

## Reduced Row Echelon Form (RREF)

Def A matrix is said to be in reduced row echelon form if

- ① The matrix is in row echelon form.
- ② The first non-zero entry in each row is the only non-zero entry in its column.

ex Determine if the following matrices are in Reduced Row Echelon Form.

①  $\begin{bmatrix} 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$  ✓ RREF.

②  $\begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{bmatrix}$  ✓ RREF

③  $\begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$  ✓ RREF

④  $\begin{bmatrix} 1 & 2 & 3 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$  ✗ RREF

⑤  $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$  ✓ RREF

ex use Gauss Jordan reduction to solve the system.

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$$-x_1 + x_2 - x_3 + 3x_4 = 0$$

$$3x_1 + x_2 - x_3 - x_4 = 0$$

$$2x_1 - x_2 - 2x_3 - x_4 = 0$$

Sol:

$$\left[ \begin{array}{cccc|c} -1 & 1 & -1 & 3 & 0 \\ 3 & 1 & -1 & -1 & 0 \\ 2 & -1 & -2 & -1 & 0 \end{array} \right] \xrightarrow{-R_1} \left[ \begin{array}{cccc|c} 1 & -1 & 1 & -3 & 0 \\ 3 & 1 & -1 & -1 & 0 \\ 2 & -1 & -2 & -1 & 0 \end{array} \right]$$

$$\left[ \begin{array}{cccc|c} 1 & -1 & 1 & -3 & 0 \\ 0 & 4 & -4 & 8 & 0 \\ 0 & 1 & -4 & 5 & 0 \end{array} \right] \xrightarrow{\frac{1}{4}R_2} \left[ \begin{array}{cccc|c} 1 & -1 & 1 & -3 & 0 \\ 0 & 1 & -1 & 2 & 0 \\ 0 & 1 & -4 & 5 & 0 \end{array} \right]$$

$$\xrightarrow{R_2+R_1} \left[ \begin{array}{cccc|c} 1 & 0 & 0 & -1 & 0 \\ 0 & 1 & -1 & 2 & 0 \\ 0 & 0 & 1 & -1 & 0 \end{array} \right]$$

$$\xrightarrow{R_3+R_2} \left[ \begin{array}{cccc|c} x_1 & x_2 & x_3 & x_4 & \\ 1 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 \end{array} \right] \leftarrow \begin{array}{l} \text{Reduced} \\ \text{Row} \\ \text{Echelon} \\ \text{Form} \end{array}$$

Free variable:  $x_4$ ,  $x_4 = \alpha$ ,  $\alpha \in \mathbb{R}$

$$x_3 - x_4 = 0 \rightarrow x_3 - \alpha = 0 \rightarrow x_3 = \alpha$$

$$x_2 + x_4 = 0 \Rightarrow x_2 + \alpha = 0 \rightarrow x_2 = -\alpha$$

$$x_1 - x_4 = 0 \Rightarrow x_1 - \alpha = 0 \Rightarrow x_1 = \alpha$$

$$\left[ \begin{array}{cccc|c} 1 & -1 & 1 & -3 & 0 \\ 0 & 1 & -1 & 2 & 0 \\ 0 & 0 & 1 & -1 & 0 \end{array} \right] \rightarrow \left[ \begin{array}{cccc|c} 1 & -1 & 1 & -3 & 0 \\ 0 & 1 & -1 & 2 & 0 \\ 0 & 0 & 1 & -1 & 0 \end{array} \right]$$

Row Echelon Form

solution is :  $(\alpha, -\alpha, \alpha, \alpha)$ .

→ this system has infinitely many solutions.