Palestine Technical University- Kadoorie (PTUK)

Mechanical Engineering Department

Summer Semester, 2023/2024

12210244: Dynamics

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Homework #: _____

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Due Date: 14^{th}, Aug. 2024

Date of Submission: _____

Exercise 1. The position coordinate of a particle which is confined to move along a straight line is given by $s = 2t^3 - 24t + 6\ m$, where s is measured in meters from a convenient origin and t is in seconds. Determine:

1. The time required for the particle to reach a velocity of 72 m/sec.
2. The acceleration of the particle when $v = 30$ m/sec.
3. The net displacement of the particle during the interval from $t = 1$ sec to $t = 4$ sec.

Ans.

$s = 2t^3 - 24t + 6$

$v = 6t^2 - 24$

$a = 12t$

1- $v = 72 = 6t^2 - 24$

$\Rightarrow 6t^2 = 96$

$t^2 = 16$

$t = 4\ sec$

2- $v = 30 = 6t^2 - 24$

$6t^2 = 54$

$t^2 = 9$

$t = 3$

$a|_{t=3} = (12)(3) = 36\ m/sec^2$

3- displacement

$= s(4) - s(1)$

$= 2(64) - 24(4) + 6 - (2 - 24 + 6)$

$= 54\ m$

Exercise 2. A particle travels along a straight line with an acceleration of $a = 10 - 0.2s\ m/sec^2$, where s is measured in meters. We need to determine the velocity of the particle when $s = 10\ m$ if $v = 5\ m/sec$ at $s = 0\ m$.

Ans.

$$a = 10 - 0.2s$$

$$a\,ds = v\,dv \Rightarrow \int_0^{10} 10 - 0.2s\,ds = \int_5^v v\,dv$$

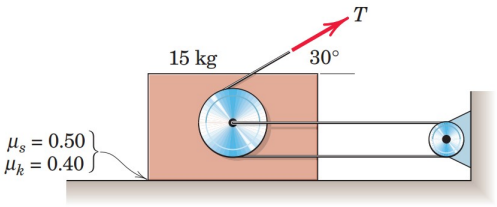
$$10s - 0.1s^2 \Big|_0^{10} = \frac{v^2}{2} \Big|_5^v$$

$$90 - 0 = \frac{v^2 - 25}{2} \Rightarrow v^2 = 205$$

$$v = 14.32$$

Exercise 3. The system is initially at rest with no slack in the cable, and the mass and friction of the pulleys are negligible. Determine the initial acceleration of the 15-kg block if:

1. $T = 20\ N$
2. $T = 30\ N$



Ans.

$\Sigma F_y = 0$

$N + T\sin 30 - mg = 0$

$N = mg - T\sin 30 = 147.15 - \frac{T}{2}$

$F_R = \mu_s N = 73.575 - \frac{T}{4}$ ↑ check

$2T + T\cos 30 = 2.87T$

$T = 20 \Rightarrow N = 137.15\ N$

$F_R = 68.575\ N$

$2.87T = 57.32$

$F_R > 2T + T\cos 30$

no motion

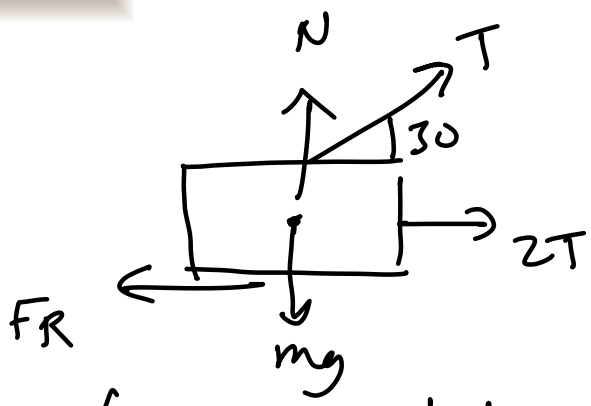
$T = 30 \Rightarrow N = 132.15\ N$

$F_R = 66.075\ N$

$2.87T = 86.1$

$F_R < 2T + T\cos 30$

There is motion



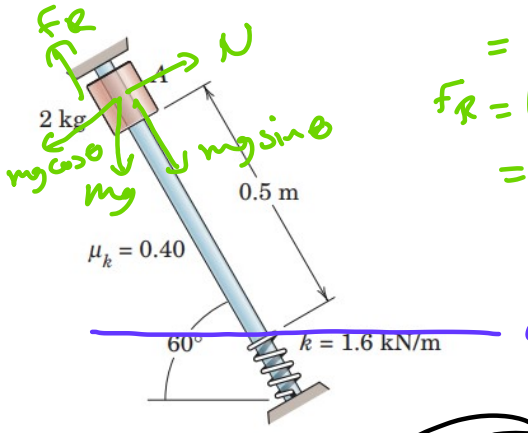
$2.87T - N\mu_k = ma$

$86.1 - (132.15)(0.4) = 15a$

$a = 2.216$

Exercise 4. The 2 kg collar is released from rest at A and slides down the inclined fixed rod in the vertical plane. The coefficient of kinetic friction is 0.40. Calculate:

1. the velocity v of the collar as it strikes the spring
2. the maximum deflection x of the spring



$N = mg\cos\theta$
$= 9.81$ N
$F_R = (9.81)(0.4)$
$= 3.924$ N

Point A

$T_A = 0$

$U_{Ag} = mg(0.5)\sin 60$
$= 8.49$

Point B

$U_{AE} = 0$

$T_B = ??$

$U_{Bg} = 0$, $U_{BE} = 0$

Ans.

$U_{A\to B} \Rightarrow -F_R(0.5)$

$U_{A\to B} = -1.962$

$\Rightarrow T_A + V_A + U_{A\to B} = T_B + V_B$

$8.49 - 1.962 = \frac{1}{2} m v_B^2$

$v_B = 2.56$ m/sec

Point C

$T_C = 0$

$U_{Cg} = -mg\,x\sin 60$

$U_{CE} = \frac{1}{2} k x^2$

$U_{A\to C} = -F_R(0.5 + x)$

$(A\to C)$

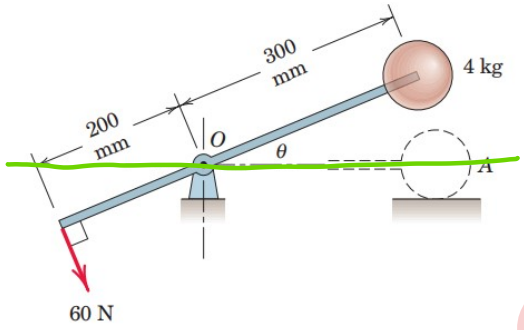
$8.49 - 3.924x - 1.962$
$= -17x + 800x^2$

$800x^2 - 13.076x - 6.528 = 0$

$x = 0.099$

$x = 99$ mm

Exercise 5. The 4 kg ball and the attached light rod rotate in the vertical plane about the fixed axis at O. If the assembly is released from rest at $\theta = 0°$ and moves under the action of the 60 N force, which is maintained normal to the rod, determine the velocity v of the ball as it approaches $\theta = 90°$. Treat the ball as a particle.



State 2

State 1

datum

Ans.

0.3

$T_2 = ??$

60 N

$V_2 = mg(0.3)$

$= 11.772$

60

$T_1 = 0$ rest

$V_1 = 0$ on datum

0.2 90

$S = r\theta$

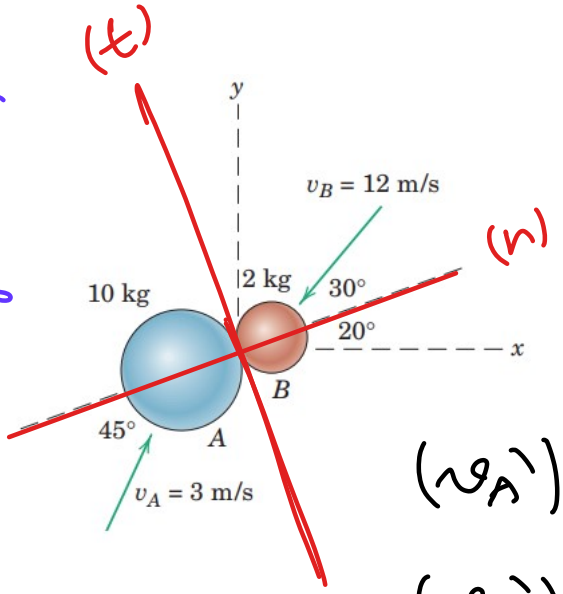
$U_{1\to 2} = 60 * \left((0.2)\frac{\pi}{2}\right) = 18.85$

$T_1 + V_1 + U_{1\to 2} = T_2 + V_2$

$0 + 18.85 = \frac{1}{2}(4)v^2 + 11.772$

$v = 1.88 \text{ m/sec}$

Exercise 6. Sphere A collides with sphere B as shown in the figure. If the coefficient of restitution is $e = 0.5$, determine the x- and y-components of the velocity of each sphere immediately after impact. Motion is confined to the x-y plane.



$(v_A)_t = 3 \sin 45 = 2.12$

$(v_B)_t = -12 \sin 30 = -6$

$(v_A)_n = 3 \cos 45 = 2.12$

$(v_A')_t = (v_A)_t = 2.12$

$(v_B')_t = (v_B)_t = -6$

Ans.

$(v_B)_n = -12 \cos 30 = -10.39$

$(10)(2.12) + (2)(-10.39) = 10(v_A')_n + 2(v_B')_n$

$10(v_A')_n + 2(v_B')_n = 0.42$ —— (1)

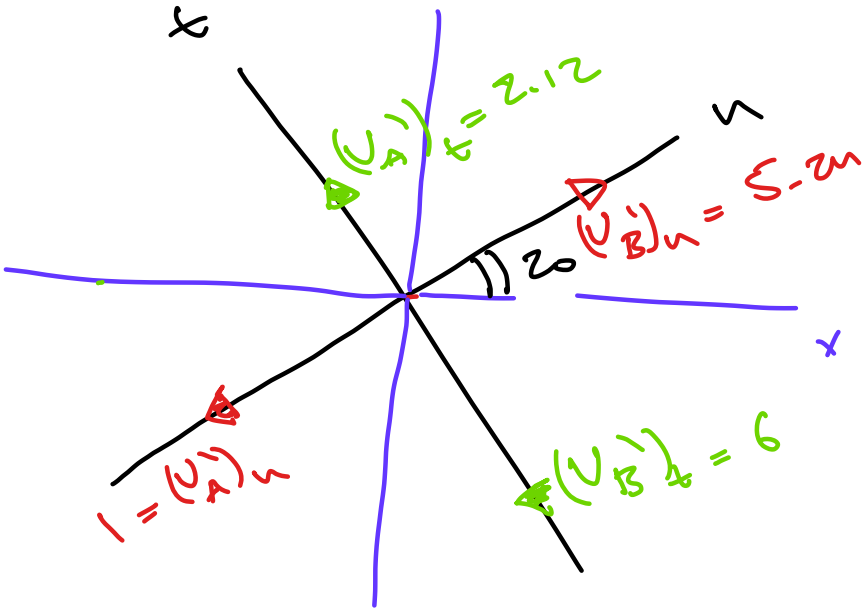
$e = 0.5 = \dfrac{(v_A')_n - (v_B')_n}{(v_B)_n - (v_A)_n} = \dfrac{(v_A')_n - (v_B')_n}{-10.39 - 2.12}$

$(v_A')_n - (v_B')_n = -6.255$ —— (2)

$(v_A')_n = -1$

$(v_B')_n = 5.24$

Ans.



$$(v_A')_x = (-1)(\cos 20) - (2.12)\sin 20$$

$$= -1.66$$

$$(v_A')_y = (-1)(\sin 20) + (2.12)(\cos 20)$$

$$= 1.65$$

$$(v_B')_x = (5.28)(\cos 20) + (6)(\sin 20)$$

$$= 7.01$$

$$(v_B')_y = (5.28)(\sin 20) - (6)(\cos 20)$$

$$= -3.83$$