2.3X

2.4 Differences between linear and nonlinear equations.

Existance and uniqueness solution

for linear equation (1st order)

$$\boxed{\frac{dy}{dt} + P(t)\,y = g(t)}\ , \quad y(t_0) = y_0$$

If $P(t)$, $g(t)$ are continuous on an open interval $I: \alpha < t < \beta$ containing the point $t = t_0$, then there exist a unique solution (function $y = \phi(t)$) satisfies the D.E. and I.C.

ex Find the interval on which the I.V.P $ty' + 2y = 4t^2$, $y(1) = 2$ has a unique solution?

Sol: نكتب المعادلة بالشكل العام ← معامل $y' = 1$

$$\div t \implies y' + \frac{2}{t}y = 4t$$

$P(t) = \dfrac{2}{t} \rightarrow$ cont. $R. - \{0\}$

$(-\infty,$ (inf)

$g(t) = 4t \rightarrow$ cont. on R

(Polynomial).

$P(t), g(t)$ are both continuous.

on $R - \{0\} = (-\infty, 0) \cup (0, \infty)$.

zero

initial condition $y(1) = 2$ → to

$\rightarrow$ so the I.V.P has a unique solution on $(0, \infty)$.

$t > 0$

Q3, Q6 → Page (72)

Determine an interval in which the solution of the given initial value problem is certain to exist.

$$y' + (\tan t)\, y = \sin t \quad , \; y(\pi) = 0$$

but $y' + y = \cot t \quad , \; y(2) = 3.$

$\sin t, \cos t \longrightarrow$ cont. on R.

$\tan t = \frac{\sin t}{\cos t} \rightarrow$ cont. on $\left(R - \{\frac{\pi}{2}, \frac{3\pi}{2}, \dots\}\right)$

Bernolli equation

$$\boxed{y' + p(t)\, y = g(t)\, y^n} \quad , \; n \geq 2$$

(non-linear).

$n = 0 \rightarrow$ linear.

$n = 1 \rightarrow$ linear.

Q28 $t^2 y' + 2ty - y^3 = 0.$ solve the D.E

Sol: this eq. is Bernolli

1 write the eq. in general form.

$$t^2 y' + 2ty = y^3 \qquad \cancel{\frac{2y}{t}}$$

$$\div t^2 \Rightarrow \boxed{y' + \underbrace{\frac{2}{t}}_{p(t)} y = \underbrace{\frac{1}{t^2}}_{g(t)} y^3} \quad , \text{Bernolli } n = 3$$

(2) divide the equation (1) by $y^n = y^3$ $[* y^{-n}]$

$\div y^3 \longrightarrow * y^{-3}$

$$\left(y' + \frac{2}{t} y = \frac{1}{t^2} y^3 \right) * y^{-3}$$

$$\boxed{y' y^{-3} + \frac{2}{t} y^{-2} = \frac{1}{t^2}}$$

(3) let $w = y^{1-n} \Rightarrow w = y^{1-3} \Rightarrow \boxed{w = y^{-2}}$

$\longrightarrow$ diff $\Rightarrow w' = -2 y^{-3} * y'$

$$\Rightarrow \boxed{\frac{w'}{-2} = y^{-3} y'}$$

(4) Substitute in eq (2)

$$\frac{w'}{-2} + \frac{2}{t} w = \frac{1}{t^2}$$

(eq in terms of $y \Rightarrow$ eq in terms w)

(5) solve the eq. in (4).

$$\frac{w'}{-2} + \frac{2}{t} w = \frac{1}{t^2}$$

first order linear differential equation $\Rightarrow$ sec. $\underline{\underline{2.1}}$

$$\frac{\acute{w}}{-2} + \frac{2}{t} w = \frac{1}{t^2} \qquad \text{solve :}$$

$* -2 \Longrightarrow \quad \acute{w} - \frac{4}{t} w = \frac{-2}{t^2} \longrightarrow$

$(1 = \acute{w}$ [illegible]

$$\mu(t) = e^{\int -\frac{4}{t} dt} = e^{-4 \ln t} = e^{\ln t^{-4}} = t^{-4}$$

$$\Rightarrow \left(\acute{w} - \frac{4}{t} w = \frac{-2}{t^2} \right) * t^{-4} \rightarrow$$

$$\left(\acute{w} - 4 t^{-1} w = -2 t^{-2} \right) * t^{-4}$$

$$\acute{w} t^{-4} - 4 t^{-5} w = -2 t^{-6}$$

$$\left(w \, t^{-4} \right)' = -2 t^{-6} \rightarrow$$

$$\int \left(w \, t^{-4} \right)' = \int -2 \, t^{-6} \, dt$$

$$w t^{-4} = \frac{-2 t^{-5}}{-5} + C \, .$$

$$\rightarrow \div t^{-4} \rightarrow w = \frac{2}{5} \frac{t^{-5}}{t^{-4}} + \frac{C}{t^{-4}}$$

$$\rightarrow w = \frac{2}{5} t^{-1} + C t^{4} \; . \; .$$

$\boxed{6}$ write the solution in terms of $y \Longrightarrow w = y^{-2}$

$$\Rightarrow y^{-2} = \frac{2}{5} \frac{1}{t} + C t^{4} \Rightarrow y^{-2} = \frac{2 + 5Ct^{5}}{5t}$$

$$\frac{1}{y^2} = \frac{2+5ct}{5t}$$ 2)

$$\frac{y^2}{1} = \frac{5t}{2+5ct}$$

$$y = \pm\sqrt{\frac{5t}{2+5ct}}$$

$$\frac{1}{t^2} =$$