

complete section 1.2

the linear system is homogeneous
if the constants on the right
hand side all zeros.

ex $2x_1 + x_2 = 0$
 $3x_1 - x_2 = 0$

$$\left[\begin{array}{cc|c} 2 & 1 & 0 \\ 3 & -1 & 0 \end{array} \right]$$

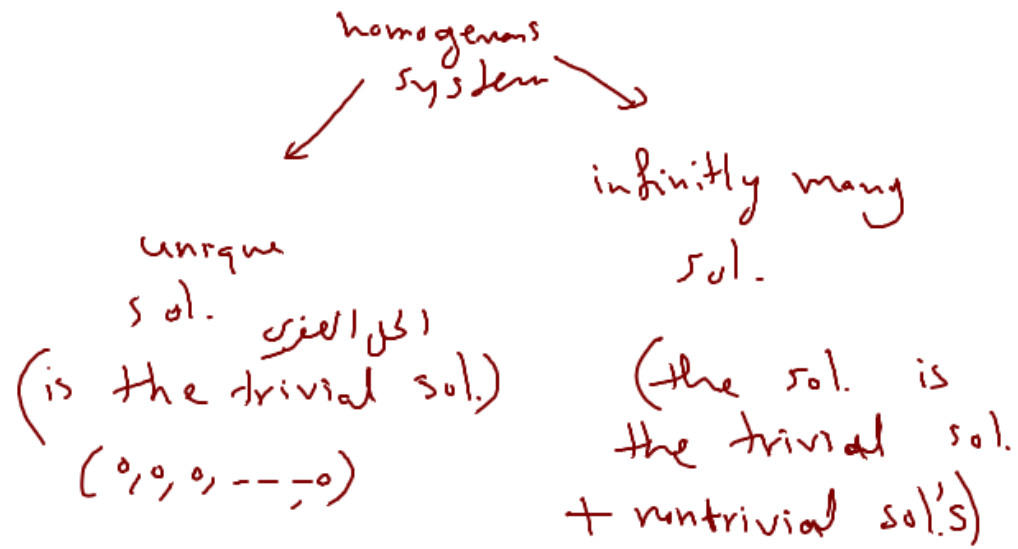
تبعاً لهن

note

The homogeneous system must be
consistent.
جب انكاملها

note

An $m \times n$ homogeneous system of
linear equations has a nontrivial
solution if $n > m$.



sys $m \times n$

$n > m \rightarrow$ free variable

$$\left[\begin{array}{cccc|c} 1 & 2 & 3 & 4 & 0 \\ 5 & 6 & 7 & 8 & 0 \end{array} \right] \quad 2 \times 4, \quad 4 > 2 \quad \text{عدد المتغيرات / عدد المعادلات}$$

\rightarrow Pivot: 2 \rightarrow 4 variables \rightarrow 2 variable free

\rightarrow infinitely many sol. \rightarrow we have non trivial sol.

ex Q8, Q9 واجب

تحت $\lambda = 0$
homogeneous

Q10 consider a linear system whose augmented matrix is of the form

$$\left[\begin{array}{ccc|c} 1 & 1 & 3 & 2 \\ 1 & 2 & 4 & 3 \\ 1 & 3 & a & b \end{array} \right]$$

or

$$\begin{aligned} x_1 + x_2 + 3x_3 &= 2 \\ x_1 + 2x_2 + 4x_3 &= 3 \\ x_1 + 3x_2 + ax_3 &= b \end{aligned}$$

- (a) For what values of (a) and (b) will the system be inconsistent
- (b) for what values of (a) and (b) will the system has a unique solution
- (c) For what values of (a) and (b) will the system has an infinitely many solutions.
- (d) For what values of (a) and (b) will the system be consistent?

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 1 & 3 & 4 & 3 \\ 1 & 3 & a & b \end{array} \right] \rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & a-3 & b-2 \end{array} \right]$$

$$\begin{array}{ccc|c} \underbrace{x_1} & \underbrace{x_2} & \underbrace{x_3} & \\ \left[\begin{array}{ccc|c} 1 & 1 & 3 & a \\ 0 & 1 & 1 & 1 \\ 0 & 0 & a-5 & b-4 \end{array} \right] \end{array}$$

(a) inconsistent:

$$\left[\begin{array}{ccc|c} 0 & 0 & 0 & \text{number} \\ & & & \neq 0 \end{array} \right]$$

$$a-5=0 \rightarrow a=5 \text{ and}$$

$$b-4 \neq 0 \rightarrow b \neq 4 \rightarrow b \in \mathbb{R} - \{4\}$$

(b) unique sol. \rightarrow no free variable.
 \rightarrow we must make x_3 lead
 (not free)

$$a-5 \neq 0 \rightarrow a \neq 5 \rightarrow a \in \mathbb{R} - \{5\}$$

and

$$b-4 \in \mathbb{R} \rightarrow b \in \mathbb{R}$$

(c) infinitely many sol: \rightarrow
 we must have a free variable.

$$\rightarrow x_3 \text{ free} \rightarrow a-5=0 \text{ \& } b-4=0$$

$$a=5 \text{ \& } b=4$$

$$(d) \left\{ \begin{array}{l} a \in \mathbb{R} - \{5\} \\ b \in \mathbb{R} \end{array} \right\} \text{ or } \left\{ \begin{array}{l} a=5 \\ b=4 \end{array} \right\}$$