



Palestine Technical University- Kadoorie (PTUK)

Mechanical Engineering Department

12210244: Dynamics

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Textbook:

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# Chapter Five: Plane Kinematics of Rigid Bodies

## Section Seven: Motion Relative to Rotating Axes

## 5 Chapter Five: Plane Kinematics of Rigid Bodies

### 5.7 Motion Relative to Rotating Axes

$$\vec{r}_A = \vec{r}_B + \vec{r} \quad \Rightarrow \quad \vec{r}_A = \vec{r}_B + (x\hat{i} + y\hat{j})$$

$$\vec{v}_A = \vec{v}_B + \vec{\omega} \times \vec{r} + \vec{v}_{rel} \quad \Rightarrow \quad \vec{v}_A = \vec{v}_B + \vec{\omega} \times \vec{r} + (\dot{x}\hat{i} + \dot{y}\hat{j})$$

$$\vec{a}_A = \vec{a}_B + \vec{\omega} \times (\vec{\omega} \times \vec{r}) + \vec{\alpha} \times \vec{r} + 2\vec{\omega} \times \vec{v}_{rel} + \vec{a}_{rel}$$

$$\vec{a}_A = \vec{a}_B + \vec{\omega} \times (\vec{\omega} \times \vec{r}) + \vec{\alpha} \times \vec{r} + 2\vec{\omega} \times \vec{v}_{rel} + (\ddot{x}\hat{i} + \ddot{y}\hat{j})$$

$$\vec{a}_{cor} = 2\vec{\omega} \times \vec{v}_{rel}$$

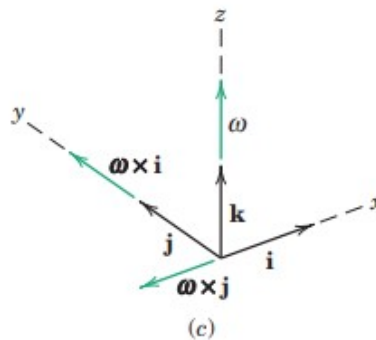
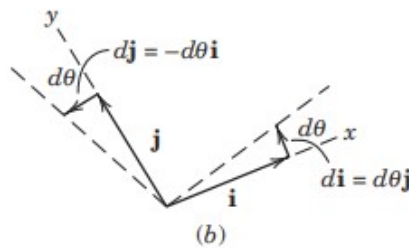
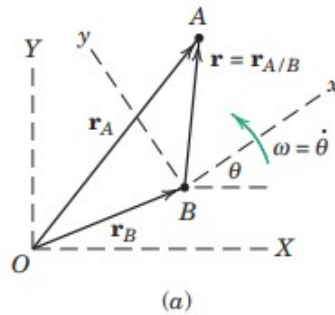


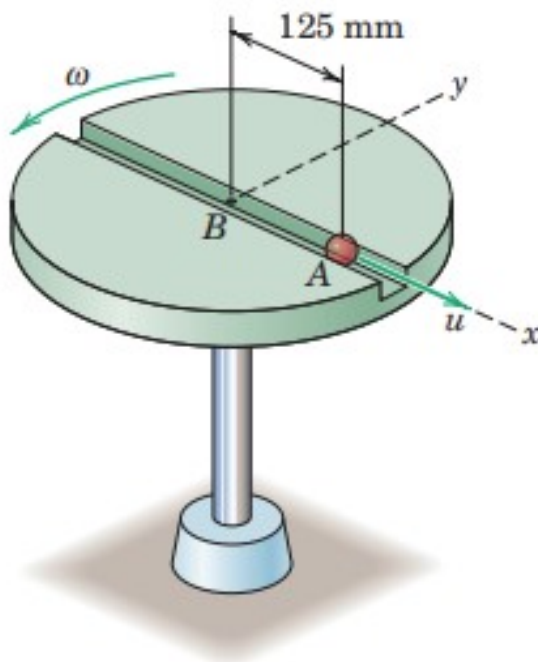
Figure 5/10

**End of Section 5.6**

## Example 1:

The disk rotates with angular speed  $\omega = 2 \text{ rad/sec}$ . The small ball  $A$  is moving along the radial slot with speed  $u = 100 \text{ mm/sec}$  relative to the disk. Determine the absolute velocity of the ball and state the angle  $\beta$  between this velocity vector and the positive x-axis.

ans.  $\vec{v}_A = 0.1\hat{i} + 0.25\hat{j} \text{ m/sec}$       $\beta = 68.2^\circ$

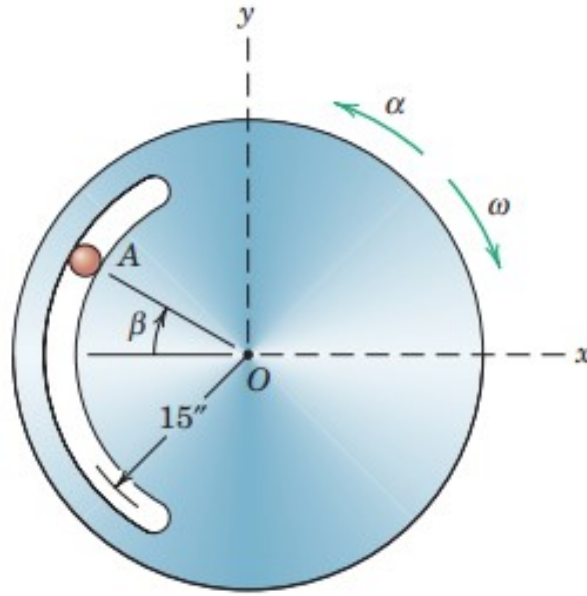


*Ans.*

## Example 2:

The disk rotates about a fixed axis through  $O$  with angular velocity  $\omega = 5 \text{ rad/sec}$  and angular acceleration  $\alpha = 3 \text{ rad/sec}^2$  in the directions shown at a certain instant. The small sphere  $A$  moves in the circular slot, and at the same instant,  $\beta = 30^\circ$ ,  $\dot{\beta} = 2 \text{ rad/sec}$  and  $\ddot{\beta} = -4 \text{ rad/sec}^2$ . Determine the absolute velocity and acceleration of  $A$  at this instant.

ans.  $v_A = 4.38\hat{i} + 7.58\hat{j} \text{ ft/sec}$       $a_A = 48.7\hat{i} - 38.2\hat{j} \text{ ft/sec}^2$

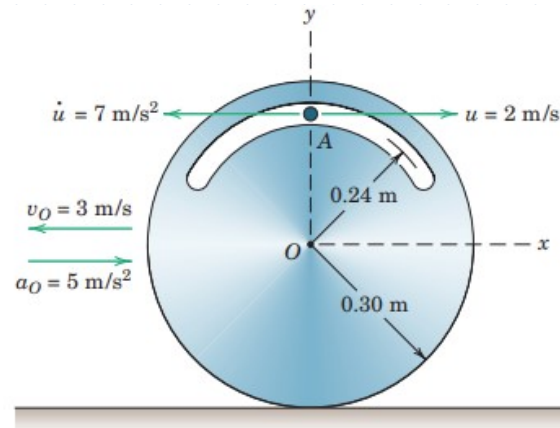


*Ans.*

## Example 3:

The disk rolls without slipping on the horizontal surface, and at the instant represented, the center  $O$  has the velocity and acceleration shown in the figure. For this instant, the particle  $A$  has the indicated speed  $u$  and time rate of change of speed  $\dot{u}$ , both relative to the disk. Determine the absolute velocity and acceleration of particle  $A$ .

$$v_A = -3.4\hat{i} \text{ m/sec} \quad a_A = 2\hat{i} - 0.667\hat{j} \text{ m/sec}^2$$



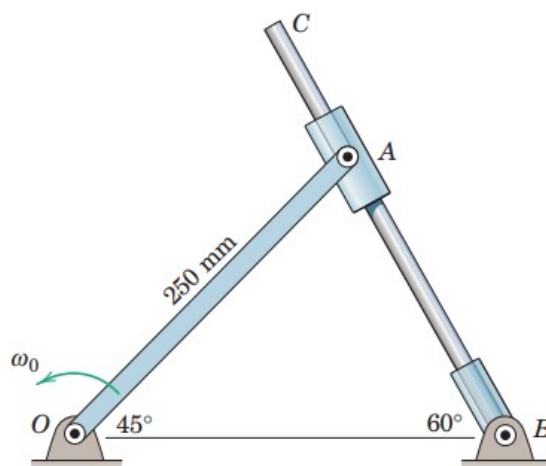


*Ans.*

## Example 4:

Bar  $OA$  has a counterclockwise angular velocity  $\omega_O = 2 \text{ rad/sec}$ . Rod  $BC$  slides freely through the pivoted collar attached to  $OA$ . Determine the angular velocity  $\omega_{BC}$  of rod  $BC$  and the velocity of collar  $A$  relative to rod  $BC$ .

ans.  $\omega_{BC} = 0.634 \text{ rad/sec CCW}$   $v_{rel} = 0.438 \text{ m/sec}$   $\theta = 120^\circ$



*Ans.*