Chapter 5

ELECTRIC FIELDS IN MATERIAL SPACE

The 12 Principles of character: (1) Honesty, (2) Understanding, (3) Compassion, (4) Appreciation, (5) Patience, (6) Discipline, (7) Fortitude, (8) Perseverance, (9) Humor, (10) Humility, (11) Generosity, (12) Respect.

—KATHRYN B. JOHNSON

5.1 INTRODUCTION

In the last chapter, we considered electrostatic fields in free space or a space that has no materials in it. Thus what we have developed so far under electrostatics may be regarded as the "vacuum" field theory. By the same token, what we shall develop in this chapter may be regarded as the theory of electric phenomena in material space. As will soon be evident, most of the formulas derived in Chapter 4 are still applicable, though some may require modification.

Just as electric fields can exist in free space, they can exist in material media. Materials are broadly classified in terms of their electrical properties as conductors and nonconductors. Nonconducting materials are usually referred to as *insulators* or *dielectrics.* A brief discussion of the electrical properties of materials in general will be given to provide a basis for understanding the concepts of conduction, electric current, and polarization. Further discussion will be on some properties of dielectric materials such as susceptibility, permittivity, linearity, isotropy, homogeneity, dielectric strength, and relaxation time. The concept of boundary conditions for electric fields existing in two different media will be introduced.

5.2 PROPERTIES OF MATERIALS

In a text of this kind, a discussion on electrical properties of materials may seem out of place. But questions such as why an electron does not leave a conductor surface, why a current-carrying wire remains uncharged, why materials behave differently in an electric field, and why waves travel with less speed in conductors than in dielectrics are easily answered by considering the electrical properties of materials. A thorough discussion on this subject is usually found in texts on physical electronics or electrical engineering. Here, a brief discussion will suffice to help us understand the mechanism by which materials influence an electric field.

In a broad sense, materials may be classified in terms of their *conductivity* σ, in mhos per meter (℧/m) or siemens per meter (S/m), as conductors and nonconductors, or technically as metals and insulators (or dielectrics). The conductivity of a material usually depends on temperature and frequency. A material with *high conductivity* ($\sigma \gg 1$) is referred to as a *metal* whereas one with *low conductivity* ($\sigma \ll 1$) is referred to as an *insulator.* A material whose conductivity lies somewhere between those of metals and insulators is called a *semiconductor.* The values of conductivity of some common materials as shown in Table B.1 in Appendix B. From this table, it is clear that materials such as copper and aluminum are metals, silicon and germanium are semiconductors, and glass and rubber are insulators.

The conductivity of metals generally increases with decrease in temperature. At temperatures near absolute zero ($T = 0°K$), some conductors exhibit infinite conductivity and are called *superconductors.* Lead and aluminum are typical examples of such metals. The conductivity of lead at 4°K is of the order of 10^{20} mhos/m. The interested reader is referred to the literature on superconductivity.[1]

We shall only be concerned with metals and insulators in this text. Microscopically, the major difference between a metal and an insulator lies in the amount of electrons available for conduction of current. Dielectric materials have few electrons available for conduction of current in contrast to metals, which have an abundance of free electrons. Further discussion on the presence of conductors and dielectrics in an electric field will be given in subsequent sections.

5.3 CONVECTION AND CONDUCTION CURRENTS

Electric voltage (or potential difference) and current are two fundamental quantities in electrical engineering. We considered potential in the last chapter. Before examining how electric field behaves in a conductor or dielectric, it is appropriate to consider electric current. Electric current is generally caused by the motion of electric charges.

The current (in amperes) through a given area is the electric charge passing through the area per unit time.

That is,

$$I = \frac{dQ}{dt} \tag{5.1}$$

Thus in a current of one ampere, charge is being transferred at a rate of one columb per second.

[1]The August 1989 issue of the *Proceedings of IEEE* was devoted to "Applications of Superconductivity."

We now introduce the concept of *current density* **J**. If current ΔI flows through a surface ΔS, the current density is

$$J_n = \frac{\Delta I}{\Delta S}$$

or

$$\Delta I = J_n \Delta S \tag{5.2}$$

assuming that the current density is perpendicular to the surface. If the current density is not normal to the surface,

$$\Delta I = \mathbf{J} \cdot \Delta \mathbf{S} \tag{5.3}$$

Thus, the total current flowing through a surface S is

$$I = \int_S \mathbf{J} \cdot d\mathbf{S} \tag{5.4}$$

Depending on how I is produced, there are different kinds of current densities: convection current density, conduction current density, and displacement current density. We will consider convection and conduction current densities here; displacement current density will be considered in Chapter 9. What we need to keep in mind is that eq. (5.4) applies to any kind of current density. Compared with the general definition of flux in eq. (3.13), eq. (5.4) shows that the current I through S is merely the flux of the current density **J**.

Convection current, as distinct from conduction current, does not involve conductors and consequently does not satisfy Ohm's law. It occurs when current flows through an insulating medium such as liquid, rarefied gas, or a vacuum. A beam of electrons in a vacuum tube, for example, is a convection current.

Consider a filament of Figure 5.1. If there is a flow of charge, of density ρ_v, at velocity $\mathbf{u} = a_y\mathbf{a}_y$, from eq. (5.1), the current through the filament is

$$\Delta I = \frac{\Delta Q}{\Delta t} = \rho_v \, \Delta S \frac{\Delta \ell}{\Delta t} = \rho_v \, \Delta S \, u_y \tag{5.5}$$

The **current density** at a given point is the current through a unit normal area at that point.

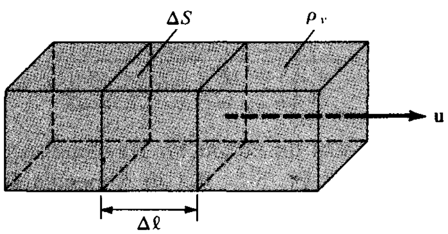


Figure 5.1 Current in a filament.

The y-directed current density J_y is given by

$$J_y = \frac{\Delta I}{\Delta S} = \rho_v u_y \tag{5.6}$$

Hence, in general

$$\boxed{\mathbf{J} = \rho_v \mathbf{u}} \tag{5.7}$$

The current I is the *convection current* and J is the *convection current density* in amperes/square meter (A/m^2).

Conduction current requires a conductor. A conductor is characterized by a large amount of free electrons that provide conduction current due an impressed electric field. When an electric field $\mathbf{E}$ is applied, the force on an electron with charge $-e$ is

$$\mathbf{F} = -e\mathbf{E} \tag{5.8}$$

Since the electron is not in free space, it will not be accelerated under the influence of the electric field. Rather, it suffers constant collision with the atomic lattice and drifts from one atom to another. If the electron with mass m is moving in an electric field $\mathbf{E}$ with an average drift velocity $\mathbf{u}$, according to Newton's law, the average change in momentum of the free electron must match the applied force. Thus,

$$\frac{m\mathbf{u}}{\tau} = -e\mathbf{E}$$

or

$$\mathbf{u} = -\frac{e\tau}{m}\mathbf{E} \tag{5.9}$$

where τ is the average time interval between collisions. This indicates that the drift velocity of the electron is directly proportional to the applied field. If there are n electrons per unit volume, the electronic charge density is given by

$$\rho_v = -ne \tag{5.10}$$

Thus the *conduction current density* is

$$\mathbf{J} = \rho_v \mathbf{u} = \frac{ne^2\tau}{m}\mathbf{E} = \sigma\mathbf{E}$$

or

$$\boxed{J = \sigma E} \tag{5.11}$$

where $\sigma = ne^2\tau/m$ is the conductivity of the conductor. As mentioned earlier, the values of σ for common materials are provided in Table B.1 in Appendix B. The relationship in eq. (5.11) is known as the point form of *Ohm's law*.

5.4 CONDUCTORS

A conductor has abundance of charge that is free to move. Consider an isolated conductor, such as shown in Figure 5.2(a). When an external electric field $\mathbf{E}_e$ is applied, the positive free charges are pushed along the same direction as the applied field, while the negative free charges move in the opposite direction. This charge migration takes place very quickly. The free charges do two things. First, they accumulate on the surface of the conductor and form an *induced surface charge.* Second, the induced charges set up an internal induced field $\mathbf{E}_i$, which cancels the externally applied field $\mathbf{E}_e$. The result is illustrated in Figure 5.2(b). This leads to an important property of a conductor:

A perfect conductor cannot contain an electrostatic field within it.

A conductor is called an *equipotential* body, implying that the potential is the same everywhere in the conductor. This is based on the fact that $\mathbf{E} = -\nabla V = 0$.

Another way of looking at this is to consider Ohm's law, $\mathbf{J} = \sigma\mathbf{E}$. To maintain a finite current density $\mathbf{J}$, in a perfect conductor ($\sigma \to \infty$), requires that the electric field inside the conductor must vanish. In other words, $\mathbf{E} \to 0$ because $\sigma \to \infty$ in a perfect conductor. If some charges are introduced in the interior of such a conductor, the charges will move to the surface and redistribute themselves quickly in such a manner that the field inside the conductor vanishes. According to Gauss's law, if $\mathbf{E} = 0$, the charge density ρ_v must be zero. We conclude again that a perfect conductor cannot contain an electrostatic field within it. Under static conditions,

$$\boxed{\mathbf{E} = 0, \qquad \rho_v = 0, \qquad V_{ab} = 0 \qquad \text{inside a conductor}} \tag{5.12}$$

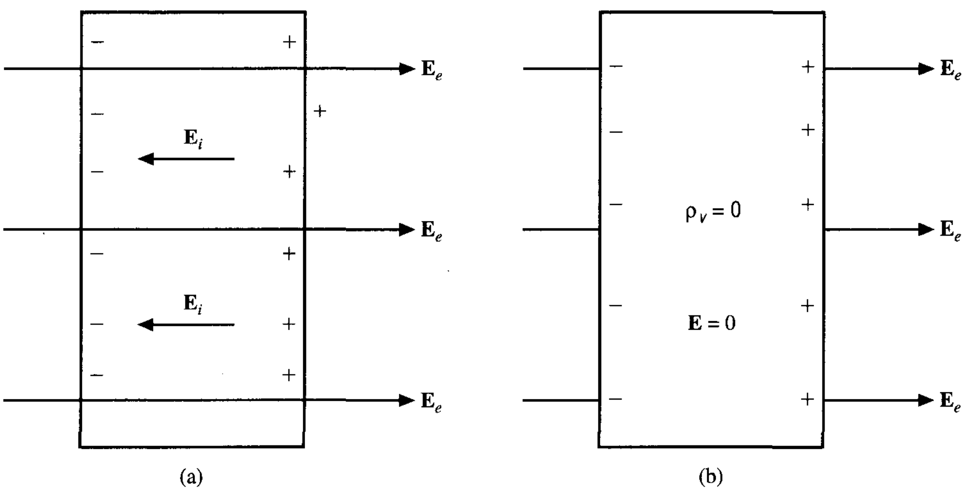


Figure 5.2 **(a)** An isolated conductor under the influence of an applied field; **(b)** a conductor has zero electric field under static conditions.

We now consider a conductor whose ends are maintained at a potential difference V, as shown in Figure 5.3. Note that in this case, $\mathbf{E} \neq 0$ inside the conductor, as in Figure 5.2. What is the difference? There is no static equilibrium in Figure 5.3 since the conductor is not isolated but wired to a source of electromotive force, which compels the free charges to move and prevents the eventual establishment of electrostatic equilibrium. Thus in the case of Figure 5.3, an electric field must exist inside the conductor to sustain the flow of current. As the electrons move, they encounter some damping forces called *resistance*. Based on Ohm's law in eq. (5.11), we will derive the resistance of the conducting material. Suppose the conductor has a *uniform* cross section S and is of length ℓ. The direction of the electric field $\mathbf{E}$ produced is the same as the direction of the flow of positive charges or current I. This direction is opposite to the direction of the flow of electrons. The electric field applied is uniform and its magnitude is given by

$$E = \frac{V}{\ell} \tag{5.13}$$

Since the conductor has a uniform cross section,

$$J = \frac{I}{S} \tag{5.14}$$

Substituting eqs. (5.11) and (5.13) into eq. (5.14) gives

$$\frac{I}{S} = \sigma E = \frac{\sigma V}{\ell} \tag{5.15}$$

Hence

$$R = \frac{V}{I} = \frac{\ell}{\sigma S}$$

or

$$\boxed{R = \frac{\rho_c \ell}{S}} \tag{5.16}$$

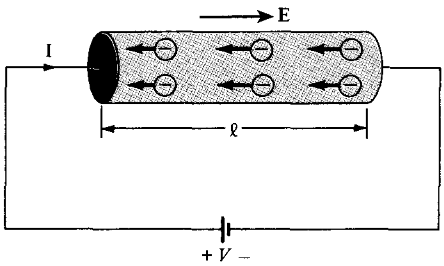


Figure 5.3 A conductor of uniform cross section under an applied **E** field.

where $\rho_c = 1/\sigma$ is the *resistivity* of the material. Equation 5.16 is useful in determining the resistance of any conductor of uniform cross section. If the cross section of the conductor is not uniform, eq. (5.16) is not applicable. However, the basic definition of resistance R as the ratio of the potential difference V between the two ends of the conductor to the current I through the conductor still applies. Therefore, applying eqs. (4.60) and (5.4) gives the resistance of a conductor of nonuniform cross section; that is,

$$R = \frac{V}{I} = \frac{\int \mathbf{E} \cdot d\mathbf{l}}{\int \sigma \mathbf{E} \cdot d\mathbf{S}} \tag{5.17}$$

Note that the negative sign before $V = -\int \mathbf{E} \cdot d\mathbf{l}$ is dropped in eq. (5.17) because $\int \mathbf{E} \cdot d\mathbf{l} < 0$ if $I > 0$. Equation (5.17) will not be utilized until we get to Section 6.5.

Power P (in watts) is defined as the rate of change of energy W (in joules) or force times velocity. Hence,

$$\int \rho_v \, dv \, \mathbf{E} \cdot \mathbf{u} = \int \mathbf{E} \cdot \rho_v \mathbf{u} \, dv$$

or

$$P = \int \mathbf{E} \cdot \mathbf{J} \, dv \tag{5.18}$$

which is known as *Joule's law.* The power density w_P (in watts/m^3) is given by the integrand in eq. (5.18); that is,

$$w_P = \frac{dP}{dv} = \mathbf{E} \cdot \mathbf{J} = \sigma \, |\mathbf{E}|^2 \tag{5.19}$$

For a conductor with uniform cross section, $dv = dS\, dl$, so eq. (5.18) becomes

$$P = \int_L E \, dl \int_S J \, dS = VI$$

or

$$P = I^2 R \tag{5.20}$$

which is the more common form of Joule's law in electric circuit theory.

EXAMPLE 5.1

If $\mathbf{J} = \frac{1}{r^3}(2 \cos \theta \, \mathbf{a}_r + \sin \theta \, \mathbf{a}_\theta)$ A/m^2, calculate the current passing through

(a) A hemispherical shell of radius 20 cm

(b) A spherical shell of radius 10 cm

Solution:

$I = \int \mathbf{J} \cdot d\mathbf{S}$, where $d\mathbf{S} = r^2 \sin\theta\, d\phi\, d\theta\, \mathbf{a}_r$ in this case.

(a) $$I = \int_{\theta=0}^{\pi/2} \int_{\phi=0}^{2\pi} \frac{1}{r^3} 2\cos\theta\, r^2 \sin\theta\, d\phi\, d\theta \Bigg|_{r=0.2}$$

$$= \frac{2}{r} 2\pi \int_{\theta=0}^{\pi/2} \sin\theta\, d(\sin\theta) \Bigg|_{r=0.2}$$

$$= \frac{4\pi}{0.2} \frac{\sin^2\theta}{2} \Bigg|_0^{\pi/2} = 10\pi = 31.4\ A$$

(b) The only difference here is that we have $0 \leq \theta \leq \pi$ instead of $0 \leq \theta \leq \pi/2$ and $r = 0.1$. Hence,

$$I = \frac{4\pi}{0.1} \frac{\sin^2\theta}{2} \Bigg|_0^{\pi} = 0$$

Alternatively, for this case

$$I = \oint \mathbf{J} \cdot d\mathbf{S} = \int \nabla \cdot \mathbf{J}\, dv = 0$$

since $\nabla \cdot \mathbf{J} = 0$.

PRACTICE EXERCISE 5.1

For the current density $\mathbf{J} = 10z \sin^2\phi\, \mathbf{a}_\rho$ A/m^2, find the current through the cylindrical surface $\rho = 2$, $1 \leq z \leq 5$ m.

Answer: 754 A.

EXAMPLE 5.2

A typical example of convective charge transport is found in the Van de Graaff generator where charge is transported on a moving belt from the base to the dome as shown in Figure 5.4. If a surface charge density 10^{-7} C/m^2 is transported at a velocity of 2 m/s, calculate the charge collected in 5 s. Take the width of the belt as 10 cm.

Solution:

If ρ_S = surface charge density, u = speed of the belt, and w = width of the belt, the current on the dome is

$$I = \rho_S u w$$

The total charge collected in $t = 5$ s is

$$Q = It = \rho_S uwt = 10^{-7} \times 2 \times 0.1 \times 5$$
$$= 100 \text{ nC}$$

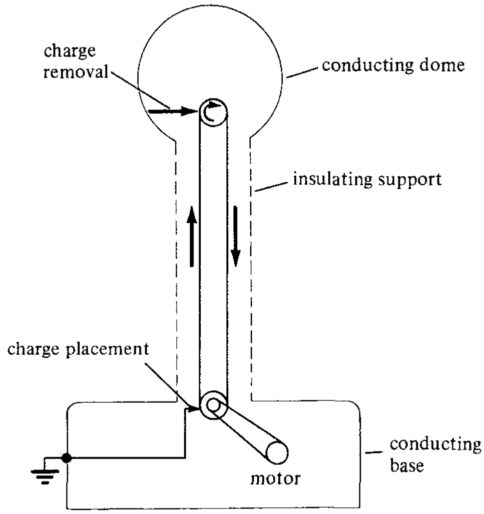


Figure 5.4 Van de Graaff generator, for Example 5.2.

PRACTICE EXERCISE 5.2

In a Van de Graaff generator, $w = 0.1$ m, $u = 10$ m/s, and the leakage paths have resistance $10^{14}\ \Omega$. If the belt carries charge $0.5\ \mu\text{C/m}^2$, find the potential difference between the dome and the base.

Answer: 50 MV.

EXAMPLE 5.3

A wire of diameter 1 mm and conductivity 5×10^7 S/m has 10^{29} free electrons/m^3 when an electric field of 10 mV/m is applied. Determine

(a) The charge density of free electrons

(b) The current density

(c) The current in the wire

(d) The drift velocity of the electrons. Take the electronic charge as $e = -1.6 \times 10^{-19}$ C.

Solution:

(In this particular problem, convection and conduction currents are the same.)

(a) $\rho_v = ne = (10^{29})(-1.6 \times 10^{-19}) = -1.6 \times 10^{10}\ \text{C/m}^3$

(b) $J = \sigma E = (5 \times 10^7)(10 \times 10^{-3}) = 500\ \text{kA/m}^2$

(c) $I = JS = (5 \times 10^5)\left(\dfrac{\pi d^2}{4}\right) = \dfrac{5\pi}{4} \cdot 10^{-6} \cdot 10^5 = 0.393$ A

(d) Since $J = \rho_v u$, $u = \dfrac{J}{\rho_v} = \dfrac{5 \times 10^5}{1.6 \times 10^{10}} = 3.125 \times 10^{-5}$ m/s.

PRACTICE EXERCISE 5.3

The free charge density in copper is 1.81×10^{10} C/m^3. For a current density of 8×10^6 A/m^2, find the electric field intensity and the drift velocity.

Answer: 0.138 V/m, 4.42×10^{-4} m/s.

EXAMPLE 5.4

A lead ($\sigma = 5 \times 10^6$ S/m) bar of square cross section has a hole bored along its length of 4 m so that its cross section becomes that of Figure 5.5. Find the resistance between the square ends.

Solution:

Since the cross section of the bar is uniform, we may apply eq. (5.16); that is,

$$R = \frac{\ell}{\sigma S}$$

where $S = d^2 - \pi r^2 = 3^2 - \pi\left(\frac{1}{2}\right)^2 = 9 - \frac{\pi}{4}$ cm^2.

Hence,

$$R = \frac{4}{5 \times 10^6 (9 - \pi/4) \times 10^{-4}} = 974\ \mu\Omega$$

PRACTICE EXERCISE 5.4

If the hole in the lead bar of Example 5.4 is completely filled with copper ($\sigma = 5.8 \times 10^6$ mhos/m), determine the resistance of the composite bar.

Answer: 876.7 $\mu\Omega$

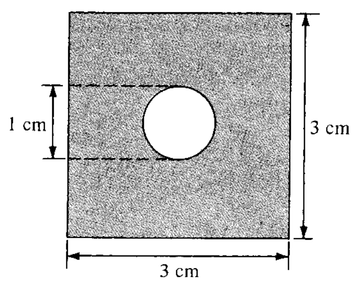


Figure 5.5 Cross section of the lead bar of Example 5.4.

5.5 POLARIZATION IN DIELECTRICS

In Section 5.2, we noticed that the main difference between a conductor and a dielectric lies in the availability of free electrons in the atomic outermost shells to conduct current. Although the charges in a dielectric are not able to move about freely, they are bound by finite forces and we may certainly expect a displacement when an external force is applied.

To understand the macroscopic effect of an electric field on a dielectric, consider an atom of the dielectric as consisting of a negative charge $-Q$ (electron cloud) and a positive charge $+Q$ (nucleus) as in Figure 5.6(a). A similar picture can be adopted for a dielectric molecule; we can treat the nuclei in molecules as point charges and the electronic structure as a single cloud of negative charge. Since we have equal amounts of positive and negative charge, the whole atom or molecule is electrically neutral. When an electric field $\mathbf{E}$ is applied, the positive charge is displaced from its equilibrium position in the direction of $\mathbf{E}$ by the force $\mathbf{F}_+ = Q\mathbf{E}$ while the negative charge is displaced in the opposite direction by the force $\mathbf{F}_- = Q\mathbf{E}$. A dipole results from the displacement of the charges and the dielectric is said to be *polarized.* In the polarized state, the electron cloud is distorted by the applied electric field $\mathbf{E}$. This distorted charge distribution is equivalent, by the principle of superposition, to the original distribution plus a dipole whose moment is

$$\mathbf{p} = Q\mathbf{d} \tag{5.21}$$

where $\mathbf{d}$ is the distance vector from $-Q$ to $+Q$ of the dipole as in Figure 5.6(b). If there are N dipoles in a volume Δv of the dielectric, the total dipole moment due to the electric field is

$$Q_1\mathbf{d}_1 + Q_2\mathbf{d}_2 + \cdots + Q_N\mathbf{d}_N = \sum_{k=1}^{N} Q_k\mathbf{d}_k \tag{5.22}$$

As a measure of intensity of the polarization, we define *polarization* $\mathbf{P}$ (in coulombs/meter square) as the dipole moment per unit volume of the dielectric; that is,

$$\boxed{\mathbf{P} = \frac{\displaystyle\lim_{\Delta v \to 0} \sum_{k=1}^{N} Q_k\mathbf{d}_k}{\Delta v}} \tag{5.23}$$

Thus we conclude that the major effect of the electric field $\mathbf{E}$ on a dielectric is the creation of dipole moments that align themselves in the direction of $\mathbf{E}$. This type of dielectric

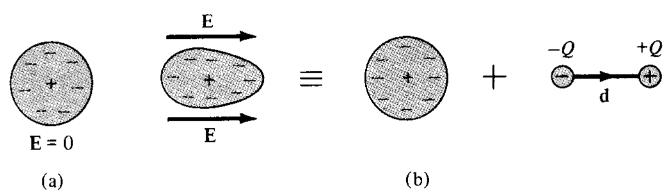


Figure 5.6 Polarization of a nonpolar atom or molecule.

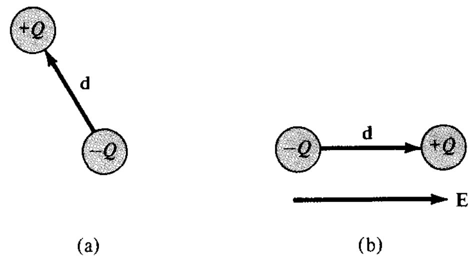


Figure 5.7 Polarization of a polar molecule: **(a)** permanent dipole (**E** = 0), **(b)** alignment of permanent dipole (**E** ≠ 0).

is said to be *nonpolar.* Examples of such dielectrics are hydrogen, oxygen, nitrogen, and the rare gases. Nonpolar dielectric molecules do not possess dipoles until the application of the electric field as we have noticed. Other types of molecules such as water, sulfur dioxide, and hydrochloric acid have built-in permanent dipoles that are randomly oriented as shown in Figure 5.7(a) and are said to be *polar.* When an electric field **E** is applied to a polar molecule, the permanent dipole experiences a torque tending to align its dipole moment parallel with **E** as in Figure 5.7(b).

Let us now calculate the field due to a polarized dielectric. Consider the dielectric material shown in Figure 5.8 as consisting of dipoles with dipole moment **P** per unit volume. According to eq. (4.80), the potential dV at an exterior point O due to the dipole moment $\mathbf{P}\, dv'$ is

$$dV = \frac{\mathbf{P} \cdot \mathbf{a}_R \, dv'}{4\pi\varepsilon_o R^2} \tag{5.24}$$

where $R^2 = (x - x')^2 + (y - y')^2 + (z - z')^2$ and R is the distance between the volume element dv' at (x', y', z') and the field point $O\ (x, y, z)$. We can transform eq. (5.24) into a form that facilitates physical interpretation. It is readily shown (see Section 7.7) that the gradient of $1/R$ with respect to the primed coordinates is

$$\nabla' = \frac{1}{R} = \frac{\mathbf{a}_R}{R^2}$$

Thus,

$$\frac{\mathbf{P} \cdot \mathbf{a}_R}{R^2} = \mathbf{P} \cdot \nabla'\left(\frac{1}{R}\right)$$

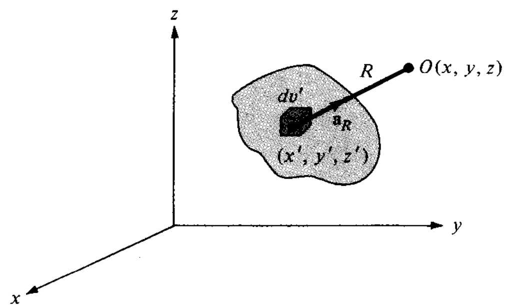


Figure 5.8 A block of dielectric material with dipole moment **p** per unit volume.

Applying the vector identity $\nabla' \cdot f\mathbf{A} = f\nabla' \cdot \mathbf{A} + \mathbf{A} \cdot \nabla' f$,

$$\frac{\mathbf{P} \cdot \mathbf{a}_R}{R^2} = \nabla' \cdot \frac{\mathbf{P}}{R} - \frac{\nabla' \cdot \mathbf{P}}{R} \tag{5.25}$$

Substituting this into eq. (5.24) and integrating over the entire volume v' of the dielectric, we obtain

$$V = \int_{v'} \frac{1}{4\pi\varepsilon_o}\left[\nabla' \cdot \frac{\mathbf{P}}{R} - \frac{1}{R}\nabla' \cdot \mathbf{P}\right] dv'$$

Applying divergence theorem to the first term leads finally to

$$V = \int_{S'} \frac{\mathbf{P} \cdot \mathbf{a}'_n}{4\pi\varepsilon_o R}\, dS' + \int_{v'} \frac{-\nabla' \cdot \mathbf{P}}{4\pi\varepsilon_o R}\, dv' \tag{5.26}$$

where $\mathbf{a}'_n$ is the outward unit normal to surface dS' of the dielectric. Comparing the two terms on the right side of eq. (5.26) with eqs. (4.68) and (4.69) shows that the two terms denote the potential due to surface and volume charge distributions with densities (upon dropping the primes)

$$\rho_{ps} = \mathbf{P} \cdot \mathbf{a}_n \tag{5.27a}$$

$$\rho_{pv} = -\nabla \cdot \mathbf{P} \tag{5.27b}$$

In other words, eq. (5.26) reveals that where polarization occurs, an equivalent volume charge density ρ_{pv} is formed throughout the dielectric while an equivalent surface charge density ρ_{ps} is formed over the surface of the dielectric. We refer to ρ_{ps} and ρ_{pv} as *bound* (or *polarization*) *surface* and *volume charge densities,* respectively, as distinct from *free* surface and volume charge densities ρ_S and ρ_v. Bound charges are those that are not free to move within the dielectric material; they are caused by the displacement that occurs on a molecular scale during polarization. Free charges are those that are capable of moving over macroscopic distance as electrons in a conductor; they are the stuff we control. The total positive bound charge on surface S bounding the dielectric is

$$Q_b = \oint \mathbf{P} \cdot d\mathbf{S} = \int \rho_{ps}\, dS \tag{5.28a}$$

while the charge that remains inside S is

$$-Q_b = \int_v \rho_{pv}\, dv = -\int_v \nabla \cdot \mathbf{P}\, dv \tag{5.28b}$$

Thus the total charge of the dielectric material remains zero, that is,

$$\text{Total charge} = \oint_S \rho_{ps}\, dS + \int_v \rho_{pv}\, dv = Q_b - Q_b = 0$$

This is expected because the dielectric was electrically neutral before polarization.

We now consider the case in which the dielectric region contains free charge. If ρ_v is the free charge volume density, the total volume charge density ρ_t is given by

$$\rho_t = \rho_v + \rho_{pv} = \nabla \cdot \varepsilon_o \mathbf{E} \tag{5.29}$$

Hence

$$\begin{aligned} \rho_v &= \nabla \cdot \varepsilon_o \mathbf{E} - \rho_{pv} \\ &= \nabla \cdot (\varepsilon_o \mathbf{E} + \mathbf{P}) \\ &= \nabla \cdot \mathbf{D} \end{aligned} \tag{5.30}$$

where

$$\boxed{\mathbf{D} = \varepsilon_o \mathbf{E} + \mathbf{P}} \tag{5.31}$$

We conclude that the net effect of the dielectric on the electric field **E** is to increase **D** inside it by amount **P**. In other words, due to the application of **E** to the dielectric material, the flux density is greater than it would be in free space. It should be noted that the definition of **D** in eq. (4.35) for free space is a special case of that in eq. (5.31) because $\mathbf{P} = 0$ in free space.

We would expect that the polarization **P** would vary directly as the applied electric field **E**. For some dielectrics, this is usually the case and we have

$$\boxed{\mathbf{P} = \chi_e \varepsilon_o \mathbf{E}} \tag{5.32}$$

where χ_e, known as the *electric susceptibility* of the material, is more or less a measure of how susceptible (or sensitive) a given dielectric is to electric fields.

5.6 DIELECTRIC CONSTANT AND STRENGTH

By substituting eq. (5.32) into eq. (5.31), we obtain

$$\mathbf{D} = \varepsilon_o(1 + \chi_e)\, \mathbf{E} = \varepsilon_o \varepsilon_r \mathbf{E} \tag{5.33}$$

or

$$\boxed{\mathbf{D} = \varepsilon \mathbf{E}} \tag{5.34}$$

where

$$\boxed{\varepsilon = \varepsilon_o \varepsilon_r} \tag{5.35}$$

and

$$\boxed{\varepsilon_r = 1 + \chi_e = \frac{\varepsilon}{\varepsilon_o}} \tag{5.36}$$

In eqs. (5.33) to (5.36), ε is called the *permittivity* of the dielectric, ε_o is the permittivity of free space, defined in eq. (4.2) as approximately $10^{-9}/36\pi$ F/m, and ε_r is called the *dielectric constant* or *relative permittivity.*

The **dielectric constant** (or **relative permittivity**) ε_r is the ratio of the permittivity of the dielectric to that of free space.

It should also be noticed that ε_r and χ_e are dimensionless whereas ε and ε_o are in farads/meter. The approximate values of the dielectric constants of some common materials are given in Table B.2 in Appendix B. The values given in Table B.2 are for static or low frequency (<1000 Hz) fields; the values may change at high frequencies. Note from the table that ε_r is always greater or equal to unity. For free space and nondielectric materials (such as metals) $\varepsilon_r = 1$.

The theory of dielectrics we have discussed so far assumes ideal dielectrics. Practically speaking, no dielectric is ideal. When the electric field in a dielectric is sufficiently large, it begins to pull electrons completely out of the molecules, and the dielectric becomes conducting. *Dielectric breakdown* is said to have occurred when a dielectric becomes conducting. Dielectric breakdown occurs in all kinds of dielectric materials (gases, liquids, or solids) and depends on the nature of the material, temperature, humidity, and the amount of time that the field is applied. The minimum value of the electric field at which dielectric breakdown occurs is called the *dielectric strength* of the dielectric material.

The **dielectric strength** is the maximum electric field that a dielectric can tolerate or withstand without breakdown.

Table B.2 also lists the dielectric strength of some common dielectrics. Since our theory of dielectrics does not apply after dielectric breakdown has taken place, we shall always assume ideal dielectric and avoid dielectric breakdown.

5.7 LINEAR, ISOTROPIC, AND HOMOGENEOUS DIELECTRICS

Although eqs. (5.24) to (5.31) are for dielectric materials in general, eqs. (5.32 to 5.34) are only for linear, isotropic materials. A material is said to be *linear* if **D** varies linearly with **E** and *nonlinear* otherwise. Materials for which ε (or σ) does not vary in the region being considered and is therefore the same at all points (i.e., independent of x, y, z) are said to be *homogeneous.* They are said to be *inhomogeneous* (or nonhomogeneous) when ε is dependent of the space coordinates. The atmosphere is a typical example of an inhomogeneous medium; its permittivity varies with altitude. Materials for which **D** and **E** are in the same direction are said to be *isotropic.* That is, isotropic dielectrics are those which have the same properties in all directions. For *anisotropic* (or nonisotropic) materials, **D**, **E**, and **P** are not parallel; ε or χ_e has nine components that are collectively referred to as a *tensor.* For example, instead of eq. (5.34), we have

$$\begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix} = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yz} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} \tag{5.37}$$

for anisotropic materials. Crystalline materials and magnetized plasma are anisotropic.

> **A dielectric material** (in which $\mathbf{D} = \varepsilon\mathbf{E}$ applies) is linear if ε does not change with the applied $\mathbf{E}$ field, homogeneous if ε does not change from point to point, and isotropic if ε does not change with direction.

The same idea holds for a conducting material in which $\mathbf{J} = \sigma\mathbf{E}$ applies. The material is linear if σ does not vary with $\mathbf{E}$, homogeneous if σ is the same at all points, and isotropic if σ does not vary with direction.

For most of the time, we will be concerned only with linear, isotropic, and homogeneous media. For such media, all formulas derived in Chapter 4 for free space can be applied by merely replacing ε_o with $\varepsilon_o\varepsilon_r$. Thus Coulomb's law of eq. (4.4), for example, becomes

$$\mathbf{F} = \frac{Q_1 Q_2}{4\pi\varepsilon_o\varepsilon_r R^2}\,\mathbf{a}_R \tag{5.38}$$

and eq. (4.96) becomes

$$W = \frac{1}{2}\int \varepsilon_o\varepsilon_r E^2\, dv \tag{5.39}$$

when applied to a dielectric medium.

EXAMPLE 5.5

A dielectric cube of side L and center at the origin has a radial polarization given by $\mathbf{P} = a\mathbf{r}$, where a is a constant and $\mathbf{r} = x\mathbf{a}_x + y\mathbf{a}_y + z\mathbf{a}_z$. Find all bound charge densities and show explicitly that the total bound charge vanishes.

Solution:

For each of the six faces of the cube, there is a surface charge ρ_{ps}. For the face located at $x = L/2$, for example,

$$\rho_{ps} = \mathbf{P}\cdot\mathbf{a}_x\Big|_{x=L/2} = ax\Big|_{x=L/2} = aL/2$$

The total bound surface charge is

$$Q_s = \int \rho_{ps}\, dS = 6\int_{-L/2}^{L/2}\int_{-L/2}^{L/2} \rho_{ps}\, dy\, dz = \frac{6aL}{2}L^2$$
$$= 3aL^3$$

The bound volume charge density is given by

$$\rho_{pv} = -\nabla \cdot \mathbf{P} = -(a + a + a) = -3a$$

and the total bound volume charge is

$$Q_v = \int \rho_{pv}\, dv = -3a \int dv = -3aL^3$$

Hence the total charge is

$$Q_t = Q_s + Q_v = 3aL^3 - 3aL^3 = 0$$

PRACTICE EXERCISE 5.5

A thin rod of cross section A extends along the x-axis from $x = 0$ to $x = L$. The polarization of the rod is along its length and is given by $P_x = ax^2 + b$. Calculate ρ_{pv} and ρ_{ps} at each end. Show explicitly that the total bound charge vanishes in this case.

Answer: 0, $-2aL$, $-b$, $aL^2 + b$, proof.

EXAMPLE 5.6

The electric field intensity in polystyrene ($\varepsilon_r = 2.55$) filling the space between the plates of a parallel-plate capacitor is 10 kV/m. The distance between the plates is 1.5 mm. Calculate:

(a) D
(b) P
(c) The surface charge density of free charge on the plates
(d) The surface density of polarization charge
(e) The potential difference between the plates

Solution:

(a) $D = \varepsilon_o \varepsilon_r E = \dfrac{10^{-9}}{36\pi} \cdot (2.55) \cdot 10^4 = 225.4 \text{ nC/m}^2$

(b) $P = \chi_e \varepsilon_o E = (1.55) \cdot \dfrac{10^{-9}}{36\pi} \cdot 10^4 = 137 \text{ nC/m}^2$

(c) $\rho_S = \mathbf{D} \cdot \mathbf{a}_n = D_n = 225.4 \text{ nC/m}^2$

(d) $\rho_{ps} = \mathbf{P} \cdot \mathbf{a}_n = P_n = 137 \text{ nC/m}^2$

(e) $V = Ed = 10^4 (1.5 \times 10^{-3}) = 15 \text{ V}$

PRACTICE EXERCISE 5.6

A parallel-plate capacitor with plate separation of 2 mm has a 1-kV voltage applied to its plates. If the space between its plates is filled with polystyrene ($\varepsilon_r = 2.55$), find **E**, **P**, and ρ_{ps}.

Answer: $500\mathbf{a}_x$ kV/m, $6.853\mathbf{a}_x$ μC/m^2, 6.853 μC/m^2.

EXAMPLE 5.7

A dielectric sphere ($\varepsilon_r = 5.7$) of radius 10 cm has a point charge 2 pC placed at its center. Calculate:

(a) The surface density of polarization charge on the surface of the sphere

(b) The force exerted by the charge on a −4-pC point charge placed on the sphere

Solution:

(a) We apply Coulomb's or Gauss's law to obtain

$$\mathbf{E} = \frac{Q}{4\pi\varepsilon_o\varepsilon_r r^2}\,\mathbf{a}_r$$

$$\mathbf{P} = \chi_e\varepsilon_o\mathbf{E} = \frac{\chi_e Q}{4\pi\varepsilon_r r^2}\,\mathbf{a}_r$$

$$\rho_{ps} = \mathbf{P}\cdot\mathbf{a}_r = \frac{(\varepsilon_r - 1)\,Q}{4\pi\varepsilon_r r^2} = \frac{(4.7)\,2\times 10^{-12}}{4\pi(5.7)\,100\times 10^{-4}}$$
$$= 13.12 \text{ pC/m}^2$$

(b) Using Coulomb's law, we have

$$\mathbf{F} = \frac{Q_1 Q_2}{4\pi\varepsilon_o\varepsilon_r r^2}\,\mathbf{a}_r = \frac{(-4)(2)\times 10^{-24}}{4\pi\cdot\dfrac{10^{-9}}{36\pi}(5.7)\,100\times 10^{-4}}\,\mathbf{a}_r$$
$$= -1.263\,\mathbf{a}_r \text{ pN}$$

PRACTICE EXERCISE 5.7

In a dielectric material, $E_x = 5$ V/m and $\mathbf{P} = \dfrac{1}{10\pi}(3\mathbf{a}_x - \mathbf{a}_y + 4\mathbf{a}_z)$ nC/m^2.

Calculate:

(a) χ_e

(b) **E**

(c) **D**

Answer: (a) 2.16, (b) $5\mathbf{a}_x - 1.67\mathbf{a}_y + 6.67\mathbf{a}_z$ V/m, (c) $139.7\mathbf{a}_x - 46.6\mathbf{a}_y + 186.3\mathbf{a}_z$ pC/m^2.

EXAMPLE 5.8

Find the force with which the plates of a parallel-plate capacitor attract each other. Also determine the pressure on the surface of the plate due to the field.

Solution:

From eq. (4.26), the electric field intensity on the surface of each plate is

$$\mathbf{E} = \frac{\rho_S}{2\varepsilon}\,\mathbf{a}_n$$

where $\mathbf{a}_n$ is a unit normal to the plate and ρ_S is the surface charge density. The total force on each plate is

$$\mathbf{F} = Q\mathbf{E} = \rho_S S \cdot \frac{\rho_S}{2\varepsilon}\,\mathbf{a}_n = \frac{\rho_S^2 S}{2\varepsilon_o\varepsilon_r}\,\mathbf{a}_n$$

or

$$F = \frac{\rho_S^2 S}{2\varepsilon} = \frac{Q^2}{2\varepsilon S}$$

The pressure of force/area is $\dfrac{\rho_S^2}{2\varepsilon_o\varepsilon_r}$.

PRACTICE EXERCISE 5.8

Shown in Figure 5.9 is a potential measuring device known as an *electrometer.* It is basically a parallel-plate capacitor with the guarded plate being suspended from a balance arm so that the force F on it is measurable in terms of weight. If S is the area of each plate, show that

$$V_1 - V_2 = \left[\frac{2\,Fd^2}{\varepsilon_o S}\right]^{1/2}$$

Answer: Proof.

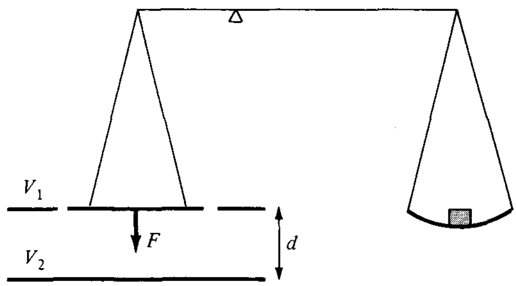


Figure 5.9 An electrometer; for Practice Exercise 5.8.

5.8 CONTINUITY EQUATION AND RELAXATION TIME

Due to the principle of charge conservation, the time rate of decrease of charge within a given volume must be equal to the net outward current flow through the closed surface of the volume. Thus current I_{out} coming out of the closed surface is

$$I_{out} = \oint \mathbf{J} \cdot d\mathbf{S} = \frac{-dQ_{in}}{dt} \tag{5.40}$$

where Q_{in} is the total charge enclosed by the closed surface. Invoking divergence theorem

$$\oint_S \mathbf{J} \cdot d\mathbf{S} = \int_v \nabla \cdot \mathbf{J}\, dv \tag{5.41}$$

But

$$\frac{-dQ_{in}}{dt} = -\frac{d}{dt}\int_v \rho_v\, dv = -\int_v \frac{\partial \rho_v}{\partial t}\, dv \tag{5.42}$$

Substituting eqs. (5.41) and (5.42) into eq. (5.40) gives

$$\int_v \nabla \cdot \mathbf{J}\, dv = -\int_v \frac{\partial \rho_v}{\partial t}\, dv$$

or

$$\boxed{\nabla \cdot \mathbf{J} = -\frac{\partial \rho_v}{\partial t}} \tag{5.43}$$

which is called the *continuity of current equation.* It must be kept in mind that the continuity equation is derived from the principle of conservation of charge and essentially states that there can be no accumulation of charge at any point. For steady currents, $\partial \rho_v / dt = 0$ and hence $\nabla \cdot \mathbf{J} = 0$ showing that the total charge leaving a volume is the same as the total charge entering it. Kirchhoff's current law follows from this.

Having discussed the continuity equation and the properties σ and ε of materials, it is appropriate to consider the effect of introducing charge at some *interior* point of a given material (conductor or dielectric). We make use of eq. (5.43) in conjunction with Ohm's law

$$\mathbf{J} = \sigma \mathbf{E} \tag{5.44}$$

and Gauss's law

$$\nabla \cdot \mathbf{E} = \frac{\rho_v}{\varepsilon} \tag{5.45}$$

Substituting eqs. (5.44) and (5.45) into eq. (5.43) yields

$$\nabla \cdot \sigma\mathbf{E} = \frac{\sigma\rho_v}{\varepsilon} = -\frac{\partial \rho_v}{\partial t}$$

or

$$\frac{\partial \rho_v}{\partial t} + \frac{\sigma}{\varepsilon}\rho_v = 0 \tag{5.46}$$

This is a homogeneous linear ordinary differential equation. By separating variables in eq. (5.46), we get

$$\frac{\partial \rho_v}{\rho_v} = -\frac{\sigma}{\varepsilon}\partial t \tag{5.47}$$

and integrating both sides gives

$$\ln \rho_v = -\frac{\sigma t}{\varepsilon} + \ln \rho_{vo}$$

where $\ln \rho_{vo}$ is a constant of integration. Thus

$$\boxed{\rho_v = \rho_{vo} e^{-t/T_r}} \tag{5.48}$$

where

$$\boxed{T_r = \frac{\varepsilon}{\sigma}} \tag{5.49}$$

In eq. 5.48, ρ_{vo} is the initial charge density (i.e., ρ_v at $t = 0$). The equation shows that as a result of introducing charge at some interior point of the material there is a decay of volume charge density ρ_v. Associated with the decay is charge movement from the interior point at which it was introduced to the surface of the material. The time constant T_r (in seconds) is known as the *relaxation time* or *rearrangement time.*

Relaxation time is the time it takes a charge placed in the interior of a material to drop to e^{-1} = 36.8 percent of its initial value.

It is short for good conductors and long for good dielectrics. For example, for copper $\sigma = 5.8 \times 10^7$ mhos/m, $\varepsilon_r = 1$, and

$$\begin{aligned} T_r &= \frac{\varepsilon_r \varepsilon_o}{\sigma} = 1 \cdot \frac{10^{-9}}{36\pi} \cdot \frac{1}{5.8 \times 10^7} \\ &= 1.53 \times 10^{-19} \text{ s} \end{aligned} \tag{5.50}$$

showing a rapid decay of charge placed inside copper. This implies that for good conductors, the relaxation time is so short that most of the charge will vanish from any interior point and appear at the surface (as surface charge). On the other hand, for fused quartz, for instance, $\sigma = 10^{-17}$ mhos/m, $\varepsilon_r = 5.0$,

$$\begin{aligned} T_r &= 5 \cdot \frac{10^{-9}}{36\pi} \cdot \frac{1}{10^{-17}} \\ &= 51.2 \text{ days} \end{aligned} \tag{5.51}$$

showing a very large relaxation time. Thus for good dielectrics, one may consider the introduced charge to remain wherever placed.

5.9 BOUNDARY CONDITIONS

So far, we have considered the existence of the electric field in a homogeneous medium. If the field exists in a region consisting of two different media, the conditions that the field must satisfy at the interface separating the media are called *boundary conditions*. These conditions are helpful in determining the field on one side of the boundary if the field on the other side is known. Obviously, the conditions will be dictated by the types of material the media are made of. We shall consider the boundary conditions at an interface separating

- dielectric (ε_{r1}) and dielectric (ε_{r2})
- conductor and dielectric
- conductor and free space

To determine the boundary conditions, we need to use Maxwell's equations:

$$\oint \mathbf{E} \cdot d\mathbf{l} = 0 \tag{5.52}$$

and

$$\oint \mathbf{D} \cdot d\mathbf{S} = Q_{\text{enc}} \tag{5.53}$$

Also we need to decompose the electric field intensity **E** into two orthogonal components:

$$\mathbf{E} = \mathbf{E}_t + \mathbf{E}_n \tag{5.54}$$

where $\mathbf{E}_t$ and $\mathbf{E}_n$ are, respectively, the tangential and normal components of **E** to the interface of interest. A similar decomposition can be done for the electric flux density **D**.

A. Dielectric–Dielectric Boundary Conditions

Consider the **E** field existing in a region consisting of two different dielectrics characterized by $\varepsilon_1 = \varepsilon_o\varepsilon_{r1}$ and $\varepsilon_2 = \varepsilon_o\varepsilon_{r2}$ as shown in Figure 5.10(a). $\mathbf{E}_1$ and $\mathbf{E}_2$ in media 1 and 2, respectively, can be decomposed as

$$\mathbf{E}_1 = \mathbf{E}_{1t} + \mathbf{E}_{1n} \tag{5.55a}$$

$$\mathbf{E}_2 = \mathbf{E}_{2t} + \mathbf{E}_{2n} \tag{5.55b}$$

We apply eq. (5.52) to the closed path *abcda* of Figure 5.10(a) assuming that the path is very small with respect to the variation of **E**. We obtain

$$0 = E_{1t}\,\Delta w - E_{1n}\frac{\Delta h}{2} - E_{2n}\frac{\Delta h}{2} - E_{2t}\,\Delta w + E_{2n}\frac{\Delta h}{2} + E_{1n}\frac{\Delta h}{2} \tag{5.56}$$

where $E_t = |\mathbf{E}_t|$ and $E_n = |\mathbf{E}_n|$. As $\Delta h \to 0$, eq. (5.56) becomes

$$\boxed{E_{1t} = E_{2t}} \tag{5.57}$$

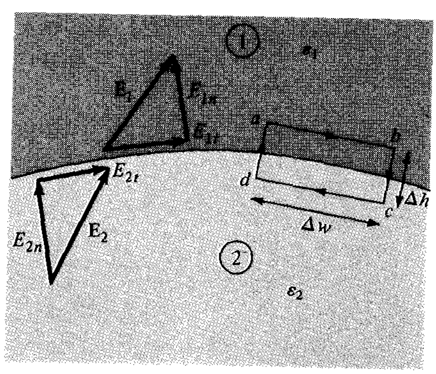
Thus the tangential components of **E** are the same on the two sides of the boundary. In other words, $\mathbf{E}_t$ undergoes no change on the boundary and it is said to be *continuous* across the boundary. Since $\mathbf{D} = \varepsilon\mathbf{E} = \mathbf{D}_t + \mathbf{D}_n$, eq. (5.57) can be written as

$$\frac{D_{1t}}{\varepsilon_1} = E_{1t} = E_{2t} = \frac{D_{2t}}{\varepsilon_2}$$

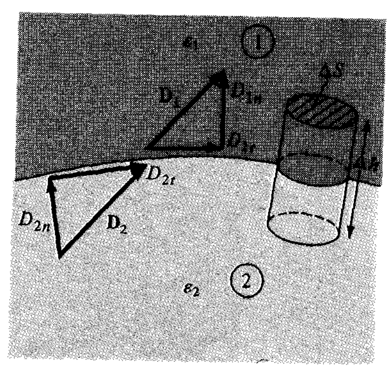
or

$$\frac{D_{1t}}{\varepsilon_1} = \frac{D_{2t}}{\varepsilon_2} \tag{5.58}$$

that is, D_t undergoes some change across the interface. Hence D_t is said to be *discontinuous* across the interface.



(a)



(b)

Figure 5.10 Dielectric–dielectric boundary.

Similarly, we apply eq. (5.53) to the pillbox (Gaussian surface) of Figure 5.10(b). Allowing $\Delta h \to 0$ gives

$$\Delta Q = \rho_S \, \Delta S = D_{1n} \, \Delta S - D_{2n} \, \Delta S$$

or

$$\boxed{D_{1n} - D_{2n} = \rho_S} \tag{5.59}$$

where ρ_S is the free charge density placed deliberately at the boundary. It should be borne in mind that eq. (5.59) is based on the assumption that **D** is directed from region 2 to region 1 and eq. (5.59) must be applied accordingly. If no free charges exist at the interface (i.e., charges are not deliberately placed there), $\rho_S = 0$ and eq. (5.59) becomes

$$\boxed{D_{1n} = D_{2n}} \tag{5.60}$$

Thus the normal component of **D** is continuous across the interface; that is, D_n undergoes no change at the boundary. Since $\mathbf{D} = \varepsilon\mathbf{E}$, eq. (5.60) can be written as

$$\varepsilon_1 E_{1n} = \varepsilon_2 E_{2n} \tag{5.61}$$

showing that the normal component of **E** is discontinuous at the boundary. Equations (5.57) and (5.59), or (5.60) are collectively referred to as *boundary conditions;* they must be satisfied by an electric field at the boundary separating two different dielectrics.

As mentioned earlier, the boundary conditions are usually applied in finding the electric field on one side of the boundary given the field on the other side. Besides this, we can use the boundary conditions to determine the "refraction" of the electric field across the interface. Consider $\mathbf{D}_1$ or $\mathbf{E}_1$ and $\mathbf{D}_2$ or $\mathbf{E}_2$ making angles θ_1 and θ_2 with the *normal* to the interface as illustrated in Figure 5.11. Using eq. (5.57), we have

$$E_1 \sin \theta_1 = E_{1t} = E_{2t} = E_2 \sin \theta_2$$

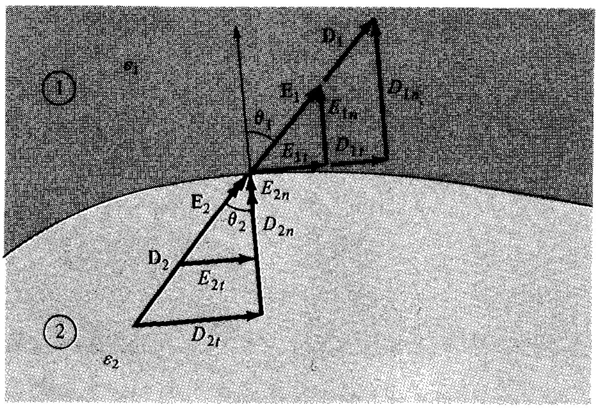


Figure 5.11 Refraction of **D** or **E** at a dielectric–dielectric boundary.

or

$$E_1 \sin \theta_1 = E_2 \sin \theta_2 \tag{5.62}$$

Similarly, by applying eq. (5.60) or (5.61), we get

$$\varepsilon_1 E_1 \cos \theta_1 = D_{1n} = D_{2n} = \varepsilon_2 E_2 \cos \theta_2$$

or

$$\varepsilon_1 E_1 \cos \theta_1 = \varepsilon_2 E_2 \cos \theta_2 \tag{5.63}$$

Dividing eq. (5.62) by eq. (5.63) gives

$$\frac{\tan \theta_1}{\varepsilon_1} = \frac{\tan \theta_2}{\varepsilon_2} \tag{5.64}$$

Since $\varepsilon_1 = \varepsilon_o \varepsilon_{r1}$ and $\varepsilon_2 = \varepsilon_o \varepsilon_{r2}$, eq. (5.64) becomes

$$\boxed{\frac{\tan \theta_1}{\tan \theta_2} = \frac{\varepsilon_{r1}}{\varepsilon_{r2}}} \tag{5.65}$$

This is the *law of refraction* of the electric field at a boundary free of charge (since $\rho_S = 0$ is assumed at the interface). Thus, in general, an interface between two dielectrics produces bending of the flux lines as a result of unequal polarization charges that accumulate on the sides of the interface.

B. Conductor–Dielectric Boundary Conditions

This is the case shown in Figure 5.12. The conductor is assumed to be perfect (i.e., $\sigma \to \infty$ or $\rho_c \to 0$). Although such a conductor is not practically realizable, we may regard conductors such as copper and silver as though they were perfect conductors.

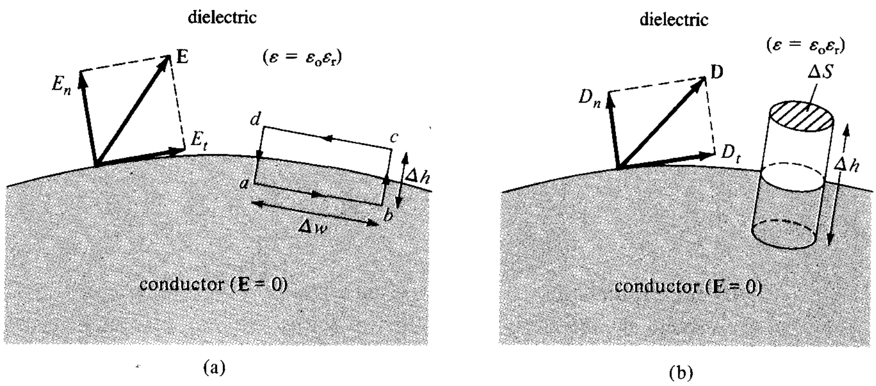


Figure 5.12 Conductor–dielectric boundary.

To determine the boundary conditions for a conductor–dielectric interface, we follow the same procedure used for dielectric–dielectric interface except that we incorporate the fact that $\mathbf{E} = 0$ inside the conductor. Applying eq. (5.52) to the closed path *abcda* of Figure 5.12(a) gives

$$0 = 0 \cdot \Delta w + 0 \cdot \frac{\Delta h}{2} + E_n \cdot \frac{\Delta h}{2} - E_t \cdot \Delta w - E_n \cdot \frac{\Delta h}{2} - 0 \cdot \frac{\Delta h}{2} \tag{5.66}$$

As $\Delta h \rightarrow 0$,

$$E_t = 0 \tag{5.67}$$

Similarly, by applying eq. (5.53) to the pillbox of Figure 5.12(b) and letting $\Delta h \rightarrow 0$, we get

$$\Delta Q = D_n \cdot \Delta S - 0 \cdot \Delta S \tag{5.68}$$

because $\mathbf{D} = \varepsilon \mathbf{E} = 0$ inside the conductor. Equation (5.68) may be written as

$$D_n = \frac{\Delta Q}{\Delta S} = \rho_S$$

or

$$D_n = \rho_S \tag{5.69}$$

Thus under static conditions, the following conclusions can be made about a perfect conductor:

1. No electric field may exist *within* a conductor; that is,

$$\boxed{\rho_v = 0, \qquad \mathbf{E} = 0} \tag{5.70}$$

2. Since $\mathbf{E} = -\nabla V = 0$, there can be no potential difference between any two points in the conductor; that is, a conductor is an equipotential body.
3. The electric field **E** can be external to the conductor and *normal* to its surface; that is

$$\boxed{D_t = \varepsilon_o \varepsilon_r E_t = 0, \qquad D_n = \varepsilon_o \varepsilon_r E_n = \rho_S} \tag{5.71}$$

An important application of the fact that $\mathbf{E} = 0$ inside a conductor is in *electrostatic screening* or *shielding*. If conductor A kept at zero potential surrounds conductor B as shown in Figure 5.13, B is said to be electrically screened by A from other electric systems, such as conductor C, outside A. Similarly, conductor C outside A is screened by A from B.

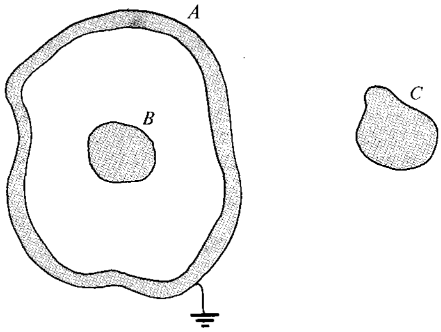


Figure 5.13 Electrostatic screening.

Thus conductor A acts like a screen or shield and the electrical conditions inside and outside the screen are completely independent of each other.

C. Conductor–Free Space Boundary Conditions

This is a special case of the conductor–dielectric conditions and is illustrated in Figure 5.14. The boundary conditions at the interface between a conductor and free space can be obtained from eq. (5.71) by replacing ε_r by 1 (because free space may be regarded as a special dielectric for which $\varepsilon_r = 1$). We expect the electric field **E** to be external to the conductor and normal to its surface. Thus the boundary conditions are

$$\boxed{D_t = \varepsilon_o E_t = 0, \qquad D_n = \varepsilon_o E_n = \rho_S} \tag{5.72}$$

It should be noted again that eq. (5.72) implies that **E** field must approach a conducting surface normally.

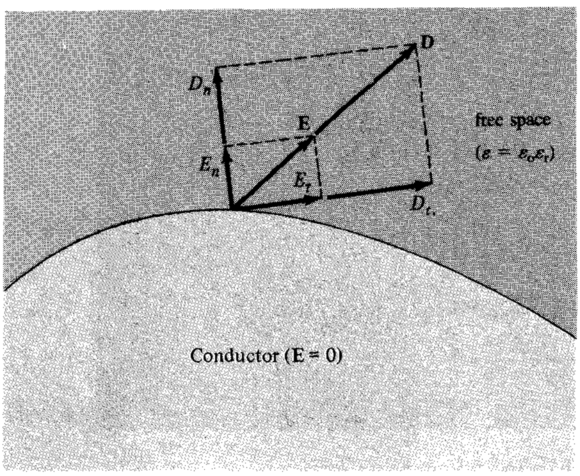


Figure 5.14 Conductor–free space boundary.

EXAMPLE 5.9

Two extensive homogeneous isotropic dielectrics meet on plane $z = 0$. For $z \geq 0$, $\varepsilon_{r1} = 4$ and for $z \leq 0$, $\varepsilon_{r2} = 3$. A uniform electric field $\mathbf{E}_1 = 5\mathbf{a}_x - 2\mathbf{a}_y + 3\mathbf{a}_z$ kV/m exists for $z \geq 0$. Find

(a) $\mathbf{E}_2$ for $z \leq 0$

(b) The angles E_1 and E_2 make with the interface

(c) The energy densities in J/m^3 in both dielectrics

(d) The energy within a cube of side 2 m centered at (3, 4, −5)

Solution:

Let the problem be as illustrated in Figure 5.15.

(a) Since $\mathbf{a}_z$ is normal to the boundary plane, we obtain the normal components as

$$E_{1n} = \mathbf{E}_1 \cdot \mathbf{a}_n = \mathbf{E}_1 \cdot \mathbf{a}_z = 3$$

$$\mathbf{E}_{1n} = 3\mathbf{a}_z$$

$$\mathbf{E}_{2n} = (\mathbf{E}_2 \cdot \mathbf{a}_z)\mathbf{a}_z$$

Also

$$\mathbf{E} = \mathbf{E}_n + \mathbf{E}_t$$

Hence,

$$\mathbf{E}_{1t} = \mathbf{E}_1 - \mathbf{E}_{1n} = 5\mathbf{a}_x - 2\mathbf{a}_y$$

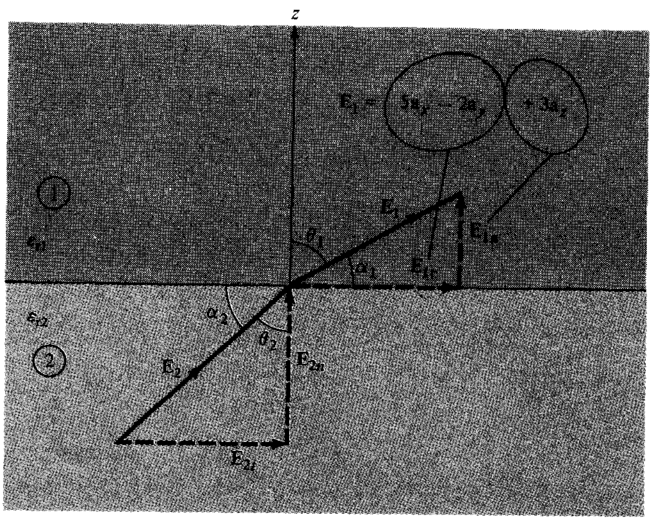


Figure 5.15 For Example 5.9.

Thus

$$\mathbf{E}_{2t} = \mathbf{E}_{1t} = 5\mathbf{a}_x - 2\mathbf{a}_y$$

Similarly,

$$\mathbf{D}_{2n} = \mathbf{D}_{1n} \rightarrow \varepsilon_{r2}\mathbf{E}_{2n} = \varepsilon_{r1}\mathbf{E}_{1n}$$

or

$$\mathbf{E}_{2n} = \frac{\varepsilon_{r1}}{\varepsilon_{r2}}\mathbf{E}_{1n} = \frac{4}{3}(3\mathbf{a}_z) = 4\mathbf{a}_z$$

Thus

$$\begin{aligned} \mathbf{E}_2 &= \mathbf{E}_{2t} + \mathbf{E}_{2n} \\ &= 5\mathbf{a}_x - 2\mathbf{a}_y + 4\mathbf{a}_z \text{ kV/m} \end{aligned}$$

(b) Let α_1 and α_2 be the angles $\mathbf{E}_1$ and $\mathbf{E}_2$ make with the interface while θ_1 and θ_2 are the angles they make with the normal to the interface as shown in Figures 5.15; that is,

$$\alpha_1 = 90 - \theta_1$$

$$\alpha_2 = 90 - \theta_2$$

Since $E_{1n} = 3$ and $E_{1t} = \sqrt{25 + 4} = \sqrt{29}$

$$\tan\theta_1 = \frac{E_{1t}}{E_{1n}} = \frac{\sqrt{29}}{3} = 1.795 \rightarrow \theta_1 = 60.9°$$

Hence,

$$\alpha_1 = 29.1°$$

Alternatively,

$$\mathbf{E}_1 \cdot \mathbf{a}_n = |\mathbf{E}_1| \cdot 1 \cdot \cos\theta_1$$

or

$$\cos\theta_1 = \frac{3}{\sqrt{38}} = 0.4867 \rightarrow \theta_1 = 60.9°$$

Similarly,

$$E_{2n} = 4 \qquad E_{2t} = E_{1t} = \sqrt{29}$$

$$\tan\theta_2 = \frac{E_{2t}}{E_{2n}} = \frac{\sqrt{29}}{4} = 1.346 \rightarrow \theta_2 = 53.4°$$

Hence,

$$\alpha_2 = 36.6°$$

Note that $\dfrac{\tan\theta_1}{\tan\theta_2} = \dfrac{\varepsilon_{r1}}{\varepsilon_{r2}}$ is satisfied.

(c) The energy densities are given by

$$w_{E1} = \frac{1}{2}\varepsilon_1|\mathbf{E}_1|^2 = \frac{1}{2}\cdot 4\cdot\frac{10^{-9}}{36\pi}\cdot(25+4+9)\times 10^6$$
$$= 672\ \mu\text{J/m}^3$$

$$w_{E2} = \frac{1}{2}\varepsilon_2|\mathbf{E}_2|^2 = \frac{1}{2}\cdot 3\cdot\frac{10^{-9}}{36\pi}(25+4+16)\times 10^6$$
$$= 597\ \mu\text{J/m}^3$$

(d) At the center $(3, 4, -5)$ of the cube of side 2 m, $z = -5 < 0$; that is, the cube is in region 2 with $2 \leq x \leq 4$, $3 \leq y \leq 5$, $-6 \leq z \leq -4$. Hence

$$W_E = \int w_{E2}\,dv = \int_{x=2}^{4}\int_{y=3}^{5}\int_{z=-6}^{-4} w_{E2}\,dz\,dy\,dz = w_{E2}(2)(2)(2)$$
$$= 597 \times 8\mu\text{J} = 4.776\ \text{mJ}$$

PRACTICE EXERCISE 5.9

A homogeneous dielectric ($\varepsilon_r = 2.5$) fills region 1 ($x \leq 0$) while region 2 ($x \geq 0$) is free space.

(a) If $\mathbf{D}_1 = 12\mathbf{a}_x - 10\mathbf{a}_y + 4\mathbf{a}_z$ nC/m^2, find $\mathbf{D}_2$ and θ_2.

(b) If $E_2 = 12$ V/m and $\theta_2 = 60°$, find E_1 and θ_1. Take θ_1 and θ_2 as defined in the previous example.

Answer: (a) $12\mathbf{a}_x - 4\mathbf{a}_y + 1.6\mathbf{a}_z$ nC/m^2, 19.75°, (b) 10.67 V/m, 77°

EXAMPLE 5.10

Region $y \leq 0$ consists of a perfect conductor while region $y \geq 0$ is a dielectric medium ($\varepsilon_{1r} = 2$) as in Figure 5.16. If there is a surface charge of 2 nC/m^2 on the conductor, determine **E** and **D** at

(a) $A(3, -2, 2)$

(b) $B(-4, 1, 5)$

Solution:

(a) Point $A(3, -2, 2)$ is in the conductor since $y = -2 < 0$ at A. Hence,

$$\mathbf{E} = 0 = \mathbf{D}$$

(b) Point $B(-4, 1, 5)$ is in the dielectric medium since $y = 1 > 0$ at B.

$$D_n = \rho_S = 2\ \text{nC/m}^2$$

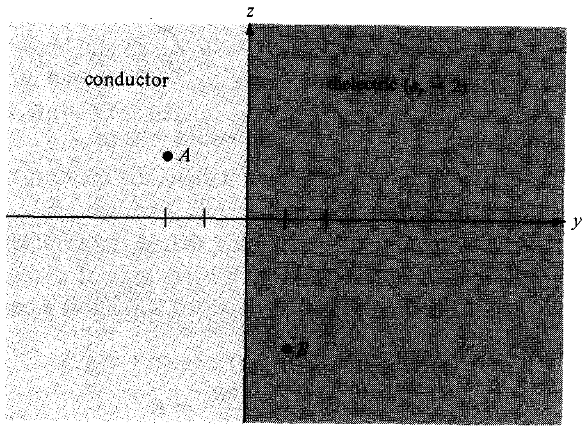


Figure 5.16 See Example 5.10.

Hence,

$$\mathbf{D} = 2\mathbf{a}_y \text{ nC/m}^2$$

and

$$\begin{aligned}\mathbf{E} &= \frac{\mathbf{D}}{\varepsilon_o\varepsilon_r} = 2 \times 10^{-9} \times \frac{36\pi}{2} \times 10^9\,\mathbf{a}_y = 36\pi\mathbf{a}_y \\ &= 113.1\mathbf{a}_y \text{ V/m}\end{aligned}$$

PRACTICE EXERCISE 5.10

It is found that $\mathbf{E} = 60\mathbf{a}_x + 20\mathbf{a}_y - 30\mathbf{a}_z$ mV/m at a particular point on the interface between air and a conducting surface. Find $\mathbf{D}$ and ρ_S at that point.

Answer: $0.531\mathbf{a}_x + 0.177\mathbf{a}_y - 0.265\mathbf{a}_z$ pC/m^2, 0.619 pC/m^2.

SUMMARY

1. Materials can be classified roughly as conductors ($\sigma \gg 1, \varepsilon_r = 1$) and dielectrics ($\sigma \ll 1, \varepsilon_r \geq 1$) in terms of their electrical properties σ and ε_r, where σ is the conductivity and ε_r is the dielectric constant or relative permittivity.
2. Electric current is the flux of electric current density through a surface; that is,

$$I = \int \mathbf{J} \cdot d\mathbf{S}$$

3. The resistance of a conductor of uniform cross section is

$$R = \frac{\ell}{\sigma S}$$

4. The macroscopic effect of polarization on a given volume of a dielectric material is to "paint" its surface with a bound charge $Q_b = \oint_S \rho_{ps}\, dS$ and leave within it an accumulation of bound charge $Q_b = \int_v \rho_{pv}\, dv$ where $\rho_{ps} = \mathbf{P} \cdot \mathbf{a}_n$ and $\rho_{pv} = -\nabla \cdot \mathbf{P}$.
5. In a dielectric medium, the **D** and **E** fields are related as $\mathbf{D} = \varepsilon\mathbf{E}$, where $\varepsilon = \varepsilon_o\varepsilon_r$ is the permittivity of the medium.
6. The electric susceptibility $\chi_e (= \varepsilon_r - 1)$ of a dielectric measures the sensitivity of the material to an electric field.
7. A dielectric material is linear if $\mathbf{D} = \varepsilon\mathbf{E}$ holds, that is, if ε is independent of **E**. It is homogeneous if ε is independent of position. It is isotropic if ε is a scalar.
8. The principle of charge conservation, the basis of Kirchhoff's current law, is stated in the continuity equation

$$\nabla \cdot \mathbf{J} + \frac{\partial \rho_v}{\partial t} = 0$$

9. The relaxation time, $T_r = \varepsilon/\sigma$, of a material is the time taken by a charge placed in its interior to decrease by a factor of $e^{-1} \simeq 37$ percent.
10. Boundary conditions must be satisfied by an electric field existing in two different media separated by an interface. For a dielectric–dielectric interface

$$E_{1t} = E_{2t}$$

$$D_{1n} - D_{2n} = \rho_S \qquad \text{or} \qquad D_{1n} = D_{2n} \qquad \text{if} \qquad \rho_S = 0$$

For a dielectric–conductor interface,

$$E_t = 0 \qquad D_n = \varepsilon E_n = \rho_S$$

because $\mathbf{E} = 0$ inside the conductor.

REVIEW QUESTIONS

5.1 Which is *not* an example of convection current?

(a) A moving charged belt
(b) Electronic movement in a vacuum tube
(c) An electron beam in a television tube
(d) Electric current flowing in a copper wire

5.2 When a steady potential difference is applied across the ends of a conducting wire,

(a) All electrons move with a constant velocity.
(b) All electrons move with a constant acceleration.
(c) The random electronic motion will, on the average, be equivalent to a constant velocity of each electron.
(d) The random electronic motion will, on the average, be equivalent to a nonzero constant acceleration of each electron.

5.3 The formula $R = \ell / (\sigma S)$ is for thin wires.

(a) True
(b) False
(c) Not necessarily

5.4 Sea water has $\varepsilon_r = 80$. Its permittivity is

(a) 81
(b) 79
(c) 5.162×10^{-10} F/m
(d) 7.074×10^{-10} F/m

5.5 Both ε_o and χ_e are dimensionless.

(a) True
(b) False

5.6 If $\nabla \cdot \mathbf{D} = \varepsilon \nabla \cdot \mathbf{E}$ and $\nabla \cdot \mathbf{J} = \sigma \nabla \cdot \mathbf{E}$ in a given material, the material is said to be

(a) Linear
(b) Homogeneous
(c) Isotropic
(d) Linear and homogeneous
(e) Linear and isotropic
(f) Isotropic and homogeneous

5.7 The relaxation time of mica ($\sigma = 10^{-15}$ mhos/m, $\varepsilon_r = 6$) is

(a) 5×10^{-10} s
(b) 10^{-6} s
(c) 5 hours
(d) 10 hours
(e) 15 hours

5.8 The uniform fields shown in Figure 5.17 are near a dielectric–dielectric boundary but on opposite sides of it. Which configurations are correct? Assume that the boundary is charge free and that $\varepsilon_2 > \varepsilon_1$.

5.9 Which of the following statements are incorrect?

(a) The conductivities of conductors and insulators vary with temperature and frequency.
(b) A conductor is an equipotential body and **E** is always tangential to the conductor.
(c) Nonpolar molecules have no permanent dipoles.
(d) In a linear dielectric, P varies linearly with E.

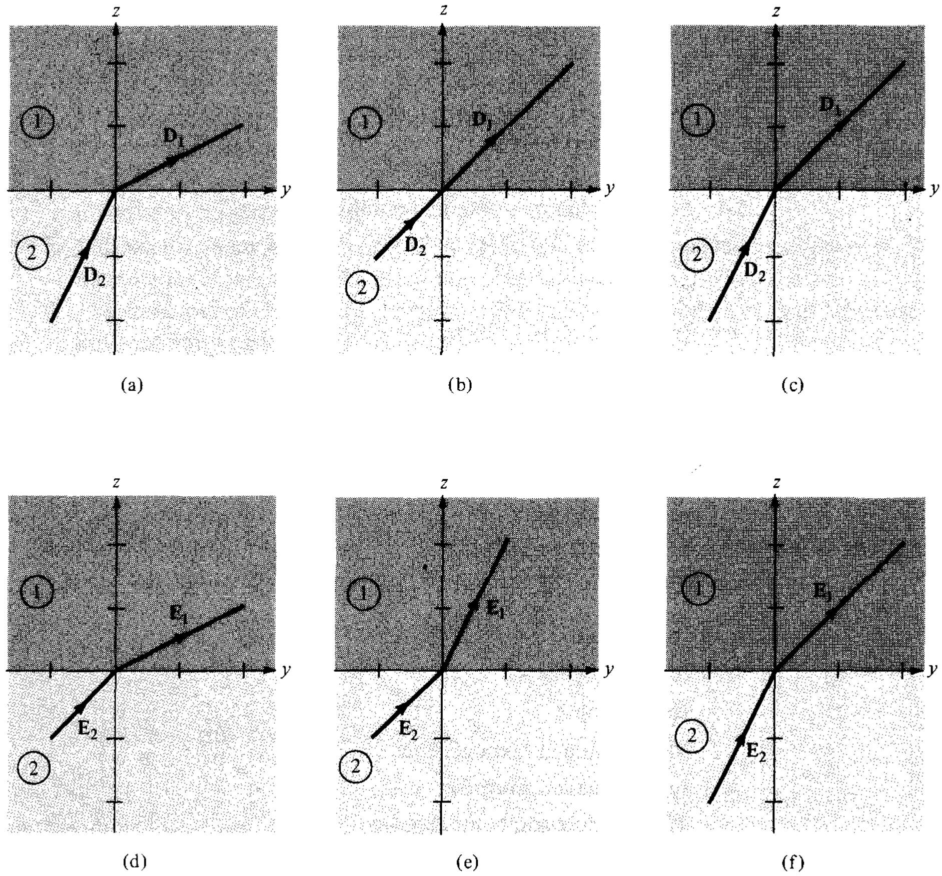


Figure 5.17 For Review Question 5.8.

5.10 The electric conditions (charge and potential) inside and outside an electric screening are completely independent of one another.

(a) True
(b) False

Answers: 5.1d, 5.2c, 5.3c, 5.4d, 5.5b, 5.6d, 5.7e, 5.8e, 5.9b, 5.10a.

PROBLEMS

5.1 In a certain region, $\mathbf{J} = 3r^2 \cos\theta\, \mathbf{a}_r - r^2 \sin\theta\, \mathbf{a}_\theta$ A/m, find the current crossing the surface defined by $\theta = 30°, 0 < \phi < 2\pi, 0 < r < 2$ m.

5.2 Determine the total current in a wire of radius 1.6 mm if $\mathbf{J} = \dfrac{500\mathbf{a}_z}{\rho}$ A/m^2.

5.3 The current density in a cylindrical conductor of radius a is

$$\mathbf{J} = 10e^{-(1-\rho/a)}\mathbf{a}_z \text{ A/m}^2$$

Find the current through the cross section of the conductor.

5.4 The charge $10^{-4}e^{-3t}$ C is removed from a sphere through a wire. Find the current in the wire at $t = 0$ and $t = 2.5$ s.

5.5 (a) Let $V = x^2y^2z$ in a region ($\varepsilon = 2\varepsilon_o$) defined by $-1 < x, y, z < 1$. Find the charge density ρ_v in the region.

(b) If the charge travels at $10^4 y\mathbf{a}_y$ m/s, determine the current crossing surface $0 < x, z < 0.5, y = 1$.

5.6 If the ends of a cylindrical bar of carbon ($\sigma = 3 \times 10^4$) of radius 5 mm and length 8 cm are maintained at a potential difference of 9 V, find: (a) the resistance of the bar, (b) the current through the bar, (c) the power dissipated in the bar.

5.7 The resistance of round long wire of diameter 3 mm is 4.04 Ω/km. If a current of 40 A flows through the wire, find

(a) The conductivity of the wire and identify the material of the wire

(b) The electric current density in the wire

5.8 A coil is made of 150 turns of copper wire wound on a cylindrical core. If the mean radius of the turns is 6.5 mm and the diameter of the wire is 0.4 mm, calculate the resistance of the coil.

5.9 A composite conductor 10 m long consists of an inner core of steel of radius 1.5 cm and an outer sheath of copper whose thickness is 0.5 cm.

(a) Determine the resistance of the conductor.

(b) If the total current in the conductor is 60 A, what current flows in each metal?

(c) Find the resistance of a solid copper conductor of the same length and cross-sectional areas as the sheath. Take the resistivities of copper and steel as 1.77×10^{-8} and 11.8×10^{-8} Ω · m, respectively.

5.10 A hollow cylinder of length 2 m has its cross section as shown in Figure 5.18. If the cylinder is made of carbon ($\sigma = 10^5$ mhos/m), determine the resistance between the ends of the cylinder. Take $a = 3$ cm, $b = 5$ cm.

5.11 At a particular temperature and pressure, a helium gas contains 5×10^{25} atoms/m³. If a 10-kV/m field applied to the gas causes an average electron cloud shift of 10^{-18} m, find the dielectric constant of helium.

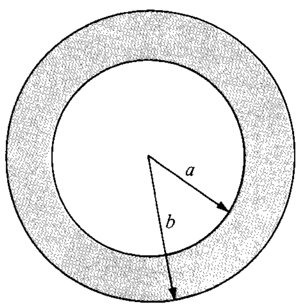


Figure 5.18 For Problems 5.10 and 5.15.

5.12 A dielectric material contains 2×10^{19} polar molecules/m^3, each of dipole moment 1.8×10^{-27} C/m. Assuming that all the dipoles are aligned in the direction of the electric field $\mathbf{E} = 10^5\,\mathbf{a}_x$ V/m, find $\mathbf{P}$ and ε_r.

5.13 In a slab of dielectric material for which $\varepsilon = 2.4\varepsilon_o$ and $V = 300z^2$ V, find: (a) $\mathbf{D}$ and ρ_v, (b) $\mathbf{P}$ and ρ_{pv}.

5.14 For $x < 0$, $\mathbf{P} = 5 \sin(\alpha y)\,\mathbf{a}_x$, where α is a constant. Find ρ_{ps} and ρ_{pv}.

5.15 Consider Figure 5.18 as a spherical dielectric shell so that $\varepsilon = \varepsilon_o\varepsilon_r$ for $a < r < b$ and $\varepsilon = \varepsilon_o$ for $0 < r < a$. If a charge Q is placed at the center of the shell, find

(a) $\mathbf{P}$ for $a < r < b$

(b) ρ_{pv} for $a < r < b$

(c) ρ_{ps} at $r = a$ and $r = b$

5.16 Two point charges when located in free space exert a force of 4.5 μN on each other. When the space between them is filled with a dielectric material, the force changes to 2 μN. Find the dielectric constant of the material and identify the material.

5.17 A conducting sphere of radius 10 cm is centered at the origin and embedded in a dielectric material with $\varepsilon = 2.5\varepsilon_o$. If the sphere carries a surface charge of 4 nC/m^2, find $\mathbf{E}$ at $(-3$ cm, 4 cm, 12 cm$)$.

5.18 At the center of a hollow dielectric sphere ($\varepsilon = \varepsilon_o\varepsilon_r$) is placed a point charge Q. If the sphere has inner radius a and outer radius b, calculate $\mathbf{D}$, $\mathbf{E}$, and $\mathbf{P}$.

5.19 A sphere of radius a and dielectric constant ε_r has a uniform charge density of ρ_o.

(a) At the center of the sphere, show that

$$V = \frac{\rho_o a}{6\varepsilon_o\varepsilon_r}(2\varepsilon_r + 1)$$

(b) Find the potential at the surface of the sphere.

5.20 For static (time-independent) fields, which of the following current densities are possible?

(a) $\mathbf{J} = 2x^3y\mathbf{a}_x + 4x^2z^2\mathbf{a}_y - 6x^2yz\mathbf{a}_z$

(b) $\mathbf{J} = xy\mathbf{a}_x + y(z + 1)\mathbf{a}_y + 2y\mathbf{a}_z$

(c) $\mathbf{J} = \dfrac{z^2}{\rho}\mathbf{a}_\rho + z\cos\phi\,\mathbf{a}_z$

(d) $\mathbf{J} = \dfrac{\sin\theta}{r^2}\mathbf{a}_r$

5.21 For an anisotropic medium

$$\begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix} = \varepsilon_o \begin{bmatrix} 4 & 1 & 1 \\ 1 & 4 & 1 \\ 1 & 1 & 4 \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix}$$

Obtain $\mathbf{D}$ for: (a) $\mathbf{E} = 10\mathbf{a}_x + 10\mathbf{a}_y$ V/m, (b) $\mathbf{E} = 10\mathbf{a}_x + 20\mathbf{a}_y - 30\mathbf{a}_z$ V/m.

5.22 If $\mathbf{J} = \dfrac{100}{\rho^2}\mathbf{a}_\rho$ A/m^2, find: (a) the rate of increase in the volume charge density, (b) the total current passing through surface defined by $\rho = 2, 0 < z < 1, 0 < \phi < 2\pi$.

5.23 Given that $\mathbf{J} = \dfrac{5e^{-10^4 t}}{r}\mathbf{a}_r$ A/m^2, at $t = 0.1$ ms, find: (a) the amount of current passing surface $r = 2$ m, (b) the charge density ρ_v on that surface.

5.24 Determine the relaxation time for each of the following medium:

(a) Hard rubber ($\sigma = 10^{-15}$ S/m, $\varepsilon = 3.1\varepsilon_o$)

(b) Mica ($\sigma = 10^{-15}$ S/m, $\varepsilon = 6\varepsilon_o$)

(c) Distilled water ($\sigma = 10^{-4}$ S/m, $\varepsilon = 80\varepsilon_o$)

5.25 The excess charge in a certain medium decreases to one-third of its initial value in 20 μs. (a) If the conductivity of the medium is 10^{-4} S/m, what is the dielectric constant of the medium? (b) What is the relaxation time? (c) After 30 μs, what fraction of the charge will remain?

5.26 Lightning strikes a dielectric sphere of radius 20 mm for which $\varepsilon_r = 2.5$, $\sigma = 5 \times 10^{-6}$ mhos/m and deposits uniformly a charge of 10 μC. Determine the initial charge density and the charge density 2 μs later.

5.27 Region 1 ($z < 0$) contains a dielectric for which $\varepsilon_r = 2.5$, while region 2 ($z > 0$) is characterized by $\varepsilon_r = 4$. Let $\mathbf{E}_1 = -30\mathbf{a}_x + 50\mathbf{a}_y + 70\mathbf{a}_z$ V/m and find: (a) $\mathbf{D}_2$, (b) $\mathbf{P}_2$, (c) the angle between $\mathbf{E}_1$ and the normal to the surface.

5.28 Given that $\mathbf{E}_1 = 10\mathbf{a}_x - 6\mathbf{a}_y + 12\mathbf{a}_z$ V/m in Figure 5.19, find: (a) $\mathbf{P}_1$, (b) $\mathbf{E}_2$ and the angle $\mathbf{E}_2$ makes with the y-axis, (c) the energy density in each region.

5.29 Two homogeneous dielectric regions 1 ($\rho \leq 4$ cm) and 2 ($\rho \geq 4$ cm) have dielectric constants 3.5 and 1.5, respectively. If $\mathbf{D}_2 = 12\mathbf{a}_\rho - 6\mathbf{a}_\phi + 9\mathbf{a}_z$ nC/m^2, calculate: (a) $\mathbf{E}_1$ and $\mathbf{D}_1$, (b) $\mathbf{P}_2$ and ρ_{pv2}, (c) the energy density for each region.

5.30 A conducting sphere of radius a is half-embedded in a liquid dielectric medium of permittivity ε_1 as in Figure 5.20. The region above the liquid is a gas of permittivity ε_2. If the total free charge on the sphere is Q, determine the electric field intensity everywhere.

***5.31** Two parallel sheets of glass ($\varepsilon_r = 8.5$) mounted vertically are separated by a uniform air gap between their inner surface. The sheets, properly sealed, are immersed in oil ($\varepsilon_r = 3.0$) as shown in Figure 5.21. A uniform electric field of strength 2000 V/m in the horizontal direction exists in the oil. Calculate the magnitude and direction of the electric field in the glass and in the enclosed air gap when (a) the field is normal to the glass surfaces, and (b) the field in the oil makes an angle of 75° with a normal to the glass surfaces. Ignore edge effects.

5.32 (a) Given that $\mathbf{E} = 15\mathbf{a}_x - 8\mathbf{a}_z$ V/m at a point on a conductor surface, what is the surface charge density at that point? Assume $\varepsilon = \varepsilon_o$.

(b) Region $y \geq 2$ is occupied by a conductor. If the surface charge on the conductor is -20 nC/m^2, find $\mathbf{D}$ just outside the conductor.

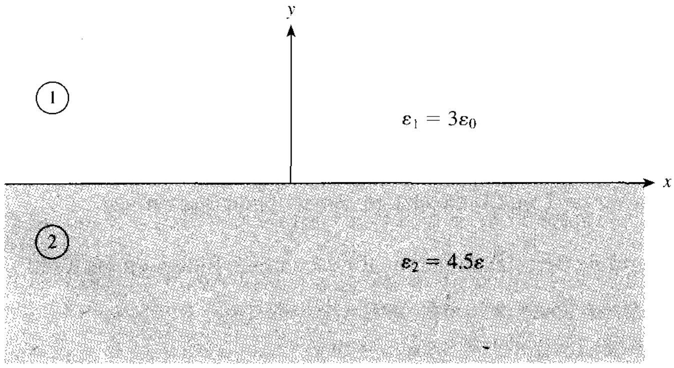


Figure 5.19 For Problem 5.28.

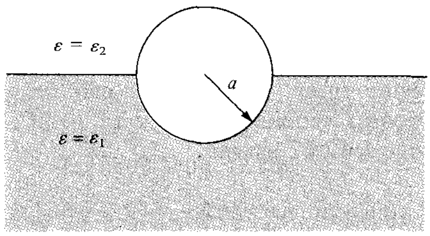


Figure 5.20 For Problem 5.30.

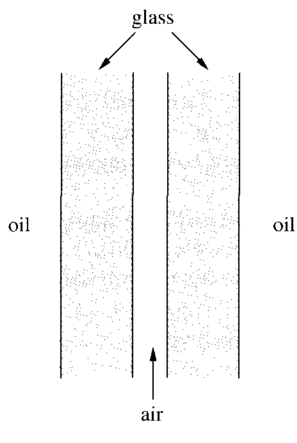


Figure 5.21 For Problem 5.31.

5.33 A silver-coated sphere of radius 5 cm carries a total charge of 12 nC uniformly distributed on its surface in free space. Calculate (a) $|\mathbf{D}|$ on the surface of the sphere, (b) $\mathbf{D}$ external to the sphere, and (c) the total energy stored in the field.