15

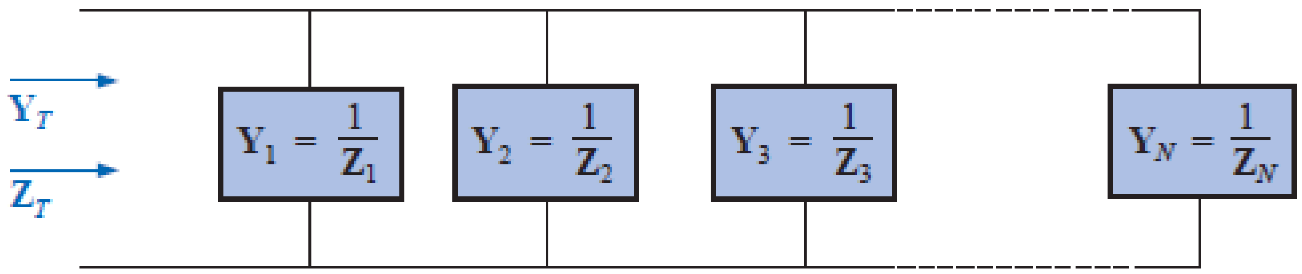
Series and Parallel ac Circuits

PARALLEL ac CIRCUITS

ADMITTANCE

we define **admittance** (**Y**) as being equal to 1/**Z**.

$$\mathbf{Y}_T = \mathbf{Y}_1 + \mathbf{Y}_2 + \mathbf{Y}_3 + \cdots + \mathbf{Y}_N \quad (15.16)$$



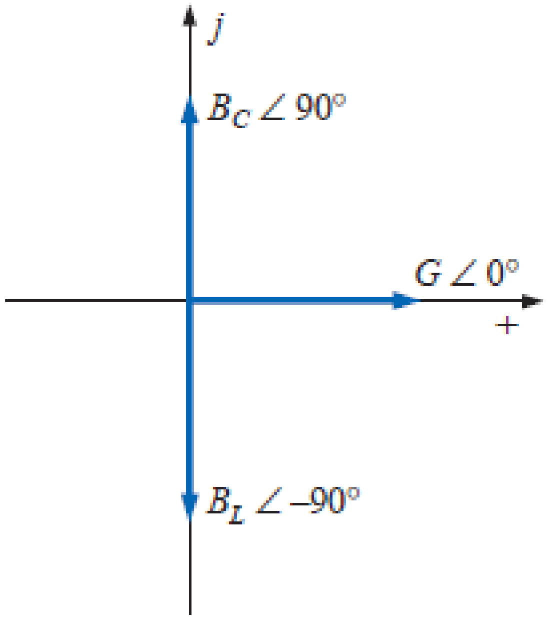


FIG. 15.55
Admittance diagram.

$$\mathbf{Y}_R = \frac{1}{\mathbf{Z}_R} = \frac{1}{R \angle 0^\circ} = G \angle 0^\circ$$

$$\mathbf{Y}_L = \frac{1}{\mathbf{Z}_L} = \frac{1}{X_L \angle 90^\circ} = \frac{1}{X_L} \angle -90^\circ$$

$$B_L = \frac{1}{X_L} \quad \text{(siemens, S)}$$

$$\mathbf{Y}_L = B_L \angle -90^\circ$$

$$\mathbf{Y}_C = \frac{1}{\mathbf{Z}_C} = \frac{1}{X_C \angle -90^\circ} = \frac{1}{X_C} \angle 90^\circ$$

$$B_C = \frac{1}{X_C} \quad \text{(siemens, S)}$$

$$\mathbf{Y}_C = B_C \angle 90^\circ$$

EXAMPLE 15.12 For the network of Fig. 15.56:

a. Find the admittance of each parallel branch.
b. Determine the input admittance.
c. Calculate the input impedance.
d. Draw the admittance diagram.

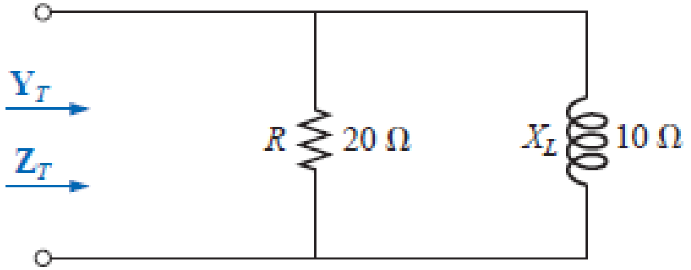


FIG. 15.56
Example 15.12.

Solutions:

a. $$\mathbf{Y}_R = G \angle 0° = \frac{1}{R} \angle 0° = \frac{1}{20\ \Omega} \angle 0°$$

$$= \mathbf{0.05\ S \angle 0° = 0.05\ S + j\ 0}$$

$$\mathbf{Y}_L = B_L \angle -90° = \frac{1}{X_L} \angle -90° = \frac{1}{10\ \Omega} \angle -90°$$

$$= \mathbf{0.1\ S \angle -90° = 0 - j\ 0.1\ S}$$

b. $$\mathbf{Y}_T = \mathbf{Y}_R + \mathbf{Y}_L = (0.05\ S + j\ 0) + (0 - j\ 0.1\ S)$$

$$= \mathbf{0.05\ S - j\ 0.1\ S} = G - jB_L$$

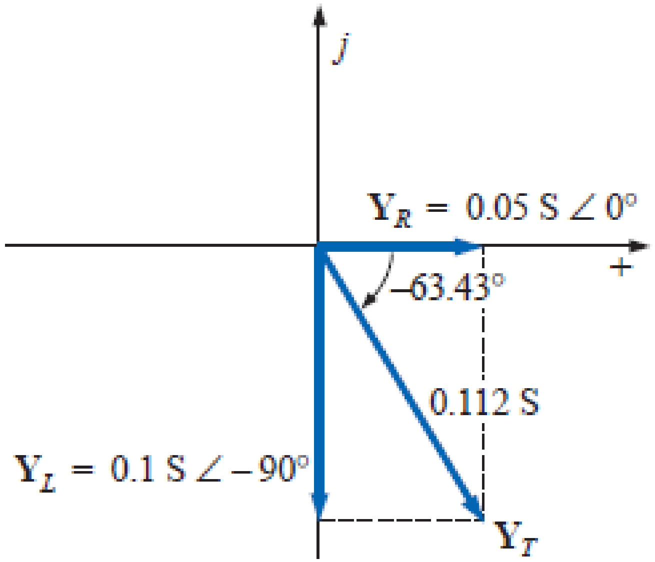
c. $$\mathbf{Z}_T = \frac{1}{\mathbf{Y}_T} = \frac{1}{0.05\ S - j\ 0.1\ S} = \frac{1}{0.112\ S \angle -63.43°}$$

$$= \mathbf{8.93\ \Omega \angle 63.43°}$$

or Eq. (15.17):

$$\mathbf{Z}_T = \frac{\mathbf{Z}_R\mathbf{Z}_L}{\mathbf{Z}_R + \mathbf{Z}_L} = \frac{(20\ \Omega \angle 0°)(10\ \Omega \angle 90°)}{20\ \Omega + j\ 10\ \Omega}$$

$$= \frac{200\ \Omega \angle 90°}{22.36 \angle 26.57°} = \mathbf{8.93\ \Omega \angle 63.43°}$$



15.8 PARALLEL ac NETWORKS

For the representative parallel ac network of Fig. 15.61, the total impedance or admittance is determined as described in the previous section, and the source current is determined by Ohm's law as follows:

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}_T} = \mathbf{E}\mathbf{Y}_T \quad (15.28)$$

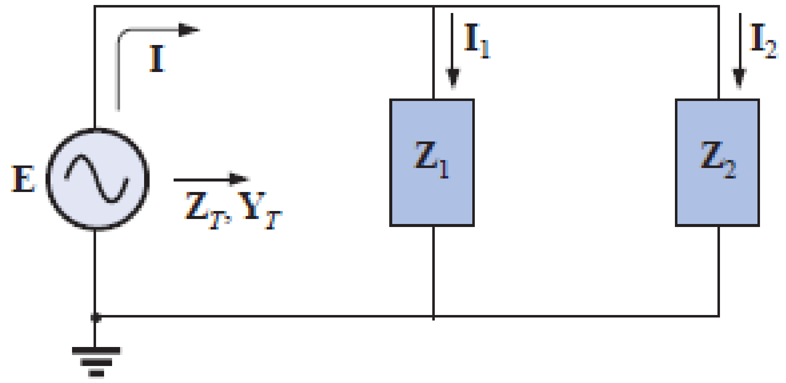


FIG. 15.61
Parallel ac network.

Since the voltage is the same across parallel elements, the current through each branch can then be found through another application of Ohm's law:

$$\mathbf{I}_1 = \frac{\mathbf{E}}{\mathbf{Z}_1} = \mathbf{E}\mathbf{Y}_1 \quad (15.29a)$$

$$\mathbf{I}_2 = \frac{\mathbf{E}}{\mathbf{Z}_2} = \mathbf{E}\mathbf{Y}_2 \quad (15.29b)$$

Kirchhoff's current law can then be applied in the same manner as employed for dc networks. However, keep in mind that we are now dealing with the algebraic manipulation of quantities that have both magnitude and direction.

$$\mathbf{I} - \mathbf{I}_1 - \mathbf{I}_2 = 0$$

or

$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 \qquad (15.30)$$

The power to the network can be determined by

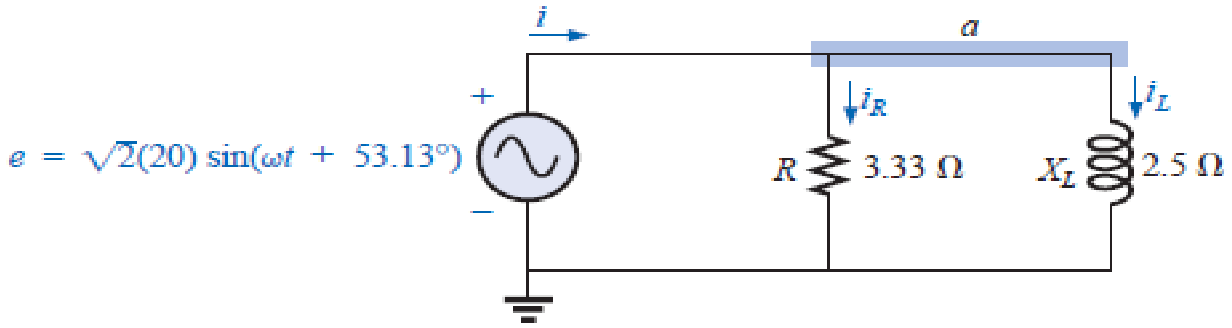
$$P = EI \cos \theta_T \qquad (15.31)$$

where θ_T is the phase angle between **E** and **I**.

Let us now look at a few examples carried out in great detail for the first exposure.

R-L

Refer to Fig. 15.62.



Phasor Notation As shown in Fig. 15.63.

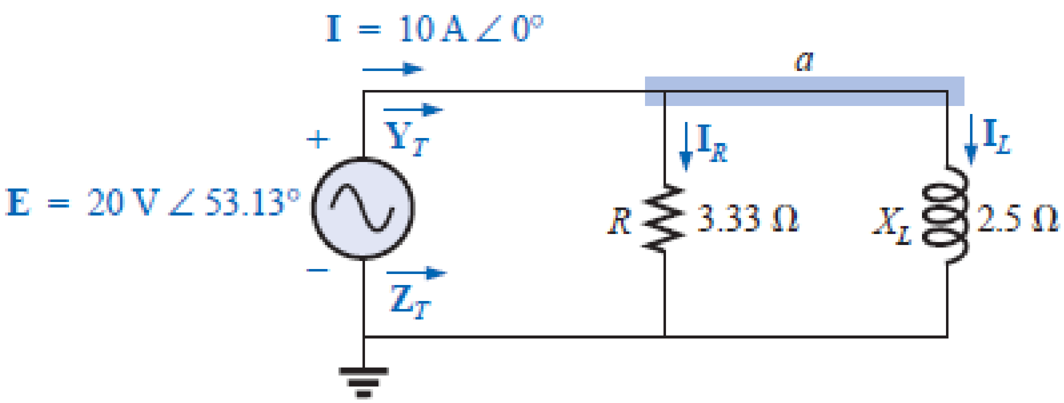


FIG. 15.63
Applying phasor notation to the network of Fig. 15.62.

Admittance diagram: As shown in Fig. 15.64.

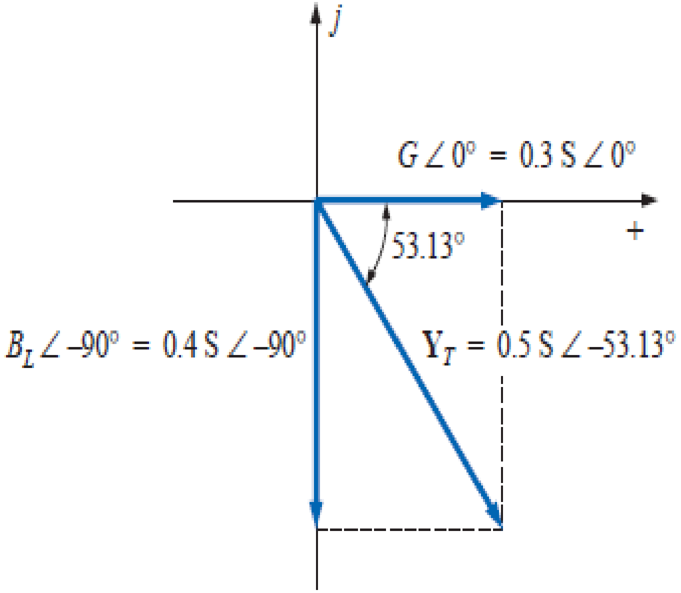


FIG. 15.64

$\mathbf{Y}_T$ and $\mathbf{Z}_T$

$$\mathbf{Y}_T = \mathbf{Y}_R + \mathbf{Y}_L$$
$$= G\angle 0° + B_L \angle -90° = \frac{1}{3.33\ \Omega}\angle 0° + \frac{1}{2.5\ \Omega}\angle -90°$$
$$= 0.3\ \text{S}\angle 0° + 0.4\ \text{S}\angle -90° = 0.3\ \text{S} - j\,0.4\ \text{S}$$
$$= \mathbf{0.5\ S}\angle\mathbf{-53.13°}$$
$$\mathbf{Z}_T = \frac{1}{\mathbf{Y}_T} = \frac{1}{0.5\ \text{S}\angle -53.13°} = \mathbf{2\ \Omega}\angle\mathbf{53.13°}$$

I

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}_T} = \mathbf{E}\mathbf{Y}_T = (20\text{ V} \angle 53.13°)(0.5\text{ S} \angle -53.13°) = \mathbf{10\text{ A} \angle 0°}$$

$\mathbf{I}_R$ and $\mathbf{I}_L$

$$\mathbf{I}_R = \frac{E \angle \theta}{R \angle 0°} = (E \angle \theta)(G \angle 0°)$$
$$= (20\text{ V} \angle 53.13°)(0.3\text{ S} \angle 0°) = \mathbf{6\text{ A} \angle 53.13°}$$
$$\mathbf{I}_L = \frac{E \angle \theta}{X_L \angle 90°} = (E \angle \theta)(B_L \angle -90°)$$
$$= (20\text{ V} \angle 53.13°)(0.4\text{ S} \angle -90°)$$
$$= \mathbf{8\text{ A} \angle -36.87°}$$

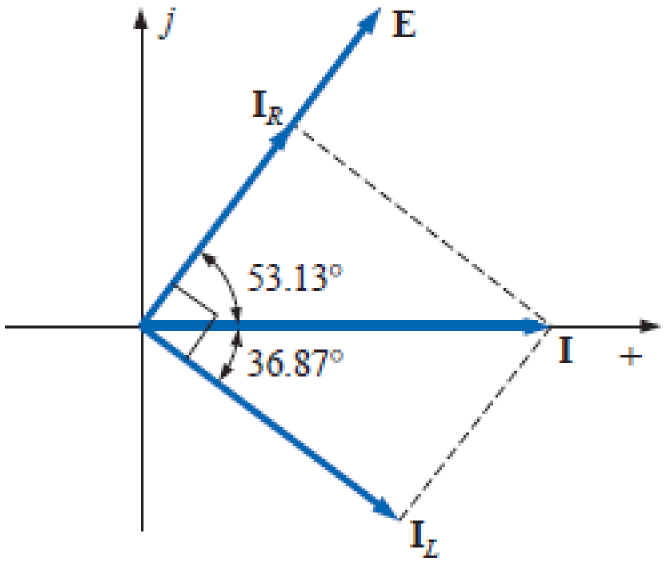


FIG. 15.65
Phasor diagram for the parallel R-L network

Power: The total power in watts delivered to the circuit is

$$P_T = EI \cos \theta_T$$
$$= (20\text{ V})(10\text{ A}) \cos 53.13° = (200\text{ W})(0.6)$$
$$= \mathbf{120\text{ W}}$$

or $$P_T = I^2R = \frac{V_R^2}{R} = V_R^2 G = (20\text{ V})^2(0.3\text{ S}) = \mathbf{120\text{ W}}$$

or, finally,

$$\begin{aligned}P_T &= P_R + P_L = EI_R\cos\theta_R + EI_L\cos\theta_L\\ &= (20\text{ V})(6\text{ A})\cos 0° + (20\text{ V})(8\text{ A})\cos 90° = 120\text{ W} + 0\\ &= \mathbf{120\ W}\end{aligned}$$

Power factor: The power factor of the circuit is

$$F_p = \cos\theta_T = \cos 53.13° = \mathbf{0.6\ lagging}$$

or, through an analysis similar to that employed for a series ac circuit,

$$\cos\theta_T = \frac{P}{EI} = \frac{E^2/R}{EI} = \frac{EG}{I} = \frac{G}{I/V} = \frac{G}{Y_T}$$

and

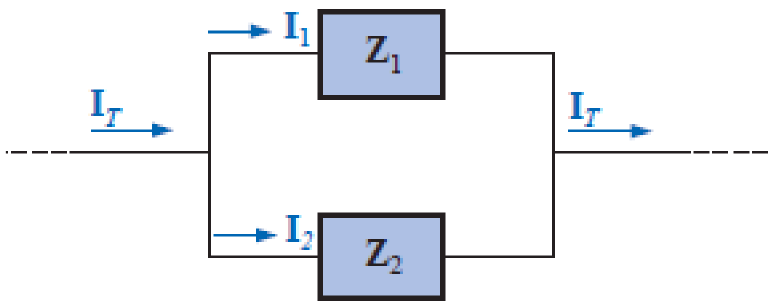
$$F_p = \cos\theta_T = \frac{G}{Y_T} \tag{15.32}$$

where G and Y_T are the magnitudes of the total conductance and admittance of the parallel network. For this case,

$$F_p = \cos\theta_T = \frac{0.3\text{ S}}{0.5\text{ S}} = \mathbf{0.6\ lagging}$$

CURRENT DIVIDER RULE

$$\mathbf{I}_1 = \frac{\mathbf{Z}_2\mathbf{I}_T}{\mathbf{Z}_1 + \mathbf{Z}_2} \quad \text{or} \quad \mathbf{I}_2 = \frac{\mathbf{Z}_1\mathbf{I}_T}{\mathbf{Z}_1 + \mathbf{Z}_2}$$



EXAMPLE 15.16 Using the current divider rule, find the current through each parallel branch of Fig. 15.78.

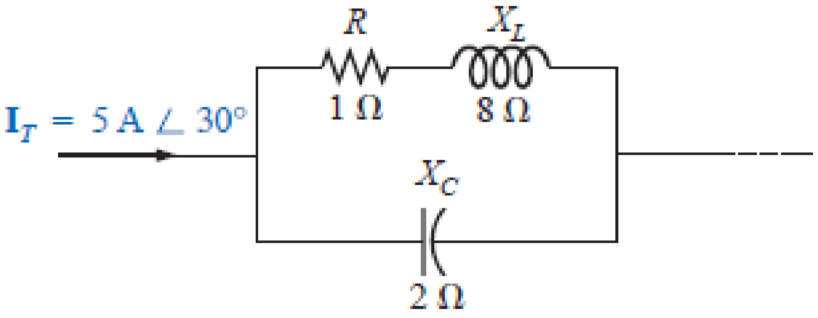


FIG. 15.78
Example 15.16.

Solution:

$$\mathbf{I}_{R\text{-}L} = \frac{\mathbf{Z}_C\mathbf{I}_T}{\mathbf{Z}_C + \mathbf{Z}_{R\text{-}L}} = \frac{(2\ \Omega \angle -90^\circ)(5\ \text{A} \angle 30^\circ)}{-j\,2\ \Omega + 1\ \Omega + j\,8\ \Omega} = \frac{10\ \text{A} \angle -60^\circ}{1 + j\,6}$$

$$= \frac{10\ \text{A} \angle -60^\circ}{6.083 \angle 80.54^\circ} \cong \mathbf{1.644\ A \angle -140.54^\circ}$$