Palestine Technical University- Kadoorie (PTUK)

Mechanical Engineering Department

Summer Semester, 2023/2024

12210244: Dynamics

Student Name: Key Solution

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Homework #:

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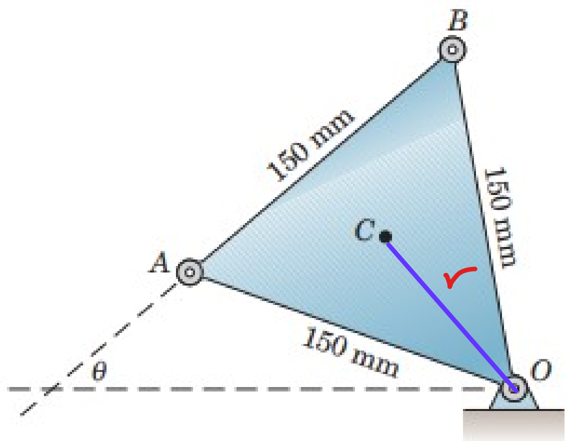
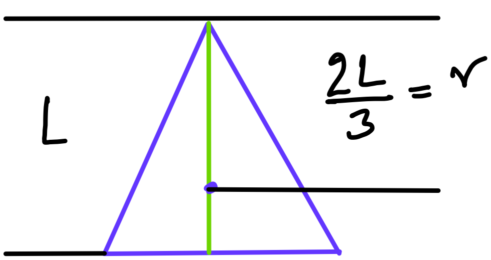
Date of Submission:

Exercise 1. The plate OAB forms an equilateral triangle which rotates counterclockwise with increasing speed about point O. If the normal and tangential components of acceleration of the centroid C at a certain instant are 80 m/sec^2 and 30 m/sec^2, respectively, determine the values of $\dot{\theta}$ and $\ddot{\theta}$ at this same instant. The angle is the angle between line AB and the fixed horizontal axis.



Ans.



$$L^2 = 150^2 - 75^2$$

$$L = 129.9 \text{ mm}$$

$$r = 86.6 \text{ mm}$$

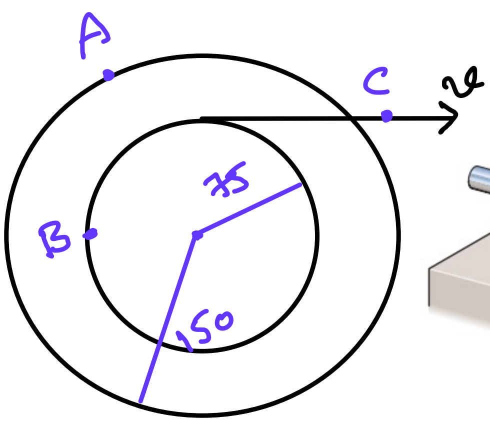
$$a_n = r\omega^2 \Rightarrow 80 = 0.0866\,\omega^2 \Rightarrow \omega = 30.4 \ \frac{rad}{sec}$$

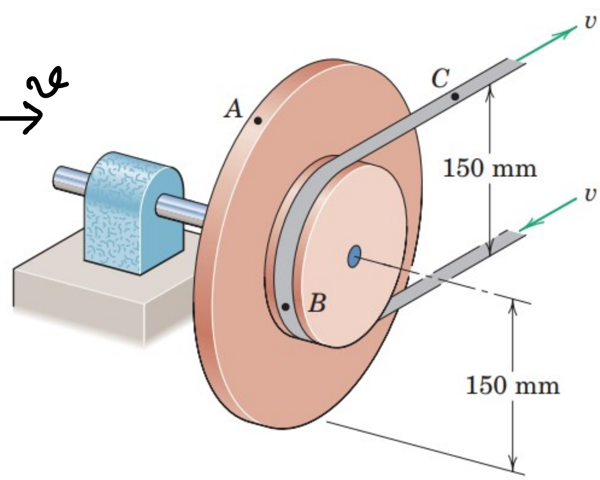
$$a_t = r\alpha \Rightarrow 30 = 0.0866\,\alpha \Rightarrow \alpha = 346.4 \ \frac{rad}{sec^2}$$

Ans.

Exercise 2. The belt-driven pulley and attached disk are rotating with increasing angular velocity. At a certain instant the speed v of the belt is 1.5 m/sec, and the total acceleration of point A is 75 m/sec^2. For this instant determine:

- the angular acceleration of the pulley and disk
- the total acceleration of point B
- the acceleration of point C on the belt





Point C

Ans.

$$v = v_C = 1.5\ m/sec \Rightarrow 1.5 = r\omega = 0.075\,\omega$$

$$\Rightarrow \omega = \frac{1.5}{0.075} = 20\ rad/sec$$

Point A

$$(a_A)_n = r\omega^2 = (0.15)(20)^2 = 60\ m/sec^2$$

$$(a_A)^2 = (a_A)_t^2 + (a_A)_n^2$$

$$(75)^2 = (a_A)_t^2 + (60)^2$$

$$a_t = 45\ m/sec^2$$

$$a_t = r\alpha$$

$$\alpha = \frac{a_t}{r} = \frac{45}{0.15}$$

$$\alpha = 300\ rad/sec^2$$

Ans.

Point B

$$(a_B)_n = r\omega^2 = 0.075\,(20)^2 = 30 \text{ m/sec}^2$$

$$(a_B)_t = r\alpha = 0.075\,(300) = 22.5 \text{ m/sec}^2$$

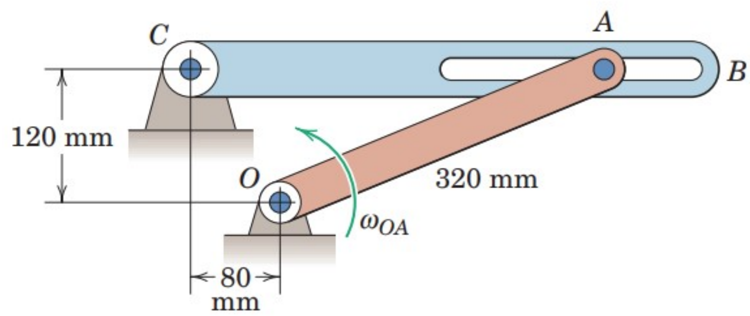
$$a_B = \sqrt{30^2 + 22.5^2} = 37.5 \text{ m/sec}^2$$

Point C

Just tangential acceleration

$$a_C = (a_B)_t = 22.5 \text{ m/sec}^2$$

Exercise 3. Link OA has an angular velocity as it passes the position shown. Determine the corresponding angular velocity of the slotted link CB. Solve by considering the relation between the infinitesimal displacements involved



Ans.

with respect to point O

$\vec{v}_A = \omega_{OA}\hat{k} \times \vec{r}_{A/O}$

$= 8\hat{k} \times (0.32\cos 22.02\,\hat{i} + 0.32\sin 22.02\,\hat{j})$

$= -0.96\hat{i} + 2.37\hat{j}$

A, 16, O, $\omega_{OA} = 8$ rad/sec

$\theta = \sin^{-1}\frac{120}{320}$

$= 22.02°$

with respect to point C

$\vec{v}_A = \omega_{BC}\hat{k} \times \vec{r}_{A/C} + v_{rel}\,\hat{i}$

$-0.96\hat{i} + 2.37\hat{j} = \omega_{BC}\hat{k} \times [(0.08 + 0.32\cos 22.02)\hat{i}$ (0.377)

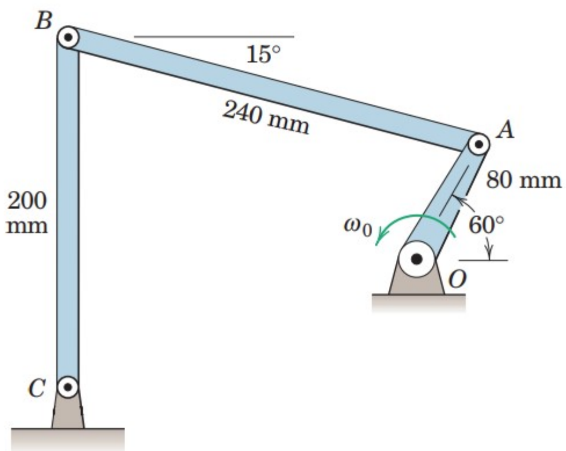
$+ v_{rel}\,\hat{i}$

$-0.96\hat{i} + 2.37\hat{j} = 0.377\,\omega_{BC}\,\hat{j} + v_{rel}\,\hat{i}$

$\Rightarrow v_{rel} = -0.96$ m/sec , $\omega_{BC} = 6.29$ rad/sec

Ans.

Exercise 4. A four-bar linkage is shown in the figure (the ground "link" OC is considered the fourth bar). If the drive link OA has a counterclockwise angular velocity $\omega_O = 10\ rad/sec$, determine the angular velocities of links AB and BC.



Ans.

with respect to point O

$\vec{v}_A = \omega_o \hat{k} \times \vec{r}_{A/O}$

$\vec{v}_A = 10\hat{k} \times (0.08 \cos 60\,\hat{i} + 0.08 \sin 60\,\hat{j})$

$= -0.69\hat{i} + 0.4j$

with respect to point A

$\vec{v}_B = \vec{v}_A + \vec{\omega}_{AB} \times \vec{r}_{B/A}$ $\quad -0.232\hat{i} + 0.062\hat{j}$

$= -0.69\hat{i} + 0.4j + \omega_{AB}\hat{k} \times (-0.24 \cos 15\,\hat{i} + 0.24 \sin 15\,j)$

$\vec{v}_B = (-0.69 - 0.062\,\omega_{AB})\,\hat{i} + (0.4 - 0.232\,\omega_{AB})j$

Ans.

with respect to point A

$$\vec{v}_B = (-0.69 - 0.062\,\omega_{AB})\,\hat{i} + (0.4 - 0.232\,\omega_{AB})\,\hat{j}$$

with respect to point C

$$\vec{v}_B = \vec{\omega}_{BC} \times \vec{r}_{B/C}$$

$$= \omega_{BC}\hat{k} \times (0.2\hat{j})$$

$$= -0.2\,\omega_{BC}\,\hat{i}$$

$$\Rightarrow (-0.69 - 0.062\,\omega_{AB})\hat{i} + (0.4 - 0.232\,\omega_{AB})\hat{j}$$

$$= -0.2\,\omega_B\,\hat{i}$$

$$j \Rightarrow \quad 0.4 - 0.232\,\omega_{AB} = 0 \Rightarrow \omega_{AB} = 1.724$$

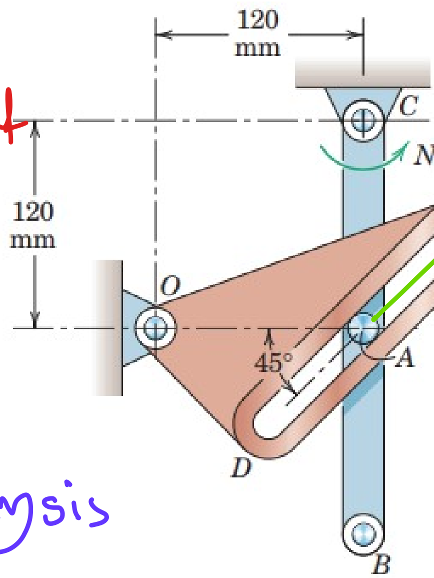
$$i \Rightarrow \quad -0.69 - 0.062(1.724) = -0.2\,(\omega_{BC})$$

$$\Rightarrow \omega_{AB} = 4 \text{ rad/sec}$$

Exercise 5. For the instant represented, link CB is rotating counterclockwise at a constant rate $\omega = 4\ rad/sec$ and its pin A causes a clockwise rotation of the slotted member ODE. Determine the angular velocity and angular acceleration of ODE for this instant.

$\omega_{BC} = 4$ Constant

$\alpha_{BC} = 0$



Velocity Analysis

Ans. with respect to point C

$$\vec{v}_A = \vec{\omega}_{BC} \times \vec{r}_{A/C} = 4\hat{k} \times (-0.12j) = 0.48\hat{i}$$

with respect to point O

$$\vec{v}_A = \vec{\omega}_{ODE} \times \vec{r}_{A/O} + \vec{v}_{rel}$$

$$= \omega_{ODE}\,\hat{k} \times (0.12\,\hat{i}) + (v\cos 45\,\hat{i} + v\sin 45\,\hat{j})$$

$$= 0.707v\,\hat{i} + (0.707\,v + 0.12\omega_{ODE})\hat{j}$$

$$i \Rightarrow 0.48 = 0.707\,v \Rightarrow v = 0.679\ m/sec$$

$$j \Rightarrow 0 = 0.707(0.679) + 0.12\,\omega_{ODE}$$

$$\Rightarrow \omega_{ODE} = -4\ rad/sec$$

Ans. Acceleration analysis

with respect to point C

$$\vec{a}_A = \vec{\omega}_{BC} \times (\vec{\omega}_{BC} \times \vec{r}_{A/C}) = 4\hat{k} \times (0.48\hat{i}) = 1.92\hat{j}$$

with respect to point O

$$\vec{a}_A = \underbrace{\vec{\omega}_{ODE} \times (\vec{\omega}_{ODE} \times \vec{r}_{A/O})}_{①} + \underbrace{\vec{\alpha}_{ODE} \times \vec{r}_{A/O}}_{②} + \underbrace{2\vec{\omega}_{ODE} \times \vec{v}_{rel}}_{③} + \underbrace{\vec{a}_{rel}}_{④}$$

$$① = -4\hat{k} \times (-4\hat{k} \times 0.12\hat{i}) = -1.92\hat{i}$$

$$② \Rightarrow \alpha_{ODE}\hat{k} \times (0.12\hat{i}) = 0.12\alpha_{ODE}\hat{j}$$

$$③ = 2[4\hat{k} \times (0.679\cos 45\hat{i} + 0.679\sin 45\hat{j})]$$

$$= 3.84\hat{i} - 3.84\hat{j}$$

$$④ = a\cos 45\hat{i} + a\sin 45\hat{j} = 0.707a\hat{i} + 0.707\hat{j}$$

$$\vec{a}_A = (0.707a + 1.92)\hat{i} + (0.707a - 3.84 + 0.12\alpha_{ODE})\hat{j}$$

$$\hat{i} \Rightarrow 0.707a + 1.92 = 0 \Rightarrow a = -2.72 \text{ m/sec}^2$$

$$\hat{j} \Rightarrow 0.707(-2.72) - 3.84 + 0.12\alpha_{ODE} = 1.92$$

$$\alpha_{ODE} = 64$$