Internal Combustion Engine 1

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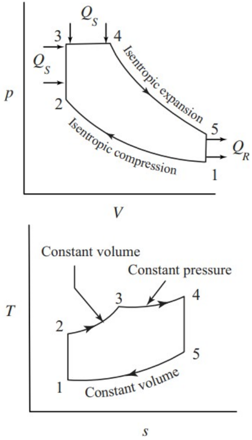
Dual Cycle

The Dual Cycle - Introduction

- In the Otto cycle, combustion is assumed at constant volume, while in the Diesel cycle, combustion is at constant pressure. However, in reality, neither cycle perfectly represents the actual combustion process.
- The Dual cycle, also known as the mixed cycle or limited pressure cycle, is a compromise between the Otto and Diesel cycles.
- In a Dual cycle, a portion of the heat is supplied at constant volume, and the remaining part at constant pressure.
- The Dual cycle on p-V and T-s diagrams.



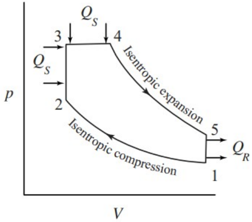
Thermal Efficiency of the Dual Cycle

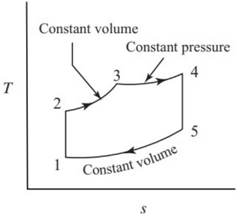
- The efficiency of the cycle can be written as:

$$\eta_{\text{Dual}} = \frac{Q_S - Q_R}{Q_S}$$

$$\eta_{\text{Dual}} = \frac{mC_v(T_3 - T_2) + mC_p(T_4 - T_3) - mC_v(T_5 - T_1)}{mC_v(T_3 - T_2) + mC_p(T_4 - T_3)}$$

$$\eta_{\text{Dual}} = 1 - \frac{T_5 - T_1}{(T_3 - T_2) + \gamma(T_4 - T_3)}$$





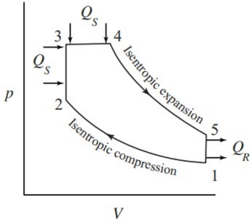
Thermal Efficiency of the Dual Cycle

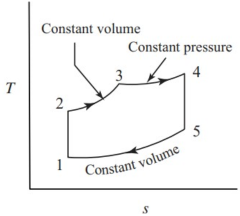
- Temperature Relations:

$$T_2 = T_1 \left(\frac{V_1}{V_2} \right)^{(\gamma-1)} = T_1 r^{(\gamma-1)}$$

$$T_3 = T_2 \left(\frac{p_3}{p_2} \right) = T_1 r_p r^{(\gamma-1)}$$

where $r_p = \frac{p_3}{p_2}$ is Pressure ratio





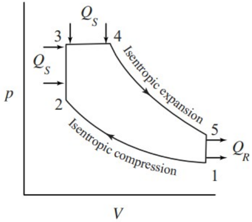
Thermal Efficiency of the Dual Cycle

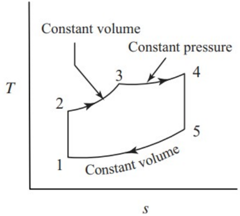
- Temperature Relations:

$$T_4 = T_3 \left(\frac{V_4}{V_3}\right) = T_1 r_c$$

$$T_4 = T_1 r_p r_c r^{(\gamma-1)}$$

$$\text{where } r_c = \frac{V_4}{V_3} \text{ is Cut-off ratio}$$





Thermal Efficiency of the Dual Cycle

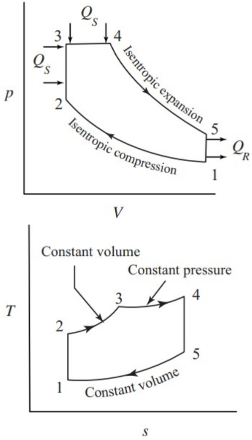
- Temperature Relations:

$$T_5 = T_4 \left(\frac{V_4}{V_5}\right)^{(\gamma-1)}$$

$$T_5 = T_1 r_p r_c r^{(\gamma-1)} \left(\frac{V_4}{V_5}\right)^{(\gamma-1)}$$

- Volume Relations:

$$\frac{V_4}{V_5} = \frac{V_4}{V_1} = \frac{V_4}{V_3} \times \frac{V_3}{V_1} = \frac{V_4}{V_3} \times \frac{V_2}{V_1} = \frac{r_c}{r}$$



Thermal Efficiency of the Dual Cycle

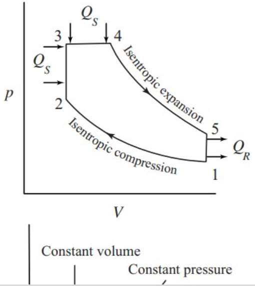
- Temperature Relations:

$$T_5 = T_1 r_p r_c r^{(\gamma-1)} \left(\frac{r_c}{r}\right)^{(\gamma-1)}$$

$$T_5 = T_1 r_p r_c^{\gamma}$$

- Substituting the temperature equations, we obtain:

$$\eta = 1 - \frac{1}{r^{(\gamma-1)}} \left[\frac{r_p r_c^{\gamma} - 1}{(r_p - 1) + r_p \gamma (r_c - 1)}\right]$$



Work Output of the Dual Cycle

$$W = p_3(V_4 - V_3) + \frac{p_4 V_4 - p_5 V_5}{\gamma - 1} - \frac{p_2 V_2 - p_1 V_1}{\gamma - 1}$$

$$W = \frac{p_1 V_1}{\gamma - 1}\left[(1-\gamma)\left(\frac{p_4 V_4}{p_1 V_1} - \frac{p_3 V_3}{p_1 V_1}\right) + \frac{p_4 V_4}{p_1 V_1} - \frac{p_5 V_5}{p_1 V_1} - \frac{p_2 V_2}{p_1 V_1} + 1\right]$$

$$W = \frac{p_1 V_1}{\gamma - 1}\left[(1-\gamma)\left(r_c r_p r^{(\gamma-1)} - r_p r^{(\gamma-1)}\right) + r_c r_p r^{(\gamma-1)} - r_p r_c^{\gamma} - r^{(\gamma-1)} + 1\right]$$

$$W = \frac{p_1 V_1}{\gamma - 1}\left[\gamma r_c r_p r^{(\gamma-1)} - \gamma r_p r^{(\gamma-1)} + r_p r^{(\gamma-1)} - r_p r_c^{\gamma} - r^{(\gamma-1)} + 1\right]$$

$$W = \frac{p_1 V_1}{\gamma - 1}\left[\gamma r_p r^{(\gamma-1)}(r_c - 1) + r^{(\gamma-1)}(r_p - 1) - (r_p r_c^{\gamma} - 1)\right]$$

Mean Effective Pressure of the Dual Cycle

$$p_m = \frac{\text{Work Output}}{\text{Swept Volume}} = \frac{W}{V_s}$$

$$p_m = \frac{1}{V_1 - V_2}\frac{p_1 V_1}{\gamma - 1}\left[\gamma r_p r^{\gamma-1}(r_c - 1) + r^{\gamma-1}(r_p - 1) - (r_p r_c^{\gamma} - 1)\right]$$

$$p_m = \frac{1}{1 - \frac{V_2}{V_1}}\frac{p_1}{\gamma - 1}\left[\gamma r_p r^{\gamma-1}(r_c - 1) + r^{\gamma-1}(r_p - 1) - (r_p r_c^{\gamma} - 1)\right]$$

$$p_m = p_1\frac{[\gamma r_p r^{\gamma}(r_c - 1) + r^{\gamma}(r_p - 1) - r(r_p r_c^{\gamma} - 1)]}{(\gamma - 1)(r - 1)}$$

This equation indicates the engine's efficiency and performance, relating the work output to the swept volume.

Example 1

An air-standard Dual cycle has a compression ratio of 10. The pressure and temperature at the beginning of compression are 1 bar and 27 °C. The maximum pressure reached is 42 bar and the maximum temperature is 1500 °C. Determine (i) the temperature at the end of constant volume heat addition (ii) cut-off ratio (iii) work done per kg of air and (iv) the cycle efficiency. Assume $C_p = 1.004$ kJ/kg K and $C_v = 0.717$ kJ/kg K for air.

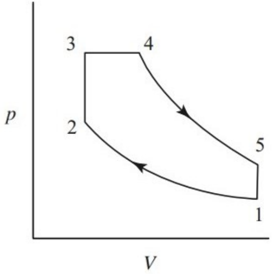
$$\frac{V_s}{V_c} = r - 1 = 9$$

$$V_s = 9V_c$$

$$\gamma = \frac{C_p}{C_v} = \frac{1.004}{0.717} = 1.4$$

Consider the process 1 – 2

$$\frac{T_2}{T_1} = r^{(\gamma-1)} = 10^{0.4} = 2.512$$



Example 1 - cont.

$$T_2 = 2.512 \times 300 = 753.6 \text{ K}$$

$$\frac{p_2}{p_1} = r^{\gamma} = 10^{1.4} = 25.12$$

$$p_2 = 25.12 \times 10^5 \text{ N/m}^2$$

Consider the process $3 - 4$

$$\frac{T_3}{T_2} = \frac{p_3}{p_2} = \frac{42}{25.12} = 1.672$$

$$T_3 = 1.672 \times 753.6 = 1260 \text{ K} = \mathbf{987^\circ \ C} \qquad \overset{\text{Ans}}{\Longleftarrow}$$

$$r_c = \frac{T_4}{T_3} = \frac{1773}{1260} = \mathbf{1.407} \qquad \overset{\text{Ans}}{\Longleftarrow}$$

Example 1 - cont.

$$\begin{aligned}
\textit{Work done/kg} &= \textit{Heat supplied - Heat rejected} \\
\textit{Heat supplied/kg} &= C_v\,(T_3 - T_2) + C_p\,(T_4 - T_3) \\
&= 0.717 \times (1260 - 753.6) + \\
&\quad 1.004 \times (1773 - 1260) = \mathbf{878.1\ kJ} \qquad \overset{\text{Ans}}{\Longleftarrow}
\end{aligned}$$

Consider the process $4-5$

$$\begin{aligned}
\frac{T_4}{T_5} &= \left(\frac{V_5}{V_4}\right)^{(\gamma-1)} = \left(\frac{r}{r_c}\right)^{(\gamma-1)} = \left(\frac{10}{1.407}\right)^{0.4} = 2.191 \\
T_5 &= \frac{T_4}{2.191} = \mathbf{809.2\ K} \qquad \overset{\text{Ans}}{\Longleftarrow}
\end{aligned}$$

Example 1 - cont.

$$\begin{aligned}
Heat\ rejected/kg &= C_v\,(T_5 - T_1) \\
&= 0.717 \times (809.2 - 300) = 365.1 \text{ kJ} \\
Work\ output/kg &= 878.1 - 365.1 = \mathbf{513\ kJ} && \overset{\textbf{Ans}}{\Longleftarrow} \\
\eta_{Dual} &= \frac{Work\ output}{Heat\ added} = \frac{513}{878.1} \\
&= 0.5842 = \mathbf{58.42\%} && \overset{\textbf{Ans}}{\Longleftarrow}
\end{aligned}$$

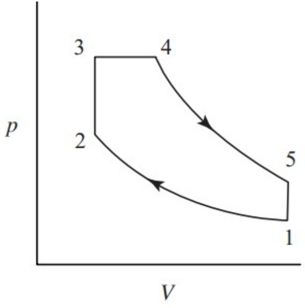
Example 2

For an engine working on the ideal Dual cycle, the compression ratio is 10 and the maximum pressure is limited to 70 bar. If the heat supplied is 1680 kJ/kg, find the pressures and temperatures at the various salient points of the cycle and the cycle efficiency. The pressure and temperature of air at the commencement of compression are 1 bar and 100 °C respectively. Assume C_p = 1.004 kJ/kg K and C_v = 0.717 kJ/kg K for air.

Example 2 - cont.

$$\frac{V_s}{V_c} = r - 1 = 9$$

$$\gamma = \frac{C_p}{C_v} = \frac{1.004}{0.717} = 1.4$$



Consider the process 1 – 2

$$\frac{p_2}{p_1} = r^{\gamma} = 10^{1.4} = 25.12$$

$$p_2 = 25.12 \times 10^5 \text{ N/m}^2$$

$$= \mathbf{25.12 \text{ bar}} \quad \overset{\text{Ans}}{\Longleftarrow}$$

$$\frac{T_2}{T_1} = r^{(\gamma-1)} = 10^{0.4} = 2.512$$

$$T_2 = 2.512 \times 373 = 936.9 \text{ K} = \mathbf{663.9^{\circ}C} \quad \overset{\text{Ans}}{\Longleftarrow}$$

Example 2 - cont.

Consider the process 2 – 3 and 3 – 4

$$\frac{T_3}{T_2} = \frac{p_3}{p_2} = \frac{70}{25.12} = 2.787$$

$$T_3 = 2.787 \times 936.9 = 2611.1 \text{ K} = \mathbf{2338^\circ\ C} \quad \stackrel{\textbf{Ans}}{\Longleftarrow}$$

Heat added during constant volume combustion

$$= C_v (T_3 - T_2) = 0.717 \times (2611.1 - 936.9)$$

$$= 1200.4 \text{ kJ/kg}$$

$$\textit{Total heat added} = 1680 \text{ kJ/kg}$$

Example 2 - cont.

Hence, heat added during constant pressure combustion

$$
\begin{aligned}
&= 1680 - 1200.4 = 479.6 \text{ kJ/kg} \\
&= C_p\,(T_4 - T_3) \\
T_4 - T_3 &= \frac{479.6}{1.004} = 477.7 \text{ K} \\
T_4 &= 477.7 + 2611.1 \\
&= 3088.8 \text{ K} = \mathbf{2815.8^\circ\ C} \qquad \stackrel{\text{Ans}}{\Longleftarrow} \\
r_c &= \frac{V_4}{V_3} = \frac{T_4}{T_3} = \frac{3088.8}{2611.1} = 1.183
\end{aligned}
$$

Example 2 - cont.

Consider the process 4 – 5

$$\frac{T_4}{T_5} = \left(\frac{r}{r_c}\right)^{(\gamma-1)} = 8.453^{0.4} = 2.35$$

$$T_5 = \frac{T_4}{2.35} = \frac{3088.8}{2.35} = 1314.4 \text{ K} = \mathbf{1041.4^\circ\ C} \stackrel{\text{Ans}}{\Longleftarrow}$$

$$\frac{p_4}{p_5} = \left(\frac{r}{r_c}\right)^{\gamma} = 19.85$$

$$p_5 = \frac{p_4}{19.85} = \frac{70 \times 10^5}{19.85}$$

$$= 3.53 \times 10^5 \text{ N/m}^2 = \mathbf{3.53\ bar} \stackrel{\text{Ans}}{\Longleftarrow}$$

Example 2 - cont.

$$\begin{aligned} \textit{Heat rejected} &= C_v(T_5 - T_1) \\ &= 0.717 \times (1314.4 - 373) = 674.98 \text{ kJ/kg} \\ \eta &= \frac{1680 - 674.98}{1680} = \mathbf{59.82\%} \qquad \underleftarrow{\textbf{Ans}} \end{aligned}$$

Example 3

An oil engine works on the Dual cycle, the heat liberated at constant pressure being twice that liberated at constant volume. The compression ratio of the engine is 8 and the expansion ratio is 5.3. But the compression and expansion processes follow the law $pV^{1.3} = C$. The pressure and temperature at the beginning of compression are 1 bar and 27 °C respectively. Assuming $C_p = 1.004$ kJ/kg K and $C_v = 0.717$ kJ/kg K for air, find the air-standard efficiency and the mean effective pressure.

Example 3 - cont.

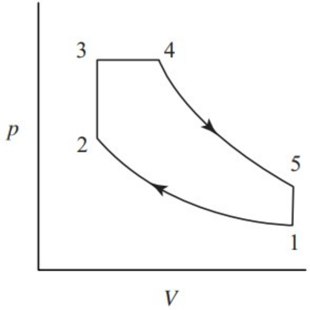
$$\gamma = \frac{C_p}{C_v} = \frac{1.004}{0.717} = 1.4$$

$$\frac{V_s}{V_c} = r - 1 = 7$$

$$V_s = 7V_c$$

$$r_e = \frac{r}{r_c} = 5.3$$

$$r_c = \frac{8}{5.3} = 1.509$$



Example 3 - cont.

$$
\begin{aligned}
mep &= = \frac{\text{Area } 12345}{V_s} \\
\text{Area } 12345 &= \text{Area under } 3-4+ \\
&\quad \text{Area under } 4-5-\text{Area under } 2-1 \\
&= p_3\left(V_3-V_4\right)+\frac{p_4V_4-p_5V_5}{n-1}- \\
&\quad \frac{p_2V_2-p_1V_1}{n-1}
\end{aligned}
$$

Example 3 - cont.

$$
\begin{aligned}
V_2 &= V_3 = V_c \\
V_1 &= V_5 = rV_c = 8\,V_c \\
V_4 &= r_c V_3 = 1.509\,V_c \\
\frac{T_2}{T_1} &= r^{(n-1)} = 8^{0.3} = 1.866 \\
T_2 &= 1.866 \times 300 = 559.82\ \mathrm{K} \\
\frac{p_2}{p_1} &= r^n = 8^{1.3} = 14.93 \\
p_2 &= 14.93 \times p_1 = 14.93 \times 10^5\ \mathrm{N/m^2}
\end{aligned}
$$

Example 3 - cont.

Heat released during constant pressure combustion

$= 2 \times$ *Heat released during constant volume combustion*

$$
\begin{aligned}
C_p (T_4 - T_3) &= 2C_v (T_3 - T_2) \\
1.004 \times (T_4 - T_3) &= 2 \times 0.717 \times (T_3 - T_2) \\
T_4 - T_3 &= 1.428 \times (T_3 - T_2) \\
\frac{T_4}{T_3} &= \frac{V_4}{V_3} = r_c = 1.509 \\
T_4 &= 1.509\, T_3
\end{aligned}
$$

Example 3 - cont.

$$
\begin{aligned}
1.509\,T_3 - T_3 &= 1.428 \times (T_3 - 559.82) \\
T_3 &= 869.88 \text{ K} \\
T_4 &= 1312.65 \text{ K} \\
\frac{p_3}{p_2} &= \frac{T_3}{T_2} = \frac{869.88}{559.82} = 1.554
\end{aligned}
$$

Example 3 - cont.

$$p_3 = 1.554 \times 14.93 \times 10^5$$
$$= 23.20 \times 10^5 \text{ N/m}^2 = p_4$$
$$\frac{T_4}{T_5} = r_e^{(n-1)} = 5.3^{0.3} = 1.649$$
$$T_5 = \frac{1312.65}{1.649} = 796.03 \text{ K}$$
$$\frac{p_4}{p_5} = r_e^n = 5.3^{1.3} = 8.741$$
$$p_5 = \frac{p_4}{8.741} = \frac{23.2 \times 10^5}{8.741}$$
$$= 2.654 \times 10^5 \text{ N/m}^2$$

Example 3 - cont.

$$
\begin{aligned}
\textit{Area 12345} &= \left[\frac{23.2 \times 1.509V_c - 2.654 \times 8V_c}{0.3} + \right. \\
&\qquad 23.2 \times (1.509V_c - V_c) - \\
&\qquad \left. \frac{14.93 \times V_c - 1 \times 8V_c}{0.3}\right] \times 10^5 \\
&= 34.63 \times V_c \times 10^5 \ \mathrm{N/m^2} \\
V_c &= \frac{V_1}{8} \\
\textit{Area 12345} &= p_m \times V_s = p_m \times 7 \times V_c \\
p_m &= \frac{34.63}{7} = 4.95 \times 10^5 \ \mathrm{N/m^2} \\
&= \mathbf{4.95\ bar} \qquad \stackrel{\text{Ans}}{\Longleftarrow}
\end{aligned}
$$

Example 3 - cont.

$$\eta = \frac{w}{q_s}$$

$$v_1 = \frac{mRT_1}{p_1} = \frac{1 \times 287 \times 300}{1 \times 10^5} = 0.861 \text{ m}^3/\text{kg}$$

$$w = 34.63 \times 10^5 \times \frac{v_1}{8} = 34.63 \times 10^5 \times \frac{0.861}{8}$$

$$= 3.727 \times 10^5 \text{ J/kg} = 372.7 \text{ kJ/kg}$$

End of Lecture 13

End of Lecture 13