

$$\underline{\text{ex}} W = \{ (a, 1), a \in \mathbb{R} \}$$

$$\underline{\text{ex}} W = \left\{ \begin{bmatrix} a \\ 1 \end{bmatrix}, a \in \mathbb{R} \right\} \text{ is it}$$

a v. space ?? use usual addition  
and scalar multiplication:

sol: (1) closed under addition

$$\begin{bmatrix} 3 \\ 1 \end{bmatrix} + \begin{bmatrix} -5 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \end{bmatrix} \notin W$$

→ it is not a vector space.

$$8 \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 24 \\ 8 \end{bmatrix} \notin W. \quad X$$

8 axioms:

$$x, y, z \in V, \alpha, \beta \in \mathbb{R}.$$

$$(A1) \quad x + y = y + x.$$

$$(A2) \quad (x + y) + z = x + (y + z)$$

(A3) there exist an element  $0$  in  $V$   
such that  $x + 0 = x$

(A4) for each  $x \in V$ ,  $-x \in V$  such that  
 $x + (-x) = 0$

$$(A5) \quad \alpha(x + y) = \alpha x + \alpha y$$

$$(A6) \quad (\alpha + \beta)x = \alpha x + \beta x$$

$$(A7) \quad (\alpha\beta)x = \alpha(\beta x)$$

$$(A8) \quad 1 \cdot x = x$$

the set together with  
the (2) operations and the  
8 axioms  $\Rightarrow$  is a vector space.

properties ( $V$  is a vector space,  $x \in V$ )

$$\textcircled{1} \underset{\substack{\downarrow \\ \text{real number}}}{0} x = \underset{\substack{\downarrow \\ \text{vector}}}{0}$$

$$\textcircled{2} x + y = 0 \rightarrow x = -y$$

$$\textcircled{3} -1x = x$$

$$\textcircled{4} x + y = x + z \rightarrow y = z$$

$$\textcircled{5} \alpha x = 0 \Rightarrow \underset{\substack{\downarrow \\ \text{scalar}}}{\alpha} = 0 \text{ or } x = \underset{\substack{\downarrow \\ \text{vector}}}{0}.$$