Fourier Series Analysis of Continuous Time Signals

ch4

- A Fourier series is an expansion of a periodic function f(x) in terms of an infinite sum of sines and cosines

Periodic Signal Representation by Fourier Series

- A continuous time signal x(t) is said to be periodic if there is a positive non-zero value of T for which

$$x(t+T) = x(t) \qquad \text{for all } t$$

- The fundamental period *T*0 of *x(t)* is the smallest positive value of *T*

$$\frac{1}{T_0} \text{ is called fundamental frequency } f_0$$

$$\omega_0 = \frac{2\pi}{T_0}$$

- The real sinusoidal signal

$$x(t) = \cos(\omega_0 t + \phi)$$

- the complex exponential signal

$$x(t) = e^{j\omega_0 t}$$

- The prerequisite for the representation of any arbitrary continuous signal x(t) in Fourier series is that it should be periodic.
- Non-periodic signals cannot be represented by Fourier series but can be represented by Fourier transform ch5

Trigonometric Fourier Series

- Consider any arbitrary continuous time signal x(t).
- This arbitrary signal can be split up as sines and cosines of fundamental frequency ω0 and all of its harmonics are expressed as given below.

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + b_n \sin n\omega_0 t$$

$$a_0 = \frac{1}{T_0} \int_{T_0} x(t)\, dt$$

$$a_n = \frac{2}{T_0} \int_{T_0} x(t) \cos n\omega_0 t\, dt$$

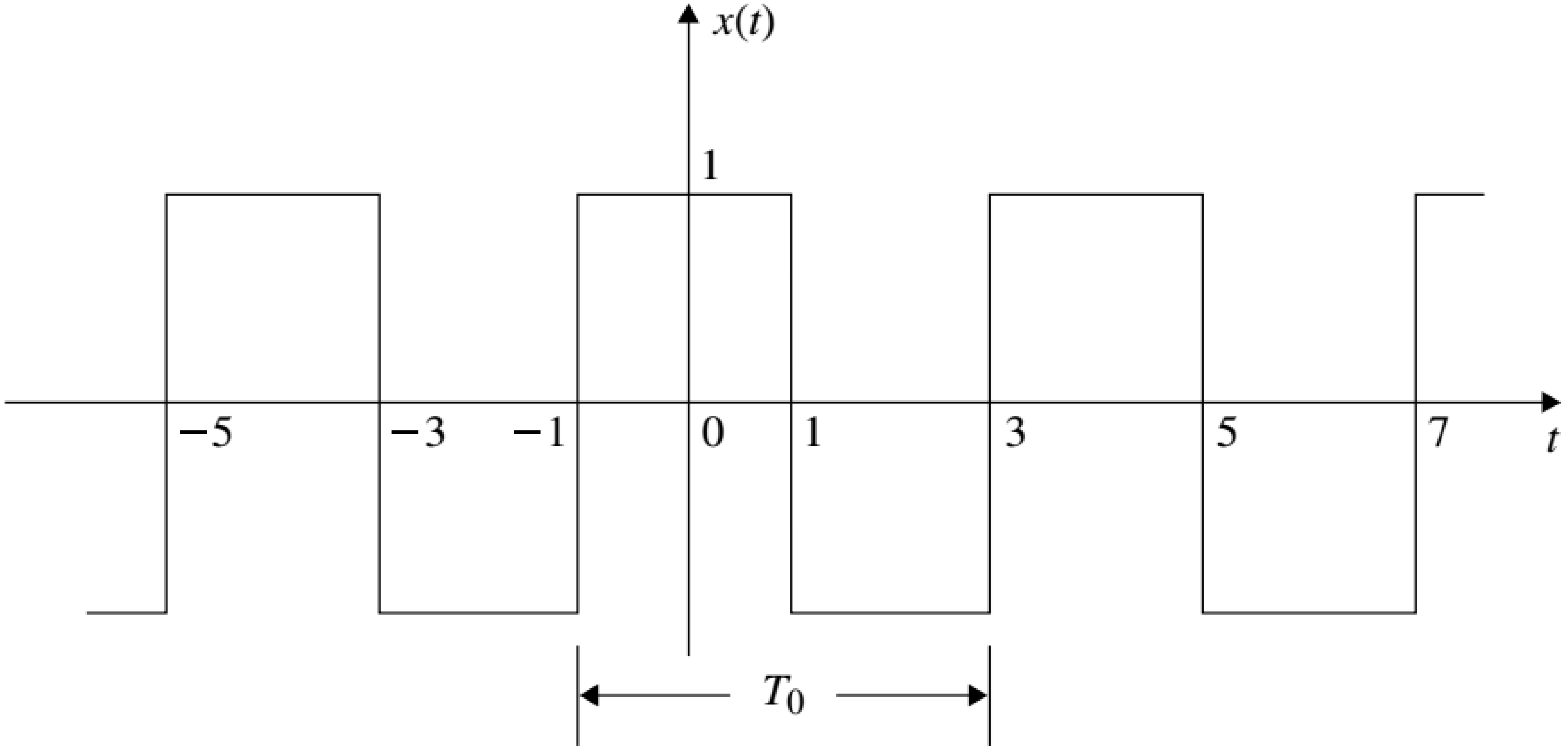
$$b_n = \frac{2}{T_0} \int_{T_0} x(t) \sin n\omega_0 t\, dt$$

$$T_0 = \frac{1}{f_0} = \frac{2\pi}{\omega_0}$$

It can be proved that

- 1. If the periodical signal x(t) is symmetrical with respect to the time axis, then the coefficient $a0 = 0$.
- 2. If the periodical signal x(t) represents an even function, only cosine terms in FS exists and therefore $bn = 0$.
- 3. If the periodical signal x(t) represents an odd function, only sine terms in FS exists and therefore $an = 0$.

Find the trigonometric Fourier series for the periodic signal shown in Fig



By investigative the figure

- it is evident that the waveform is symmetrical with respect to the

time axis t. Hence, a0 = 0.

- By folding x(t) across the vertical axis, it is observed that x(t) = x(−t) which shows that the function of the signal is even. Hence, bn = 0.
- it is easily obtained that the fundamental period T0 = 4 seconds

$$\omega_0 = \frac{2\pi}{T_0} = \frac{\pi}{2}$$

$$x(t) = a_0 + \sum_{n=1}^{\infty} [a_n \cos n\omega_0 t + b_n \sin n\omega_0 t]$$

$$\begin{aligned} x(t) &= 1 \qquad \text{for} \quad -1 \le t \le 1 \\ &= -1 \qquad \text{for} \quad 1 \le t \le 3 \end{aligned}$$

Substituting $a_0 = 0$ and $b_n = 0$, and $\omega_0 = \frac{\pi}{2}$

$$x(t) = \sum_{n=1}^{\infty} a_n \cos \frac{n\pi}{2} t$$

$$
\begin{aligned}
a_n &= \frac{2}{T_0}\int_{-1}^{3} x(t)\cos\left(\frac{n\pi}{2}t\right)dt \\
&= \frac{1}{2}\left[\int_{-1}^{1}\cos\frac{n\pi}{2}t\,dt + \int_{1}^{3}(-1)\cos\frac{n\pi}{2}t\,dt\right] \\
&= \frac{1}{2}\left[\left\{\frac{2}{n\pi}\sin\frac{n\pi}{2}t\right\}_{-1}^{1} - \left\{\frac{2}{n\pi}\sin\frac{n\pi}{2}t\right\}_{1}^{3}\right] \\
&= \frac{1}{n\pi}\left[\sin\frac{n\pi}{2} + \sin\frac{n\pi}{2} + \sin\frac{n\pi}{2} + \sin\frac{n\pi}{2}\right] \\
&= \frac{4}{n\pi}\sin\frac{n\pi}{2}
\end{aligned}
$$

$$= 0 \qquad \text{for } n = \text{even}$$

$$= \frac{4}{n\pi} \qquad \text{for } n = 1, 5, 9, 13, \ldots$$

$$= -\frac{4}{n\pi} \qquad \text{for } n = 3, 7, 11, 15, \ldots$$

$$x(t) = \sum_{n=1}^{\infty} a_n \cos \frac{n\pi}{2} t$$

$$\boxed{x(t) = \frac{4}{\pi}\left[\cos \frac{\pi}{2}t - \frac{1}{3}\cos \frac{3\pi}{2}t + \frac{1}{5}\cos \frac{5\pi}{2}t - \frac{1}{7}\cos \frac{7\pi}{2}t\right]}$$