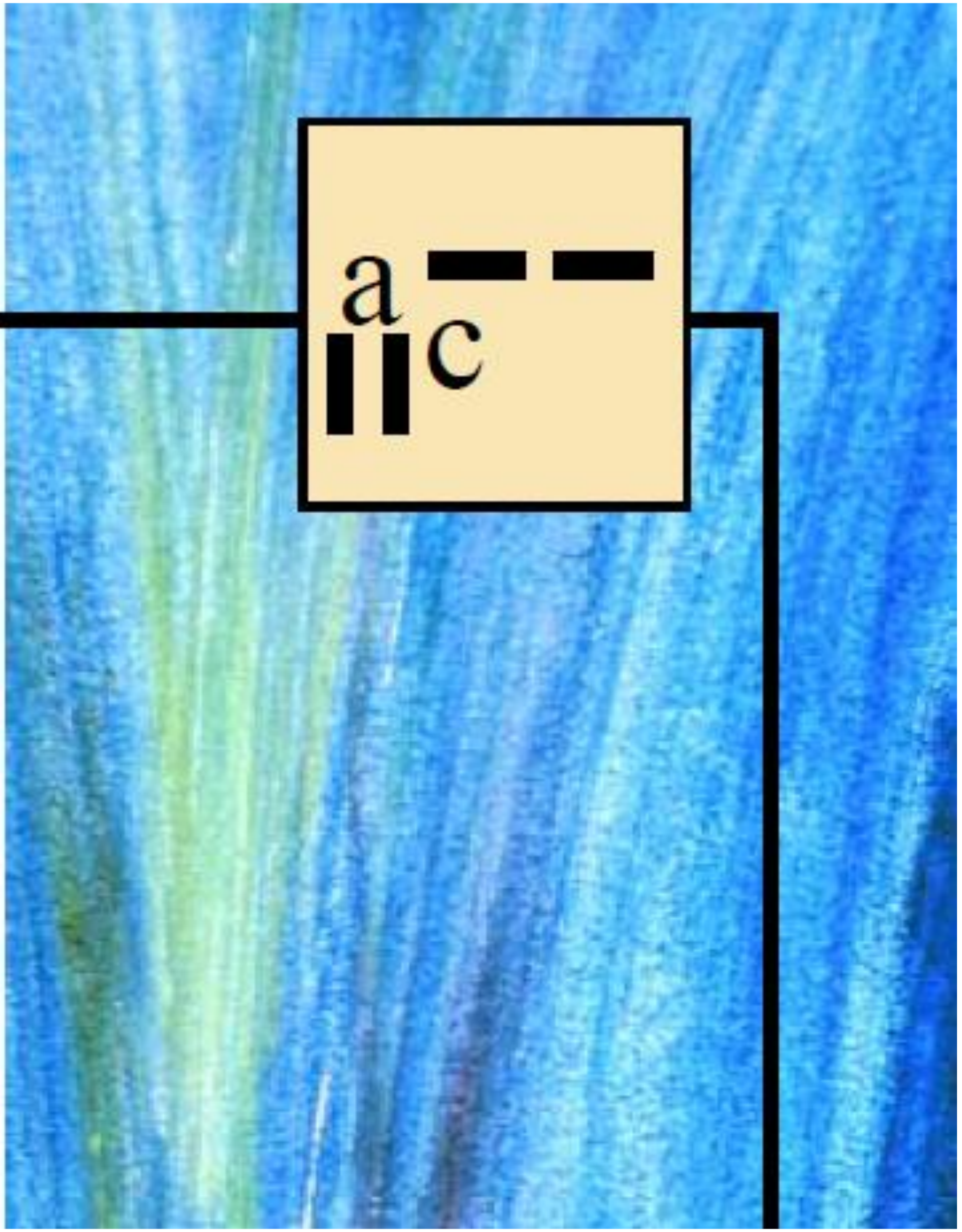
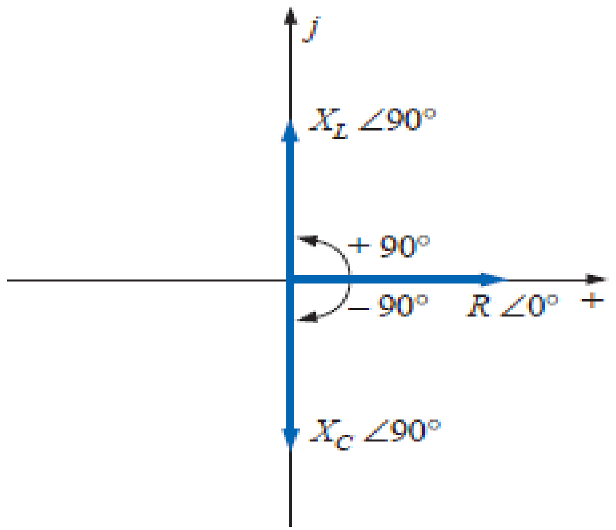
15

Series and Parallel ac Circuits



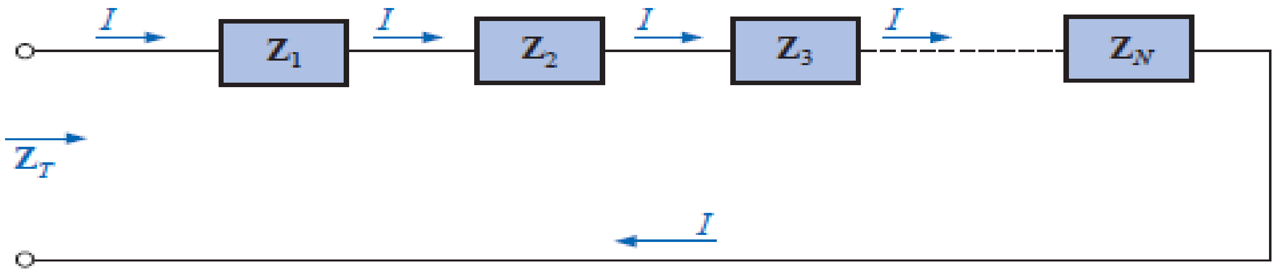
Z for all elements



15.3 SERIES CONFIGURATION

The overall properties of series ac circuits (Fig. 15.20) are the same as those for dc circuits. For instance, the total impedance of a system is the sum of the individual impedances:

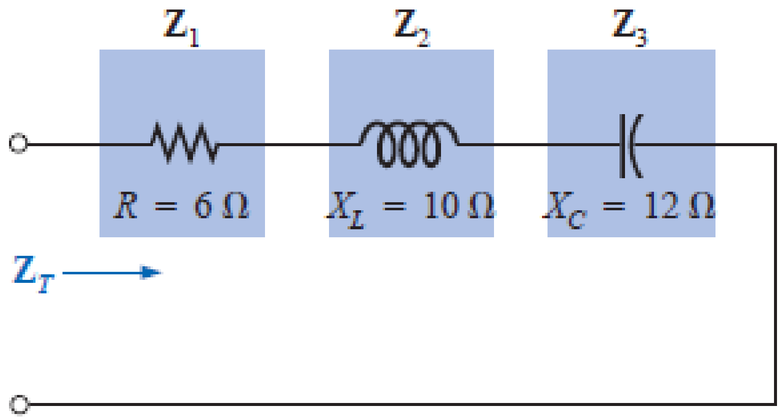
$$\mathbf{Z}_T = \mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_3 + \cdots + \mathbf{Z}_N \quad (15.4)$$

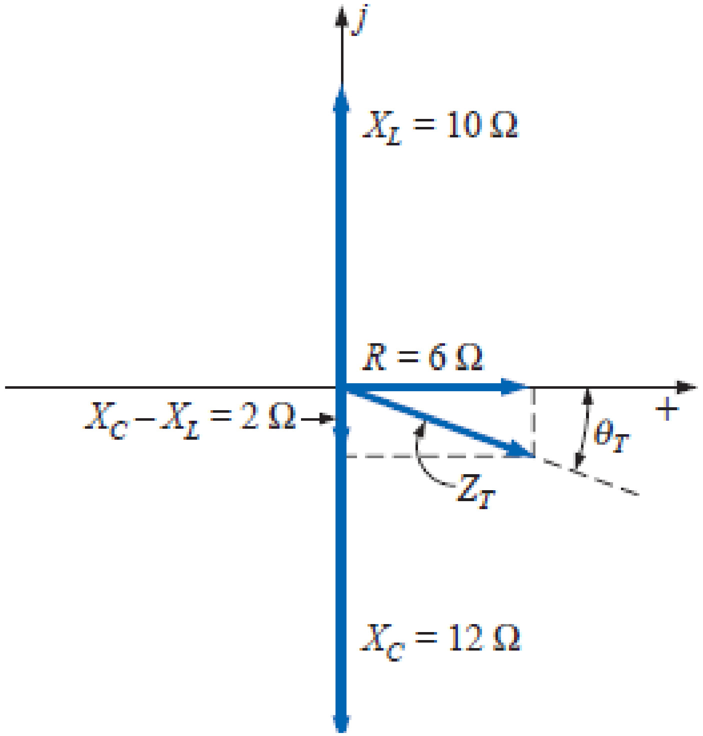


EXAMPLE 15.8 Determine the input impedance to the series networl of Fig. 15.23. Draw the impedance diagram.

Solution:

$$\mathbf{Z}_T = \mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_3$$
$$= R\angle 0° + X_L\angle 90° + X_C\angle -90°$$
$$= R + jX_L - jX_C$$
$$= R + j(X_L - X_C) = 6\ \Omega + j(10\ \Omega - 12\ \Omega) = 6\ \Omega - j2\ \Omega$$
$$\mathbf{Z}_T = \mathbf{6.325\ \Omega \angle -18.43°}$$





Series Ac network

$$\mathbf{Z}_T = \mathbf{Z}_1 + \mathbf{Z}_2$$

and

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}_T} \qquad (15.5)$$

The voltage across each element can then be found by another application of Ohm's law:

$$\mathbf{V}_1 = \mathbf{I}\mathbf{Z}_1 \qquad (15.6a)$$

$$\mathbf{V}_2 = \mathbf{I}\mathbf{Z}_2 \qquad (15.6b)$$

Kirchhoff's voltage law can then be applied in the same manner as it is employed for dc circuits. However, keep in mind that we are now dealing with the algebraic manipulation of quantities that have both magnitude and direction.

$$\mathbf{E} - \mathbf{V}_1 - \mathbf{V}_2 = 0$$

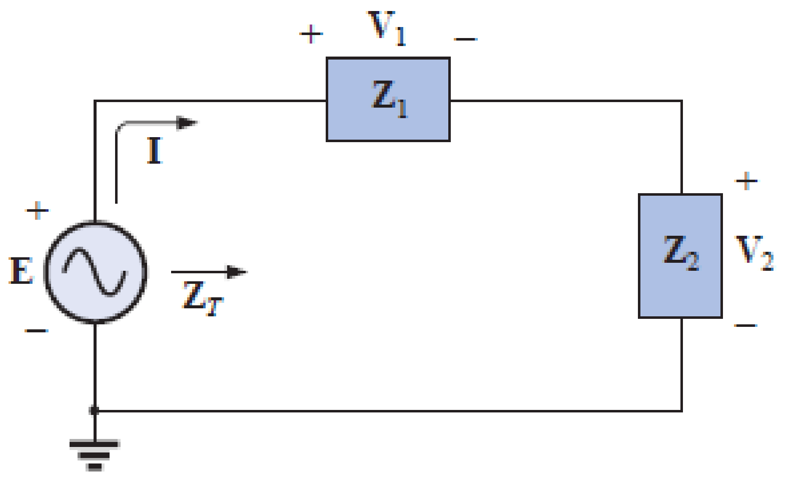
or

$$\mathbf{E} = \mathbf{V}_1 + \mathbf{V}_2 \qquad (15.7)$$

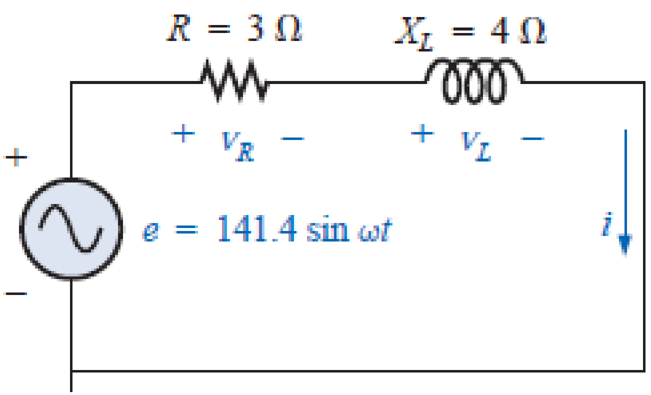
The power to the circuit can be determined by

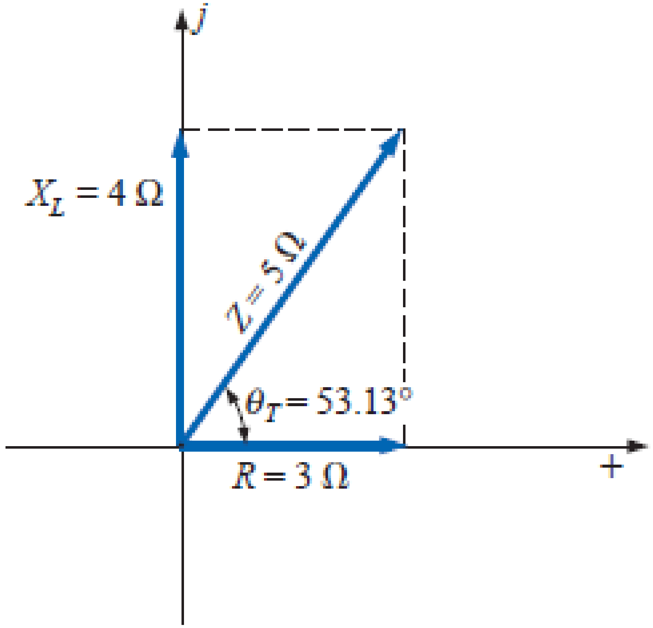
$$P = EI\cos\theta_T \qquad (15.8)$$

where θ_T is the phase angle between $\mathbf{E}$ and $\mathbf{I}$.



R-L





Phasor Notation

$$e = 141.4 \sin \omega t \Rightarrow \mathbf{E} = 100 \text{ V} \angle 0^\circ$$

Note Fig. 15.27.

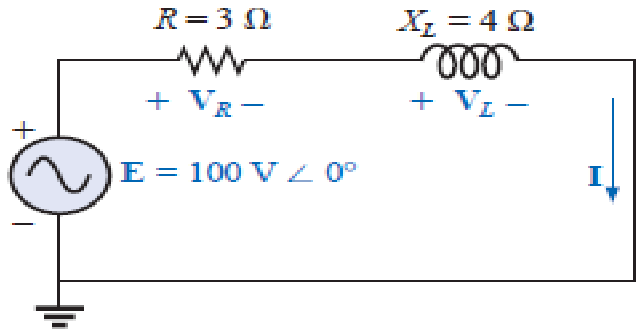


FIG. 15.27
Applying phasor notation to the network of Fig. 15.26.

$\mathbf{Z}_T$

$$\mathbf{Z}_T = \mathbf{Z}_1 + \mathbf{Z}_2 = 3\ \Omega \angle 0^\circ + 4\ \Omega \angle 90^\circ = 3\ \Omega + j4\ \Omega$$

and
$$\mathbf{Z}_T = 5\ \Omega \angle 53.13^\circ$$

Impedance diagram: See Fig. 15.28.

I

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}_T} = \frac{100 \text{ V} \angle 0^\circ}{5\ \Omega \angle 53.13^\circ} = \mathbf{20\ A \angle -53.13^\circ}$$

$\mathbf{V}_R$ and $\mathbf{V}_L$

Ohm's law:

$$\mathbf{V}_R = \mathbf{I}\mathbf{Z}_R = (20\text{ A}\angle -53.13^\circ)(3\ \Omega \angle 0^\circ) = \mathbf{60\ V}\angle\mathbf{-53.13^\circ}$$

$$\mathbf{V}_L = \mathbf{I}\mathbf{Z}_L = (20\text{ A}\angle -53.13^\circ)(4\ \Omega \angle 90^\circ) = \mathbf{80\ V}\angle\mathbf{36.87^\circ}$$

Kirchhoff's voltage law:

$$\Sigma_{\circlearrowright} \mathbf{V} = \mathbf{E} - \mathbf{V}_R - \mathbf{V}_L = 0$$

or

$$\mathbf{E} = \mathbf{V}_R + \mathbf{V}_L$$

In rectangular form,

$$\mathbf{V}_R = 60\text{ V}\angle -53.13^\circ = 36\text{ V} - j\,48\text{ V}$$

$$\mathbf{V}_L = 80\text{ V}\angle +36.87^\circ = 64\text{ V} + j\,48\text{ V}$$

and

$$\mathbf{E} = \mathbf{V}_R + \mathbf{V}_L = (36\text{ V} - j\,48\text{ V}) + (64\text{ V} + j\,48\text{ V}) = 100\text{ V} + j\,0 = 100\text{ V}\angle 0^\circ$$

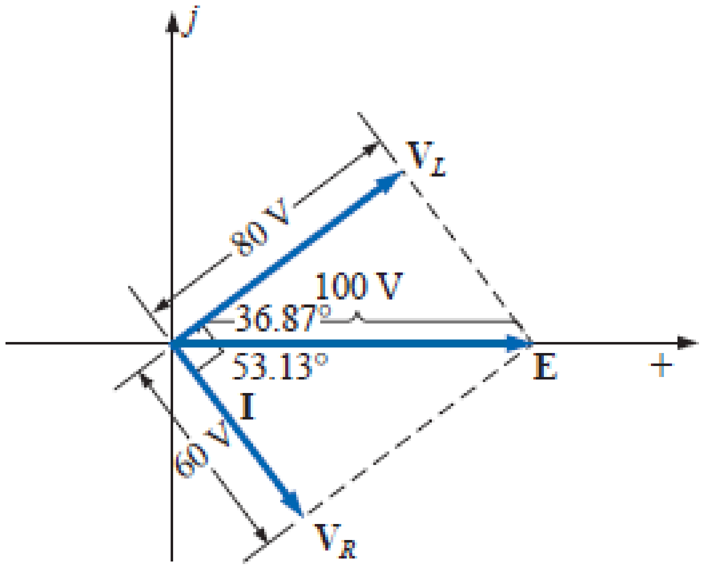


FIG. 15.29
Phasor diagram for the series R-L circuit of Fig. 15.26.

Power: The total power in watts delivered to the circuit is

$$\begin{aligned} P_T &= EI\cos\theta_T \\ &= (100\text{ V})(20\text{ A})\cos 53.13^\circ = (2000\text{ W})(0.6) \\ &= \mathbf{1200\ W} \end{aligned}$$

where E and I are effective values and θ_T is the phase angle between E and I, or

$$\begin{aligned} P_T &= I^2R \\ &= (20\text{ A})^2(3\ \Omega) = (400)(3) \\ &= \mathbf{1200\ W} \end{aligned}$$

where I is the effective value, or, finally,

$$\begin{aligned} P_T = P_R + P_L &= V_R I\cos\theta_R + V_L I\cos\theta_L \\ &= (60\text{ V})(20\text{ A})\cos 0^\circ + (80\text{ V})(20\text{ A})\cos 90^\circ \\ &= 1200\text{ W} + 0 \\ &= \mathbf{1200\ W} \end{aligned}$$

$$\cos\theta = \frac{P}{EI} = \frac{I^2R}{EI} = \frac{IR}{E} = \frac{R}{E/I} = \frac{R}{Z_T}$$

we find

$$\boxed{F_p = \cos\theta_T = \frac{R}{Z_T}}$$

$$F_p = \cos\theta = \frac{R}{Z_T} = \frac{3\ \Omega}{5\ \Omega} = \mathbf{0.6\ lagging}$$

R-L-C

Refer to Fig. 15.35.

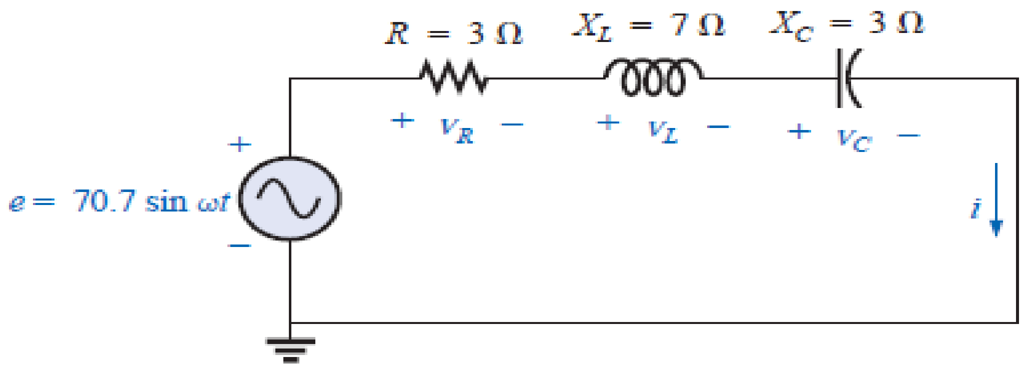
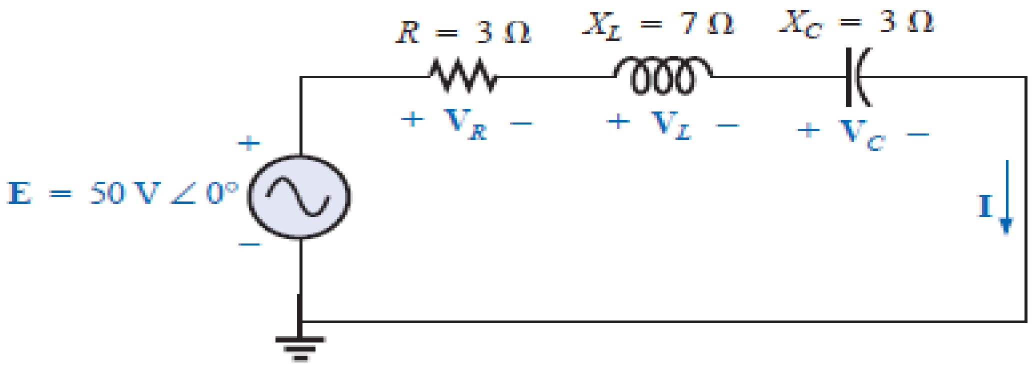
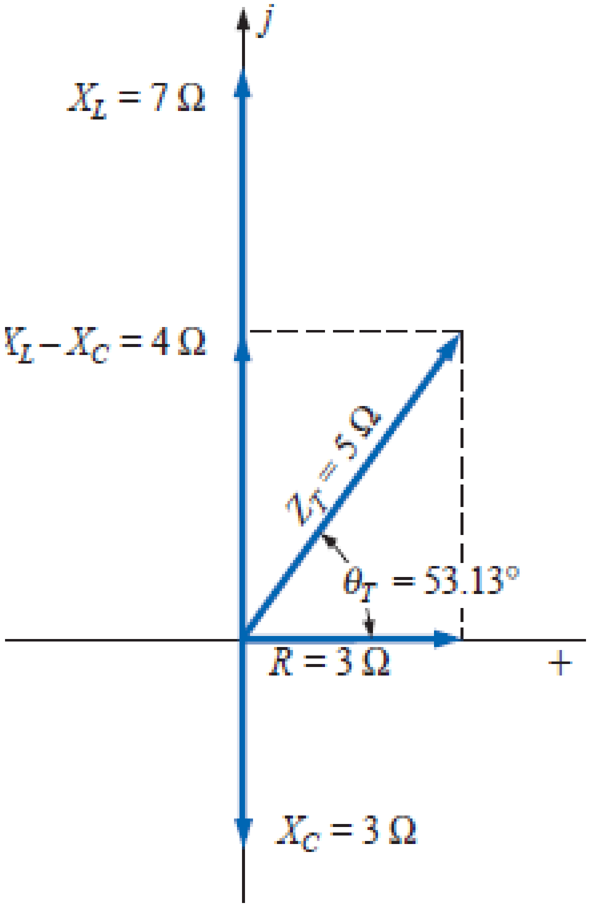


FIG. 15.35
Series R-L-C ac circuit.

Phasor Notation As shown in Fig. 15.36.





$\mathbf{Z}_T$

$$\mathbf{Z}_T = \mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_3 = R\ \angle 0° + X_L\ \angle 90° + X_C\ \angle -90°$$
$$= 3\ \Omega + j\,7\ \Omega - j\,3\ \Omega = 3\ \Omega + j\,4\ \Omega$$

and $$\mathbf{Z}_T = \mathbf{5\ \Omega\ \angle 53.13°}$$

Impedance diagram: As shown in Fig. 15.37.

I

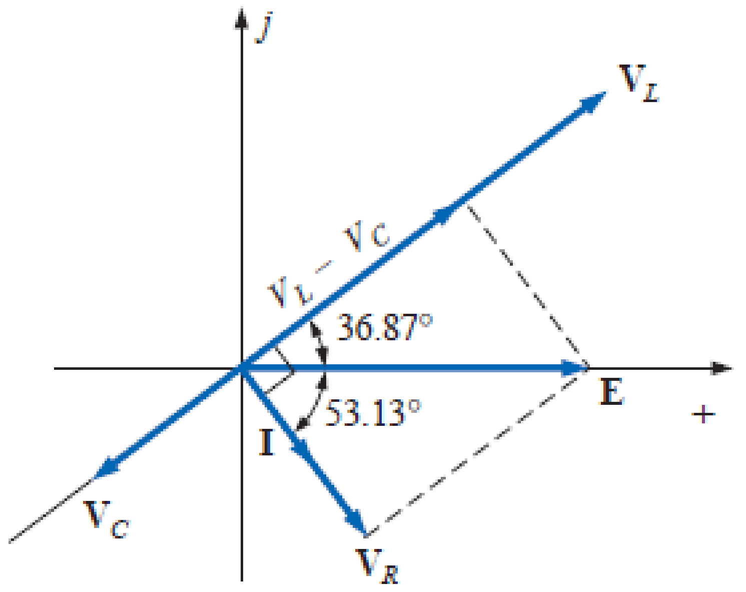
$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}_T} = \frac{50\ \text{V}\ \angle 0°}{5\ \Omega\ \angle 53.13°} = \mathbf{10\ A\ \angle -53.13°}$$

$\mathbf{V}_R$, $\mathbf{V}_L$, and $\mathbf{V}_C$

$$\mathbf{V}_R = \mathbf{I}\mathbf{Z}_R = (I\ \angle\theta)(R\ \angle 0°) = (10\ \text{A}\ \angle -53.13°)(3\ \Omega\ \angle 0°)$$
$$= \mathbf{30\ V\ \angle -53.13°}$$

$$\mathbf{V}_L = \mathbf{I}\mathbf{Z}_L = (I\ \angle\theta)(X_L\ \angle 90°) = (10\ \text{A}\ \angle -53.13°)(7\ \Omega\ \angle 90°)$$
$$= \mathbf{70\ V\ \angle 36.87°}$$

$$\mathbf{V}_C = \mathbf{I}\mathbf{Z}_C = (I\ \angle\theta)(X_C\ \angle -90°) = (10\ \text{A}\ \angle -53.13°)(3\ \Omega\ \angle -90°)$$
$$= \mathbf{30\ V\ \angle -143.13°}$$



Power: The total power in watts delivered to the circuit is

$$P_T = EI\cos\theta_T = (50\text{ V})(10\text{ A})\cos 53.13° = (500)(0.6) = \mathbf{300\ W}$$

or $$P_T = I^2R = (10\text{ A})^2(3\ \Omega) = (100)(3) = \mathbf{300\ W}$$

or

$$\begin{aligned} P_T &= P_R + P_L + P_C \\ &= V_RI\cos\theta_R + V_LI\cos\theta_L + V_CI\cos\theta_C \\ &= (30\text{ V})(10\text{ A})\cos 0° + (70\text{ V})(10\text{ A})\cos 90° + (30\text{ V})(10\text{ A})\cos 90° \\ &= (30\text{ V})(10\text{ A}) + 0 + 0 = \mathbf{300\ W} \end{aligned}$$

Power factor: The power factor of the circuit is

$$F_p = \cos\theta_T = \cos 53.13° = \mathbf{0.6\ lagging}$$

Using Eq. (15.9), we obtain

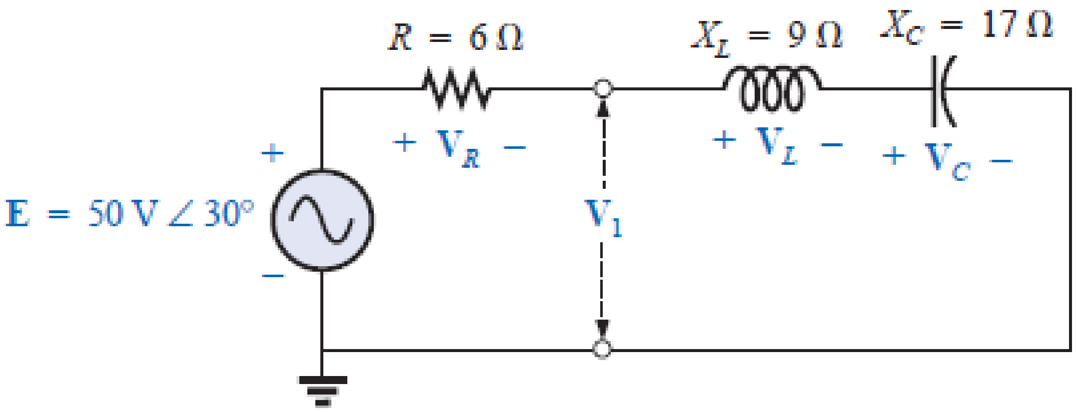
$$F_p = \cos\theta = \frac{R}{Z_T} = \frac{3\ \Omega}{5\ \Omega} = \mathbf{0.6\ lagging}$$

15.4 VOLTAGE DIVIDER RULE

The basic format for the **voltage divider rule** in ac circuits is exactly the same as that for dc circuits:

$$\mathbf{V}_x = \frac{\mathbf{Z}_x\mathbf{E}}{\mathbf{Z}_T} \qquad (15.10)$$

EXAMPLE 15.10 Using the voltage divider rule, find the unknown voltages $\mathbf{V}_R$, $\mathbf{V}_L$, $\mathbf{V}_C$, and $\mathbf{V}_1$ for the circuit of Fig. 15.41.



Solution:

$$\mathbf{V}_R = \frac{\mathbf{Z}_R\mathbf{E}}{\mathbf{Z}_R + \mathbf{Z}_L + \mathbf{Z}_C} = \frac{(6\ \Omega\ \angle 0^\circ)(50\ \text{V}\ \angle 30^\circ)}{6\ \Omega\ \angle 0^\circ + 9\ \Omega\ \angle 90^\circ + 17\ \Omega\ \angle -90^\circ}$$

$$= \frac{300\ \angle 30^\circ}{6 + j\,9 - j\,17} = \frac{300\ \angle 30^\circ}{6 - j\,8}$$

$$= \frac{300\ \angle 30^\circ}{10\ \angle -53.13^\circ} = \mathbf{30\ V\ \angle 83.13^\circ}$$

$$\mathbf{V}_L = \frac{\mathbf{Z}_L\mathbf{E}}{\mathbf{Z}_T} = \frac{(9\ \Omega\ \angle 90^\circ)(50\ \text{V}\ \angle 30^\circ)}{10\ \Omega\ \angle -53.13^\circ} = \frac{450\ \text{V}\angle 120^\circ}{10\ \angle -53.13^\circ}$$

$$= \mathbf{45\ V\ \angle 173.13^\circ}$$

$$\mathbf{V}_C = \frac{\mathbf{Z}_C\mathbf{E}}{\mathbf{Z}_T} = \frac{(17\ \Omega\ \angle -90^\circ)(50\ \text{V}\ \angle 30^\circ)}{10\ \Omega\ \angle -53.13^\circ} = \frac{850\ \text{V}\angle -60^\circ}{10\ \angle -53^\circ}$$

$$= \mathbf{85\ V\ \angle -6.87^\circ}$$

$$\mathbf{V}_1 = \frac{(\mathbf{Z}_L + \mathbf{Z}_C)\mathbf{E}}{\mathbf{Z}_T} = \frac{(9\ \Omega\ \angle 90^\circ + 17\ \Omega\ \angle -90^\circ)(50\ \text{V}\ \angle 30^\circ)}{10\ \Omega\ \angle -53.13^\circ}$$

$$= \frac{(8\ \angle -90^\circ)(50\ \angle 30^\circ)}{10\ \angle -53.13^\circ}$$

$$= \frac{400\ \angle -60^\circ}{10\ \angle -53.13^\circ} = \mathbf{40\ V\ \angle -6.87^\circ}$$