

# Engineering Economy

[5-2]

## Present Worth Analysis

# Present Worth Analysis of Different-life Alternatives

- When the present worth method is used to compare **mutually exclusive** alternatives that have **different** lives, then the PW of the alternatives must be compared over the **same number of years and end at the same time**
- A **fair** comparison can be made only when the PW values represent costs (and receipts) **associated with equal periods**
- The equal-period requirement can be satisfied by comparing the alternatives over a period of time equal to the **least common multiple (LCM) of their lives**

# Present Worth Analysis of Different-life Alternatives – The LCM Approach

- The LCM approach automatically makes the cash flows for all alternatives extend to the same time period
- For example, alternatives with expected lives of 2 and 3 years are compared over a 6-year time period

# The LCM Approach – Example

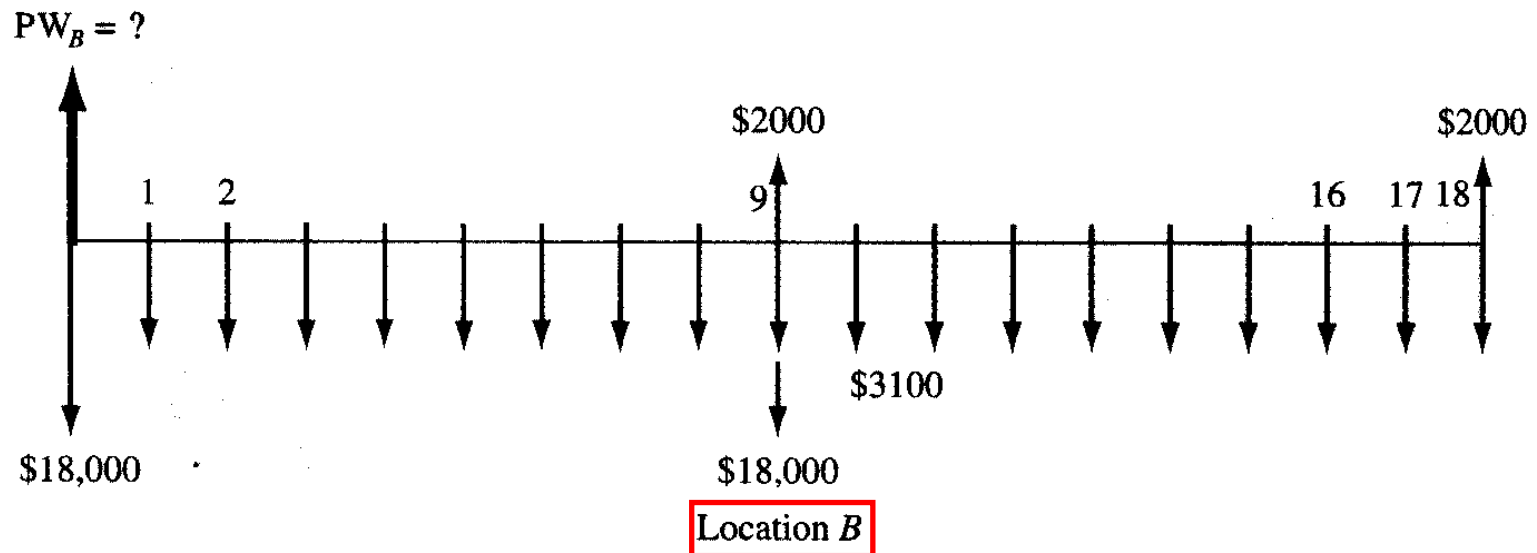
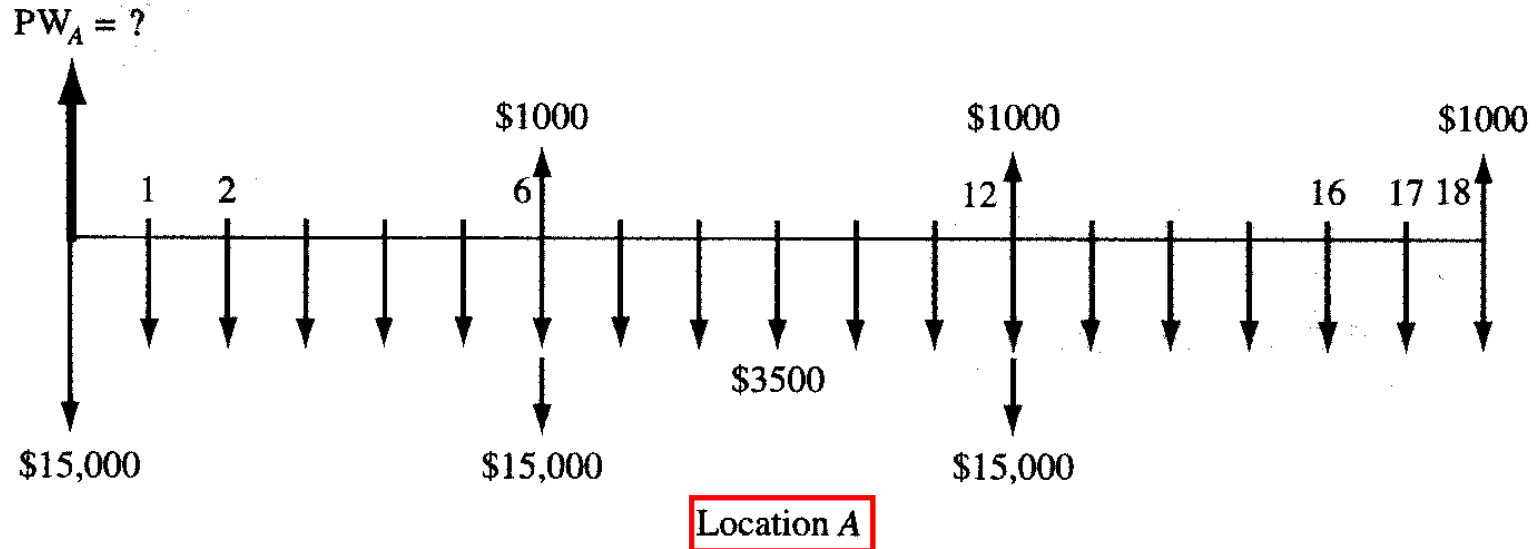
- A project engineer is assigned to start up a new office in a city where a 6-year contract has been finalized
- Two lease options are available, each with a first cost, annual lease cost, and deposit-return estimates as shown below:

	Location A	Location B
First cost, \$	-15,000	-18,000
Annual lease cost, \$	-3,500	-3,100
Deposit return, \$	1,000	2,000
Lease term, years	6	9

# The LCM Approach – Example

- **Determine** which lease option should be selected on the basis of a present worth comparison, if the MARR is 15% per year
- Since the leases have different lives, compare them over the LCM of 18 years
- Repeat the first cost in year 0 of each new cycle
- Calculate **PW** at 15% over 18 years

# The LCM Approach – Example



# The LCM Approach – Example

- $PW_A = -15,000 - 15,000(P/F, 15\%, 6) + 1,000(P/F, 15\%, 6) - 15,000(P/F, 15\%, 12) + 1,000(P/F, 15\%, 12) + 1,000(P/F, 15\%, 18) - 3,500(P/A, 15\%, 18) = \$ -45,036$
- $PW_B = -18,000 - 18,000(P/F, 15\%, 9) + 2,000(P/F, 15\%, 9) + 2,000(P/F, 15\%, 18) - 3,100(P/A, 15\%, 18) = \$ -41,384$
- Location B is selected, since it costs less in PW terms; that is, the  $PW_B$  value is numerically larger than  $PW_A$

# Future Worth Analysis

- The **future worth** of an alternative (FW) can be used to compare alternatives
- Once the FW value is determined, the selection guidelines are the same as with PW analysis
- For one alternative, if  $FW \geq 0$  means the MARR is met or exceeded
- For two mutually exclusive alternatives, select the one with the numerically larger FW value



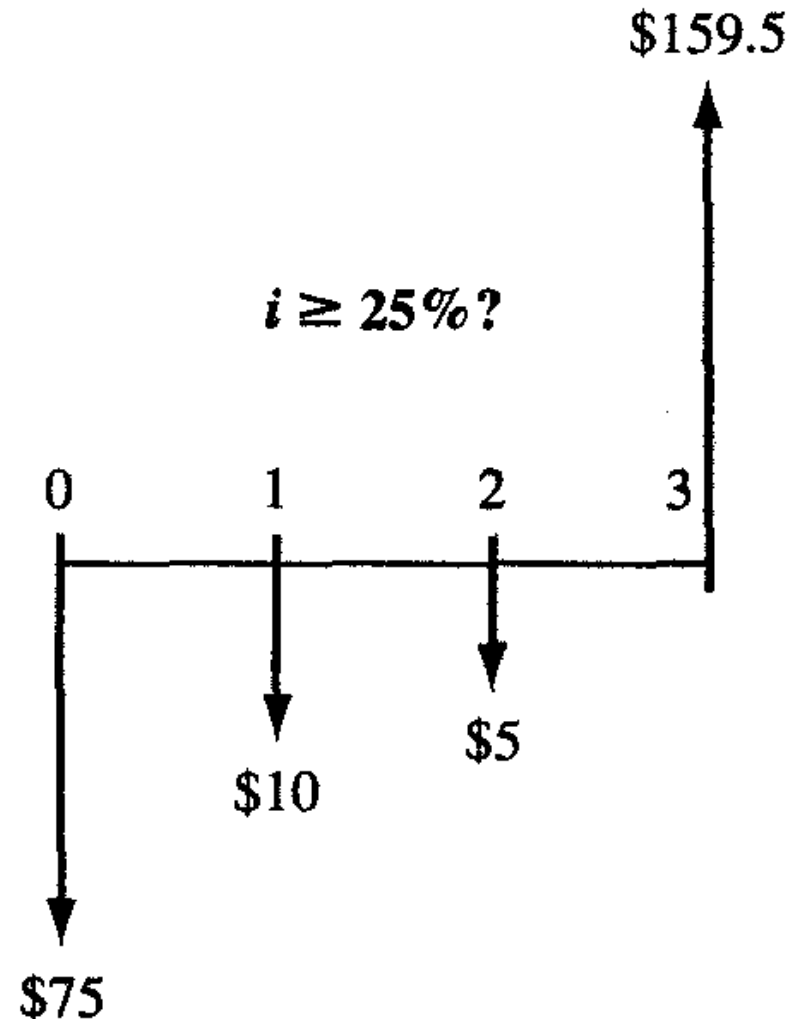
# Future Worth Analysis – Example

- A company purchased a store chain for \$75 million three years ago
- There was a net loss of \$10 million **at the end of year 1** of ownership
- Net cash flow is increasing with an arithmetic gradient of \$+5 million per year starting the second year, and this pattern is expected to continue for the foreseeable future. Expected MARR of 25% per year
- [1] The company has just been offered \$159.5 million to sell the store. Use FW analysis to determine if the MARR will be realized at this selling price
- [2] If the company continues to own the chain, what selling price must be obtained at the end of 5 years of ownership to make the MARR?

# Future Worth Analysis – Example

[1] Find the future worth in year 3 at  $i = 25\%$  per year and an offer price of \$159.5 million

- $FW = -75(F/P, 25\%, 3) - 10(F/P, 25\%, 2) - 5(F/P, 25\%, 1) + 159.5 = -168.36 + 159.5 = \$ -8.86$  million
- No, the MARR of 25% will not be realized if the \$159.5 million offer is accepted

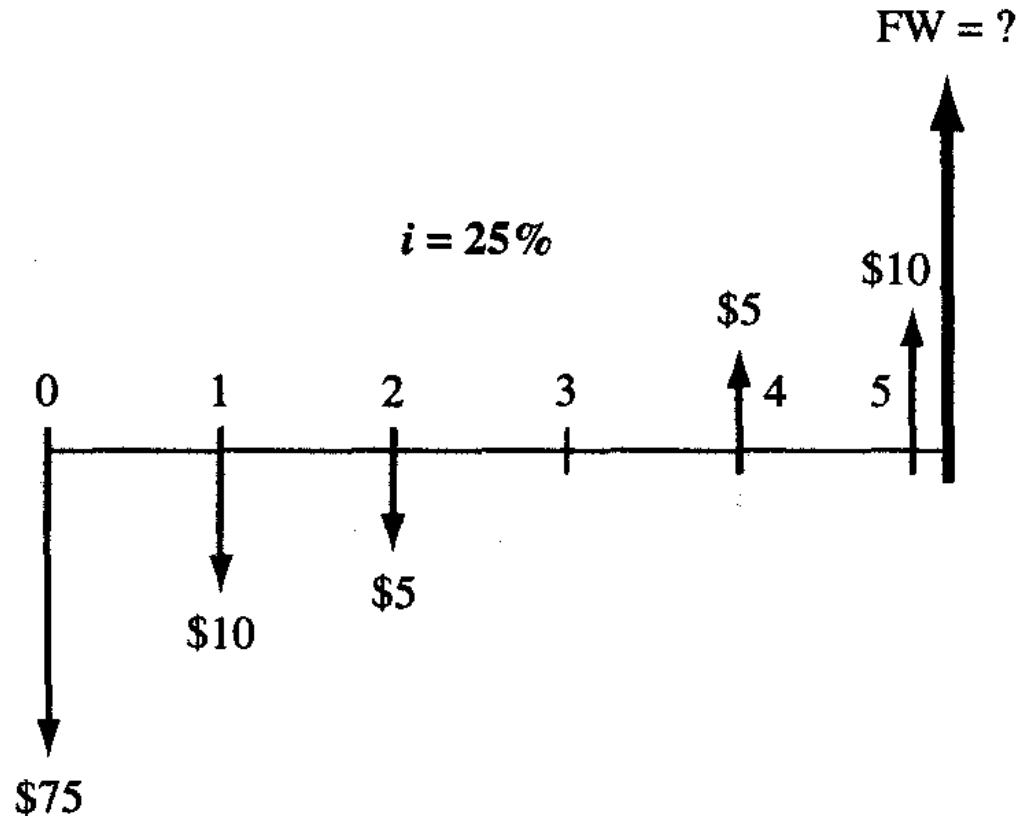


# Future Worth Analysis – Example

[2] Determine the future worth 5 years from now at 25% per year

- $$FW = -75(F/P, 25\%, 5) - 10(F/A, 25\%, 5) + 5(A/G, 25\%, 5)(F/A, 25\%, 5) = \$ -246.81 \text{ million}$$

- The offer must be for at least \$246.81 million to **make** the MARR



# Capitalized Cost Calculation and Analysis

- Capitalized cost (CC) is the present worth of an alternative that will last forever
- Public sector projects such as bridges, dams, irrigation systems, and railroads fall into this category
- **Capitalized cost** is the **present sum** of money that would need to be set aside now, at some interest rate, to yield the funds required to provide the service (or whatever) indefinitely.
- To accomplish this, the money set aside for future expenditures must not decline.
- The interest received on the money set aside can be spent, but not the principal.

# Capitalized Cost Calculation and Analysis

- Thus, for any initial present sum  $P$ , there can be an end-of-period withdrawal of  $A$  equal to  $iP$  each period.
- These withdrawals may continue forever without diminishing the initial sum  $P$ .
- This gives us the basic relationship:

$$\text{For } n = \infty, \quad A = Pi$$

# Capitalized Cost Calculation and Analysis

- The **CC value** can be computed from the following equation:

$$CC = \frac{A}{i}$$

where **A** is the annual worth

- If \$10,000 earns 20% per year, compounded annually, the ***maximum*** amount of money that can be withdrawn at the end of every year for ***eternity*** is \$2,000, or the interest accumulated each year
- This leaves the original \$10,000 to earn interest so that another \$2,000 ***will be accumulated the next year***

# Capitalized Cost Calculation and Analysis

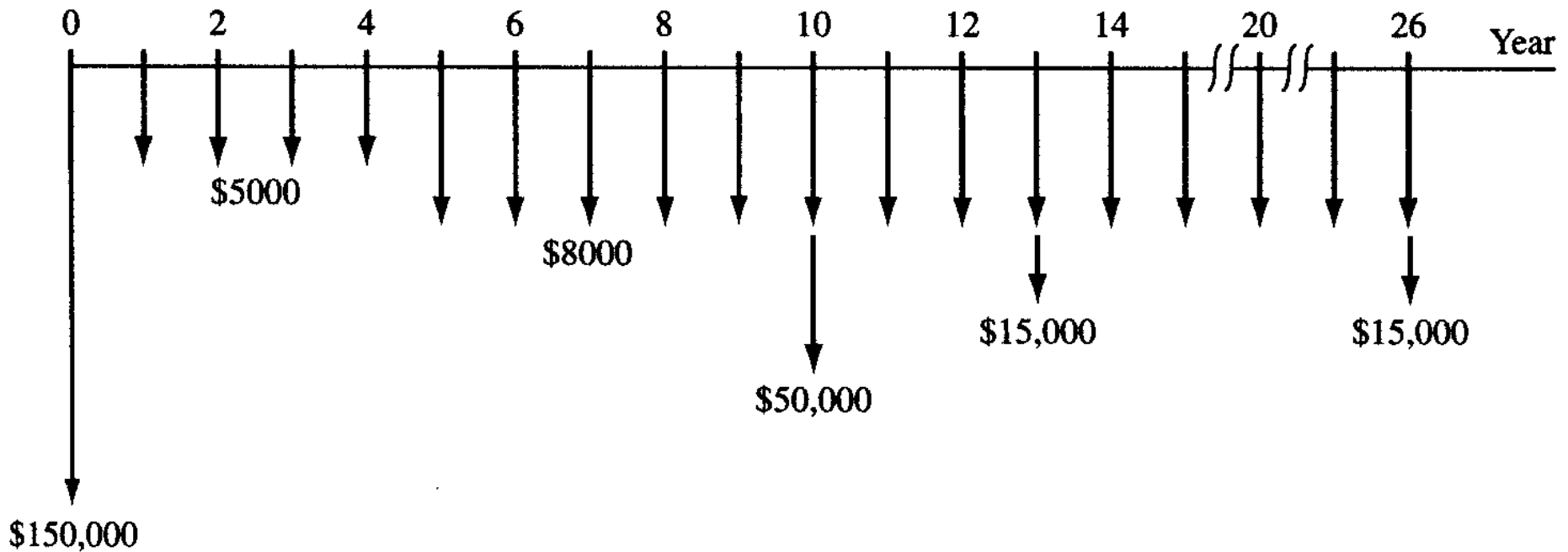
## Example

- A new computer system will be used for the *indefinite* future, find the equivalent value (a) now and (b) for each year hereafter
- The system has an installed cost of \$150,000 and an additional cost of \$50,000 after 10 years. The annual maintenance cost is \$5,000 for the first 4 years and \$8,000 thereafter. In addition, it is expected to be a *recurring* major upgrade cost of \$15,000 every 13 years
- Assume that  $i = 5\%$  per year

# Capitalized Cost Calculation and Analysis

## Example

$i = 5\%$  per year





# Capitalized Cost Calculation and Analysis

## Example

1. Draw the cash flow diagram
2. Find the present worth of the nonrecurring costs of \$150,000 now and \$50,000 in year 10 at  $i = 5\%$ . Label this **CC1** where  $CC1 = -150,000 - 50,000(P/F, 5\%, 10) = \$ -180,695$
3. Convert the recurring cost of \$15,000 every 13 years into an annual worth  $A1$  for the first 13 years.  $A1 = -15,000(A/F, 5\%, 13) = \$ -847$ . The same value,  $A1 = \$ -847$ , applies to all the other 13-year periods as well

# Capitalized Cost Calculation and Analysis

## Example

4. The capitalized cost for the two annual maintenance cost series can be determined by considering a series of \$ -5,000 from now to infinity and find the present worth of \$ -8,000 - (\$ -5,000) = \$ -3,000 from year 5 on. The annual cost (A2) is \$ -5,000 forever

The capitalized cost **CC2** of \$ -3,000 from year 5 to infinity is found using **the present value at year 4 (A/i)** and then multiply this by the P/F factor →

$$CC2 = (-3,000/0.05)(P/F, 5\%, 4) = \$ -49,362$$

The two annual cost series are converted into a capitalized cost **CC3** →

$$CC3 = (A1 + A2)/i = (-847 + (-5,000))/0.05 = \$ -116,940$$

# Capitalized Cost Calculation and Analysis

## Example

5. The total capitalized cost CC is obtained by adding the three CC values =  $-180,695 - 49,362 - 116,940 = \$ -346,997$

# Capitalized Cost Calculation and Analysis Procedure

1. Draw a cash flow diagram showing all nonrecurring (one-time) cash flows and at least two cycles of all recurring (periodic) cash flows
2. Find the present worth of all **nonrecurring** amounts. This is their CC value
3. Find the equivalent uniform annual worth (A value) **through one life cycle of all recurring amounts**. This is the same value in all succeeding life cycles. Add this to all other uniform amounts occurring in years 1 through infinity and the result is the total equivalent uniform annual worth (AW)
4. Divide the AW obtained in step 3 by the interest rate  $i$  to obtain a CC value
5. Add the CC values obtained in steps 2 and 4

# Payback Period Analysis

- The payback period ( $n_p$ ) is the estimated time in years it will take for the estimated revenues and other economic benefits to recover the **initial investment**
- The payback period analysis should provide initial screening or supplemental information in conjunction with an analysis performed using present worth or another method
- In order to find  $n_p$ , the **initial investment** should equal the present worth of net present value of all **estimated cash flows**

# Payback Period Analysis

## Example [1]

- An engineering firm has just approved an \$18 million design contract
- The services are expected to generate new annual net cash flows of \$3 million
- Contract period is 10 years
- If  $i = 15\%$ , compute the payback period

# Payback Period Analysis

## Example [1]

- The initial investment = \$18 million
- The present value of the expected return =  $A(P/A, i, n)$   
when considering that  $A = \$3$  million  $\rightarrow 3(P/A, 15\%, n)$
- The two amounts ought to be equal  $\rightarrow 18 = 3(P/A, 15\%, n)$ .  
This gives a value of  $n_p = 16.47$  years

# Payback Period Analysis

## Example [2]

- Two machines are being considered for purchase by a company. Machine 2 is expected to be **versatile** and technologically advanced enough to provide **net income** longer than machine 1
- The quality manager used a rate of return of 15% per year and a PC-based economic analysis package. The manager recommended machine 1 because **it has a shorter payback period of 6.57 years** at  $i = 15\%$ . **Is this right?**

	Machine 1	Machine 2
First cost, \$	12,000	8,000
Annual NCF, \$	3,000	1,000 (years 1-5) 3,000 (years 6-14)
Maximum life, years	7	14



# Payback Period Analysis

## Example [2]

- For Machine 1

$12,000 = 3,000(P/A, 15\%, n_p)$  which gives a payback period of *6.57 years*

- For Machine 2

$8,000 = 1,000(P/A, 15\%, 5) + 3,000(P/A, 15\%, n_p - 5) \times (P/F, 15\%, 5)$  which gives a payback period of *9.52 years*

# Payback Period Analysis

## Example [2]

- If we would like to use PW analysis to compare the machines, then the following procedure ought to be considered:

- For Machine 1

$$PW1 = -12,000 - 12,000(P/F,15\%,7) + 3,000(P/A,15\%,14) = \$663$$

- For Machine 2

$$PW2 = -8,000 + 1,000(P/A,15\%,5) + 3,000(P/A,15\%,9)(P/F,15\%,5) = \$2,470$$

- Machine 2 is selected since its PW value is larger