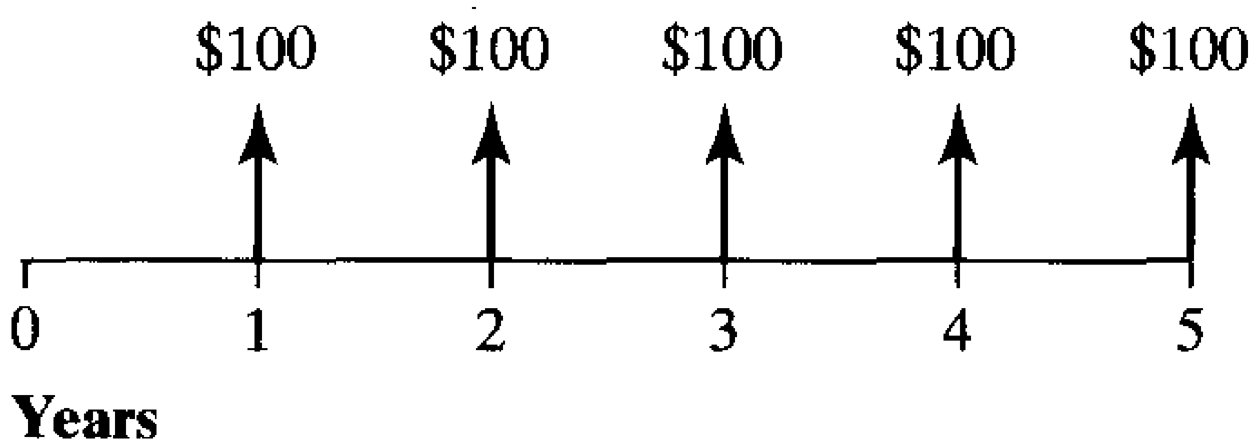
You have
P find F

You have
F find P

Equal (Uniform) Series

- Includes transactions arranged as a series of equal cash flows at regular intervals, known as an equal payment series (or uniform series)

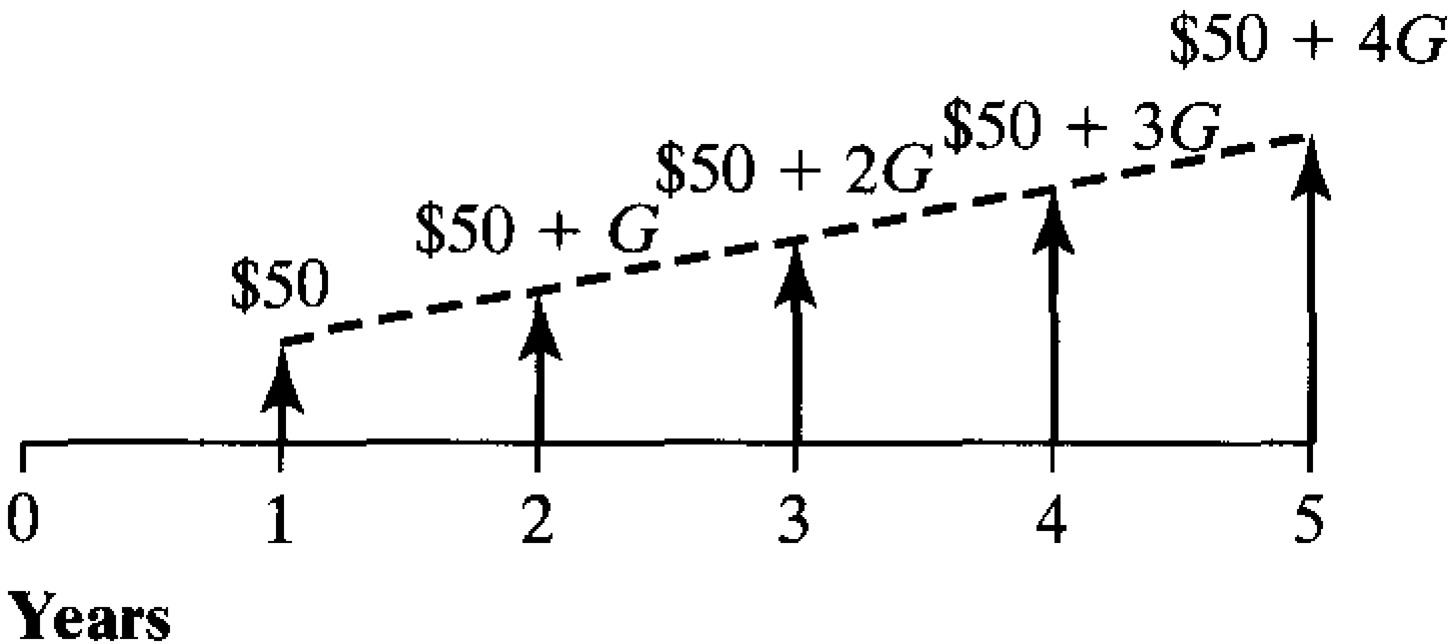
- The equal-cash-flow formulas deal with the equivalence relations P, F, and A (where A is the constant amount of the cash flows in the series)



Linear Gradient Series

- A common pattern of variation occurs when each cash flow in a series increases (or decreases) by a fixed amount
- A five-year loan repayment plan might specify, for example, a series of annual payments that increase by $500 each year
- We call this type a *linear gradient series* because its cash flow diagram produces an ascending (or descending) straight line
- In addition to using P, F, and A, the formulas employed in such problems involve a constant amount G of the change in each cash flow

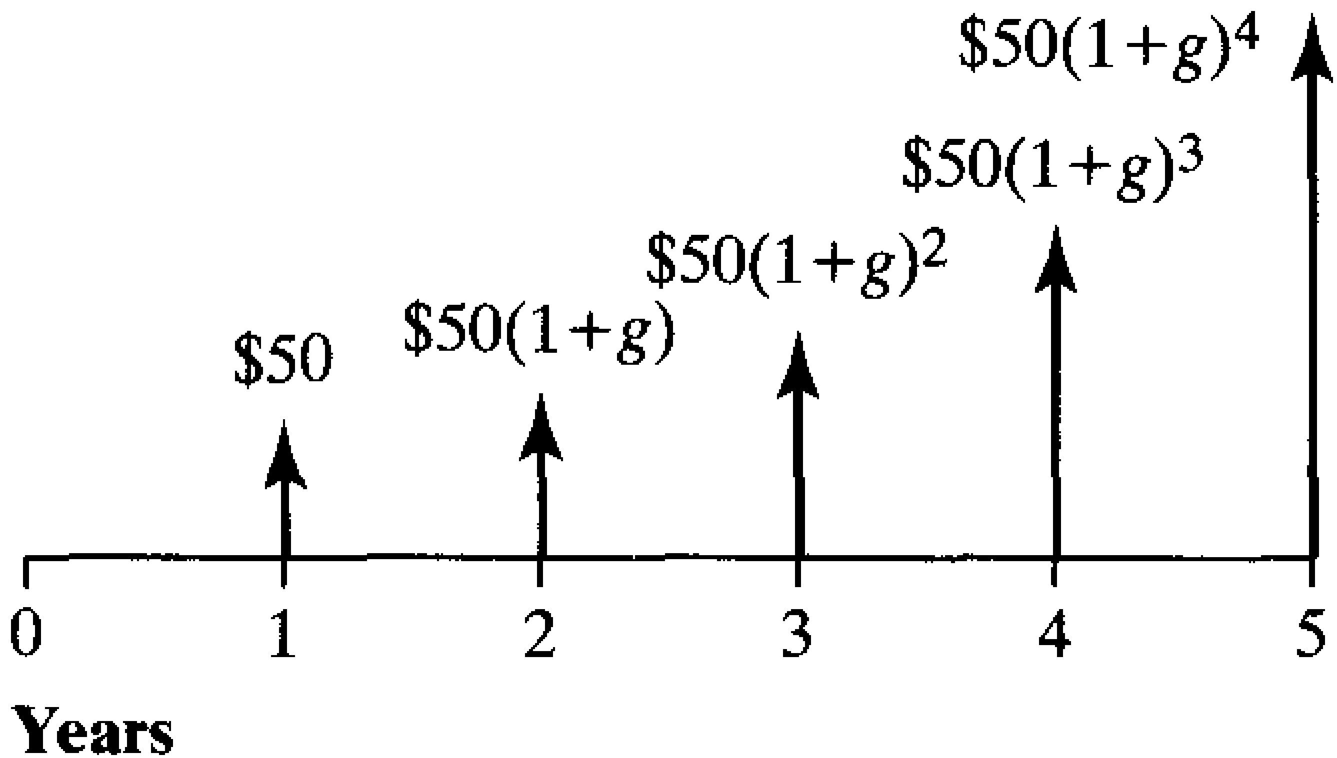
Linear Gradient Series



Geometric Gradient Series

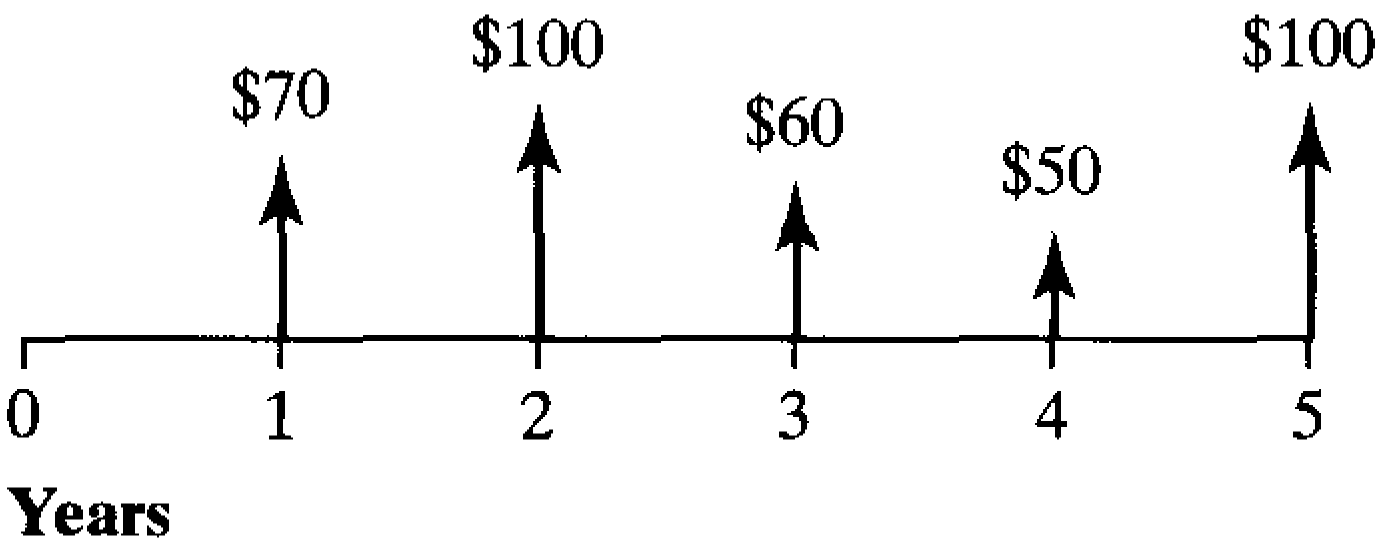
- This type is formed when the series in a cash flow is determined not by some fixed amount like $500, but by some fixed rate, expressed as a percentage
- The curving gradient in the diagram is named the *geometric gradient series*
- In the formulas dealing with such series, the rate of change is represented by a lowercase g

Geometric Gradient Series



Irregular (Mixed) Series

A series of cash flows may be irregular, in that it does not exhibit a regular overall pattern



Single-Payment Factors

- We know that the amount of money F accumulated after n years from a present worth P with interest compounded one time per year is given by the following equation → $F = P(1+i)^n$

- The factor $(1+i)^n$ is called the single-payment compound amount factor (SPCAF) and is usually referred to as the F/P factor

- The factor P/F is known as the single-payment present worth factor (SPPWF)

Single-Payment Factors

- Note that single payment means that only one payment or receipt is involved

- A standard notation has been adopted for all the economic factors and is always in the general form (X/Y,i,n)

- The letter X represents what is sought, while the letter Y represents what is given

- For example, F/P means find F when P is given

- Thus, (F/P,6%,20) represents the ***factor*** that is used to calculate the future amount F accumulated in 20 periods if the interest rate is 6% per period. P is given

Single-Payment Factors

Factor Notation	Factor Name	Find/Given	Standard Notation Equation	Equation with Factor Formula	Excel Functions
$(F/P,i,n)$	Single-payment compound amount	F/P	$F = P(F/P,i,n)$	$F = P(1+i)^n$	FV(i%,n,,P)
$(P/F,i,n)$	Single-payment present worth	P/F	$P = F(P/F,i,n)$	$P = F[1/(1+i)^n]$	PV(i%,n,,F)

The value of (P/F,5%,10) → $P = F[1/(1+i)^n] = 0.6139$

6%	TABLE 11 Discrete Cash Flow: Compound Interest Factors							6%
	Single Payments		Uniform Series Payments				Arithmetic Gradients	
n	Compound Amount F/P	Present Worth P/F	Sinking Fund A/F	Compound Amount F/A	Capital Recovery A/P	Present Worth P/A	Gradient Present Worth P/G	Gradient Uniform Series A/G
1	1.0600	0.9434	1.00000	1.0000	1.06000	0.9434		
2	1.1236	0.8900	0.48544	2.0600	0.54544	1.8334	0.8900	0.4854
3	1.1910	0.8396	0.31411	3.1836	0.37411	2.6730	2.5692	0.9612
4	1.2625	0.7921	0.22859	4.3746	0.28859	3.4651	4.9455	1.4272
5	1.3382	0.7473	0.17740	5.6371	0.23740	4.2124	7.9345	1.8836
6	1.4185	0.7050	0.14336	6.9753	0.20336	4.9173	11.4594	2.3304
7	1.5036	0.6651	0.11914	8.3938	0.17914	5.5824	15.4497	2.7676
8	1.5938	0.6274	0.10104	9.8975	0.16104	6.2098	19.8416	3.1952
9	1.6895	0.5919	0.08702	11.4913	0.14702	6.8017	24.5768	3.6133
10	1.7908	0.5584	0.07587	13.1808	0.13587	7.3601	29.6023	4.0220
11	1.8983	0.5268	0.06679	14.9716	0.12679	7.8869	34.8702	4.4213
12	2.0122	0.4970	0.05928	16.8699	0.11928	8.3838	40.3369	4.8113
13	2.1329	0.4688	0.05296	18.8821	0.11296	8.8527	45.9629	5.1920
14	2.2609	0.4423	0.04758	21.0151	0.10758	9.2950	51.7128	5.5635
15	2.3966	0.4173	0.04296	23.2760	0.10296	9.7122	57.5546	5.9260

Single-Payment Factors

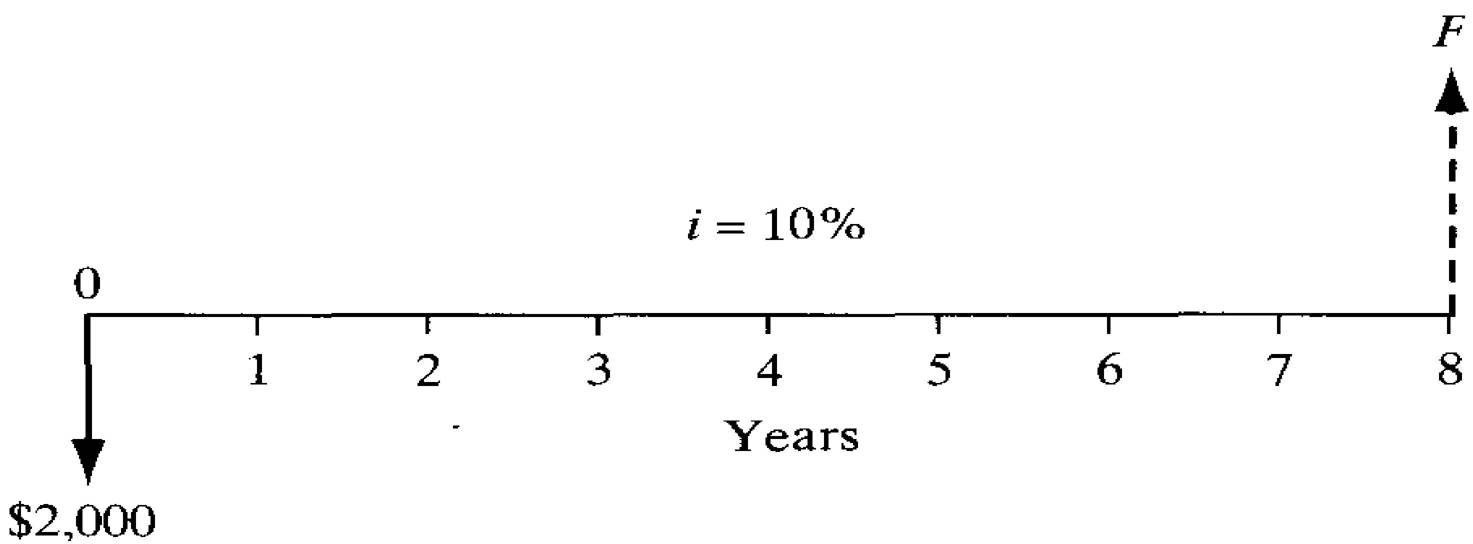
Example

If you had \$2,000 now and invested it at 10%, how much would it be worth in eight years?

$F = P(1+i)^n = \$2{,}000 \times (1+0.1)^8 = \$4{,}287.18$

<u>Or</u>

$F = P(F/P,i,n) = 2{,}000(F/P,10\%,8)$

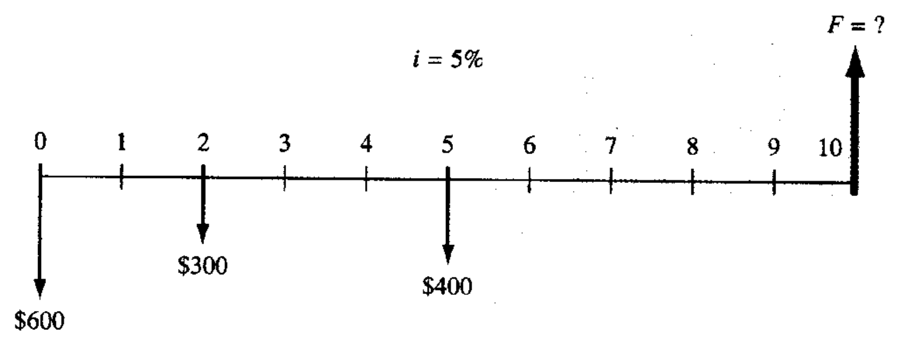


Single-Payment Factors
Example

- The office supplies for an engineering firm for different years were as follows:
  Year 0: \$600; Year 2: \$300; and Year 5: \$400
- What is the equivalent value in year 10 if the interest rate is 5% per year?

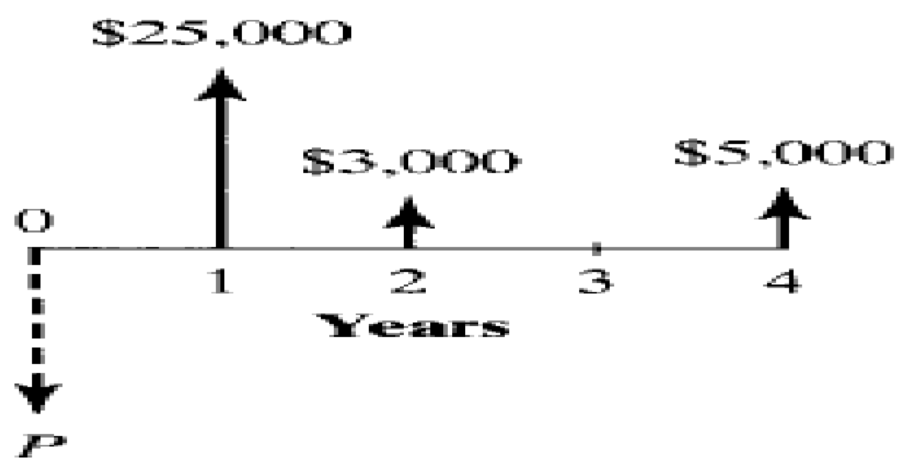
- Draw the cash flow diagram for the values \$600, \$300, and \$400
- Use *F/P* factors to find F in year 10
- F = 600(F/P,5%,10) + 300(F/P,5%,8) + 400(F/P,5%,5) =

$$600 \times (1.6289) + 300 \times (1.4775) + 400 \times (1.2763) = \$1931.11$$



Example
Decomposition and Superposition

- A company wishes to set aside money now to invest over the next four years. The company can earn 10% on a lump sum deposited now, and it wishes to *withdraw* the money in the following increments:
  - Year 1: $25,000
  - Year 2: $3,000
  - Year 3: No expenses
  - Year 4: $5,000



- How much money must be deposited now to cover the anticipated payments over the next 4 years?

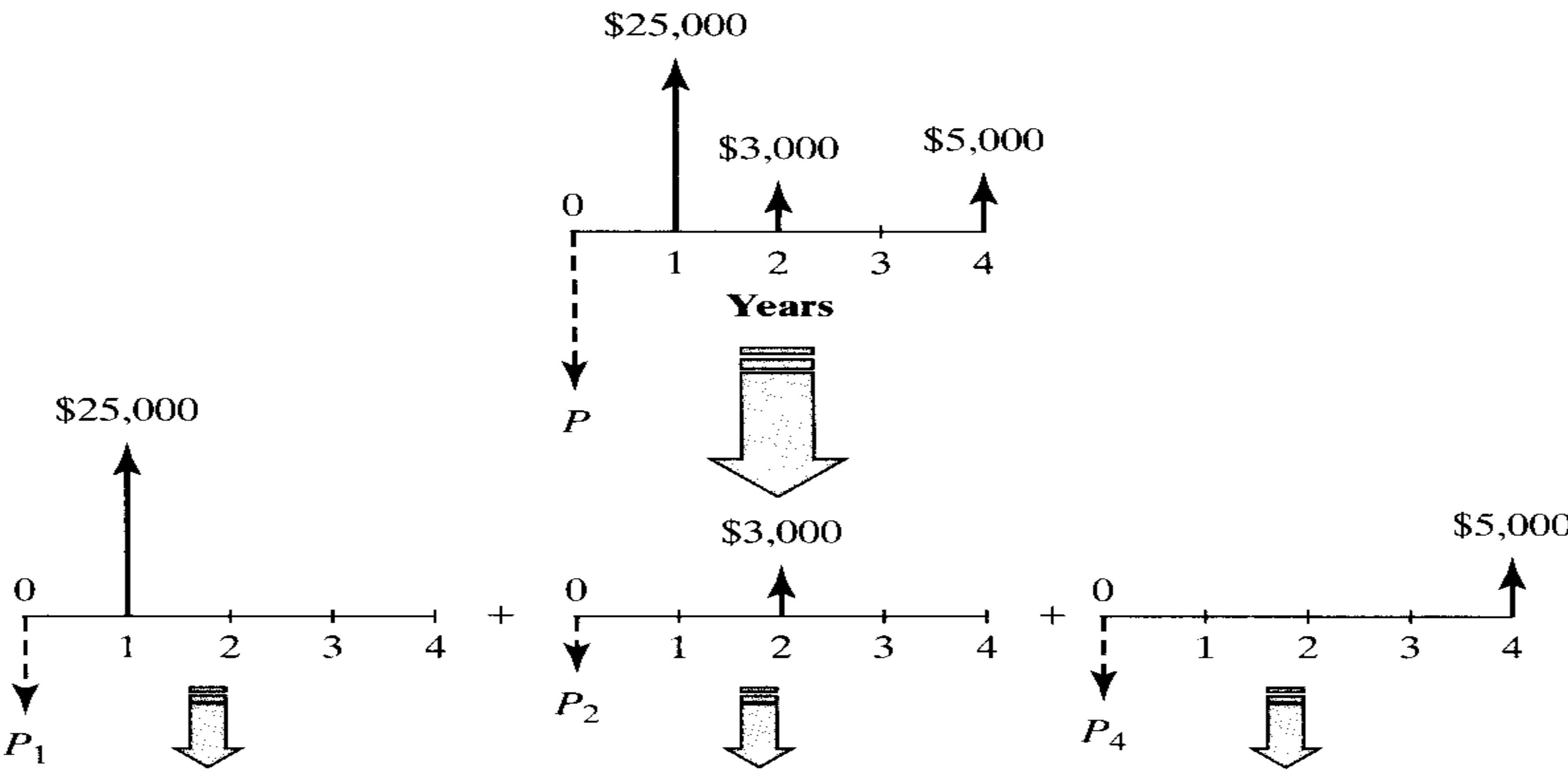
Example
Decomposition and Superposition

- Apparently, one way to deal with an uneven series of cash flows is to calculate the equivalent present value of each single cash flow and to sum the present values to find P

- That is, the cash flow is broken into three components (decomposition) and later all the three present values are summed up (superposition)

Example

Decomposition and Superposition



$P_1 = \$25{,}000(P/F, 10\%, 1)$
$= \$22{,}727$

$P_2 = \$3{,}000(P/F, 10\%, 2)$
$= \$2{,}479$

$P_4 = \$5{,}000(P/F, 10\%, 4)$
$= \$3{,}415$

$$\boxed{P = P_1 + P_2 + P_4 = \$28{,}622}$$

Example

Decomposition and Superposition

- To see if the needed \$28,622 is sufficient, let's calculate the balance at the end of each year
- If you deposit \$28,622 now, it will grow to (1.10)(\$28,622), or \$31,484, at the end of year 1. From this balance, you pay out \$25,000
- The remaining balance, \$6,484, will again grow to (1.10)(\$6,484), or \$7,132, at the end of year 2. Now you make the second payment (\$3,000) out of this balance, which will leave you with only \$4,132 at the end of year 2
- Since no payment occurs in year 3, the balance will grow to $\$(1.10)^2(\$4{,}132)$, or \$5,000, at the end of year 4
- The final withdrawal in the amount of \$5,000 will deplete the balance completely