2.2 Properties of determinants

What are the effects of the elementary Row operations (I, II, III) on the value of determinant.

Row operation (I)

Interchanging two rows (or columns) of a matrix : changes the sign of the determinant.

ex
$$\begin{vmatrix} 3 & 4 \\ 5 & 8 \end{vmatrix} = 4$$

$$\begin{vmatrix} 5 & 8 \\ 3 & 4 \end{vmatrix} = -4$$

Row operation (II).

Multiplying a single row or column of a matrix by a scalar has the effect of multiplying the value of the determinant by that scalar.

ex
$$\begin{vmatrix} 2 & 5 & 4 \\ 3 & 1 & 2 \\ 5 & 4 & 6 \end{vmatrix} = -16$$

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$$\xRightarrow{\boxed{*10}}$$

$$\begin{vmatrix} 20 & 50 & 40 \\ 3 & 1 & 2 \\ 5 & 4 & 6 \end{vmatrix} = -16 * 10 = -160.$$

$$A = \begin{bmatrix} 2 & 5 & 4 \\ 3 & 1 & 2 \\ 5 & 4 & 6 \end{bmatrix} \rightarrow \det(A) = -16$$

$$10A = \begin{bmatrix} 20 & 50 & 40 \\ 30 & 10 & 20 \\ 50 & 40 & 60 \end{bmatrix} \rightarrow \det(10A) = -16 * 10 * 10 * 10 = -16000$$

Thm

(*) A is $n \times n$ matrix:

(1) $\det(\alpha A) = \alpha^n \det(A)$

(2) $\det(A^T) = \det(A)$.

(3) $\det(A^{-1}) = \dfrac{1}{\det(A)}$

(*) $\det(AB) = \det(A) * \det(B)$

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let A, B 3×3 matrices, $\det(AB) = 10$, $\det(A) = 5$

find (1) $\det(B)$.

sol:
$$\det(AB) = \det(A)\det(B)$$
$$10 = 5 \det(B)$$
$$\boxed{2 = \det(B)}$$

② $\det(B^{-1}) = \frac{1}{\det(B)} = \frac{1}{2}$

③ $\det(2B^{-1}) = 2^3 \det(B^{-1})$

$= 8 * \frac{1}{2} = 4$

④ $\det(4A^T) = 4^3 \det(A^T)$

$= 64 * \det(A)$

$= 64 * 5 = 320$

⑤ $\det(-A^{-1}) = (-1)^3 \det(A^{-1}) = -1 * \frac{1}{5}$

$= -\frac{1}{5}$

$\det(-A) \neq -\det(A)$ If A $n \times n$ matrix

⑥ $\det(2A^{-1}B^T) = 2^3 \det(A^{-1}B^T)$

$= 8 \det(A^{-1}) * \det(B^T)$

$= 8 * \frac{1}{5} * 2$

$= \frac{16}{5}$.

Row Operation (III)

Adding a multiple of one row (or column) to another doesn't change the value of the determinant.

ex $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & I \end{vmatrix} = 10$

find $\begin{vmatrix} 5g & 5h & 5I \\ d & e & f \\ a+2d & b+2e & c+2f \end{vmatrix} =$

$10 * -1 * 5 = -50$

بدل 1 و 3

ex Find det using Row operation.

$\begin{vmatrix} 2 & 1 & 3 \\ 4 & 2 & 1 \\ 6 & -3 & 4 \end{vmatrix} \xrightarrow[-3R_1+R_3]{-2R_1+R_2} \begin{vmatrix} 2 & 1 & 3 \\ 0 & 0 & -5 \\ 0 & -6 & -5 \end{vmatrix}$

لم [illegible]

$\xrightarrow{R_{23}} (-1) * \begin{vmatrix} 2 & 1 & 3 \\ 0 & -6 & -5 \\ 0 & 0 & -5 \end{vmatrix} = -1 * 2 * -6 * -5$

$= -60$

triangular matrix