Engineering Economy

[3]
Combining Factors
Examples

General

- Most estimated cash flow series do not fit exactly the series for which the *factors* and equations were developed earlier

- Therefore, it is necessary to combine the equations

- However, there are several ways to address a particular sequence of cash flows in order to determine the present, future, or annual worth

- Different ways to address this will be explained herein through a set of solved examples

Combining Factors
The Different Cases

1. Shifted series: Determine P, F, or A of a uniform series starting at a time other than period 1

2. Calculate P, F, or A of randomly placed single amounts and uniform series

3. Make equivalence calculations for cash flows involving shifted arithmetic or geometric gradients

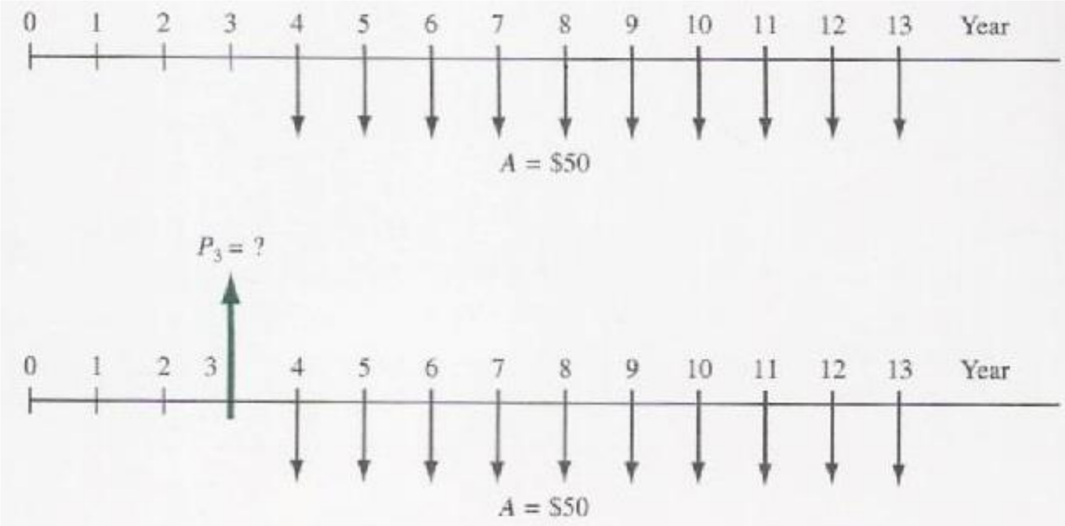
4. Make equivalence calculations for cash flows involving decreasing arithmetic gradients (refer to section [2-3])

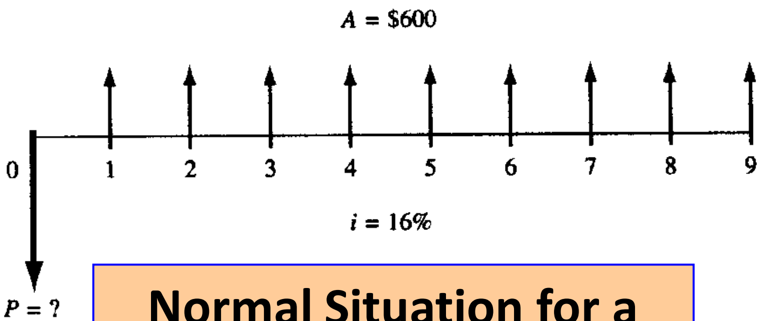
Shifted Uniform Series
Example – 1

- When dealing with uniform series, the normal situation is to have the series begins at the *end of period 1*

- Or, the present worth is always located one period prior to the first uniform series amount when using the P/A factor

- When we have a situation that the payment does not start at the end of period 1, then the series is called "shifted series"





Normal Situation for a nine-period uniform annual payments

Shifted Uniform Series

Example – 1

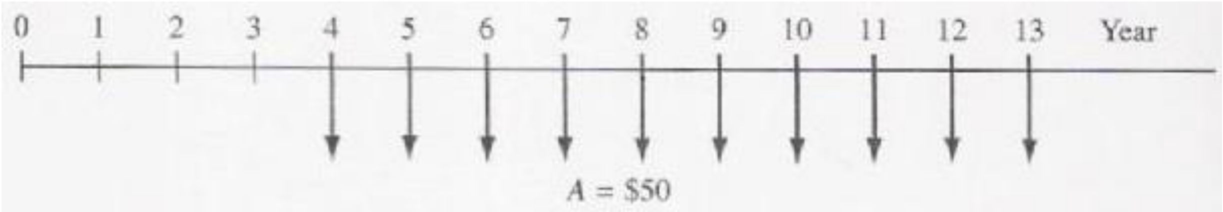
- When we have shifted uniform series, then **P** can be determined by any of the following methods:

  [1] Use the P/F factor to find the present worth of each disbursement at year 0 and add them up

  [2] Use the F/P factor to find the future worth of each disbursement in year 13, add them, and then find the present worth of the total P=F(P/F,i,13)

  [3] Use the F/A factor to find the future amount F=A(F/A,i,10) and then compute the present worth using P=F(P/F,i,13)

  [4] Use the P/A factor to compute the present worth (located in year 3 not 0) and then find the present worth in year 0 by using the (P/F,i,3) factor



Shifted Uniform Series
Example – 1

- Assume that the interest rate is 8%

Year	F	P
0	0	$0.00
1	0	$0.00
2	0	$0.00
3	0	$0.00
4	50	$36.75
5	50	$34.03
6	50	$31.51
7	50	$29.17
8	50	$27.01
9	50	$25.01
10	50	$23.16
11	50	$21.44
12	50	$19.86
13	50	$18.38
Total	-	$266.33

$P = F[1/(1+i)^n]$

$P = F(P/F,i,n)$

Find the present worth corresponding to the future values

$P = F(P/F,8\%,7)$

Shifted Uniform Series

Example – 1

Year	P	F
0	0	$0.00
1	0	$0.00
2	0	$0.00
3	0	$0.00
4	50	$99.95
5	50	$92.55
6	50	$85.69
7	50	$79.34
8	50	$73.47
9	50	$68.02
10	50	$62.99
11	50	$58.32
12	50	$54.00
13	50	$50.00
Total	-	$724.33
	P	$266.33

$$F = P(1+i)^n$$

Find the future worth corresponding to the present values, sum up these future values and convert back to find the present value

$$F = P(F/P,8\%,4)$$

Shifted Uniform Series

Example – 1

$$\left[\frac{(1+i)^n - 1}{i}\right]$$

F/A	(F/A,i,10)	14.49
	F13	$724.33
	P0	$266.33

$$F = A\left[\frac{(1+i)^n - 1}{i}\right]$$

P = F(P/F,8%,13)

Shifted Uniform Series

Example – 1

$$\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$$

P/A	(P/A,i,10)	6.71
	P3	$335.50
	P0	$266.33

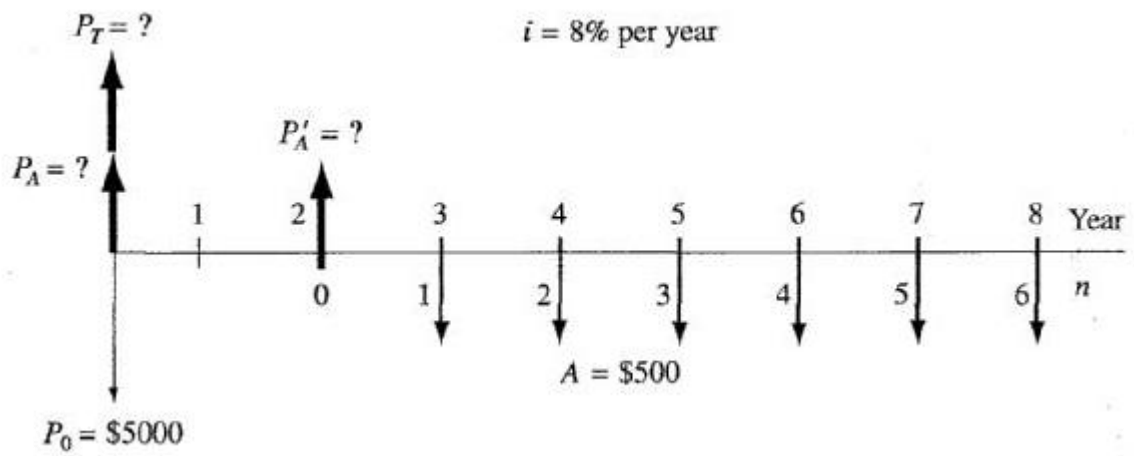
$$P = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$$

P = F(P/F,8%,3)

Shifted Uniform Series
Example – 2

- An engineering technology group just *purchased* a software for \$5,000 now and annual payments of \$500 per year for 6 years starting 3 years from now for *annual upgrades*

- What is the *present worth* of the payments if the interest rate is 8% per year?



Shifted Uniform Series
Example – 2

- Find the value of P'_A for the shifted series
  $P'_A = \$500(P/A,8\%,6)$

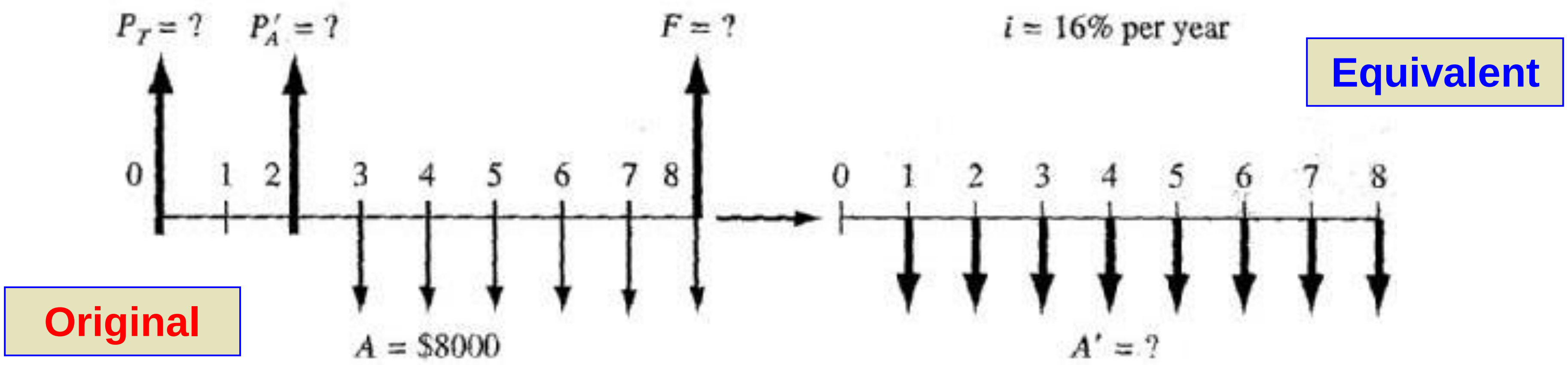
- Since P'_A is located in year 2, now find P_A in year 0
  $P_A = P'_A(P/F,8\%,2)$

- The total present worth is determined by adding P_A and the initial payment P_0 in year 0
  $P_T = P_0 + P_A = 5{,}000 + 500(P/A,8\%,6)(P/F,8\%,2)$
  $= 5{,}000 + 500(4.6229)(0.8573) = \$6{,}981.6$

Shifted Uniform Series
Example – 3

- Recalibration of sensitive measuring devices costs $8,000 per year

- If the machine will be recalibrated for each of 6 years starting 3 years after purchase, calculate the 8-year equivalent uniform series at 16% per year



Shifted Uniform Series
Example – 3

- To solve this question, first calculate P'_A from the uniform series A, and then find the P_T value. After that, compute A' based on the value of P_T

- $P'_A = 8{,}000(P/A,16\%,6)$

- $P_T = P'_A(P/F,16\%,2) = 8{,}000(P/A,16\%,6)(P/F,16\%,2)$

  $= 8{,}000(3.6847)(0.7432) = \$21{,}907.75$

- The equivalent series A' for 8 years can now be determined via the A/P factor:

  $A' = P_T(A/P,16\%,8) = \$5{,}043.60$

Shifted Uniform Series
Example – 3

- An alternative way to solve this question is by finding the future worth F in year 8 based on the uniform series A. Then use this future worth value to find the uniform series A'

- F = 8,000(F/A,16%,6) = \$71,820

- The A/F factor is used to obtain A' over all 8 years
  A' = F(A/F,16%,8) = \$5,043.2

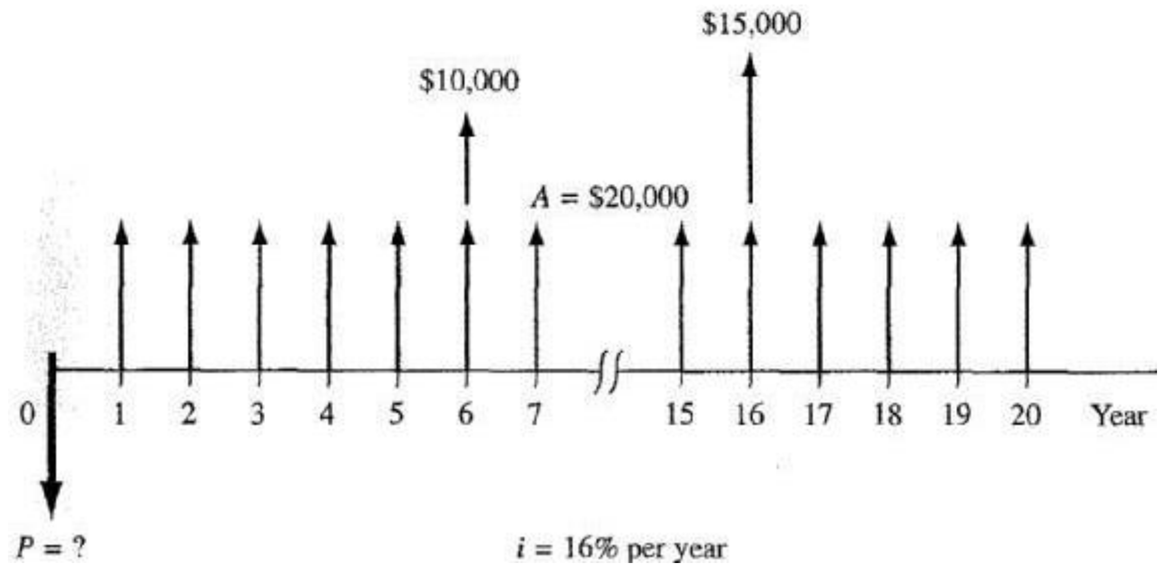
Uniform Series and Randomly Placed Amounts

- When cash flows include both a uniform series and randomly placed single amounts, the procedure to find the present worth value would be as follows:

  - Find the present worth for the uniform series using the P/A factor
  - Find the present worth for the single amounts using the P/F factor

Uniform Series and Randomly Placed Amounts

Example



- To find the present worth, do the following:

  [1] Find P for the 20-year uniform series = 20,000(P/A,16%,20)

  [2] Find P for the $10,000 amount = 10,000(P/F,16%,6)

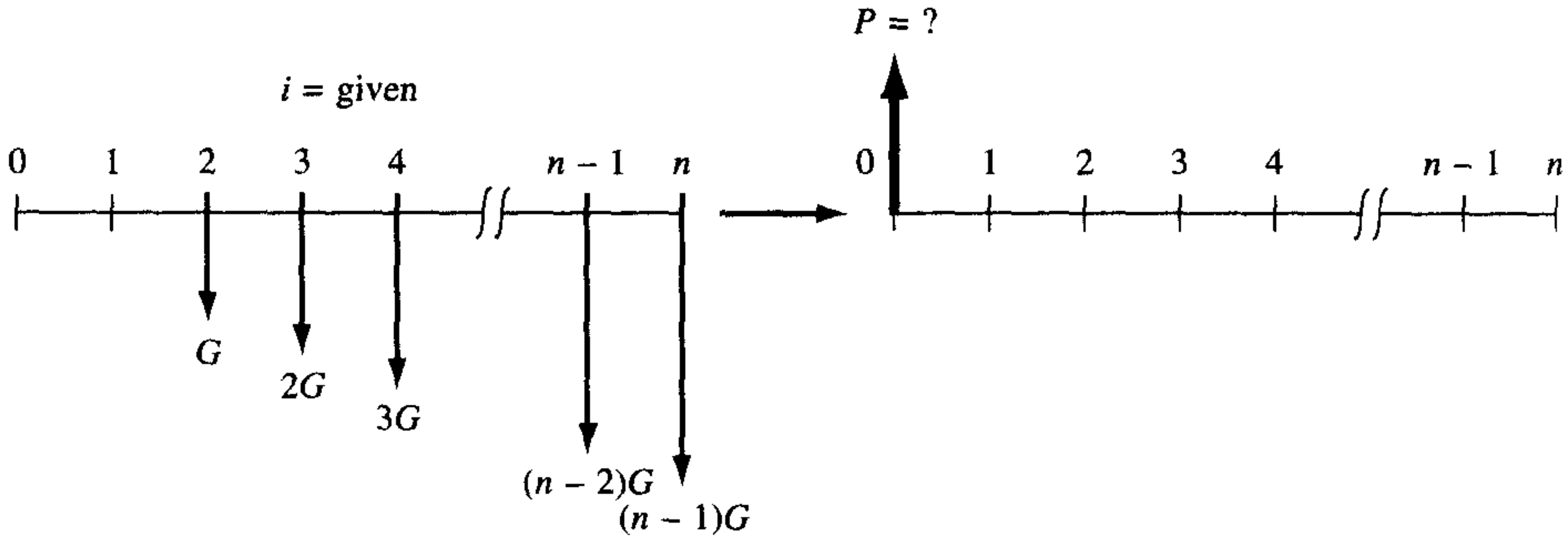
  [3] Find P for the $15,000 amount = 15,000(P/F,16%,16)

- Sum up the three values and this equals $124,075

Shifted Gradients

- To find the present worth of an arithmetic gradient series, we use the relation P = G(P/G,i,n)

- Just keep in mind that the *present worth* of an arithmetic gradient will always be located two periods before the gradient starts



Shifted Gradients

- A gradient that starts at any time that is not the end of second year is called a shifted gradient

- When having shifted gradients, then we can resort to renumbering the time scale

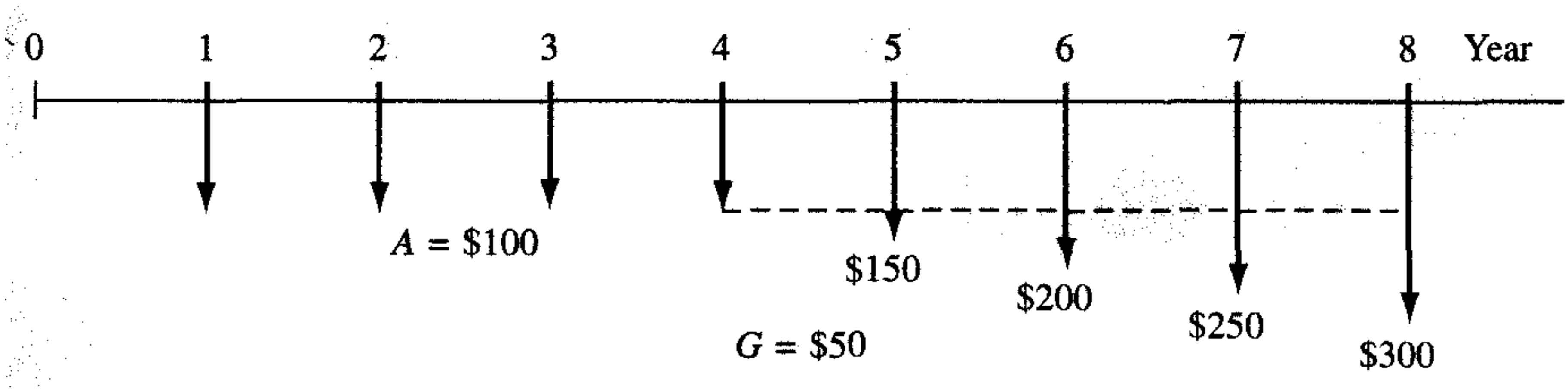
- The period in which the gradient first appears is labeled period **2** and the value n is obtained accordingly

Shifted Gradients

Example – 1

- An engineer has tracked the average inspection cost for 8 years. The cost average was steady at $100 for the first four years but have increased consistently by $50 for each of the last 4 years

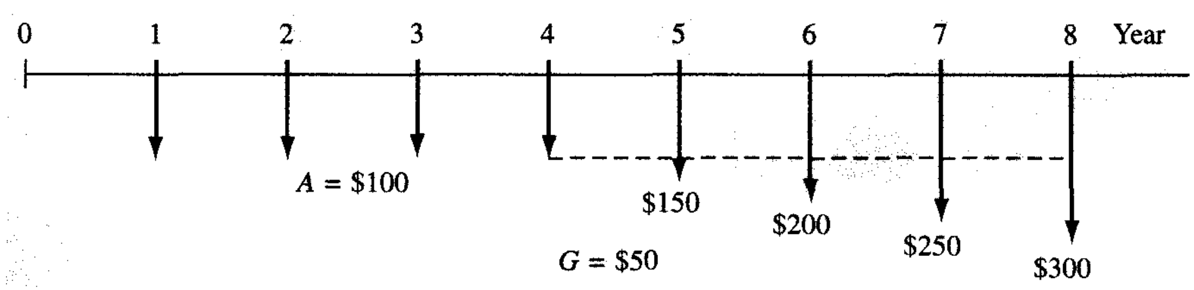
- What is the total present worth (in year 0)?



Shifted Gradients

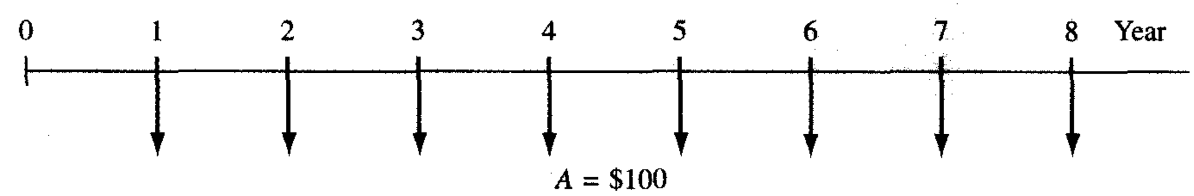
Example – 1

To solve it, you need to decompose it as follows:



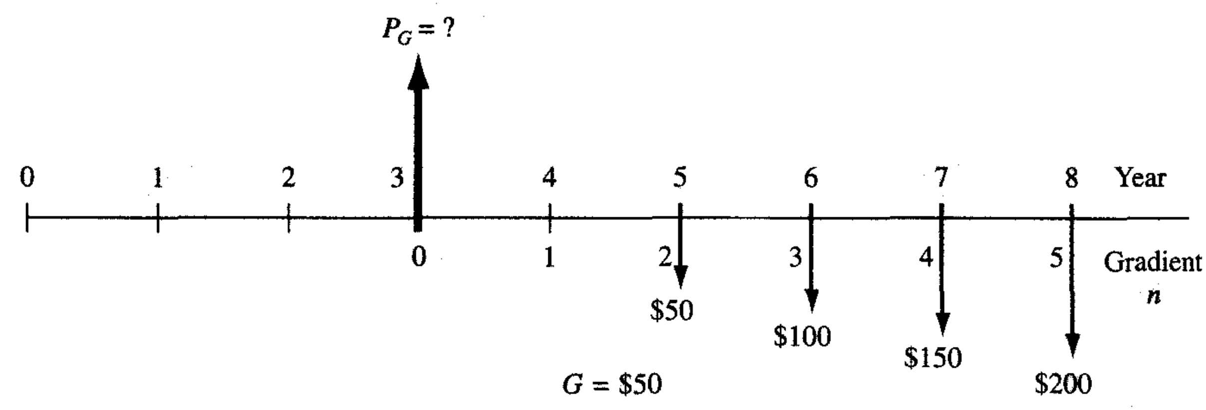
Original cash flow diagram

$$P = P_A + P_G$$



Uniform series cash flow diagram

$$P_A = 100(P/A,i,8)$$



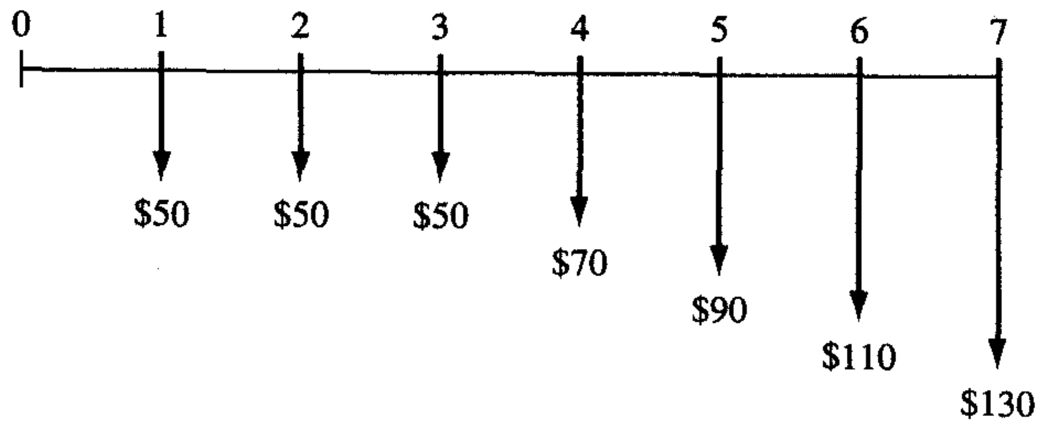
Arithmetic gradient cash flow diagram

$$P'_G = 50(P/g,I,5)$$

$$P_G = [50(P/G,i,5)](P/F,i,3)$$

Shifted Gradients
Example – 2

- Compute the *equivalent uniform* annual series in years 1 through 7



- Find the present worth for the arithmetic gradient series at year 2: $P_G = 20(P/G,i,5)$
- $P_0 = P_G(P/F,i,2)$
- Annualize the P_0: $A_G = P_0(A/P,i,7)$
- Add the base amount = $50 + A_G$