chapter "4"

Linear transformations تحويلات خطية

4.1 Definition and examples

Def: $L : V \longrightarrow W$

A mapping "L" from a vector space "V" into a vector space "W" is said to be alinear transformation if

$$L(\alpha v_1 + \beta v_2) = \alpha L(v_1) + \beta L(v_2),$$

$\alpha_1, \alpha_2 \in R$ (scalars), $v_1, v_2 \in V \Rightarrow L(v_1), L(v_2) \in W$

OR

$$\left[\begin{array}{l} L(v_1 + v_2) = L(v_1) + L(v_2) \\ L(\alpha v) = \alpha L(v) \end{array}\right\}$$ مهم

note If $L : V \longrightarrow V$ is alinear transformation the "L" is called alinear operator.

ex Determine if $L(\vec{x}) = 3x$, $x \in R^2$ is alinear operator

Sol: $L : R^2 \longrightarrow R^2$.

$L(x) = 3x$, $\Rightarrow$ $v_1 \longrightarrow L(v_1) = 3v_1$

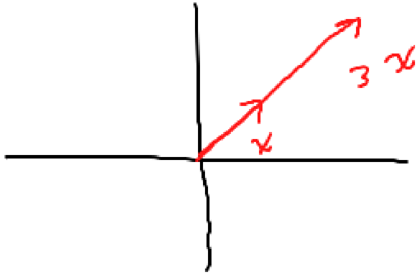
$v_2 \longrightarrow L(v_2) = 3v_2$

$$L(\alpha_1 v_1 + \alpha_2 v_2) \overset{??}{=} \alpha_1 L(v_1) + \alpha_2 L(v_2).$$

$$L(\underline{\underline{\alpha_1 v_1 + \alpha_2 v_2}}) = 3(\alpha_1 v_1 + \alpha_2 v_2)$$

$$= \alpha_1(3v_1) + \alpha_2(3v_2) = \alpha_1 L(v_1) + \alpha_2 L(v_2)$$

$\rightarrow L$ is alinear operator.
(or transformation)



<u>OR</u> حل مطول

$$L: \mathbb{R}^2 \rightarrow \mathbb{R}^2$$

$$v_1 = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \rightarrow L(v_1) = 3\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} =$$

$$v_2 = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \rightarrow L(v_2) = 3\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} =$$

$$L(\alpha v_1 + \beta v_2) = L\left(\alpha \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \beta \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}\right)$$

$$= L\left(\begin{bmatrix} \alpha x_1 + \beta y_1 \\ \alpha x_2 + \beta y_2 \end{bmatrix}\right) = 3\begin{bmatrix} \alpha x_1 + \beta y_1 \\ \alpha x_2 + \beta y_2 \end{bmatrix}$$

$$= \alpha\, 3\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \beta\, 3\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \alpha L(v_1) + \beta L(v_2).$$

ex consider the mapping

$L(x) = x_1 e_1$ for each $x \in R^2$.

is L a linear operator ?

Sol: $L : R^2 \longrightarrow R^2$

$L(x) = x_1 e_1 \Rightarrow x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$L\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = x_1 e_1 = x_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} x_1 \\ 0 \end{bmatrix}$

$v_1 = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \rightarrow L(v_1) = \begin{bmatrix} x_1 \\ 0 \end{bmatrix}$

$v_2 = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \rightarrow L(v_2) = \begin{bmatrix} y_1 \\ 0 \end{bmatrix}$.

$L(\alpha v_1 + \beta v_2) \overset{??}{=} \alpha L(v_1) + \beta L(v_2)$.

$L(\alpha v_1 + \beta v_2) = L\left(\alpha \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \beta \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}\right)$

$= L\left(\begin{bmatrix} \alpha x_1 + \beta y_1 \\ \alpha x_2 + \beta y_2 \end{bmatrix}\right) = \begin{bmatrix} \alpha x_1 + \beta y_1 \\ 0 \end{bmatrix}$

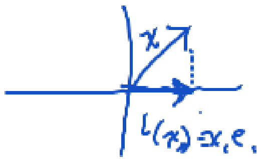
$= \alpha \begin{bmatrix} x_1 \\ 0 \end{bmatrix} + \beta \begin{bmatrix} y_1 \\ 0 \end{bmatrix} = \alpha L(v_1) + \beta L(v_2)$.

L is a linear transformation

OR $L(\alpha v_1 + \beta v_2) = (\alpha x_1 + \beta y_1) e_1$

$= \alpha e_1 x_1 + \beta e_1 y_1$

$= \alpha L(v_1) + \beta L(v_2)$



ex The mapping $M : R^2 \to R$

defined by $M(x) = \sqrt{x_1^2 + x_2^2}$

is M a linear transformation,

sol: $L(v_1 + v_2) = L(v_1)$