

Exp. 1 :
Time response of first order system

Example of system, measured system and control system

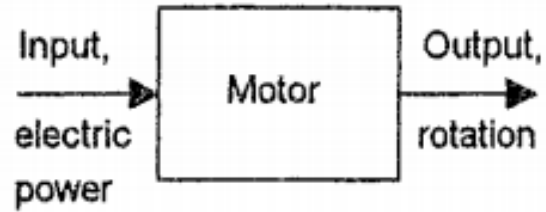


Fig. 1.1 An example of a system

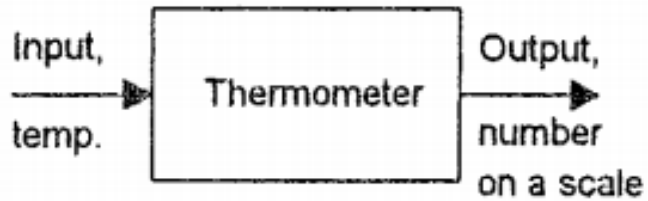


Fig. 1.2 An example of a measurement system

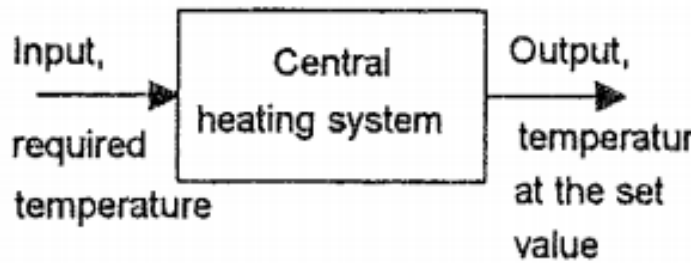


Fig. 1.3 An example of a control system

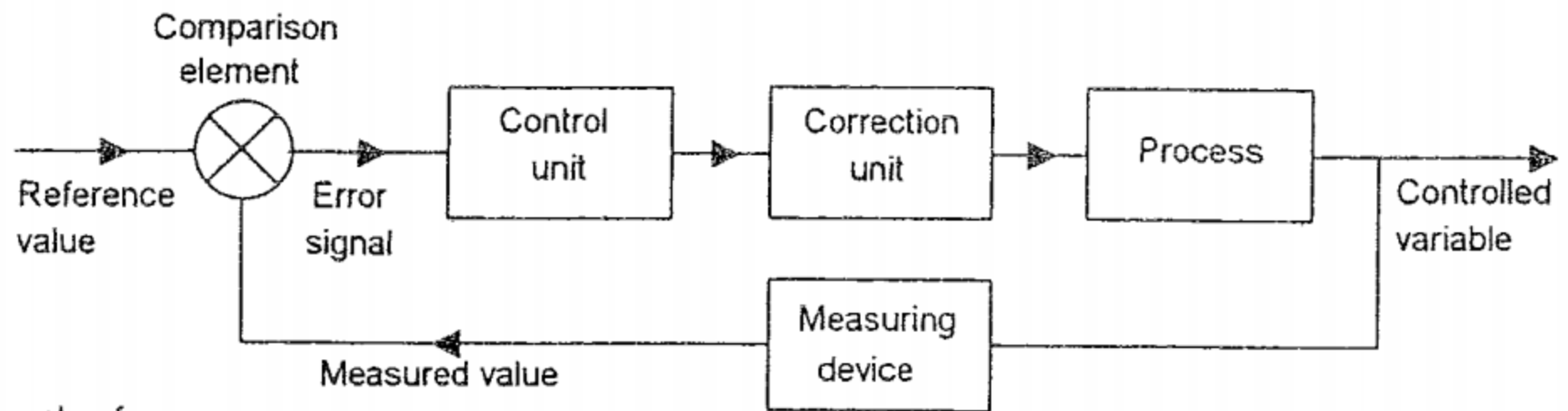


Fig. 1.9 The elements of a closed-loop control system

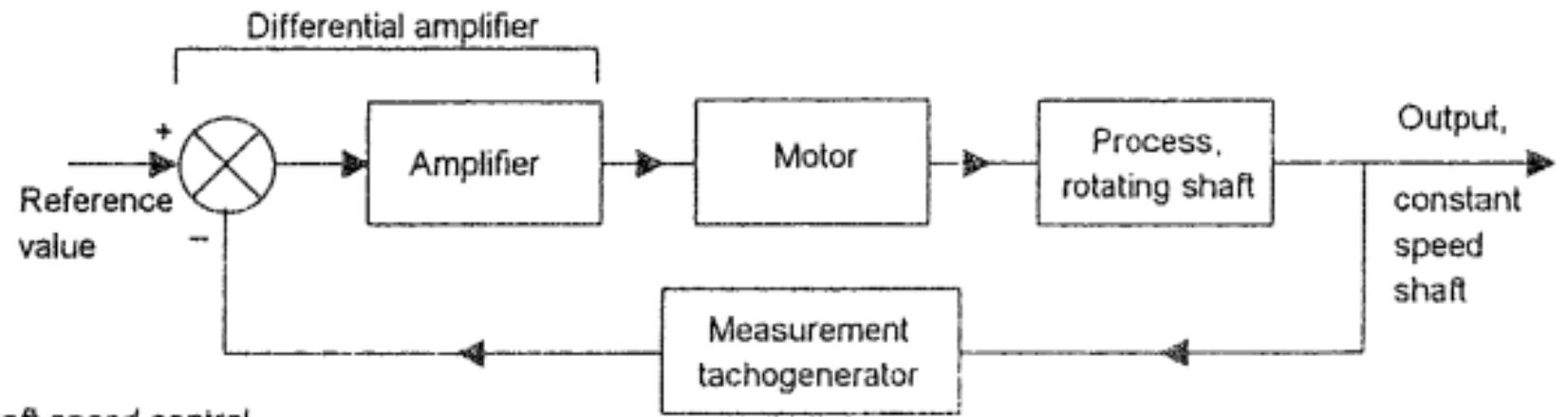
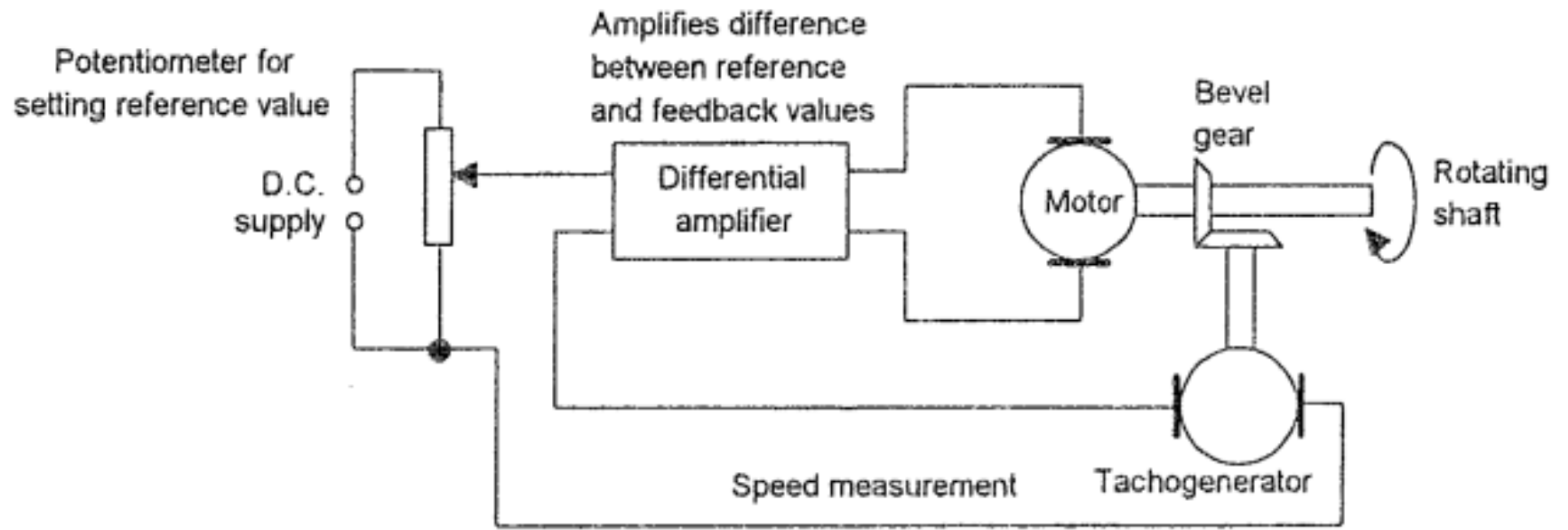


Fig. 1.11 Shaft speed control

Figure 1.11
A servomechanism:
a remote antenna-
positioning system.

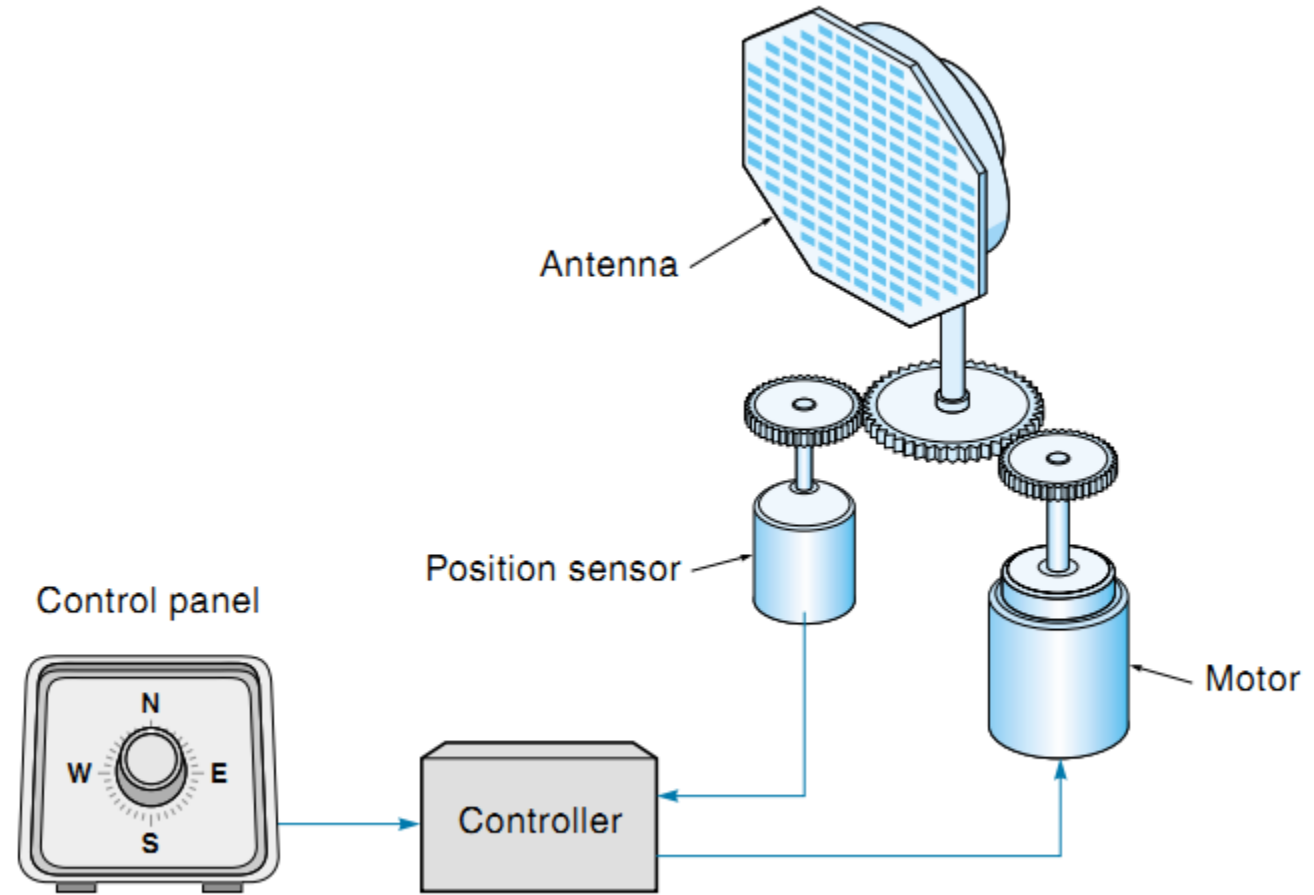
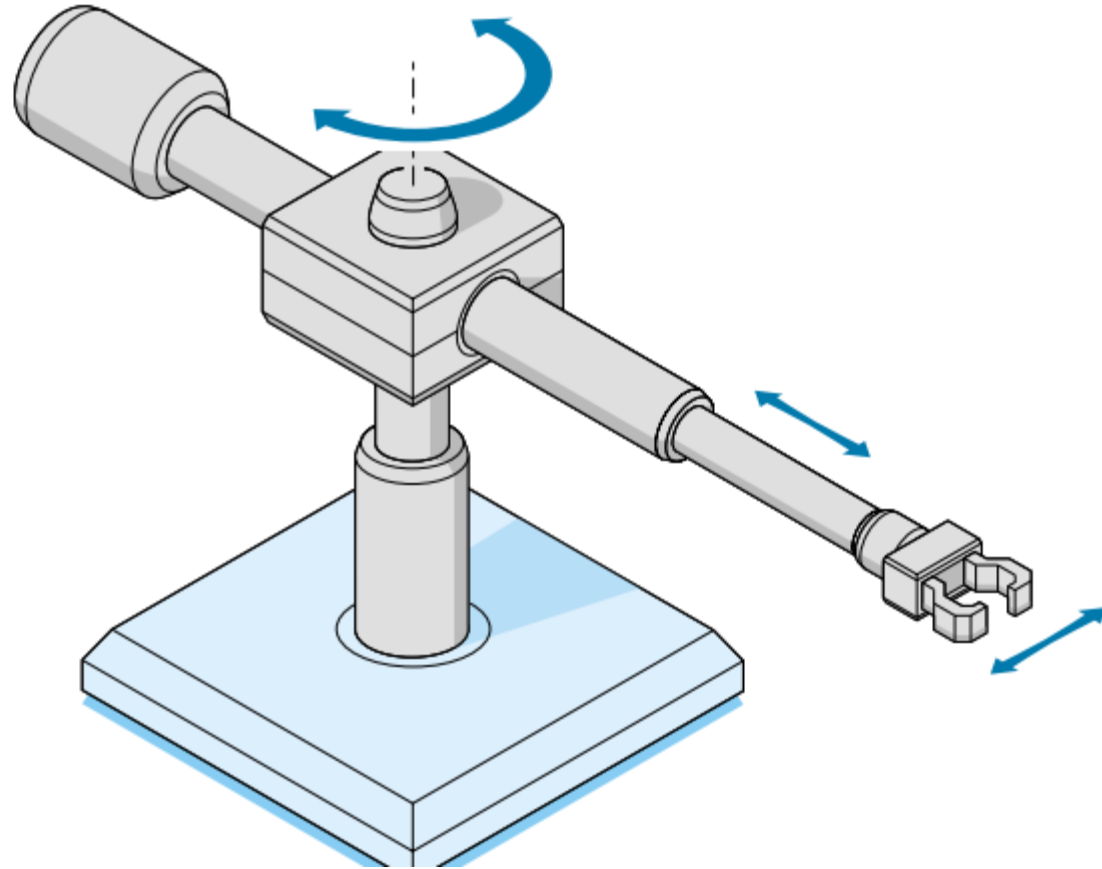


Figure 1.13
A pick-and-place robot.



Response of input system

the response of any system to an input is not instantaneous for example kettle , if you switch on kettle its take some time for water in the kettle to reach boiling point

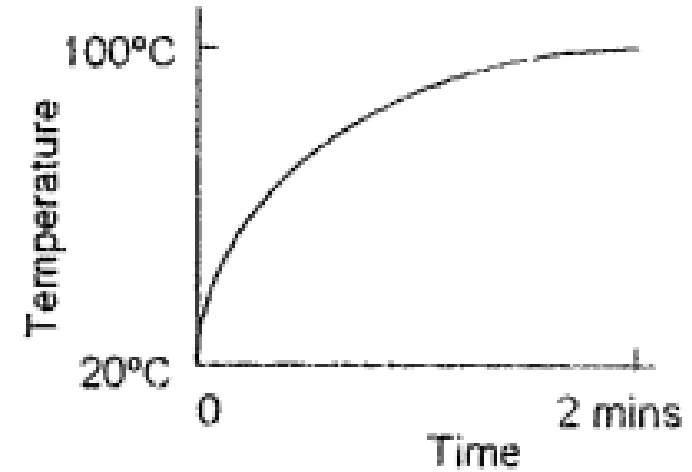
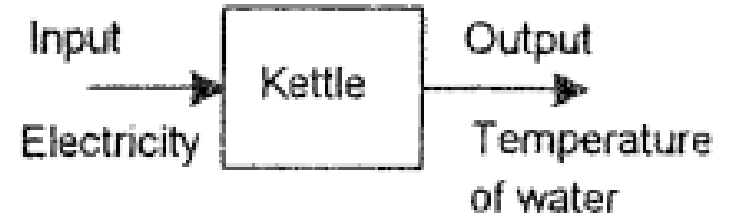


Fig. 1.18 The response to an input for a kettle system

Response of input system

Systems according to order are divided into :

First order second order third order and so on

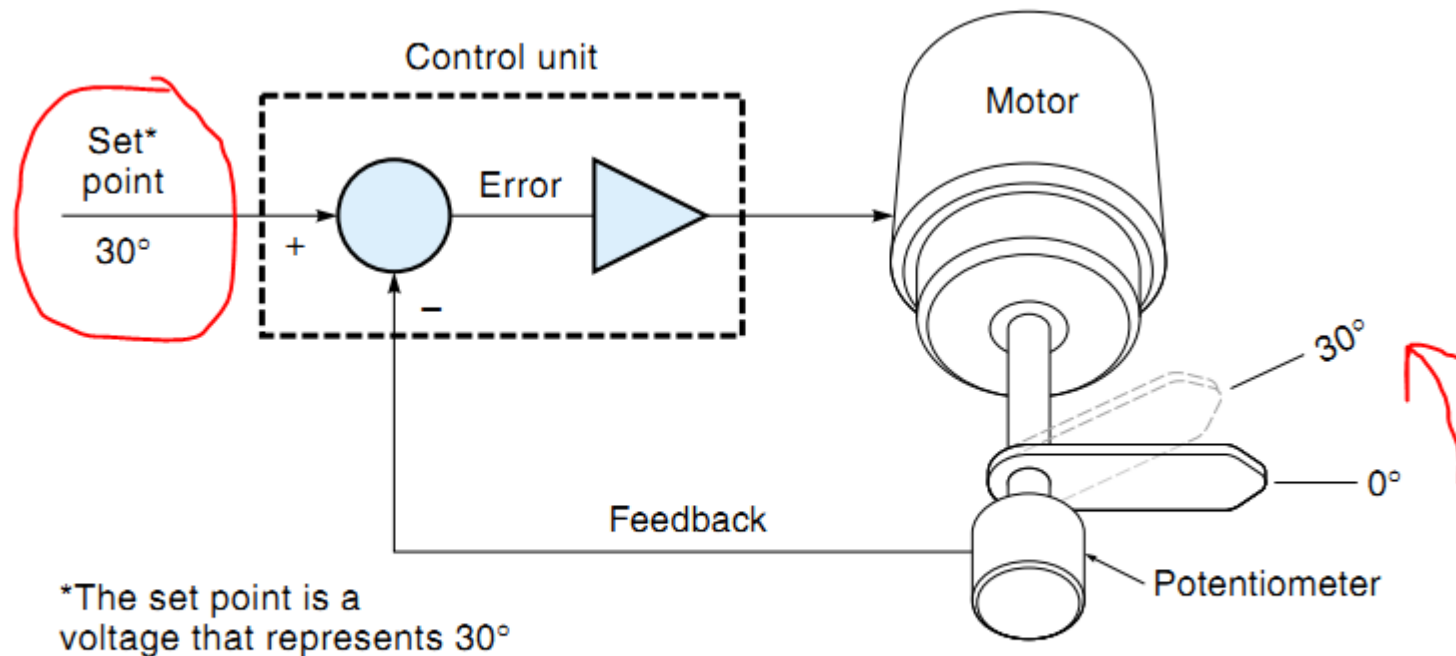
First order system

consider a system where the relationship between the input and the output is in the form of first-order differential equation

Transfer Function :

A **transfer function** (TF) is a mathematical relationship between the input and output of a control system component. Specifically, the transfer function is defined as the output divided by the input, expressed as

$$\text{TF} = \frac{\text{output}}{\text{input}} \quad (1.1)$$



*The set point is a voltage that represents 30°

(b) A simple closed-loop position system (Example 1.2)

EXAMPLE 1.3

A potentiometer is used as a position sensor [see Figure 1.3(b)]. The pot is configured in such a way that 0° of rotation yields 0 V and 300° yields 10 V. Find the transfer function of the pot.

SOLUTION

The transfer function is output divided by input. In this case, the input to the pot is “position in degrees,” and output is volts:

$$TF = \frac{\text{output}}{\text{input}} = \frac{10 \text{ V}}{300^\circ} = 0.0333 \text{ V/deg}$$

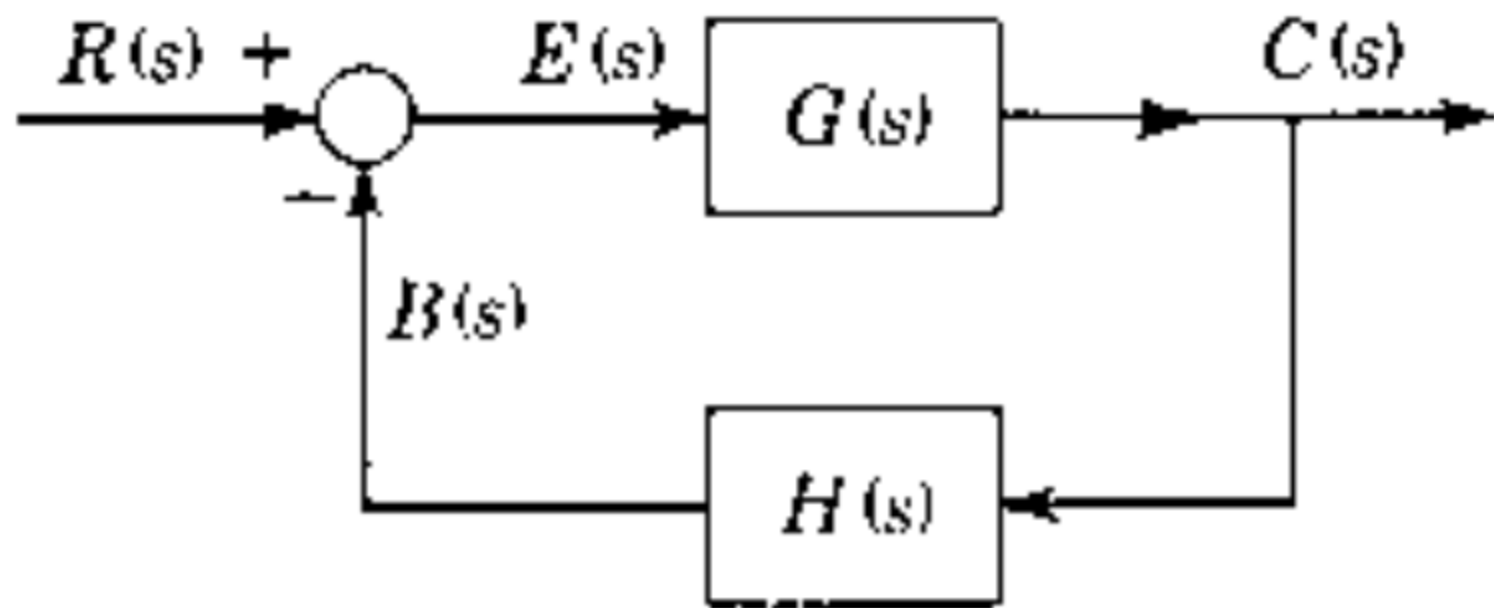
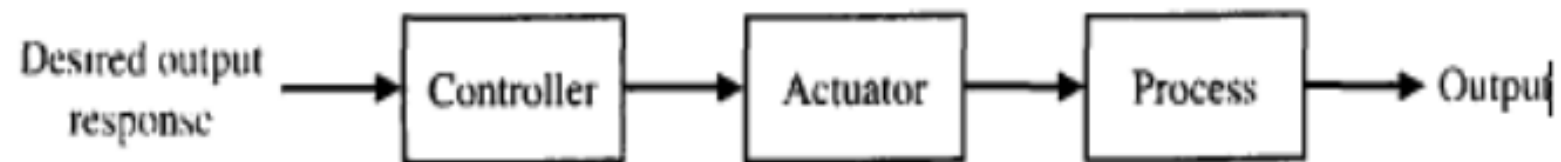
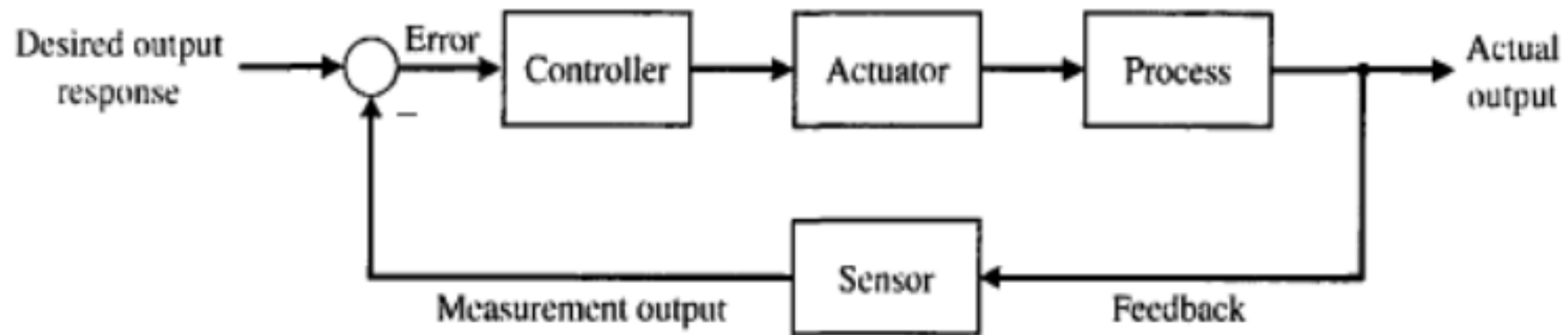


FIGURE 6.1 Simple feedback system.

1- Open loop control system



2- Closed loop control system



Open and a closed loop system

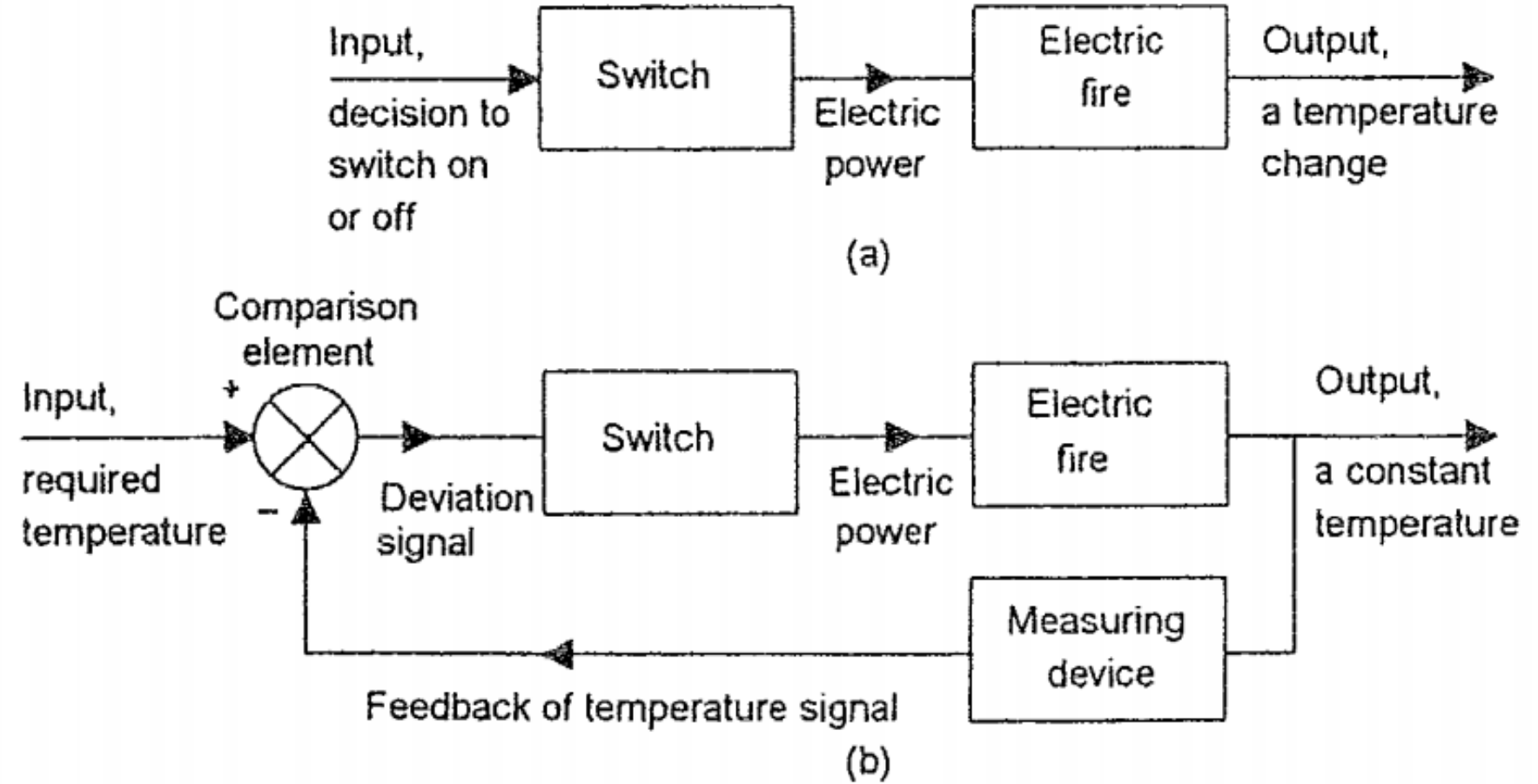
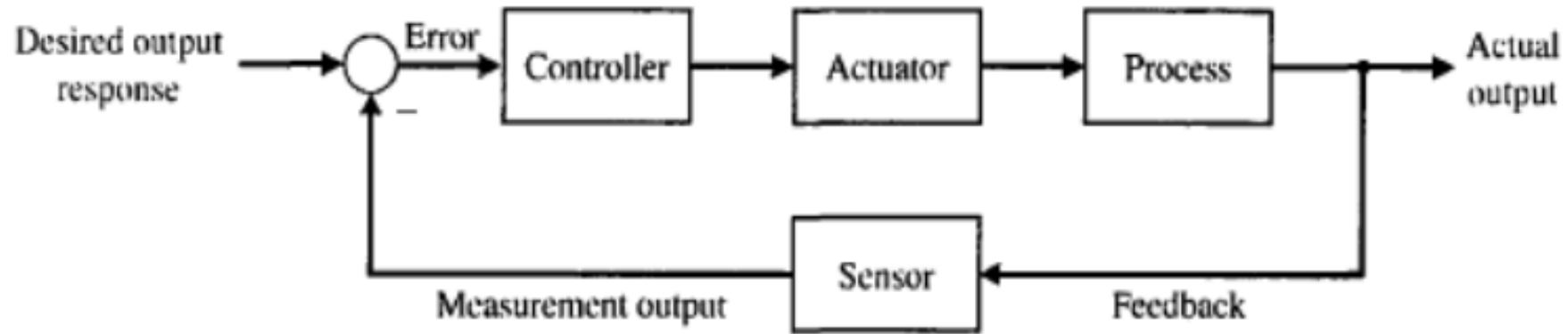


Fig. 1.8 Heating a room:
(a) an open-loop system,
(b) a closed-loop system



Where the total transfer function = $T(s) = \frac{\text{open loop system}}{1 + \text{open loop system} * \text{feedback signal}} = \frac{G(s)}{1 + G(s) * H(s)}$

The error signal can be calculated using the following expression

Error = input – output = input – transfer function *input =input*[1- transfer function] =R(s)[1-T(s)]

The common input type used in control system

Input type	Time domain	Laplace domain
Impulse input	$\delta(t)$	1
Step input	A	$\frac{A}{S}$
Ramp input	$A*t$	$\frac{A}{S^2}$
Parabolic input	$A*t^2$	$\frac{A}{S^3}$

**since A is a constant represent the input value

Systems according to order are dived in to:

First order system, second order system, third order system, etc.....

$$G(s) = \frac{K}{\tau \cdot s + 1} \quad (\text{the general form for the transfer function of the First Order System})$$

Where ****K = gain = $\frac{\text{output}}{\text{input}}$** **** τ = time constant**

$$\text{Transfer function} = G(s) = \frac{C(s)}{R(s)} = \frac{\text{output}}{\text{input}}$$

For step input with value A

$$C(s) = G(s) \cdot R(s) = \frac{K}{\tau \cdot s + 1} * \frac{A}{s} \quad (\text{Laplace response equation})$$

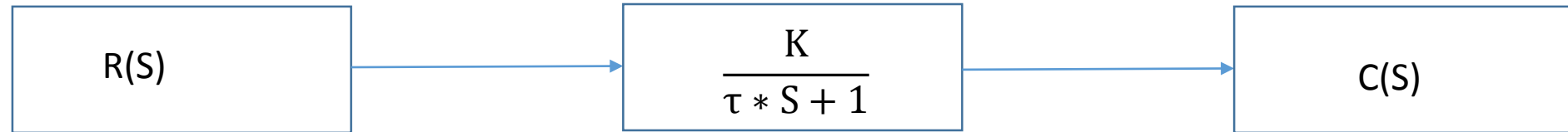
$$C(t) = K * A(1 - e^{-\frac{t}{\tau}}) \quad (\text{Time response equation})$$

$$E(t) = \text{input} - \text{output} = A - C(t) \quad (\text{error signal})$$

$$C(t)\% = \frac{C(t)}{A} * 100\% \quad (\text{output percentage value})$$

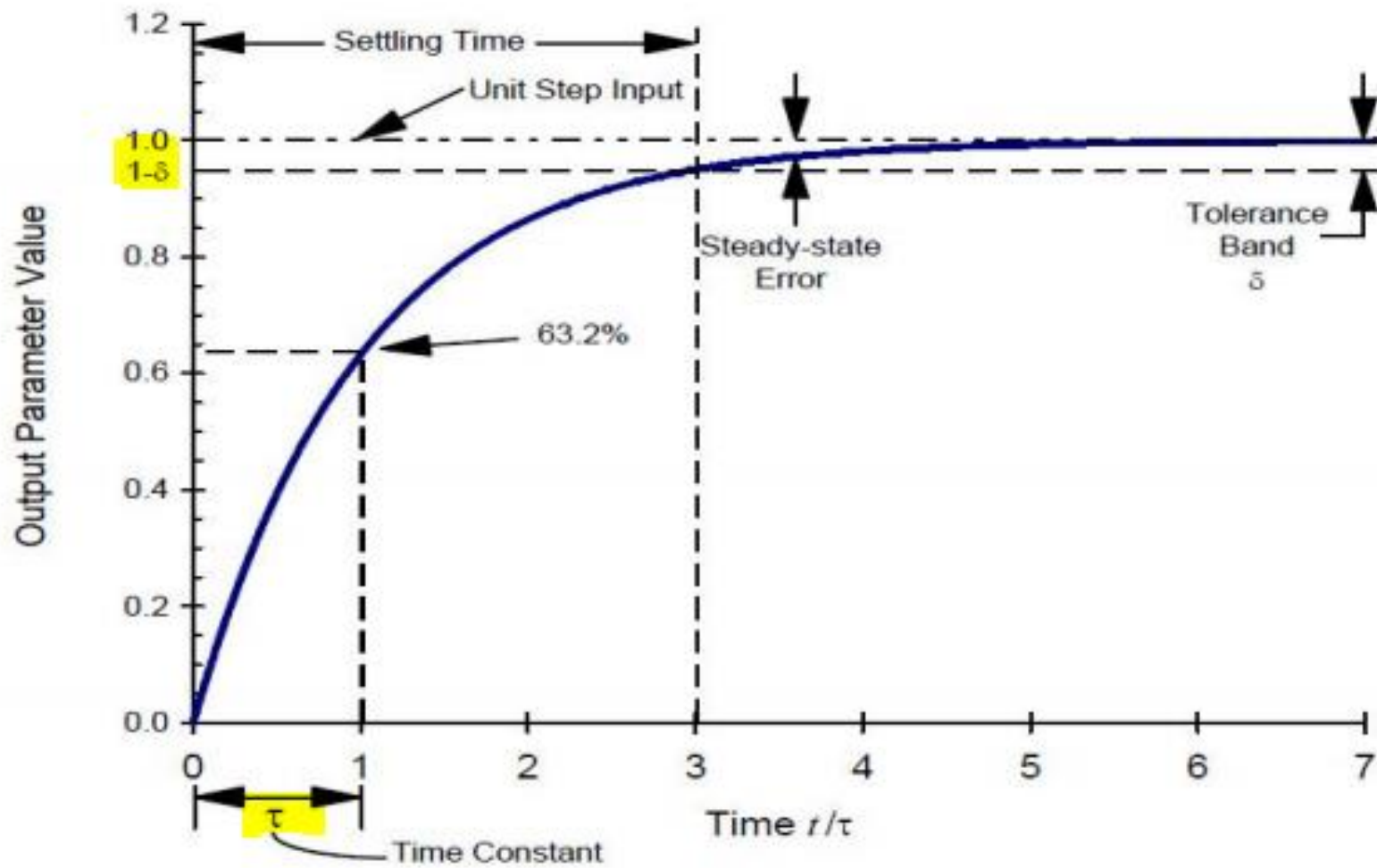
$$E(t)\% = \frac{E(t)}{A} * 100\% \quad (\text{error percentage value})$$

First order system



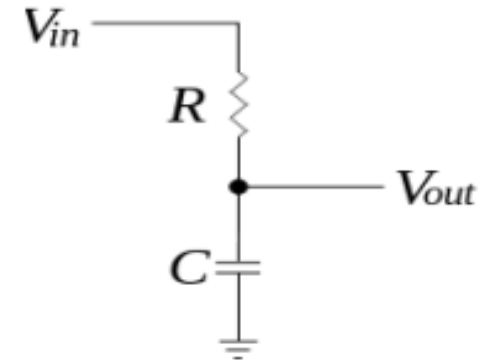
$$\text{T.F} = C(S) / R(S) = \frac{K}{\tau * S + 1}$$

The output response for First Order System is shown below:



Mathematical modeling for First Order System:

Example: Resistor & Capacitor at Series



$$-V_{in} + I_R * R + V_0 = 0 \quad (\text{Kirchhoff Voltage Loop})$$

$$I_R = I_C = C * \frac{dV_c}{dt}$$

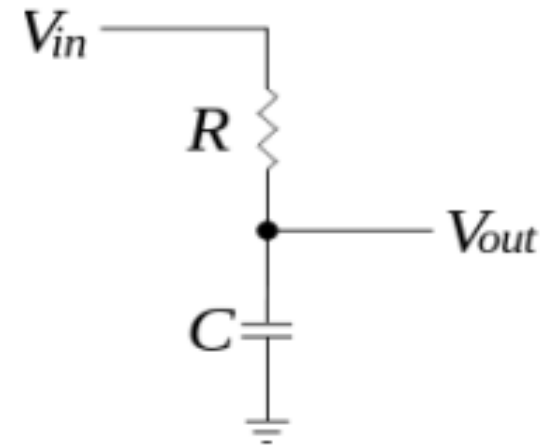
$$-V_{in} + C * R * \frac{dV_c}{dt} + V_{out} = 0$$

$$\text{But } V_c = V_{out} \text{ and } \frac{d}{dt} = S$$

$$-V_{in}(s) + C * R * S * V_{out}(s) + V_{out}(s) = 0 \quad (\text{transfer to Laplace domain})$$

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{1}{R * C * S + 1} \quad (\text{transfer function})$$

In a previous example if all are equal **1 Mega ohm** and capacitor equal **1 microfarad** and V_{in} is step input = **5 volt** ,find the transfer function ?



$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{1}{R * C * S + 1} \text{ (transfer function)}$$

$$\mathbf{T.F} = \frac{1}{1 \text{maga ohm} * 1 \text{ microfarad} * S + 1}$$

$$\mathbf{T.F} = \frac{1}{1 * S + 1}$$

1. steady state value (Y_{ss}) which is the final value that the system reach and still on

2. find the time constant (τ)

Time constant = the value of time that the response reach 0.632 from its final value (Y_{ss})

3. find the settling time (T_s) at 5%

Settling time at 5% = the value of time that the response reach 0.95 from its final value (Y_{ss})