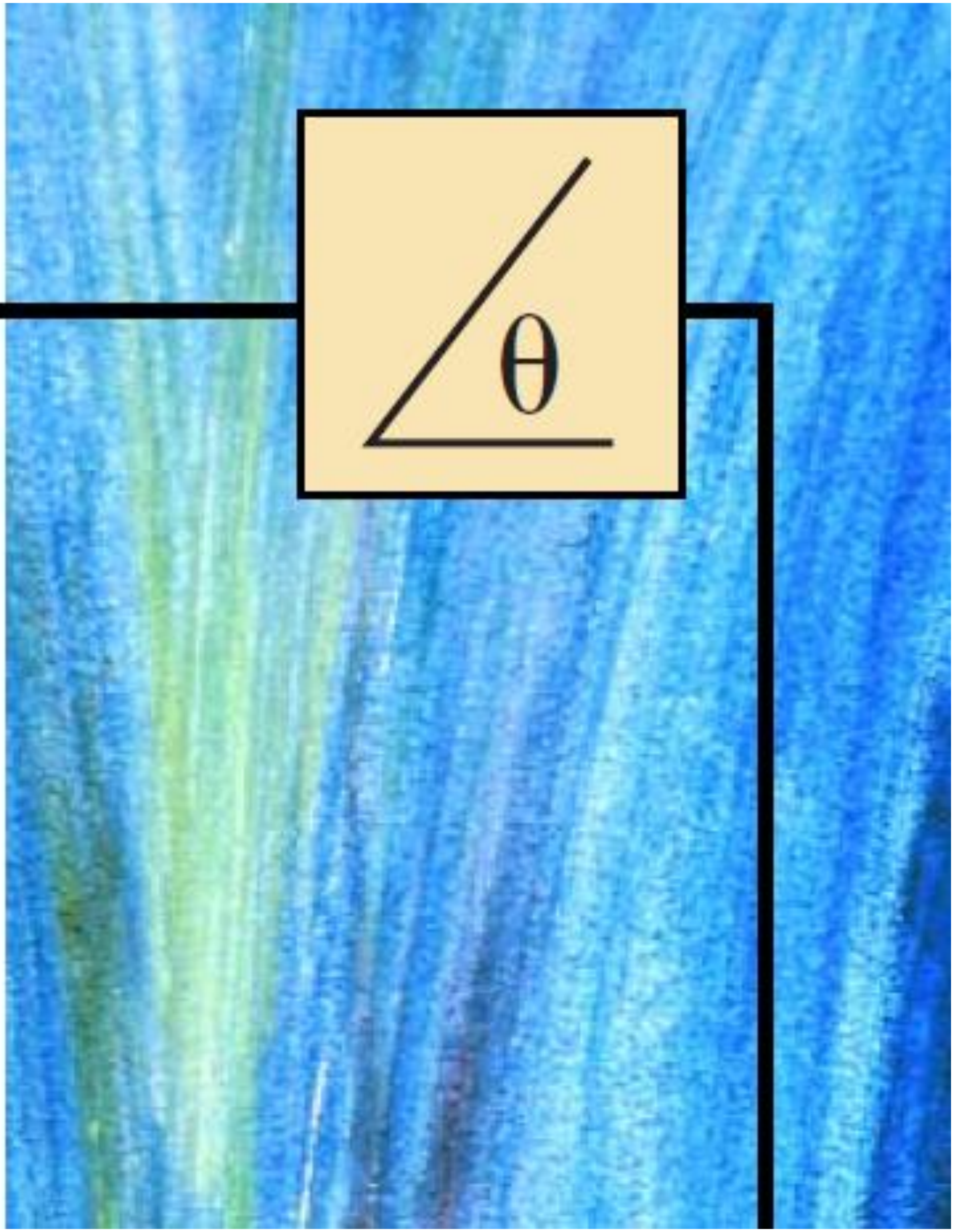
# 14

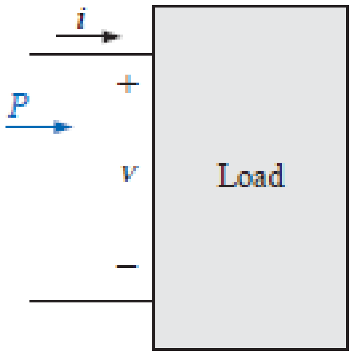
# The Basic Elements and Phasors



## 14.5 AVERAGE POWER AND POWER FACTOR

$$v = V_m \sin(\omega t + \theta_v)$$

$$i = I_m \sin(\omega t + \theta_i)$$




Average power

$$P = \frac{V_m I_m}{2} \cos \theta \qquad \text{(watts, W)}$$

$$\theta = |\theta_v - \theta_i|$$

where $P$ is the average power in watts. This equation can also be written

$$P = \left(\frac{V_m}{\sqrt{2}}\right)\left(\frac{I_m}{\sqrt{2}}\right) \cos \theta$$

or, since $V_{\text{eff}} = \frac{V_m}{\sqrt{2}}$ and $I_{\text{eff}} = \frac{I_m}{\sqrt{2}}$

Equation (14.14) becomes

$$P = V_{\text{eff}} I_{\text{eff}} \cos \theta \qquad \textbf{(14.15)}$$

## Resistor

In a purely resistive circuit, since $v$ and $i$ are in phase, $|\theta_v - \theta_i| = \theta = 0°$, and $\cos\theta = \cos 0° = 1$, so that

$$P = \frac{V_m I_m}{2} = V_{\text{eff}} I_{\text{eff}} \qquad \text{(W)}$$

Or, since $$I_{\text{eff}} = \frac{V_{\text{eff}}}{R}$$

then $$P = \frac{V_{\text{eff}}^2}{R} = I_{\text{eff}}^2 R \qquad \text{(W)}$$

## Inductor

In a purely inductive circuit, since $v$ leads $i$ by 90°, $|\theta_v - \theta_i| = \theta = |-90°| = 90°$. Therefore,

$$P = \frac{V_m I_m}{2} \cos 90° = \frac{V_m I_m}{2}(0) = \mathbf{0\ W}$$

*The average power or power dissipated by the ideal inductor (no associated resistance) is zero watts.*

## Capacitor

In a purely capacitive circuit, since $i$ leads $v$ by 90°, $|\theta_v - \theta_i| = \theta = |-90°| = 90°$. Therefore,

$$P = \frac{V_m I_m}{2}\cos(90°) = \frac{V_m I_m}{2}(0) = 0\text{ W}$$

*The average power or power dissipated by the ideal capacitor (no associated resistance) is zero watts.*

---

**EXAMPLE 14.10** Find the average power dissipated in a network whose input current and voltage are the following:

$$i = 5\sin(\omega t + 40°)$$
$$v = 10\sin(\omega t + 40°)$$

***Solution:*** Since $v$ and $i$ are in phase, the circuit appears to be purely resistive at the input terminals. Therefore,

$$P = \frac{V_m I_m}{2} = \frac{(10\text{ V})(5\text{ A})}{2} = \mathbf{25\text{ W}}$$

or

$$R = \frac{V_m}{I_m} = \frac{10\text{ V}}{5\text{ A}} = 2\ \Omega$$

and

$$P = \frac{V_{\text{eff}}^2}{R} = \frac{[(0.707)(10\text{ V})]^2}{2} = \mathbf{25\text{ W}}$$

or

$$P = I_{\text{eff}}^2 R = [(0.707)(5\text{ A})]^2(2) = \mathbf{25\text{ W}}$$

---

**EXAMPLE 14.11** Determine the average power delivered to networks having the following input voltage and current:

a. $v = 100 \sin(\omega t + 40°)$
$i = 20 \sin(\omega t + 70°)$

b. $v = 150 \sin(\omega t - 70°)$
$i = 3 \sin(\omega t - 50°)$

***Solutions:***

a. $V_m = 100, \quad \theta_v = 40°$
$I_m = 20, \quad \theta_i = 70°$
$\theta = |\theta_v - \theta_i| = |40° - 70°| = |-30°| = 30°$
and

$$P = \frac{V_m I_m}{2} \cos \theta = \frac{(100\text{ V})(20\text{ A})}{2} \cos(30°) = (1000\text{ W})(0.866)$$
$$= \mathbf{866\text{ W}}$$

b. $V_m = 150\text{ V}, \quad \theta_v = -70°$
$I_m = 3\text{ A}, \quad \theta_i = -50°$
$\theta = |\theta_v - \theta_i| = |-70° - (-50°)|$
$= |-70° + 50°| = |-20°| = 20°$
and

$$P = \frac{V_m I_m}{2} \cos \theta = \frac{(150\text{ V})(3\text{ A})}{2} \cos(20°) = (225\text{ W})(0.9397)$$
$$= \mathbf{211.43\text{ W}}$$

# Power Factor

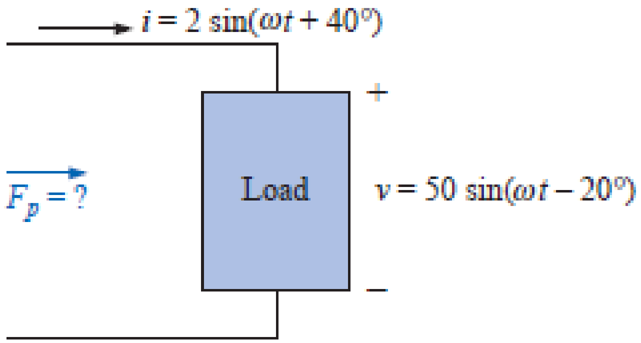
In the equation $P = (V_m I_m/2)\cos\theta$, the factor that has significant control over the delivered power level is the $\cos\theta$. No matter how large the voltage or current, if $\cos\theta = 0$, the power is zero; if $\cos\theta = 1$, the power delivered is a maximum. Since it has such control, the expression was given the name **power factor** and is defined by

$$\text{Power factor} = F_p = \cos\theta \qquad \textbf{(14.18)}$$

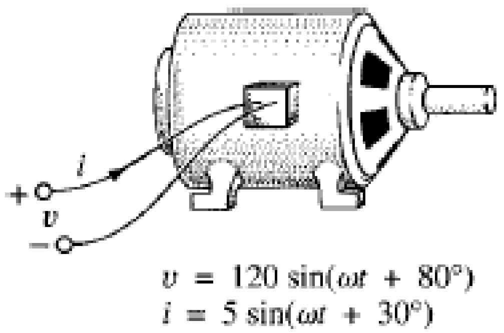
Power factor is said to be leading if I leads V
And said to be lagging if I lagges v

**EXAMPLE 14.12** Determine the power factors of the following loads, and indicate whether they are leading or lagging:

a. $F_p = \cos\theta = \cos|40° - (-20°)| = \cos 60° = \mathbf{0.5\ leading}$

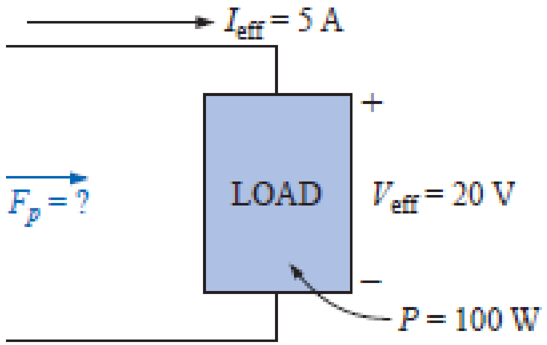


b. $F_p = \cos\theta\,|80° - 30°| = \cos 50° = \mathbf{0.6428\ lagging}$




c. $F_p = \cos\theta = \dfrac{P}{V_{\text{eff}} I_{\text{eff}}} = \dfrac{100\ \text{W}}{(20\ \text{V})(5\ \text{A})} = \dfrac{100\ \text{W}}{100\ \text{W}} = \mathbf{1}$

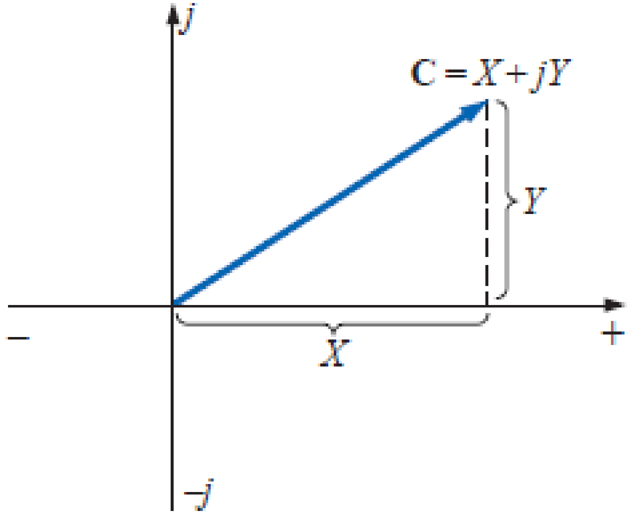
The load is resistive, and $F_p$ is neither leading nor lagging.

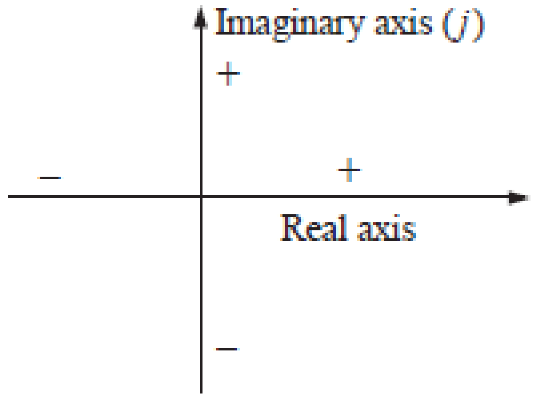


COMPLEX NUMBERS

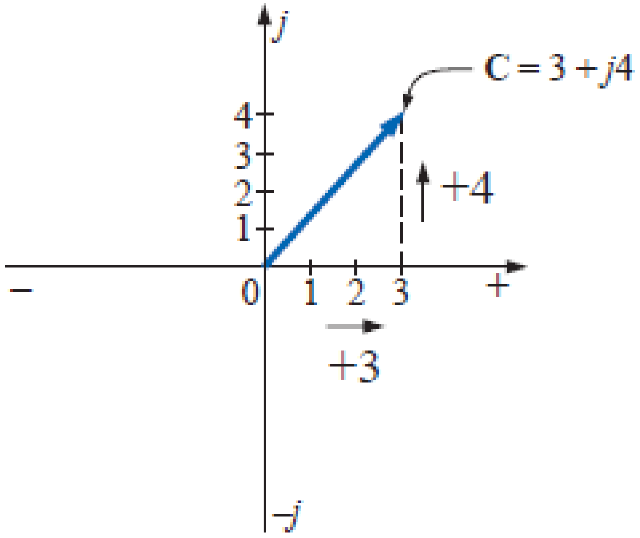
## 14.7 RECTANGULAR FORM

$$\mathbf{C} = X + jY$$




$$\mathbf{C} = 3 + j4$$

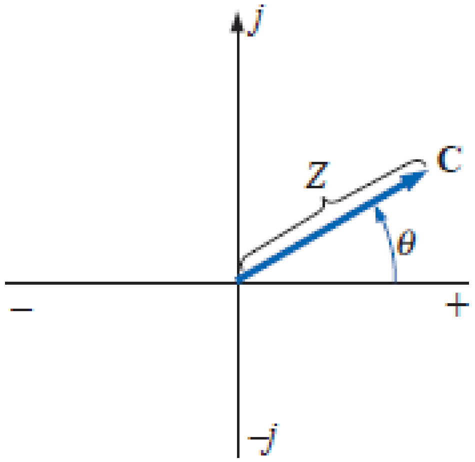


## 14.8 POLAR FORM

The format for the **polar form** is

$$\mathbf{C} = Z \angle \theta$$

with the letter $Z$ chosen from the sequence $X$, $Y$, $Z$.



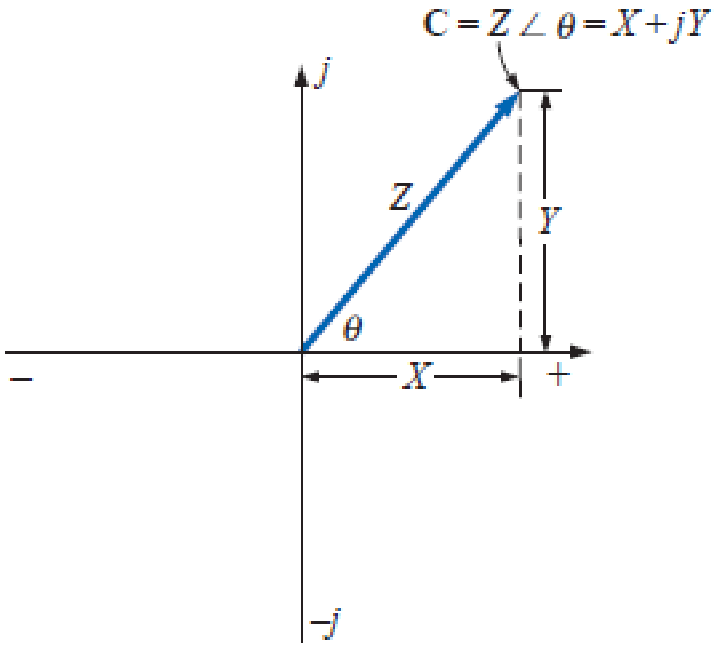

## 14.9 CONVERSION BETWEEN FORMS

The two forms are related by the following equations, as illustrated in Fig. 14.45.

### Rectangular to Polar

$$Z = \sqrt{X^2 + Y^2} \tag{14.23}$$

$$\theta = \tan^{-1} \frac{Y}{X} \tag{14.24}$$



## Polar to Rectangular

$$X = Z \cos \theta \quad (14.25)$$

$$Y = Z \sin \theta \quad (14.26)$$

**EXAMPLE 14.15** Convert the following from rectangular to p
form:

$$\mathbf{C} = 3 + j\,4 \qquad \text{(Fig. 14.46)}$$

***Solution:***

$$Z = \sqrt{(3)^2 + (4)^2} = \sqrt{25} = 5$$

$$\theta = \tan^{-1}\left(\frac{4}{3}\right) = 53.13^\circ$$

and $$\mathbf{C} = 5 \angle 53.13^\circ$$