1.3 Matrix Arithmetic

matrix:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ \vdots & & & & \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}_{m\times n}$$

a_{ij}: $i \to$ row, $j \to$ column.

, order = $m \times n$ (size)

vectors:

row vector: $\tilde{x} = (x_1, x_2, x_3, x_4)$

ex $\begin{bmatrix} 2 & 3 & 4 & 5 \end{bmatrix}$ or $(2,3,4,5)$

$\tilde{a}_i = (a_{i1}, a_{i2}, a_{i3}, \ldots, a_{in})_{1\times n}$ ← row vector of A, $i = 1, 2, \ldots, m$

(i)th row

column vector: $x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}_{n\times 1}$

ex $\begin{bmatrix} 1 \\ 5 \\ 7 \end{bmatrix}$, $a_j = \begin{bmatrix} a_{1j} \\ a_{2j} \\ \vdots \\ a_{mj} \end{bmatrix}_{m\times 1}$, $j = 1, \ldots, n$

ex $A = \begin{bmatrix} 3 & 2 & 4 \\ -1 & 8 & 5 \end{bmatrix}$

then $a_1 = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$, $a_2 = \begin{bmatrix} 2 \\ 8 \end{bmatrix}$, $a_3 = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$

$\tilde{a}_1 = (3, 2, 4)$, $\tilde{a}_2 = (-1, 8, 5)$

Equality

Two $m \times n$ matrices A, B are said to be equal if $a_{ij} = b_{ij}$ for each (i) and (j)

ex $\begin{bmatrix} 1 & 2 \\ 3 & x^2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ y & 4 \end{bmatrix} \Rightarrow y = 3$

$x^2 = 4 \rightarrow x = \pm 2$

Scalar Multiplication

vector متجه

scalar قيمة ثابتة $\Rightarrow$ ع الأرقام عدد حقيقي

بدون اتجاه.

ex $A = \begin{bmatrix} 4 & 8 & 6 \\ -2 & 0 & 10 \end{bmatrix}$

then

$\frac{1}{2} A = \begin{bmatrix} 2 & 4 & 3 \\ -1 & 0 & 5 \end{bmatrix}$,

$3A = \begin{bmatrix} 12 & 24 & 18 \\ -6 & 0 & 30 \end{bmatrix}$

Matrix addition

the matrices must have the same order (size).

ex
$$\begin{bmatrix} 1 & 3 \\ 5 & 6 \end{bmatrix}_{2\times 2} + \begin{bmatrix} -1 & -7 \\ 8 & 0 \end{bmatrix}_{2\times 2} = \begin{bmatrix} 0 & -4 \\ 13 & 6 \end{bmatrix}$$

ex $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 0 & 1 \\ 3 & -3 & -3 \end{bmatrix}$

$$3A - B = \begin{bmatrix} 3 & 6 & 9 \\ 12 & 15 & 18 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 1 \\ 3 & -3 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 6 & 8 \\ 9 & 18 & 21 \end{bmatrix}$$

Matrix Multiplication

$$\underset{m\times n}{A} * \underset{n\times r}{B} = C_{m\times r}$$

شرط عملية الضرب

number columns in the first matrix = number of rows in the second matrix.

اذا لم تتحقق $\Rightarrow$ undefined

غير معرف.

ex If $A = \begin{bmatrix} 3 & -2 \\ 2 & 4 \\ 1 & -3 \end{bmatrix}$, $B = \begin{bmatrix} -2 & 1 & 3 \\ 4 & 1 & 6 \end{bmatrix}$

3×2 $\qquad$ 2×3

find AB, BA

sol:

$$\underset{3\times2}{A}\ \underset{2\times3}{B} = \begin{bmatrix} 3 & -2 \\ 2 & 4 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} -2 & 1 & 3 \\ 4 & 1 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} (3*-2)+(-2*4) & (3*1)+(-2*1) & (3*3)+(-2*6) \\ & & \\ & & \end{bmatrix}$$