complete sec. 1.3

ex If $A = \begin{bmatrix} 3 & -2 \\ 2 & 4 \\ 1 & -3 \end{bmatrix}_{3\times 2}$, $B = \begin{bmatrix} -2 & 1 & 3 \\ 4 & 1 & 6 \end{bmatrix}_{2\times 3}$

Find: AB, BA.

Sol:

$$\underset{3\times 2}{A}\ \underset{2\times 3}{B} = \begin{bmatrix} 3 & -2 \\ 2 & 4 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} -2 & 1 & 3 \\ 4 & 1 & 6 \end{bmatrix}$$

$3*1+-2*1$

$3*3+-2*6$

$$= \begin{bmatrix} (3*-2)+(-2*4) & 3+-2 & 9+-12 \\ -4+16 & 2+4 & 6+24 \\ -2+-12 & 1+-3 & 3+-18 \end{bmatrix}$$

$$= \begin{bmatrix} -14 & 1 & -3 \\ 12 & 6 & 30 \\ -14 & -2 & -15 \end{bmatrix}_{3\times 3}$$

$$\underset{2\times 3}{B}\ \underset{3\times 2}{A} = \begin{bmatrix} -2 & 1 & 3 \\ 4 & 1 & 6 \end{bmatrix} \begin{bmatrix} 3 & -2 \\ 2 & 4 \\ 1 & -3 \end{bmatrix} =$$

$$\begin{bmatrix} -6+2+3 & 4+4-9 \\ 12+2+6 & -8+4-18 \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ 20 & -22 \end{bmatrix}_{2\times 2}$$

In general In matrices

$AB \neq BA$ (multiplication is not commutative) ex $A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 3 & -1 \\ 2 & 0 & 7 \end{bmatrix}$

$C = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, $D = [1, 4, 7]$.

find:

(1) $\underset{2\times2}{A}\underset{1\times3}{D}$ = multiplication is undefined.

(2) $\underset{3\times1}{C}\underset{1\times3}{D} = [\quad]_{3\times3}$

(3) $\underset{1\times3}{D}\underset{3\times1}{C} = [\quad]_{1\times1}$

(4) AB ✓

(5) BA ✗ not defined. impossible.

Linear System

$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \cdots + a_{1n}x_n = b_1$

$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \cdots + a_{2n}x_n = b_1$

$\vdots$

$a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 \cdots + a_{mn}x_n = b_m$

variable (unknowns) = $x_1, \ldots x_n$, eq: m

matrix equation: $AX = b$

coefficient matrix

matrix of unknowns

constant matrix.

$$\underset{m\times n}{\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}} \underset{n\times1}{\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}} = \underset{m\times1}{\begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}}$$

$$\underset{m\times n}{A}\ \underset{n\times 1}{X} = \underset{m\times 1}{b} \qquad \text{matrix equation (eq).}$$

$$x_1 \begin{bmatrix} a_{11} \\ a_{21} \\ a_{31} \\ \vdots \\ a_{m1} \end{bmatrix} + x_2 \begin{bmatrix} a_{12} \\ a_{22} \\ a_{32} \\ \vdots \\ a_{m2} \end{bmatrix} + \cdots = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}$$

A

$$x_1 a_1 + x_2 a_2 + \cdots x_n a_n = b \quad \text{matrix eq.}$$

$$R^m = R^{m\times 1}$$

$$\begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_m \end{bmatrix} \in$$

ex

$$2x_1 + 3x_2 - 2x_3 = 5$$

$$5x_1 - 4x_2 + 2x_3 = 6$$

write matrix eq...

$$x_1 \begin{bmatrix} 2 \\ 5 \end{bmatrix} + x_2 \begin{bmatrix} 3 \\ -4 \end{bmatrix} + x_3 \begin{bmatrix} -2 \\ 2 \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \end{bmatrix}$$

Def: If $a_1, a_2, \ldots a_n$ are vectors in R^m and $c_1, c_2, \ldots c_n$ scalar, then the sum of the form

scalar vector

$$c_1 a_1 + c_2 a_2 + \cdots + c_n a_n$$

is said to be a linear combination of the vectors $a_1, a_2, \ldots, a_n$

complete ex:-

$2x_1+3x_2-2x_3=5$

$5x_1-4x_2+2x_3=6$

$$x_1\begin{bmatrix}2\\5\end{bmatrix}+x_2\begin{bmatrix}3\\-4\end{bmatrix}+x_3\begin{bmatrix}-2\\2\end{bmatrix}=\begin{bmatrix}5\\6\end{bmatrix}.$$

If we choose $x_1=2,\ x_2=3,\ x_3=4$

$x_1, x_2, x_3 \to$ solution of the sys.

we find them $\left[\begin{array}{ccc|c}2 & 3 & -2 & 5\\ 5 & -4 & 2 & 6\end{array}\right] \to$ sol. infinitly

$$\underline{2}\begin{bmatrix}2\\5\end{bmatrix}+\underline{3}\begin{bmatrix}3\\-4\end{bmatrix}+\underline{4}\begin{bmatrix}-2\\2\end{bmatrix}=\begin{bmatrix}5\\6\end{bmatrix}$$

column vector of (A) $\qquad$ b

Thus $\begin{bmatrix}5\\6\end{bmatrix}$ is alinear combination of the vectors $\begin{bmatrix}2\\5\end{bmatrix}, \begin{bmatrix}3\\-4\end{bmatrix}, \begin{bmatrix}-2\\2\end{bmatrix}$.

Thm (Consistency theorem of linear System)

Alinear system $Ax=b$ is consistent if and only if (b) can be writen as alinear combination of column vectors of A.

$\Rightarrow$ <u>الكرة</u> $Ax=b$ is inconsistent $\Leftrightarrow$ b <u>can't</u> be written as alinear - - -

ex $x_1 + 2x_2 = 1$

$2x_1 + 4x_2 = 1$

can we write $b = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ as a linear combination of column vectors of A: $\begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \end{bmatrix}$.

sol: solution $\longrightarrow$ b can written

no solution $\rightarrow$ b can't be written.

find the sol: $\left[\begin{array}{cc|c} 1 & 2 & 1 \\ 2 & 4 & 1 \end{array}\right]$

$\rightarrow \left[\begin{array}{cc|c} 1 & 2 & 1 \\ 0 & 0 & -1 \end{array}\right] \rightarrow$

this system is inconsistent

$\rightarrow$ using thm $\rightarrow$ $b = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ can't be written as a linear combination of $\begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \end{bmatrix}$.

The Transpose of a matrix

Def: The transpose of a $m \times n$ matrix A is the $n \times m$ matrix B defined, $b_{ji} = a_{ij}$

$j = 1, \ldots, n$

$i = 1, \ldots, m$

transpose: A^T

ex $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}_{2 \times 3} \longrightarrow A^T = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}_{3 \times 2}$

ex $B = \begin{bmatrix} 1 & -1 \\ 3 & 0 \end{bmatrix}_{2 \times 2} \longrightarrow B^T = \begin{bmatrix} 1 & 3 \\ -1 & 0 \end{bmatrix}_{2 \times 2}$

Rules

1. $(A^T)^T = A$
2. $(\alpha A)^T = \alpha A^T$
3. $(A + B)^T = A^T + B^T$
4. $(AB)^T = B^T A^T \leftarrow$ انتباه