# Engineering Economy

## [2-2]
## Time Value of Money

### Uniform Series

# Present-Worth Factor
## Equal Payment Uniform Series

- What would you have to *invest* now P in order to withdraw A dollars at the end of each of the next n periods?

- In this case, it is the P/A factor used to calculate the equivalent P value in year 0 for a uniform series of A values beginning at the end of period 1 and extending for n periods

$$P = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$$

- The term in brackets is the conversion factor known as the uniform-series present worth factor (USPWF)

# Capital Recovery Factor
## Equal Payment Uniform Series

- To reverse the situation, the present worth P is known and the equivalent uniform-series amount A is sought

- The first A value occurs at the end of period 1, that is, one period after P occurs

$$A = P\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right]$$

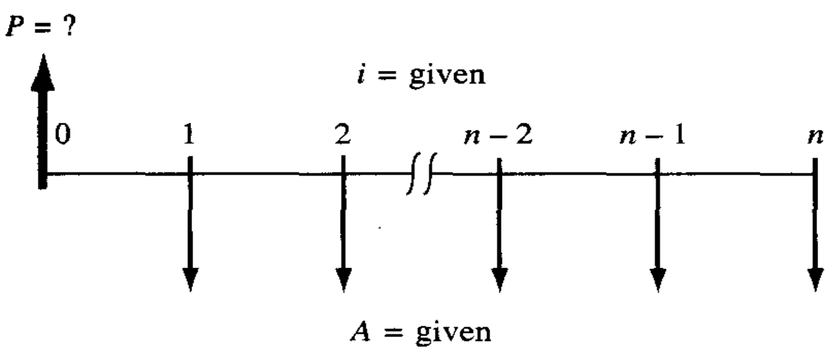
- The term in brackets is called the capital recovery factor (CRF), or A/P factor. It is like investing P now and getting the equivalent through annual uniform n number of equal payments (A)

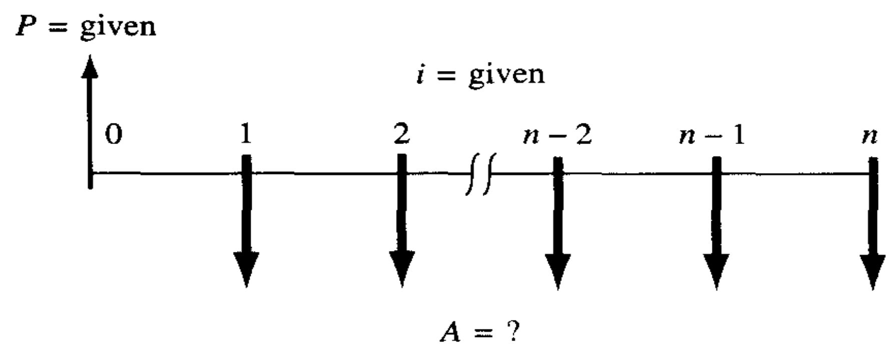
# Present-Worth and Capital Recovery Factor

## Equal Payment Uniform Series

| Factor Notation | Factor Name | Find/Given | Factor Formula | Standard Notation Equation | Excel Function |
|---|---|---|---|---|---|
| $(P/A,i,n)$ | Uniform-series present worth | $P/A$ | $\frac{(1+i)^n-1}{i(1+i)^n}$ | $P = A(P/A,i,n)$ | PV($i$%,$n$,$A$) |
| $(A/P,i,n)$ | Capital recovery | $A/P$ | $\frac{i(1+i)^n}{(1+i)^n-1}$ | $A = P(A/P,i,n)$ | PMT($i$%,$n$,$P$) |

If i = 15% and n = 25 years, the P/A factor value is (P/A,15%,25) = 6.4641



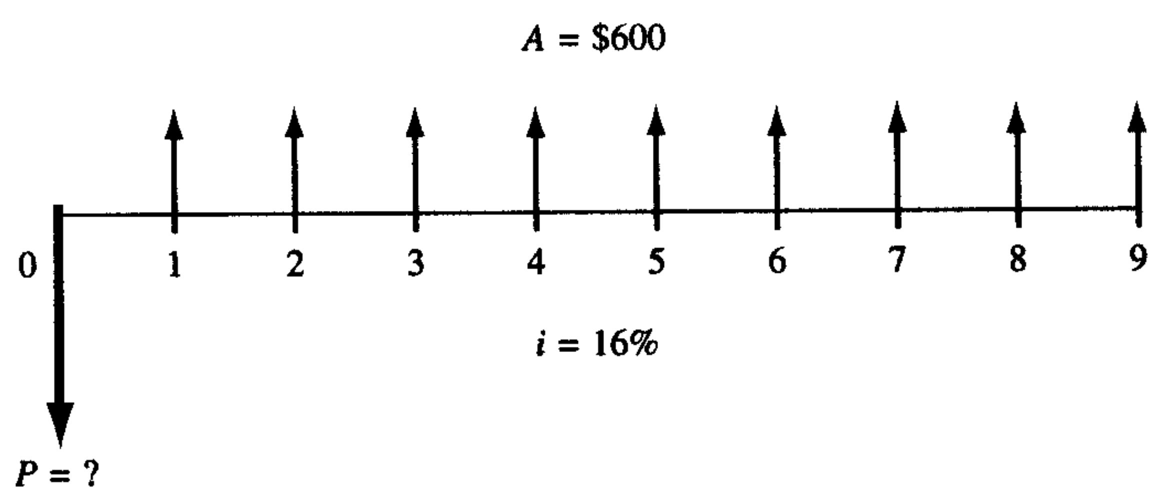

*Very important to note that there is no A payment at t = 0 but only P payment*

# Present-Worth Factor
## Equal Payment Uniform Series

- How much money should you be willing to pay now for a guaranteed $600 per year for 9 years starting next year, at a rate of return of 16% per year?

- The present worth is:

  $P = 600(P/A,16\%,9) = 600 \times (4.6065) = \$2,763.90$



# Present-Worth Factor
## Equal Payment Uniform Series

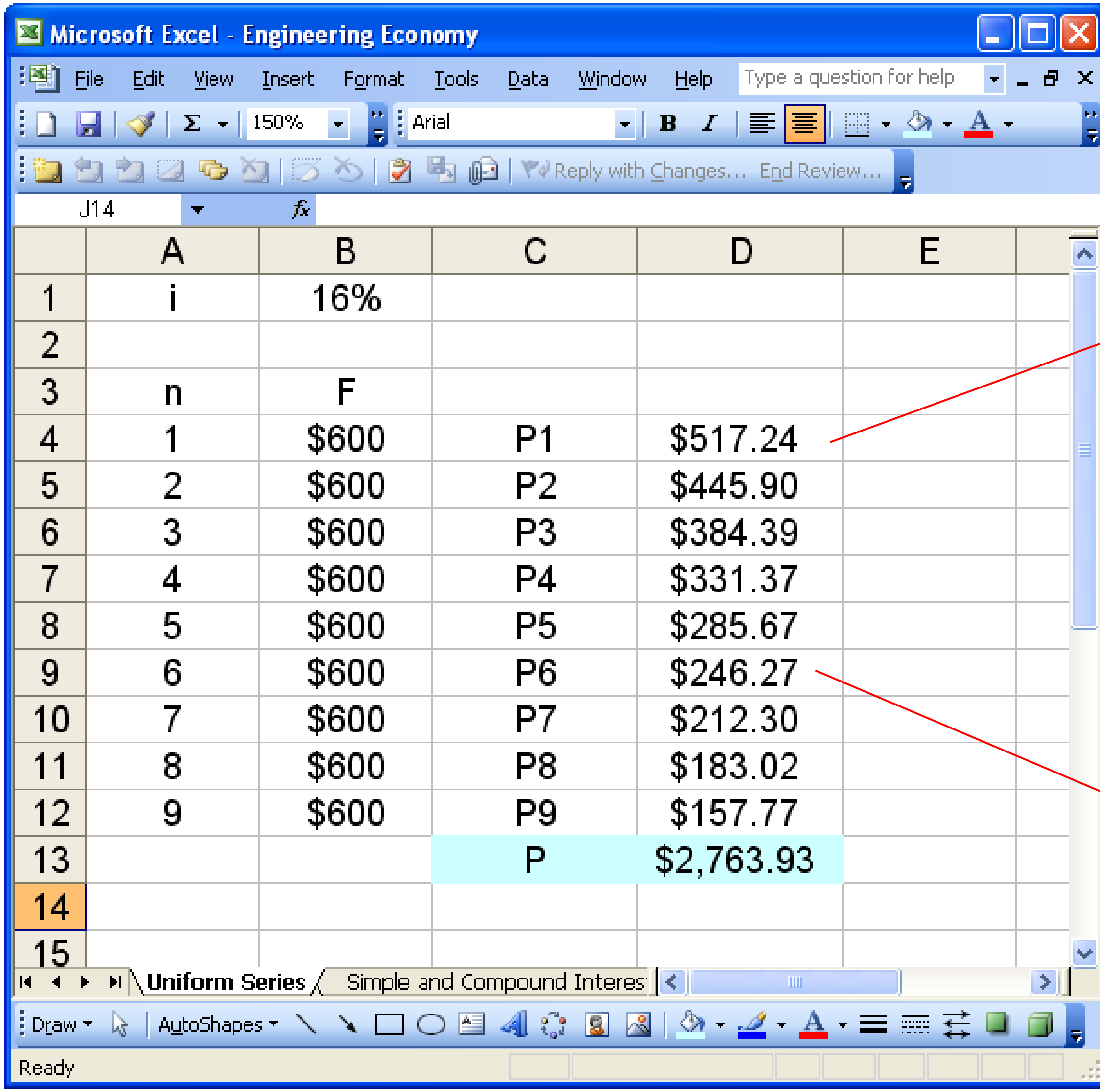
- We can solve the previous example in the following way using the superposition theory

- Simply assume each $600 dollar due by the end of each year is the future value of a present value (at time = 0)

- Thereafter, sum up all these present values to arrive at the total present value that yield the equal payments of $600 at the end of each year

# Present-Worth Factor

## Equal Payment Uniform Series




| | A | B | C | D |
|---|---|---|---|---|
| 1 | i | 16% | | |
| 2 | | | | |
| 3 | n | F | | |
| 4 | 1 | $600 | P1 | $517.24 |
| 5 | 2 | $600 | P2 | $445.90 |
| 6 | 3 | $600 | P3 | $384.39 |
| 7 | 4 | $600 | P4 | $331.37 |
| 8 | 5 | $600 | P5 | $285.67 |
| 9 | 6 | $600 | P6 | $246.27 |
| 10 | 7 | $600 | P7 | $212.30 |
| 11 | 8 | $600 | P8 | $183.02 |
| 12 | 9 | $600 | P9 | $157.77 |
| 13 | | | P | $2,763.93 |

**PV(16%,1,,600)**

**PV(16%,6,,600)**

# Sinking Fund Factor and
# Uniform Series Compound Amount

- Suppose we are interested in the future amount F of a fund to which we contribute A dollars each period and on which we earn interest at a rate of i per period

$$A = F\left[\frac{i}{(1+i)^n - 1}\right] \rightarrow \textbf{(A/F,i,n)}$$

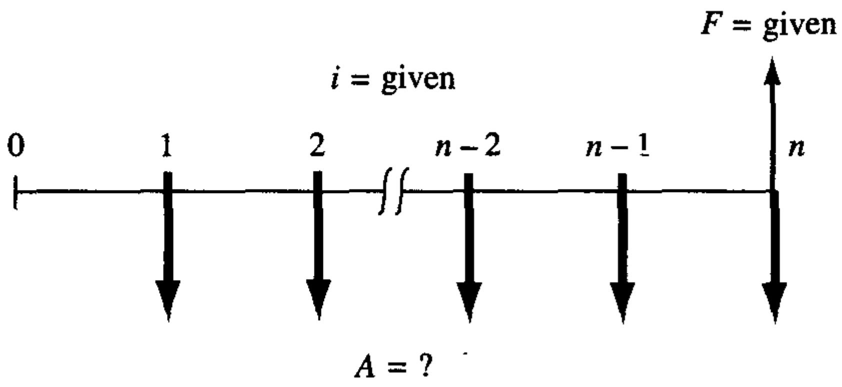
- The expression in brackets is the A/F or sinking fund factor

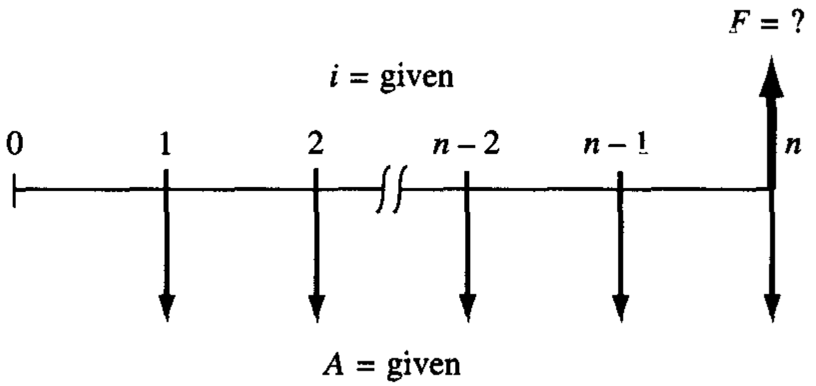
- The above equation can be rearranged to find F for a stated A series in periods 1 through n

$$F = A\left[\frac{(1+i)^n - 1}{i}\right] \rightarrow$$

**Uniform-series compound amount factor (F/A,i,n)**

# Sinking Fund Factor and
# Uniform Series Compound Amount

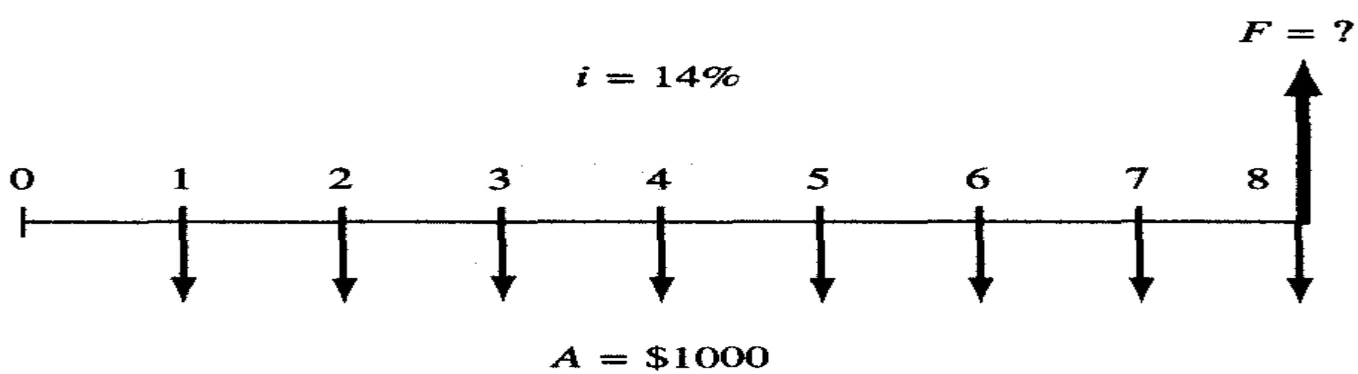


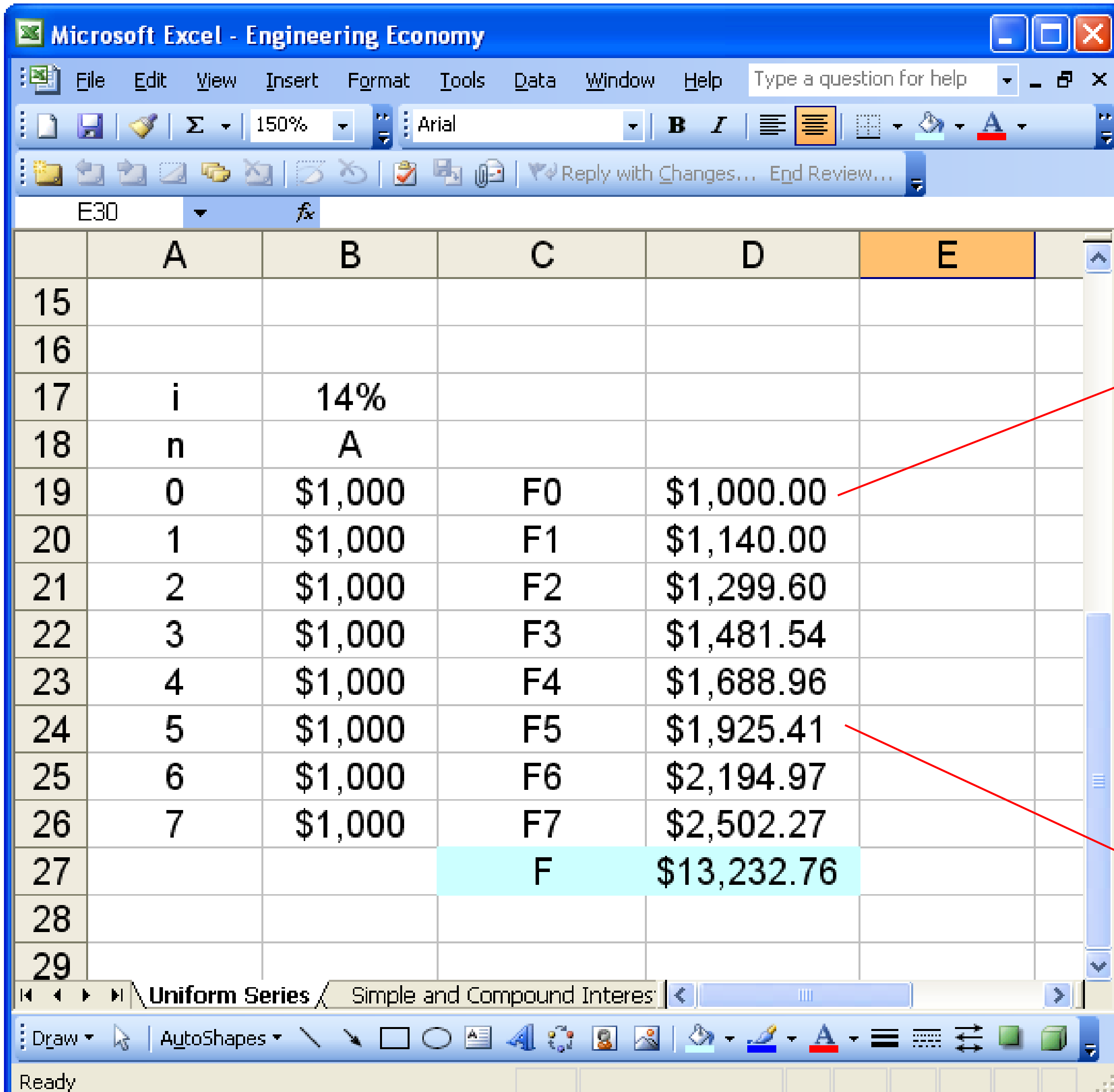

*Note that A payments start at the beginning of the second year*

# Sinking Fund Factor and Uniform Series Compound Amount

- What is the equivalent future worth of one thousand dollar of investment each year for 8 years starting 1 year from now with an interest rate of 14%?
- You need to find out the value of F
- The cash flow diagram shows the annual payments
- F = 1,000 × (F/A,14%,8) = $13,232.8



# Sinking Fund Factor and
# Uniform Series Compound Amount



| | A | B | C | D | E |
|---|---|---|---|---|---|
| 15 | | | | | |
| 16 | | | | | |
| 17 | i | 14% | | | |
| 18 | n | A | | | |
| 19 | 0 | $1,000 | F0 | $1,000.00 | |
| 20 | 1 | $1,000 | F1 | $1,140.00 | |
| 21 | 2 | $1,000 | F2 | $1,299.60 | |
| 22 | 3 | $1,000 | F3 | $1,481.54 | |
| 23 | 4 | $1,000 | F4 | $1,688.96 | |
| 24 | 5 | $1,000 | F5 | $1,925.41 | |
| 25 | 6 | $1,000 | F6 | $2,194.97 | |
| 26 | 7 | $1,000 | F7 | $2,502.27 | |
| 27 | | | F | $13,232.76 | |
| 28 | | | | | |
| 29 | | | | | |

**FV(14%,0,,1000)**

**FV(14%,5,,1000)**