chapter:2 First order Differential Equations.

2.1 Linear first order D.E

first order differential eq. is of the form: $\frac{dy}{dt} = f(t,y)$.

aim we want to find a function $y = \Phi(t)$ that satisfies the D.E (solution of the D.E)

using several methods depending on the type of the equation.

1. linear equation (2.1)
2. Seprable equation (2.2)
3. Bernolli equation (2.4)
4. exact equation (2.6)

1 linear equation:

$$\boxed{\frac{dy}{dt} + P(t)\,y = g(t)}$$

, $P(t), g(t)$ are given functions of t., to solve it use integrating factor $[\mu(t)]$

ex $\frac{dy}{dt} + 2y = 3$

solve the Differential equation.

sol: first order $\to y'$ + linear.

$y' + 2y = 3 \longrightarrow g(t) = 3$

$\hookrightarrow P(t) = 2$

$\boxed{1}$ write the equation in general form $\to$ نقسم على معامل y' حتى يصير $= 1$

$y' + 2y = 3$

$(f.g)' = f'g + g'f$

$\boxed{2}$ find integrating factor

$\mu(t) = e^{\int P(t)\,dt}$

$= e^{\int 2\,dt} = e^{2t}$

$\boxed{3}$ multiply the eq in (step 1) by $\mu(t)$:-

$(y' + 2y = 3) * e^{2t}$

$y'e^{2t} + 2e^{2t}y = 3e^{2t}$

$\boxed{4}$ $(f.g)' \Rightarrow (y \cdot e^{2t})' = 3e^{2t}$

(5) Integrate.

$$(e^{2t}y)' = 3e^{2t} \Rightarrow \int (e^{2t}y)' = \int 3e^{2t}\,dt$$

$$e^{2t}y = 3\frac{e^{2t}}{2} + C$$

(6) write in terms of $y \Rightarrow y = ??$

$\div e^{2t}$

$$\Rightarrow y = \frac{3}{2}\frac{e^{2t}}{e^{2t}} + \frac{C}{e^{2t}} \Rightarrow y = \frac{3}{2} + \frac{C}{e^{2t}}$$

$$\rightarrow y = \frac{3}{2} + Ce^{-2t}.$$

(7) find "C" if we have initial condition.

ex solve the I.V.P :-

$$ty' + 2y = 4t^2, \quad y(1) = 2$$

$y(1)=2$ → initial condition

sol:

D.E + I.C $\Rightarrow$ I.V.P

(1) $\div t \Rightarrow \frac{ty'}{t} + \frac{2}{t}y = \frac{4t^2}{t}$

$$\boxed{\int \frac{1}{x}\,dx = \ln|x| + C}$$

$$\rightarrow y' + \frac{2}{t}y = 4t$$

(2) integrating factor

$$\mu(t) = e^{\int \frac{2}{t}\,dt} = e^{2\int \frac{1}{t}\,dt}$$

$$= e^{2\ln t} = e^{\ln t^2} = t^2$$

③ $\left(y' + \frac{2}{t} y = 4t\right) * t^2$

$y' t^2 + 2t y = 4t^3$

④ $(y t^2)' = 4t^3$

⑤ $\int (y t^2)' = \int 4t^3 \, dt$

$\Leftrightarrow \quad y t^2 = \frac{4t^4}{4} + C \Rightarrow y t^2 = t^4 + C$

⑥ $\div t^2 \Rightarrow \boxed{y = t^2 + \frac{c}{t^2}}$

⑦ Find c $\Longrightarrow$ $y(1) = 2$

$2 = (1)^2 + \frac{c}{(1)^2} \Rightarrow 2 = 1 + c \rightarrow c = 1$

$\rightarrow \boxed{y = t^2 + \frac{1}{t^2}}$

$\underline{Q}$ $t^3 y' + 2t^2 y = t^2 \sin t$, $y(\pi) = 0$

$\underline{sol}$: linear. eq $\Rightarrow$

① $\div t^3 \longrightarrow$ $\boxed{y' + \frac{2}{t} y = \frac{\sin t}{t}}$ $\longleftarrow$ الشكل العام

② $\mu(t) = e^{\int \frac{2}{t} dt} = e^{2\ln t} = e^{\ln t^2} = t^2$

③ $\left(y' + \frac{2}{t} y = \frac{\sin t}{t}\right) t^2$

$y' t^2 + 2t\, y = t \sin t$

④ $(y t^2)' = t \sin t$

⑤ $\int (y t^2)' = \int t \sin t \, dt$

$y t^2 = \text{use by Parts}$

$\int \underline{t}\, \underline{\sin t} \, dt =$

$-t\cos t + \sin t + C$

	$\int$
t (+)	$\sin t$
1 (−)	$-\cos t$
0 (+)	$-\sin t$

$y t^2 = -t\cos t + \sin t \; c$

⑥ $\div t^2 \Rightarrow y = -\frac{\cos t}{t} + \frac{\sin t}{t^2} + \frac{c}{t^2}$

find $c \Rightarrow y(\pi)=0$

$$y = -\frac{\cos t}{t} + \frac{\sin t}{t^2} + \frac{c}{t^2}$$

$$0 = \frac{-\cos\pi}{\pi} + \frac{\overset{=0}{\sin\pi}}{\pi^2} + \frac{c}{\pi^2}$$

$$0 = \frac{-(-1)}{\pi} + 0 + \frac{c}{\pi^2}$$

$$0 = \frac{1}{\pi} + \frac{c}{\pi^2}$$

$$0 = \frac{\pi}{\pi^2} + \frac{c}{\pi^2} \Rightarrow \frac{c+\pi}{\pi^2} = 0$$

$$\rightarrow c+\pi = 0 \rightarrow c = -\pi$$

$\Rightarrow$ solution:

$$y = -\frac{\cos t}{t} + \frac{\sin t}{t^2} + \frac{-\pi}{t^2}$$