Engineering Economy

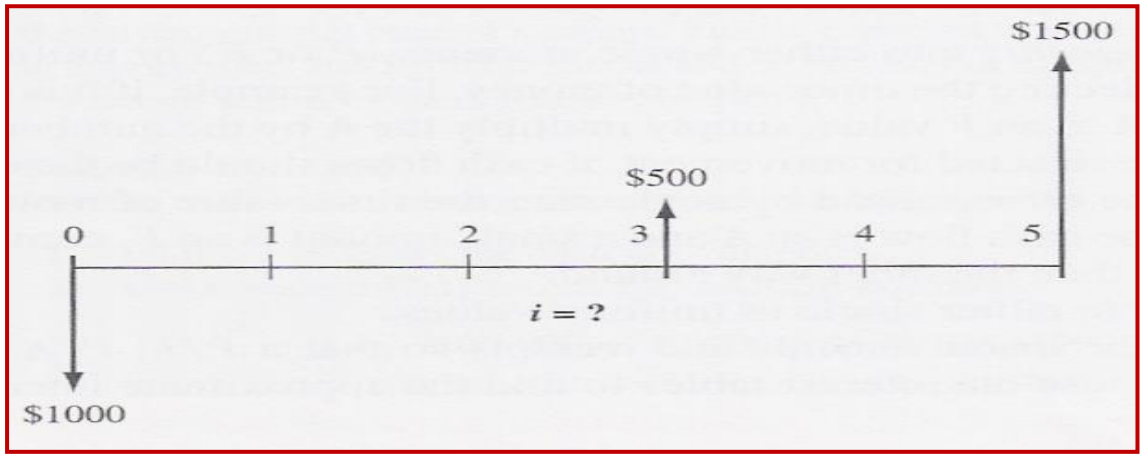
[7-2]
Rate of Return Analysis
Single Alternatives

Multiple Rate of Return Values

- We have so far come across net cash flows that change once during the investment period

- That is, we started with a negative value at the very beginning (initial payment) and the cash flows became positives (incomes)

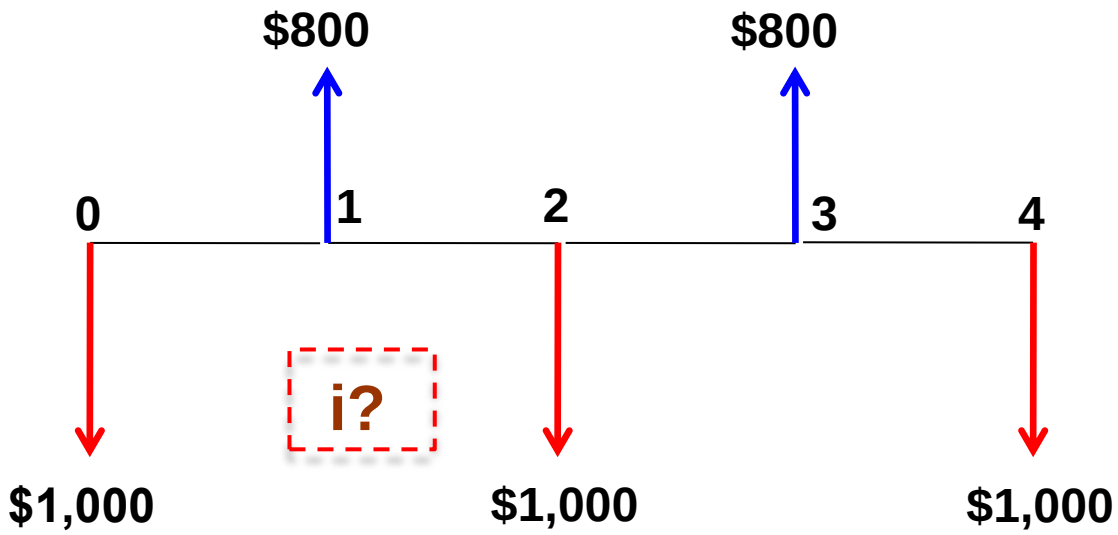
- This situation is called conventional (simple) cash flow series



Multiple Rate of Return Values

- In many situations, the net cash flows switch between positive and negative values from year to another so that we have more than one sign change

- Such series is called non-conventional (non-simple)



Multiple Rate of Return Values

Type of Series	Sign on Net Cash Flow 0	1	2	3	4	5	6	Number of Sign Changes
Conventional	−	+	+	+	+	+	+	1
Conventional	−	−	−	+	+	+	+	1
Conventional	+	+	+	+	+	−	−	1
Nonconventional	−	+	+	+	−	−	−	2
Nonconventional	+	+	−	−	−	+	+	2
Nonconventional	−	+	−	−	+	+	+	3

Examples of *conventional* and *non-conventional* net cash flows for a 6-year project

Multiple Rate of Return Values

- When there is *more than one* sign change in the net cash flows, it is possible that there will be multiple i* values in the -100% to plus infinity range

- A test should be performed on the non-conventional series to determine if there is one unique or multiple i* values

- Rule of signs: total number of *real-number roots* is always less than or equal to the number of sign changes in the series

Multiple Rate of Return Values

Example [1]

Calculate the i* values for the following cash flows:

Year	0	1	2	3
$	+2,000	-500	-8,100	+6,800

Multiple Rate of Return Values
Example [1]

PW =

$2{,}000 - 500(P/F,i,1) - 8{,}100(P/F,i,2) + 6{,}800(P/F,i,3) = 0$

i	PW
0%	$200.00
5%	$50.97
10%	-$39.82
15%	-$88.44
20%	-$106.48
25%	-$102.40
30%	-$82.39
35%	-$51.01
40%	-$11.66
45%	$33.13
50%	$81.48

- If we examine the sign change we find out the following sequence:

  \+ — — +

  0 to 1 to 1 to 2

- So, we have at most two potential rates of return values

Multiple Rate of Return Values

Example [1]

$i^*_1 = 7.47\%$

and

$i^*_2 = 41.35\%$