# Engineering Economy

## [7-1]
## Rate of Return Analysis
### Single Alternatives

# Definition of Rate of Return
## Example [1]

- A bank lent a newly graduated engineer $1,000 at i = 10% per year for 4 years. From the bank's perspective (the lender), the investment in this young engineer is expected to produce an equivalent net cash flow of *$315.47* for each of 4 years

- Compute the amount of the unrecovered investment for each of the 4 years using the rate of return on the unrecovered balance

# Definition of Rate of Return
## Example [1]

| Year | Beginning unrecovered balance | Interest on unrecovered balance | Cash flow | Recovered amount | Ending unreovered amount |
|---|---|---|---|---|---|
| 0 | - | - | -$1,000.00 | - | -$1,000.00 |
| 1 | -$1,000.00 | $100.00 | $315.47 | $215.47 | -$784.53 |
| 2 | -$784.53 | $78.45 | $315.47 | $237.02 | -$547.51 |
| 3 | -$547.51 | $54.75 | $315.47 | $260.72 | -$286.79 |
| 4 | -$286.79 | $28.68 | $315.47 | $286.79 | $0.00 |
| **Total** | | **$261.88** | **$261.88** | **$1,000.00** | |

# Definition of Rate of Return

- There are two perspectives when interpreting the rate of return (interest rate)

- SUPPOSE THAT YOU BORROWED SOME MONEY
  - From your perspective: the interest rate is applied to the unpaid balance so that the total loan amount and interest are paid in full exactly with the last loan payment
  - From the perspective of the lender: there is unrecovered balance at each time period and the interest rate is the return on this unrecovered balance so that the amount lent and the interest are recovered exactly with the last receipt

# Definition of Rate of Return

- The rate of return is expressed as a *percent per period*

- The numerical value of *i* can range from -100% to infinity. That is a return of i = -100% means the entire amount is lost

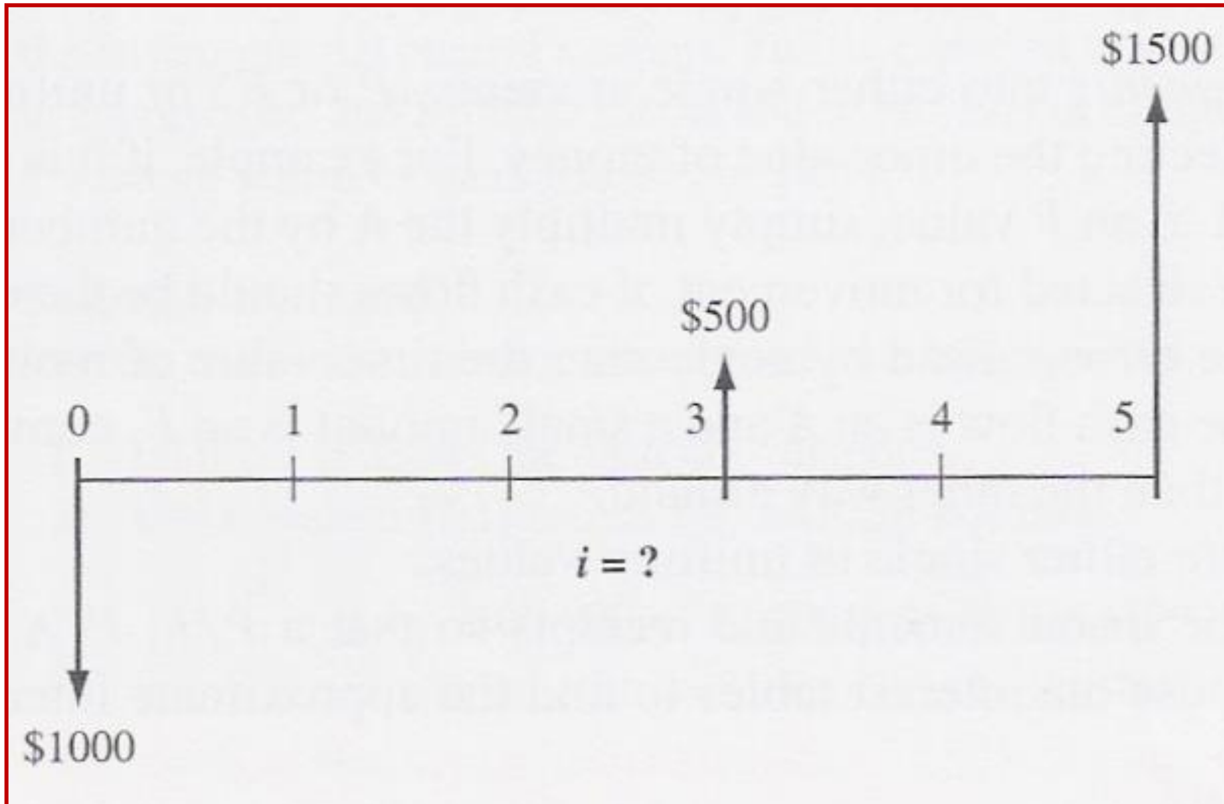
# Calculation of Rate of Return
## Example [2]

- Consider a scenario where you deposited a $1,000 in a savings account that pays $500 in the third year and $1,500 in the fifth year

- *What is the interest rate that **yields** such payments?*

# Calculation of Rate of Return
## Example [2]

- We know that the *outflow* is $1,000
- This in turn should economically be equivalent to the *two inflow* values of $500 and $1,500
- That is the time value of money should be equal



# Calculation of Rate of Return
## Example [2]

- In other words, "$\sum PW = 0$" for all the cash flow values

- For this example, we have $\sum PW = -1{,}000 + 500(P/F, i^*, 3) + 1{,}500(P/F, i^*, 5) = 0$

- $i^*$ is the value of $i$ that makes the above equation equals zero

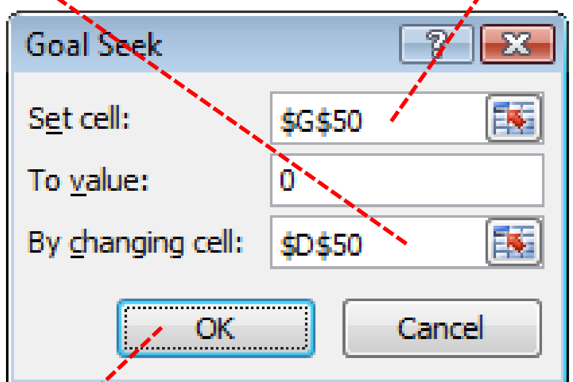
- $-1{,}000 + 500(1+i^*)^{-3} + 1{,}500(1+i^*)^{-5} = 0 \rightarrow i^* = 16.9\%$

- If we use trial-and-error method or a spreadsheet, we can find the value of $i^*$

# Calculation of Rate of Return

## Example [2]

| | D | E | F | G |
|---|---|---|---|---|
| 48 | | | | |
| 49 | i | (P/F,i,3) | (P/F,i,5) | ∑PW |
| 50 | 10.00% | 0.75 | 0.62 | $305.0 |
| 51 | | | | |

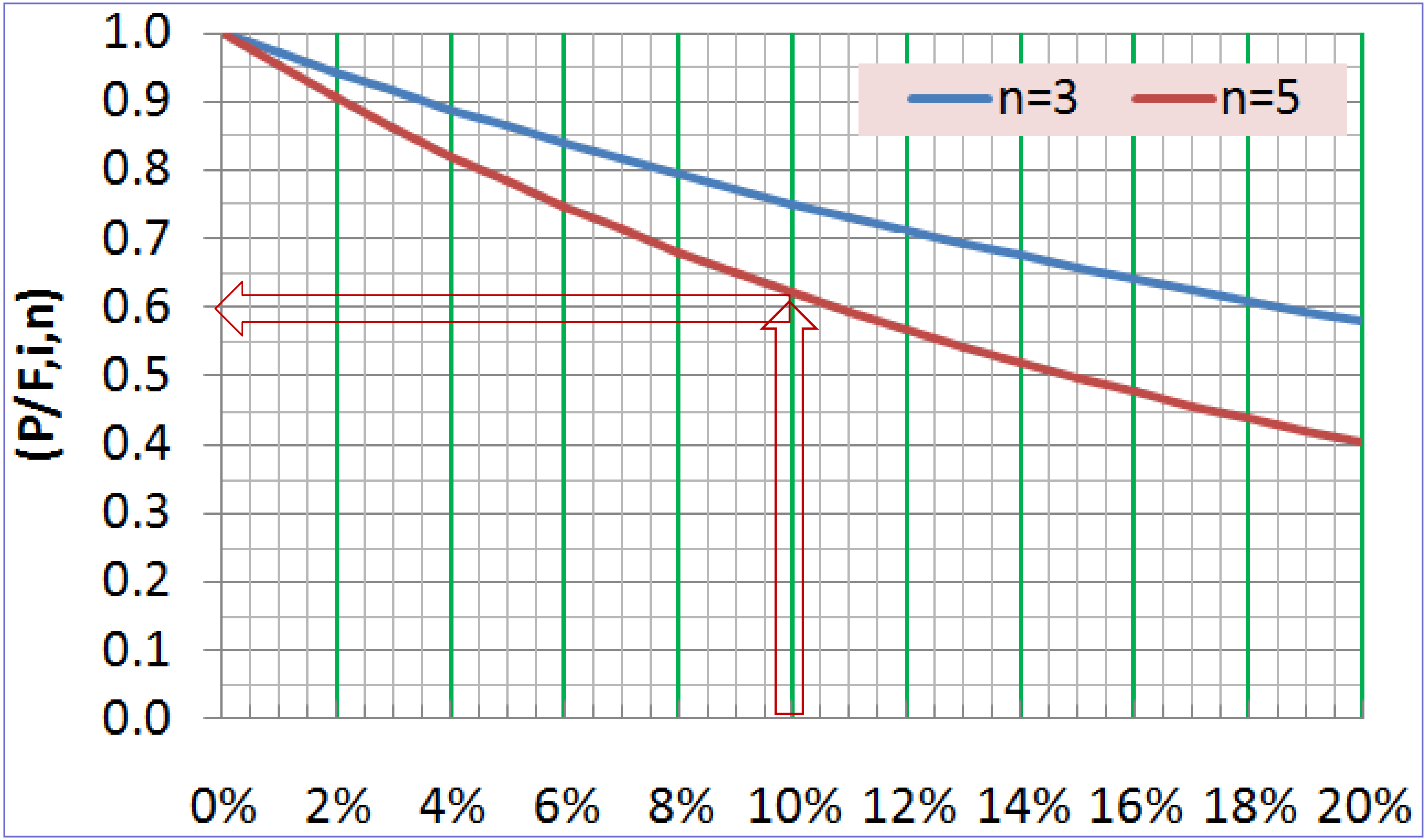



| i | (P/F,i,3) | (P/F,i,5) | ∑PW |
|---|---|---|---|
| 16.90% | 0.6259377 | 0.4580208 | $0.0 |

You can use "Goal Seek" option in Excel

# Calculation of Rate of Return

## Example [2]




This figure shows the values of (P/F,i,n) for different i values for n=3 and n=5

# Calculation of Rate of Return
## Example [2]

- For instance, if you choose i = 10% this gives a (P/F,10%,3) = 0.75 and (P/F,10%,5) = 0.62

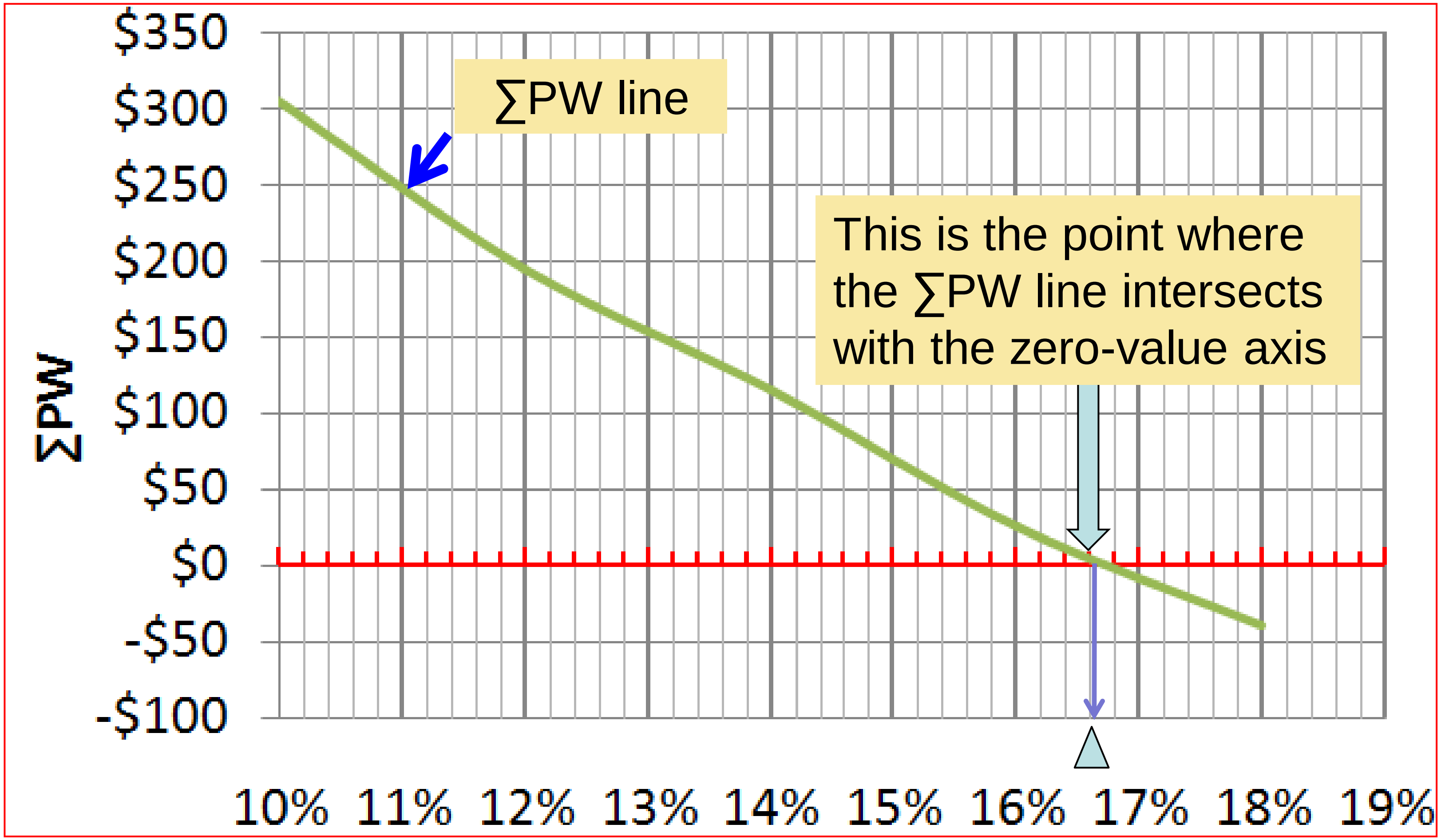
- Check for i = 10% the value of ∑PW = -1,000 + 500 × 0.75 + 1,500 × 0.62 = $305

- Redo the same thing with different values of i

| i | (P/F,i,3) | (P/F,i,5) | ∑PW |
|---|---|---|---|
| 10.00% | 0.75 | 0.62 | $305.0 |
| 12.00% | 0.71 | 0.56 | $195.0 |
| 14.00% | 0.67 | 0.52 | $115.0 |
| 16.00% | 0.64 | 0.47 | $25.0 |
| 18.00% | 0.6 | 0.44 | -$40.0 |

# Calculation of Rate of Return

## Example [2]




- ✓ This depicts the variability of ∑PW with the interest rate
- ✓ i = i* when intersection with *the zero line* occurs

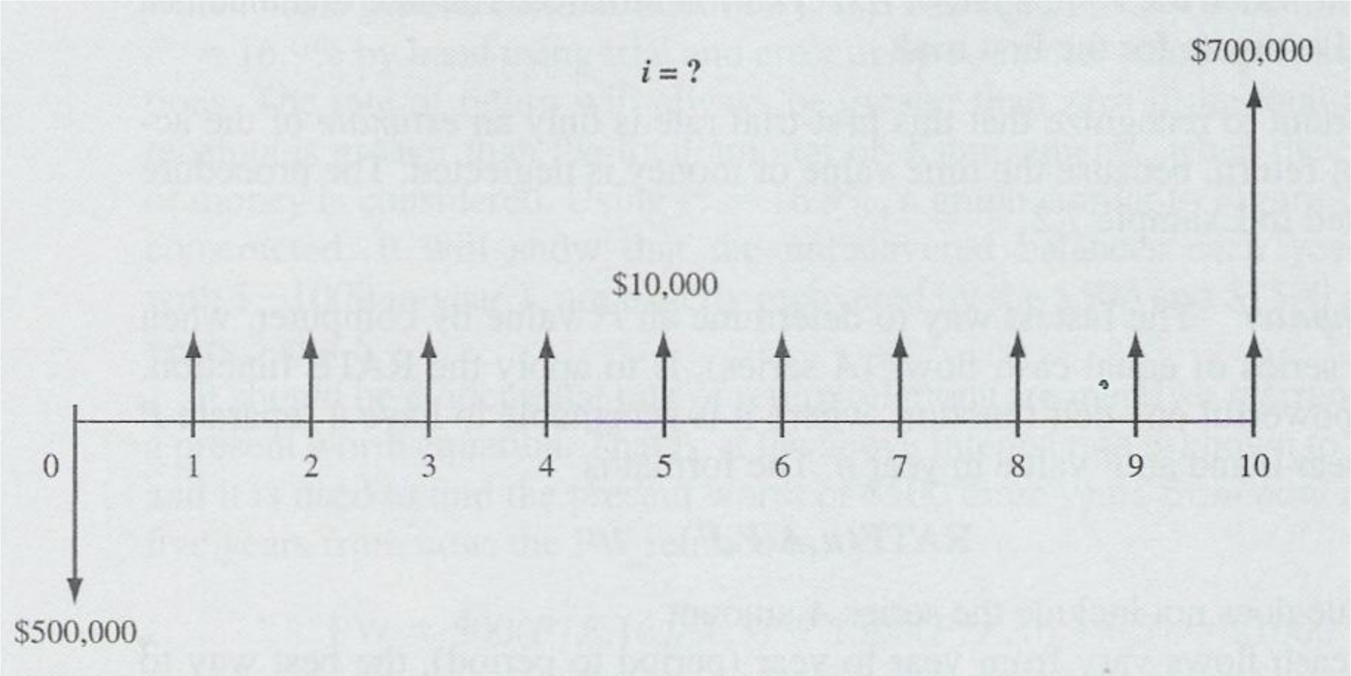
# Calculation of Rate of Return
## Example [3]

- An engineering firm has requested that \$500,000 be spent now on software and hardware to improve the efficiency of the environmental control systems of a plant

- This is expected to save \$10,000 per year for 10 years in energy costs and \$700,000 at the end of 10 years in equipment refurbishment costs

- *Find the rate of return*

# Calculation of Rate of Return
## Example [3]




- We know that ∑PW ought to equal zero if the cash flows to be economically equivalent. As such,

- 0 = -500,000 + 10,000(P/A,i*,10) + 700,000(P/F,i*,10)

# Calculation of Rate of Return
## Example [3]

- The point is to arrive at a good estimate for an initial value of i

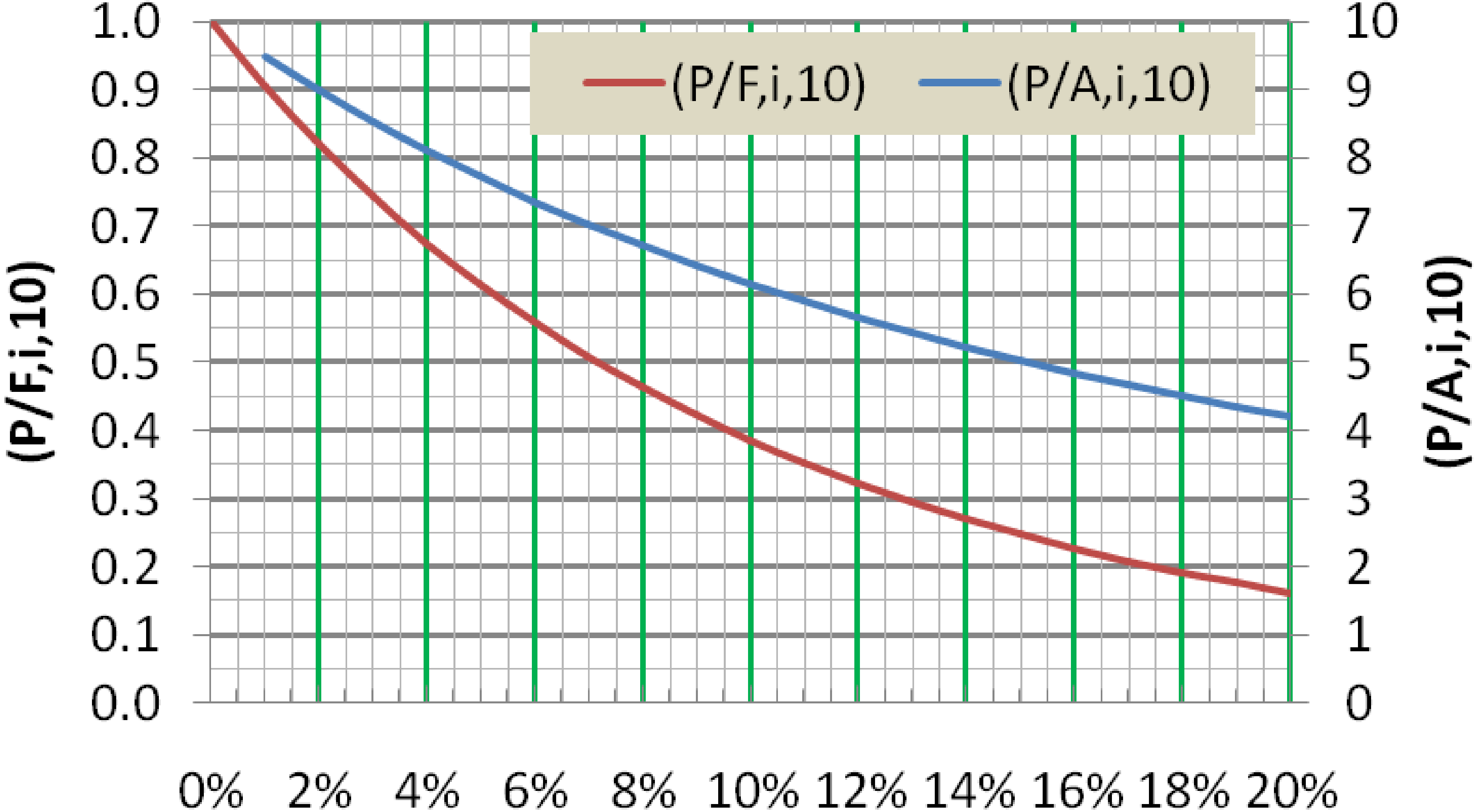
- If we *look over* the cash flow diagram we easily notice that the bulk comes from the $700,000 cash flow. If we neglect the time value of money for the annual series then:

- $-500{,}000 + (700{,}000 + 10 \times 10{,}000)(P/F,i,10) \approx 0$

- This means that $(P/F,i,10) = 0.625$

# Calculation of Rate of Return

## Example [3]



# Calculation of Rate of Return

## Example [3]

| i | (P/F,i,10) | (P/A,i,10) |
|---|---|---|
| 0.00% | 1 | - |
| 1.00% | 0.905 | 9.471 |
| 2.00% | 0.820 | 8.983 |
| 3.00% | 0.744 | 8.530 |
| 4.00% | 0.676 | 8.111 |
| 5.00% | 0.614 | 7.722 |
| 6.00% | 0.558 | 7.360 |
| 7.00% | 0.508 | 7.024 |
| 8.00% | 0.463 | 6.710 |
| 9.00% | 0.422 | 6.418 |
| 10.00% | 0.386 | 6.145 |
| 11.00% | 0.352 | 5.889 |
| 12.00% | 0.322 | 5.650 |
| 13.00% | 0.295 | 5.426 |
| 14.00% | 0.270 | 5.216 |
| 15.00% | 0.247 | 5.019 |
| 16.00% | 0.227 | 4.833 |
| 17.00% | 0.208 | 4.659 |
| 18.00% | 0.191 | 4.494 |
| 19.00% | 0.176 | 4.339 |
| 20.00% | 0.162 | 4.192 |

- Either by using the chart or the table aside, we can easily figure out that a factor of 0.625 yields a value of interest rate between 4% and 5%
- Now, let us plug in i = 4% and i = 5%
- This is our equation now:

  0 = -500,000 + 10,000(P/A,i*,10) + 700,00(P/F,i*,10)

# Calculation of Rate of Return
## Example [3]

- For i = 4% → ∑PW = -500,000 + 10,000(P/A,4%,10) + 700,00(P/F,4%,10) = $54,003
- For i = 5% → ∑PW = -500,000 + 10,000(P/A,5%,10) + 700,00(P/F,5%,10) = $6,956
- Apparently, ∑PW > 0 which means that we still need to increase the interest rate
- For i = 6%, ∑PW = $-35,522 which signifies that 6%>i*>5%

| i | (P/F,i,10) | (P/A,i,10) | ∑PW |
|---|---|---|---|
| 4.00% | 0.676 | 8.110896 | $54,003.9 |
| 5.00% | 0.614 | 7.721735 | $6,956.6 |
| 6.00% | 0.558 | 7.360087 | -$35,522.8 |

# Calculation of Rate of Return
## Example [3]

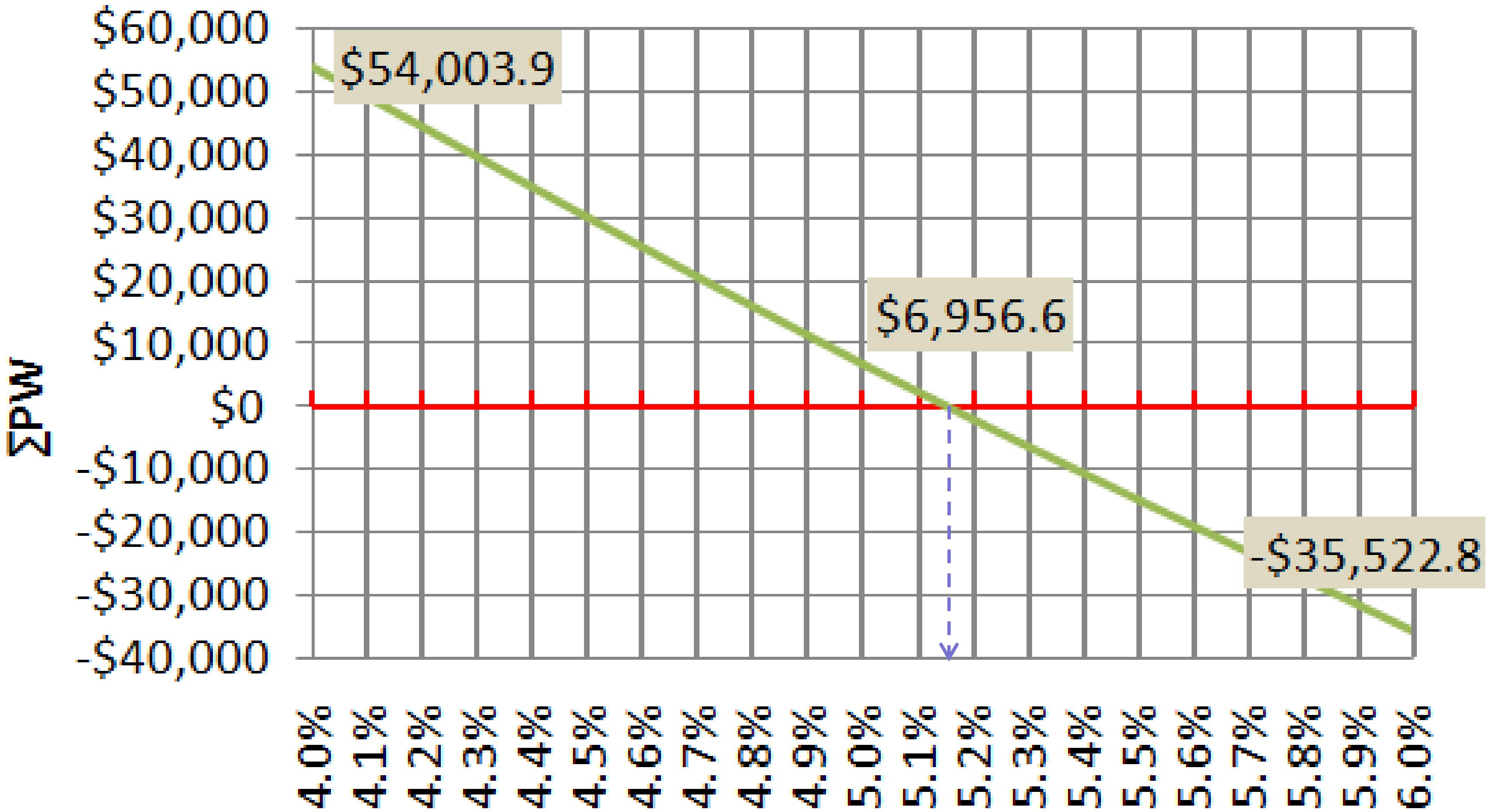
- Now, we need to find the actual $i^*$ value

- To do so, linearly interpolate between 5% and 6%

- $i^* = 5\% + [1\% \times 6{,}956]/[6{,}956-(-35{,}522)] = \underline{5.16\%}$

# Calculation of Rate of Return

## Example [3]



# Calculation of Rate of Return

## Example [3] – Using Excel

| | |
|---|---|
| 0 | -$500,000 |
| 1 | $10,000 |
| 2 | $10,000 |
| 3 | $10,000 |
| 4 | $10,000 |
| 5 | $10,000 |
| 6 | $10,000 |
| 7 | $10,000 |
| 8 | $10,000 |
| 9 | $10,000 |
| 10 | $710,000 |
| | |
| | =IRR(J108:J118) |

| | |
|---|---|
| 0 | -$500,000 |
| 1 | $10,000 |
| 2 | $10,000 |
| 3 | $10,000 |
| 4 | $10,000 |
| 5 | $10,000 |
| 6 | $10,000 |
| 7 | $10,000 |
| 8 | $10,000 |
| 9 | $10,000 |
| 10 | $710,000 |
| | |
| | 5.157% |

# Calculation of Rate of Return
## Example [4] – Using Excel

Banks 1 and 2 offer you the following deals 1 and 2 respectively:

- Deal 1

  Invest $2,000 today. At the end of years 1, 2, and 3 get $100, $100, and $500. At the end of year 4, get $2,200

- Deal 2

  Invest $2,000 today. At the end of years 1, 2, and 3 get $100, $100, and $100. At the end of year 4, get $2,000

- Which deal is the best?

# Calculation of Rate of Return

## Example [4] – Using Excel

Deal 1

$$2000 - [100/(1+i)^1 + 100/(1+i)^2 + 500/(1+i)^3 + 2200/(1+i)]^4 = 0$$

i = 10.784 %

Deal 2

$$2000 - [100/(1+i)^1 + 100/(1+i)^2 + 100/(1+i)^3 + 2000/(1+i)^4] = 0$$

i = 3.819%

*Apparently Deal 1 is better yet in the next chapter we will know more about comparing alternatives using rate of return analysis*

| Year | Deal 1 | Deal 2 |
|---|---|---|
| 0 | -$2,000 | -$2,000 |
| 1 | $100 | $100 |
| 2 | $100 | $100 |
| 3 | $500 | $100 |
| 4 | $2,200 | $2,000 |

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| Year | Deal 1 | Deal 2 |
|---|---|---|
| 0 | -$2,000 | -$2,000 |
| 1 | $100 | $100 |
| 2 | $100 | $100 |
| 3 | $500 | $100 |
| 4 | $2,200 | $2,000 |
| i | 10.784% | 3.819% |