# Showing that *f* is one-to-one or onto

Suppose that $f : A \to B$.

*To show that $f$ is injective* Show that if $f(x) = f(y)$ for arbitrary $x, y \in A$ with $x \neq y$, then $x = y$.

*To show that $f$ is not injective* Find particular elements $x, y \in A$ such that $x \neq y$ and $f(x) = f(y)$.

*To show that $f$ is surjective* Consider an arbitrary element $y \in B$ and find an element $x \in A$ such that $f(x) = y$.

*To show that $f$ is not surjective* Find a particular $y \in B$ such that $f(x) \neq y$ for all $x \in A$.

# Showing that *f* is one-to-one or onto

**Example 1**: Let $f$ be the function from $\{a,b,c,d\}$ to $\{1,2,3\}$ defined by $f(a) = 3$, $f(b) = 2$, $f(c) = 1$, and $f(d) = 3$. Is $f$ an onto function?

**Solution**: Yes, $f$ is onto since all three elements of the codomain are images of elements in the domain. If the codomain were changed to $\{1,2,3,4\}$, $f$ would not be onto.

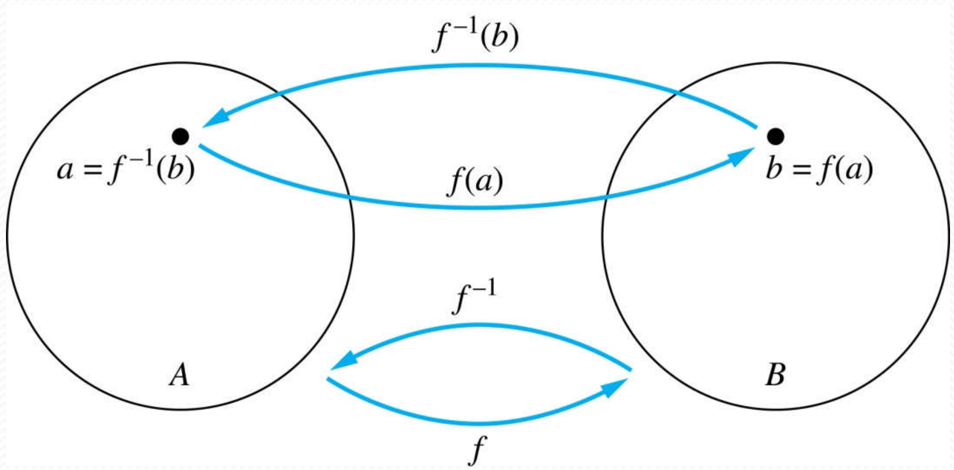
**Example 2**: Is the function $f(x) = x^2$ from the set of integers to the set of integers onto?

**Solution**: No, $f$ is not onto because there is no integer $x$ with $x^2 = -1$, for example.

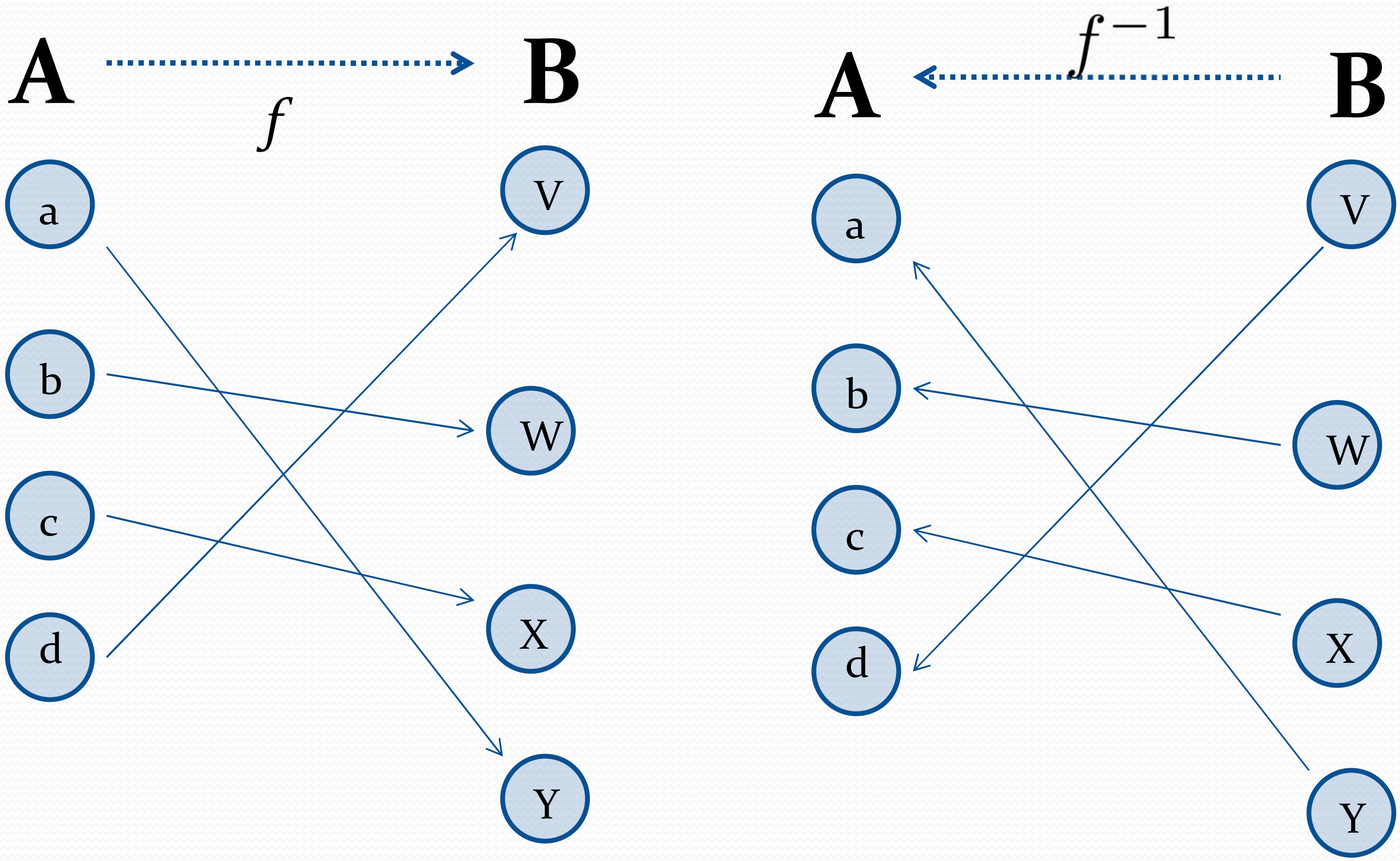
# Inverse Functions

**Definition**: Let $f$ be a bijection from $A$ to $B$. Then the *inverse* of $f$, denoted $f^{-1}$, is the function from $B$ to $A$ defined as $f^{-1}(y) = x$ iff $f(x) = y$

No inverse exists unless $f$ is a bijection. Why?



# Inverse Functions



# Questions

**Example 1**: Let $f$ be the function from $\{a,b,c\}$ to $\{1,2,3\}$ such that $f(a) = 2$, $f(b) = 3$, and $f(c) = 1$. Is $f$ invertible and if so what is its inverse?

**Solution**: The function $f$ is invertible because it is a one-to-one correspondence. The inverse function $f^{-1}$ reverses the correspondence given by $f$, so $f^{-1}(1) = c$, $f^{-1}(2) = a$, and $f^{-1}(3) = b$.

# Questions

**Example 2**: Let $f$: **Z** $\rightarrow$ **Z** be such that $f(x) = x + 1$. Is $f$ invertible, and if so, what is its inverse?

**Solution**: The function $f$ is invertible because it is a one-to-one correspondence. The inverse function $f^{-1}$ reverses the correspondence so $f^{-1}(y) = y - 1$.

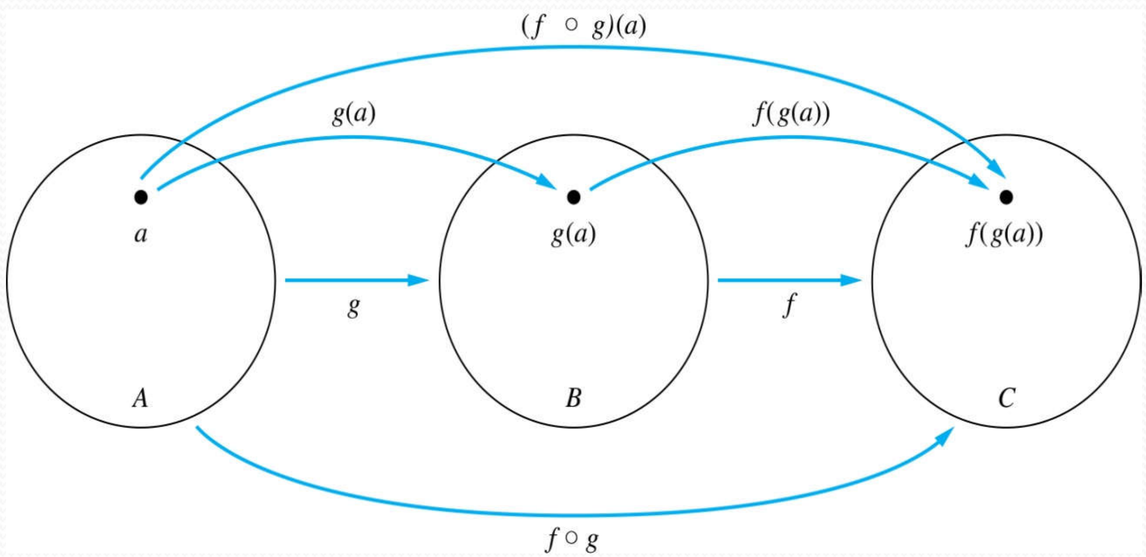
# Questions

**Example 3**: Let *f*: **R** → **R** be such that $f(x) = x^2$. Is *f* invertible, and if so, what is its inverse?

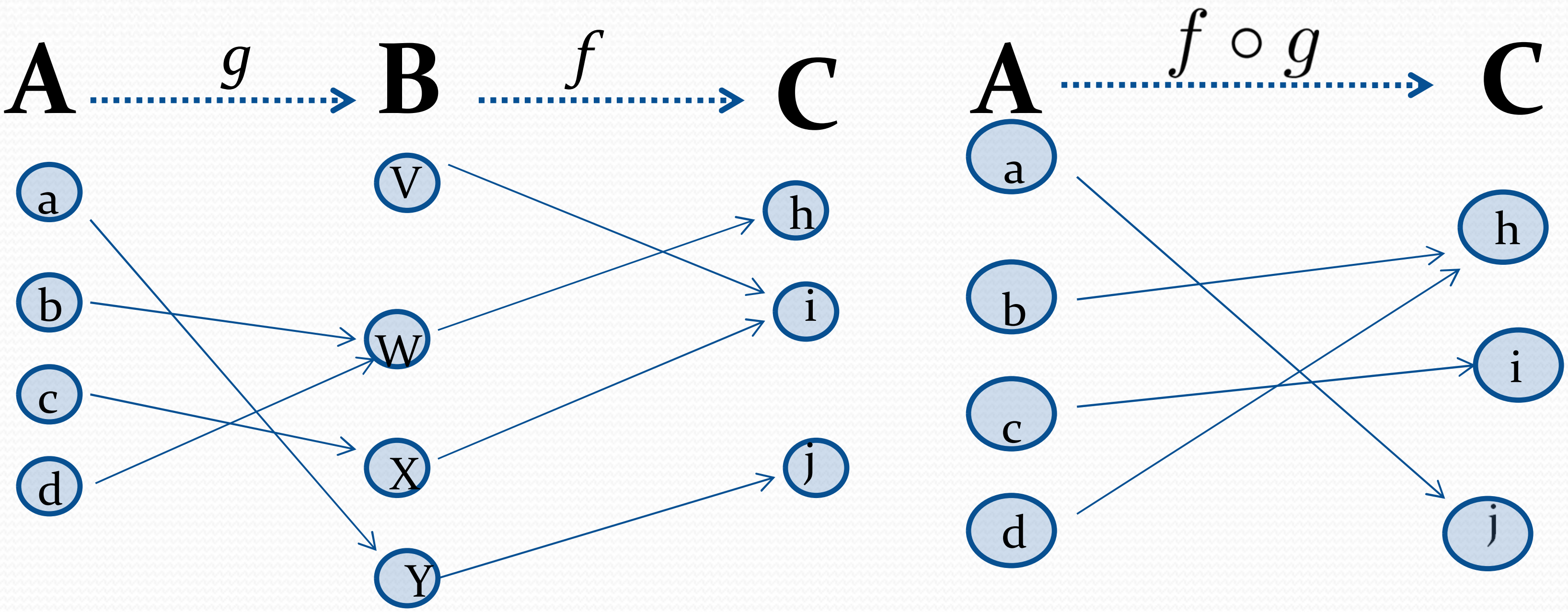
**Solution**: The function *f* is not invertible because it is not one-to-one .

# Composition

- **Definition**: Let $f$: $B \rightarrow C$, $g$: $A \rightarrow B$. The *composition of f with g*, denoted $f \circ g$ is the function from $A$ to $C$ defined by $$f \circ g(x) = f(g(x))$$



# Composition



# Composition

**Example 1**: If $f(x) = x^2$ and $g(x) = 2x + 1$ , then

$$f(g(x)) = (2x + 1)^2$$

and

$$g(f(x)) = 2x^2 + 1$$

# Composition Questions

**Example 2**: Let $g$ be the function from the set $\{a,b,c\}$ to itself such that $g(a) = b$, $g(b) = c$, and $g(c) = a$. Let $f$ be the function from the set $\{a,b,c\}$ to the set $\{1,2,3\}$ such that $f(a) = 3$, $f(b) = 2$, and $f(c) = 1$.

What is the composition of $f$ and $g$, and what is the composition of $g$ and $f$.

**Solution:** The composition $f \circ g$ is defined by

$f \circ g\ (a) = f(g(a)) = f(b) = 2.$

$f \circ g\ (b) = f(g(b)) = f(c) = 1.$

$f \circ g\ (c) = f(g(c)) = f(a) = 3.$

Note that $g \circ f$ is not defined, because the range of $f$ is not a subset of the domain of $g$.

# Composition Questions

**Example 2**: Let f and g be functions from the set of integers to the set of integers defined by $f(x) = 2x + 3$ and $g(x) = 3x + 2$.

What is the composition of $f$ and $g$, and also the composition of $g$ and $f$ ?

**Solution:**

$$f \circ g\ (x) = f(g(x)) = f(3x + 2) = 2(3x + 2) + 3 = 6x + 7$$

$$g \circ f\ (x) = g(f(x)) = g(2x + 3) = 3(2x + 3) + 2 = 6x + 11$$