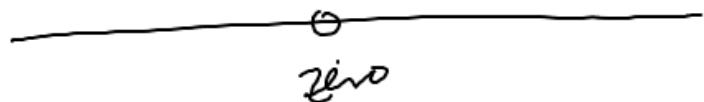


$$P(t) = \frac{2}{t} \rightarrow \text{cont. } \mathbb{R} - \{0\}$$

$$g(t) = 4t \rightarrow \text{cont. on } \mathbb{R} \quad \begin{matrix} \downarrow \\ \text{(exists)} \end{matrix}$$

(Polynomial).

$P(t), g(t)$ are both continuous.
on $\mathbb{R} - \{0\} = (-\infty, 0) \cup (0, \infty)$.



initial condition $y(1) = 2$ \xrightarrow{t}

\rightarrow so the I.V.P has a unique
solution on $(0, \infty)$.

$$\underline{t > 0}$$

Q3, Q6 \rightarrow Page (72)

Determine an interval in which the solution of the given initial value problem is certain to exist.

$$\dot{y} + (\tan t) y = \sin t, \quad y(\pi) = 0$$

$$\ln t \dot{y} + y = \cot t, \quad y(2) = 3.$$

$\sin t, \cos t \rightarrow$ contn on \mathbb{R} .

$\tan t = \frac{\sin t}{\cos t} \rightarrow$ contn on $(\mathbb{R} - \{\frac{\pi}{2}, \frac{3\pi}{2}, \dots\})$

Bernoulli equation

$$\dot{y} + p(t)y = g(t)y^n, \quad n \geq 2 \quad (\text{non-linear}).$$

$n=0 \rightarrow$ linear.

$n=1 \rightarrow$ linear.

Q28 $t^2 \dot{y} + 2ty - y^3 = 0$. solve the DE

Sol: this eq. is Bernoulli

1 write the eq. in general form.

$$t^2 \dot{y} + 2ty = y^3$$

$$\div t^2 \Rightarrow \dot{y} + \frac{2}{t}y = \frac{1}{t^2}y^3, \quad \text{Bernoulli } n=3$$

$\leftarrow p(t)$ $\rightarrow g(t)$

(2) divide the equation⁽¹⁾ by $y^n = y^3$
 $\div y^3 \rightarrow * y^{-3}$ [$* \bar{y}^n$]

$$\left(\bar{y} + \frac{2}{t} y = \frac{1}{t^2} y^3 \right) * y^{-3}$$

$$\bar{y} y^{-3} + \frac{2}{t} y^{-2} = \frac{1}{t^2}$$

(3) let $\omega = y^{1-n} \Rightarrow \omega = y^{1-3} \Rightarrow \omega = y^{-2}$

\rightarrow diff $\Rightarrow \bar{\omega} = -2 \bar{y}^{-3} * \bar{y}$

$$\Rightarrow \frac{\bar{\omega}}{-2} = \bar{y}^{-3} \bar{y}$$

(4) Substitute in eq (2)

$$\frac{\bar{\omega}}{-2} + \frac{2}{t} \omega = \frac{1}{t^2}$$

(eq in terms of $y \Rightarrow$ eq in terms ω)

(5) solve the eq. in (4).

$$\frac{\bar{\omega}}{-2} + \frac{2}{t} \omega = \frac{1}{t^2}$$

first order linear differential
equation \Rightarrow sec. 2.1

$$\frac{\dot{\omega}}{-2} + \frac{2}{t} \omega = \frac{1}{t^2} \quad \text{solve:}$$

$$\begin{aligned} & \times -2 \Rightarrow \dot{\omega} - \frac{4}{t} \omega = \frac{-2}{t^2} \rightarrow \\ & (\text{I} = \omega \text{ for } \text{I} = \omega) \end{aligned}$$

$$\mu(t) = e^{\int -\frac{4}{t} dt} = e^{-4 \ln t} = e^{\ln t^{-4}} = t^{-4}$$

$$\Rightarrow \left(\dot{\omega} - \frac{4}{t} \omega = \frac{-2}{t^2} \right) \times t^{-4} \rightarrow$$

$$\left(\dot{\omega} - 4t^{-1} \omega = -2t^{-2} \right) \times t^{-4}$$

$$\dot{\omega} t^{-4} - 4t^{-5} \omega = -2t^{-6}$$

$$\left(\omega t^{-4} \right)' = -2t^{-6} \rightarrow$$

$$\int \left(\omega t^{-4} \right)' = \int -2t^{-6} dt$$

$$\omega t^{-4} = \frac{-2t^{-5}}{-5} + C$$

$$\rightarrow \div t^{-4} \rightarrow \omega = \frac{2}{5} \frac{t^{-5}}{t^{-4}} + \frac{C}{t^{-4}}$$

$$\rightarrow \omega = \frac{2}{5} t^{-1} + C t^4$$

ⓧ write the solution in terms

$$\text{of } y \Rightarrow \omega = y^{-2}$$

$$\rightarrow y^{-2} = \frac{2}{5} \frac{1}{t} + C t^4 \Rightarrow y^{-2} = \frac{2 + 5Ct^5}{5t}$$

$$\frac{1}{y^2} = \frac{2 + 5ct}{5t} \quad \Rightarrow$$

$$\frac{y^2}{1} = \frac{5t}{2 + 5ct}$$

$$y = \sqrt{\frac{5t}{2 + 5ct}}$$

$$\frac{1}{t^2} =$$