"Divided Difference Method"

We want to approximate $f(x)$ by a polynomial ($P(x)$)
s.t. $P(x_k) = f(x_k)$, $k = 0, \dots, n$.

تفسير المطلوب

معناه
نريد أن نقرب الاقتران الصعب إلى كثير حدود
لذلك تعطى قيم من $0, \dots, n$ ⇐ $(n+1)$ قيمة
يكون الاقتران مساويا لكثير الحدود عندها.

نفرض أن

كثير حدود $P(x) =$

$$P(x) = a_0 + a_1(x - x_0) + a_2(x - x_0)(x - x_1) + \dots + a_n(x - x_0)(x - x_1)\cdots(x - x_{n-1}).$$

عوض في $P(x) \Leftarrow$

$$\boxed{f(x_0)} = P(x_0) = \boxed{a_0}$$

$$P(x_1) = f(x_1) = a_0 + a_1(x_1 - x_0) + 0 + \dots + 0.$$

$$\Rightarrow \boxed{a_1 = \frac{f(x_1) - f(x_0)}{x_1 - x_0}} = f[x_0, x_1]$$

هذا الشيء اسمه divided difference

$$a_2 = f[x_0, x_1, x_2] = \frac{f[x_1, x_2] - f[x_0, x_1]}{x_2 - x_0}$$

In general $a_n = f[x_0, x_1, \dots, x_n]$

$$= \frac{f[x_1, \dots, x_n] - f[x_0, \dots, x_{n-1}]}{x_n - x_0}$$

Ex: Let $f(x) = \cos(x)$, $x_0 = 0.2$, $x_1 = 0.3$, $x_2 = 0.4$

Find $\boxed{1}$ $f[x_0, x_1]$ $\boxed{2}$ $f[x_1, x_2]$.

$\boxed{3}$ find a quadratic polynomial $p(x)$ by divided difference formula (use 3-digit chopping).

Sl. $\boxed{1}$

$$f[x_0, x_1] = \frac{f(x_1) - f(x_0)}{x_1 - x_0} = \frac{\cos 0.3 - \cos 0.2}{0.1}$$

$$= \frac{0.955 - 0.980}{0.1} = \boxed{-0.24}$$

$\boxed{2}$

$$f[x_1, x_2] = \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{\cos 0.4 - \cos 0.3}{0.4 - 0.3} = \boxed{-0.34}$$

$\boxed{3}$

$$a_0 = f(x_0) = \cos 0.2 = 0.98$$

$$a_1 = \boxed{-0.24}$$ وجدناها فوق

$$a_2 = f[x_0, x_1, x_2] = \frac{f[x_1, x_2] - f[x_0, x_1]}{x_2 - x_0}$$

$$= \frac{-0.34 - (-0.24)}{0.4 - 0.2} = \boxed{-0.5}$$

$\Rightarrow$ The quadratic polynomial is

$$P(x) = a_0 + a_1(x - x_0) + a_2(x - x_0)(x - x_1)$$

$$= 0.98 + (-0.24)(x - 0.2) +$$

$$+ (-0.5)(x - 0.2)(x - 0.3).$$

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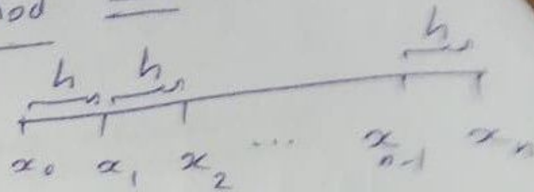
Divided Difference Method تابع

من الآن فصاعدا

$h = \Delta x = x_i - x_{i-1}$



Def^n: $\Delta f(x_0) = f(x_1) - f(x_0)$, $\Delta f(x_1) = f(x_2) - f(x_1)$

$\Delta f(x_2) = f(x_3) - f(x_2) \cdots \Rightarrow \boxed{\Delta f(x_n) = f(x_{n+1}) - f(x_n)}$

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Ex: Find $\boxed{\Delta^2 f(x_0)} = \Delta(\Delta f(x_0)) = \Delta(f(x_1) - f(x_0))$

$= \Delta f(x_1) - \Delta f(x_0) = f(x_2) - f(x_1) - (f(x_1) - f(x_0))$

$= \boxed{f(x_2) - 2f(x_1) + f(x_0)}$ #

In general $\boxed{\Delta^n f(x_0) = \Delta^{n-1} f(x_1) - \Delta^{n-1} f(x_0)} \Leftarrow$ قاعدة

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Ex: Show that $\boxed{a_n = \dfrac{\Delta^n f(x_0)}{n!\, h^n}}$ حيث a_n هي المعامل في كثير الحدود

sl: $a_0 = f(x_0)$, $a_1 = f[x_0 - x_1] = \dfrac{f(x_1) - f(x_0)}{x_1 - x_0}$

$\Rightarrow \boxed{a_1 = \dfrac{\Delta f(x_0)}{h}}$

$a_2 = f[x_0, x_1, x_2] = \dfrac{f[x_1, x_2] - f[x_0, x_1]}{x_2 - x_0}$

$= \left(\dfrac{f(x_2) - f(x_1)}{x_2 - x_1} - \dfrac{f(x_1) - f(x_0)}{x_1 - x_0}\right) / h$ ربط الحدود تبع

$a_2 = \dfrac{\Delta f(x_1) - \Delta f(x_0)}{2h^2} = \boxed{\dfrac{\Delta^2 f(x_0)}{2h^2}}$ ...

$\Rightarrow a_n = \dfrac{\Delta^n f(x_0)}{n!\, h^n}$ #