Review chapter "3"

1 note:

(a) Let u, v are subspaces of a vector space W, then $u \cap v$ is also a subspace of W.

(b) let u, v are subspaces of a vector space W, then $u \cup v$ not necessarily be a subspace of W.

2 Page 125

Q1 (d) Determine whether the following set form subspaces of R^2.
$\{ (x_1, x_2)^T, \ |x_1| = |x_2| \}$.

sol:

$S = \{ (a,a)^T, (a,-a)^T, (-a,a)^T, (-a,-a)^T, \ldots \} \quad a \in R$

$x, y \in S \xrightarrow{??} x+y \in S$.

$\begin{bmatrix} a \\ a \end{bmatrix} + \begin{bmatrix} a \\ -a \end{bmatrix} = \begin{bmatrix} 2a \\ 0 \end{bmatrix} \notin S \rightarrow S$ is not a subspace of R^2.

ex $(1,1)^T, (1,-1)^T \in S \rightarrow (1,1)+(1,-1) = (2,0) \notin S$

Q2 (a) Determine whether the following sets form subspaces of R^3.
$\{ (x_1, x_2, x_3) \quad x_1 + x_3 = 1 \}$

sol:

$(1,3,0) \in S \neq \phi$

$X = (x_1, x_2, x_3), \quad x_1 + x_3 = 1$

$Y = (y_1, y_2, y_3), \quad y_1 + y_3 = 1$

(1) $X, Y \in S \xrightarrow{??} X+Y \in S$

$X+Y = (x_1+y_1, x_2+y_2, x_3+y_3)^T$, but $x_1+y_1+x_3+y_3 = \overbrace{x_1+x_3}^{1} + \overbrace{y_1+y_3}^{1} = 2$

$\rightarrow X+Y \notin S \longrightarrow S$ is not a subspace of R^3.

Q3 Determine whether the following are subspaces of $R^{2\times 2}$.

(a) the set of 2×2 diagonal matrices.

* $\begin{bmatrix} 0 & 0 \\ 0 & 2 \end{bmatrix} \in S \neq \phi$
* $X = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}, \ Y = \begin{bmatrix} c & 0 \\ 0 & d \end{bmatrix}$

$X, Y \in S \xrightarrow{??} X+Y \in S$ — $X+Y = \begin{bmatrix} a+c & 0 \\ 0 & b+d \end{bmatrix} \in S$ ✓

$X \in S, \ \alpha \in \mathbb{R} \xrightarrow{??} \alpha X \in S$ — $\alpha X = \begin{bmatrix} \alpha a & 0 \\ 0 & \alpha b \end{bmatrix} \in S$ ✓

S is a subspace of $R^{2\times 2}$.

ⓑ the set of all 2x2 triangular matrices.

$X = \begin{bmatrix} a & b \\ 0 & c \end{bmatrix}$, $Y = \begin{bmatrix} d & 0 \\ e & f \end{bmatrix}$

$X, Y \in S \xrightarrow{??} X + Y \in S$ $\quad X+Y = \begin{bmatrix} a+d & b \\ e & f+c \end{bmatrix} \notin S \rightarrow$ S is not a subspace of R^{2x2}

3) Example (12) - Page 124 -

Is the vectors $1-x^2$, $x+2$, x^2 span P_3 ?

sol:

مجموعة كثير الحدود التي درجتها أقل من (n)

P_n : the set of all polynomials of degree less than (n).

P_3 : مجموعة كثير الحدود التي درجتها أقل من (3)

وهي في هذا السؤال ثلاثة اقترانات $x^2 / x+2 / 1-x^2$

نأخذ أي vector عشوائي في P_3 (تكون من الدرجة الثانية) : $(ax^2 + bx + c) \in P_3$

$$ax^2 + bx + c \stackrel{??}{=} \alpha_1(1-x^2) + \alpha_2(x+2) + \alpha_3(x^2).$$

هل تستطيع ايجاد قيم $\alpha_1, \alpha_2, \alpha_3$??

$$ax^2 + bx + c = \alpha_1 - \alpha_1 x^2 + \alpha_2 x + 2\alpha_2 + \alpha_3 x^2$$

$$ax^2 + bx + c = (\alpha_3 - \alpha_1) x^2 + \alpha_2 x + (\alpha_1 + 2\alpha_2)$$

نقارن المعاملات :

$$\left.\begin{aligned} \alpha_3 - \alpha_1 &= a \\ \alpha_2 &= b \\ \alpha_1 + 2\alpha_2 &= c \end{aligned}\right]$$

solve this system. $\Rightarrow \left[\begin{array}{ccc|c} -1 & 0 & 1 & a \\ 0 & 1 & 0 & b \\ 1 & 2 & 0 & c \end{array}\right] \Rightarrow$ if it is consistent $\Rightarrow$ so span ??

$$\left[\begin{array}{ccc|c} 1 & 0 & -1 & -a \\ 0 & 1 & 0 & b \\ 1 & 2 & 0 & c \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & -1 & -a \\ 0 & 1 & 0 & b \\ 0 & 2 & 1 & a+c \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & -1 & -a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & a+c-2b \end{array}\right]$$

$\alpha_3 = a + c - 2b$

$\alpha_2 = b$

$\alpha_1 - \alpha_3 = -a \rightarrow \alpha_1 - (a+c-2b) = -a \rightarrow \alpha_1 = -2b + c$

$\rightarrow$ the sys. consistent $\rightarrow x^2, x+2, 1-x^2$ span P_3.

واجب (H.W)

check independence of $1-x^2, x+2, x^2$

4 T/F   الإجابات (1-T, 2-F, 3-T, 4-T)

1-(T/F) If A is an $m\times n$ matrix, the dimension of the row space of A equals the dimension of the column space of A.

2-(T/F) Two row equivalent matrices have the same columnspace.

3-(T/F) Row space of (A) = column space of (A^T).

4-(T/F) column space of A = Row space of (A^T).

5- Question "14" Page 160

Let A be a 4x4 matrix with reduced row echelon form given by

$$U=\begin{bmatrix}1&0&2&1\\0&1&1&4\\0&0&0&0\\0&0&0&0\end{bmatrix}.\quad \text{If } a_1=\begin{bmatrix}-3\\5\\2\\1\end{bmatrix},\ a_2=\begin{bmatrix}4\\-3\\7\\-1\end{bmatrix}$$

determine a_3, a_4, A.

Sol:

note that: the matrices A, U have the same dependency relations

أي ان العلاقة بين الاعمدة في المصفوفة A هي نفس العلاقة بين الاعمدة في المصفوفة Row Echelon form المتعلقة بـ A ، والمصفوفة Reduced row Echelon form المتعلقة بـ A .

نلاحظ في هذا السؤال ان المصفوفة U هي Reduced Row Echelon . لذلك سهل ان نستنتج فوراً العلاقة بين الاعمدة ، حيث أن

$$a_3 = 2a_1 + a_2$$

وبالتالي يكون في المصفوفة A نفس العلاقة ← لذلك نستطيع ايجاد العمود الثالث في A

$$a_3 = 2a_1+a_2 = 2\begin{bmatrix}-3\\5\\2\\1\end{bmatrix}+\begin{bmatrix}4\\-3\\7\\1\end{bmatrix}=\begin{bmatrix}-2\\7\\11\\3\end{bmatrix}.$$

وايضاً من (U) نجد أن $a_4 = a_1 + 4a_2$ ← ومنه نجد العمود الرابع في A

$$a_4 = a_1+4a_2=\begin{bmatrix}-3\\5\\2\\1\end{bmatrix}+4\begin{bmatrix}4\\-3\\7\\-1\end{bmatrix}=\begin{bmatrix}13\\-7\\30\\-3\end{bmatrix}\Rightarrow A=\begin{bmatrix}-3&4&-2&13\\5&-3&7&-7\\2&7&11&30\\1&-1&3&-3\end{bmatrix}$$

ملاحظة: اذا كان يعطى في سؤال Row Echelon حول إلى Reduced Row لأنها اسهل في ايجاد العلاقة بين الأعمدة.

Complete ex (12) - Page 124.

check independence of. $1-x^2$, $x+2$, x^2.

sol:

$c_1 v_1 + c_2 v_2 + c_3 v_3 = 0$

$c_1(1-x^2) + c_2(x+2) + c_3(x^2) = 0$

$c_1 - c_1 x^2 + c_2 x + 2c_2 + c_3 x^2 = 0$

$(-c_1 + c_3)x^2 + c_2 x + (2c_2 + c_1) = 0$

ملاحظة، ما بنقدر نحل
هنا (نشتغل) على
ال det يعني dep.

$-c_1 + c_3 = 0$

$c_2 = 0$

$2c_2 + c_1 = 0$

حول ال sys. in matrices

$$\Rightarrow \left[\begin{array}{ccc|c} -1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 2 & 0 & 0 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 \end{array}\right]$$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \quad \left.\begin{array}{l} c_3 = 0 \\ c_2 = 0 \\ c_1 - c_3 = 0 \rightarrow c_1 = 0 \end{array}\right]$$

$c_1, c_2, c_3 = 0$

$\rightarrow$ the vectors are linearly independent.

Q can we write v_1 as a linear combination of v_2, v_3 ??

no, because v_1, v_2, v_3 are Linearly independent

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Q8/ Page 137 / (b) check independence of $\underset{v_1}{2}$, $\underset{v_2}{x^2}$, $\underset{v_3}{x}$, $\underset{v_4}{2x+3}$ in P_3

sol,

$c_1 v_1 + c_2 v_2 + c_3 v_3 + c_4 v_4 = 0$

$c_1(2) + c_2(x^2) + c_3(x) + c_4(2x+3) = 0$

$2c_1 + c_2 x^2 + c_3 x + 2c_4 x + 3c_4 = 0 \Rightarrow c_2 x^2 + (c_3 + 2c_4)x + 2c_1 + 3c_4 = 0$

$\Rightarrow c_2 = 0$

$c_3 + 2c_4 = 0$

$2c_1 + 3c_4 = 0$

$$\Longrightarrow \left[\begin{array}{cccc|c} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 \\ 2 & 0 & 0 & 3 & 0 \end{array}\right] \rightarrow \left[\begin{array}{cccc|c} 2 & 0 & 0 & 3 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 \end{array}\right]$$

$$\rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 0 & \frac{3}{2} & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 \end{array}\right]$$

$\rightarrow \boxed{c_4 = \alpha}$ (free) $\rightarrow c_3 + 2c_4 = 0 \rightarrow c_3 + 2\alpha = 0$ $\boxed{c_3 = -2\alpha}$

$\rightarrow \boxed{c_2 = 0}$

$\rightarrow c_1 + \frac{3}{2}c_4 = 0 \rightarrow c_1 + \frac{3}{2}\alpha = 0 \rightarrow \boxed{c_1 = -\frac{3}{2}\alpha}$

$\rightarrow v_1, v_2, v_3, v_4$ are Linearly dependent.

Q can we write v_1 as a Linear combination of v_2, v_3, v_4 ?

Yes, because v_1, v_2, v_3, v_4 are Linearly dependent.

so write it :$\rightarrow$

$c_1 v_1 + c_2 v_2 + c_3 v_3 + c_4 v_4 = 0$ Let $\boxed{\alpha = 1}$ $\rightarrow$ $c_4 = 1$, $c_3 = -2$, $c_2 = 0$, $c_1 = -\frac{3}{2}$

$-\frac{3}{2} v_1 + 0 v_2 + -2 v_3 + 1 v_4 = 0$

$\frac{-3}{2} v_1 - 2v_3 + v_4 = 0$

بما أنه المطلوب هو كتابة v_1 على شكل تركيب خطي من المتجهات، لازم نجعلها بالطرف لوحدها.

$\frac{-3}{2} v_1 = 2v_3 - v_4 \quad * -\frac{2}{3}$

$v_1 = -\frac{4}{3} v_3 + \frac{2}{3} v_4$

Questions : chapter "1" and "2"

Q $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, find det(A) ? Is A singular ?

solution : $\det(A) = \cos^2\theta + \sin^2\theta = 1 \rightarrow \det(A) = 1$

$\det(A) = 1 \neq 0 \rightarrow$ A is non-singular.

Q If A is 5X3 matrix, and $b = 2a_1 + 5a_2$.
Is this system consistent or inconsistent.

sol: $5X3 \rightarrow 3$ columns $\rightarrow [a_1 \; a_2 \; a_3]$ $\leftarrow$ column vectors of A.

we can write (b) as a linear combination of columns vectors of a.

$b = 2a_1 + 5a_2 + 0a_3 \rightarrow$ from them $\rightarrow$ ~~the~~ the system

$Ax = b$ is consistent.