Palestine Technical University- Kadoorie (PTUK)

Mechanical Engineering Department

12210244: Dynamics

Summer Semester, 2023/2024

This is an explanation of the Dynamics course

offered at Palestine Technical University - Kadoorie

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Textbook:

Engineering Mechanics: Dynamics, 7th Edition

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Chapter Three: Kinetics of Particles

Section Five: Curvilinear Motion

3 Chapter Three: Kinetics of Particles

3.5 Curvilinear Motion

1. Rectangular coordinates

$$\sum \boldsymbol{F_x} = \boldsymbol{m a_x}$$
$$\sum \boldsymbol{F_y} = \boldsymbol{m a_y}$$
$$\boldsymbol{a_x} = \boldsymbol{\ddot{x}}$$
$$\boldsymbol{a_y} = \boldsymbol{\ddot{y}}$$

2. Normal and tangential coordinates

$$\sum \boldsymbol{F_n} = \boldsymbol{m a_n}$$
$$\sum \boldsymbol{F_t} = \boldsymbol{m a_t}$$
$$\boldsymbol{a_n} = \boldsymbol{\rho \dot{\theta}^2} = \frac{\boldsymbol{v^2}}{\boldsymbol{\rho}}$$
$$\boldsymbol{a_t} = \boldsymbol{\dot{v}}$$

3. Polar coordinates

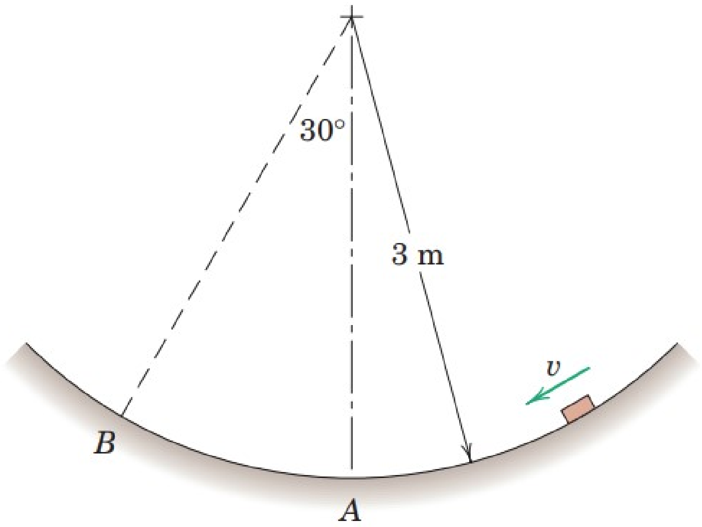
$$\sum \boldsymbol{F_r} = \boldsymbol{m a_r}$$
$$\sum \boldsymbol{F_\theta} = \boldsymbol{m a_\theta}$$
$$\boldsymbol{a_r} = \boldsymbol{\ddot{r} - r\dot{\theta}^2}$$
$$\boldsymbol{a_\theta} = \boldsymbol{r\ddot{\theta} + 2\dot{r}\dot{\theta}}$$

End of Section 3.5

Example 1:

The small 0.6-kg block slides with a small amount of friction on the circular path of radius 3 m in the vertical plane. If the speed of the block is 5 m/s as it passes point A and 4 m/s as it passes point B, determine the normal force exerted on the block by the surface at each of these two locations.

ans. $N_A = 10.9\ N$ $N_B = 8.3\ N$

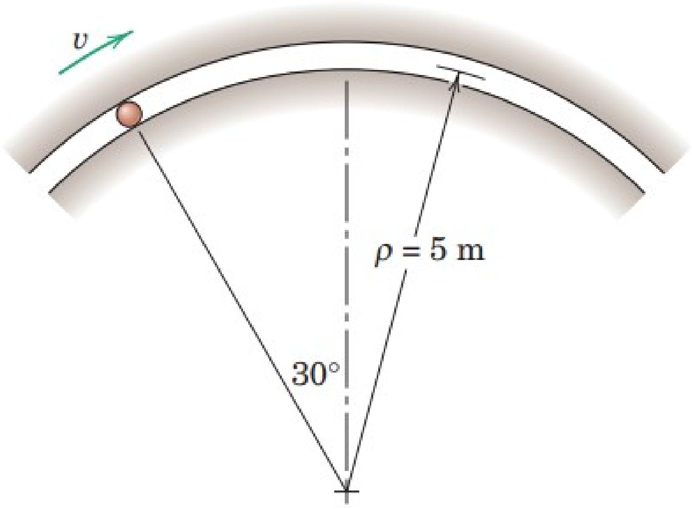


Ans.

Example 2:

The 0.1-kg particle has a speed $v = 10\ m/s$ as it passes the 30^circ position shown. The coefficient of kinetic friction between the particle and the vertical plane track is $\mu_k = 0.2$. Determine the magnitude of the total force exerted by the track on the particle. What is the deceleration of the particle?

$ans.$ $N = 1.15\ N$ $a_t = -7.21\ m/sec^2$



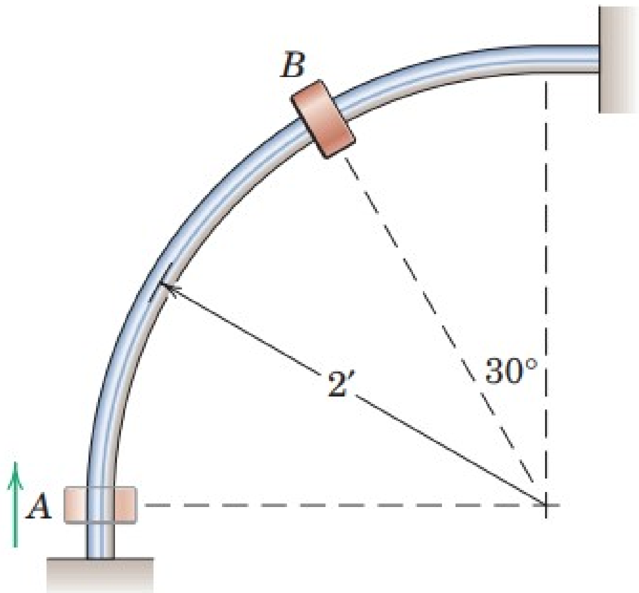
Ans.

Example 3:

A 2 lb slider is propelled upward at A along the fixed curved bar which lies in a vertical plane. If the slider is observed to have a speed of 10 ft/sec as it passes position B, determine

1. the magnitude N of the force exerted by the fixed rod on the slider
2. the rate at which the speed of the slider is decreasing. Assume that friction is negligible

ans. 1) $N = 1.374\ Ib$ $a_t = -16.1\ ft/sec^2$

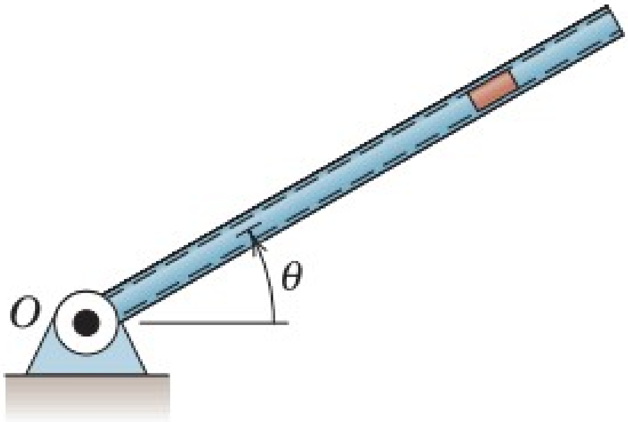


Ans.

Example 4:

The hollow tube is pivoted about a horizontal axis through point O and is made to rotate in the vertical plane with a constant counterclockwise angular velocity $\dot{\theta} = 3\ rad/sec$. If a 0.2 lb particle is sliding in the tube toward O with a velocity of 4 ft/sec relative to the tube when the position $\theta = 30°$ is passed, calculate the magnitude N of the normal force exerted by the wall of the tube on the particle at this instant.

$ans.$ $N = 0.0241\ Ib$



Ans.

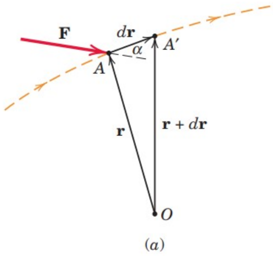
Chapter Three: Kinetics of Particles

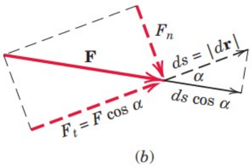
Section Six: Work and Kinetic Energy

3 Chapter Three: Kinetics of Particles

3.6 Work and Kinetic Energy

Definition of Work





$$dU = \vec{F}\cdot d\vec{r}$$

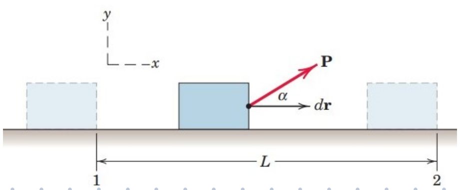
$$dU = F_t\ ds$$

$$U = \int_1^2 \vec{F}\cdot d\vec{r} = \int_1^2 (F_x\ dx + F_y\ dy + F_z\ dz)$$

$$U = \int_1^2 F_t\ ds$$

Examples of Work

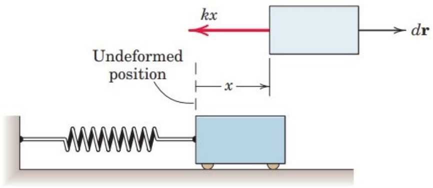
1. Work Associated with a Constant External Force



$$U_{1\to 2} = \int_1^2 \vec{F}\cdot d\vec{r} \Rightarrow U_{1\to 2} = \int_1^2 \left[(P\ cos(\alpha)\hat{i} + (P\ sin(\alpha)\hat{j}\right]\cdot dx\hat{i}$$

$$U_{1\to 2} = \int_{x_1}^{x_2} P\ cos(\alpha)dx = P\ cos(\alpha)\ (x_2 - x_1) = P\ L\ cos(\alpha)$$

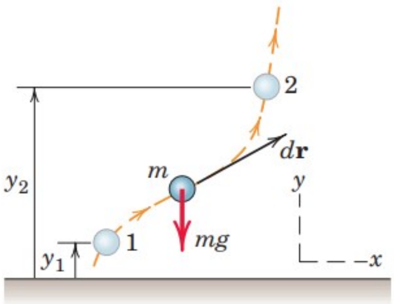
2. Work Associated with a Spring Force



$$\boldsymbol{U_{1\to 2} = \int_1^2 \vec{F}\cdot d\vec{r} \Rightarrow U_{1\to 2} = \int_1^2 \left[(-k\ x\ \hat{i})\right]\cdot dx\hat{i}}$$

$$\boldsymbol{U_{1\to 2} = -\int_{x_1}^{x_2} k\ x\ dx = \frac{1}{2}\ k\ (x_1^2 - x_2^2)}$$

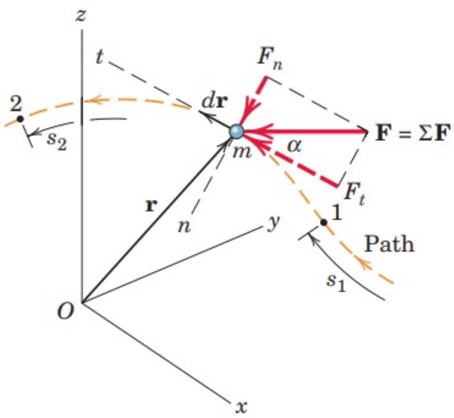
3. Work Associated with Weight



$$\boldsymbol{U_{1\to 2} = \int_1^2 \vec{F}\cdot d\vec{r} \Rightarrow U_{1\to 2} = \int_1^2 \left[(-m\ g\ \hat{j})\right]\cdot (dx\hat{i} + dy\hat{j})}$$

$$\boldsymbol{U_{1\to 2} = -m\ g\ \int_{y_1}^{y_2}\ dy = -m\ g\ (y_2 - y_1)}$$

4. Work and Curvilinear Motion



$$\boldsymbol{U_{1\to 2} = \int_1^2 \vec{F}\cdot d\vec{r} = \int_{s_1}^{s_2} F_t\, ds}$$

$$\boldsymbol{U_{1\to 2} = \int_1^2 m\cdot \vec{a}\; d\vec{r} = \int_{s_1}^{s_2} m\cdot a_t\; ds = \int_{v_1}^{v_2} m\cdot v\; dv = \frac{1}{2}m(v_2^2 - v_1^2)}$$

5. Principle of Work and Kinetic Energy

$$\boldsymbol{T = \frac{1}{2}\, m\; v^2}$$

$$\boldsymbol{U_{1\to 2} = T_2 - T_1 = \Delta T}$$

$$\boldsymbol{T_1 + U_{1\to 2} = T_2}$$

6. Power

$$\boldsymbol{P = \vec{F}\cdot\vec{v}}$$

7. Efficiency

$$\boldsymbol{e_m = \frac{P_{out}}{P_{in}}}$$

End of Section 3.6

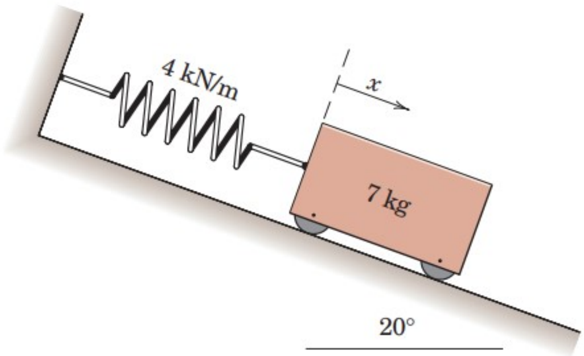
Example 1:

The spring is unstretched when $x = 0$. If the body moves from the initial position $x_1 = 100\ mm$ to the final position $x_2 = 200\ mm$,

(a) determine the work done by the spring on the body,

(b) determine the work done on the body by its weight.

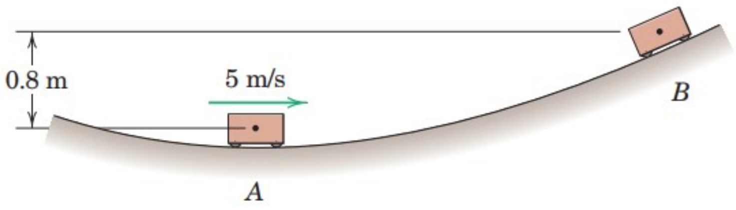
ans. $U_s = -60\ J$ $U_w = 2.35\ J$



Ans.

Example 2:

The small body has a speed $v_A = 5\ m/sec$ at point A. Neglecting friction, determine its speed v_B at point B after it has risen $0.8\ m$. Is knowledge of the shape of the track necessary?
ans. $v_B = 3.05\ m/sec$

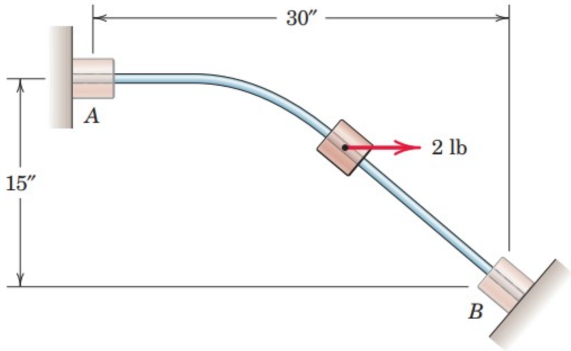


Ans.

Example 3:

The 1.5 lb collar slides with negligible friction on the fixed rod in the vertical plane. If the collar starts from rest at A under the action of the constant 2 lb horizontal force, calculate its velocity v_B as it hits the stop at B.

ans. $v_B = 17.18\ ft/sec$

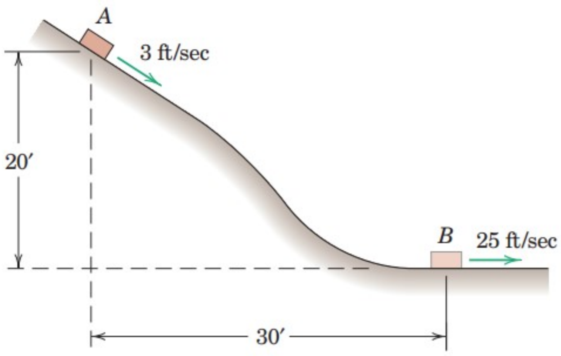


Ans.

Example 4:

The 64.4 lb crate slides down the curved path in the vertical plane. If the crate has a velocity of $v_A = 3\ ft/sec$ down the incline at A and a velocity of $v_B = 25\ ft/sec$ at B, compute the work U_f done on the crate by friction during the motion from A to B.

ans. $U_f = -672\ ft.Ib$



Ans.