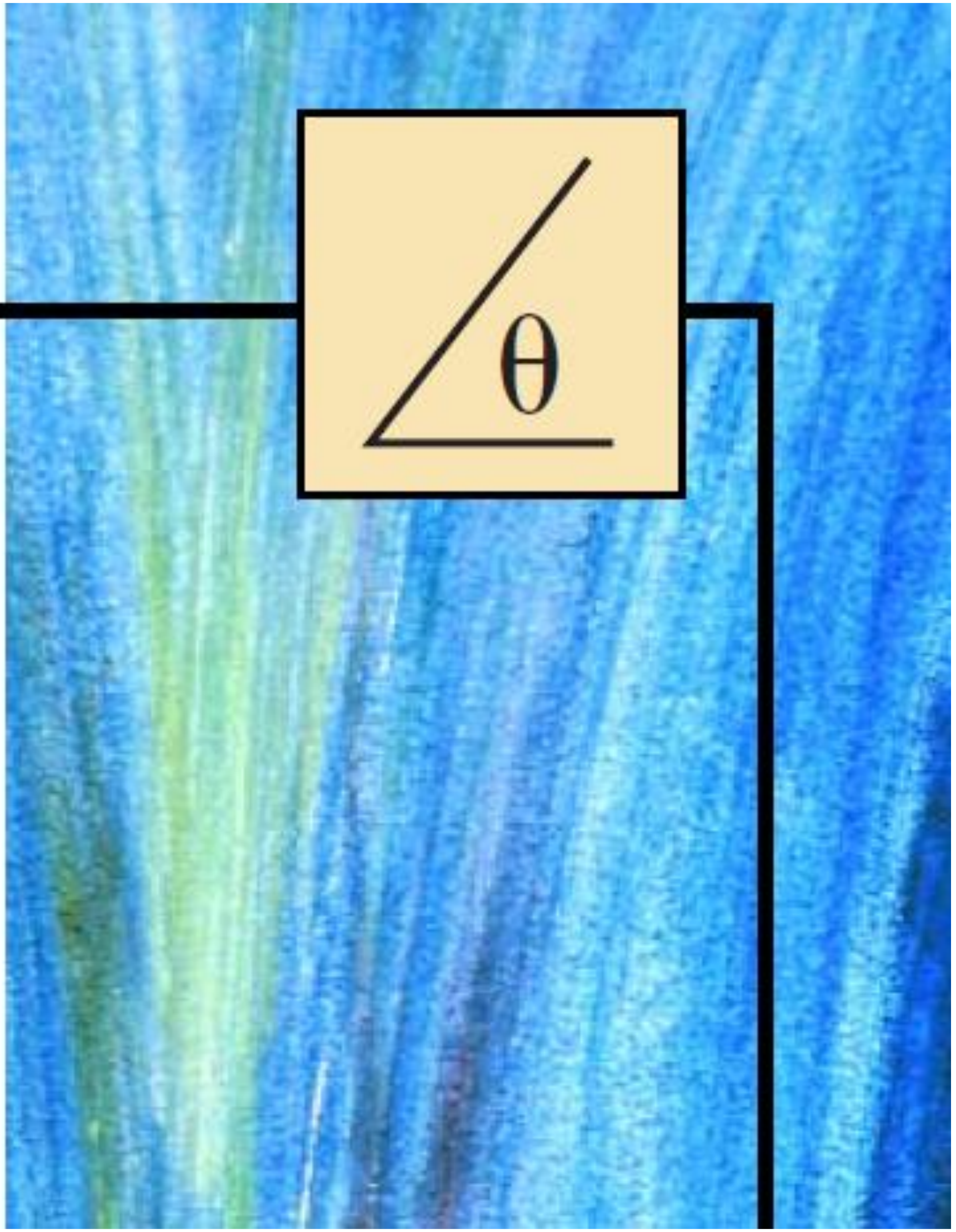
14

The Basic Elements and Phasors



EXAMPLE 14.16 Convert the following from polar to rectangular form:

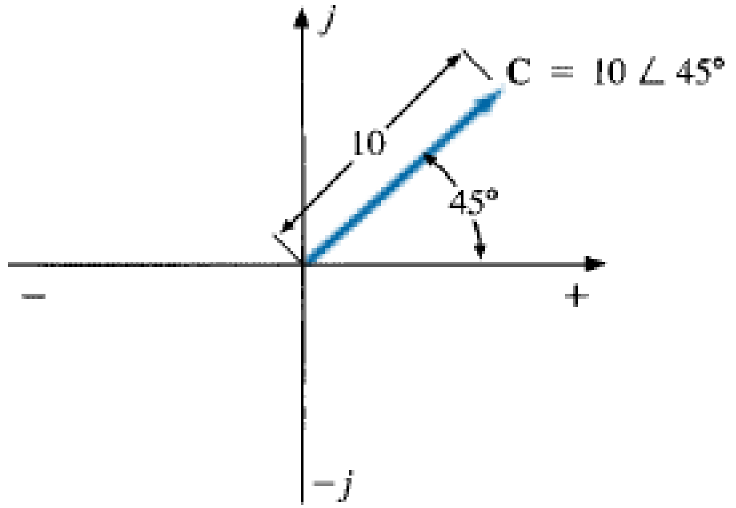
$$\mathbf{C} = 10 \angle 45^\circ \qquad \text{(Fig. 14.47)}$$

Solution:

$$X = 10 \cos 45^\circ = (10)(0.707) = 7.07$$

$$Y = 10 \sin 45^\circ = (10)(0.707) = 7.07$$

and $$\mathbf{C} = \mathbf{7.07 + j7.07}$$



Mathematical operation :

Addition

To add two or more complex numbers, simply add the real and imaginary parts separately. For example, if

$$\mathbf{C}_1 = \pm X_1 \pm j\,Y_1 \quad \text{and} \quad \mathbf{C}_2 = \pm X_2 \pm j\,Y_2$$

then $$\mathbf{C}_1 + \mathbf{C}_2 = (\pm X_1 \pm X_2) + j\,(\pm Y_1 \pm Y_2) \qquad \textbf{(14.30)}$$

EXAMPLE 14.19

a. Add $\mathbf{C}_1 = 2 + j4$ and $\mathbf{C}_2 = 3 + j1$.

b. Add $\mathbf{C}_1 = 3 + j6$ and $\mathbf{C}_2 = -6 + j3$.

Solutions:

a. By Eq. (14.30),

$$\mathbf{C}_1 + \mathbf{C}_2 = (2 + 3) + j(4 + 1) = \mathbf{5 + j5}$$

Note Fig. 14.52. An alternative method is

$$\begin{array}{r} 2 + j4 \\ \underline{3 + j1} \\ \downarrow \quad \downarrow \\ \mathbf{5 + j5} \end{array}$$

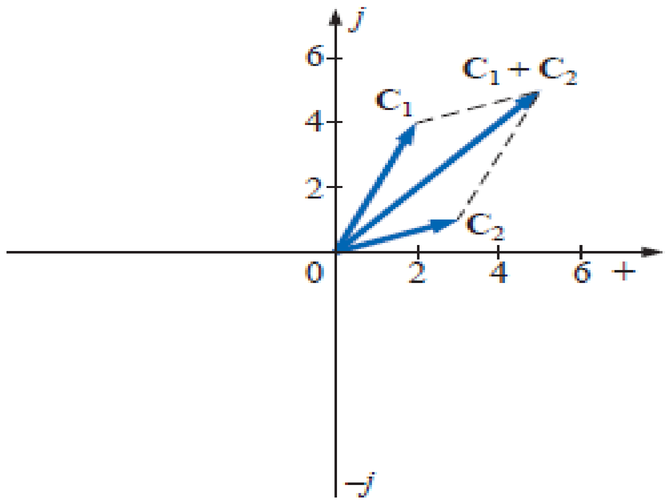


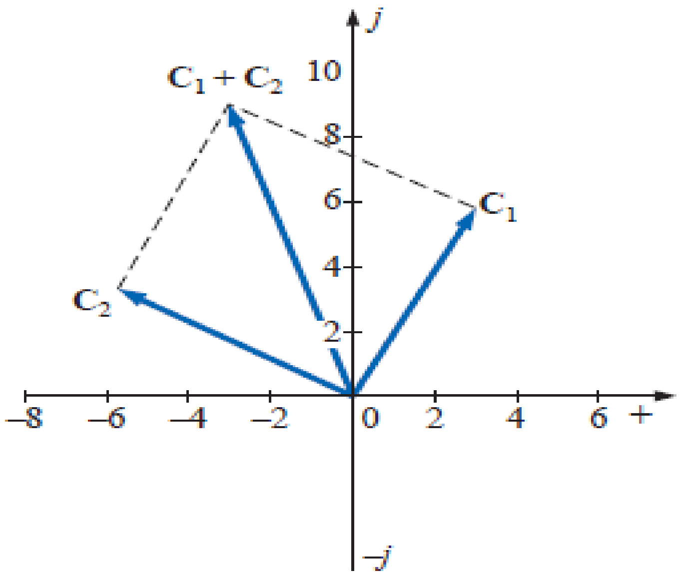
FIG. 14.52
Example 14.19(a).

b. By Eq. (14.30),

$$\mathbf{C}_1 + \mathbf{C}_2 = (3 - 6) + j(6 + 3) = \mathbf{-3 + j9}$$

Note Fig. 14.53. An alternative method is

$$\begin{array}{r} 3 + j6 \\ \underline{-6 + j3} \\ \downarrow \quad \downarrow \\ \mathbf{-3 + j9} \end{array}$$



Subtraction

In subtraction, the real and imaginary parts are again considered separately. For example, if

$$\mathbf{C}_1 = \pm X_1 \pm j\,Y_1 \quad \text{and} \quad \mathbf{C}_2 = \pm X_2 \pm j\,Y_2$$

then

$$\boxed{\mathbf{C}_1 - \mathbf{C}_2 = [\pm X_2 - (\pm X_2)] + j[\pm Y_1 - (\pm Y_2)]} \qquad \textbf{(14.31)}$$

EXAMPLE 14.20

a. Subtract $\mathbf{C}_2 = 1 + j\,4$ from $\mathbf{C}_1 = 4 + j\,6$.
b. Subtract $\mathbf{C}_2 = -2 + j\,5$ from $\mathbf{C}_1 = +3 + j\,3$.

Solutions:

a. By Eq. (14.31),

$$\mathbf{C}_1 - \mathbf{C}_2 = (4 - 1) + j\,(6 - 4) = \mathbf{3 + j2}$$

Note Fig. 14.54. An alternative method is

$$\begin{array}{c} 4 + j\,6 \\ \underline{-(1 + j\,4)} \\ \downarrow \quad \downarrow \\ \mathbf{3 + j2} \end{array}$$

b. By Eq. (14.31),

$$\mathbf{C}_1 - \mathbf{C}_2 = [3 - (-2)] + j(3 - 5) = \mathbf{5 - j2}$$

Note Fig. 14.55. An alternative method is

$$\begin{array}{r} 3 + j3 \\ \underline{-(-2 + j5)} \\ \downarrow \quad \downarrow \\ \mathbf{5 - j2} \end{array}$$

Multiplication

In *polar* form, the magnitudes are multiplied and the angles added algebraically. For example, for

$$\mathbf{C}_1 = Z_1 \angle \theta_1 \quad \text{and} \quad \mathbf{C}_2 = Z_2 \angle \theta_2$$

we write

$$\boxed{\mathbf{C}_1 \cdot \mathbf{C}_2 = Z_1 Z_2 \,\underline{/\theta_1 + \theta_2}} \tag{14.33}$$

EXAMPLE 14.23

a. Find $\mathbf{C}_1 \cdot \mathbf{C}_2$ if

$$\mathbf{C}_1 = 5 \angle 20^\circ \quad \text{and} \quad \mathbf{C}_2 = 10 \angle 30^\circ$$

b. Find $\mathbf{C}_1 \cdot \mathbf{C}_2$ if

$$\mathbf{C}_1 = 2 \angle -40^\circ \quad \text{and} \quad \mathbf{C}_2 = 7 \angle +120^\circ$$

Solutions:

a. $\mathbf{C}_1 \cdot \mathbf{C}_2 = (5 \angle 20^\circ)(10 \angle 30^\circ) = (5)(10) \,/20^\circ + 30^\circ = \mathbf{50 \angle 50^\circ}$

b. $\mathbf{C}_1 \cdot \mathbf{C}_2 = (2 \angle -40^\circ)(7 \angle +120^\circ) = (2)(7) \,/-40^\circ + 120^\circ$
$= \mathbf{14 \angle +80^\circ}$

To multiply a complex number in rectangular form by a real number requires that both the real part and the imaginary part be multiplied by the real number. For example,

$$(10)(2 + j3) = 20 + j30$$

and $\quad 50 \angle 0^\circ(0 + j6) = j300 = 300 \angle 90^\circ$

Division

In *polar* form, division is accomplished by simply dividing the magnitude of the numerator by the magnitude of the denominator and subtracting the angle of the denominator from that of the numerator. That is, for

$$\mathbf{C}_1 = Z_1 \angle \theta_1 \quad \text{and} \quad \mathbf{C}_2 = Z_2 \angle \theta_2$$

we write

$$\frac{\mathbf{C}_1}{\mathbf{C}_2} = \frac{Z_1}{Z_2} \angle \theta_1 - \theta_2 \qquad (14.35)$$

EXAMPLE 14.25

a. Find $\mathbf{C}_1/\mathbf{C}_2$ if $\mathbf{C}_1 = 15 \angle 10°$ and $\mathbf{C}_2 = 2 \angle 7°$.

b. Find $\mathbf{C}_1/\mathbf{C}_2$ if $\mathbf{C}_1 = 8 \angle 120°$ and $\mathbf{C}_2 = 16 \angle -50°$.

Solutions:

a. $\dfrac{\mathbf{C}_1}{\mathbf{C}_2} = \dfrac{15 \angle 10°}{2 \angle 7°} = \dfrac{15}{2} \angle 10° - 7° = \mathbf{7.5 \angle 3°}$

b. $\dfrac{\mathbf{C}_1}{\mathbf{C}_2} = \dfrac{8 \angle 120^\circ}{16 \angle -50^\circ} = \dfrac{8}{16} \underline{/120^\circ - (-50^\circ)} = \mathbf{0.5} \angle \mathbf{170^\circ}$

14.12 PHASORS

If we have voltage or current in time domain we can convert it to complex (phasor) domain as

$$v(t) = v_m.\sin(wt + \theta_v) \rightarrow V = \frac{v_m}{\sqrt{2}} \angle \theta_v$$

$$i(t) = I_m.\sin(wt + \theta_i) \rightarrow I = \frac{I_m}{\sqrt{2}} \angle \theta_i$$

EXAMPLE 14.31 Find the input voltage of the circuit of Fig. 14.65 if

$$\left. \begin{aligned} v_a &= 50 \sin(377t + 30^\circ) \\ v_b &= 30 \sin(377t + 60^\circ) \end{aligned} \right\} f = 60 \text{ Hz}$$

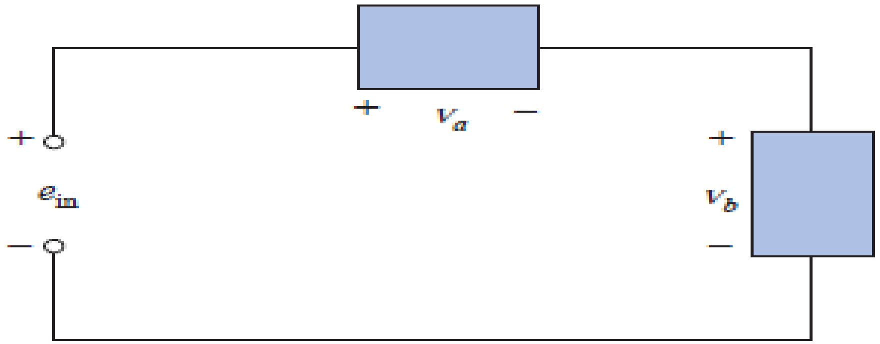


FIG. 14.65
Example 14.31.

Solution: Applying Kirchhoff's voltage law, we have

$$e_{in} = v_a + v_b$$

Converting from the time to the phasor domain yields

$$v_a = 50 \sin(377t + 30°) \Rightarrow \mathbf{V}_a = 35.35 \text{ V} \angle 30°$$
$$v_b = 30 \sin(377t + 60°) \Rightarrow \mathbf{V}_b = 21.21 \text{ V} \angle 60°$$

Converting from polar to rectangular form for addition yields

$$\mathbf{V}_a = 35.35 \text{ V} \angle 30° = 30.61 \text{ V} + j\,17.68 \text{ V}$$
$$\mathbf{V}_b = 21.21 \text{ V} \angle 60° = 10.61 \text{ V} + j\,18.37 \text{ V}$$

Then

$$\mathbf{E}_{in} = \mathbf{V}_a + \mathbf{V}_b = (30.61 \text{ V} + j\,17.68 \text{ V}) + (10.61 \text{ V} + j\,18.37 \text{ V})$$
$$= 41.22 \text{ V} + j\,36.05 \text{ V}$$

Converting from rectangular to polar form, we have

$$\mathbf{E}_{in} = 41.22 \text{ V} + j\,36.05 \text{ V} = 54.76 \text{ V} \angle 41.17°$$

Converting from the phasor to the time domain, we obtain

$$\mathbf{E}_{in} = 54.76 \text{ V} \angle 41.17° \Rightarrow e_{in} = \sqrt{2}(54.76) \sin(377t + 41.17°)$$

and $$e_{in} = \mathbf{77.43 \sin(377t + 41.17°)}$$

EXAMPLE 14.32 Determine the current i_2 for the network of Fig. 14.67.

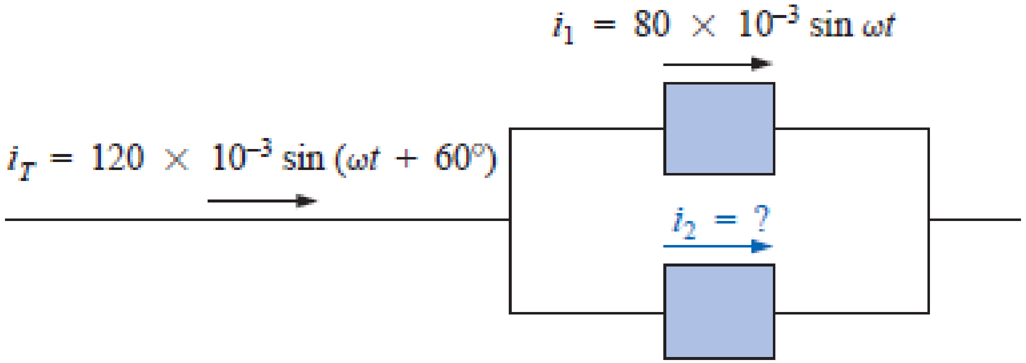


FIG. 14.67
Example 14.32.

Solution: Applying Kirchhoff's current law, we obtain

$$i_T = i_1 + i_2 \quad \text{or} \quad i_2 = i_T - i_1$$

Converting from the time to the phasor domain yields

$$i_T = 120 \times 10^{-3} \sin(\omega t + 60°) \Rightarrow 84.84 \text{ mA} \angle 60°$$
$$i_1 = 80 \times 10^{-3} \sin \omega t \Rightarrow 56.56 \text{ mA} \angle 0°$$

Converting from polar to rectangular form for subtraction yields

$$\mathbf{I}_T = 84.84 \text{ mA} \angle 60° = 42.42 \text{ mA} + j\,73.47 \text{ mA}$$
$$\mathbf{I}_1 = 56.56 \text{ mA} \angle 0° = 56.56 \text{ mA} + j\,0$$

Then

$$\begin{aligned}\mathbf{I}_2 &= \mathbf{I}_T - \mathbf{I}_1 \\ &= (42.42 \text{ mA} + j\,73.47 \text{ mA}) - (56.56 \text{ mA} + j\,0)\end{aligned}$$

and
$$\mathbf{I}_2 = -14.14 \text{ mA} + j\,73.47 \text{ mA}$$

Converting from rectangular to polar form, we have

$$\mathbf{I}_2 = 74.82 \text{ mA} \angle 100.89°$$

Converting from the phasor to the time domain, we have

$$\begin{aligned}\mathbf{I}_2 &= 74.82 \text{ mA} \angle 100.89° \Rightarrow \\ i_2 &= \sqrt{2}(74.82 \times 10^{-3}) \sin(\omega t + 100.89°)\end{aligned}$$

and
$$i_2 = \mathbf{105.8 \times 10^{-3} \sin(\omega t + 100.89°)}$$