Palestine Technical University- Kadoorie (PTUK)

Mechanical Engineering Department

12210244: Dynamics

Summer Semester, 2023/2024

This is an explanation of the Dynamics course

offered at Palestine Technical University - Kadoorie

Prepared by:

Dr. Hammam Daraghma

Textbook:

Engineering Mechanics: Dynamics, 7th Edition

Author:

J.L. Meriam and L.G. Kraige, 2013

Chapter Six: Plane Kinetics of Rigid Bodies

Section One: Introduction

6 Chapter Six: Plane Kinetics of Rigid Bodies

- **Section A: Force, Mass, and Acceleration**
  - 6/1 Introduction
  - 6/2 General Equations of Motion
  - 6/3 Translation
  - 6/4 Fixed-Axis Rotation
  - 6/5 General Plane Motion
- **Section B: Work and Energy**
  - 6/6 Work-Energy Relations
  - 6/7 Acceleration from Work-Energy
- **Section C: Impulse and Momentum**
  - 6/8 Impulse-Momentum Equations

6.1 Introduction

- The chapter explores the relationship between external forces and rigid body motion.
- It builds on kinematic relationships from earlier chapters.
- Plane motion is defined for bodies with symmetrical dimensions.
- It requires force and moment equations to describe motion.
- The chapter covers translational, rotational, and general plane motions.

End of Section 6.1

6.2 General Equations of Motion

General rigid body in three dimensions

$$\sum \vec{\boldsymbol{F}} = m\vec{\boldsymbol{a}}_{\boldsymbol{G}}$$

$$\sum \vec{\boldsymbol{M}}_{\boldsymbol{G}} = \dot{\vec{\boldsymbol{H}}}_{\boldsymbol{G}}$$

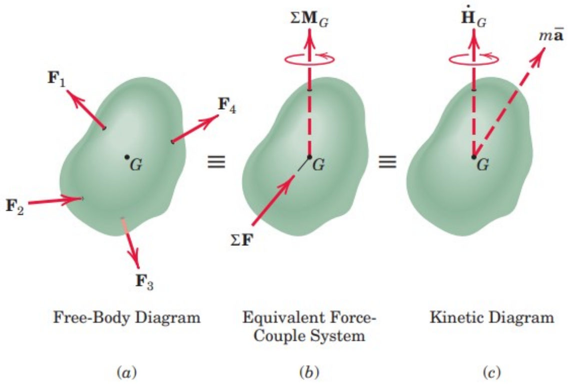


Figure 6/1

Plane-Motion Equations

Center of gravity is center of rotation

$$\sum \vec{\boldsymbol{F}} = m\vec{\boldsymbol{a}}_{\boldsymbol{G}}$$

$$\sum \vec{\boldsymbol{M}}_{\boldsymbol{G}} = \dot{\vec{\boldsymbol{H}}}_{\boldsymbol{G}} = I_G\dot{\vec{\boldsymbol{\omega}}} = I_G\vec{\boldsymbol{\alpha}}$$

$$I_G = K^2 m$$

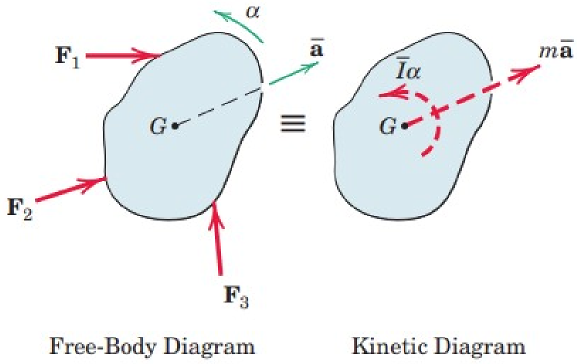


Figure 6/4

Center of gravity <u>is not</u> center of rotation

$$\sum \vec{F} = m\vec{a}_G$$

$$\sum \vec{M}_P = \dot{\vec{H}}_P = I_P\dot{\vec{\omega}} = I_P\vec{\alpha}$$

$$I_P = I_G + md^2$$

$$I_G = K^2 m$$

$$\sum \vec{M}_P = I_G\vec{\alpha} + m\vec{a}_G d$$

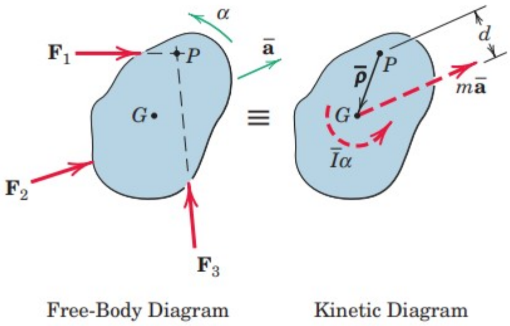


Figure 6/5

Systems of Interconnected Bodies

$$\sum \vec{F} = \sum m\vec{a}_G$$

$$\sum \vec{M}_P = \sum I_G\vec{\alpha} + \sum m\vec{a}_G d$$

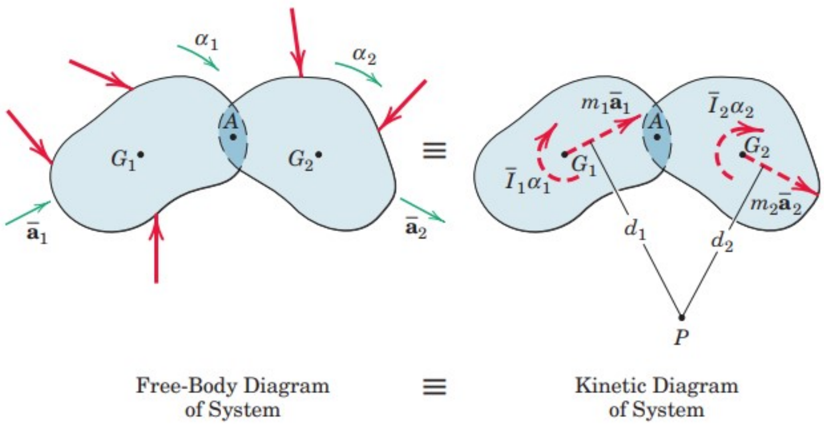


Figure 6/7

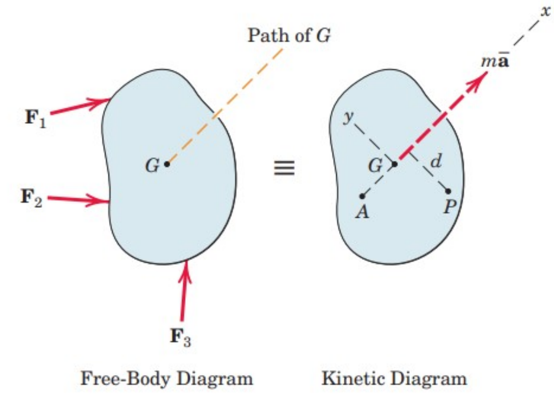
End of Section 6.2

6.3 Translation

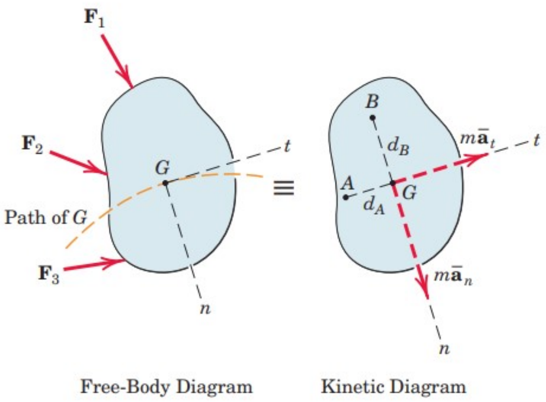
$$\sum \vec{\boldsymbol{F}} = m\vec{\boldsymbol{a}}_{\boldsymbol{G}}$$

$$\sum \vec{\boldsymbol{M}}_{\boldsymbol{G}} = I_G\vec{\boldsymbol{\alpha}} = 0$$

$$\sum \vec{\boldsymbol{M}}_{\boldsymbol{P}} = m\vec{\boldsymbol{a}}_{\boldsymbol{G}}d$$



(*a*) Rectilinear Translation
($\alpha = 0,\ \omega = 0$)



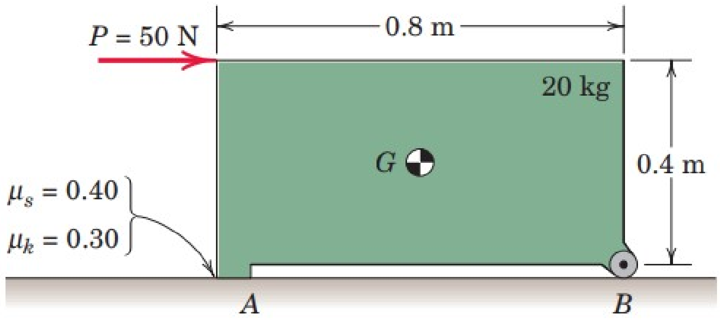
(*b*) Curvilinear Translation
($\alpha = 0,\ \omega = 0$)

Figure 6/8

End of Section 6.3

Example 1:

Determine the acceleration of the initially stationary 20 kg body when the 50 N force P is applied as shown. The small wheels at B are ideal, and the feet at A are small.
ans. $a = 1.306\ m/sec^2$

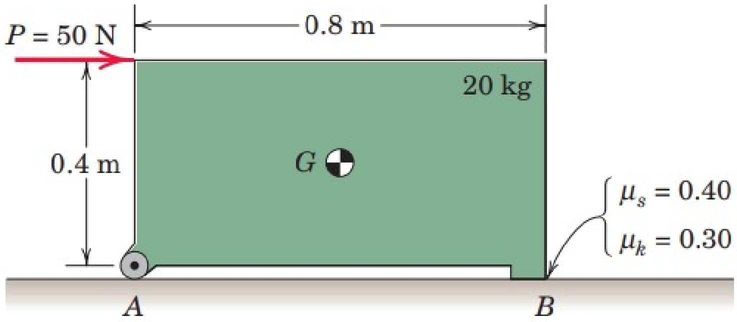


Ans.

Example 2:

Repeat the previous problem for the case when the wheels and feet have been reversed as shown in the figure for this problem. Compare your answer to the stated result for the previous problem.

ans. $a = 0.706\ m/sec^2$

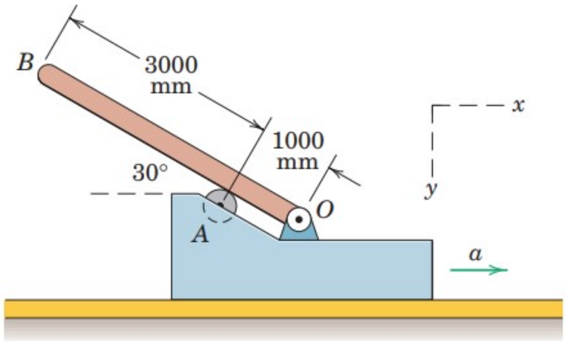


Ans.

Example 3:

The uniform 30 kg bar OB is secured to the accelerating frame in the position from the horizontal by the hinge at O and roller at A. If the horizontal acceleration of the frame is compute the force on the roller and the $x-$ and $y-$ components of the force supported by the pin at O.

ans. $F_A = 1110\ N$ $O_x = 45\ N\ right$ $O_y = 667\ N\ down$

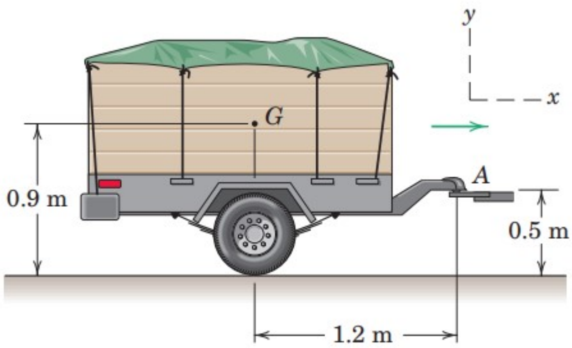


Ans.

Example 4:

The loaded trailer has a mass of 900 kg with center of mass at G and is attached at A to a rear-bumper hitch. If the car and trailer reach a velocity of 60 km/hr on a level road in a distance of 30 m from rest with constant acceleration, compute the vertical component of the force supported by the hitch at A. Neglect the small friction force exerted on the relatively light wheels.

ans. $F_A = 1389\ N\ down$



Ans.