Internal Combustion Engine 1

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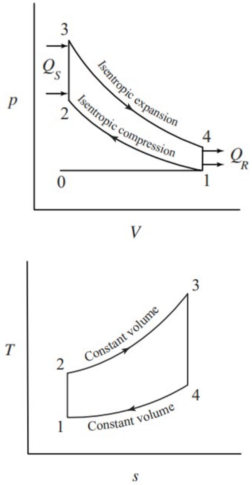
Otto Cycle

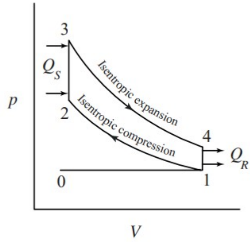
Introduction

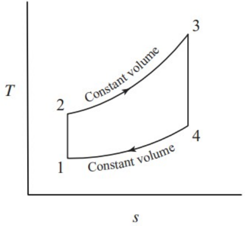
- The Carnot cycle is impractical due to high pressure and volume ratios with low mean effective pressure.
- Nicolaus Otto (1876) introduced a constant-volume heat addition cycle, forming the basis for modern spark-ignition engines.
- The cycle is depicted in p-V and T-s diagrams.



Introductions - cont.

- At full throttle:
  - Processes 0→1 and 1→0 are suction and exhaust.
  - Process 1→2 is isentropic compression.
  - Process 2→3 represents constant volume heat addition (spark-ignition).
  - Processes 3→4 is isentropic expansion.
  - Processes 4→1 represents constant volume heat rejection (Exhaust).





Isentropic Processes

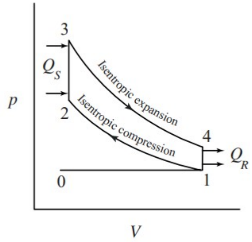
- **Isentropic Processes:**

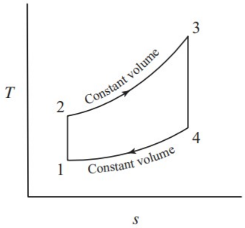
$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} \qquad \frac{T_3}{T_4} = \left(\frac{V_4}{V_3}\right)^{\gamma-1}$$

$$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1} \qquad T_3 = T_4\left(\frac{V_4}{V_3}\right)^{\gamma-1}$$

- **Since** $\frac{V_1}{V_2} = \frac{V_4}{V_3} = r$**:**

$$T_3 = T_4 \cdot r^{\gamma-1}, \qquad T_2 = T_1 \cdot r^{\gamma-1}$$



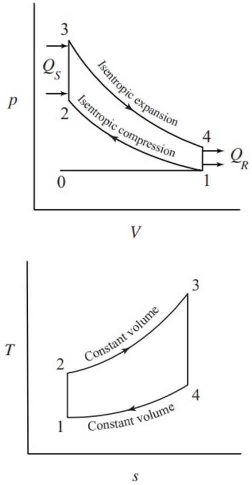


Constant volume heat addition and rejection

- **Heat Supplied and Rejected:**

$$Q_S = mC_v(T_3 - T_2)$$

$$Q_R = mC_v(T_4 - T_1)$$



Thermodynamic Efficiency Analysis

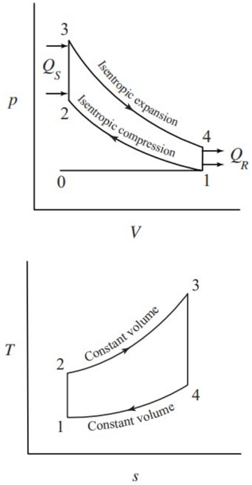
- **Thermal Efficiency Formula:**

$$\eta_{\text{Otto}} = \frac{Q_S - Q_R}{Q_S}$$

$$\eta_{\text{Otto}} = \frac{mC_v(T_3 - T_2) - mC_v(T_4 - T_1)}{mC_v(T_3 - T_2)}$$

- **Simplified Expression:**

$$\eta_{\text{Otto}} = 1 - \frac{T_4 - T_1}{T_3 - T_2}$$



Otto Cycle Efficiency

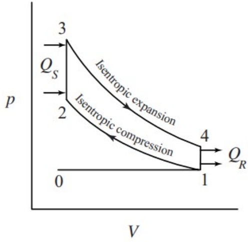
Substituting for T_3 and T_2 in terms of T_4 and T_1:

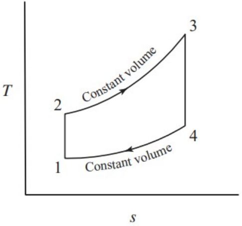
$$T_3 = T_4 \cdot r^{\gamma-1}, \quad T_2 = T_1 \cdot r^{\gamma-1}$$

$$\eta_{\text{Otto}} = 1 - \frac{T_4 - T_1}{T_4 \cdot r^{\gamma-1} - T_1 \cdot r^{\gamma-1}}$$

$$\eta_{\text{Otto}} = 1 - \frac{T_4 - T_1}{(r^{\gamma-1}) \cdot (T_4 - T_1)} = 1 - \frac{1}{r^{\gamma-1}}$$

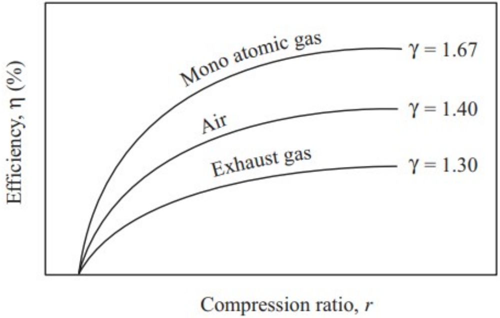
$$\eta_{\text{Otto}} = 1 - \frac{1}{r^{\gamma-1}}$$





Effect of γ and r

- Efficiency increases with higher compression ratio r.
- Efficiency is independent of heat supplied and pressure ratio.
- Higher γ values improve efficiency.
- The effect of γ and r on efficiency, shown in the following figure:



Net Work Output

- **Net Work Output for Otto Cycle:**

$$W = \frac{p_3 V_3 - p_4 V_4}{\gamma - 1} - \frac{p_2 V_2 - p_1 V_1}{\gamma - 1}$$

- **Pressure Ratios:**

$$\frac{p_2}{p_1} = \frac{p_3}{p_4} = r^{\gamma}$$

$$\frac{p_3}{p_2} = \frac{p_4}{p_1} = r_p$$

- **Volume Relations:**

$$V_1 = rV_2 \quad \text{and} \quad V_4 = rV_3$$

Work Output Simplification

Substituting in Work Output Formula:

$$W = \frac{p_1 V_1}{\gamma - 1}\left(\frac{p_3 V_3}{p_1 V_1} - \frac{p_4 V_4}{p_1 V_1} - \frac{p_2 V_2}{p_1 V_1} + 1\right)$$

$$= \frac{p_1 V_1}{\gamma - 1}\left(\frac{r_p r^{\gamma}}{r} - r_p - \frac{r^{\gamma}}{r} + 1\right)$$

$$= \frac{p_1 V_1}{\gamma - 1}\left(r_p - 1\right)\left(r^{(\gamma-1)} - 1\right)$$

Mean Effective Pressure

Mean Effective Pressure Formula:

$$MEP = \frac{\text{Work output}}{\text{Swept volume}}$$

$$\text{Swept volume} = V_1 - V_2 = V_2(r-1)$$

$$MEP = \frac{1}{\gamma - 1} \frac{p_1 V_1 (r_p - 1)(r^{(\gamma-1)} - 1)}{V_2(r-1)}$$

$$MEP = \frac{p_1 r (r_p - 1)(r^{(\gamma-1)} - 1)}{(\gamma - 1)(r-1)}$$

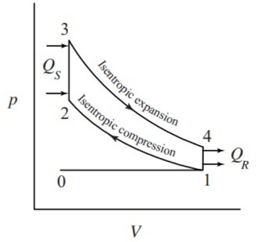
Implications of Pressure Ratio

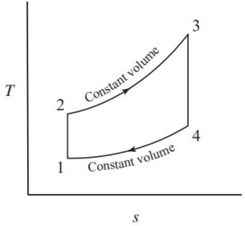
Work Output Proportional to Pressure Ratio:

$$\text{Work output} \propto p_r$$

Effect of Compression Ratio:

- As the compression ratio increases, both the mean effective pressure and thermal efficiency increase.
- The efficiency is independent of the heat supplied and pressure ratio.





Example 1

An engine working on Otto cycle has the following conditions : Pressure at the beginning of compression is 1 bar and pressure at the end of compression is 11 bar. Calculate the compression ratio and air-standard efficiency of the engine. Assume $\gamma = 1.4$.

$$r = \frac{V_1}{V_2} = \left(\frac{p_2}{p_1}\right)^{\left(\frac{1}{\gamma}\right)} = 11^{\frac{1}{1.4}} = \mathbf{5.54} \quad \stackrel{\textbf{Ans}}{\Longleftarrow}$$

$$\eta_{\text{air-std}} = 1 - \frac{1}{r^{\gamma-1}} = 1 - \left(\frac{1}{5.54}\right)^{0.4}$$

$$= 0.496 = \mathbf{49.6\%} \quad \stackrel{\textbf{Ans}}{\Longleftarrow}$$

Example 2

In an engine working on ideal Otto cycle the temperatures at the beginning and end of compression are 50 °C and 373 °C. Find the compression ratio and the air-standard efficiency of the engine.

$$r = \frac{V_1}{V_2} = \left(\frac{T_1}{T_2}\right)^{\frac{1}{\gamma-1}} = \left(\frac{646}{323}\right)^{\frac{1}{0.4}} = \mathbf{5.66} \qquad \overset{\text{Ans}}{\Longleftarrow}$$

$$\eta_{Otto} = 1 - \frac{1}{r^{\gamma-1}} = 1 - \frac{T_1}{T_2}$$

$$= 1 - \frac{323}{646} = 0.5 = \mathbf{50\%} \qquad \overset{\text{Ans}}{\Longleftarrow}$$

Example 3

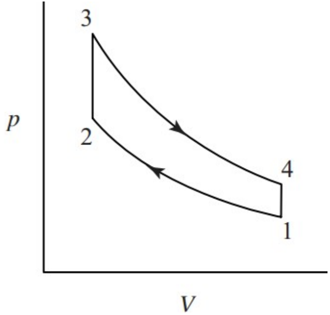
In an Otto cycle air at 17 °C and 1 bar is compressed adiabatically until the pressure is 15 bar. Heat is added at constant volume until the pressure rises to 40 bar. Calculate the air-standard efficiency, the compression ratio and the mean effective pressure for the cycle. Assume $C_v = 0.717$ kJ/kg K and $R = 8.314$ kJ/kmol K.

Consider the process 1 – 2

$$p_1 V_1^\gamma = p_2 V_2^\gamma$$

$$\frac{V_1}{V_2} = r = \left(\frac{p_2}{p_1}\right)^{\frac{1}{\gamma}}$$

$$= \left(\frac{15}{1}\right)^{\frac{1}{1.4}} = 6.91$$



Example 3 - cont.

$$\eta = 1 - \left(\frac{1}{r}\right)^{\gamma-1}$$

$$= 1 - \left(\frac{1}{6.91}\right)^{0.4} = 0.539 = \mathbf{53.9\%}$$

$$T_2 = \frac{p_2 V_2}{p_1 V_1} T_1 = \frac{15}{1} \times \frac{1}{6.91} \times 290 = 629.5 \text{ K}$$

Consider the process 2 – 3

$$T_3 = \frac{p_3 T_2}{p_2} = \frac{40}{15} \times 629.5 = 1678.7$$

$$\textit{Heat supplied} = C_v(T_3 - T_2)$$

$$= 0.717 \times (1678.7 - 629.5) = 752.3 \text{ kJ/kg}$$

$$\textit{Work done} = \eta \times q_s = 0.539 \times 752.3 = 405.5 \text{ kJ/kg}$$

Example 3 - cont.

$$p_m = \frac{\text{Work done}}{\text{Swept volume}}$$

$$v_1 = \frac{V_1}{m} = M\frac{RT_1}{p_1} = \frac{8314 \times 290}{29 \times 1 \times 10^5} = 0.8314 \text{ m}^3/\text{kg}$$

$$v_1 - v_2 = \frac{5.91}{6.91} \times 0.8314 = 0.711 \text{ m}^3/\text{kg}$$

$$p_m = \frac{405.5}{0.711} \times 10^3 = 5.70 \times 10^5 \text{ N/m}^2$$

$$= \mathbf{5.70 \ bar} \qquad \underline{\text{Ans}} \Longleftarrow$$

Example 4

A spark-ignition engine working on ideal Otto cycle has the compression ratio 6. The initial pressure and temperature of air are 1 bar and 37 °C. The maximum pressure in the cycle is 30 bar. For unit mass flow, calculate (i) p, V and T at various salient points of the cycle and (ii) the ratio of heat supplied to the heat rejected. Assume $\gamma = 1.4$ and $R = 8.314$ kJ/kmol K.

Example 4 - cont.

Consider point 1,

$$n = \frac{m}{M} = \frac{1}{29}$$

$$V_1 = \frac{nRT_1}{p_1} = \frac{1 \times 8314 \times 310}{29 \times 10^5} = \mathbf{0.889\ m^3} \quad \overset{\textbf{Ans}}{\Longleftarrow}$$

Consider point 2,

$$p_2 = p_1\ r^{\gamma} = 10^5 \times 6^{1.4} = 12.3 \times 10^5\ \mathrm{N/m^2} = \mathbf{12.3\ bar} \quad \overset{\textbf{Ans}}{\Longleftarrow}$$

Example 4 - cont.

$$V_2 = \frac{V_1}{6} = \frac{0.889}{6}$$

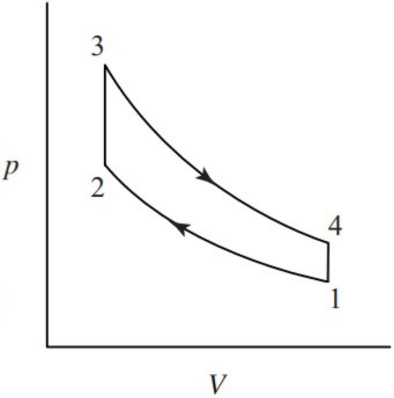
$$= \mathbf{0.148\ m^3} \quad \stackrel{\textbf{Ans}}{\Longleftarrow}$$

$$T_2 = \frac{p_2 V_2}{p_1 V_1} T_1$$

$$= \frac{12.3 \times 10^5 \times 0.148 \times 310}{1 \times 10^5 \times 0.889}$$

$$= 634.8\ \text{K}$$

$$= \mathbf{361.8^\circ\ C} \quad \stackrel{\textbf{Ans}}{\Longleftarrow}$$



Example 4 - cont.

Consider point 3,

$$V_3 = V_2 = \mathbf{0.148\ m^3} \quad \stackrel{\text{Ans}}{\Longleftarrow}$$

$$p_3 = 30 \times 10^5\ \text{N/m}^2 = \mathbf{30\ bar} \quad \stackrel{\text{Ans}}{\Longleftarrow}$$

$$\frac{p_3}{T_3} = \frac{p_2}{T_2}$$

$$T_3 = \frac{30 \times 10^5}{12.3 \times 10^5} \times 634.8$$

$$= 1548\ \text{K} = \mathbf{1275^\circ\ C} \quad \stackrel{\text{Ans}}{\Longleftarrow}$$

Example 4 - cont.

Consider point 4,

$$p_3 V_3^{\gamma} = p_4 V_4^{\gamma}$$

$$p_4 = p_3 \left(\frac{V_3}{V_4}\right)^{\gamma} = 30 \times 10^5 \left(\frac{1}{6}\right)^{1.4}$$

$$= 2.44 \times 10^5 \ \mathrm{N/m^2} = \mathbf{2.44\ bar} \quad \stackrel{\textbf{Ans}}{\Longleftarrow}$$

$$V_4 = V_1 = \mathbf{0.889\ m^3} \quad \stackrel{\textbf{Ans}}{\Longleftarrow}$$

$$T_4 = T_1 \frac{p_4}{p_1} = 310 \times \frac{2.44 \times 10^5}{1 \times 10^5}$$

$$= 756.4\ \mathrm{K} = \mathbf{483.4^{\circ}\ C} \quad \stackrel{\textbf{Ans}}{\Longleftarrow}$$

Example 4 - cont.

For unit mass,

$$C_v = \frac{R}{M(\gamma-1)} \frac{8.314}{29 \times 0.4} = 0.717 \text{ kJ/kg K}$$

$$\begin{aligned} \textit{Heat supplied} &= C_v(T_3 - T_2) \\ &= 0.717 \times (1548 - 635.5) = 654.3 \text{ kJ} \end{aligned}$$

$$\begin{aligned} \textit{Heat rejected} &= C_v(T_4 - T_1) \\ &= 0.717 \times (756.4 - 310) = 320.1 \text{ kJ} \end{aligned}$$

$$\frac{\textit{Heat supplied}}{\textit{Heat rejected}} = \frac{654.3}{320.1} = \mathbf{2.04} \qquad \stackrel{\textbf{Ans}}{\Longleftarrow}$$

End of Lecture 11