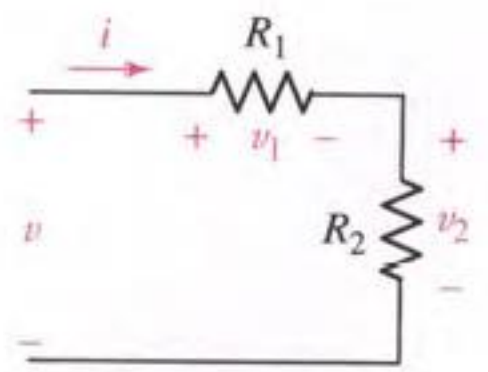
Chapter 3
Voltage and current laws

VOLTAGE AND CURRENT DIVISION

Voltage divider rule (VDR) for resistors in series

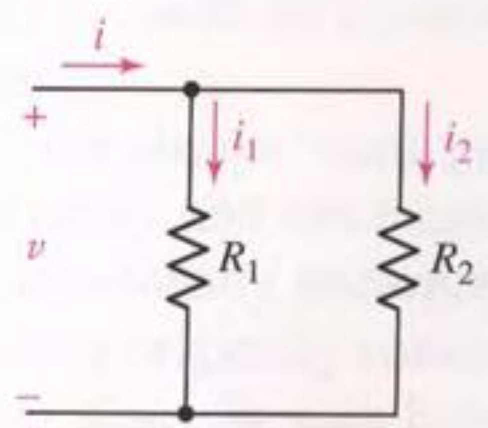
$$v_k = \frac{R_k}{R_1 + R_2 + \cdots + R_N} v$$



Current divider rule (CDR) : only for two parallel resistors

$$i_2 = i \frac{R_1}{R_1 + R_2}$$

$$i_1 = i \frac{R_2}{R_1 + R_2}$$

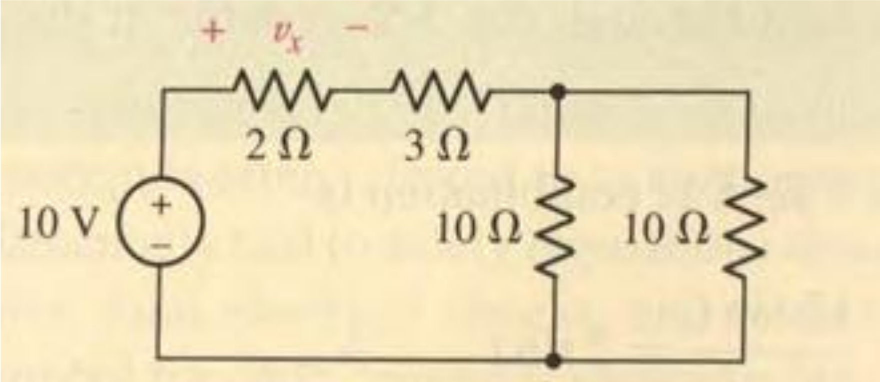


Question

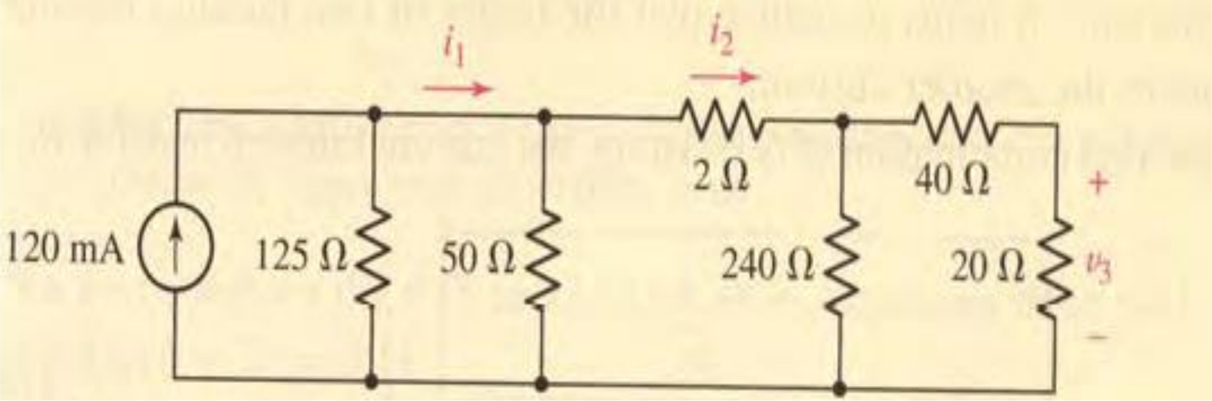
Using VDR find Vx

$$R_{T1} = 5\Omega$$

$$Vx = \frac{10(2)}{2+3+5} = \frac{20}{10} = 2v$$



Question : find i1, i2, v3



$$R_{t1} = \frac{240(40+20)}{240+(40+20)} = \frac{240(60)}{240+60} = \frac{14400}{300} = 48\Omega$$

$$R_{t2} = 2 + R_{t1} = 2 + 48 = 50\Omega$$

$$R_{t3} = \frac{50(50)}{50+50} = 25\Omega$$

CDR

$$i_1 = \frac{120m(125)}{125+25} = \frac{15}{150} = 0.1A = 100mA$$

$$i_2 = \frac{i_1(50)}{50+50} = \frac{100mA(50)}{100} = 50mA$$

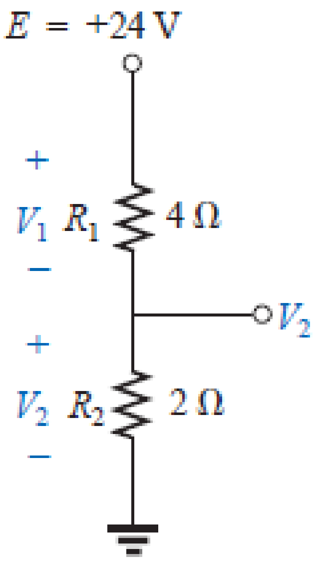
$$i_3 = \frac{i_2(240)}{240+60} = \frac{50mA(240)}{300} = 40mA$$

$$v_3 = 20(i_3) = 20(40mA) = 800mv = 0.8v$$

EXAMPLE Using the voltage divider rule, determine the voltages V_1 and V_2 of Fig.

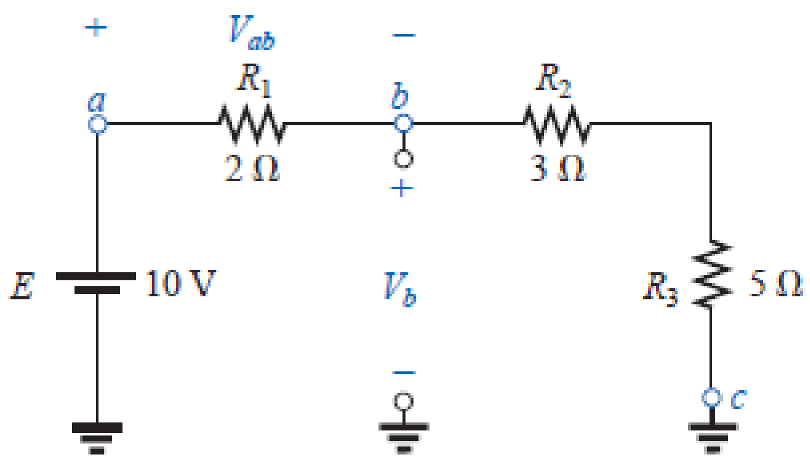
$$V_1 = \frac{R_1E}{R_1 + R_2} = \frac{(4\ \Omega)(24\ \text{V})}{4\ \Omega + 2\ \Omega} = \mathbf{16\ V}$$

$$V_2 = \frac{R_2E}{R_1 + R_2} = \frac{(2\ \Omega)(24\ \text{V})}{4\ \Omega + 2\ \Omega} = \mathbf{8\ V}$$



EXAMPLE For the network of Fig.

a. Calculate V_{ab}.
b. Determine V_b.



Solutions:

a. Voltage divider rule:

$$V_{ab} = \frac{R_1E}{R_T} = \frac{(2\ \Omega)(10\ \text{V})}{2\ \Omega + 3\ \Omega + 5\ \Omega} = \mathbf{+2\ V}$$

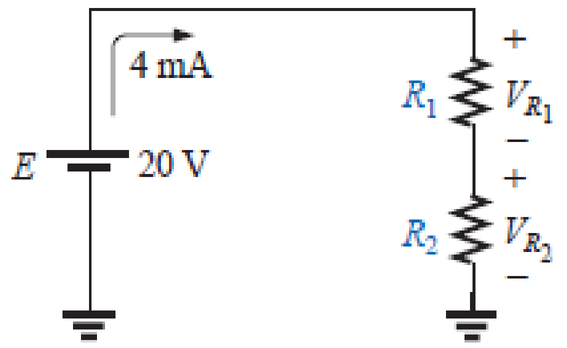
b. Voltage divider rule:

$$V_b = V_{R_2} + V_{R_3} = \frac{(R_2 + R_3)E}{R_T} = \frac{(3\ \Omega + 5\ \Omega)(10\ \text{V})}{10\ \Omega} = \mathbf{8\ V}$$

or $$V_b = V_a - V_{ab} = E - V_{ab} = 10\ \text{V} - 2\ \text{V} = \mathbf{8\ V}$$

c. V_c = ground potential = **0 V**

EXAMPLE Design the voltage divider of Fig. such that $V_{R_1} = 4V_{R_2}$.



Solution: The total resistance is defined by

$$R_T = \frac{E}{I} = \frac{20\text{ V}}{4\text{ mA}} = 5\text{ k}\Omega$$

Since $V_{R_1} = 4V_{R_2}$,

$$R_1 = 4R_2$$

Thus

$$R_T = R_1 + R_2 = 4R_2 + R_2 = 5R_2$$

and

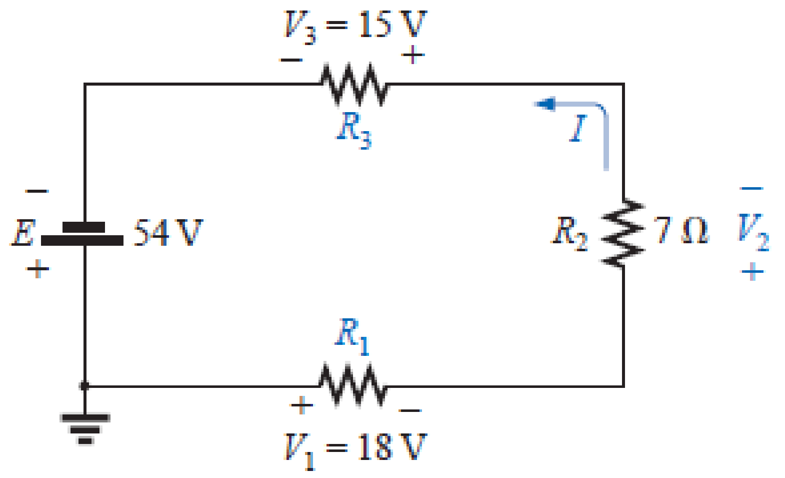
$$5R_2 = 5\text{ k}\Omega$$
$$R_2 = \mathbf{1\text{ k}\Omega}$$

and

$$R_1 = 4R_2 = \mathbf{4\text{ k}\Omega}$$

EXAMPLE

a. Determine V_2 using Kirchhoff's voltage law.
b. Determine I.
c. Find R_1 and R_3.



Solutions:

a. Kirchhoff's voltage law (clockwise direction):

$$-E + V_3 + V_2 + V_1 = 0$$

or

$$E = V_1 + V_2 + V_3$$

and $V_2 = E - V_1 - V_3 = 54\text{ V} - 18\text{ V} - 15\text{ V} = \mathbf{21\text{ V}}$

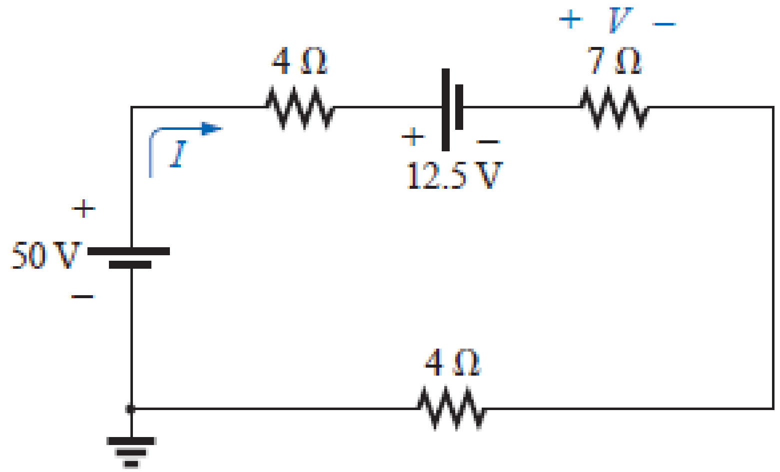
b. $I = \dfrac{V_2}{R_2} = \dfrac{21\text{ V}}{7\ \Omega} = \mathbf{3\text{ A}}$

c. $R_1 = \dfrac{V_1}{I} = \dfrac{18\text{ V}}{3\text{ A}} = \mathbf{6\ \Omega}$

$R_3 = \dfrac{V_3}{I} = \dfrac{15\text{ V}}{3\text{ A}} = \mathbf{5\ \Omega}$

EXAMPLE

Determine I and the voltage across the 7-Ω resistor

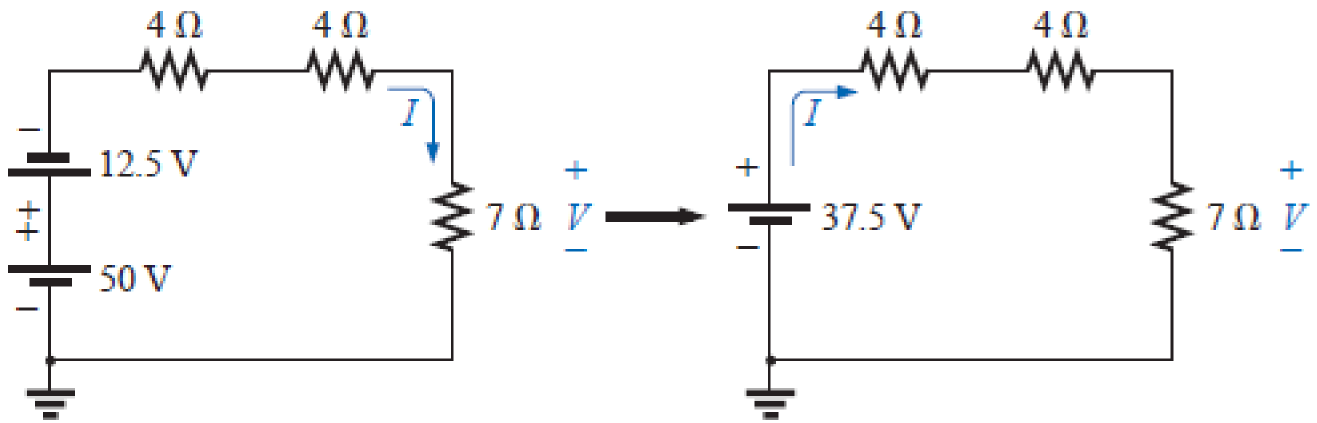


Solution: The network is redrawn in Fig. 5.22.

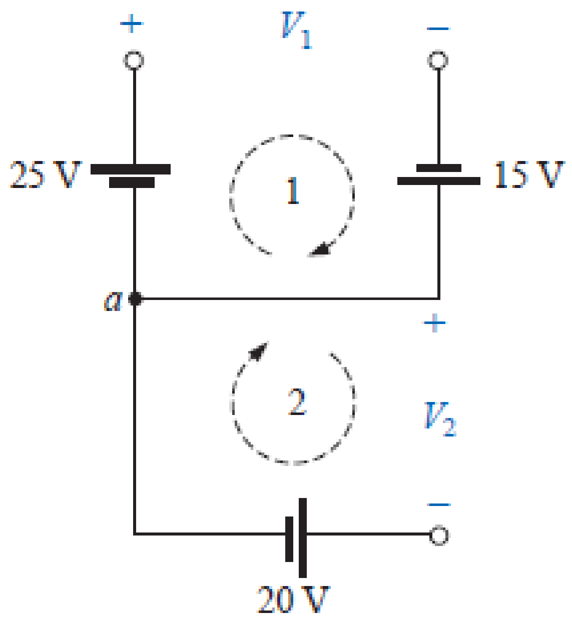
$$R_T = (2)(4\ \Omega) + 7\ \Omega = 15\ \Omega$$

$$I = \frac{E}{R_T} = \frac{37.5\ \text{V}}{15\ \Omega} = \mathbf{2.5\ A}$$

$$V_{7\Omega} = IR = (2.5\ \text{A})(7\ \Omega) = \mathbf{17.5\ V}$$



EXAMPLE Find V_1 and V_2 for the network



Solution: For path 1, starting at point a in a clockwise direction:

$$+25\ \text{V} - V_1 + 15\ \text{V} = 0$$

and $$V_1 = \mathbf{40\ V}$$

For path 2, starting at point a in a clockwise direction:

$$-V_2 - 20\ \text{V} = 0$$

and $$V_2 = \mathbf{-20\ V}$$

EXAMPLE

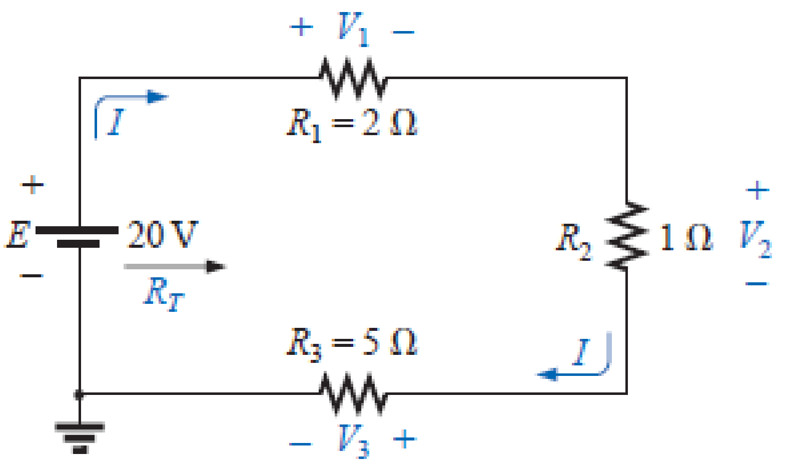
a. Find the total resistance

b. Calculate the source current I_s.

c. Determine the voltages V_1, V_2, and V_3.

d. Calculate the power dissipated by R_1, R_2, and R_3.

e. Determine the power delivered by the source, and compare it to the sum of the power levels of part (d).



Solutions:

a. $R_T = R_1 + R_2 + R_3 = 2\ \Omega + 1\ \Omega + 5\ \Omega = \mathbf{8\ \Omega}$

b. $I_s = \dfrac{E}{R_T} = \dfrac{20\text{ V}}{8\ \Omega} = \mathbf{2.5\ A}$

c. $V_1 = IR_1 = (2.5\text{ A})(2\ \Omega) = \mathbf{5\ V}$
$V_2 = IR_2 = (2.5\text{ A})(1\ \Omega) = \mathbf{2.5\ V}$
$V_3 = IR_3 = (2.5\text{ A})(5\ \Omega) = \mathbf{12.5\ V}$

d. $P_1 = V_1I_1 = (5\text{ V})(2.5\text{ A}) = \mathbf{12.5\ W}$
$P_2 = I_2^2R_2 = (2.5\text{ A})^2(1\ \Omega) = \mathbf{6.25\ W}$
$P_3 = V_3^2/R_3 = (12.5\text{ V})^2/5\ \Omega = \mathbf{31.25\ W}$

e. $P_{del} = EI = (20\text{ V})(2.5\text{ A}) = \mathbf{50\ W}$
$P_{del} = P_1 + P_2 + P_3$
$50\text{ W} = 12.5\text{ W} + 6.25\text{ W} + 31.25\text{ W}$
$50\text{ W} = 50\text{ W}$ (checks)