2.2 Seperable equation

The general form of first order D.E

$$\frac{dy}{dx} = f(x,y) \Rightarrow$$

$$M(x,y) + N(x,y)\frac{dy}{dx} = 0$$

$$M(x) + N(y)\frac{dy}{dx} = 0$$

$$M(x)\,dx + N(y)\,dy = 0$$

seperable equation

$M(x)$: function of x only (تعتمد على x فقط)

$N(y)$: function of y only (تعتمد على y فقط)

to solve it $\Rightarrow$ integrate both sides.

ex Show that $\acute{y} = \frac{x^2}{1-y^2}$ is seperable, then solve it ?

sol: $\acute{y} = \frac{x^2}{1-y^2} \Rightarrow \frac{dy}{dx} = \frac{x^2}{1-y^2}$

$$(1-y^2)\,dy = x^2\,dx \Rightarrow \text{seperable equation}$$

$(1-y^2)$: تعتمد على y فقط (y متغير)

x^2: تعتمد على x فقط (x متغير)

to solve it integrate.

$$\int 1-y^2 \, dy = \int x^2 \, dx$$

$$y - \frac{y^3}{3} = \frac{x^3}{3} + C \quad , \; c : \text{constant}$$

→ implicit solution. — حل ضمني — implicitly

$y =$ x [illegible] ✗

⊛ explicit solution. — حل صريح

$y =$ x [illegible] ✓

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ex $\dfrac{dy}{dx} = \dfrac{3x^2 + 4x + 2}{2(y-1)} \; , \; y(0) = -1$

solve the I.V.P

Sol:

$$(3x^2 + 4x + 2)\, dx = 2(y-1)\, dy$$

fun. of x only $M(x)$ — fun. of y only $N(y)$

→ seperable equation.

$$\int 3x^2 + 4x + 2 \, dx = 2\int y - 1 \, dy.$$

$$\frac{3x^3}{3} + \frac{4x^2}{2} + 2x + C = 2\left(\frac{y^2}{2} - y\right)$$

$$x^3 + 2x^2 + 2x + C = y^2 - 2y$$

find (C) $\Rightarrow$ $y(0) = -1$

$x^3 + 2x^2 + 2x + C = y^2 - 2y$

$0 + 2(0) + 2(0) + C = (-1)^2 - 2(-1)$

$C = 1 + 2 \Rightarrow C = 3$

$\Rightarrow x^3 + 2x^2 + 2x + 3 = y^2 - 2y.$

$\Longrightarrow$ completing square on y.

$\pm \left(y \cdot 2 \cdot \frac{1}{2}\right)^2 \Rightarrow \left(-2 \cdot \frac{1}{2}\right)^2 = (-1)^2 = 1$

$y^2 - 2y \underline{\pm 1} = x^3 + 2x^2 + 2x + 3 + 1$

$(y-1)^2 = x^3 + 2x^2 + 2x + 4$

take $\sqrt{\ }$

$y - 1 = \pm \sqrt{x^3 + 2x^2 + 2x + 4}$

$y = 1 \pm \sqrt{x^3 + 2x^2 + 2x + 4}.$

use $y(0) = -1 \Rightarrow y(0) = 1 \pm \sqrt{0+0+0+4}$

$y(0) = 1 \pm 2$

$-1 = 1 - 2$

use $(-)$

$y = 1 - \sqrt{x^3 + 2x^2 + 2x + 4}.$

ex $\frac{dy}{dx} = \frac{y\cos x}{1+2y^2}$, $y(0)=1$

solve the D.E

sol:

$$(1+2y^2)\,dy = y\cos x\,dx$$

$\div y \Rightarrow$

$$\frac{1+2y^2}{y}\,dy = \cos x\,dx$$

$$\frac{\frac{1}{y}+2y}{\text{نهاية}}\,dy = \frac{\cos x}{\text{نهاية}}\,dx$$

$\rightarrow$ seperable eq.

$\Rightarrow$ integrate.

$$\int \frac{1}{y} + 2y\,dy = \int \cos x\,dx$$

$$\ln|y| + \frac{2y^2}{2} = \sin x + C.$$

$y(0)=1 \Rightarrow \ln|1| + (1)^2 = \sin 0 + C$

$0+1 = 0+C \Rightarrow C=1$

$\rightarrow \ln|y| + y^2 = \sin x + 1.$

implicitly.

ex $\frac{dy}{dx} = \frac{y-4x}{x-y}$

is it seperable eq ??

$$(x-y)\,dy = (y-4x)\,dx.$$

not seperable. ✗ ✗