Palestine Technical University- Kadoorie (PTUK)

Mechanical Engineering Department

Summer Semester, 2023/2024

12210244: Dynamics

Midterm Exam

Student Name: Key solution

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Due Date: 14th, Aug. 2024

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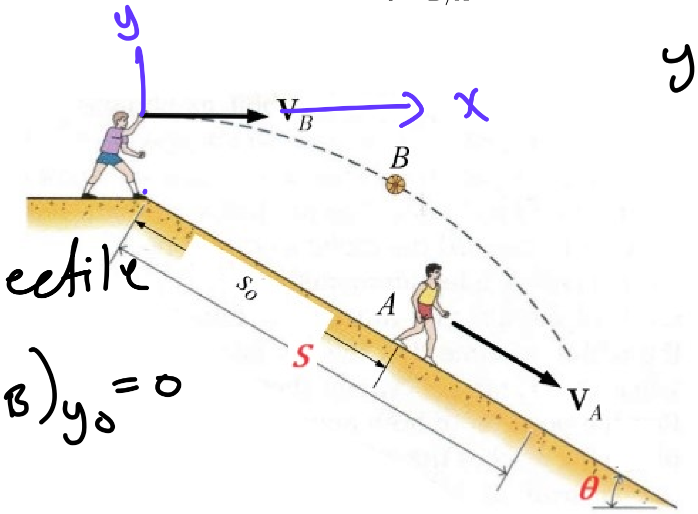
Two boys are playing catch on a hill inclined at an angle of $\theta = 30°$, as depicted in the figure below. The first boy throws a ball horizontally with an initial speed $\boldsymbol{v_B}$. The second boy, starting from a position $\boldsymbol{s_0} = \mathbf{20}$ m, begins to run at a constant speed of $\boldsymbol{v_A} = \mathbf{10}$ m/s. Determine the following:

1. The initial speed of the ball, $\boldsymbol{v_B}$, that will allow the second boy to catch it while running.
2. The distance $\boldsymbol{s}$ at which the second boy will catch the ball.
3. The relative velocity of the ball with respect to the second boy, $\boldsymbol{v}_{B/A}$, at the moment he catches it.
4. The magnitude and direction of the relative velocity $\boldsymbol{v}_{B/A}$.

$$y_B = \overset{\text{zero}}{\cancel{(y_B)_0}} + \overset{\text{zero}}{\cancel{(v_B)_{y_0}}}\, t - \frac{1}{2} g t^2$$

$$y_B = -\frac{g}{2} t^2$$



Ball is projectile

$(v_B)_{x_0} = ??$, $(v_B)_{y_0} = 0$

$x_0 = 0$, $y_0 = 0$

$x_B = (v_B)_{x_0} * t$, $y_B = 4.905\, t^2$

2nd boy moving in straight line with constant velocity

$(v_A)_x = v_A \cos\theta = 10 \cos 30 = 8.66$ m/sec

$(v_A)_y = v_A \sin\theta = 10 \sin 30 = 5$ m/sec

$x_A = (s_0 + v_A t) \cos\theta = (20 + 10\, t) \cos 30$

$x_A = 17.32 + 8.66\, t$

For the 2nd boy to catch the ball

$x_A = x_B \Rightarrow \boxed{(v_B)_{x_0}\, t = 17.32 + 8.66\, t}$ — ①

ans.

$$y_A = (s_0 + (U_A)_y t)\sin\theta$$

$$= -(20+5t)\sin 30 \quad = -10 - 5t$$

For the 2nd boy to catch the ball

$$y_A = B_B \Rightarrow -4.905t^2 = -10 - 5t \quad —— ②$$

$$4.905t^2 - 5t - 10 = 0 \Rightarrow t = 2.025$$

$$\boxed{Eq\text{-}1} \Rightarrow (v_B)_{x_0} t = 17.32 + 8.66\,t$$

A) $$(v_B)_{x_0} = \frac{17.32 + 8.66(2.025)}{2.025} = 17.21 \text{ m/sec}$$

B) $$S = S_0 + U_A t = 20 + (10)(2.025) = 40.25 \text{ m}$$

C) $$v_A = 8.66\hat{i} - 5\hat{j} \quad , \quad v_B = 17.21\hat{i} - (9.81)(2.025)\hat{j}$$
$$= 17.21\hat{i} - 19.86\hat{j}$$

$$v_{B/A} = v_B - v_A = 8.55\hat{i} - 14.86\hat{j}$$

D) $$|v_{B/A}| = 17.14 \text{ m/sec}$$

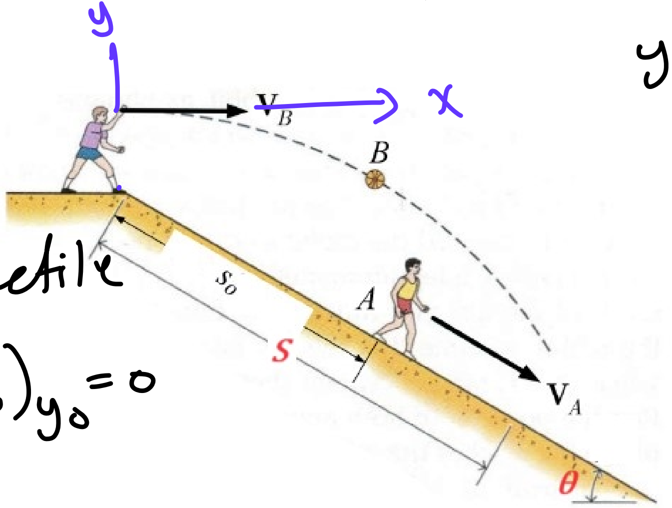
$$\theta = \tan^{-1}\frac{-14.86}{8.55} = -60.1^\circ$$

Two boys are playing catch on a hill inclined at an angle of $\theta = 45°$, as depicted in the figure below. The first boy throws a ball horizontally with an initial speed $\boldsymbol{v_B}$. The second boy, starting from a position $\boldsymbol{s_0} = \mathbf{30}$ m, begins to run at a constant speed of $\boldsymbol{v_A} = \mathbf{15}$ m/s. Determine the following:

1. The initial speed of the ball, $\boldsymbol{v_B}$, that will allow the second boy to catch it while running.
2. The distance $\boldsymbol{s}$ at which the second boy will catch the ball.
3. The relative velocity of the ball with respect to the second boy, $\boldsymbol{v}_{B/A}$, at the moment he catches it.
4. The magnitude and direction of the relative velocity $\boldsymbol{v}_{B/A}$.

$$y_B = \overset{zero}{\cancel{(y_B)_0}} + \overset{zero}{\cancel{(v_B)_{y_0}}}\, t - \frac{1}{2} g t^2$$

$$y_B = -\frac{g}{2} t^2$$



Ball is projectile

$(v_B)_{x_0} = ??$, $(v_B)_{y_0} = 0$

$x_o = 0$, $y_o = 0$

$x_B = (V_B)_{x_0} * t$, $y_B = 4.905\, t^2$

2nd boy moving in straight line with constant velocity

$(v_A)_x = v_A \cos\theta = 15 \cos 45 = 10.6$ m/sec

$(V_A)_y = U_A \sin\theta = 15 \sin 45 = 10.6$ m/sec

$x_A = (s_o + V_A t) \cos\theta = (30 + 15\, t) \cos 45$

$x_A = 21.21 + 10.6\, t$

For the 2nd boy to catch the ball

$x_A = x_B \Rightarrow \boxed{(v_B)_{x_0}\, t = 21.21 + 10.6 t}$ —— ①

ans.

$$y_A = -(s_0 + (v_A)_g\, t)\sin\theta$$

$$= -(30 + 15t)\sin 45 \quad = -21.2 - 10.6t$$

For the 2nd boy to catch the ball

$$y_A = B_B \Rightarrow -4.905t^2 = -21.2 - 10.6t \quad \text{———} \quad ②$$

$$4.905t^2 - 16.6t - 21.2 = 0 \Rightarrow t = 3.42$$

$$\boxed{Eq\text{-}1} \Rightarrow (v_B)_{x_0} t = 21.21 + 10.6\,t$$

A) $$(v_B)_{x_0} = \frac{21.21 + 10.6(3.42)}{3.42} = 16.8 \text{ m/sec}$$

B) $$S = S_0 + v_A t = 30 + (15)(3.42) = 81.38 \text{ m}$$

C) $$v_A = 10.6\,\hat{i} - 10.6\,j, \quad v_B = 16.8\,\hat{i} - (9.81)(3.42)\,j$$

$$= 16.8\,\hat{i} - 33.6\,\hat{j}$$

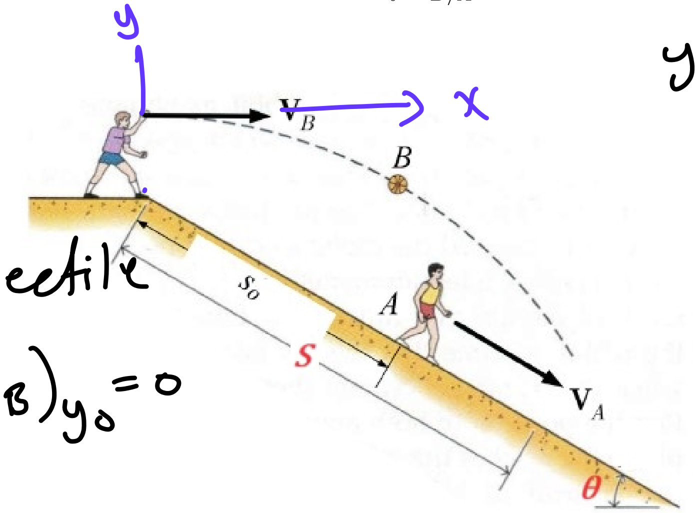
$$v_{B/A} = v_B - v_A = 6.2\,\hat{i} - 23\,\hat{j}$$

D) $$|v_{B/A}| = 23.82 \text{ m/sec}$$

$$\theta = \tan^{-1}\frac{-23}{6.2} = -74.9^\circ$$

Two boys are playing catch on a hill inclined at an angle of $\theta = 60°$, as depicted in the figure below. The first boy throws a ball horizontally with an initial speed $\boldsymbol{v_B}$. The second boy, starting from a position $\boldsymbol{s_0 = 10}$ m, begins to run at a constant speed of $\boldsymbol{v_A = 5}$ m/s. Determine the following:

1. The initial speed of the ball, $\boldsymbol{v_B}$, that will allow the second boy to catch it while running.
2. The distance $\boldsymbol{s}$ at which the second boy will catch the ball.
3. The relative velocity of the ball with respect to the second boy, $\boldsymbol{v}_{B/A}$, at the moment he catches it.
4. The magnitude and direction of the relative velocity $\boldsymbol{v}_{B/A}$.



$y_B = \cancel{(y_B)_0} + \cancel{(v_B)_{y_0}} t - \frac{1}{2} g t^2$

zero zero

$y_B = -\frac{g}{2} t^2$

Ball is projectile

$(v_B)_{x_0} = ??$, $(v_B)_{y_0} = 0$

$x_0 = 0$, $y_0 = 0$

$x_B = (v_B)_{x_0} * t$, $y_B = 4.905 t^2$

2nd boy moving in straight line with constant velocity

$(v_A)_x = v_A \cos\theta = 5 \cos 60 = 2.5$ m/sec

$(v_A)_y = v_A \sin\theta = 5 \sin 60 = 4.33$ m/sec

$x_A = (s_0 + v_A t) \cos\theta = (10 + 5 t) \cos 60$

$x_A = 5 + 2.5 t$

For the 2nd boy to catch the ball

$x_A = x_B \Rightarrow \boxed{(v_B)_{x_0} t = 5 + 2.5 t}$ — ①

ans.

$$y_A = (s_0 + (v_A)_y t)\sin\theta$$

$$= -(10 + 5t)\sin 60 = -8.66 - 4.33t$$

For the 2nd boy to catch the ball

$$y_A = B_B \Rightarrow -4.905t^2 = -8.66 - 4.33t \quad \text{—} \; ②$$

$$4.905t^2 - 4.33t - 8.66 = 0 \Rightarrow \quad t = 1.84 \text{ sec}$$

$\boxed{\text{Eq-1}}$

$$\Rightarrow (v_B)_{x_0} t = 5 + 2.5t$$

A) $$(v_B)_{x_0} = \frac{5 + 2.5(1.84)}{1.84} = 5.22 \text{ m/sec}$$

B) $$S = S_0 + U_A t = 10 + (5)(1.84) = 19.2 \text{ m}$$

C) $$v_A = 2.5\hat{i} - 4.33\hat{j}, \quad v_B = 5.22\hat{i} - (9.81)(1.84)\hat{j}$$

$$= 5.22\hat{i} - 18.1\hat{j}$$

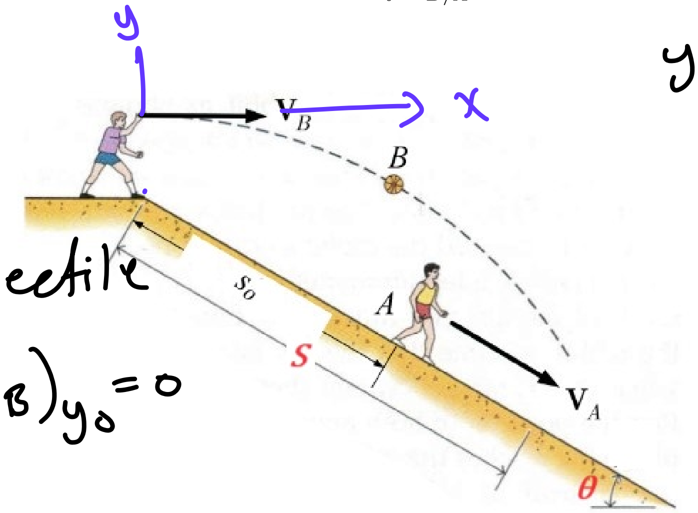
$$v_{B/A} = v_B - v_A = 2.72\hat{i} - 13.77\hat{j}$$

D) $$|v_{B/A}| = 14.03 \text{ m/sec}$$

$$\theta = \tan^{-1}\frac{-13.77}{2.72} = -78.8^\circ$$

Two boys are playing catch on a hill inclined at an angle of $\theta = 30°$, as depicted in the figure below. The first boy throws a ball horizontally with an initial speed $\boldsymbol{v_B}$. The second boy, starting from a position $\boldsymbol{s_0} = \mathbf{40}$ m, begins to run at a constant speed of $\boldsymbol{v_A} = \mathbf{20}$ m/s. Determine the following:

1. The initial speed of the ball, $\boldsymbol{v_B}$, that will allow the second boy to catch it while running.
2. The distance $\boldsymbol{s}$ at which the second boy will catch the ball.
3. The relative velocity of the ball with respect to the second boy, $\boldsymbol{v}_{B/A}$, at the moment he catches it.
4. The magnitude and direction of the relative velocity $\boldsymbol{v}_{B/A}$.



$$y_B = \overset{\text{zero}}{\cancel{(y_B)_0}} + \overset{\text{zero}}{\cancel{(v_B)_{y_0}}} t - \frac{1}{2} g t^2$$

$$y_B = -\frac{g}{2} t^2$$

Ball is projectile

$(v_B)_{x_0} = ??$, $(v_B)_{y_0} = 0$

$x_0 = 0$, $y_0 = 0$

$x_B = (v_B)_{x_0} * t$, $y_B = 4.905 t^2$

2nd boy moving in straight line with constant velocity

$(v_A)_x = v_A \cos\theta = 20 \cos 30 = 17.32$ m/sec

$(v_A)_y = v_A \sin\theta = 20 \sin 30 = 10$ m/sec

$x_A = (s_0 + v_A t) \cos\theta = (40 + 20 t) \cos 30$

$x_A = 34.64 + 17.32t$

For the 2nd boy to catch the ball

$x_A = x_B \Rightarrow \boxed{(v_B)_{x_0} t = 34.64 + 17.32t}$ — ①

ans.

$$y_A = (S_0 + (v_A)_y\, t)\sin\theta$$

$$= {}^-(40 + 20t)\sin 30 \quad = -20 - 10\,t$$

For the 2nd boy to catch the ball

$$y_A = B_B \Rightarrow -4.905\,t^2 = {}^-20 - 10\,t \quad \text{——} \quad ②$$

$$4.905\,t^2 - 10t - 20 = 0 \Rightarrow t = 3.28 \text{ sec}$$

$$\boxed{\text{Eq-1}} \Rightarrow (v_B)_{x_0} t = 34.64 + 13.32\,t$$

A) $(v_B)_{x_0} = \dfrac{34.64 + 17.32\,(3.28)}{3.28} = 27.88$ m/sec

B) $S = S_0 + v_A\,t = 40 + (20)(3.28) = 105.63$ m

C) $v_A = 17.32\,\hat{i} - 10\,\hat{j}$, $v_B = 27.88\,\hat{i} - (9.81)(3.28)\hat{j}$

$= 27.88\,\hat{i} - 32.17\,\hat{j}$

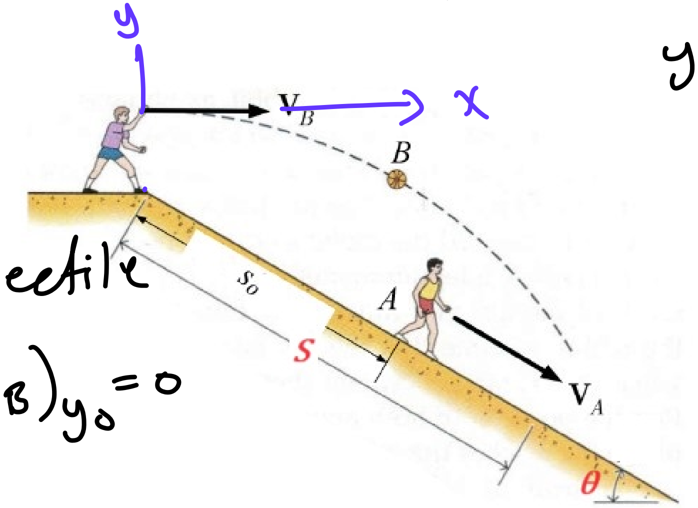
$$v_{B/A} = v_B - v_A = 10.56\,\hat{i} - 22.17\,\hat{j}$$

D) $|v_{B/A}| = 24.57$ m/sec

$$\theta = \tan^{-1}\frac{-22.17}{10.56} = -64.5^\circ$$

Two boys are playing catch on a hill inclined at an angle of $\theta = 45°$, as depicted in the figure below. The first boy throws a ball horizontally with an initial speed $\boldsymbol{v_B}$. The second boy, starting from a position $\boldsymbol{s_0} = \mathbf{50}$ m, begins to run at a constant speed of $\boldsymbol{v_A} = \mathbf{25}$ m/s. Determine the following:

1. The initial speed of the ball, $\boldsymbol{v_B}$, that will allow the second boy to catch it while running.
2. The distance $\boldsymbol{s}$ at which the second boy will catch the ball.
3. The relative velocity of the ball with respect to the second boy, $\boldsymbol{v}_{B/A}$, at the moment he catches it.
4. The magnitude and direction of the relative velocity $\boldsymbol{v}_{B/A}$.



$$y_B = \cancel{(y_B)_0} + \cancel{(v_B)_{y_0}} t - \frac{1}{2} g t^2$$

Zero Zero

$$y_B = -\frac{g}{2} t^2$$

Ball is projectile

$(v_B)_{x_0} = ??$, $(v_B)_{y_0} = 0$

$x_0 = 0$, $y_0 = 0$

$x_B = (v_B)_{x_0} * t$, $y_B = 4.905 t^2$

2nd boy moving in straight line with Constant velocity

$(v_A)_x = v_A \cos\theta = 25 \cos 45 = 17.68$ m/sec

$(v_A)_y = v_A \sin\theta = 25 \sin 45 = 17.68$ m/sec

$x_A = (s_0 + v_A t) \cos\theta = (50 + 25 t) \cos 45$

$x_A = 35.36 + 17.68 t$

For the 2nd boy to catch the ball

$x_A = x_B \Rightarrow \boxed{(v_B)_{x_0} t = 35.36 + 17.68 t}$ — ①

ans.

$$y_A = (s_0 + (v_A)_y t)\sin\theta$$

$$= -(50 + 25t)\sin 45 = -35.36 - 17.68t$$

For the 2nd boy to catch the ball

$y_A = B_B \Rightarrow -4.905t^2 = -35.36 - 17.68t$ — ②

$4.905t^2 - 17.68t - 35.36 = 0 \Rightarrow t = 5.04$ sec

$\boxed{Eq\text{-}1} \Rightarrow (v_B)_{x_0} t = 35.36 + 17.68t$

A) $(v_B)_{x_0} = \frac{35.36 + 17.68(5.04)}{5.04} = 24.7$ m/sec

B) $S = S_0 + v_A t = 50 + (25)(5.04) = 176$ m

C) $v_A = 17.68\hat{i} - 17.68\hat{j}$, $v_B = 24.7\hat{i} - (9.81)(5.04)j$

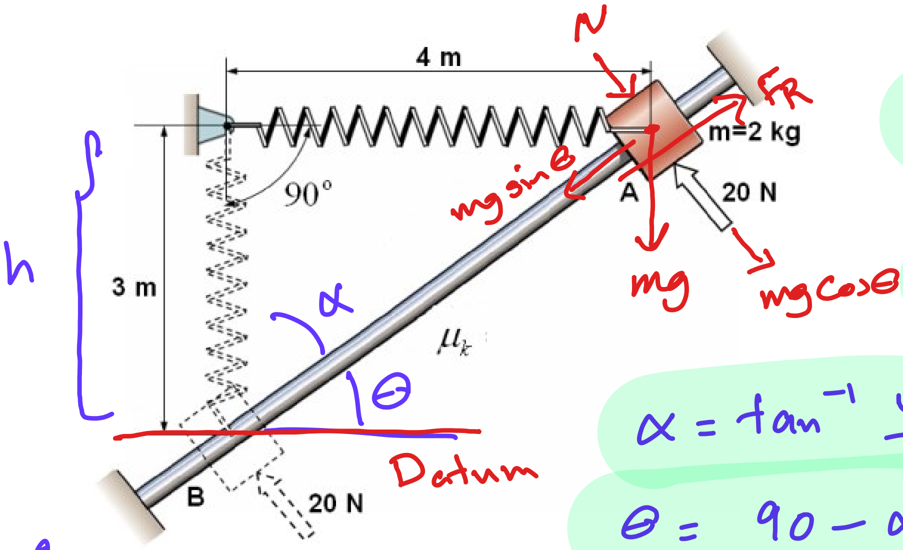
$= 24.7\hat{i} - 49.4\hat{j}$

$v_{B/A} = v_B - v_A = 7.02\hat{i} - 31.72\hat{j}$

D) $|v_{B/A}| = 32.49$ m/sec

$\theta = \tan^{-1}\frac{-31.72}{7.02} = -77.52$

The **2** kg collar is released from rest at A and slides on inclined rod as shown. The coefficient of kinetic friction is between the collar and the rod is **0.2**. The spring of stiffness, $k = 30\ N/m$ has an unstretched length of **2.5** m. Calculate the velocity of collar at position B.



$x_A = 4 - 2.5$
$= 1.5$

$x_B = 3 - 2.5$
$= 0.5$

$\alpha = \tan^{-1} \frac{4}{3} = 53.13$

$\theta = 90 - \alpha = 36.87$

$T_A = 0$

$V_{AE} = \frac{1}{2} K x_A^2$
$= \frac{1}{2}(30)(1.5)^2$
$= 33.75$

$V_{AG} = + mgh$
$= 2(9.81)(3)$
$= 58.86$

$-N + 20 - mg\cos\theta = 0$

$N = 20 - mg \cos 36.87$

$N = 4.3$

$\Rightarrow FR = \mu_k N$
$= (0.2)(4.3)$

$F_R = 0.86\ N$

$T_B = \frac{1}{2} m v_B^2 = v_B^2$

$V_{BE} = \frac{1}{2} K x_B^2$
$= \frac{1}{2}(30)(0.5)^2$
$= 3.75$

$V_{BG} = 0$

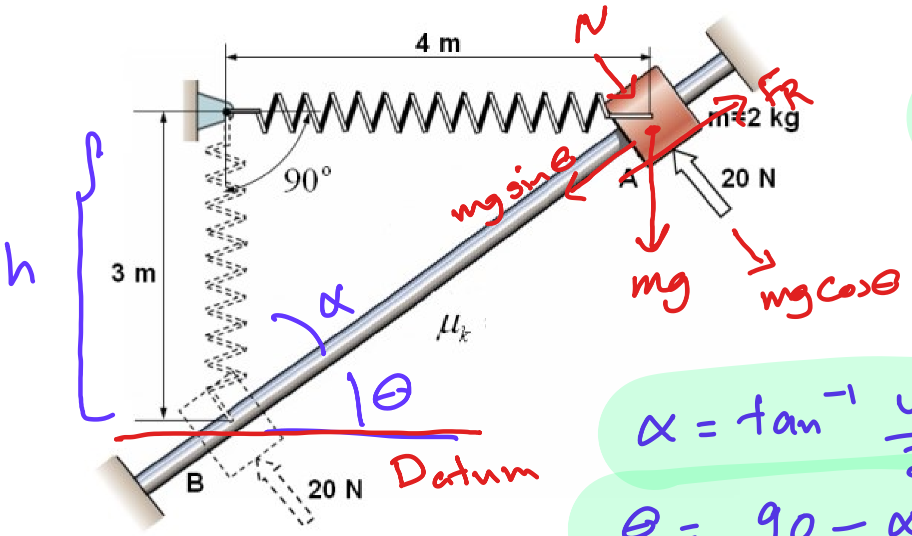
$U_{A \to B} = -FR\, d$
$= -(0.86)(5)$
$= 4.3$

$d = \sqrt{3^2 + 4^2}$
$= \sqrt{25}$
$= 5$

$0 + 33.75 + 58.86 - 4.3 = v_B^2 + 3.75 + 0 \Big\} \Rightarrow v_B = 9.19 \frac{m}{s}$

ans.

The **2** $\boldsymbol{kg}$ collar is released from rest at $\boldsymbol{A}$ and slides on inclined rod as shown. The coefficient of kinetic friction is between the collar and the rod is **0.3**. The spring of stiffness, $\boldsymbol{k = 40\ N/m}$ has an unstretched length of **1.5** $\boldsymbol{m}$. Calculate the velocity of collar at position $\boldsymbol{B}$.



$x_A = 4 - 1.5$
$= 2.5$

$x_B = 3 - 1.5$
$= 1.5$

$\alpha = \tan^{-1}\frac{4}{3} = 53.13$

$\theta = 90 - \alpha = 36.87$

$T_A = 0$

$V_{AE} = \frac{1}{2}Kx_A^2$
$= \frac{1}{2}(40)(2.5)^2$
$= 125$

$V_{AG} = +mgh$
$= 2(9.81)(3)$
$= 58.86$

$-N + 20 - mg\cos\theta = 0$

$N = 20 - mg\cos 36.87$

$N = 4.3$

$\Rightarrow FR = \mu_k N$
$= (0.3)(4.3)$

$F_R = 1.29\ N$

$T_B = \frac{1}{2}mv_B^2 = v_B^2$

$V_{BE} = \frac{1}{2}Kx_B^2$
$= \frac{1}{2}(40)(1.5)^2$
$= 45$

$V_{BG} = 0$

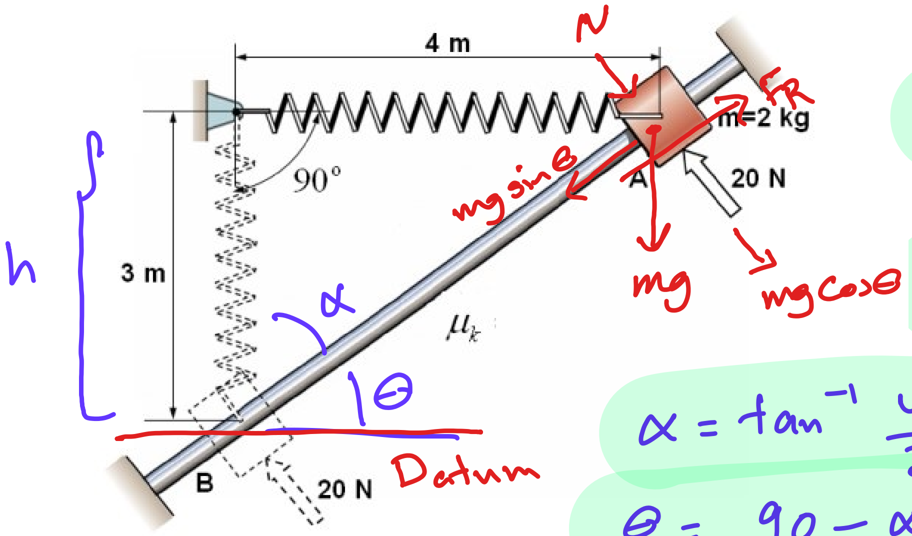
$U_{A\to B} = -F_R d$
$= (1.29)(5)$
$= -6.45$

$d = \sqrt{3^2 + 4^2}$
$= \sqrt{25}$
$= 5$

$0 + 125 + 58.86 - 6.45 = v_B^2 + 45 + 0 \Big\} \Rightarrow v_B = 11.5\ \frac{m}{s}$

$ans.$

The **2 *kg*** collar is released from rest at ***A*** and slides on inclined rod as shown. The coefficient of kinetic friction is between the collar and the rod is **0.25**. The spring of stiffness, ***k* = 35 *N/m*** has an unstretched length of **2 *m***. Calculate the velocity of collar at position ***B***.



$x_A = 4 - 2$
$= 2$

$x_B = 3 - 2$
$= 1$

$\alpha = \tan^{-1} \frac{4}{3} = 53.13$

$\theta = 90 - \alpha = 36.87$

$T_A = 0$

$V_{AE} = \frac{1}{2} K x_A^2$
$= \frac{1}{2}(35)(2)^2$
$= 70$

$V_{AG} = + mgh$
$= 2(9.81)(3)$
$= 58.86$

$-N + 20 - mg \cos\theta = 0$
$N = 20 - mg \cos 36.87$
$N = 4.3$

$\Rightarrow F_R = \mu_k N$
$= (0.25)(4.3)$
$F_R = 1.075 N$

$T_B = \frac{1}{2} m v_B^2 = v_B^2$

$V_{BE} = \frac{1}{2} K x_B^2$
$= \frac{1}{2}(35)(1)^2$
$= 17.5$

$V_{BG} = 0$

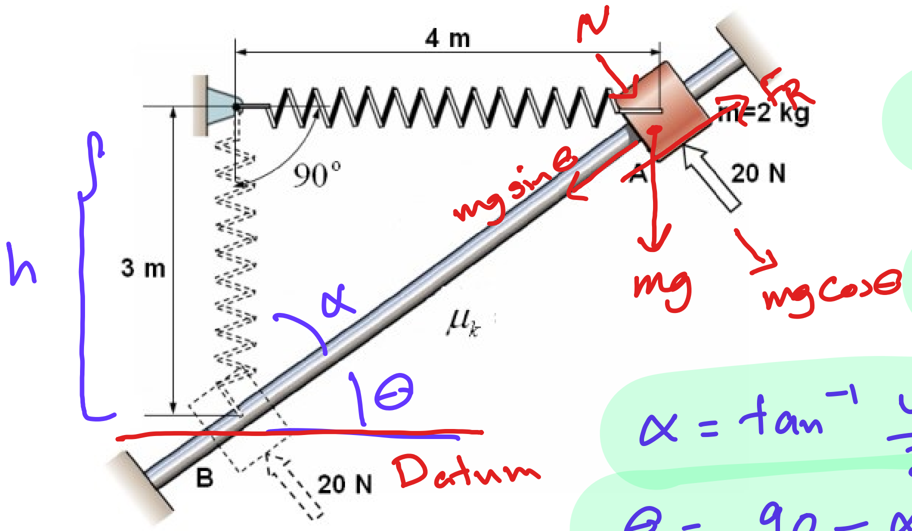
$U_{A\to B} = -F_R d$
$= (1.075)(5)$
$= -5.375$

$d = \sqrt{3^2 + 4^2}$
$= \sqrt{25}$
$= 5$

$0 + 70 + 58.86 - 5.375 = v_B^2 + 17.5 + 0 \Big\} \Rightarrow v_B = 10.29 \frac{m}{s}$

$ans.$

The **2 *kg*** collar is released from rest at ***A*** and slides on inclined rod as shown. The coefficient of kinetic friction is between the collar and the rod is **0.3**. The spring of stiffness, $\boldsymbol{k = 35\ N/m}$ has an unstretched length of **2.5 *m***. Calculate the velocity of collar at position ***B***.



$x_A = 4 - 2.5$
$= 1.5$

$x_B = 3 - 2.5$
$= 0.5$

$\alpha = \tan^{-1}\frac{4}{3} = 53.13$

$\theta = 90 - \alpha = 36.87$

$T_A = 0$

$V_{AE} = \frac{1}{2} K x_A^2$
$= \frac{1}{2}(35)(1.5)^2$
$= 39.375$

$V_{AG} = + mgh$
$= 2(9.81)(3)$
$= 58.86$

$-N + 20 - mg\cos\theta = 0$

$N = 20 - mg\cos 36.87$

$N = 4.3$

$\Rightarrow FR = \mu_k N$
$= (0.3)(4.3)$
$FR = 1.29\ N$

$T_B = \frac{1}{2} m v_B^2 = v_B^2$

$V_{BE} = \frac{1}{2} K x_B^2$
$= \frac{1}{2}(35)(0.5)^2$
$= 4.375$

$V_{BG} = 0$

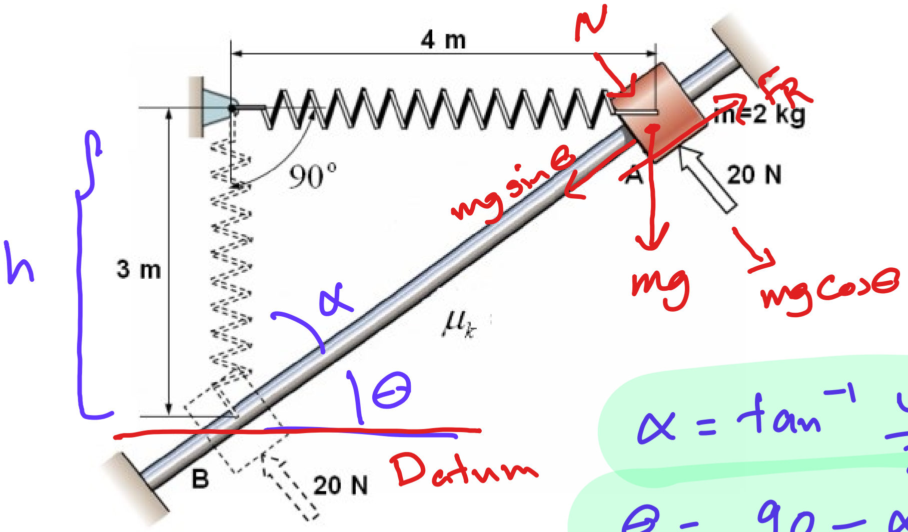
$U_{A\to B} = -FR\, d$
$= (1.29)(5)$
$= -6.45$

$d = \sqrt{3^2 + 4^2}$
$= \sqrt{25}$
$= 5$

$0 + 39.375 + 58.86 - 6.45 = v_B^2 + 4.375 + 0 \Big\} \Rightarrow v_B = 9.35\ \frac{m}{s}$

$ans.$

The **2 *kg*** collar is released from rest at ***A*** and slides on inclined rod as shown. The coefficient of kinetic friction is between the collar and the rod is **0.35**. The spring of stiffness, $k = 40\ N/m$ has an unstretched length of **3.5 *m***. Calculate the velocity of collar at position ***B***.



$X_A = 4 - 3.5$
$= 0.5$

$X_B = 3 - 3.5$
$= -0.5$

$\alpha = \tan^{-1}\frac{4}{3} = 53.13$

$\theta = 90 - \alpha = 36.87$

$T_A = 0$

$V_{AE} = \frac{1}{2}Kx_A^2$
$= \frac{1}{2}(40)(0.5)^2$
$= 5$

$V_{AG} = +mgh$
$= 2(9.81)(3)$
$= 58.86$

$-N + 20 - mg\cos\theta = 0$

$N = 20 - mg\cos 36.87$

$N = 4.3$

$\Rightarrow F_R = \mu_k N$
$= (0.35)(4.3)$

$F_R = 1.505\ N$

$T_B = \frac{1}{2}mv_B^2 = v_B^2$

$V_{BE} = \frac{1}{2}K x_B^2$
$= \frac{1}{2}(40)(0.5)^2$
$= 5$

$V_{BG} = 0$

$U_{A\to B} = -F_R d$
$= -(1.505)(5)$
$= -7.525$

$d = \sqrt{3^2 + 4^2}$
$= \sqrt{25}$
$= 5$

$0 + 5 + 58.86 - 7.525 = v_B^2 + 5 + 0 \Rightarrow v_B = 7.16\ \frac{m}{s}$

$ans.$