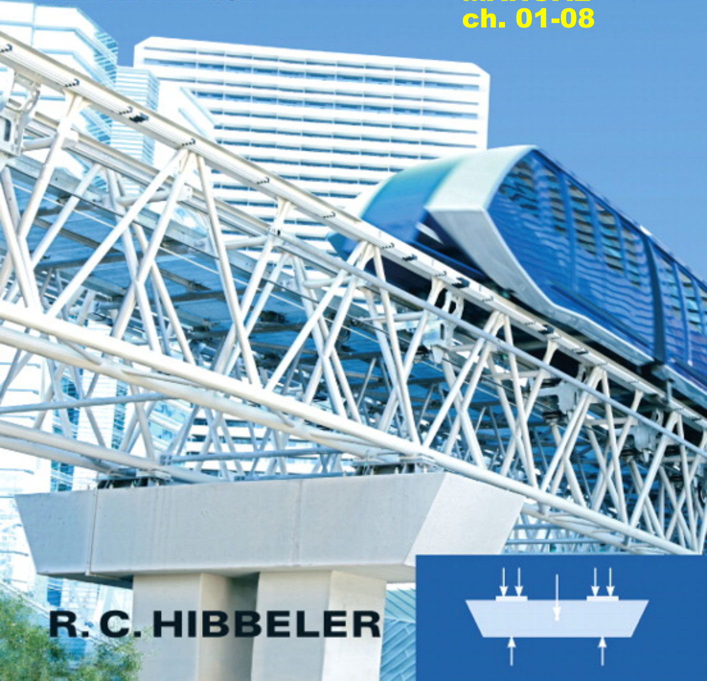
ENGINEERING MECHANICS
Statics

THIRTEENTH EDITION

INSTRUCTOR'S SOLUTIONS MANUAL
ch. 01-08



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SOLUTION MANUAL CONTENTS

1–1.

Round off the following numbers to three significant figures: (a) 58 342 m, (b) 68.534 s, (c) 2553 N, and (d) 7555 kg.

SOLUTION

a) 58.3 km b) 68.5 s c) 2.55 kN d) 7.56 Mg **Ans.**

1–2.

Wood has a density of 4.70 slug/ft^3. What is its density expressed in SI units?

SOLUTION

$$(4.70\ \text{slug/ft}^3)\left\{\frac{(1\ \text{ft}^3)(14.59\ \text{kg})}{(0.3048\ \text{m})^3(1\ \text{slug})}\right\} = 2.42\ \text{Mg/m}^3$$ **Ans.**

1–3.

Represent each of the following combinations of units in the correct SI form using an appropriate prefix: (a) kN/μs, (b) Mg/mN, and (c) MN/(kg · ms).

SOLUTION

a) $\text{kN}/\mu\text{s} = \dfrac{(10^3)\ \text{N}}{(10^{-6})\ \text{s}} = \dfrac{(10^9)\ \text{N}}{\text{s}} = \text{GN/s}$ **Ans.**

b) $\text{Mg/mN} = \dfrac{(10^6)\ \text{g}}{(10^{-3})\ \text{N}} = \dfrac{(10^9)\ \text{g}}{\text{N}} = \text{Gg/N}$ **Ans.**

c) $\text{MN}/(\text{kg}\cdot\text{ms}) = \dfrac{(10^6)\ \text{N}}{\text{kg}\cdot(10^{-3})\ \text{s}} = \dfrac{(10^9)\ \text{N}}{\text{kg}\cdot\text{s}} = \text{GN}/(\text{kg}\cdot\text{s})$ **Ans.**

***1–4.**

Represent each of the following combinations of units in the correct SI form using an appropriate prefix: (a) m/ms, (b) μkm, (c) ks/mg, and (d) km $\cdot$ μN.

SOLUTION

a) $\text{m/ms} = \left(\dfrac{\text{m}}{(10)^{-3}\ \text{s}}\right) = \left(\dfrac{(10)^{3}\ \text{m}}{\text{s}}\right) = \text{km/s}$ **Ans.**

b) $\mu\text{km} = (10)^{-6}(10)^{3}\ \text{m} = (10)^{-3}\ \text{m} = \text{mm}$ **Ans.**

c) $\text{ks/mg} = \left(\dfrac{(10)^{3}\ \text{s}}{(10)^{-6}\ \text{kg}}\right) = \left(\dfrac{(10)^{9}\ \text{s}}{\text{kg}}\right) = \text{Gs/kg}$ **Ans.**

d) $\text{km} \cdot \mu\text{N} = [(10)^{3}\ \text{m}][(10)^{-6}\ \text{N}] = (10)^{-3}\ \text{m} \cdot \text{N} = \text{mm} \cdot \text{N}$ **Ans.**

1–5.

Represent each of the following quantities in the correct SI form using an appropriate prefix: (a) 0.000 431 kg, (b) $35.3(10^3)$ N, and (c) 0.005 32 km.

SOLUTION

a) $0.000\,431 \text{ kg} = 0.000\,431\left(10^3\right) \text{ g} = 0.431 \text{ g}$ **Ans.**

b) $35.3\left(10^3\right) \text{ N} = 35.3 \text{ kN}$ **Ans.**

c) $0.005\,32 \text{ km} = 0.005\,32\left(10^3\right) \text{ m} = 5.32 \text{ m}$ **Ans.**

1–6.

If a car is traveling at 55 mi/h, determine its speed in kilometers per hour and meters per second.

SOLUTION

$$55 \text{ mi/h} = \left(\frac{55 \text{ mi}}{1 \text{ h}}\right)\left(\frac{5280 \text{ ft}}{1 \text{ mi}}\right)\left(\frac{0.3048 \text{ m}}{1 \text{ ft}}\right)\left(\frac{1 \text{ km}}{1000 \text{ m}}\right)$$

$$= 88.5 \text{ km/h}$$ **Ans.**

$$88.5 \text{ km/h} = \left(\frac{88.5 \text{ km}}{1 \text{ h}}\right)\left(\frac{1000 \text{ m}}{1 \text{ km}}\right)\left(\frac{1 \text{ h}}{3600 \text{ s}}\right) = 24.6 \text{ m/s}$$ **Ans.**

1–7.

The *pascal* (Pa) is actually a very small unit of pressure. To show this, convert $1 \text{ Pa} = 1 \text{ N/m}^2$ to lb/ft^2. Atmospheric pressure at sea level is 14.7 lb/in^2. How many pascals is this?

SOLUTION

Using Table 1–2, we have

$$1 \text{ Pa} = \frac{1 \text{ N}}{\text{m}^2}\left(\frac{1 \text{ lb}}{4.4482 \text{ N}}\right)\left(\frac{0.3048^2 \text{ m}^2}{1 \text{ ft}^2}\right) = 20.9\left(10^{-3}\right) \text{ lb/ft}^2 \qquad \textbf{Ans.}$$

$$1 \text{ ATM} = \frac{14.7 \text{ lb}}{\text{in}^2}\left(\frac{4.448 \text{ N}}{1 \text{ lb}}\right)\left(\frac{144 \text{ in}^2}{1 \text{ ft}^2}\right)\left(\frac{1 \text{ ft}^2}{0.3048^2 \text{ m}^2}\right)$$

$$= 101.3\left(10^3\right) \text{ N/m}^2$$

$$= 101 \text{ kPa} \qquad \textbf{Ans.}$$

***1–8.**

The specific weight (wt./vol.) of brass is 520 lb/ft^3. Determine its density (mass/vol.) in SI units. Use an appropriate prefix.

SOLUTION

$$520 \text{ lb/ft}^3 = \left(\frac{520 \text{ lb}}{\text{ft}^3}\right)\left(\frac{1 \text{ ft}}{0.3048 \text{ m}}\right)^3\left(\frac{4.448 \text{ N}}{1 \text{ lb}}\right)\left(\frac{1 \text{ kg}}{9.81 \text{ N}}\right)$$

$$= 8.33 \text{ Mg/m}^3$$ **Ans.**

1–9.

A rocket has a mass of $250(10^3)$ slugs on earth. Specify (a) its mass in SI units and (b) its weight in SI units. If the rocket is on the moon, where the acceleration due to gravity is $g_m = 5.30\ \text{ft/s}^2$, determine to three significant figures (c) its weight in SI units and (d) its mass in SI units.

SOLUTION

Using Table 1–2 and applying Eq. 1–3, we have

a) $$250\left(10^3\right)\text{ slugs} = \left[250\left(10^3\right)\text{ slugs}\right]\left(\frac{14.59\text{ kg}}{1\text{ slugs}}\right)$$

$$= 3.6475\left(10^6\right)\text{ kg}$$

$$= 3.65\text{ Gg}$$ **Ans.**

b) $$W_e = mg = \left[3.6475\left(10^6\right)\text{ kg}\right]\left(9.81\text{ m/s}^2\right)$$

$$= 35.792\left(10^6\right)\text{ kg}\cdot\text{m/s}^2$$

$$= 35.8\text{ MN}$$ **Ans.**

c) $$W_m = mg_m = \left[250\left(10^3\right)\text{ slugs}\right]\left(5.30\text{ ft/s}^2\right)$$

$$= \left[1.325\left(10^6\right)\text{ lb}\right]\left(\frac{4.448\text{ N}}{1\text{ lb}}\right)$$

$$= 5.894\left(10^6\right)\text{ N} = 5.89\text{ MN}$$ **Ans.**

Or

$$W_m = W_e\left(\frac{g_m}{g}\right) = (35.792\text{ MN})\left(\frac{5.30\text{ ft/s}^2}{32.2\text{ ft/s}^2}\right) = 5.89\text{ MN}$$

d) Since the mass is independent of its location, then

$$m_m = m_e = 3.65\left(10^6\right)\text{ kg} = 3.65\text{ Gg}$$ **Ans.**

1–10.

Evaluate each of the following to three significant figures and express each answer in SI units using an appropriate prefix: (a) (0.631 Mm)/$(8.60\text{ kg})^2$, (b) $(35\text{ mm})^2(48\text{ kg})^3$.

SOLUTION

a) $$(0.631\text{ Mm})/(8.60\text{ kg})^2 = \left(\frac{0.631\left(10^6\right)\text{ m}}{(8.60)^2\text{ kg}^2}\right) = \frac{8532\text{ m}}{\text{kg}^2}$$

$$= 8.53\left(10^3\right)\text{ m/kg}^2 = 8.53\text{ km/kg}^2$$ **Ans.**

b) $$(35\text{ mm})^2(48\text{ kg})^3 = \left[35\left(10^{-3}\right)\text{ m}\right]^2(48\text{ kg})^3 = 135\text{ m}^2\cdot\text{kg}^3$$ **Ans.**

1–11.

Evaluate each of the following to three significant figures and express each answer in Sl units using an appropriate prefix: (a) 354 mg(45 km)/(0.0356 kN), (b) (0.004 53 Mg) (201 ms), and (c) 435 MN/23.2 mm.

SOLUTION

a) $(354\text{ mg})(45\text{ km})/(0.0356\text{ kN}) = \dfrac{\left[354\left(10^{-3}\right)\text{ g}\right]\left[45\left(10^{3}\right)\text{ m}\right]}{0.0356\left(10^{3}\right)\text{ N}}$

$$= \frac{0.447\left(10^{3}\right)\text{ g}\cdot\text{m}}{\text{N}}$$

$$= 0.447\text{ kg}\cdot\text{m/N}$$ **Ans.**

b) $(0.00453\text{ Mg})(201\text{ ms}) = \left[4.53\left(10^{-3}\right)\left(10^{3}\right)\text{ kg}\right]\left[201\left(10^{-3}\right)\text{ s}\right]$

$$= 0.911\text{ kg}\cdot\text{s}$$ **Ans.**

c) $435\text{ MN}/23.2\text{ mm} = \dfrac{435\left(10^{6}\right)\text{ N}}{23.2\left(10^{-3}\right)\text{ m}} = \dfrac{18.75\left(10^{9}\right)\text{ N}}{\text{m}} = 18.8\text{ GN/m}$ **Ans.**

***1–12.**

Convert each of the following and express the answer using an appropriate prefix: (a) 175 lb/ft^3 to kN/m^3, (b) 6 ft/h to mm/s, and (c) 835 lb · ft to kN · m.

SOLUTION

a) $175\ \text{lb/ft}^3 = \left(\dfrac{175\ \text{lb}}{\text{ft}^3}\right)\left(\dfrac{1\ \text{ft}}{0.3048\ \text{m}}\right)^3\left(\dfrac{4.448\ \text{N}}{1\ \text{lb}}\right)$

$= \left(\dfrac{27.5(10)^3\ \text{N}}{\text{m}^3}\right) = 27.5\ \text{kN/m}^3$ **Ans.**

b) $6\ \text{ft/h} = \left(\dfrac{6\ \text{ft}}{1\ \text{h}}\right)\left(\dfrac{0.3048\ \text{m}}{1\ \text{ft}}\right)\left(\dfrac{1\ \text{h}}{3600\ \text{s}}\right)$

$= 0.508(10)^{-3}\ \text{m/s} = 0.508\ \text{mm/s}$ **Ans.**

c) $835\ \text{lb}\cdot\text{ft} = (835\ \text{lb}\cdot\text{ft})\left(\dfrac{4.448\ \text{N}}{1\ \text{lb}}\right)\left(\dfrac{0.3048\ \text{m}}{1\ \text{ft}}\right)$

$= 1.13(10)^3\ \text{N}\cdot\text{m} = 1.13\ \text{kN}\cdot\text{m}$ **Ans.**

1–13.

Convert each of the following to three significant figures: (a) $20\ \text{lb}\cdot\text{ft}$ to $\text{N}\cdot\text{m}$, (b) $450\ \text{lb/ft}^3$ to kN/m^3, and (c) 15 ft/h to mm/s.

SOLUTION

Using Table 1–2, we have

a) $$20\ \text{lb}\cdot\text{ft} = (20\ \text{lb}\cdot\text{ft})\left(\frac{4.448\ \text{N}}{1\ \text{lb}}\right)\left(\frac{0.3048\ \text{m}}{1\ \text{ft}}\right)$$

$$= 27.1\ \text{N}\cdot\text{m}$$ **Ans.**

b) $$450\ \text{lb/ft}^3 = \left(\frac{450\ \text{lb}}{1\ \text{ft}^3}\right)\left(\frac{4.448\ \text{N}}{1\ \text{lb}}\right)\left(\frac{1\ \text{kN}}{1000\ \text{N}}\right)\left(\frac{1\ \text{ft}^3}{0.3048^3\ \text{m}^3}\right)$$

$$= 70.7\ \text{kN/m}^3$$ **Ans.**

c) $$15\ \text{ft/h} = \left(\frac{15\ \text{ft}}{1\ \text{h}}\right)\left(\frac{304.8\ \text{mm}}{1\ \text{ft}}\right)\left(\frac{1\ \text{h}}{3600\ \text{s}}\right) = 1.27\ \text{mm/s}$$ **Ans.**

1–14.

Evaluate each of the following and express with an appropriate prefix: (a) $(430\text{ kg})^2$, (b) $(0.002\text{ mg})^2$, and (c) $(230\text{ m})^3$.

SOLUTION

a) $(430\text{ kg})^2 = 0.185(10^6)\text{ kg}^2 = 0.185\text{ Mg}^2$ **Ans.**

b) $(0.002\text{ mg})^2 = [2(10^{-6})\text{ g}]^2 = 4\ \mu\text{g}^2$ **Ans.**

c) $(230\text{ m})^3 = [0.23(10^3)\text{ m}]^3 = 0.0122\text{ km}^3$ **Ans.**

1–15.

Determine the mass of an object that has a weight of (a) 20 mN, (b) 150 kN, and (c) 60 MN. Express the answer to three significant figures.

SOLUTION

Applying Eq. 1–3, we have

a) $m = \dfrac{W}{g} = \dfrac{20\left(10^{-3}\right)\ \text{kg}\cdot\text{m/s}^2}{9.81\ \text{m/s}^2} = 2.04\ \text{g}$ **Ans.**

b) $m = \dfrac{W}{g} = \dfrac{150\left(10^{3}\right)\ \text{kg}\cdot\text{m/s}^2}{9.81\ \text{m/s}^2} = 15.3\ \text{Mg}$ **Ans.**

c) $m = \dfrac{W}{g} = \dfrac{60\left(10^{6}\right)\ \text{kg}\cdot\text{m/s}^2}{9.81\ \text{m/s}^2} = 6.12\ \text{Gg}$ **Ans.**

***1–16.**

What is the weight in newtons of an object that has a mass of: (a) 10 kg, (b) 0.5 g, and (c) 4.50 Mg? Express the result to three significant figures. Use an appropriate prefix.

SOLUTION

a) $W = \left(9.81 \text{ m/s}^2\right)(10 \text{ kg}) = 98.1 \text{ N}$ **Ans.**

b) $W = \left(9.81 \text{ m/s}^2\right)(0.5 \text{ g})(10^{-3} \text{ kg/g}) = 4.90 \text{ mN}$ **Ans.**

c) $W = \left(9.81 \text{ m/s}^2\right)(4.5 \text{ Mg})\left(10^3 \text{ kg/Mg}\right) = 44.1 \text{ kN}$ **Ans.**

1–17.

If an object has a mass of 40 slugs, determine its mass in kilograms.

SOLUTION

$$40 \text{ slugs } (14.59 \text{ kg/slug}) = 584 \text{ kg}$$ **Ans.**

1–18.

Using the SI system of units, show that Eq. 1–2 is a dimensionally homogeneous equation which gives F in newtons. Determine to three significant figures the gravitational force acting between two spheres that are touching each other. The mass of each sphere is 200 kg and the radius is 300 mm.

SOLUTION

Using Eq. 1–2,

$$F = G\frac{m_1 m_2}{r^2}$$

$$\text{N} = \left(\frac{\text{m}^3}{\text{kg}\cdot\text{s}^2}\right)\left(\frac{\text{kg}\cdot\text{kg}}{\text{m}^2}\right) = \frac{\text{kg}\cdot\text{m}}{\text{s}^2} \qquad \textbf{\textit{(Q.E.D.)}}$$

$$F = G\frac{m_1 m_2}{r^2}$$

$$= 66.73\left(10^{-12}\right)\left[\frac{200(200)}{0.6^2}\right]$$

$$= 7.41\left(10^{-6}\right)\text{ N} = 7.41\ \mu\text{N} \qquad \textbf{Ans.}$$

1–19.

Water has a density of 1.94 slug/ft^3. What is the density expressed in SI units? Express the answer to three significant figures.

SOLUTION

$$\rho_w = \left(\frac{1.94 \text{ slug}}{1 \text{ ft}^3}\right)\left(\frac{14.59 \text{ kg}}{1 \text{ slug}}\right)\left(\frac{1 \text{ ft}^3}{0.3048^3 \text{ m}^3}\right)$$

$$= 999.6 \text{ kg/m}^3 = 1.00 \text{ Mg/m}^3 \qquad \textbf{Ans.}$$

***1–20.**

Two particles have a mass of 8 kg and 12 kg, respectively. If they are 800 mm apart, determine the force of gravity acting between them. Compare this result with the weight of each particle.

SOLUTION

$$F = G\frac{m_1 m_2}{r^2}$$

Where $G = 66.73\left(10^{-12}\right)\ \text{m}^3/(\text{kg}\cdot\text{s}^2)$

$$F = 66.73\left(10^{-12}\right)\left[\frac{8(12)}{(0.8)^2}\right] = 10.0\left(10^{-9}\right)\ \text{N} = 10.0\ \text{nN}$$ **Ans.**

$$W_1 = 8(9.81) = 78.5\ \text{N}$$ **Ans.**

$$W_2 = 12(9.81) = 118\ \text{N}$$ **Ans.**

1–21.

If a man weighs 155 lb on earth, specify (a) his mass in slugs, (b) his mass in kilograms, and (c) his weight in newtons. If the man is on the moon, where the acceleration due to gravity is $g_m = 5.30\ \text{ft/s}^2$, determine (d) his weight in pounds, and (e) his mass in kilograms.

SOLUTION

a) $m = \dfrac{155}{32.2} = 4.81\ \text{slug}$ **Ans.**

b) $m = 155\left[\dfrac{14.59\ \text{kg}}{32.2}\right] = 70.2\ \text{kg}$ **Ans.**

c) $W = 155(4.4482) = 689\ \text{N}$ **Ans.**

d) $W = 155\left[\dfrac{5.30}{32.2}\right] = 25.5\ \text{lb}$ **Ans.**

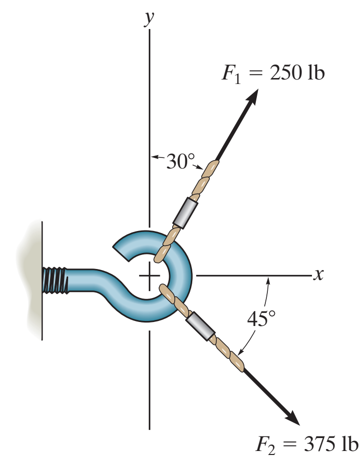
e) $m = 155\left[\dfrac{14.59\ \text{kg}}{32.2}\right] = 70.2\ \text{kg}$ **Ans.**

Also,

$m = 25.5\left[\dfrac{14.59\ \text{kg}}{5.30}\right] = 70.2\ \text{kg}$ **Ans.**

2–1.

Determine the magnitude of the resultant force $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$ and its direction, measured counterclockwise from the positive x axis.



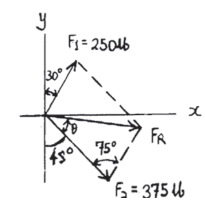
SOLUTION

$$F_R = \sqrt{(250)^2 + (375)^2 - 2(250)(375)\cos 75^\circ} = 393.2 = 393 \text{ lb}$$ **Ans.**

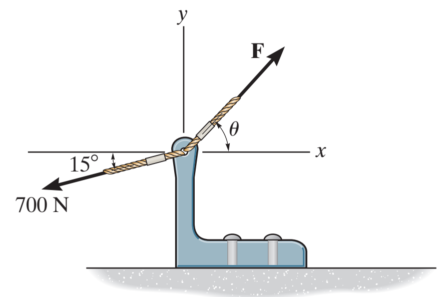
$$\frac{393.2}{\sin 75^\circ} = \frac{250}{\sin \theta}$$

$$\theta = 37.89^\circ$$

$$\phi = 360^\circ - 45^\circ + 37.89^\circ = 353^\circ$$ **Ans.**

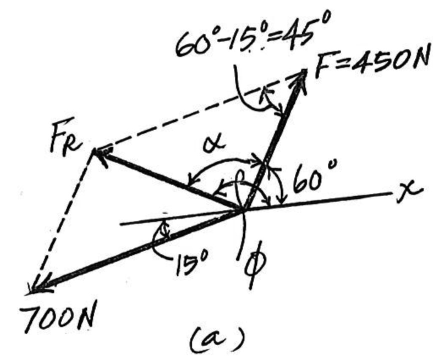


2–2.
If $\theta = 60°$ and $F = 450$ N, determine the magnitude of the resultant force and its direction, measured counterclockwise from the positive x axis.



SOLUTION

The parallelogram law of addition and the triangular rule are shown in Figs. *a* and *b*, respectively.



Applying the law of consines to Fig. *b*,

$$F_R = \sqrt{700^2 + 450^2 - 2(700)(450)\cos 45°}$$

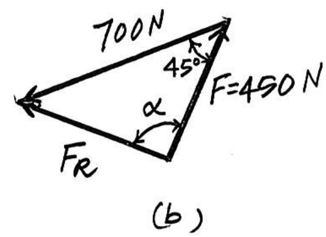
$$= 497.01\text{ N} = 497\text{ N} \qquad \textbf{Ans.}$$

This yields

$$\frac{\sin\alpha}{700} = \frac{\sin 45°}{497.01} \qquad \alpha = 95.19°$$

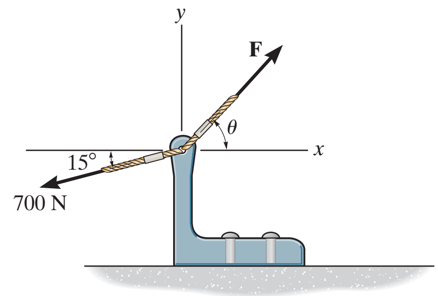
Thus, the direction of angle ϕ of $\mathbf{F}_R$ measured counterclockwise from the positive x axis, is

$$\phi = \alpha + 60° = 95.19° + 60° = 155° \qquad \textbf{Ans.}$$



2–3.

If the magnitude of the resultant force is to be 500 N, directed along the positive y axis, determine the magnitude of force $\mathbf{F}$ and its direction θ.



SOLUTION

The parallelogram law of addition and the triangular rule are shown in Figs. a and b, respectively.

Applying the law of cosines to Fig. b,

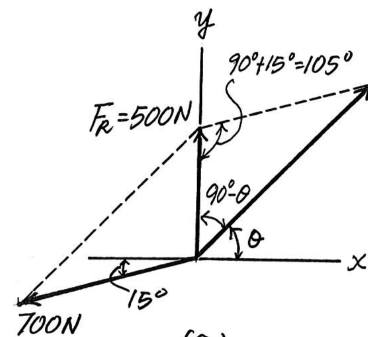
$$F = \sqrt{500^2 + 700^2 - 2(500)(700)\cos 105^\circ}$$

$$= 959.78\text{ N} = 960\text{ N} \qquad \textbf{Ans.}$$

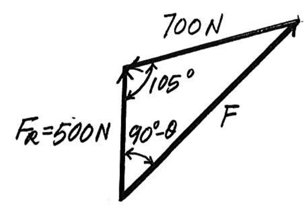
Applying the law of sines to Fig. b, and using this result, yields

$$\frac{\sin(90^\circ + \theta)}{700} = \frac{\sin 105^\circ}{959.78}$$

$$\theta = 45.2^\circ \qquad \textbf{Ans.}$$



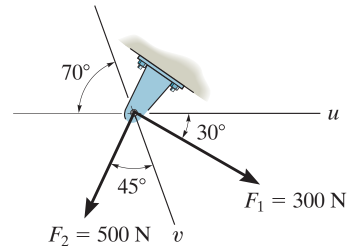
(a)



(b)

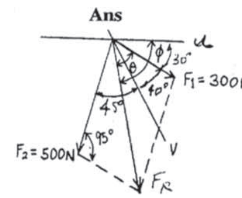
***2–4.**

Determine the magnitude of the resultant force $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$ and its direction, measured clockwise from the positive u axis.



SOLUTION

$$F_R = \sqrt{(300)^2 + (500)^2 - 2(300)(500)\cos 95^\circ} = 605.1 = 605 \text{ N} \qquad \textbf{Ans.}$$



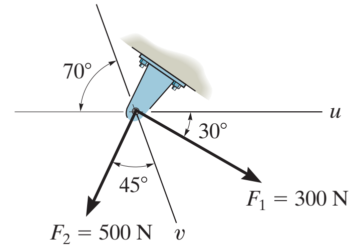
$$\frac{605.1}{\sin 95^\circ} = \frac{500}{\sin \theta}$$

$$\theta = 55.40^\circ$$

$$\phi = 55.40^\circ + 30^\circ = 85.4^\circ \qquad \textbf{Ans.}$$

2–5.

Resolve the force $\mathbf{F}_1$ into components acting along the u and v axes and determine the magnitudes of the components.



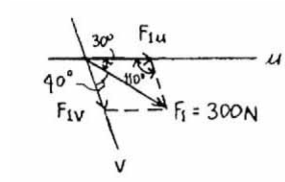
SOLUTION

$$\frac{F_{1u}}{\sin 40^\circ} = \frac{300}{\sin 110^\circ}$$

$$F_{1u} = 205 \text{ N}$$ **Ans.**

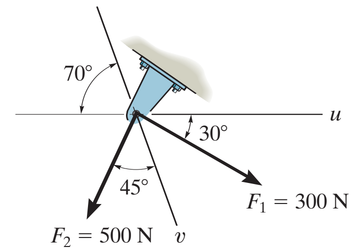
$$\frac{F_{1v}}{\sin 30^\circ} = \frac{300}{\sin 110^\circ}$$

$$F_{1v} = 160 \text{ N}$$ **Ans.**



2–6.

Resolve the force $\mathbf{F}_2$ into components acting along the u and v axes and determine the magnitudes of the components.



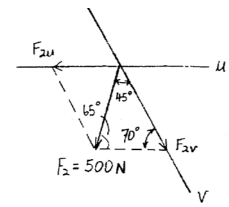
SOLUTION

$$\frac{F_{2u}}{\sin 45^\circ} = \frac{500}{\sin 70^\circ}$$

$$F_{2u} = 376 \text{ N}$$ **Ans.**

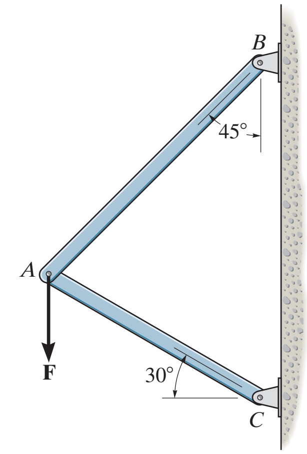
$$\frac{F_{2v}}{\sin 65^\circ} = \frac{500}{\sin 70^\circ}$$

$$F_{2v} = 482 \text{ N}$$ **Ans.**



2–7.

The vertical force **F** acts downward at A on the two-membered frame. Determine the magnitudes of the two components of **F** directed along the axes of AB and AC. Set $F = 500$ N.



SOLUTION

Parallelogram Law: The parallelogram law of addition is shown in Fig. *a*.

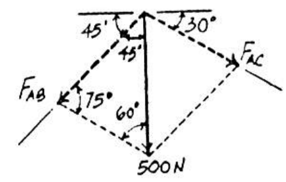
Trigonometry: Using the law of sines (Fig. *b*), we have

$$\frac{F_{AB}}{\sin 60^\circ} = \frac{500}{\sin 75^\circ}$$

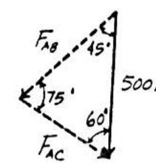
$$F_{AB} = 448 \text{ N} \qquad \textbf{Ans.}$$

$$\frac{F_{AC}}{\sin 45^\circ} = \frac{500}{\sin 75^\circ}$$

$$F_{AC} = 366 \text{ N} \qquad \textbf{Ans.}$$



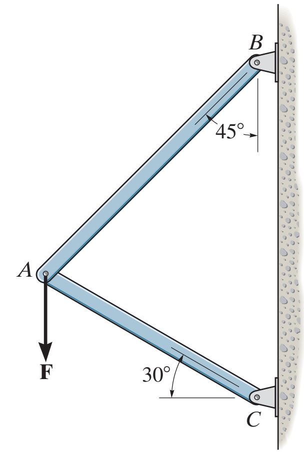
(a)



(b)

***2–8.**

Solve Prob. 2-7 with $F = 350$ lb.



SOLUTION

Parallelogram Law: The parallelogram law of addition is shown in Fig. *a*.

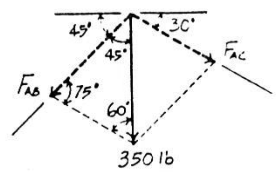
Trigonometry: Using the law of sines (Fig. *b*), we have

$$\frac{F_{AB}}{\sin 60^\circ} = \frac{350}{\sin 75^\circ}$$

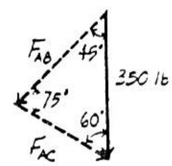
$$F_{AB} = 314 \text{ lb} \qquad \textbf{Ans.}$$

$$\frac{F_{AC}}{\sin 45^\circ} = \frac{350}{\sin 75^\circ}$$

$$F_{AC} = 256 \text{ lb} \qquad \textbf{Ans.}$$



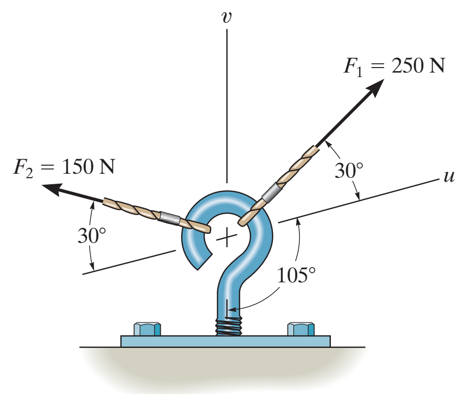
(a)



(b)

2–9.

Resolve $\mathbf{F}_1$ into components along the u and v axes and determine the magnitudes of these components.

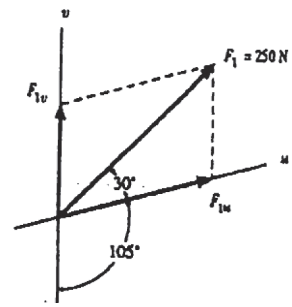


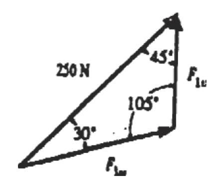
SOLUTION

Sine law:

$$\frac{F_{1v}}{\sin 30^\circ} = \frac{250}{\sin 105^\circ} \qquad F_{1v} = 129\ \text{N} \qquad \textbf{Ans.}$$

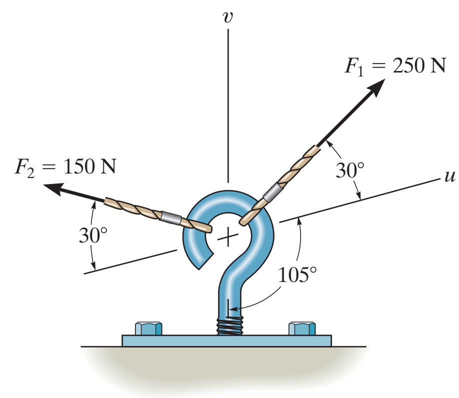
$$\frac{F_{1u}}{\sin 45^\circ} = \frac{250}{\sin 105^\circ} \qquad F_{1u} = 183\ \text{N} \qquad \textbf{Ans.}$$





2–10.

Resolve $\mathbf{F}_2$ into components along the u and v axes and determine the magnitudes of these components.

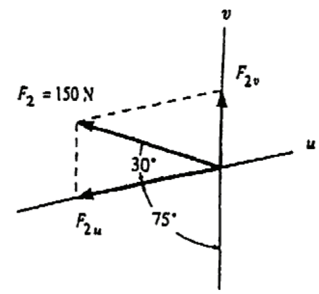


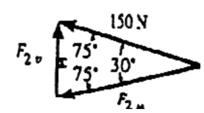
SOLUTION

Sine law:

$$\frac{F_{2v}}{\sin 30^\circ} = \frac{150}{\sin 75^\circ} \qquad F_{2v} = 77.6 \text{ N} \qquad \textbf{Ans.}$$

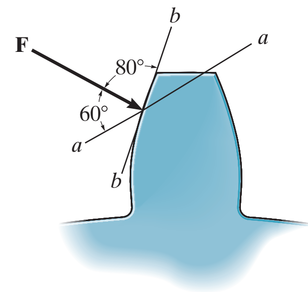
$$\frac{F_{2u}}{\sin 75^\circ} = \frac{150}{\sin 75^\circ} \qquad F_{2u} = 150 \text{ N} \qquad \textbf{Ans.}$$





2–11.

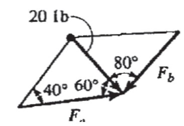
The force acting on the gear tooth is $F = 20$ lb. Resolve this force into two components acting along the lines *aa* and *bb*.



SOLUTION

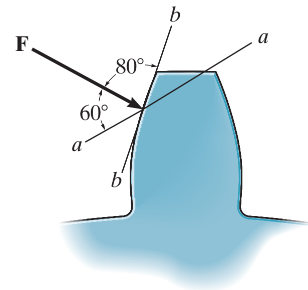
$$\frac{20}{\sin 40^\circ} = \frac{F_a}{\sin 80^\circ}; \qquad F_a = 30.6 \text{ lb}$$ **Ans.**

$$\frac{20}{\sin 40^\circ} = \frac{F_b}{\sin 60^\circ}; \qquad F_b = 26.9 \text{ lb}$$ **Ans.**



***2–12.**

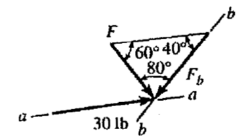
The component of force **F** acting along line *aa* is required to be 30 lb. Determine the magnitude of **F** and its component along line *bb*.



SOLUTION

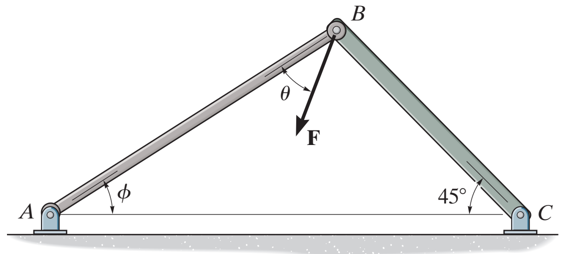
$$\frac{30}{\sin 80^\circ} = \frac{F}{\sin 40^\circ}; \qquad F = 19.6\text{ lb} \qquad \textbf{Ans.}$$

$$\frac{30}{\sin 80^\circ} = \frac{F_b}{\sin 60^\circ}; \qquad F_b = 26.4\text{ lb} \qquad \textbf{Ans.}$$



2–13.

Force **F** acts on the frame such that its component acting along member AB is 650 lb, directed from B towards A, and the component acting along member BC is 500 lb, directed from B towards C. Determine the magnitude of **F** and its direction θ. Set $\phi = 60°$.



SOLUTION

The parallelogram law of addition and triangular rule are shown in Figs. *a* and *b*, respectively.

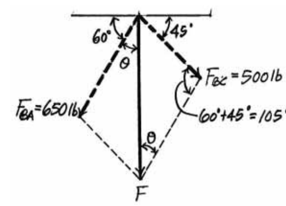
Applying the law of cosines to Fig. *b*,

$$F = \sqrt{500^2 + 650^2 - 2(500)(650)\cos 105°}$$

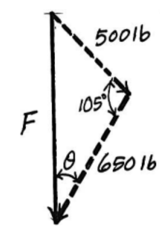
$$= 916.91 \text{ lb} = 917 \text{ lb} \qquad \textbf{Ans.}$$

Using this result and applying the law of sines to Fig. *b*, yields

$$\frac{\sin\theta}{500} = \frac{\sin 105°}{916.91} \qquad \theta = 31.8° \qquad \textbf{Ans.}$$



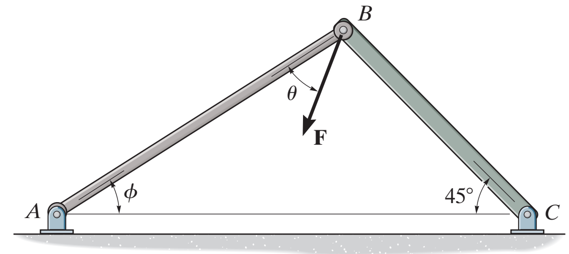
(a)



(b)

2–14.

Force **F** acts on the frame such that its component acting along member AB is 650 lb, directed from B towards A. Determine the required angle ϕ $(0^\circ \leq \phi \leq 90^\circ)$ and the component acting along member BC. Set $F = 850$ lb and $\theta = 30^\circ$.



SOLUTION

The parallelogram law of addition and the triangular rule are shown in Figs. *a* and *b*, respectively.

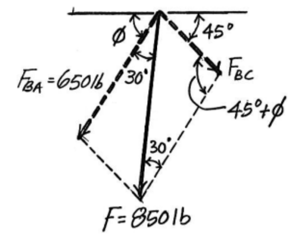
Applying the law of cosines to Fig. *b*,

$$F_{BC} = \sqrt{850^2 + 650^2 - 2(850)(650)\cos 30^\circ}$$

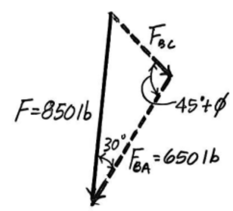
$$= 433.64 \text{ lb} = 434 \text{ lb} \qquad \textbf{Ans.}$$

Using this result and applying the sine law to Fig. *b*, yields

$$\frac{\sin(45^\circ + \phi)}{850} = \frac{\sin 30^\circ}{433.64} \qquad \phi = 56.5^\circ \qquad \textbf{Ans.}$$



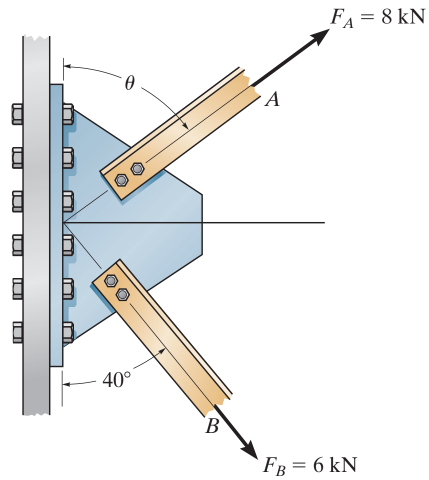
(a)



(b)

2–15.

The plate is subjected to the two forces at A and B as shown. If $\theta = 60°$, determine the magnitude of the resultant of these two forces and its direction measured clockwise from the horizontal.



SOLUTION

Parallelogram Law: The parallelogram law of addition is shown in Fig. *a*.

Trigonometry: Using law of cosines (Fig. *b*), we have

$$F_R = \sqrt{8^2 + 6^2 - 2(8)(6)\cos 100°}$$

$$= 10.80 \text{ kN} = 10.8 \text{ kN} \qquad \textbf{Ans.}$$

The angle θ can be determined using law of sines (Fig. *b*).

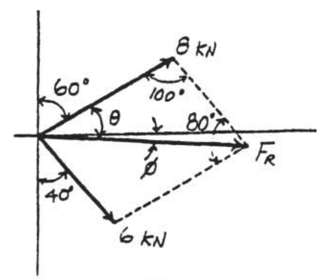
$$\frac{\sin\theta}{6} = \frac{\sin 100°}{10.80}$$

$$\sin\theta = 0.5470$$

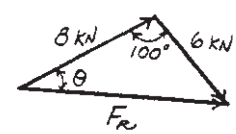
$$\theta = 33.16°$$

Thus, the direction ϕ of $\mathbf{F}_R$ measured from the x axis is

$$\phi = 33.16° - 30° = 3.16° \qquad \textbf{Ans.}$$



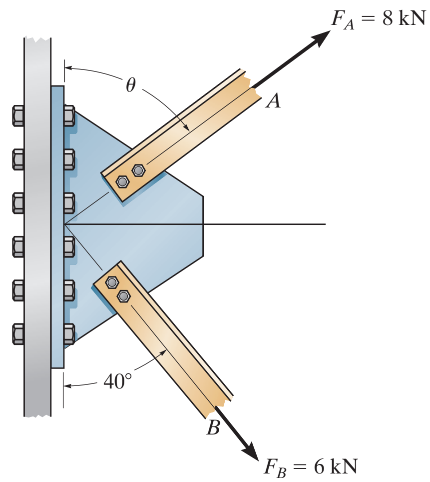
(a)



(b)

***2–16.**

Determine the angle of θ for connecting member A to the plate so that the resultant force of $\mathbf{F}_A$ and $\mathbf{F}_B$ is directed horizontally to the right. Also, what is the magnitude of the resultant force?



SOLUTION

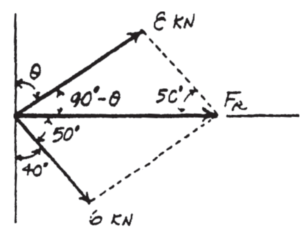
Parallelogram Law: The parallelogram law of addition is shown in Fig. *a*.

Trigonometry: Using law of sines (Fig .*b*), we have

$$\frac{\sin(90° - \theta)}{6} = \frac{\sin 50°}{8}$$

$$\sin(90° - \theta) = 0.5745$$

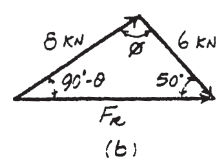
$$\theta = 54.93° = 54.9° \qquad \textbf{Ans.}$$



From the triangle, $\phi = 180° - (90° - 54.93°) - 50° = 94.93°$. Thus, using law of cosines, the magnitude of $\mathbf{F}_R$ is

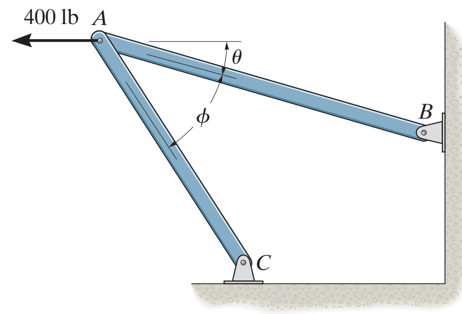
$$F_R = \sqrt{8^2 + 6^2 - 2(8)(6)\cos 94.93°}$$

$$= 10.4 \text{ kN} \qquad \textbf{Ans.}$$



2–17.

Determine the design angle θ ($0° \le \theta \le 90°$) for strut AB so that the 400-lb horizontal force has a component of 500 lb directed from A towards C. What is the component of force acting along member AB? Take $\phi = 40°$.



SOLUTION

Parallelogram Law: The parallelogram law of addition is shown in Fig. *a*.

Trigonometry: Using law of sines (Fig. *b*), we have

$$\frac{\sin\theta}{500} = \frac{\sin 40°}{400}$$

$$\sin\theta = 0.8035$$

$$\theta = 53.46° = 53.5° \qquad \textbf{Ans.}$$

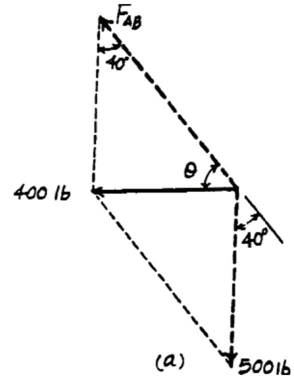
Thus,

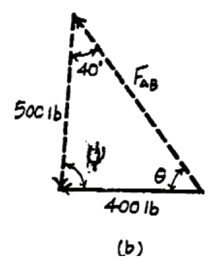
$$\psi = 180° - 40° - 53.46° = 86.54°$$

Using law of sines (Fig. *b*)

$$\frac{F_{AB}}{\sin 86.54°} = \frac{400}{\sin 40°}$$

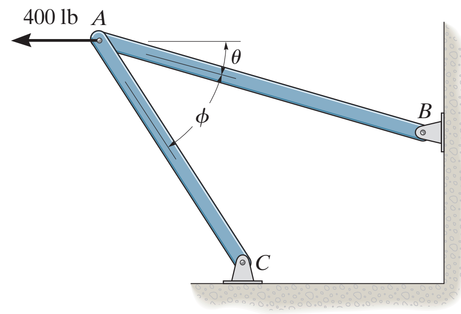
$$F_{AB} = 621 \text{ lb} \qquad \textbf{Ans.}$$





2–18.

Determine the design angle $\phi\ (0^\circ \le \phi \le 90^\circ)$ between struts AB and AC so that the 400-lb horizontal force has a component of 600 lb which acts up to the left, in the same direction as from B towards A. Take $\theta = 30^\circ$.



SOLUTION

Parallelogram Law: The parallelogram law of addition is shown in Fig. a.

Trigonometry: Using law of cosines (Fig. b), we have

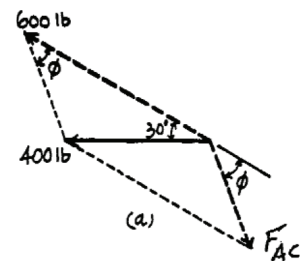
$$F_{AC} = \sqrt{400^2 + 600^2 - 2(400)(600)\cos 30^\circ} = 322.97 \text{ lb}$$

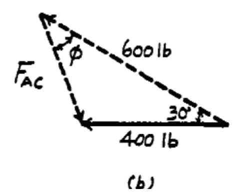
The angle ϕ can be determined using law of sines (Fig. b).

$$\frac{\sin\phi}{400} = \frac{\sin 30^\circ}{322.97}$$

$$\sin\phi = 0.6193$$

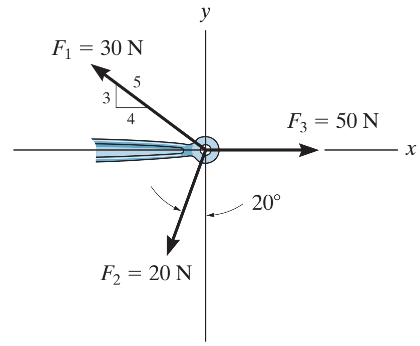
$$\phi = 38.3^\circ \qquad \textbf{Ans.}$$





2–19.

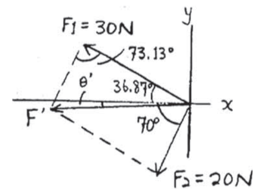
Determine the magnitude and direction of the resultant $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$ of the three forces by first finding the resultant $\mathbf{F}' = \mathbf{F}_1 + \mathbf{F}_2$ and then forming $\mathbf{F}_R = \mathbf{F}' + \mathbf{F}_3$.



SOLUTION

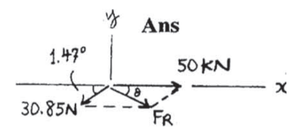
$$F' = \sqrt{(20)^2 + (30)^2 - 2(20)(30)\cos 73.13°} = 30.85 \text{ N}$$

$$\frac{30.85}{\sin 73.13°} = \frac{30}{\sin (70° - \theta')}; \qquad \theta' = 1.47°$$



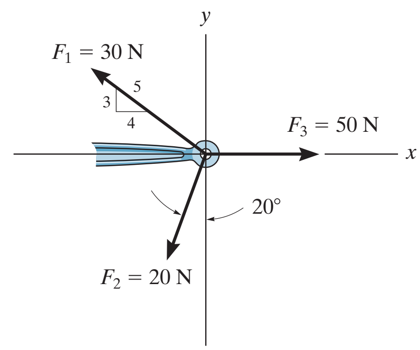
$$F_R = \sqrt{(30.85)^2 + (50)^2 - 2(30.85)(50)\cos 1.47°} = 19.18 = 19.2 \text{ N}$$ **Ans.**

$$\frac{19.18}{\sin 1.47°} = \frac{30.85}{\sin \theta}; \qquad \theta = 2.37° ⦪$$ **Ans.**



***2–20.**

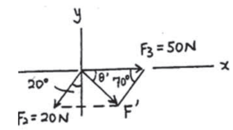
Determine the magnitude and direction of the resultant $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$ of the three forces by first finding the resultant $\mathbf{F}' = \mathbf{F}_2 + \mathbf{F}_3$ and then forming $\mathbf{F}_R = \mathbf{F}' + \mathbf{F}_1$.



SOLUTION

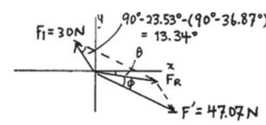
$$F' = \sqrt{(20)^2 + (50)^2 - 2(20)(50)\cos 70^\circ} = 47.07\text{ N}$$

$$\frac{20}{\sin\theta'} = \frac{47.07}{\sin 70^\circ}; \qquad \theta' = 23.53^\circ$$



$$F_R = \sqrt{(47.07)^2 + (30)^2 - 2(47.07)(30)\cos 13.34^\circ} = 19.18 = 19.2\text{ N} \qquad \textbf{Ans.}$$

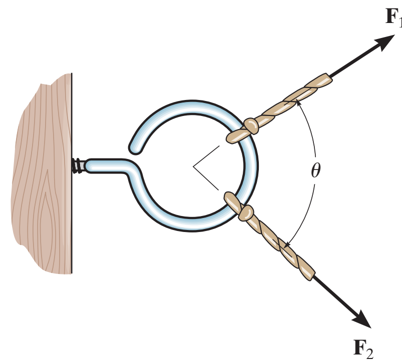
$$\frac{19.18}{\sin 13.34^\circ} = \frac{30}{\sin\phi}; \qquad \phi = 21.15^\circ$$



$$\theta = 23.53^\circ - 21.15^\circ = 2.37^\circ \qquad \textbf{Ans.}$$

2–21.

Two forces act on the screw eye. If $F_1 = 400$ N and $F_2 = 600$ N, determine the angle $\theta(0° \leq \theta \leq 180°)$ between them, so that the resultant force has a magnitude of $F_R = 800$ N.



SOLUTION

The parallelogram law of addition and triangular rule are shown in Figs. *a* and *b*, respectively. Applying law of cosines to Fig. *b*,

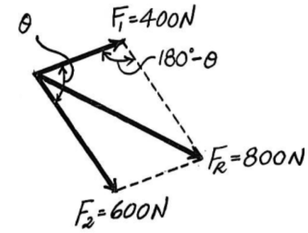
$$800 = \sqrt{400^2 + 600^2 - 2(400)(600)\cos(180° - \theta°)}$$

$$800^2 = 400^2 + 600^2 - 480000\cos(180° - \theta)$$

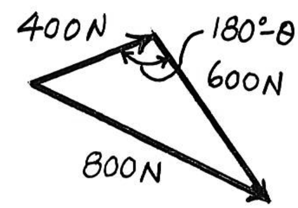
$$\cos(180° - \theta) = -0.25$$

$$180° - \theta = 104.48$$

$$\theta = 75.52° = 75.5° \qquad \textbf{Ans.}$$



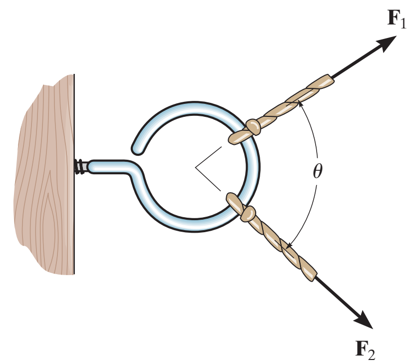
(a)



(b)

2–22.

Two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ act on the screw eye. If their lines of action are at an angle θ apart and the magnitude of each force is $F_1 = F_2 = F$, determine the magnitude of the resultant force $\mathbf{F}_R$ and the angle between $\mathbf{F}_R$ and $\mathbf{F}_1$.



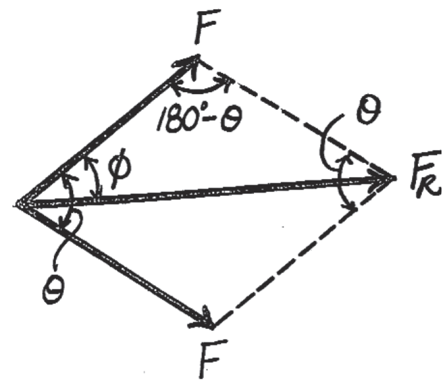
SOLUTION

$$\frac{F}{\sin \phi} = \frac{F}{\sin (\theta - \phi)}$$

$$\sin (\theta - \phi) = \sin \phi$$

$$\theta - \phi = \phi$$

$$\phi = \frac{\theta}{2} \qquad \textbf{Ans.}$$



(a)

$$F_R = \sqrt{(F)^2 + (F)^2 - 2(F)(F)\cos(180^\circ - \theta)}$$

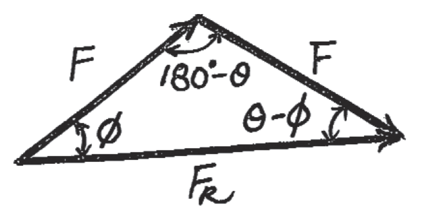
Since $\cos(180^\circ - \theta) = -\cos\theta$

$$F_R = F\left(\sqrt{2}\right)\sqrt{1 + \cos\theta}$$

Since $\cos\left(\frac{\theta}{2}\right) = \sqrt{\frac{1 + \cos\theta}{2}}$

Then

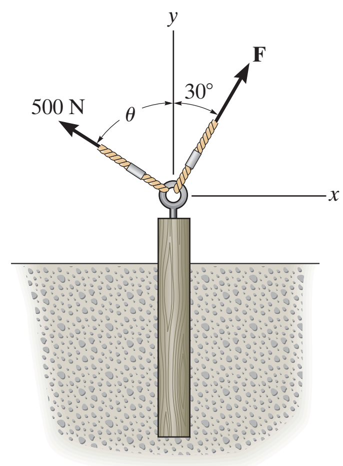
$$F_R = 2F\cos\left(\frac{\theta}{2}\right) \qquad \textbf{Ans.}$$



(b)

2–23.

Two forces act on the screw eye. If $F = 600\text{ N}$, determine the magnitude of the resultant force and the angle θ if the resultant force is directed vertically upward.



SOLUTION

The parallelogram law of addition and triangular rule are shown in Figs. *a* and *b* respectively. Applying law of sines to Fig. *b*,

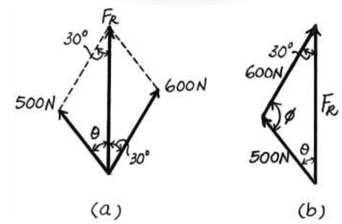
$$\frac{\sin\theta}{600} = \frac{\sin 30^\circ}{500}; \quad \sin\theta = 0.6 \quad \theta = 36.87^\circ = 36.9^\circ \qquad \textbf{Ans.}$$

Using the result of θ,

$$\phi = 180^\circ - 30^\circ - 36.87^\circ = 113.13^\circ$$

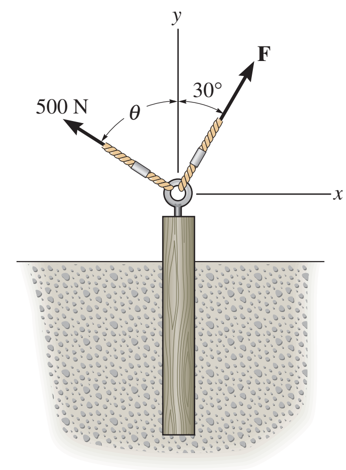
Again, applying law of sines using the result of ϕ,

$$\frac{F_R}{\sin 113.13^\circ} = \frac{500}{\sin 30^\circ}; \qquad F_R = 919.61\text{ N} = 920\text{ N} \qquad \textbf{Ans.}$$



***2–24.**

Two forces are applied at the end of a screw eye in order to remove the post. Determine the angle θ $(0° \le \theta \le 90°)$ and the magnitude of force **F** so that the resultant force acting on the post is directed vertically upward and has a magnitude of 750 N.



SOLUTION

Parallelogram Law: The parallelogram law of addition is shown in Fig. *a*.

Trigonometry: Using law of sines (Fig. *b*), we have

$$\frac{\sin \phi}{750} = \frac{\sin 30°}{500}$$

$$\sin \phi = 0.750$$

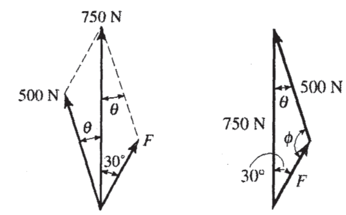
$$\phi = 131.41° \text{ (By observation, } \phi > 90°)$$

Thus,

$$\theta = 180° - 30° - 131.41° = 18.59° = 18.6° \qquad \textbf{Ans.}$$

$$\frac{F}{\sin 18.59°} = \frac{500}{\sin 30°}$$

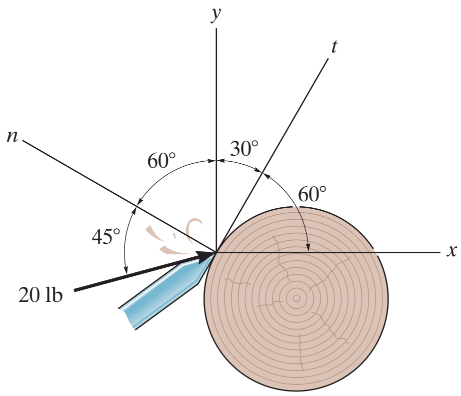
$$F = 319 \text{ N} \qquad \textbf{Ans.}$$



(a) (b)

2–25.

The chisel exerts a force of 20 lb on the wood dowel rod which is turning in a lathe. Resolve this force into components acting (a) along the n and t axes and (b) along the x and y axes.



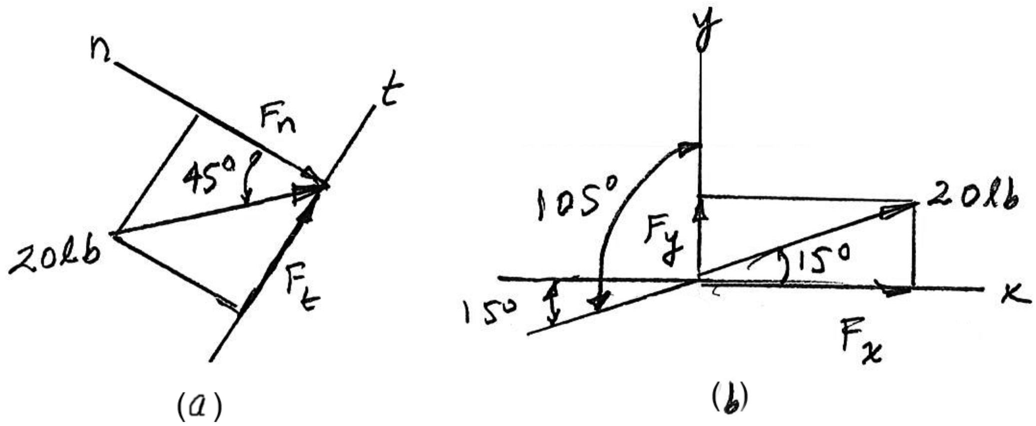
SOLUTION

a) $F_n = -20 \cos 45° = -14.1 \text{ lb}$ **Ans.**

$F_t = 20 \sin 45° = 14.1 \text{ lb}$ **Ans.**

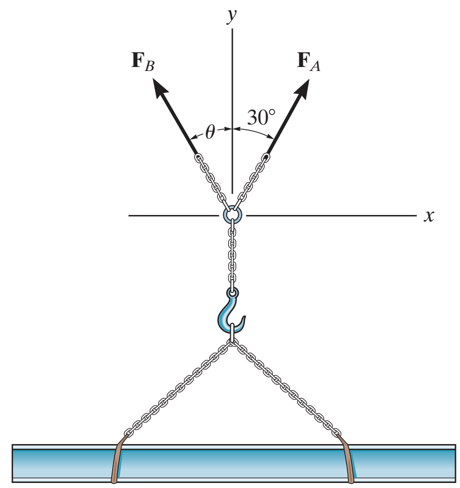
b) $F_x = 20 \cos 15° = 19.3 \text{ lb}$ **Ans.**

$F_y = 20 \sin 15° = 5.18 \text{ lb}$ **Ans.**



2–26.

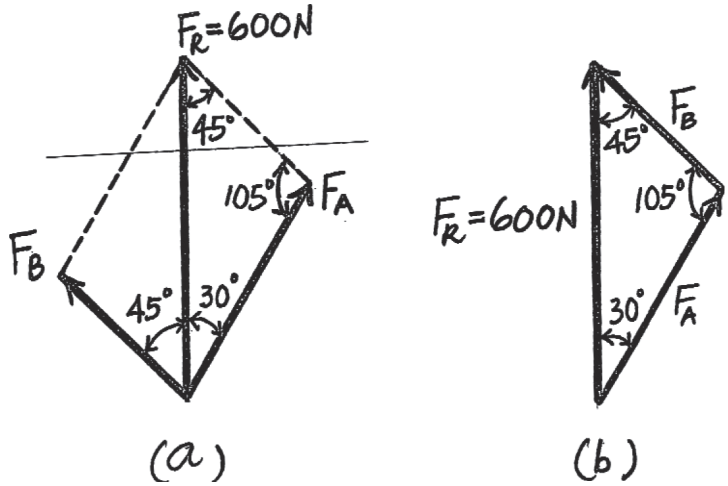
The beam is to be hoisted using two chains. Determine the magnitudes of forces $\mathbf{F}_A$ and $\mathbf{F}_B$ acting on each chain in order to develop a resultant force of 600 N directed along the positive y axis. Set $\theta = 45°$.



SOLUTION

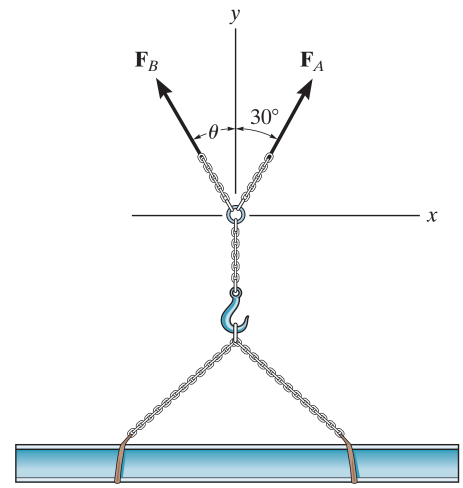
$$\frac{F_A}{\sin 45°} = \frac{600}{\sin 105°}; \qquad F_A = 439 \text{ N} \qquad \textbf{Ans.}$$

$$\frac{F_B}{\sin 30°} = \frac{600}{\sin 105°}; \qquad F_B = 311 \text{ N} \qquad \textbf{Ans.}$$



2–27.

The beam is to be hoisted using two chains. If the resultant force is to be 600 N directed along the positive y axis, determine the magnitudes of forces $\mathbf{F}_A$ and $\mathbf{F}_B$ acting on each chain and the angle θ of $\mathbf{F}_B$ so that the magnitude of $\mathbf{F}_B$ is a *minimum*. $\mathbf{F}_A$ acts at 30° from the y axis, as shown.



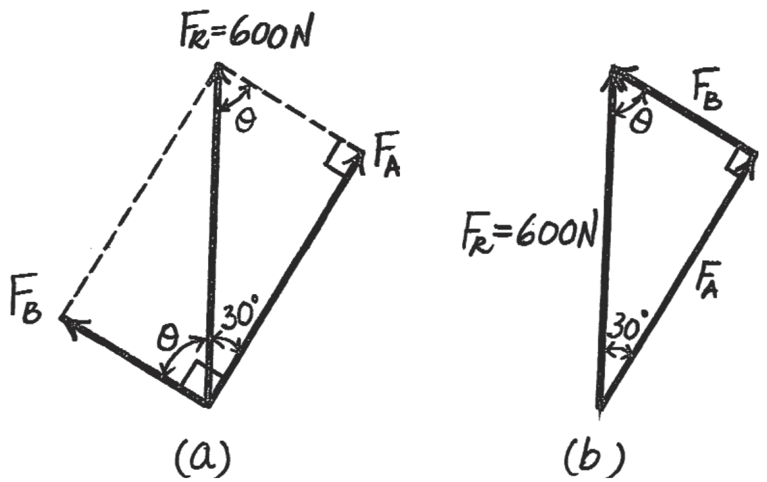
SOLUTION

For minimum F_B, require

$$\theta = 60^\circ \qquad \textbf{Ans.}$$

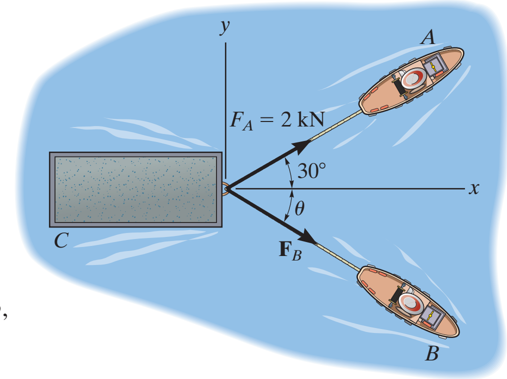
$$F_A = 600 \cos 30^\circ = 520 \text{ N} \qquad \textbf{Ans.}$$

$$F_B = 600 \sin 30^\circ = 300 \text{ N} \qquad \textbf{Ans.}$$



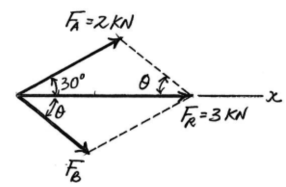
***2–28.**

If the resultant force of the two tugboats is 3 kN, directed along the positive x axis, determine the required magnitude of force $\mathbf{F}_B$ and its direction θ.

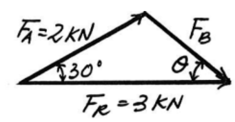


SOLUTION

The parallelogram law of addition and the triangular rule are shown in Figs. *a* and *b*, respectively.



(a)



(b)

Applying the law of cosines to Fig. *b*,

$$F_B = \sqrt{2^2 + 3^2 - 2(2)(3)\cos 30^\circ}$$

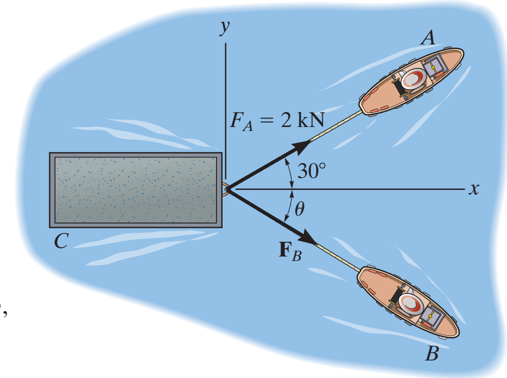
$$= 1.615\text{kN} = 1.61 \text{ kN}$$ **Ans.**

Using this result and applying the law of sines to Fig. *b*, yields

$$\frac{\sin\theta}{2} = \frac{\sin 30^\circ}{1.615} \qquad \theta = 38.3^\circ$$ **Ans.**

2–29.

If $F_B = 3$ kN and $\theta = 45°$, determine the magnitude of the resultant force of the two tugboats and its direction measured clockwise from the positive x axis.



SOLUTION

The parallelogram law of addition and the triangular rule are shown in Figs. *a* and *b*, respectively.

Applying the law of cosines to Fig. *b*,

$$F_R = \sqrt{2^2 + 3^2 - 2(2)(3)\cos 105°}$$

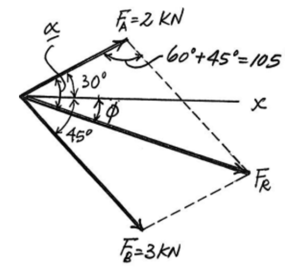
$$= 4.013 \text{ kN} = 4.01 \text{ kN} \qquad \textbf{Ans.}$$

Using this result and applying the law of sines to Fig. *b,* yields

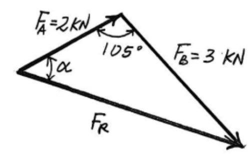
$$\frac{\sin \alpha}{3} = \frac{\sin 105°}{4.013} \qquad \alpha = 46.22°$$

Thus, the direction angle ϕ of $\mathbf{F}_R$, measured clockwise from the positive x axis, is

$$\phi = \alpha - 30° = 46.22° - 30° = 16.2° \qquad \textbf{Ans.}$$



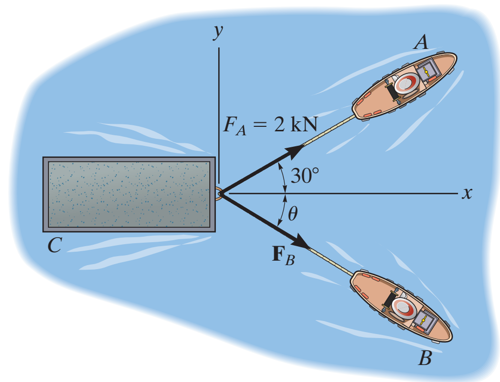
(a)



(b)

2–30.

If the resultant force of the two tugboats is required to be directed towards the positive x axis, and F_B is to be a minimum, determine the magnitude of $\mathbf{F}_R$ and $\mathbf{F}_B$ and the angle θ.



SOLUTION

For $\mathbf{F}_B$ to be minimum, it has to be directed perpendicular to $\mathbf{F}_R$. Thus,

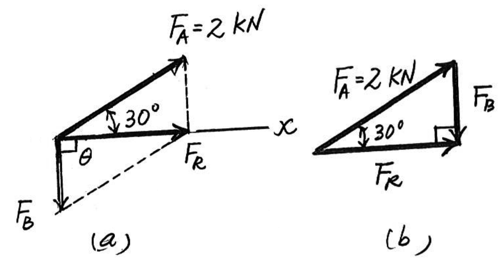
$$\theta = 90° \qquad \textbf{Ans.}$$

The parallelogram law of addition and triangular rule are shown in Figs. *a* and *b*, respectively.

By applying simple trigonometry to Fig. *b*,

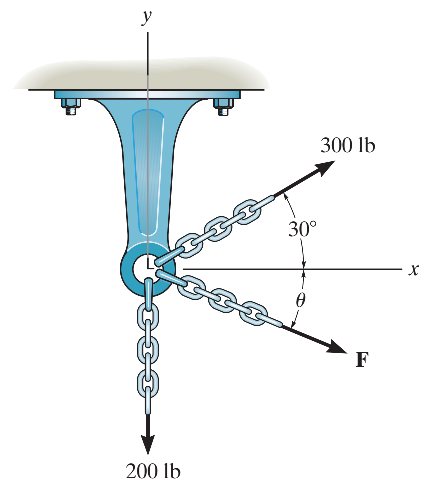
$$F_B = 2 \sin 30° = 1 \text{ kN} \qquad \textbf{Ans.}$$

$$F_R = 2 \cos 30° = 1.73 \text{ kN} \qquad \textbf{Ans.}$$



2–31.

Three chains act on the bracket such that they create a resultant force having a magnitude of 500 lb. If two of the chains are subjected to known forces, as shown, determine the angle θ of the third chain measured clockwise from the positive x axis, so that the magnitude of force **F** in this chain is a *minimum*. All forces lie in the x–y plane. What is the magnitude of **F**? *Hint*: First find the resultant of the two known forces. Force **F** acts in this direction.



SOLUTION

Cosine law:

$$F_{R1} = \sqrt{300^2 + 200^2 - 2(300)(200)\cos 60^\circ} = 264.6 \text{ lb}$$

Sine law:

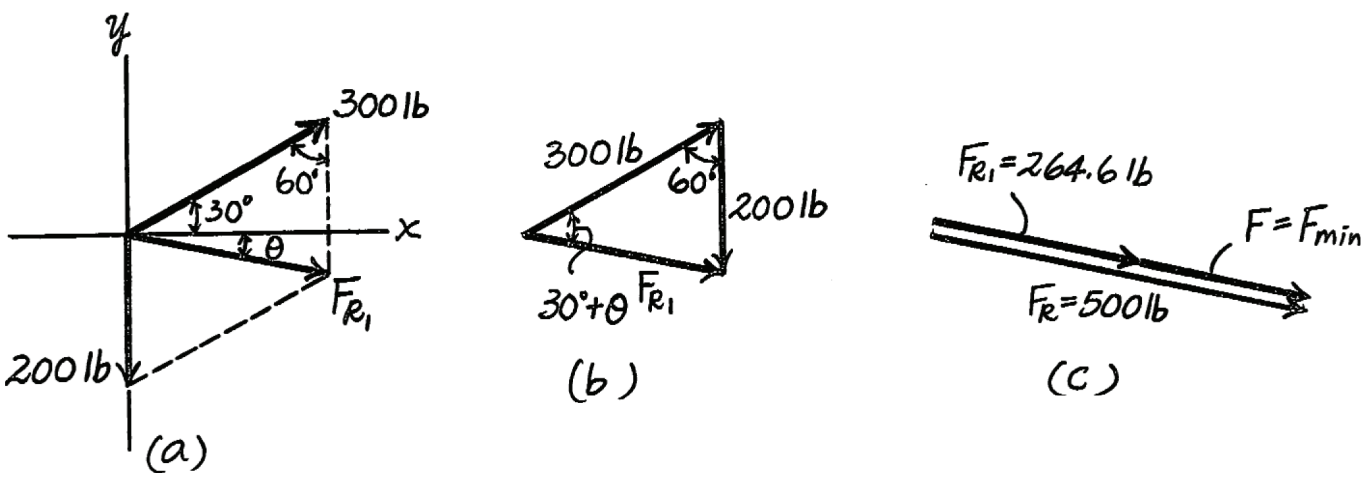
$$\frac{\sin(30^\circ + \theta)}{200} = \frac{\sin 60^\circ}{264.6} \qquad \theta = 10.9^\circ \qquad \textbf{Ans.}$$

When **F** is directed along $\mathbf{F}_{R1}$, F will be minimum to create the resultant force.

$$F_R = F_{R1} + F$$

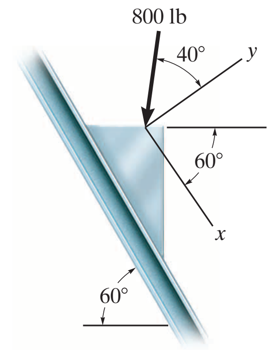
$$500 = 264.6 + F_{\min}$$

$$F_{\min} = 235 \text{ lb} \qquad \textbf{Ans.}$$



***2–32.**

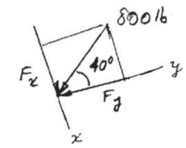
Determine the x and y components of the 800-lb force.



SOLUTION

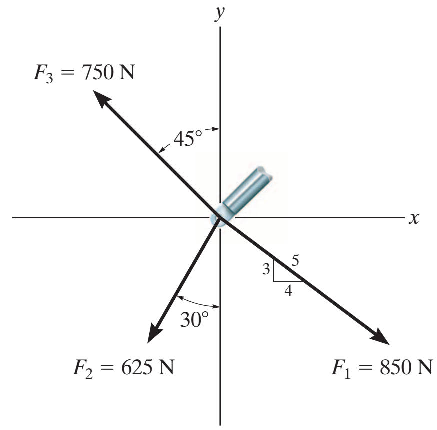
$$F_x = 800 \sin 40^\circ = 514 \text{ lb}$$ **Ans.**

$$F_y = -800 \cos 40^\circ = -613 \text{ lb}$$ **Ans.**



2–33.

Determine the magnitude of the resultant force and its direction, measured counterclockwise from the positive x axis.



SOLUTION

$\xrightarrow{+} F_{R_x} = \Sigma F_x;$ $\quad F_{R_x} = \frac{4}{5}(850) - 625 \sin 30° - 750 \sin 45° = -162.8 \text{ N}$

$+\uparrow F_{R_y} = \Sigma F_y;$ $\quad F_{Ry} = -\frac{3}{5}(850) - 625 \cos 30° + 750 \cos 45° = -520.9 \text{ N}$

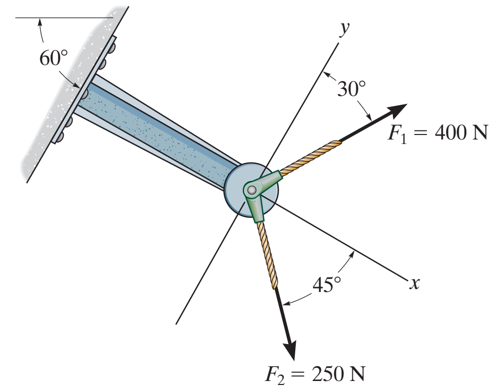
$$F_R = \sqrt{(-162.8)^2 + (-520.9)^2} = 546 \text{ N}$$ **Ans.**

$$\phi = \tan^{-1}\left[\frac{-520.9}{-162.8}\right] = 72.64°$$

$$\theta = 180° + 72.64° = 253°$$ **Ans.**

2–34.

Resolve $\mathbf{F}_1$ and $\mathbf{F}_2$ into their x and y components.



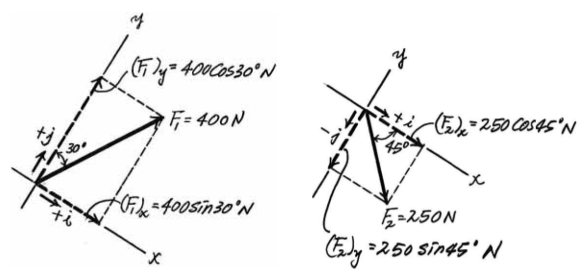
SOLUTION

$$\mathbf{F}_1 = \{400 \sin 30°(+\mathbf{i}) + 400 \cos 30°(+\mathbf{j})\}\text{ N}$$

$$= \{200\mathbf{i} + 346\mathbf{j}\}\text{ N} \qquad \textbf{Ans.}$$

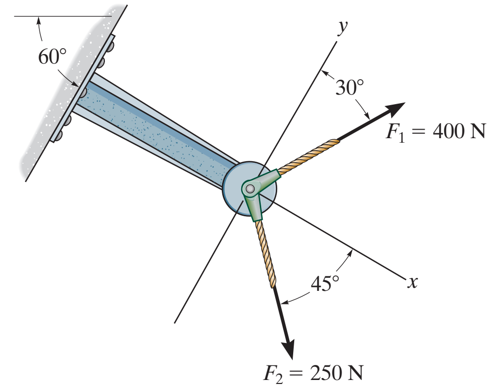
$$\mathbf{F}_2 = \{250 \cos 45°(+\mathbf{i}) + 250 \sin 45°(-\mathbf{j})\}\text{ N}$$

$$= \{177\mathbf{i} + 177\mathbf{j}\}\text{ N} \qquad \textbf{Ans.}$$



2–35.

Determine the magnitude of the resultant force and its direction measured counterclockwise from the positive x axis.



SOLUTION

Rectangular Components: By referring to Fig. a, the x and y components of $\mathbf{F}_1$ and $\mathbf{F}_2$ can be written as

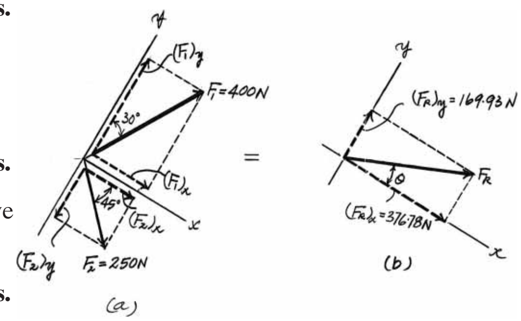
$$(F_1)_x = 400 \sin 30^\circ = 200 \text{ N} \qquad (F_1)_y = 400 \cos 30^\circ = 346.41 \text{ N}$$

$$(F_2)_x = 250 \cos 45^\circ = 176.78 \text{ N} \qquad (F_2)_y = 250 \sin 45^\circ = 176.78 \text{ N}$$

Resultant Force: Summing the force components algebraically along the x and y axes, we have

$$\overset{+}{\rightarrow} \Sigma(F_R)_x = \Sigma F_x; \qquad (F_R)_x = 200 + 176.78 = 376.78 \text{ N} \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma(F_R)_y = \Sigma F_y; \qquad (F_R)_y = 346.41 - 176.78 = 169.63 \text{ N} \uparrow$$



The magnitude of the resultant force $\mathbf{F}_R$ is

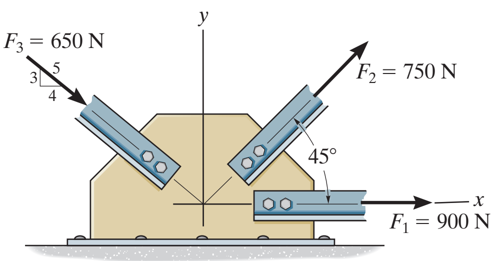
$$F_R = \sqrt{(F_R)_x^{\,2} + (F_R)_y^{\,2}} = \sqrt{376.78^2 + 169.63^2} = 413 \text{ N} \qquad \textbf{Ans.}$$

The direction angle θ of $\mathbf{F}_R$, Fig. b, measured counterclockwise from the positive axis, is

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left(\frac{169.63}{376.78}\right) = 24.2^\circ \qquad \textbf{Ans.}$$

***2–36.**

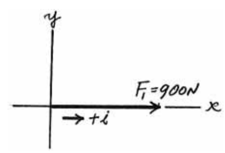
Resolve each force acting on the gusset plate into its x and y components, and express each force as a Cartesian vector.

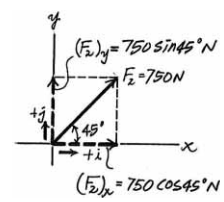


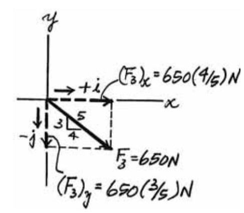
$$\mathbf{F}_1 = \{900(+\mathbf{i})\} = \{900\mathbf{i}\}\ \text{N}$$ **Ans.**

$$\mathbf{F}_2 = \{750\cos 45°(+\mathbf{i}) + 750\sin 45°(+\mathbf{j})\}\ \text{N}$$
$$= \{530\mathbf{i} + 530\mathbf{j}\}\ \text{N}$$ **Ans.**

$$\mathbf{F}_3 = \left\{650\left(\frac{4}{5}\right)(+\mathbf{i}) + 650\left(\frac{3}{5}\right)(-\mathbf{j})\right\}\ \text{N}$$
$$= \{520\,\mathbf{i} - 390\mathbf{j})\}\ \text{N}$$ **Ans.**

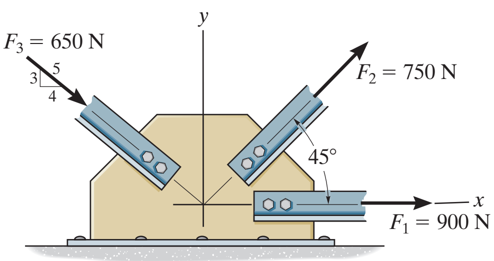






2–37.

Determine the magnitude of the resultant force acting on the plate and its direction, measured counterclockwise from the positive x axis.



SOLUTION

Rectangular Components: By referring to Fig. a, the x and y components of $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ can be written as

$$(F_1)_x = 900 \text{ N} \qquad (F_1)_y = 0$$

$$(F_2)_x = 750 \cos 45° = 530.33 \text{ N} \qquad (F_2)_y = 750 \sin 45° = 530.33 \text{ N}$$

$$(F_3)_x = 650\left(\frac{4}{5}\right) = 520 \text{ N} \qquad (F_3)_y = 650\left(\frac{3}{5}\right) = 390 \text{ N}$$

Resultant Force: Summing the force components algebraically along the x and y axes, we have

$$\overset{+}{\rightarrow} \Sigma(F_R)_x = \Sigma F_x; \qquad (F_R)_x = 900 + 530.33 + 520 = 1950.33 \text{ N} \rightarrow$$

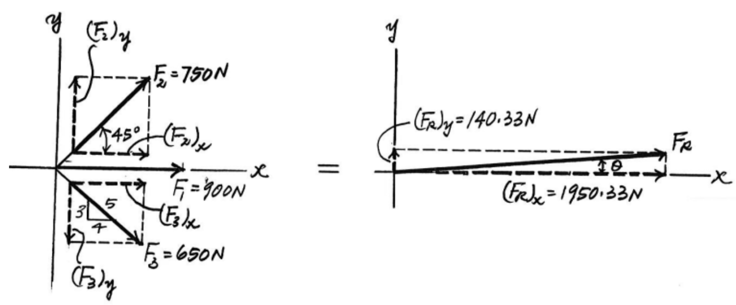
$$+\uparrow \Sigma(F_R)_y = \Sigma F_y; \qquad (F_R)_y = 530.33 - 390 = 140.33 \text{ N} \uparrow$$

The magnitude of the resultant force $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x^{\,2} + (F_R)_y^{\,2}} = \sqrt{1950.33^2 + 140.33^2} = 1955 \text{ N} = 1.96 \text{ kN} \quad \textbf{Ans.}$$

The direction angle θ of $\mathbf{F}_R$, measured clockwise from the positive x axis, is

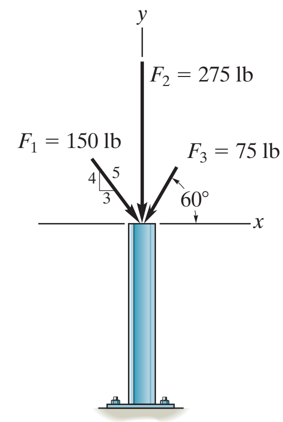
$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left(\frac{140.33}{1950.33}\right) = 4.12° \quad \textbf{Ans.}$$



(a)

2–38.

Express each of the three forces acting on the column in Cartesian vector form and compute the magnitude of the resultant force.



SOLUTION

$$\mathbf{F}_1 = 150\left(\frac{3}{5}\right)\mathbf{i} - 150\left(\frac{4}{5}\right)\mathbf{j}$$

$$\mathbf{F}_1 = \{90\mathbf{i} - 120\mathbf{j}\}\text{ lb}$$ **Ans.**

$$\mathbf{F}_2 = \{-275\mathbf{j}\}\text{ lb}$$ **Ans.**

$$\mathbf{F}_3 = -75\cos 60^\circ\mathbf{i} - 75\sin 60^\circ\mathbf{j}$$

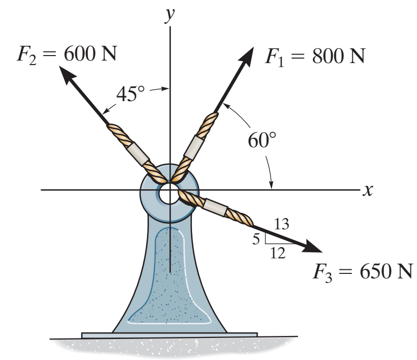
$$\mathbf{F}_3 = \{-37.5\mathbf{i} - 65.0\mathbf{j}\}\text{ lb}$$ **Ans.**

$$\mathbf{F}_R = \Sigma\mathbf{F} = \{52.5\mathbf{i} - 460\mathbf{j}\}\text{ lb}$$

$$\mathbf{F}_R = \sqrt{(52.5)^2 + (-460)^2} = 463\text{ lb}$$ **Ans.**

2–39.

Resolve each force acting on the support into its x and y components, and express each force as a Cartesian vector.



SOLUTION

$$\mathbf{F}_1 = \{800 \cos 60°(+\mathbf{i}) + 800 \sin 60°(+\mathbf{j})\}\ \text{N}$$

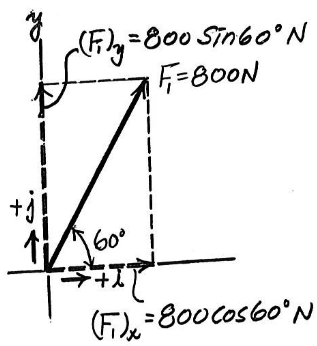
$$= \{400\mathbf{i} + 693\mathbf{j}\}\ \text{N}$$ **Ans.**

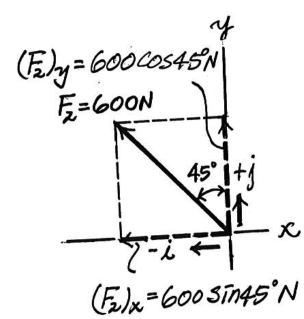
$$\mathbf{F}_2 = \{600 \sin 45°(-\mathbf{i}) + 600 \cos 45°(+\mathbf{j})\}\ \text{N}$$ **Ans.**

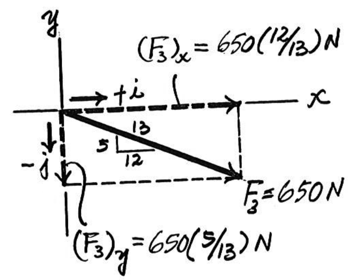
$$= \{-424\mathbf{i} + 424\mathbf{j}\}\ \text{N}$$

$$\mathbf{F}_3 = \left\{650\left(\frac{12}{13}\right)(+\mathbf{i}) + 650\left(\frac{5}{13}\right)(-\mathbf{j})\right\}\ \text{N}$$ **Ans.**

$$= \{600\mathbf{i} - 250\mathbf{j}\}\ \text{N}$$

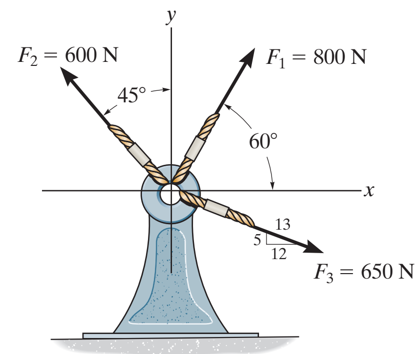






***2–40.**

Determine the magnitude of the resultant force and its direction θ, measured counterclockwise from the positive x axis.



SOLUTION

Rectangular Components: By referring to Fig. *a*, the x and y components of $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ can be written as

$$(F_1)_x = 800 \cos 60° = 400 \text{ N} \qquad (F_1)_y = 800 \sin 60° = 692.82 \text{ N}$$

$$(F_2)_x = 600 \sin 45° = 424.26 \text{ N} \qquad (F_2)_y = 600 \cos 45° = 424.26 \text{ N}$$

$$(F_3)_x = 650\left(\frac{12}{13}\right) = 600 \text{ N} \qquad (F_3)_y = 650\left(\frac{5}{13}\right) = 250 \text{ N}$$

Resultant Force: Summing the force components algebraically along the x and y axes, we have

$$\overset{+}{\rightarrow} \Sigma(F_R)_x = \Sigma F_x; \qquad (F_R)_x = 400 - 424.26 + 600 = 575.74 \text{ N} \rightarrow$$

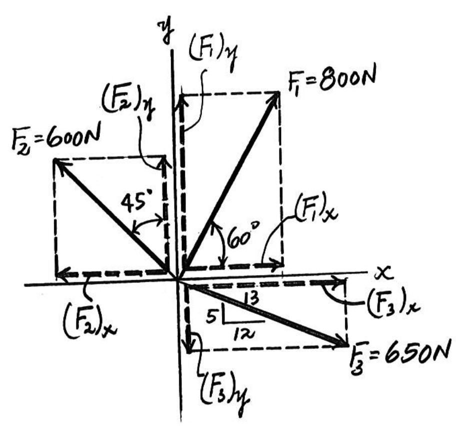
$$+\uparrow \Sigma(F_R)_y = \Sigma F_y; \qquad (F_R)_y = -692.82 + 424.26 - 250 = 867.08 \text{ N} \uparrow$$

The magnitude of the resultant force $\mathbf{F}_R$ is

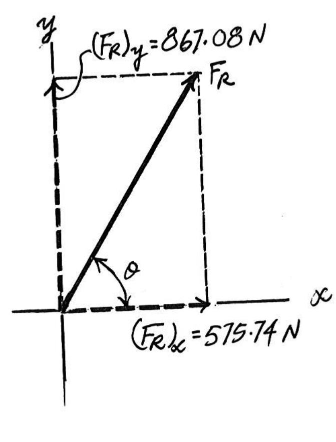
$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{575.74^2 + 867.08^2} = 1041 \text{ N} = 1.04 \text{ kN} \quad \textbf{Ans.}$$

The direction angle θ of $\mathbf{F}_R$, Fig. *b*, measured counterclockwise from the positive x axis, is

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left(\frac{867.08}{575.74}\right) = 56.4° \quad \textbf{Ans.}$$



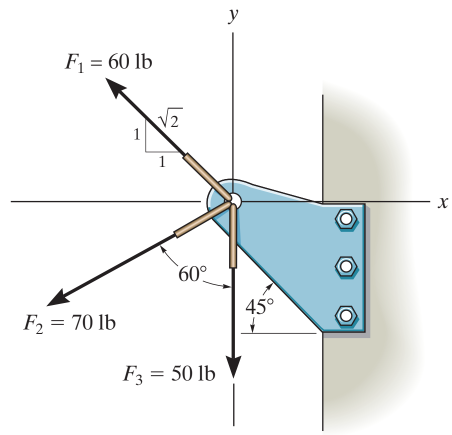
(a)



(b)

2–41.

Determine the magnitude of the resultant force and its direction measured counterclockwise from the positive x axis.



SOLUTION

$$\mathbf{F}_1 = -60\left(\frac{1}{\sqrt{2}}\right)\mathbf{i} + 60\left(\frac{1}{\sqrt{2}}\right)\mathbf{j} = \{-42.43\mathbf{i} + 42.43\mathbf{j}\} \text{ lb}$$

$$\mathbf{F}_2 = -70 \sin 60°\mathbf{i} - 70 \cos 60°\mathbf{j} = \{-60.62\mathbf{i} - 35\mathbf{j}\} \text{ lb}$$

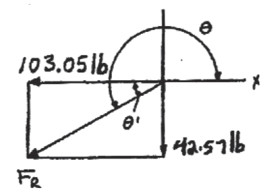
$$\mathbf{F}_3 = \{-50\,\mathbf{j}\} \text{ lb}$$

$$\mathbf{F}_R = \Sigma\mathbf{F} = \{-103.05\,\mathbf{i} - 42.57\,\mathbf{j}\} \text{ lb}$$

$$F_R = \sqrt{(-103.05)^2 + (-42.57)^2} = 111 \text{ lb}$$ **Ans.**

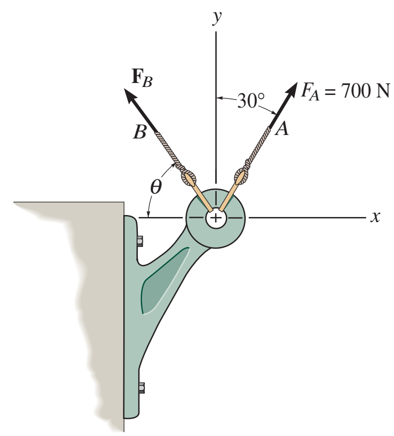
$$\theta' = \tan^{-1}\left(\frac{42.57}{103.05}\right) = 22.4°$$

$$\theta = 180° + 22.4° = 202°$$ **Ans.**



2–42.

Determine the magnitude and orientation θ of $\mathbf{F}_B$ so that the resultant force is directed along the positive y axis and has a magnitude of 1500 N.



SOLUTION

Scalar Notation: Summing the force components algebraically, we have

$\overset{+}{\rightarrow} F_{R_x} = \Sigma F_x;\qquad 0 = 700 \sin 30° - F_B \cos \theta$

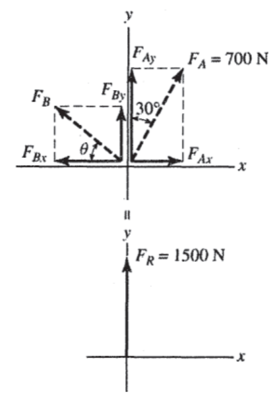
$$F_B \cos \theta = 350 \qquad \textbf{(1)}$$

$+\uparrow F_{R_y} = \Sigma F_y;\qquad 1500 = 700 \cos 30° + F_B \sin \theta$

$$F_B \sin \theta = 893.8 \qquad \textbf{(2)}$$

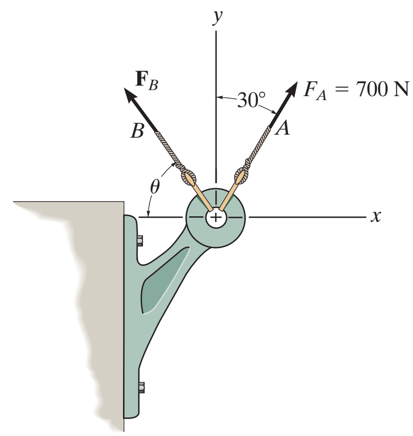
Solving Eq. (1) and (2) yields

$$\theta = 68.6° \quad F_B = 960 \text{ N} \qquad \textbf{Ans.}$$



2–43.

Determine the magnitude and orientation, measured counterclockwise from the positive y axis, of the resultant force acting on the bracket, if $F_B = 600$ N and $\theta = 20°$.



SOLUTION

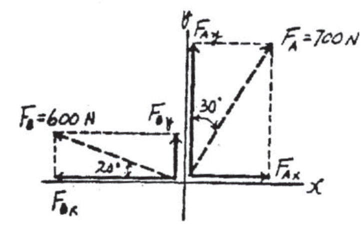
Scalar Notation: Summing the force components algebraically, we have

$$\xrightarrow{+} F_{R_x} = \Sigma F_x; \qquad F_{R_x} = 700 \sin 30° - 600 \cos 20°$$

$$= -213.8 \text{ N} = 213.8 \text{ N} \leftarrow$$

$$+\uparrow F_{R_y} = \Sigma F_y; \qquad F_{R_y} = 700 \cos 30° + 600 \sin 20°$$

$$= 811.4 \text{ N} \uparrow$$

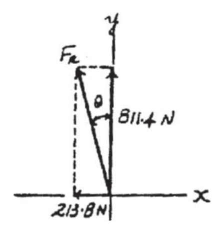


The magnitude of the resultant force $\mathbf{F}_R$ is

$$F_R = \sqrt{F_{R_x}^2 + F_{R_y}^2} = \sqrt{213.8^2 + 811.4^2} = 839 \text{ N} \qquad \textbf{Ans.}$$

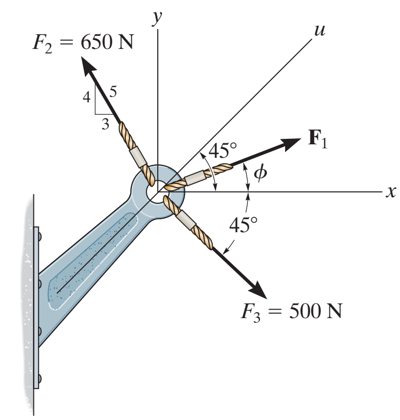
The direction angle θ measured counterclockwise from the positive y axis is

$$\theta = \tan^{-1}\frac{F_{R_x}}{F_{R_y}} = \tan^{-1}\left(\frac{213.8}{811.4}\right) = 14.8° \qquad \textbf{Ans.}$$



***2–44.**

The magnitude of the resultant force acting on the bracket is to be 400 N. Determine the magnitude of $\mathbf{F}_1$ if $\phi = 30°$.



SOLUTION

Rectangular Components: By referring to Fig. *a*, the *x* and *y* components of $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ can be written as

$$(F_1)_x = F_1 \cos 30° = 0.8660F_1 \qquad (F_1)_y = F_1 \sin 30° = 0.5F_1$$

$$(F_2)_x = 650\left(\frac{3}{5}\right) = 390 \text{ N} \qquad (F_2)_y = 650\left(\frac{4}{5}\right) = 520 \text{ N}$$

$$(F_3)_x = 500 \cos 45° = 353.55 \text{ N} \qquad (F_3)_y = 500 \sin 45° = 353.55 \text{ N}$$

Resultant Force: Summing the force components algebraically along the *x* and *y* axes, we have

$$\overset{+}{\rightarrow} \Sigma(F_R)_x = \Sigma F_x; \qquad (F_R)_x = 0.8660F_1 - 390 + 353.55 = 0.8660F_1 - 36.45$$

$$+\uparrow \Sigma(F_R)_y = \Sigma F_y; \qquad (F_R)_y = 0.5F_1 + 520 - 353.55 = 0.5F_1 + 166.45$$

Since the magnitude of the resultant force is $\mathbf{F}_R = 400$ N, we can write

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2}$$

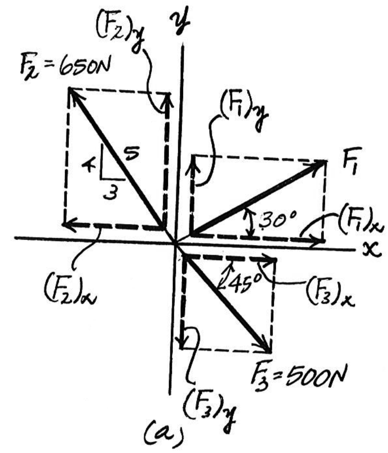
$$400 = \sqrt{(0.8660F_1 - 36.45)^2 + (0.5F_1 + 166.45)^2}$$

$$F_1^2 + 103.32F_1 - 130967.17 = 0 \qquad \textbf{Ans.}$$

Solving,

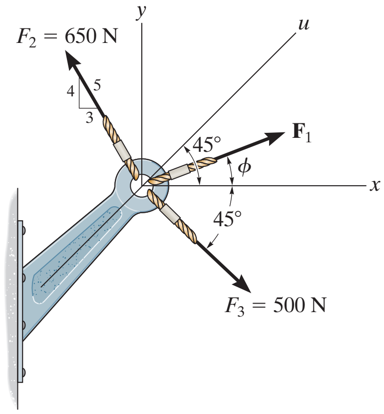
$$F_1 = 314 \text{ N} \qquad \text{or} \qquad F_1 = -417 \text{ N} \qquad \textbf{Ans.}$$

The negative sign indicates that $\mathbf{F}_1 = 417$ N must act in the opposite sense to that shown in the figure.



2–45.

If the resultant force acting on the bracket is to be directed along the positive u axis, and the magnitude of $\mathbf{F}_1$ is required to be *minimum*, determine the magnitudes of the resultant force and $\mathbf{F}_1$.



SOLUTION

Rectangular Components: By referring to Figs. a and b, the x and y components of $\mathbf{F}_1$, $\mathbf{F}_2$, $\mathbf{F}_3$, and $\mathbf{F}_R$ can be written as

$$(F_1)_x = F_1 \cos\phi \qquad (F_1)_y = F_1 \sin\phi$$

$$(F_2)_x = 650\left(\frac{3}{5}\right) = 390\text{ N} \qquad (F_2)_y = 650\left(\frac{4}{5}\right) = 520\text{ N}$$

$$(F_3)_x = 500\cos 45^\circ = 353.55\text{ N} \qquad (F_3)_y = 500\sin 45^\circ = 353.55\text{ N}$$

$$(F_R)_x = F_R\cos 45^\circ = 0.7071F_R \qquad (F_R)_y = F_R\sin 45^\circ = 0.7071F_R$$

Resultant Force: Summing the force components algebraically along the x and y axes, we have

$$\overset{+}{\rightarrow}\Sigma(F_R)_x = \Sigma F_x; \qquad 0.7071F_R = F_1\cos\phi - 390 + 353.55 \qquad \textbf{(1)}$$

$$+\uparrow\Sigma(F_R)_y = \Sigma F_y; \qquad 0.7071F_R = F_1\sin\phi + 520 - 353.55 \qquad \textbf{(2)}$$

Eliminating F_R from Eqs. (1) and (2), yields

$$F_1 = \frac{202.89}{\cos\phi - \sin\phi} \qquad \textbf{(3)}$$

The first derivative of Eq. (3) is

$$\frac{dF_1}{d\phi} = \frac{\sin\phi + \cos\phi}{(\cos\phi - \sin\phi)^2} \qquad \textbf{(4)}$$

The second derivative of Eq. (3) is

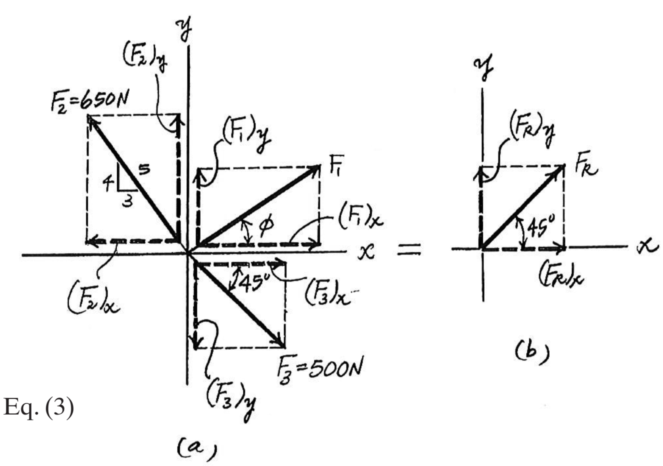
$$\frac{d^2F_1}{d\phi^2} = \frac{2(\sin\phi + \cos\phi)^2}{(\cos\phi - \sin\phi)^3} + \frac{1}{\cos\phi - \sin\phi} \qquad \textbf{(5)}$$

For $\mathbf{F}_1$ to be minimum, $\dfrac{dF_1}{d\phi} = 0$. Thus, from Eq. (4)

$$\sin\phi + \cos\phi = 0$$

$$\tan\phi = -1$$

$$\phi = -45^\circ$$



Substituting $\phi = -45^\circ$ into Eq. (5), yields

$$\frac{d^2F_1}{d\phi^2} = 0.7071 > 0$$

This shows that $\phi = -45^\circ$ indeed produces minimum F_1. Thus, from Eq. (3)

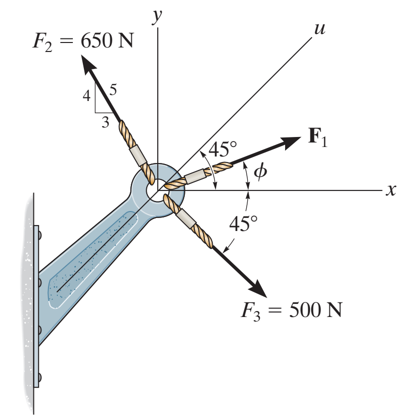
$$F_1 = \frac{202.89}{\cos(-45^\circ) - \sin(-45^\circ)} = 143.47\text{ N} = 143\text{ N} \qquad \textbf{Ans.}$$

Substituting $\phi = -45^\circ$ and $F_1 = 143.47$ N into either Eq. (1) or Eq. (2), yields

$$F_R = 919\text{ N} \qquad \textbf{Ans.}$$

2–46.

If the magnitude of the resultant force acting on the bracket is 600 N, directed along the positive u axis, determine the magnitude of **F** and its direction ϕ.



SOLUTION

Rectangular Components: By referring to Figs. a and b, the x and y components of $\mathbf{F}_1$, $\mathbf{F}_2$, $\mathbf{F}_3$, and $\mathbf{F}_R$ can be written as

$$(F_1)_x = F_1 \cos \phi \qquad (F_1)_y = F_1 \sin \phi$$

$$(F_2)_x = 650\left(\frac{3}{5}\right) = 390 \text{ N} \qquad (F_2)_y = 650\left(\frac{4}{5}\right) = 520 \text{ N}$$

$$(F_3)_x = 500 \cos 45° = 353.55 \text{ N} \qquad (F_3)_y = 500 \cos 45° = 353.55 \text{ N}$$

$$(F_R)_x = 600 \cos 45° = 424.26 \text{ N} \qquad (F_R)_y = 600 \sin 45° = 424.26 \text{ N}$$

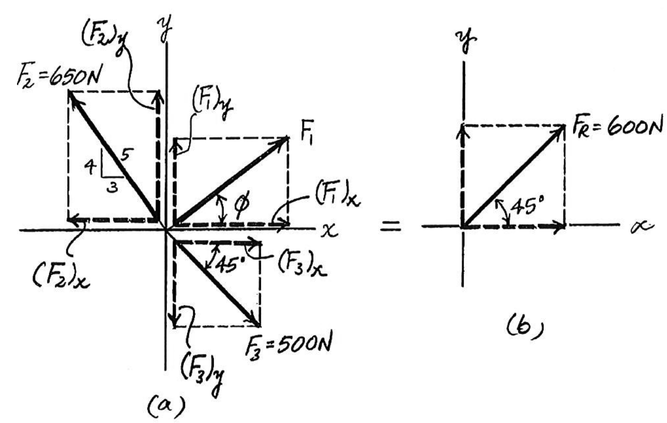
Resultant Force: Summing the force components algebraically along the x and y axes, we have

$$\overset{+}{\rightarrow} \Sigma(F_R)_x = \Sigma F_x; \qquad 424.26 = F_1 \cos \phi - 390 + 353.55 \qquad \textbf{(1)}$$
$$F_1 \cos \phi = 460.71$$

$$+\uparrow \Sigma(F_R)_y = \Sigma F_y; \qquad 424.26 = F_1 \sin \phi + 520 - 353.55 \qquad \textbf{(2)}$$
$$F_1 \sin \phi = 257.82$$

Solving Eqs. (1) and (2), yields

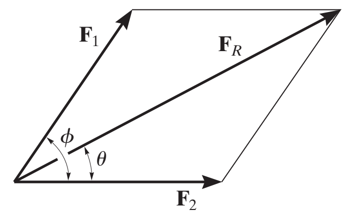
$$\phi = 29.2° \qquad F_1 = 528 \text{ N} \qquad \textbf{Ans.}$$



2–47.

Determine the magnitude and direction θ of the resultant force $\mathbf{F}_R$. Express the result in terms of the magnitudes of the components $\mathbf{F}_1$ and $\mathbf{F}_2$ and the angle ϕ.

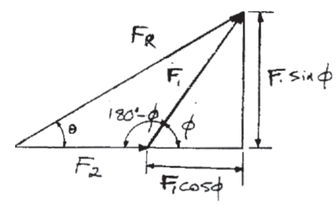
SOLUTION



$$F_R^2 = F_1^2 + F_2^2 - 2F_1F_2\cos(180° - \phi)$$

Since $\cos(180° - \phi) = -\cos\phi$,

$$F_R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\phi}$$ **Ans.**



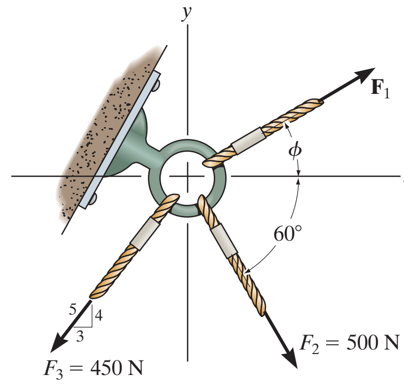
From the figure,

$$\tan\theta = \frac{F_1\sin\phi}{F_2 + F_1\cos\phi}$$

$$\theta = \tan^{-1}\left(\frac{F_1\sin\phi}{F_2 + F_1\cos\phi}\right)$$ **Ans.**

***2–48.**

If $F_1 = 600$ N and $\phi = 30°$, determine the magnitude of the resultant force acting on the eyebolt and its direction measured clockwise from the positive x axis.



SOLUTION

Rectangular Components: By referring to Fig. *a*, the x and y components of each force can be written as

$$(F_1)_x = 600 \cos 30° = 519.62 \text{ N} \quad (F_1)_y = 600 \sin 30° = 300 \text{ N}$$

$$(F_2)_x = 500 \cos 60° = 250 \text{ N} \quad (F_2)_y = 500 \sin 60° = 433.01 \text{ N}$$

$$(F_3)_x = 450\left(\frac{3}{5}\right) = 270 \text{ N} \quad (F_3)_y = 450\left(\frac{4}{5}\right) = 360 \text{ N}$$

Resultant Force: Summing the force components algebraically along the x and y axes,

$$\overset{+}{\rightarrow} \Sigma(F_R)_x = \Sigma F_x; \quad (F_R)_x = 519.62 + 250 - 270 = 499.62 \text{ N} \rightarrow$$

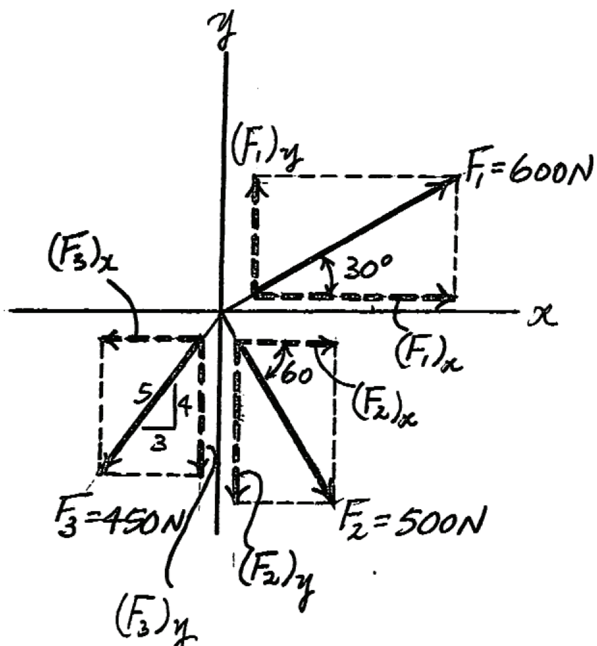
$$+\uparrow \Sigma(F_R)_y = \Sigma F_y; \quad (F_R)_y = 300 - 433.01 - 360 = -493.01 \text{ N} = 493.01 \text{ N} \downarrow$$

The magnitude of the resultant force $\mathbf{F}_R$ is

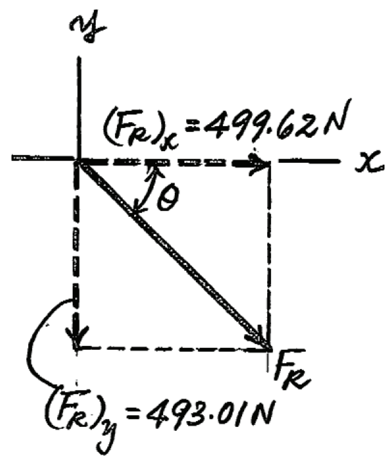
$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{499.62^2 + 493.01^2} = 701.91 \text{ N} = 702 \text{ N}$$ **Ans.**

The direction angle θ of $\mathbf{F}_R$, Fig. *b*, measured clockwise from the x axis, is

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left(\frac{493.01}{499.62}\right) = 44.6°$$ **Ans.**



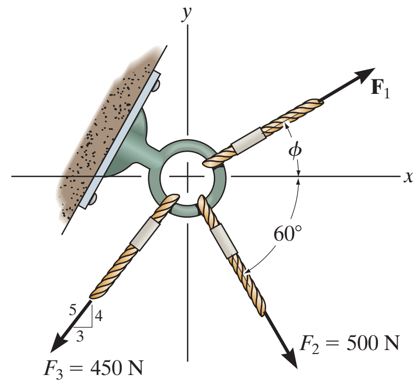
(a)



(b)

2–49.

If the magnitude of the resultant force acting on the eyebolt is 600 N and its direction measured clockwise from the positive x axis is $\theta = 30°$, determine the magnitude of $\mathbf{F}_1$ and the angle ϕ.



SOLUTION

Rectangular Components: By referring to Figs. a and b, the x and y components of $\mathbf{F}_1$, $\mathbf{F}_2$, $\mathbf{F}_3$, and $\mathbf{F}_R$ can be written as

$$(F_1)_x = F_1 \cos\phi \qquad (F_1)_y = F_1 \sin\phi$$

$$(F_2)_x = 500\cos 60° = 250 \text{ N} \qquad (F_2)_y = 500 \sin 60° = 433.01 \text{ N}$$

$$(F_3)_x = 450\left(\frac{3}{5}\right) = 270 \text{ N} \qquad (F_3)_y = 450\left(\frac{4}{5}\right) = 360 \text{ N}$$

$$(F_R)_x = 600\cos 30° = 519.62 \text{ N} \qquad (F_R)_y = 600 \sin 30° = 300 \text{ N}$$

Resultant Force: Summing the force components algebraically along the x and y axes,

$$\overset{+}{\rightarrow} \Sigma(F_R)_x = \Sigma F_x; \qquad 519.62 = F_1\cos\phi + 250 - 270$$

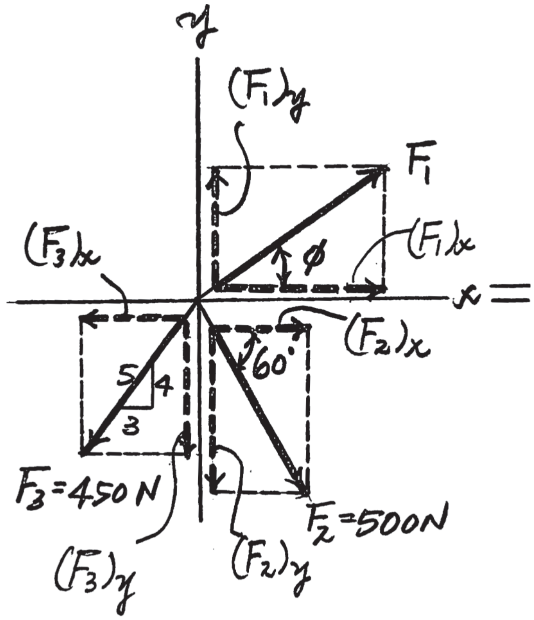
$$F_1 \cos\phi = 539.62 \qquad \textbf{(1)}$$

$$+\uparrow \Sigma(F_R)_y = \Sigma F_y; \quad -300 = F_1 \sin\phi - 433.01 - 360$$

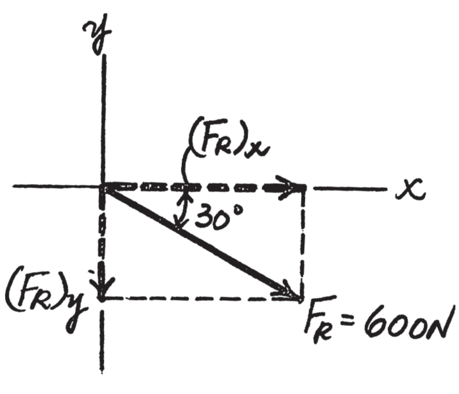
$$F_1 \sin\phi = 493.01 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2), yields

$$\phi = 42.4° \qquad F_1 = 731 \text{ N} \qquad \textbf{Ans.}$$



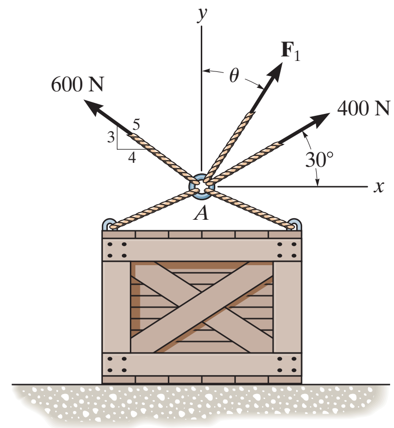
(a)



(b)

2–50.

Determine the magnitude of $\mathbf{F}_1$ and its direction θ so that the resultant force is directed vertically upward and has a magnitude of 800 N.



SOLUTION

Scalar Notation: Summing the force components algebraically, we have

$$\overset{+}{\rightarrow} F_{R_x} = \Sigma F_x; \qquad F_{R_x} = 0 = F_1 \sin\theta + 400\cos 30^\circ - 600\left(\frac{4}{5}\right)$$

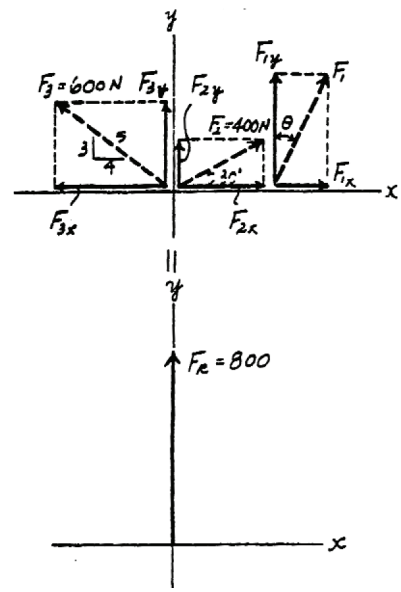
$$F_1 \sin\theta = 133.6 \qquad \textbf{(1)}$$

$$+\uparrow F_{R_y} = \Sigma F_y; \qquad F_{R_y} = 800 = F_1 \cos\theta + 400\sin 30^\circ + 600\left(\frac{3}{5}\right)$$

$$F_1 \cos\theta = 240 \qquad \textbf{(2)}$$

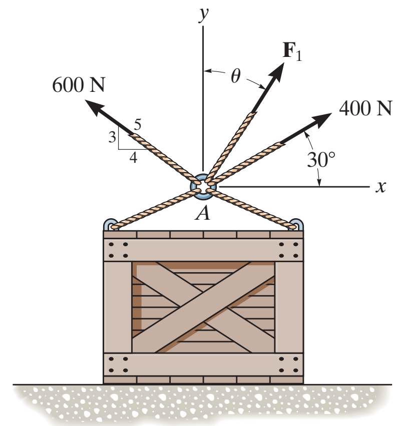
Solving Eqs. (1) and (2) yields

$$\theta = 29.1^\circ \qquad F_1 = 275\ \text{N} \qquad \textbf{Ans.}$$



2–51.

Determine the magnitude and direction measured counterclockwise from the positive x axis of the resultant force of the three forces acting on the ring A. Take $F_1 = 500$ N and $\theta = 20°$.



SOLUTION

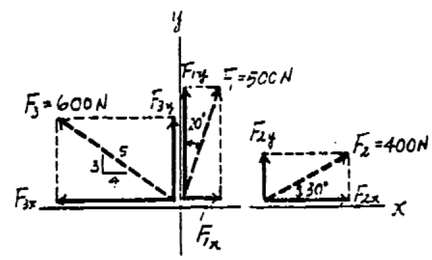
Scalar Notation: Summing the force components algebraically, we have

$$\overset{+}{\rightarrow} F_{R_x} = \Sigma F_x; \qquad F_{R_x} = 500 \sin 20° + 400 \cos 30° - 600\left(\frac{4}{5}\right)$$

$$= 37.42 \text{ N} \rightarrow$$

$$+\uparrow F_{R_y} = \Sigma F_y; \qquad F_{R_y} = 500 \cos 20° + 400 \sin 30° + 600\left(\frac{3}{5}\right)$$

$$= 1029.8 \text{ N} \uparrow$$

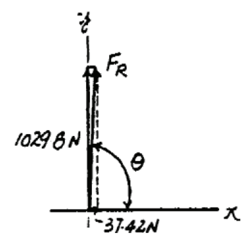


The magnitude of the resultant force $\mathbf{F}_R$ is

$$F_R = \sqrt{F_{R_x}^2 + F_{R_y}^2} = \sqrt{37.42^2 + 1029.8^2} = 1030.5 \text{ N} = 1.03 \text{ kN} \qquad \textbf{Ans.}$$

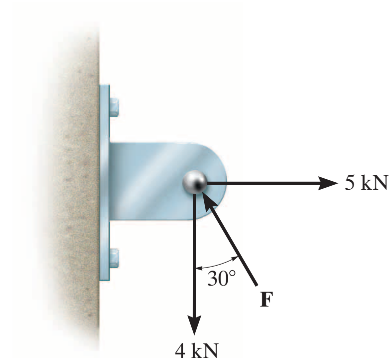
The direction angle θ measured counterclockwise from positive x axis is

$$\theta = \tan^{-1}\frac{F_{R_y}}{F_{R_x}} = \tan^{-1}\left(\frac{1029.8}{37.42}\right) = 87.9° \qquad \textbf{Ans.}$$



***2–52.**

Determine the magnitude of force **F** so that the resultant $\mathbf{F}_R$ of the three forces is as small as possible. What is the minimum magnitude of $\mathbf{F}_R$?

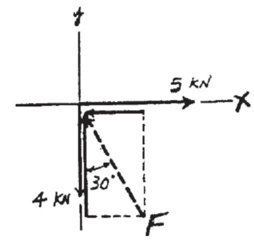


SOLUTION

Scalar Notation: Summing the force components algebraically, we have

$$\overset{+}{\rightarrow} F_{R_x} = \Sigma F_x; \qquad F_{R_x} = 5 - F\sin 30° = 5 - 0.50F \rightarrow$$

$$+\uparrow F_{R_y} = \Sigma F_y; \qquad F_{R_y} = F\cos 30° - 4 = 0.8660F - 4 \uparrow$$



The magnitude of the resultant force $\mathbf{F}_R$ is

$$F_R = \sqrt{F_{R_x}^2 + F_{R_y}^2}$$

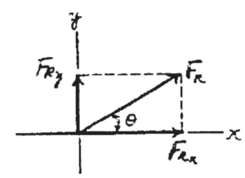
$$= \sqrt{(5 - 0.50F)^2 + (0.8660F - 4)^2}$$

$$= \sqrt{F^2 - 11.93F + 41} \qquad \textbf{(1)}$$

$$F_R^2 = F^2 - 11.93F + 41$$

$$2F_R\frac{dF_R}{dF} = 2F - 11.93 \qquad \textbf{(2)}$$

$$\left(F_R\frac{d^2F_R}{dF^2} + \frac{dF_R}{dF} \times \frac{dF_R}{dF}\right) = 1 \qquad \textbf{(3)}$$



In order to obtain the *minimum* resultant force $\mathbf{F}_R$, $\frac{dF_R}{dF} = 0$. From Eq. (2)

$$2F_R\frac{dF_R}{dF} = 2F - 11.93 = 0$$

$$F = 5.964 \text{ kN} = 5.96 \text{ kN} \qquad \textbf{Ans.}$$

Substituting $F = 5.964$ kN into Eq. (1), we have

$$F_R = \sqrt{5.964^2 - 11.93(5.964) + 41}$$

$$= 2.330 \text{ kN} = 2.33 \text{ kN} \qquad \textbf{Ans.}$$

Substituting $F_R = 2.330$ kN with $\frac{dF_R}{dF} = 0$ into Eq. (3), we have

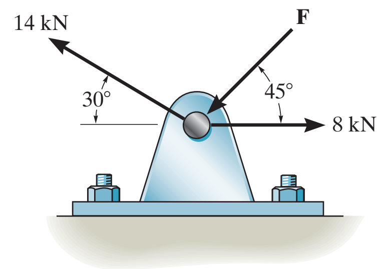
$$\left[(2.330)\frac{d^2F_R}{dF^2} + 0\right] = 1$$

$$\frac{d^2F_R}{dF^2} = 0.429 > 0$$

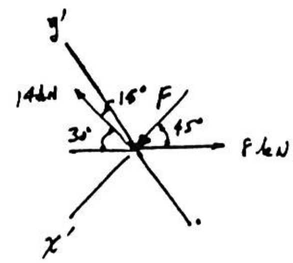
Hence, $F = 5.96$ kN is indeed producing a minimum resultant force.

2–53.

Determine the magnitude of force **F** so that the resultant force of the three forces is as small as possible. What is the magnitude of the resultant force?



SOLUTION



$\overset{+}{\rightarrow} F_{Rx} = \Sigma F_x;$ $\quad F_{Rz} = 8 - F\cos 45° - 14\cos 30°$

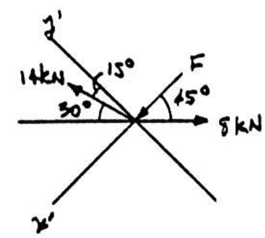
$= -4.1244 - F\cos 45°$

$+\uparrow F_{Ry} = \Sigma F_y;$ $\quad F_{Ry} = -F\sin 45° + 14\sin 30°$

$= 7 - F\sin 45°$

$$F_R^2 = (-4.1244 - F\cos 45°)^2 + (7 - F\sin 45°)^2 \qquad \textbf{(1)}$$

$$2F_R\frac{dF_R}{dF} = 2(-4.1244 - F\cos 45°)(-\cos 45°) + 2(7 - F\sin 45°)(-\sin 45°) = 0$$



$F = 2.03\text{ kN}$ **Ans.**

From Eq. (1); $\quad F_R = 7.87\text{ kN}$ **Ans.**

Also, from the figure require

$(F_R)_{x'} = 0 = \Sigma F_{x'};$ $\quad F + 14\sin 15° - 8\cos 45° = 0$

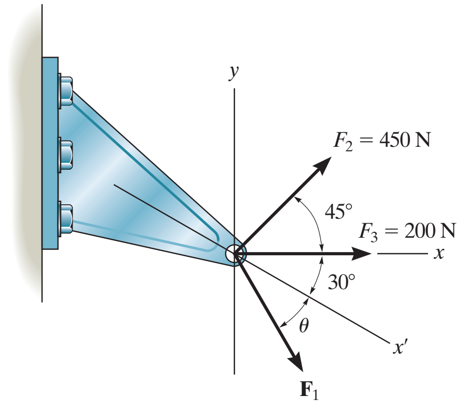
$F = 2.03\text{ kN}$ **Ans.**

$(F_R)_{y'} = \Sigma F_{y'};$ $\quad F_R = 14\cos 15° - 8\sin 45°$

$F_R = 7.87\text{ kN}$ **Ans.**

2–54.

Three forces act on the bracket. Determine the magnitude and direction θ of $\mathbf{F}_1$ so that the resultant force is directed along the positive x' axis and has a magnitude of 1 kN.



SOLUTION

$\xrightarrow{+} F_{Rx} = \Sigma F_x; \qquad 1000 \cos 30^\circ = 200 + 450 \cos 45^\circ + F_1 \cos(\theta + 30^\circ)$

$+\uparrow F_{Ry} = \Sigma F_y; \qquad -1000 \sin 30^\circ = 450 \sin 45^\circ - F_1 \sin(\theta + 30^\circ)$

$F_1 \sin(\theta + 30^\circ) = 818.198$

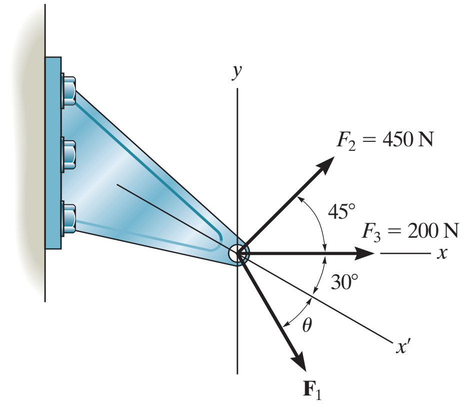
$F_1 \cos(\theta + 30^\circ) = 347.827$

$\theta + 30^\circ = 66.97^\circ, \quad \theta = 37.0^\circ$ **Ans.**

$F_1 = 889$ N **Ans.**

2–55.

If $F_1 = 300$ N and $\theta = 20°$, determine the magnitude and direction, measured counterclockwise from the x' axis, of the resultant force of the three forces acting on the bracket.



SOLUTION

$\overset{+}{\rightarrow} F_{Rx} = \Sigma F_x;\quad F_{Rx} = 300 \cos 50° + 200 + 450 \cos 45° = 711.03 \text{ N}$

$+\uparrow F_{Ry} = \Sigma F_y;\quad F_{Ry} = -300 \sin 50° + 450 \sin 45° = 88.38 \text{ N}$

$F_R = \sqrt{(711.03)^2 + (88.38)^2} = 717 \text{ N}$ **Ans.**

$\phi' \text{ (angle from } x \text{ axis)} = \tan^{-1}\left[\frac{88.38}{711.03}\right]$

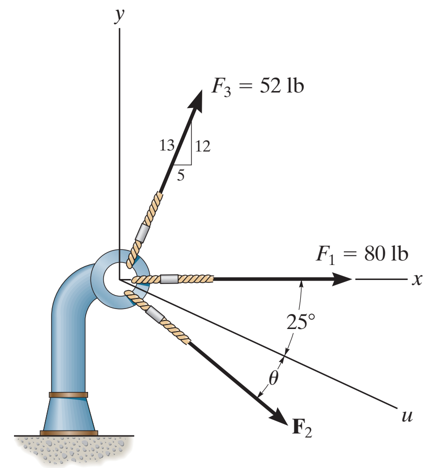
$\phi' = 7.10°$

$\phi \text{ (angle from } x' \text{ axis)} = 30° + 7.10°$

$\phi = 37.1°$ **Ans.**

***2–56.**

Three forces act on the bracket. Determine the magnitude and direction θ of $\mathbf{F}_2$ so that the resultant force is directed along the positive u axis and has a magnitude of 50 lb.



SOLUTION

Scalar Notation: Summing the force components algebraically, we have

$$\overset{+}{\rightarrow} F_{R_x} = \Sigma F_x; \qquad 50 \cos 25° = 80 + 52\left(\frac{5}{13}\right) + F_2 \cos(25° + \theta)$$

$$F_2 \cos(25° + \theta) = -54.684 \qquad \textbf{(1)}$$

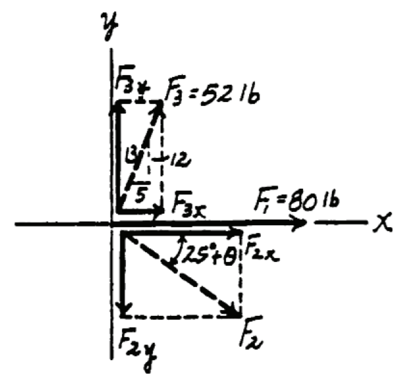
$$+\uparrow F_{R_y} = \Sigma F_y; \qquad -50 \sin 25° = 52\left(\frac{12}{13}\right) - F_2 \sin(25° + \theta)$$

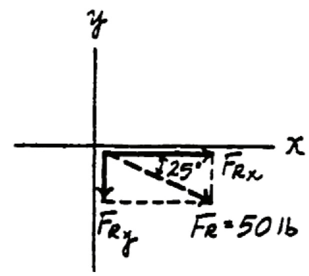
$$F_2 \sin(25° + \theta) = 69.131 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$25° + \theta = 128.35° \qquad \theta = 103° \qquad \textbf{Ans.}$$

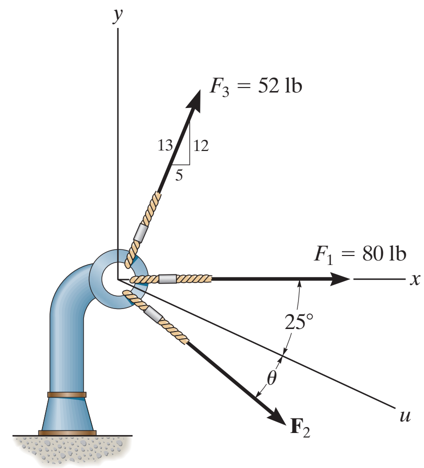
$$F_2 = 88.1 \text{ lb} \qquad \textbf{Ans.}$$





2–57.

If $F_2 = 150$ lb and $\theta = 55°$, determine the magnitude and direction, measured clockwise from the positive x axis, of the resultant force of the three forces acting on the bracket.



SOLUTION

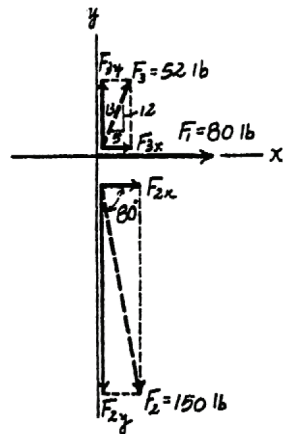
Scalar Notation: Summing the force components algebraically, we have

$$\overset{+}{\rightarrow} F_{R_x} = \Sigma F_x; \qquad F_{R_x} = 80 + 52\left(\frac{5}{13}\right) + 150 \cos 80°$$

$$= 126.05 \text{ lb} \rightarrow$$

$$+\uparrow F_{R_y} = \Sigma F_y; \qquad F_{R_y} = 52\left(\frac{12}{13}\right) - 150 \sin 80°$$

$$= -99.72 \text{ lb} = 99.72 \text{ lb} \downarrow$$

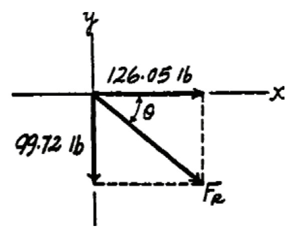


The magnitude of the resultant force $\mathbf{F}_R$ is

$$F_R = \sqrt{F_{R_x}^2 + F_{R_y}^2} = \sqrt{126.05^2 + 99.72^2} = 161 \text{ lb} \qquad \textbf{Ans.}$$

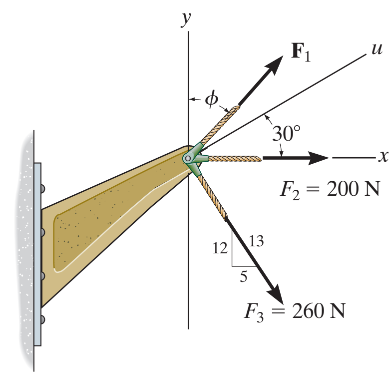
The direction angle θ measured clockwise from positive x axis is

$$\theta = \tan^{-1}\frac{F_{R_y}}{F_{R_x}} = \tan^{-1}\left(\frac{99.72}{126.05}\right) = 38.3° \qquad \textbf{Ans.}$$



2–58.

If the magnitude of the resultant force acting on the bracket is to be 450 N directed along the positive u axis, determine the magnitude of $\mathbf{F}_1$ and its direction ϕ.



SOLUTION

Rectangular Components: By referring to Fig. a, the x and y components of $\mathbf{F}_1$, $\mathbf{F}_2$, $\mathbf{F}_3$, and $\mathbf{F}_R$ can be written as

$(F_1)_x = F_1 \sin \phi \qquad (F_1)_y = F_1 \cos \phi$

$(F_2)_x = 200 \text{ N} \qquad (F_2)_y = 0$

$(F_3)_x = 260\left(\dfrac{5}{13}\right) = 100 \text{ N} \qquad (F_3)_y = 260\left(\dfrac{12}{13}\right) = 240 \text{ N}$

$(F_R)_x = 450 \cos 30° = 389.71 \text{ N} \qquad (F_R)_y = 450 \sin 30° = 225 \text{ N}$

Resultant Force: Summing the force components algebraically along the x and y axes,

$\overset{+}{\rightarrow} \Sigma(F_R)_x = \Sigma F_x; \quad 389.71 = F_1 \sin \phi + 200 + 100$

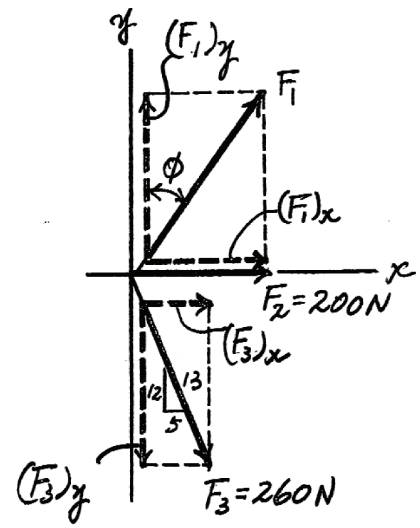
$$F_1 \sin \phi = 89.71 \qquad \textbf{(1)}$$

$+\uparrow \Sigma(F_R)_y = \Sigma F_y; \quad 225 = F_1 \cos \phi - 240$

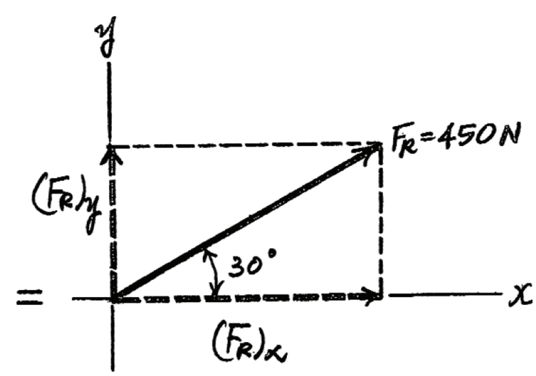
$$F_1 \cos \phi = 465 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2), yields

$$\phi = 10.9° \qquad F_1 = 474 \text{ N} \qquad \textbf{Ans.}$$



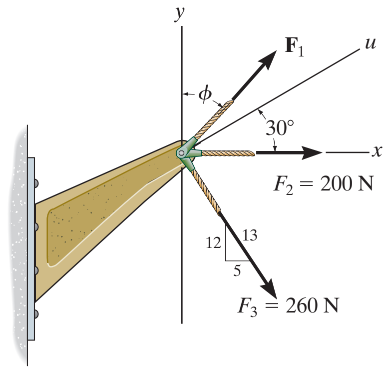
(a)



(b)

2–59.

If the resultant force acting on the bracket is required to be a minimum, determine the magnitudes of $\mathbf{F}_1$ and the resultant force. Set $\phi = 30°$.



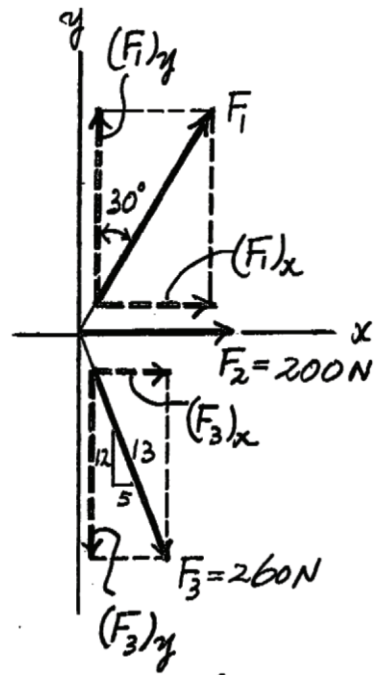
SOLUTION

Rectangular Components: By referring to Fig. *a*, the *x* and *y* components of $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ can be written as

$$(F_1)_x = F_1 \sin 30° = 0.5F_1 \qquad (F_1)_y = F_1 \cos 30° = 0.8660F_1$$

$$(F_2)_x = 200 \text{ N} \qquad (F_2)_y = 0$$

$$(F_3)_x = 260\left(\frac{5}{13}\right) = 100 \text{ N} \qquad (F_3)_y = 260\left(\frac{12}{13}\right) = 240 \text{ N}$$



(a)

Resultant Force: Summing the force components algebraically along the *x* and *y* axes,

$$\overset{+}{\rightarrow} \Sigma(F_R)_x = \Sigma F_x; \quad (F_R)_x = 0.5F_1 + 200 + 100 = 0.5F_1 + 300$$

$$+\uparrow \Sigma(F_R)_y = \Sigma F_y; \quad (F_R)_y = 0.8660F_1 - 240$$

The magnitude of the resultant force $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2}$$

$$= \sqrt{(0.5F_1 + 300)^2 + (0.8660F_1 - 240)^2}$$

$$= \sqrt{F_1^2 - 115.69F_1 + 147\,600} \qquad \textbf{(1)}$$

Thus,

$$F_R^2 = F_1^2 - 115.69F_1 + 147\,600 \qquad \textbf{(2)}$$

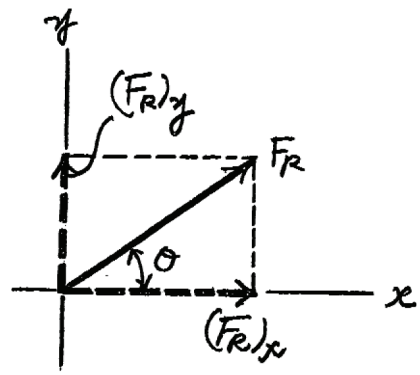
The first derivative of Eq. (2) is

$$2F_R\frac{dF_R}{dF_1} = 2F_1 - 115.69 \qquad \textbf{(3)}$$

For $\mathbf{F}_R$ to be minimum, $\frac{dF_R}{dF_1} = 0$. Thus, from Eq. (3)

$$2F_R\frac{dF_R}{dF_1} = 2F_1 - 115.69 = 0$$

$$F_1 = 57.846 \text{ N} = 57.8 \text{ N} \qquad \textbf{Ans.}$$



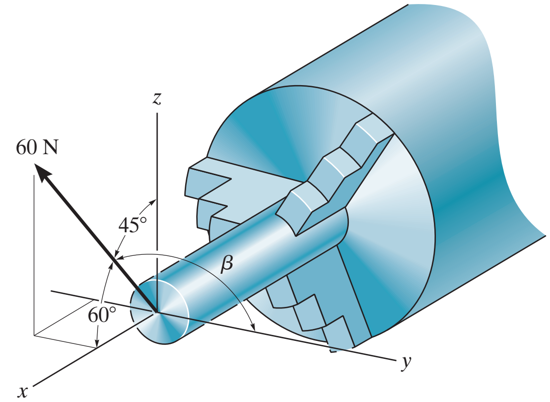
(b)

from Eq. (1),

$$F_R = \sqrt{(57.846)^2 - 115.69(57.846) + 147\,600} = 380 \text{ N} \qquad \textbf{Ans.}$$

***2–60.**

The stock mounted on the lathe is subjected to a force of 60 N. Determine the coordinate direction angle β and express the force as a Cartesian vector.



SOLUTION

$$1 = \sqrt{\cos^2\alpha + \cos^2\beta + \cos^2\gamma}$$

$$1 = \cos^2 60° + \cos^2\beta + \cos^2 45°$$

$$\cos\beta = \pm 0.5$$

$$\beta = 60°, 120°$$

Use

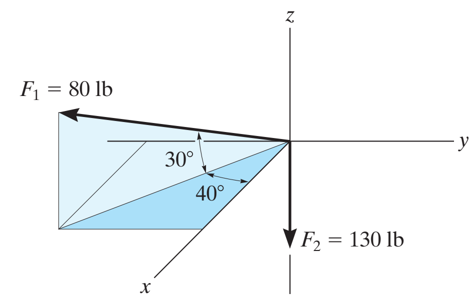
$$\beta = 120°$$ **Ans.**

$$F = 60\text{ N}(\cos 60°\mathbf{i} + \cos 120°\mathbf{j} + \cos 45°\mathbf{k})$$

$$= \{30\mathbf{i} - 30\mathbf{j} + 42.4\mathbf{k}\}\text{ N}$$ **Ans.**

2–61.

Determine the magnitude and coordinate direction angles of the resultant force and sketch this vector on the coordinate system.



SOLUTION

$$\mathbf{F}_1 = \{80 \cos 30^\circ \cos 40^\circ \mathbf{i} - 80 \cos 30^\circ \sin 40^\circ \mathbf{j} + 80 \sin 30^\circ \mathbf{k}\} \text{ lb}$$

$$\mathbf{F}_1 = \{53.1\mathbf{i} - 44.5\mathbf{j} + 40\mathbf{k}\} \text{ lb}$$

$$\mathbf{F}_2 = \{-130\mathbf{k}\} \text{ lb}$$

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$

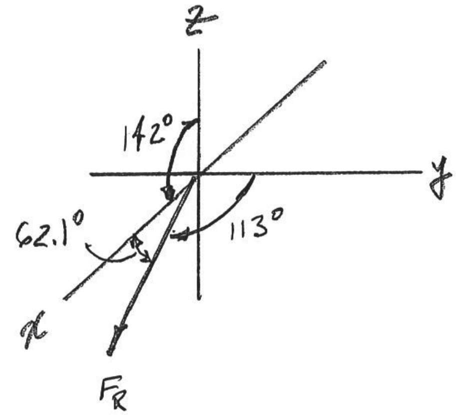
$$\mathbf{F}_R = \{53.1\mathbf{i} - 44.5\mathbf{j} - 90.0\mathbf{k}\} \text{ lb}$$

$$F_R = \sqrt{(53.1)^2 + (-44.5)^2 + (-90.0)^2} = 114 \text{ lb}$$ **Ans.**

$$\alpha = \cos^{-1}\left(\frac{53.1}{113.6}\right) = 62.1^\circ$$ **Ans.**

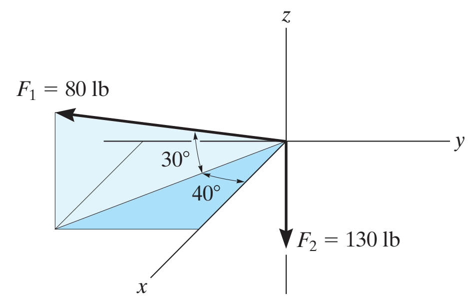
$$\beta = \cos^{-1}\left(\frac{-44.5}{113.6}\right) = 113^\circ$$ **Ans.**

$$\gamma = \cos^{-1}\left(\frac{-90.0}{113.6}\right) = 142^\circ$$ **Ans.**



2–62.

Specify the coordinate direction angles of $\mathbf{F}_1$ and $\mathbf{F}_2$ and express each force as a Cartesian vector.



SOLUTION

$\mathbf{F}_1 = \{80 \cos 30° \cos 40° \mathbf{i} - 80 \cos 30° \sin 40° \mathbf{j} + 80 \sin 30° \mathbf{k}\}$ lb

$\mathbf{F}_1 = \{53.1\mathbf{i} - 44.5\mathbf{j} + 40\mathbf{k}\}$ lb **Ans.**

$\alpha_1 = \cos^{-1}\left(\frac{53.1}{80}\right) = 48.4°$ **Ans.**

$\beta_1 = \cos^{-1}\left(\frac{-44.5}{80}\right) = 124°$ **Ans.**

$\gamma_1 = \cos^{-1}\left(\frac{40}{80}\right) = 60°$ **Ans.**

$\mathbf{F}_2 = \{-130\mathbf{k}\}$ lb **Ans.**

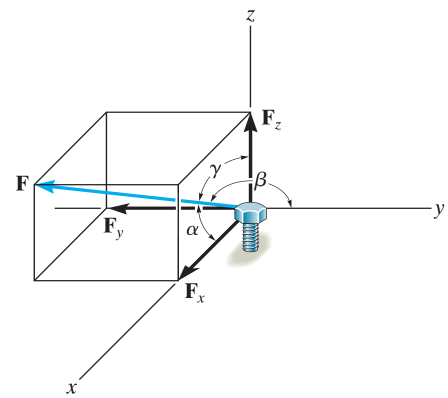
$\alpha_2 = \cos^{-1}\left(\frac{0}{130}\right) = 90°$ **Ans.**

$\beta_2 = \cos^{-1}\left(\frac{0}{130}\right) = 90°$ **Ans.**

$\gamma_2 = \cos^{-1}\left(\frac{-130}{130}\right) = 180°$ **Ans.**

2–63.

The bolt is subjected to the force **F**, which has components acting along the x, y, z axes as shown. If the magnitude of **F** is 80 N, and $\alpha = 60°$ and $\gamma = 45°$, determine the magnitudes of its components.



SOLUTION

$$\cos\beta = \sqrt{1 - \cos^2\alpha - \cos^2\gamma}$$

$$= \sqrt{1 - \cos^2 60° - \cos^2 45°}$$

$$\beta = 120°$$

$$F_x = |80 \cos 60°| = 40 \text{ N}$$ **Ans.**

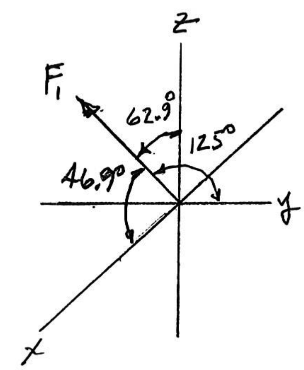
$$F_y = |80 \cos 120°| = 40 \text{ N}$$ **Ans.**

$$F_z = |80 \cos 45°| = 56.6 \text{ N}$$ **Ans.**

***2–64.**

Determine the magnitude and coordinate direction angles of $\mathbf{F}_1 = \{60\mathbf{i} - 50\mathbf{j} + 40\mathbf{k}\}$ N and $\mathbf{F}_2 = \{-40\mathbf{i} - 85\mathbf{j} + 30\mathbf{k}\}$ N. Sketch each force on an x, y, z reference frame.

SOLUTION



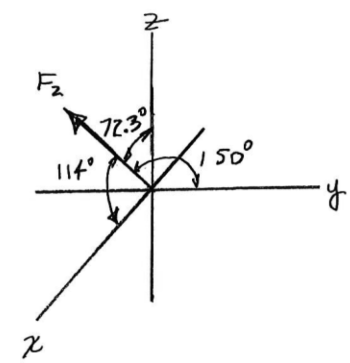
$\mathbf{F}_1 = 60\,\mathbf{i} - 50\,\mathbf{j} + 40\,\mathbf{k}$

$F_1 = \sqrt{(60)^2 + (-50)^2 + (40)^2} = 87.7496 = 87.7\text{ N}$ **Ans.**

$\alpha_1 = \cos^{-1}\left(\dfrac{60}{87.7496}\right) = 46.9°$ **Ans.**

$\beta_1 = \cos^{-1}\left(\dfrac{-50}{87.7496}\right) = 125°$ **Ans.**

$\gamma_1 = \cos^{-1}\left(\dfrac{40}{87.7496}\right) = 62.9°$ **Ans.**



$\mathbf{F}_2 = -40\,\mathbf{i} - 85\,\mathbf{j} + 30\,\mathbf{k}$

$F_2 = \sqrt{(-40)^2 + (-85)^2 + (30)^2} = 98.615 = 98.6\text{ N}$ **Ans.**

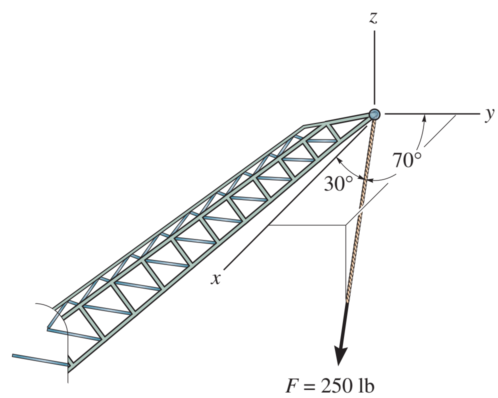
$\alpha_2 = \cos^{-1}\left(\dfrac{-40}{98.615}\right) = 114°$ **Ans.**

$\beta_2 = \cos^{-1}\left(\dfrac{-85}{98.615}\right) = 150°$ **Ans.**

$\gamma_2 = \cos^{-1}\left(\dfrac{30}{98.615}\right) = 72.3°$ **Ans.**

2–65.

The cable at the end of the crane boom exerts a force of 250 lb on the boom as shown. Express **F** as a Cartesian vector.



SOLUTION

Cartesian Vector Notation: With $\alpha = 30^\circ$ and $\beta = 70^\circ$, the third coordinate direction angle γ can be determined using Eq. 2–8.

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$$

$$\cos^2 30^\circ + \cos^2 70^\circ + \cos^2 \gamma = 1$$

$$\cos \gamma = \pm 0.3647$$

$$\gamma = 68.61^\circ \text{ or } 111.39^\circ$$

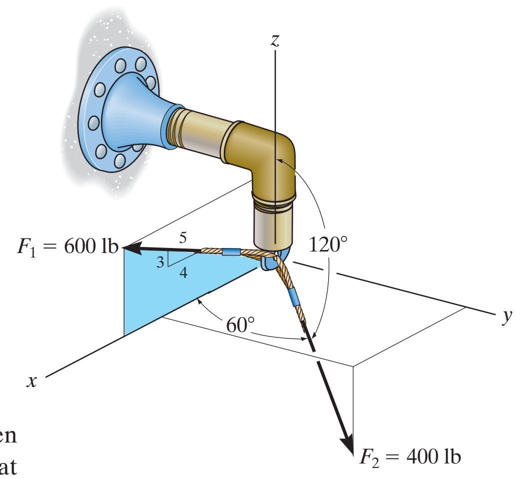
By inspection, $\gamma = 111.39^\circ$ since the force **F** is directed in negative octant.

$$\mathbf{F} = 250\{\cos 30^\circ \mathbf{i} + \cos 70^\circ \mathbf{j} + \cos 111.39^\circ\} \text{ lb}$$

$$= \{217\mathbf{i} + 85.5\mathbf{j} - 91.2\mathbf{k}\} \text{ lb}$$ **Ans.**

2–66.

Express each force acting on the pipe assembly in Cartesian vector form.



SOLUTION

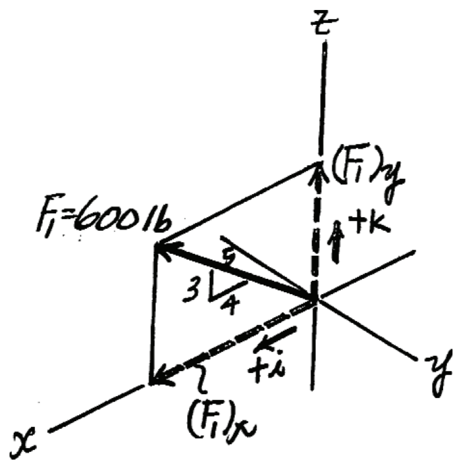
Rectangular Components: Since $\cos^2\alpha_2 + \cos^2\beta_2 + \cos^2\gamma_2 = 1$, then $\cos\beta_2 = \pm\sqrt{1 - \cos^2 60° - \cos^2 120°} = \pm 0.7071$. However, it is required that $\beta_2 < 90°$, thus, $\beta_2 = \cos^{-1}(0.7071) = 45°$. By resolving $\mathbf{F}_1$ and $\mathbf{F}_2$ into their x, y, and z components, as shown in Figs. *a* and *b*, respectively, $\mathbf{F}_1$ and $\mathbf{F}_2$ can be expressed in Cartesian vector form, as

$$\mathbf{F}_1 = 600\left(\frac{4}{5}\right)(+\mathbf{i}) + 0\mathbf{j} + 600\left(\frac{3}{5}\right)(+\mathbf{k})$$

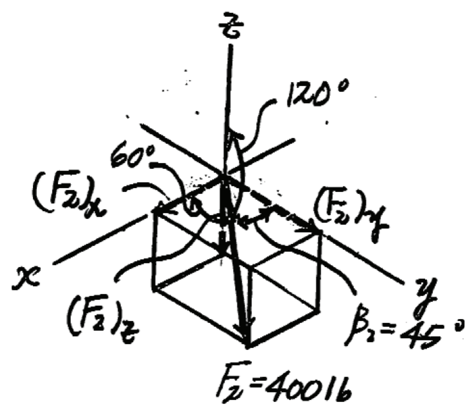
$$= [480\mathbf{i} + 360\mathbf{k}]\text{ lb} \qquad \textbf{Ans.}$$

$$\mathbf{F}_2 = 400\cos 60°\mathbf{i} + 400\cos 45°\mathbf{j} + 400\cos 120°\mathbf{k}$$

$$= [200\mathbf{i} + 283\mathbf{j} - 200\mathbf{k}]\text{ lb} \qquad \textbf{Ans.}$$



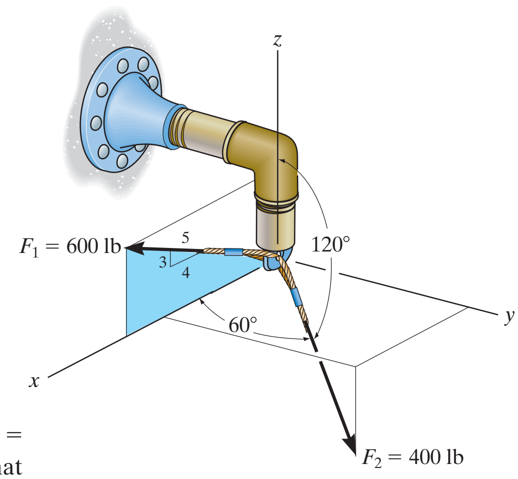
(a)



(b)

2–67.

Determine the magnitude and direction of the resultant force acting on the pipe assembly.



SOLUTION

Force Vectors: Since $\cos^2\alpha_2 + \cos^2\beta_2 + \cos^2\gamma_2 = 1$, then $\cos\gamma_2 = \pm\sqrt{1 - \cos^2 60° - \cos^2 120°} = \pm\, 0.7071$. However, it is required that $\beta_2 < 90°$, thus, $\beta_2 = \cos^{-1}(0.7071) = 45°$. By resolving $\mathbf{F}_1$ and $\mathbf{F}_2$ into their x, y, and z components, as shown in Figs. *a* and *b*, respectively, $\mathbf{F}_1$ and $\mathbf{F}_2$ can be expressed in Cartesian vector form, as

$$\mathbf{F}_1 = 600\left(\frac{4}{5}\right)(+\mathbf{i}) + 0\mathbf{j} + 600\left(\frac{3}{5}\right)(+\mathbf{k})$$

$$= \{480\mathbf{i} + 360\mathbf{k}\}\text{ lb}$$

$$\mathbf{F}_2 = 400\cos 60°\mathbf{i} + 400\cos 45°\mathbf{j} + 400\cos 120°\mathbf{k}$$

$$= \{200\mathbf{i} + 282.84\mathbf{j} - 200\mathbf{k}\}\text{ lb}$$

Resultant Force: By adding $\mathbf{F}_1$ and $\mathbf{F}_2$ vectorally, we obtain $\mathbf{F}_R$.

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$

$$= (480\mathbf{i} + 360\mathbf{k}) + (200\mathbf{i} + 282.84\mathbf{j} - 200\mathbf{k})$$

$$= \{680\mathbf{i} + 282.84\mathbf{j} + 160\mathbf{k}\}\text{ lb}$$

The magnitude of $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2 + (F_R)_z^2}$$

$$= \sqrt{680^2 + 282.84^2 + 160^2} = 753.66\text{ lb} = 754\text{ lb} \qquad \textbf{Ans.}$$

The coordinate direction angles of $\mathbf{F}_R$ are

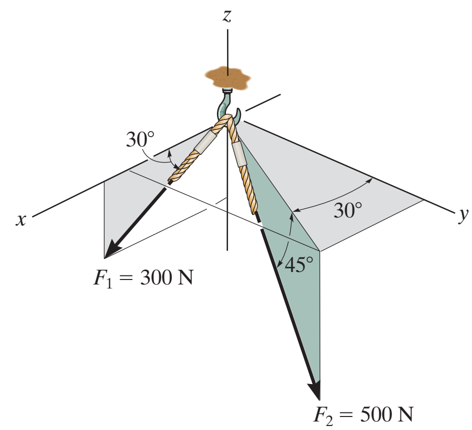
$$\alpha = \cos^{-1}\left[\frac{(F_R)_x}{F_R}\right] = \cos^{-1}\left(\frac{680}{753.66}\right) = 25.5° \qquad \textbf{Ans.}$$

$$\beta = \cos^{-1}\left[\frac{(F_R)_y}{F_R}\right] = \cos^{-1}\left(\frac{282.84}{753.66}\right) = 68.0° \qquad \textbf{Ans.}$$

$$\gamma = \cos^{-1}\left[\frac{(F_R)_z}{F_R}\right] = \cos^{-1}\left(\frac{160}{753.66}\right) = 77.7° \qquad \textbf{Ans.}$$

***2–68.**

Express each force as a Cartesian vector.



SOLUTION

Rectangular Components: By referring to Figs. *a* and *b*, the *x*, *y*, and *z* components of $\mathbf{F}_1$ and $\mathbf{F}_2$ can be written as

$(F_1)_x = 300 \cos 30° = 259.8 \text{ N}$ $\quad$ $(F_2)_x = 500 \cos 45° \sin 30° = 176.78 \text{ N}$

$(F_1)_y = 0$ $\quad$ $(F_2)_y = 500 \cos 45° \cos 30° = 306.19 \text{ N}$

$(F_1)_t = 300 \sin 30° = 150 \text{ N}$ $\quad$ $(F_2)_z = 500 \sin 45° = 353.55 \text{ N}$

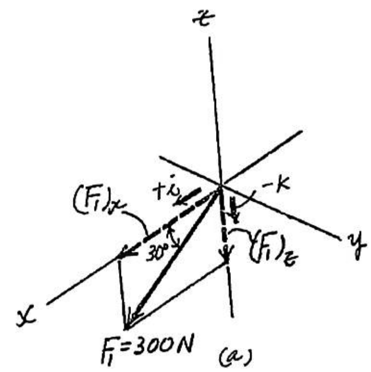
Thus, $\mathbf{F}_1$ and $\mathbf{F}_2$ can be written in Cartesian vector form as

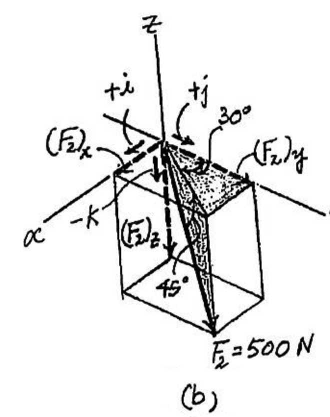
$$\mathbf{F}_1 = 259.81(+\mathbf{i}) + 0\mathbf{j} + 150(-\mathbf{k})$$

$$= \{260\mathbf{i} - 150\mathbf{k}\} \text{ N}$$ **Ans.**

$$\mathbf{F}_2 = 176.78(+\mathbf{i}) + 306.19(+\mathbf{j}) + 353.55(-\mathbf{k})$$

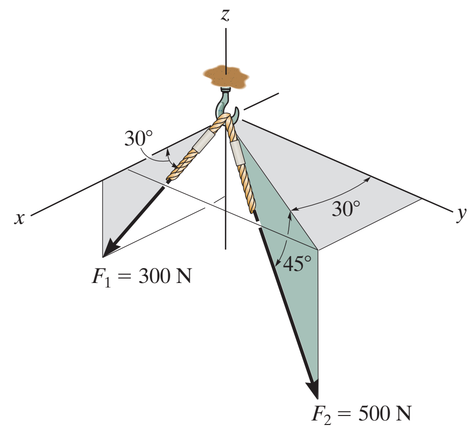
$$= 2\{177\mathbf{i} + 306\mathbf{j} - 354\mathbf{k}\} \text{ N}$$ **Ans.**





2–69.

Determine the magnitude and coordinate direction angles of the resultant force acting on the hook.

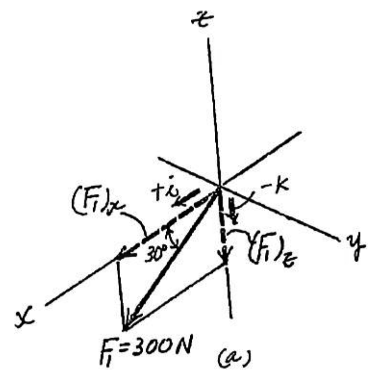


SOLUTION

Force Vectors: By resolving $\mathbf{F}_1$ and $\mathbf{F}_2$ into their x, y, and z components, as shown in Figs. *a* and *b*, respectively, $\mathbf{F}_1$ and $\mathbf{F}_2$ can be expessed in Cartesian vector form as

$$\mathbf{F}_1 = 300 \cos 30°(+\mathbf{i}) + 0\mathbf{j} + 300 \sin 30°(-\mathbf{k})$$
$$= \{259.81\mathbf{i} - 150\mathbf{k}\} \text{ N}$$

$$\mathbf{F}_2 = 500 \cos 45° \sin 30°(+\mathbf{i}) + 500 \cos 45° \cos 30°(+\mathbf{j}) + 500 \sin 45°(-\mathbf{k})$$
$$= \{176.78\mathbf{i} - 306.19\mathbf{j} - 353.55\mathbf{k}\} \text{ N}$$

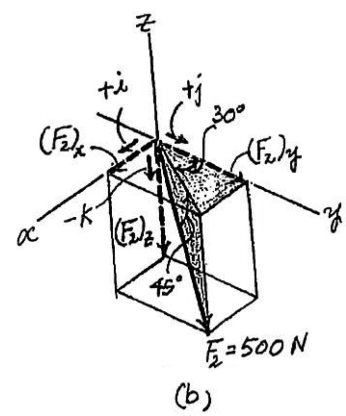


Resultant Force: The resultant force acting on the hook can be obtained by vectorally adding $\mathbf{F}_1$ and $\mathbf{F}_2$. Thus,

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$
$$= (259.81\mathbf{i} - 150\mathbf{k}) + (176.78\mathbf{i} + 306.19\mathbf{j} - 353.55\mathbf{k})$$
$$= \{436.58\mathbf{i}) + 306.19\mathbf{j} - 503.55\mathbf{k}\} \text{ N}$$

The magnitude of $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x{}^2 + (F_R)_y{}^2 (F_R)_z{}^2}$$
$$= \sqrt{(436.58)^2 + (306.19)^2 + (-503.55)^2} = 733.43 \text{ N} = 733 \text{ N} \qquad \textbf{Ans.}$$



The coordinate direction angles of $\mathbf{F}_R$ are

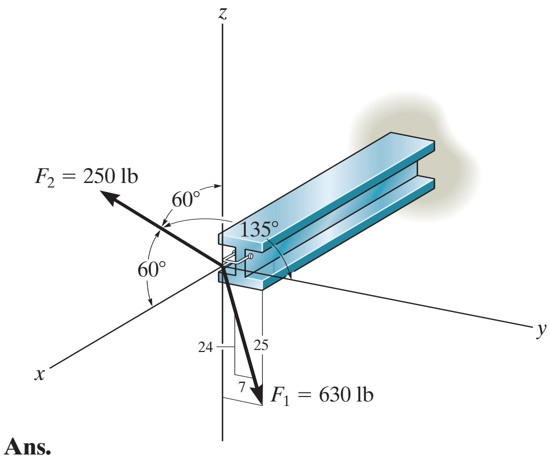
$$\theta_x = \cos^{-1}\left[\frac{(F_R)_x}{F_R}\right] = \cos^{-1}\left(\frac{436.58}{733.43}\right) = 53.5° \qquad \textbf{Ans.}$$

$$\theta_y = \cos^{-1}\left[\frac{(F_R)_y}{F_R}\right] = \cos^{-1}\left(\frac{306.19}{733.43}\right) = 65.3° \qquad \textbf{Ans.}$$

$$\theta_z = \cos^{-1}\left[\frac{(F_R)_z}{F_R}\right] = \cos^{-1}\left(\frac{-503.55}{733.43}\right) = 133° \qquad \textbf{Ans.}$$

2–70.

The beam is subjected to the two forces shown. Express each force in Cartesian vector form and determine the magnitude and coordinate direction angles of the resultant force.



SOLUTION

$$\mathbf{F}_1 = 630\left(\frac{7}{25}\right)\mathbf{j} - 630\left(\frac{24}{25}\right)\mathbf{k}$$

$$\mathbf{F}_1 = (176.4\,\mathbf{j} - 604.8\mathbf{k})$$

$$\mathbf{F}_1 = \{176\mathbf{j} - 605\mathbf{k}\}\text{ lb}$$ **Ans.**

$$\mathbf{F}_2 = 250\cos 60°\mathbf{i} + 250\cos 135°\mathbf{j} + 250\cos 60°\mathbf{k}$$

$$\mathbf{F}_2 = (125\mathbf{i} - 176.777\mathbf{j} + 125\mathbf{k})$$

$$\mathbf{F}_2 = \{125\mathbf{i} - 177\mathbf{j} + 125\mathbf{k}\}\text{ lb}$$ **Ans.**

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$

$$\mathbf{F}_R = 125\mathbf{i} - 0.3767\mathbf{j} - 479.8\mathbf{k}$$

$$\mathbf{F}_R = \{125\mathbf{i} - 0.377\mathbf{j} - 480\mathbf{k}\}\text{ lb}$$

$$F_R = \sqrt{(125)^2 + (-0.3767)^2 + (-479.8)^2} = 495.82$$

$$= 496\text{ lb}$$ **Ans.**

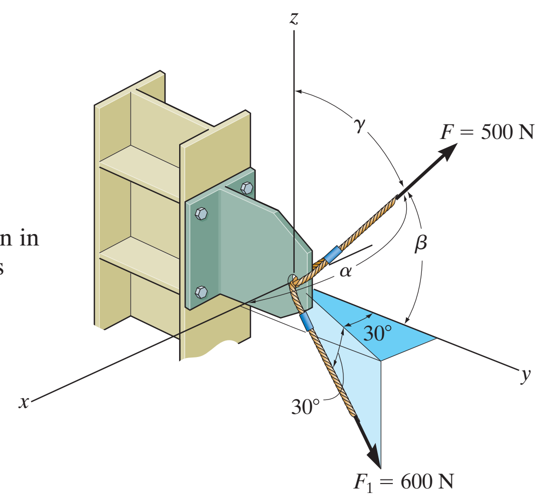
$$\alpha = \cos^{-1}\left(\frac{125}{495.82}\right) = 75.4°$$ **Ans.**

$$\beta = \cos^{-1}\left(\frac{-0.3767}{495.82}\right) = 90.0°$$ **Ans.**

$$\gamma = \cos^{-1}\left(\frac{-479.8}{495.82}\right) = 165°$$ **Ans.**

2–71.

If the resultant force acting on the bracket is directed along the positive y axis, determine the magnitude of the resultant force and the coordinate direction angles of $\mathbf{F}$ so that $\beta < 90°$.



SOLUTION

Force Vectors: By resolving $\mathbf{F}_1$ and $\mathbf{F}$ into their x, y, and z components, as shown in Figs. a and b, respectively, $\mathbf{F}_1$ and $\mathbf{F}$ can be expressed in Cartesian vector form as

$$\mathbf{F}_1 = 600\cos 30°\sin 30°(+\mathbf{i}) + 600\cos 30°\cos 30°(+\mathbf{j}) + 600\sin 30°(-\mathbf{k})$$

$$= \{259.81\mathbf{i} + 450\mathbf{j} - 300\mathbf{k}\}\ \text{N}$$

$$\mathbf{F} = 500\cos\alpha\mathbf{i} + 500\cos\beta\mathbf{j} + 500\cos\gamma\mathbf{k}$$

Since the resultant force $\mathbf{F}_R$ is directed towards the positive y axis, then

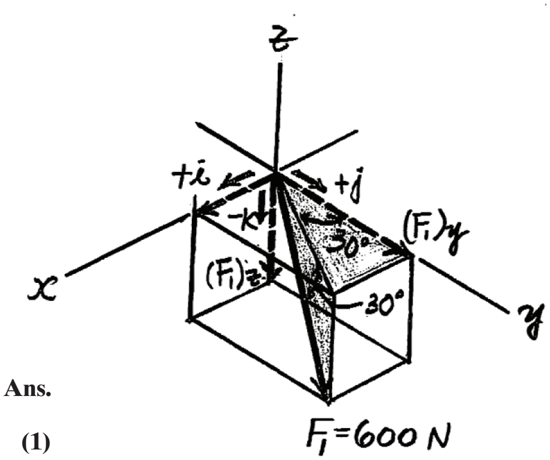
$$\mathbf{F}_R = F_R\,\mathbf{j}$$

Resultant Force:

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}$$

$$F_R\,\mathbf{j} = (259.81\mathbf{i} + 450\mathbf{j} - 300\mathbf{k}) + (500\cos\alpha\mathbf{i} + 500\cos\beta\mathbf{j} + 500\cos\gamma\mathbf{k})$$

$$F_R\,\mathbf{j} = (259.81 + 500\cos\alpha)\mathbf{i} + (450 + 500\cos\beta)\mathbf{j} + (500\cos\gamma - 300)\mathbf{k}$$



(a)

Equating the **i**, **j**, and **k** components,

$$0 = 259.81 + 500\cos\alpha$$

$$\alpha = 121.31° = 121° \qquad \textbf{Ans.}$$

$$F_R = 450 + 500\cos\beta \qquad \textbf{(1)}$$

$$0 = 500\cos\gamma - 300$$

$$\gamma = 53.13° = 53.1° \qquad \textbf{Ans.}$$

However, since $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$, $\alpha = 121.31°$, and $\gamma = 53.13°$,

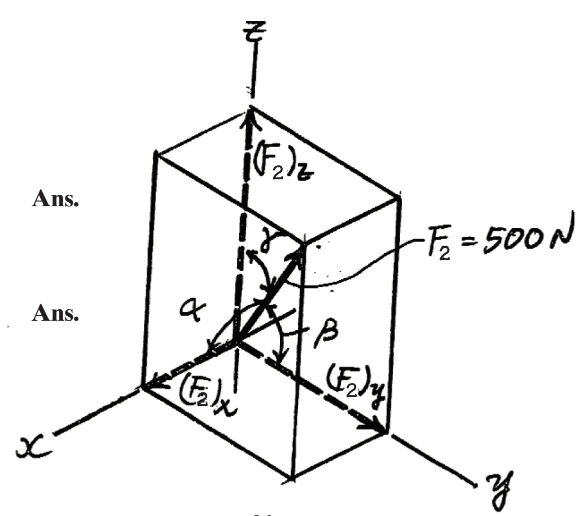
$$\cos\beta = \pm\sqrt{1 - \cos^2 121.31° - \cos^2 53.13°} = \pm 0.6083$$

If we substitute $\cos\beta = 0.6083$ into Eq. (1),

$$F_R = 450 + 500(0.6083) = 754\ \text{N} \qquad \textbf{Ans.}$$

and

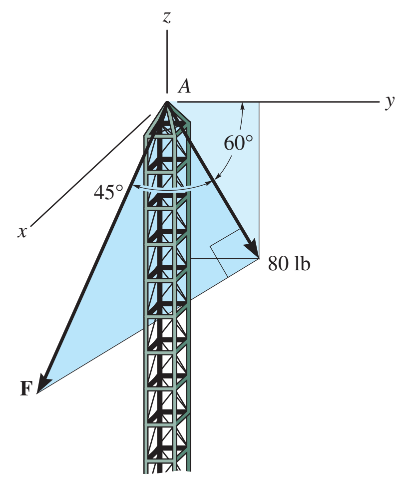
$$\beta = \cos^{-1}(0.6083) = 52.5° \qquad \textbf{Ans.}$$



(b)

***2–72.**

A force **F** is applied at the top of the tower at A. If it acts in the direction shown such that one of its components lying in the shaded y-z plane has a magnitude of 80 lb, determine its magnitude F and coordinate direction angles α, β, γ.



SOLUTION

Cartesian Vector Notation: The magnitude of force **F** is

$$F \cos 45° = 80 \qquad F = 113.14 \text{ lb} = 113 \text{ lb} \qquad \textbf{Ans.}$$

Thus,

$$\mathbf{F} = \{113.14 \sin 45°\mathbf{i} + 80 \cos 60°\mathbf{j} - 80 \sin 60°\mathbf{k}\} \text{ lb}$$

$$= \{80.0\mathbf{i} + 40.0\mathbf{j} - 69.28\mathbf{k}\} \text{ lb}$$

The coordinate direction angles are

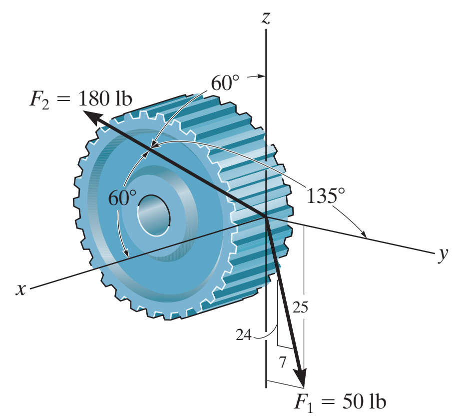
$$\cos \alpha = \frac{F_x}{F} = \frac{80.0}{113.14} \qquad \alpha = 45.0° \qquad \textbf{Ans.}$$

$$\cos \beta = \frac{F_y}{F} = \frac{40.0}{113.14} \qquad \beta = 69.3° \qquad \textbf{Ans.}$$

$$\cos \gamma = \frac{F_z}{F} = \frac{-69.28}{113.14} \qquad \gamma = 128° \qquad \textbf{Ans.}$$

2–73.

The spur gear is subjected to the two forces caused by contact with other gears. Express each force as a Cartesian vector.



SOLUTION

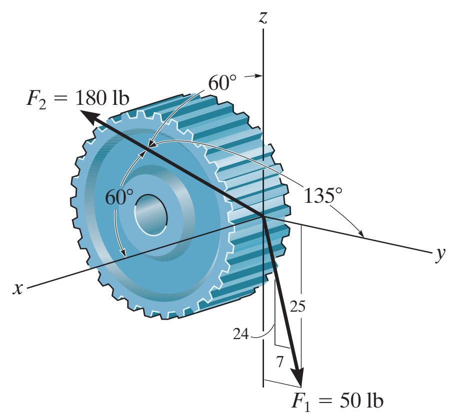
$$\mathbf{F}_1 = \frac{7}{25}(50)\mathbf{j} - \frac{24}{25}(50)\mathbf{k} = \{14.0\mathbf{j} - 48.0\mathbf{k}\}\text{ lb}$$ **Ans.**

$$\mathbf{F}_2 = 180\cos 60^\circ\mathbf{i} + 180\cos 135^\circ\mathbf{j} + 180\cos 60^\circ\mathbf{k}$$

$$= \{90\mathbf{i} - 127\mathbf{j} + 90\mathbf{k}\}\text{ lb}$$ **Ans.**

2–74.

The spur gear is subjected to the two forces caused by contact with other gears. Determine the resultant of the two forces and express the result as a Cartesian vector.



SOLUTION

$$F_{Rx} = 180 \cos 60° = 90$$

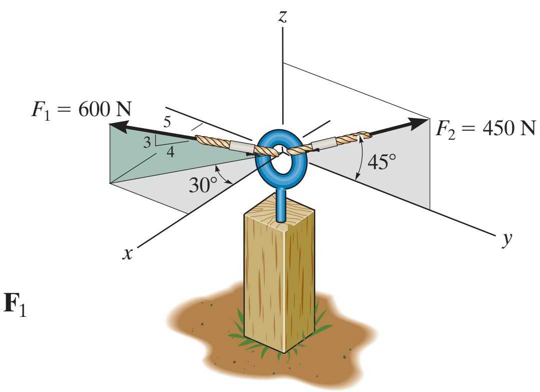
$$F_{Ry} = \frac{7}{25}(50) + 180 \cos 135° = -113$$

$$F_{Rz} = -\frac{24}{25}(50) + 180 \cos 60° = 42$$

$$\mathbf{F}_R = \{90\mathbf{i} - 113\mathbf{j} + 42\mathbf{k}\} \text{ lb}$$ **Ans.**

2–75.

Determine the coordinate direction angles of force $\mathbf{F}_1$.



SOLUTION

Rectangular Components: By referring to Figs. *a*, the *x*, *y*, and *z* components of $\mathbf{F}_1$ can be written as

$$(F_1)_x = 600\left(\frac{4}{5}\right)\cos 30^\circ \text{ N} \qquad (F_1)_y = 600\left(\frac{4}{5}\right)\sin 30^\circ \text{ N} \qquad (F_1)_z = 600\left(\frac{3}{5}\right) \text{ N}$$

Thus, $\mathbf{F}_1$ expressed in Cartesian vector form can be written as

$$\mathbf{F}_1 = 600\left\{\frac{4}{5}\cos 30^\circ(+\mathbf{i}) + \frac{4}{5}\sin 30^\circ(-\mathbf{j}) + \frac{3}{5}(+\mathbf{k})\right\} \text{N}$$
$$= 600[0.6928\mathbf{i} - 0.4\mathbf{j} + 0.6\mathbf{k}] \text{ N}$$

Therefore, the unit vector for $\mathbf{F}_1$ is given by

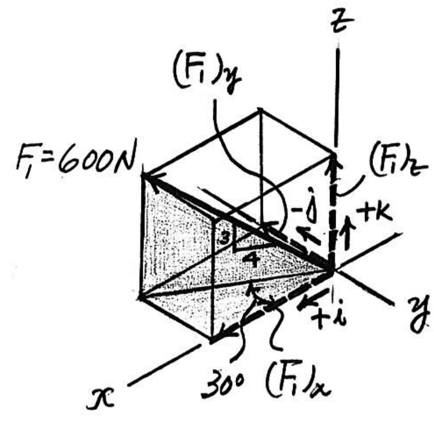
$$\mathbf{u}_{F_1} = \frac{\mathbf{F}_1}{F_1} = \frac{600(0.6928\mathbf{i} - 0.4\mathbf{j} + 0.6\mathbf{k})}{600} = 0.6928\mathbf{i} - 0.4\mathbf{j} + 0.6\mathbf{k}$$

The coordinate direction angles of $\mathbf{F}_1$ are

$$\alpha = \cos^{-1}(u_{F_1})_x = \cos^{-1}(0.6928) = 46.1^\circ$$ **Ans.**

$$\beta = \cos^{-1}(u_{F_1})_y = \cos^{-1}(-0.4) = 114^\circ$$ **Ans.**

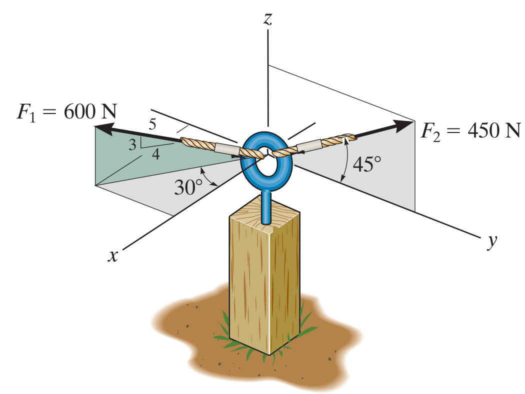
$$\gamma = \cos^{-1}(u_{F_1})_z = \cos^{-1}(0.6) = 53.1^\circ$$ **Ans.**



(a)

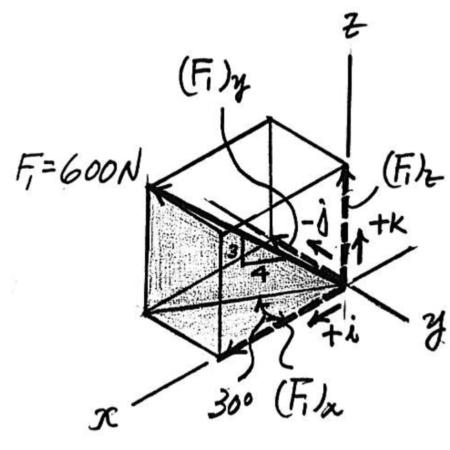
***2–76.**

Determine the magnitude and coordinate direction angles of the resultant force acting on the eyebolt.



SOLUTION

Force Vectors: By resolving $\mathbf{F}_1$ and $\mathbf{F}_2$ into their x, y, and z components, as shown in Figs. a and b, respectively, they are expressed in Cartesian vector form as



(a)

$$\mathbf{F}_1 = 600\left(\frac{4}{5}\right)\cos 30°(+\mathbf{i}) + 600\left(\frac{4}{5}\right)\sin 30°(-\mathbf{j}) + 600\left(\frac{3}{5}\right)(+\mathbf{k})$$

$$= \{415.69\mathbf{i} - 240\mathbf{j} + 360\mathbf{k}\}\text{ N}$$

$$\mathbf{F}_2 = 0\mathbf{i} + 450\cos 45°(+\mathbf{j}) + 450\sin 45°(+\mathbf{k})$$

$$= \{318.20\mathbf{j} + 318.20\mathbf{k}\}\text{ N}$$

Resultant Force: The resultant force acting on the eyebolt can be obtained by vectorally adding $\mathbf{F}_1$ and $\mathbf{F}_2$. Thus,

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$

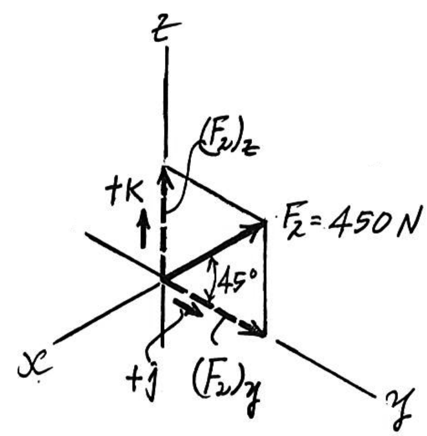
$$= (415.69\mathbf{i} - 240\mathbf{j} + 360\mathbf{k}) + (318.20\mathbf{j} + 318.20\mathbf{k})$$

$$= \{415.69\mathbf{i} + 78.20\mathbf{j} + 678.20\mathbf{k}\}\text{ N}$$

The magnitude of $\mathbf{F}_R$ is given by

$$F_R = \sqrt{(F_R)_x{}^2 + (F_R)_y{}^2 + (F_R)_z{}^2}$$

$$= \sqrt{(415.69)^2 + (78.20)^2 + (678.20)^2} = 799.29\text{ N} = 799\text{ N} \qquad \textbf{Ans.}$$



(b)

The coordinate direction angles of $\mathbf{F}_R$ are

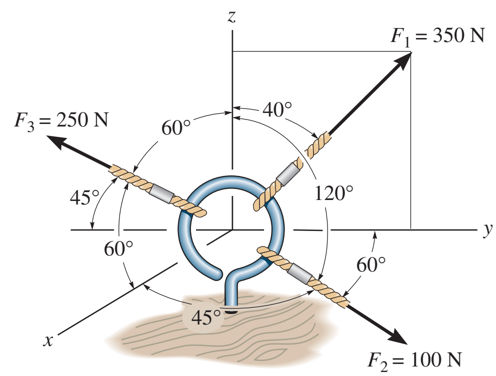
$$\alpha = \cos^{-1}\left[\frac{(F_R)_x}{F_R}\right] = \cos^{-1}\left(\frac{415.69}{799.29}\right) = 58.7° \qquad \textbf{Ans.}$$

$$\beta = \cos^{-1}\left[\frac{(F_R)_y}{F_R}\right] = \cos^{-1}\left(\frac{78.20}{799.29}\right) = 84.4° \qquad \textbf{Ans.}$$

$$\gamma = \cos^{-1}\left[\frac{(F_R)_z}{F_R}\right] = \cos^{-1}\left(\frac{678.20}{799.29}\right) = 32.0° \qquad \textbf{Ans.}$$

2–77.

The cables attached to the screw eye are subjected to the three forces shown. Express each force in Cartesian vector form and determine the magnitude and coordinate direction angles of the resultant force.



SOLUTION

Cartesian Vector Notation:

$$\mathbf{F}_1 = 350\{\sin 40°\mathbf{j} + \cos 40°\mathbf{k}\}\text{ N}$$

$$= \{224.98\mathbf{j} + 268.12\mathbf{k}\}\text{ N}$$

$$= \{225\mathbf{j} + 268\mathbf{k}\}\text{ N}$$ **Ans.**

$$\mathbf{F}_2 = 100\{\cos 45°\mathbf{i} + \cos 60°\mathbf{j} + \cos 120°\mathbf{k}\}\text{ N}$$

$$= \{70.71\mathbf{i} + 50.0\mathbf{j} - 50.0\mathbf{k}\}\text{ N}$$

$$= \{70.7\mathbf{i} + 50.0\mathbf{j} - 50.0\mathbf{k}\}\text{ N}$$ **Ans.**

$$\mathbf{F}_3 = 250\{\cos 60°\mathbf{i} + \cos 135°\mathbf{j} + \cos 60°\mathbf{k}\}\text{ N}$$

$$= \{125.0\mathbf{i} - 176.78\mathbf{j} + 125.0\mathbf{k}\}\text{ N}$$

$$= \{125\mathbf{i} - 177\mathbf{j} + 125\mathbf{k}\}\text{ N}$$ **Ans.**

Resultant Force:

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$$

$$= \{(70.71 + 125.0)\mathbf{i} + (224.98 + 50.0 - 176.78)\mathbf{j} + (268.12 - 50.0 + 125.0)\mathbf{k}\}\text{ N}$$

$$= \{195.71\mathbf{i} + 98.20\mathbf{j} + 343.12\mathbf{k}\}\text{ N}$$

The magnitude of the resultant force is

$$F_R = \sqrt{F_{R_x}^2 + F_{R_y}^2 + F_{R_z}^2}$$

$$= \sqrt{195.71^2 + 98.20^2 + 343.12^2}$$

$$= 407.03\text{ N} = 407\text{ N}$$ **Ans.**

The coordinate direction angles are

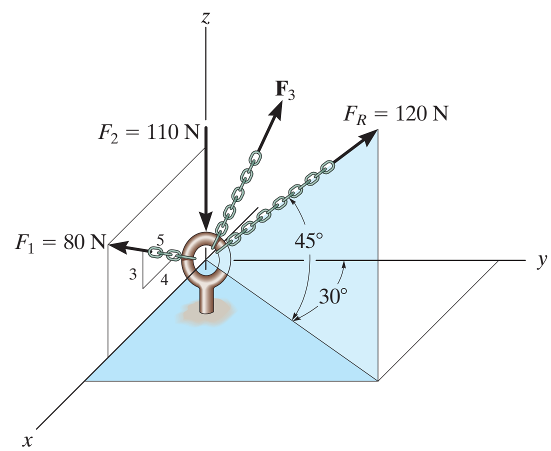
$$\cos\alpha = \frac{F_{R_x}}{F_R} = \frac{195.71}{407.03} \qquad \alpha = 61.3°$$ **Ans.**

$$\cos\beta = \frac{F_{R_y}}{F_R} = \frac{98.20}{407.03} \qquad \beta = 76.0°$$ **Ans.**

$$\cos\gamma = \frac{F_{R_z}}{F_R} = \frac{343.12}{407.03} \qquad \gamma = 32.5°$$ **Ans.**

2–78.

Three forces act on the ring. If the resultant force $\mathbf{F}_R$ has a magnitude and direction as shown, determine the magnitude and the coordinate direction angles of force $\mathbf{F}_3$.



SOLUTION

Cartesian Vector Notation:

$$\mathbf{F}_R = 120\{\cos 45°\sin 30°\mathbf{i} + \cos 45°\cos 30°\mathbf{j} + \sin 45°\mathbf{k}\}\text{ N}$$

$$= \{42.43\mathbf{i} + 73.48\mathbf{j} + 84.85\mathbf{k}\}\text{ N}$$

$$\mathbf{F}_1 = 80\left\{\frac{4}{5}\mathbf{i} + \frac{3}{5}\mathbf{k}\right\}\text{ N} = \{64.0\mathbf{i} + 48.0\mathbf{k}\}\text{ N}$$

$$\mathbf{F}_2 = \{-110\mathbf{k}\}\text{ N}$$

$$\mathbf{F}_3 = \{F_{3_x}\mathbf{i} + F_{3_y}\mathbf{j} + F_{3_z}\mathbf{k}\}\text{ N}$$

Resultant Force:

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$$

$$\{42.43\mathbf{i} + 73.48\mathbf{j} + 84.85\mathbf{k}\} = \left\{\left(64.0 + F_{3_x}\right)\mathbf{i} + F_{3_y}\mathbf{j} + \left(48.0 - 110 + F_{3_z}\right)\mathbf{k}\right\}$$

Equating **i**, **j** and **k** components, we have

$$64.0 + F_{3_x} = 42.43 \qquad F_{3_x} = -21.57\text{ N}$$

$$F_{3_y} = 73.48\text{ N}$$

$$48.0 - 110 + F_{3_z} = 84.85 \qquad F_{3_z} = 146.85\text{ N}$$

The magnitude of force $\mathbf{F}_3$ is

$$F_3 = \sqrt{F_{3_x}^2 + F_{3_y}^2 + F_{3_z}^2}$$

$$= \sqrt{(-21.57)^2 + 73.48^2 + 146.85^2}$$

$$= 165.62\text{ N} = 166\text{ N}$$ **Ans.**

The coordinate direction angles for $\mathbf{F}_3$ are

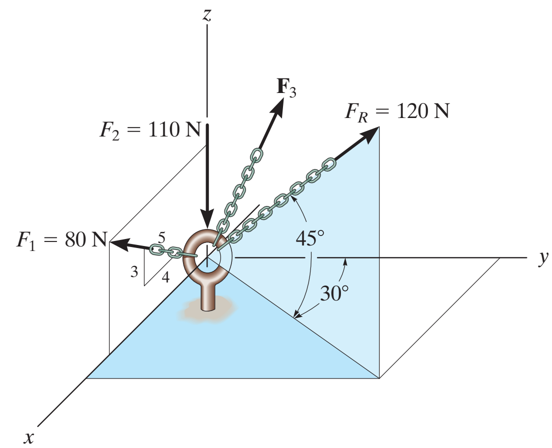
$$\cos\alpha = \frac{F_{3_x}}{F_3} = \frac{-21.57}{165.62} \qquad \alpha = 97.5°$$ **Ans.**

$$\cos\beta = \frac{F_{3_y}}{F_3} = \frac{73.48}{165.62} \qquad \beta = 63.7°$$ **Ans.**

$$\cos\gamma = = \frac{F_{3_z}}{F_3} = \frac{146.85}{165.62} \qquad \gamma = 27.5°$$ **Ans.**

2–79.

Determine the coordinate direction angles of $\mathbf{F}_1$ and $\mathbf{F}_R$.



SOLUTION

Unit Vector of $\mathbf{F}_1$ ***and*** $\mathbf{F}_R$***:***

$$\mathbf{u}_{F_1} = \frac{4}{5}\mathbf{i} + \frac{3}{5}\mathbf{k} = 0.8\mathbf{i} + 0.6\mathbf{k}$$

$$\mathbf{u}_R = \cos 45^\circ \sin 30^\circ\mathbf{i} + \cos 45^\circ \cos 30^\circ\mathbf{j} + \sin 45^\circ\mathbf{k}$$

$$= 0.3536\mathbf{i} + 0.6124\mathbf{j} + 0.7071\mathbf{k}$$

Thus, the coordinate direction angles $\mathbf{F}_1$ and $\mathbf{F}_R$ are

$\cos \alpha_{F_1} = 0.8 \qquad \alpha_{F_1} = 36.9^\circ$ **Ans.**

$\cos \beta_{F_1} = 0 \qquad \beta_{F_1} = 90.0^\circ$ **Ans.**

$\cos \gamma_{F_1} = 0.6 \qquad \gamma_{F_1} = 53.1^\circ$ **Ans.**

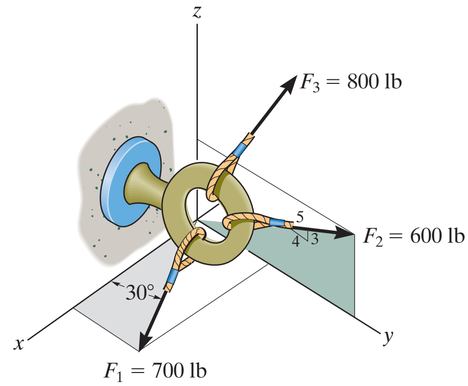
$\cos \alpha_R = 0.3536 \qquad \alpha_R = 69.3^\circ$ **Ans.**

$\cos \beta_R = 0.6124 \qquad \beta_R = 52.2^\circ$ **Ans.**

$\cos \gamma_R = 0.7071 \qquad \gamma_R = 45.0^\circ$ **Ans.**

***2–80.**

If the coordinate direction angles for $\mathbf{F}_3$ are $\alpha_3 = 120°$, $\beta_3 = 45°$ and $\gamma_3 = 60°$, determine the magnitude and coordinate direction angles of the resultant force acting on the eyebolt.



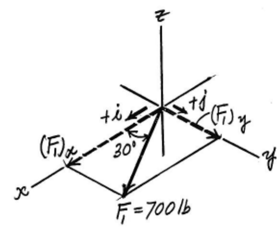
SOLUTION

Force Vectors: By resolving $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ into their x, y, and z components, as shown in Figs. *a*, *b*, and *c*, respectively, $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ can be expressed in Cartesian vector form as

$$\mathbf{F}_1 = 700\cos 30°(+\mathbf{i}) + 700\sin 30°(+\mathbf{j}) = \{606.22\mathbf{i} + 350\mathbf{j}\}\text{ lb}$$

$$\mathbf{F}_2 = 0\mathbf{i} + 600\left(\frac{4}{5}\right)(+\mathbf{j}) + 600\left(\frac{3}{5}\right)(+\mathbf{k}) = \{480\mathbf{j} + 360\mathbf{k}\}\text{ lb}$$

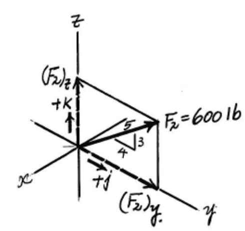
$$\mathbf{F}_3 = 800\cos 120°\mathbf{i} + 800\cos 45°\mathbf{j} + 800\cos 60°\mathbf{k} = [-400\mathbf{i} + 565.69\mathbf{j} + 400\mathbf{k}]\text{ lb}$$



(a)

Resultant Force: By adding $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ vectorally, we obtain $\mathbf{F}_R$. Thus,

$$\begin{aligned}\mathbf{F}_R &= \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 \\ &= (606.22\mathbf{i} + 350\mathbf{j}) + (480\mathbf{j} + 360\mathbf{k}) + (-400\mathbf{i} + 565.69\mathbf{j} + 400\mathbf{k}) \\ &= [206.22\mathbf{i} + 1395.69\mathbf{j} + 760\mathbf{k}]\text{ lb}\end{aligned}$$



(b)

The magnitude of $\mathbf{F}_R$ is

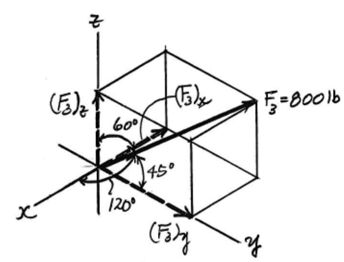
$$\begin{aligned}F_R &= \sqrt{(F_R)_x^{\,2} + (F_R)_y^{\,2} + (F_R)_z^{\,2}} \\ &= \sqrt{(206.22)^2 + (1395.69)^2 + (760)^2} = 1602.52\text{ lb} = 1.60\text{ kip} \qquad \textbf{Ans.}\end{aligned}$$

The coordinate direction angles of $\mathbf{F}_R$ are

$$\alpha = \cos^{-1}\left[\frac{(F_R)_x}{F_R}\right] = \cos^{-1}\left(\frac{206.22}{1602.52}\right) = 82.6° \qquad \textbf{Ans.}$$

$$\beta = \cos^{-1}\left[\frac{(F_R)_y}{F_R}\right] = \cos^{-1}\left(\frac{1395.69}{1602.52}\right) = 29.4° \qquad \textbf{Ans.}$$

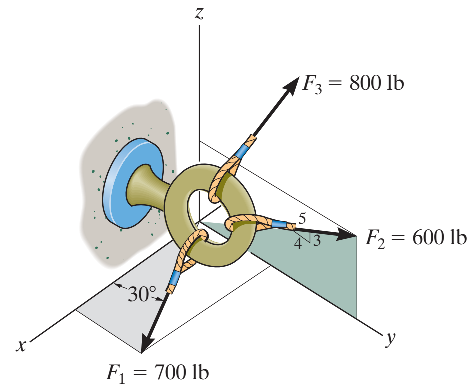
$$\gamma = \cos^{-1}\left[\frac{(F_R)_z}{F_R}\right] = \cos^{-1}\left(\frac{760}{1602.52}\right) = 61.7° \qquad \textbf{Ans.}$$



(c)

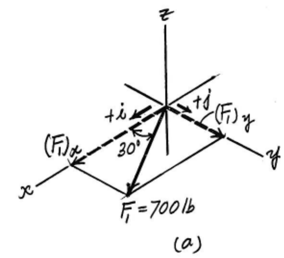
2–81.

If the coordinate direction angles for $\mathbf{F}_3$ are $\alpha_3 = 120°$, $\beta_3 = 45°$ and $\gamma_3 = 60°$, determine the magnitude and coordinate direction angles of the resultant force acting on the eyebolt.



SOLUTION

Force Vectors: By resolving $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ into their x, y, and z components, as shown in Figs. a, b, and c, respectively, $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ can be expressed in Cartesian vector form as



$$\mathbf{F}_1 = 700 \cos 30°(+\mathbf{i}) + 700 \sin 30°(+\mathbf{j}) = \{606.22\mathbf{i} + 350\mathbf{j}\} \text{ lb}$$

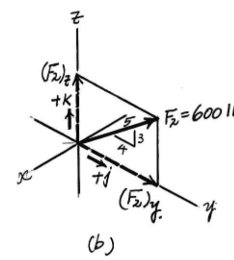
$$\mathbf{F}_2 = 0\mathbf{i} + 600\left(\frac{4}{5}\right)(+\mathbf{j}) + 600\left(\frac{3}{5}\right)(+\mathbf{k}) = \{480\mathbf{j} + 360\mathbf{k}\} \text{ lb}$$

$$\mathbf{F}_3 = 800 \cos 120°\mathbf{i} + 800 \cos 45°\mathbf{j} + 800 \cos 60°\mathbf{k} = \{-400\mathbf{i} + 565.69\mathbf{j} + 400\mathbf{k}\} \text{ lb}$$

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$$

$$= 606.22\mathbf{i} + 350\mathbf{j} + 480\mathbf{j} + 360\mathbf{k} - 400\mathbf{i} + 565.69\mathbf{j} + 400\mathbf{k}$$

$$= \{206.22\mathbf{i} + 1395.69\mathbf{j} + 760\mathbf{k}\} \text{ lb}$$



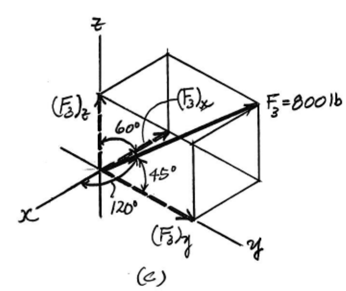
$$F_R = \sqrt{(206.22)^2 + (1395.69)^2 + (760)^2}$$

$$= 1602.52 \text{ lb} = 1.60 \text{ kip} \qquad \textbf{Ans.}$$

$$\alpha = \cos^{-1}\left(\frac{206.22}{1602.52}\right) = 82.6° \qquad \textbf{Ans.}$$

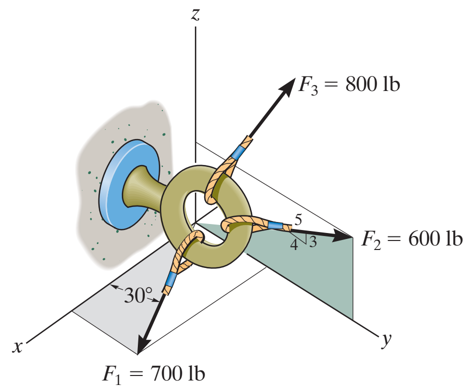
$$\beta = \cos^{-1}\left(\frac{1395.69}{1602.52}\right) = 29.4° \qquad \textbf{Ans.}$$

$$\gamma = \cos^{-1}\left(\frac{760}{1602.52}\right) = 61.7° \qquad \textbf{Ans.}$$



2–82.

If the direction of the resultant force acting on the eyebolt is defined by the unit vector $\mathbf{u}_{F_R} = \cos 30°\mathbf{j} + \sin 30°\mathbf{k}$, determine the coordinate direction angles of $\mathbf{F}_3$ and the magnitude of $\mathbf{F}_R$.



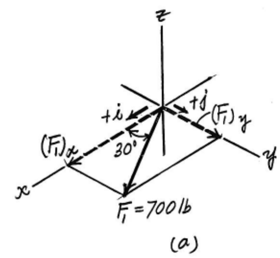
SOLUTION

Force Vectors: By resolving $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ into their x, y, and z components, as shown in Figs. a, b, and c, respectively, $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ can be expressed in Cartesian vector form as

$$\mathbf{F}_1 = 700 \cos 30°(+\mathbf{i}) + 700 \sin 30°(+\mathbf{j}) = \{606.22\mathbf{i} + 350\mathbf{j}\} \text{ lb}$$

$$\mathbf{F}_2 = 0\mathbf{i} + 600\left(\frac{4}{5}\right)(+\mathbf{j}) + 600\left(\frac{3}{5}\right)(+\mathbf{k}) = \{480\mathbf{j} + 360\mathbf{k}\} \text{ lb}$$

$$\mathbf{F}_3 = 800 \cos \alpha_3\mathbf{i} + 800 \cos \beta_3\mathbf{j} + 800 \cos \gamma_3\mathbf{k}$$



Since the direction of $\mathbf{F}_R$ is defined by $\mathbf{u}_{F_R} = \cos 30°\mathbf{j} + \sin 30°\mathbf{k}$, it can be written in Cartesian vector form as

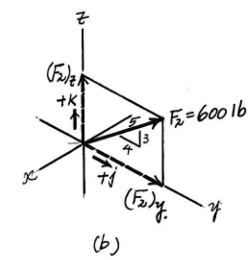
$$\mathbf{F}_R = F_R\mathbf{u}_{F_R} = F_R(\cos 30°\mathbf{j} + \sin 30°\mathbf{k}) = 0.8660F_R\,\mathbf{j} + 0.5F_R\,\mathbf{k}$$

Resultant Force: By adding $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ vectorally, we obtain $\mathbf{F}_R$. Thus,

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$$

$$0.8660F_R\,\mathbf{j} + 0.5F_R\,\mathbf{k} = (606.22\mathbf{i} + 350\mathbf{j}) + (480\mathbf{j} + 360\mathbf{k}) + (800 \cos \alpha_3\mathbf{i} + 800 \cos \beta_3\mathbf{j} + 800 \cos \gamma_3\mathbf{k})$$

$$0.8660F_R\,\mathbf{j} + 0.5F_R\,\mathbf{k} = (606.22 + 800 \cos \alpha_3)\mathbf{i} + (350 + 480 + 800 \cos \beta_3)\mathbf{j} + (360 + 800 \cos \gamma_3)\mathbf{k}$$

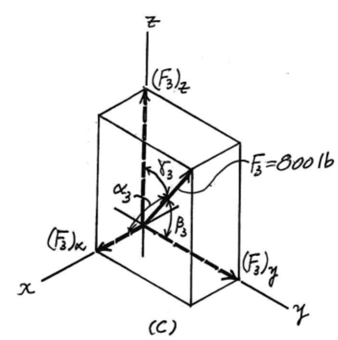


Equating the **i**, **j**, and **k** components, we have

$$0 = 606.22 + 800 \cos \alpha_3$$
$$800 \cos \alpha_3 = -606.22 \qquad \textbf{(1)}$$

$$0.8660F_R = 350 + 480 + 800 \cos \beta_3$$
$$800 \cos \beta_3 = 0.8660F_R - 830 \qquad \textbf{(2)}$$

$$0.5F_R = 360 + 800 \cos \gamma_3$$
$$800 \cos \gamma_3 = 0.5F_R - 360 \qquad \textbf{(3)}$$



Squaring and then adding Eqs. (1), (2), and (3), yields

$$800^2\,[\cos^2 \alpha_3 + \cos^2 \beta_3 + \cos^2 \gamma_3] = F_R{}^2 - 1797.60F_R + 1{,}186{,}000 \qquad \textbf{(4)}$$

However, $\cos^2 \alpha_3 + \cos^2 \beta_3 + \cos^2 \gamma_3 = 1$. Thus, from Eq. (4)

$$F_R{}^2 - 1797.60F_R + 546{,}000 = 0$$

Solving the above quadratic equation, we have two positive roots

$F_R = 387.09 \text{ N} = 387 \text{ N}$ **Ans.**

$F_R = 1410.51 \text{ N} = 1.41 \text{ kN}$ **Ans.**

From Eq. (1),

$\alpha_3 = 139°$ **Ans.**

Substituting $F_R = 387.09$ N into Eqs. (2), and (3), yields

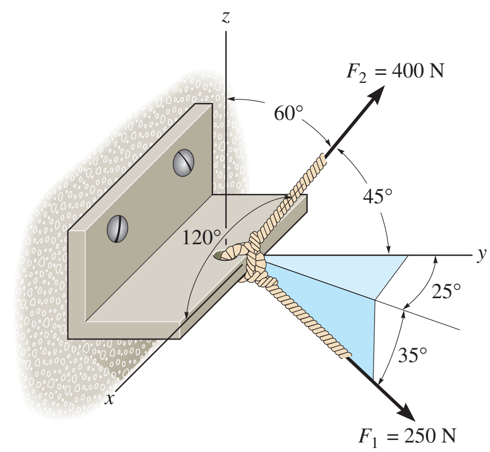
$\beta_3 = 128°$ $\qquad \gamma_3 = 102°$ **Ans.**

Substituting $F_R = 1410.51$ N into Eqs. (2), and (3), yields

$\beta_3 = 60.7°$ $\qquad \gamma_3 = 64.4°$ **Ans.**

2–83.

The bracket is subjected to the two forces shown. Express each force in Cartesian vector form and then determine the resultant force $\mathbf{F}_R$. Find the magnitude and coordinate direction angles of the resultant force.



SOLUTION

Cartesian Vector Notation:

$\mathbf{F}_1 = 250\{\cos 35° \sin 25°\mathbf{i} + \cos 35° \cos 25°\mathbf{j} - \sin 35°\mathbf{k}\}$ N

$= \{86.55\mathbf{i} + 185.60\mathbf{j} - 143.39\mathbf{k}\}$ N

$= \{86.5\mathbf{i} + 186\mathbf{j} - 143\mathbf{k}\}$ N **Ans.**

$\mathbf{F}_2 = 400\{\cos 120°\mathbf{i} + \cos 45°\mathbf{j} + \cos 60°\mathbf{k}\}$ N

$= \{-200.0\mathbf{i} + 282.84\mathbf{j} + 200.0\mathbf{k}\}$ N

$= \{-200\mathbf{i} + 283\mathbf{j} + 200\mathbf{k}\}$ N **Ans.**

Resultant Force:

$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$

$= \{(86.55 - 200.0)\mathbf{i} + (185.60 + 282.84)\mathbf{j} + (-143.39 + 200.0)\,\mathbf{k}\}$

$= \{-113.45\mathbf{i} + 468.44\mathbf{j} + 56.61\mathbf{k}\}$ N

$= \{-113\mathbf{i} + 468\mathbf{j} + 56.6\mathbf{k}\}$ N **Ans.**

The magnitude of the resultant force is

$$F_R = \sqrt{F_{R_x}^2 + F_{R_y}^2 + F_{R_z}^2}$$

$$= \sqrt{(-113.45)^2 + 468.44^2 + 56.61^2}$$

$$= 485.30 \text{ N} = 485 \text{ N}$$ **Ans.**

The coordinate direction angles are

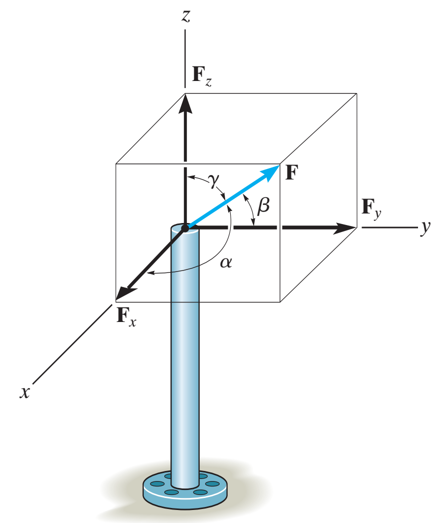
$$\cos\alpha = \frac{F_{R_x}}{F_R} = \frac{-113.45}{485.30} \qquad \alpha = 104°$$ **Ans.**

$$\cos\beta = \frac{F_{R_y}}{F_R} = \frac{468.44}{485.30} \qquad \beta = 15.1°$$ **Ans.**

$$\cos\gamma = \frac{F_{R_z}}{F_R} = \frac{56.61}{485.30} \qquad \gamma = 83.3°$$ **Ans.**

***2–84.**

The pole is subjected to the force **F**, which has components acting along the *x, y, z* axes as shown. If the magnitude of **F** is 3 kN, $\beta = 30°$, and $\gamma = 75°$, determine the magnitudes of its three components.



SOLUTION

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

$$\cos^2\alpha + \cos^2 30° + \cos^2 75° = 1$$

$$\alpha = 64.67°$$

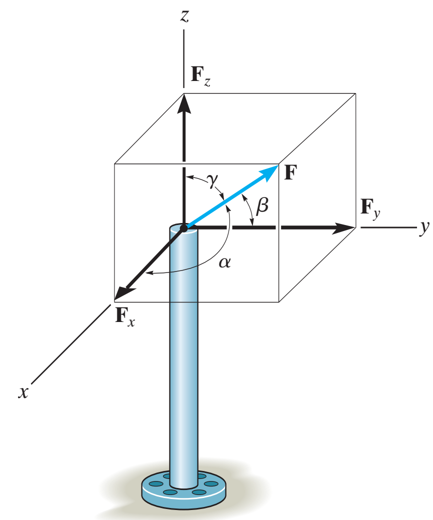
$$F_x = 3\cos 64.67° = 1.28 \text{ kN}$$ **Ans.**

$$F_y = 3\cos 30° = 2.60 \text{ kN}$$ **Ans.**

$$F_z = 3\cos 75° = 0.776 \text{ kN}$$ **Ans.**

2–85.

The pole is subjected to the force **F** which has components $F_x = 1.5$ kN and $F_z = 1.25$ kN. If $\beta = 75°$, determine the magnitudes of **F** and $\mathbf{F}_y$.



SOLUTION

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

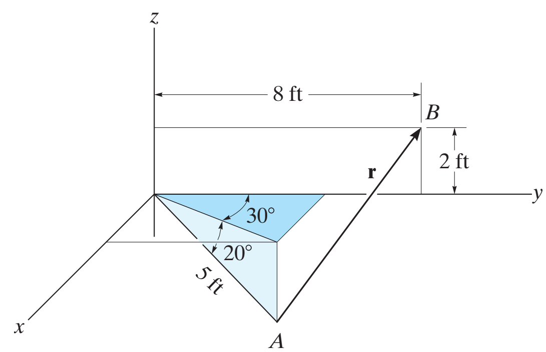
$$\left(\frac{1.5}{F}\right)^2 + \cos^2 75° + \left(\frac{1.25}{F}\right)^2 = 1$$

$$F = 2.02 \text{ kN} \qquad \textbf{Ans.}$$

$$F_y = 2.02 \cos 75° = 0.523 \text{ kN} \qquad \textbf{Ans.}$$

2–86.

Express the position vector **r** in Cartesian vector form; then determine its magnitude and coordinate direction angles.



SOLUTION

$$\mathbf{r} = (-5\cos 20^\circ \sin 30^\circ)\mathbf{i} + (8 - 5\cos 20^\circ \cos 30^\circ)\mathbf{j} + (2 + 5\sin 20^\circ)\mathbf{k}$$

$$\mathbf{r} = \{-2.35\mathbf{i} + 3.93\mathbf{j} + 3.71\mathbf{k}\}\text{ ft}$$ **Ans.**

$$r = \sqrt{(-2.35)^2 + (3.93)^2 + (3.71)^2} = 5.89\text{ ft}$$ **Ans.**

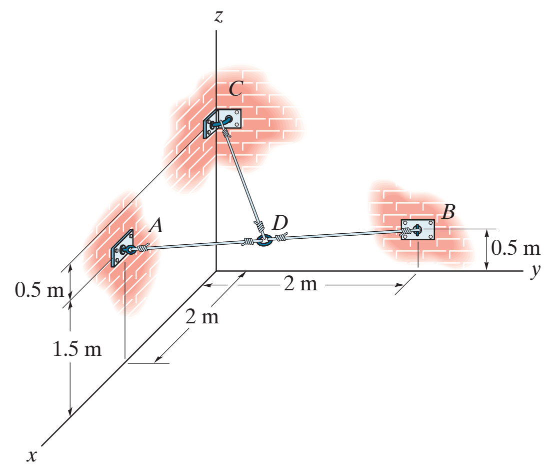
$$\alpha = \cos^{-1}\left(\frac{-2.35}{5.89}\right) = 113^\circ$$ **Ans.**

$$\beta = \cos^{-1}\left(\frac{3.93}{5.89}\right) = 48.2^\circ$$ **Ans.**

$$\gamma = \cos^{-1}\left(\frac{3.71}{5.89}\right) = 51.0^\circ$$ **Ans.**

2–87.

Determine the lengths of wires *AD*, *BD*, and *CD*. The ring at *D* is midway between *A* and *B*.



SOLUTION

$$D\left(\frac{2+0}{2}, \frac{0+2}{2}, \frac{1.5+0.5}{2}\right)\text{ m} = D(1, 1, 1)\text{ m}$$

$$\mathbf{r}_{AD} = (1-2)\mathbf{i} + (1-0)\mathbf{j} + (1-1.5)\mathbf{k}$$

$$= -1\mathbf{i} + 1\mathbf{j} - 0.5\mathbf{k}$$

$$\mathbf{r}_{BD} = (1-0)\mathbf{i} + (1-2)\mathbf{j} + (1-0.5)\mathbf{k}$$

$$= 1\mathbf{i} - 1\mathbf{j} + 0.5\mathbf{k}$$

$$\mathbf{r}_{CD} = (1-0)\mathbf{i} + (1-0)\mathbf{j} + (1-2)\mathbf{k}$$

$$= 1\mathbf{i} + 1\mathbf{j} - 1\mathbf{k}$$

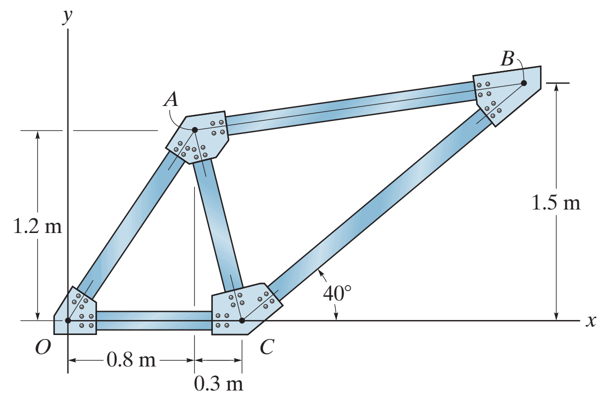
$$r_{AD} = \sqrt{(-1)^2 + 1^2 + (-0.5)^2} = 1.50\text{ m}$$ **Ans.**

$$r_{BD} = \sqrt{1^2 + (-1)^2 + 0.5^2} = 1.50\text{ m}$$ **Ans.**

$$r_{CD} = \sqrt{1^2 + 1^2 + (-1)^2} = 1.73\text{ m}$$ **Ans.**

***2–88.**

Determine the length of member AB of the truss by first establishing a Cartesian position vector from A to B and then determining its magnitude.



SOLUTION

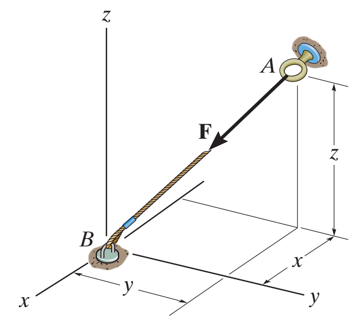
$$\mathbf{r}_{AB} = (1.1) = \frac{1.5}{\tan 40^\circ} - 0.80)\mathbf{i} + (1.5 - 1.2)\mathbf{j}$$

$$\mathbf{r}_{AB} = \{2.09\mathbf{i} + 0.3\mathbf{j}\}\text{ m}$$

$$\mathbf{r}_{AB} = \sqrt{(2.09)^2 + (0.3)^2} = 2.11\text{ m}$$ **Ans.**

2–89.

If $\mathbf{F} = \{350\mathbf{i} - 250\mathbf{j} - 450\mathbf{k}\}$ N and cable AB is 9 m long, determine the x, y, z coordinates of point A.



SOLUTION

Position Vector: The position vector $\mathbf{r}_{AB}$, directed from point A to point B, is given by

$$\mathbf{r}_{AB} = [0 - (-x)]\mathbf{i} + (0 - y)\mathbf{j} + (0 - z)\mathbf{k}$$

$$= x\mathbf{i} - y\mathbf{j} - z\mathbf{k}$$

Unit Vector: Knowing the magnitude of $\mathbf{r}_{AB}$ is 9 m, the unit vector for $\mathbf{r}_{AB}$ is given by

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{x\mathbf{i} - y\mathbf{j} - z\mathbf{k}}{9}$$

The unit vector for force $\mathbf{F}$ is

$$\mathbf{u}_F = \frac{\mathbf{F}}{F} = \frac{350\mathbf{i} - 250\mathbf{j} - 450\mathbf{k}}{\sqrt{350^2 + (-250)^2 + (-450)^2}} = 0.5623\mathbf{i} - 0.4016\mathbf{j} - 0.7229\mathbf{k}$$

Since force $\mathbf{F}$ is also directed from point A to point B, then

$$\mathbf{u}_{AB} = \mathbf{u}_F$$

$$\frac{x\mathbf{i} - y\mathbf{j} - z\mathbf{k}}{9} = 0.5623\mathbf{i} - 0.4016\mathbf{j} - 0.7229\mathbf{k}$$

Equating the $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ components,

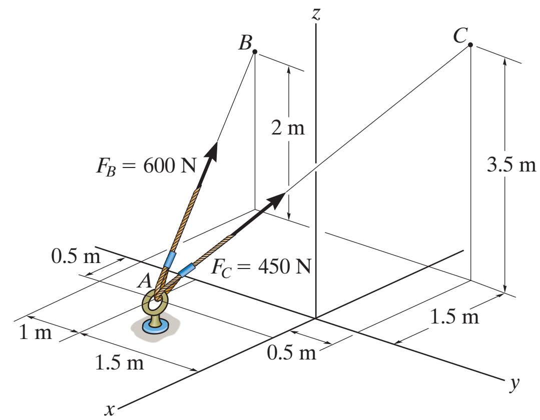
$$\frac{x}{9} = 0.5623 \qquad x = 5.06 \text{ m}$$ **Ans.**

$$\frac{-y}{9} = -0.4016 \qquad y = 3.61 \text{ m}$$ **Ans.**

$$\frac{-z}{9} = 0.7229 \qquad z = 6.51 \text{ m}$$ **Ans.**

2–90.

Express $\mathbf{F}_B$ and $\mathbf{F}_C$ in Cartesian vector form.



SOLUTION

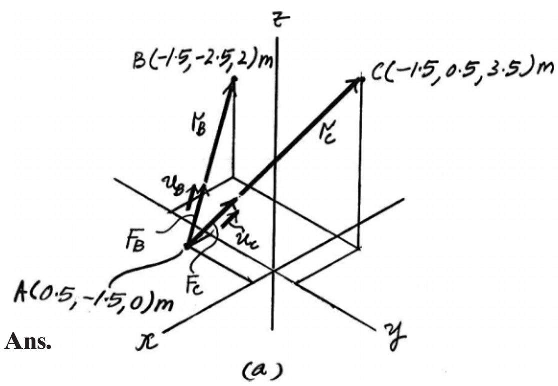
Force Vectors: The unit vectors $\mathbf{u}_B$ and $\mathbf{u}_C$ of $\mathbf{F}_B$ and $\mathbf{F}_C$ must be determined first. From Fig. *a*

$$\mathbf{u}_B = \frac{\mathbf{r}_B}{r_B} = \frac{(-1.5 - 0.5)\mathbf{i} + [-2.5 - (-1.5)]\mathbf{j} + (2 - 0)\mathbf{k}}{\sqrt{(-1.5 - 0.5)^2 + [-2.5 - (-1.5)]^2 + (2 - 0)^2}}$$

$$= -\frac{2}{3}\mathbf{i} - \frac{1}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}$$

$$\mathbf{u}_C = \frac{\mathbf{r}_C}{r_C} = \frac{(-1.5 - 0.5)\mathbf{i} + [0.5 - (-1.5)]\mathbf{j} + (3.5 - 0)\mathbf{k}}{\sqrt{(-1.5 - 0.5)^2 + [0.5 - (-1.5)]^2 + (3.5 - 0)^2}}$$

$$= -\frac{4}{9}\mathbf{i} + \frac{4}{9}\mathbf{j} + \frac{7}{9}\mathbf{k}$$



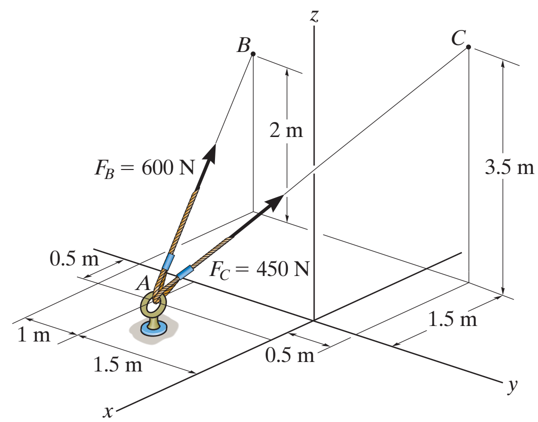
Thus, the force vectors $\mathbf{F}_B$ and $\mathbf{F}_C$ are given by

$$\mathbf{F}_B = F_B\mathbf{u}_B = 600\left(-\frac{2}{3}\mathbf{i} - \frac{1}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}\right) = \{-400\mathbf{i} - 200\mathbf{j} + 400\mathbf{k}\}\text{ N}$$ **Ans.**

$$\mathbf{F}_C = F_C\mathbf{u}_C = 450\left(-\frac{4}{9}\mathbf{i} + \frac{4}{9}\mathbf{j} + \frac{7}{9}\mathbf{k}\right) = \{-200\mathbf{i} + 200\mathbf{j} + 350\mathbf{k}\}\text{ N}$$ **Ans.**

2–91.

Determine the magnitude and coordinate direction angles of the resultant force acting at A.



SOLUTION

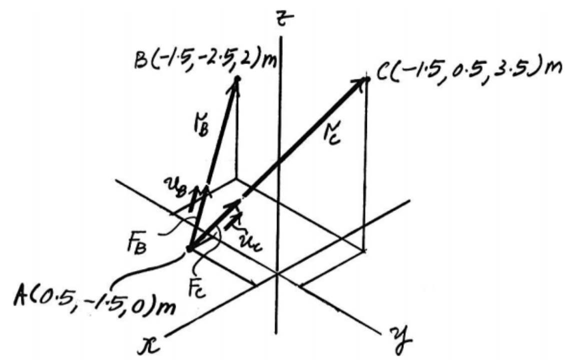
Force Vectors: The unit vectors $\mathbf{u}_B$ and $\mathbf{u}_C$ of $\mathbf{F}_B$ and $\mathbf{F}_C$ must be determined first. From Fig. *a*

$$\mathbf{u}_B = \frac{\mathbf{r}_B}{r_B} = \frac{(-1.5 - 0.5)\mathbf{i} + [-2.5 - (-1.5)]\mathbf{j} + (2 - 0)\mathbf{k}}{\sqrt{(-1.5 - 0.5)^2 + [-2.5 - (-1.5)]^2 + (2 - 0)^2}}$$

$$= -\frac{2}{3}\mathbf{i} - \frac{1}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}$$

$$\mathbf{u}_C = \frac{\mathbf{r}_C}{r_C} = \frac{(-1.5 - 0.5)\mathbf{i} + [0.5 - (-1.5)]\mathbf{j} + (3.5 - 0)\mathbf{k}}{\sqrt{(-1.5 - 0.5)^2 + [0.5 - (-1.5)]^2 + (3.5 - 0)^2}}$$

$$= -\frac{4}{9}\mathbf{i} + \frac{4}{9}\mathbf{j} + \frac{7}{9}\mathbf{k}$$



(a)

Thus, the force vectors $\mathbf{F}_B$ and $\mathbf{F}_C$ are given by

$$\mathbf{F}_B = F_B\mathbf{u}_B = 600\left(-\frac{2}{3}\mathbf{i} - \frac{1}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}\right) = \{-400\mathbf{i} - 200\mathbf{j} + 400\mathbf{k}\}\text{ N}$$

$$\mathbf{F}_C = F_C\mathbf{u}_C = 450\left(-\frac{4}{9}\mathbf{i} + \frac{4}{9}\mathbf{j} + \frac{7}{9}\mathbf{k}\right) = \{-200\mathbf{i} + 200\mathbf{j} + 350\mathbf{k}\}\text{ N}$$

Resultant Force:

$$\mathbf{F}_R = \mathbf{F}_B + \mathbf{F}_C = (-400\mathbf{i} - 200\mathbf{j} + 400\mathbf{k}) + (-200\mathbf{i} + 200\mathbf{j} + 350\mathbf{k})$$

$$= \{-600\mathbf{i} + 750\mathbf{k}\}\text{ N}$$

The magnitude of $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x{}^2 + (F_R)_y{}^2 + (F_R)_z{}^2}$$

$$= \sqrt{(-600)^2 + 0^2 + 750^2} = 960.47\text{ N} = 960\text{ N}$$

The coordinate direction angles of $\mathbf{F}_R$ are

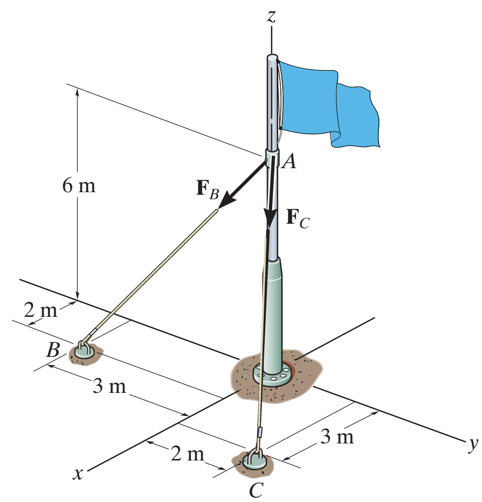
$$\alpha = \cos^{-1}\left[\frac{(F_R)_x}{F_R}\right] = \cos^{-1}\left(\frac{-600}{960.47}\right) = 129°$$ **Ans.**

$$\beta = \cos^{-1}\left[\frac{(F_R)_y}{F_R}\right] = \cos^{-1}\left(\frac{0}{960.47}\right) = 90°$$ **Ans.**

$$\gamma = \cos^{-1}\left[\frac{(F_R)_z}{F_R}\right] = \cos^{-1}\left(\frac{760}{960.47}\right) = 38.7°$$ **Ans.**

***2–92.**

If $F_B = 560$ N and $F_C = 700$ N, determine the magnitude and coordinate direction angles of the resultant force acting on the flag pole.



SOLUTION

Force Vectors: The unit vectors $\mathbf{u}_B$ and $\mathbf{u}_C$ of $\mathbf{F}_B$ and $\mathbf{F}_C$ must be determined first. From Fig. *a*

$$\mathbf{u}_B = \frac{\mathbf{r}_B}{r_B} = \frac{(2-0)\mathbf{i} + (-3-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(2-0)^2 + (-3-0)^2 + (0-6)^2}} = \frac{2}{7}\mathbf{i} - \frac{3}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$$

$$\mathbf{u}_C = \frac{\mathbf{r}_C}{r_C} = \frac{(3-0)\mathbf{i} + (2-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(3-0)^2 + (2-0)^2 + (0-6)^2}} = \frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$$

Thus, the force vectors $\mathbf{F}_B$ and $\mathbf{F}_C$ are given by

$$\mathbf{F}_B = F_B\mathbf{u}_B = 560\left(\frac{2}{7}\mathbf{i} - \frac{3}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right) = \{160\mathbf{i} - 240\mathbf{j} - 480\mathbf{k}\}\text{ N}$$

$$\mathbf{F}_C = F_C\mathbf{u}_C = 700\left(\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right) = \{300\mathbf{i} + 200\mathbf{j} - 600\mathbf{k}\}\text{ N}$$

Resultant Force:

$$\mathbf{F}_R = \mathbf{F}_B + \mathbf{F}_C = (160\mathbf{i} - 240\mathbf{j} - 480\mathbf{k}) + (300\mathbf{i} + 200\mathbf{j} - 600\mathbf{k})$$

$$= \{460\mathbf{i} - 40\mathbf{j} + 1080\mathbf{k}\}\text{ N}$$

The magnitude of $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x{}^2 + (F_R)_y{}^2 + (F_R)_z{}^2}$$

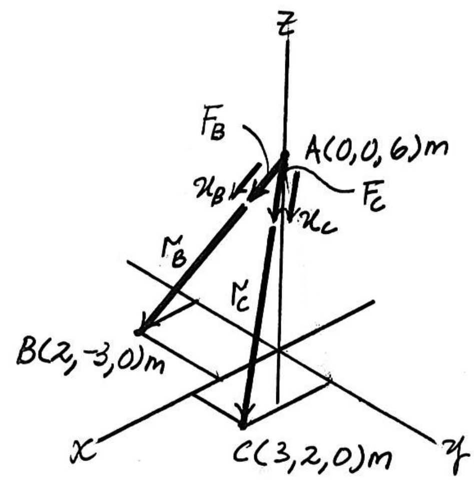
$$= \sqrt{(460)^2 + (-40)^2 + (-1080)^2} = 1174.56\text{ N} = 1.17\text{ kN}$$ **Ans.**

The coordinate direction angles of $\mathbf{F}_R$ are

$$\alpha = \cos^{-1}\left[\frac{(F_R)_x}{F_R}\right] = \cos^{-1}\left(\frac{460}{1174.56}\right) = 66.9°$$ **Ans.**

$$\beta = \cos^{-1}\left[\frac{(F_R)_y}{F_R}\right] = \cos^{-1}\left(\frac{-40}{1174.56}\right) = 92.0°$$ **Ans.**

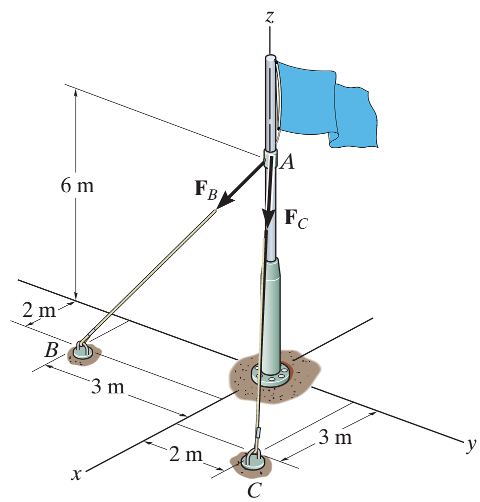
$$\gamma = \cos^{-1}\left[\frac{(F_R)_z}{F_R}\right] = \cos^{-1}\left(\frac{-1080}{1174.56}\right) = 157°$$ **Ans.**



(a)

2–93.

If $F_B = 700$ N, and $F_C = 560$ N, determine the magnitude and coordinate direction angles of the resultant force acting on the flag pole.



SOLUTION

Force Vectors: The unit vectors $\mathbf{u}_B$ and $\mathbf{u}_C$ of $\mathbf{F}_B$ and $\mathbf{F}_C$ must be determined first. From Fig. *a*

$$\mathbf{u}_B = \frac{\mathbf{r}_B}{r_B} = \frac{(2-0)\mathbf{i} + (-3-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(2-0)^2 + (-3-0)^2 + (0-6)^2}} = \frac{2}{7}\mathbf{i} - \frac{3}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$$

$$\mathbf{u}_C = \frac{\mathbf{r}_C}{r_C} = \frac{(3-0)\mathbf{i} + (2-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(3-0)^2 + (2-0)^2 + (0-6)^2}} = \frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$$

Thus, the force vectors $\mathbf{F}_B$ and $\mathbf{F}_C$ are given by

$$\mathbf{F}_B = F_B\mathbf{u}_B = 700\left(\frac{2}{7}\mathbf{i} - \frac{3}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right) = \{200\mathbf{i} - 300\mathbf{j} - 600\mathbf{k}\}\text{ N}$$

$$\mathbf{F}_C = F_C\mathbf{u}_C = 560\left(\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right) = \{240\mathbf{i} + 160\mathbf{j} - 480\mathbf{k}\}\text{ N}$$

Resultant Force:

$$\mathbf{F}_R = \mathbf{F}_B + \mathbf{F}_C = (200\mathbf{i} - 300\mathbf{j} - 600\mathbf{k}) + (240\mathbf{i} + 160\mathbf{j} - 480\mathbf{k})$$

$$= \{440\mathbf{i} - 140\mathbf{j} - 1080\mathbf{k}\}\text{ N}$$

The magnitude of $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x{}^2 + (F_R)_y{}^2 + (F_R)_z{}^2}$$

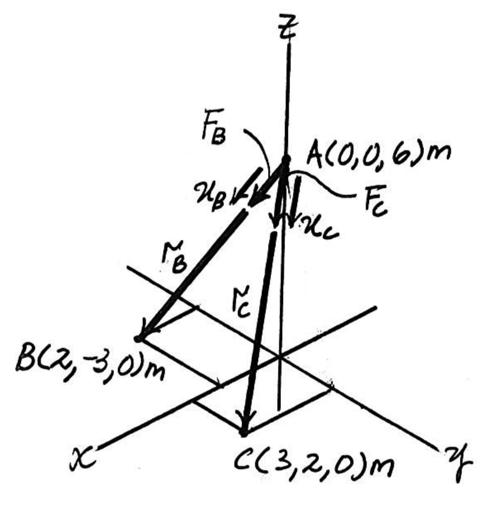
$$= \sqrt{(440)^2 + (-140)^2 + (-1080)^2} = 1174.56\text{ N} = 1.17\text{ kN}$$ **Ans.**

The coordinate direction angles of $\mathbf{F}_R$ are

$$\alpha = \cos^{-1}\left[\frac{(F_R)_x}{F_R}\right] = \cos^{-1}\left(\frac{440}{1174.56}\right) = 68.0°$$ **Ans.**

$$\beta = \cos^{-1}\left[\frac{(F_R)_y}{F_R}\right] = \cos^{-1}\left(\frac{-140}{1174.56}\right) = 96.8°$$ **Ans.**

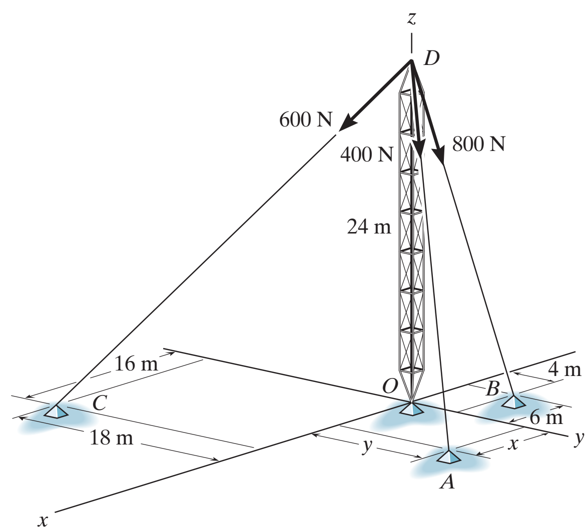
$$\gamma = \cos^{-1}\left[\frac{(F_R)_z}{F_R}\right] = \cos^{-1}\left(\frac{-1080}{1174.56}\right) = 157°$$ **Ans.**



(a)

2–94.

The tower is held in place by three cables. If the force of each cable acting on the tower is shown, determine the magnitude and coordinate direction angles α, β, γ of the resultant force. Take $x = 20$ m, $y = 15$ m.



SOLUTION

$$\mathbf{F}_{DA} = 400\left(\frac{20}{34.66}\mathbf{i} + \frac{15}{34.66}\mathbf{j} - \frac{24}{34.66}\mathbf{k}\right) \text{N}$$

$$\mathbf{F}_{DB} = 800\left(\frac{-6}{25.06}\mathbf{i} + \frac{4}{25.06}\mathbf{j} - \frac{24}{25.06}\mathbf{k}\right) \text{N}$$

$$\mathbf{F}_{DC} = 600\left(\frac{16}{34}\mathbf{i} - \frac{18}{34}\mathbf{j} - \frac{24}{34}\mathbf{k}\right) \text{N}$$

$$\mathbf{F}_R = \mathbf{F}_{DA} + \mathbf{F}_{DB} + \mathbf{F}_{DC}$$

$$= \{321.66\mathbf{i} - 16.82\mathbf{j} - 1466.71\mathbf{k}\} \text{ N}$$

$$F_R = \sqrt{(321.66)^2 + (-16.82)^2 + (-1466.71)^2}$$

$$= 1501.66 \text{ N} = 1.50 \text{ kN}$$ **Ans.**

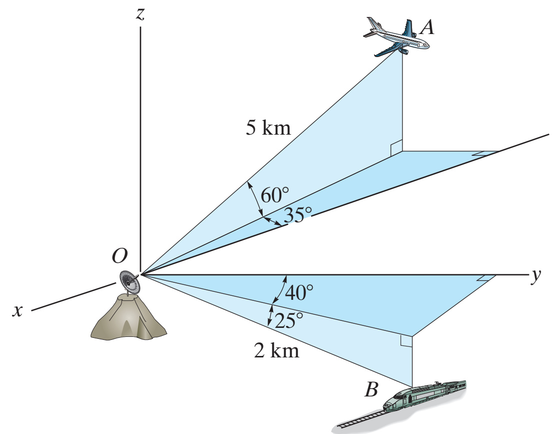
$$\alpha = \cos^{-1}\left(\frac{321.66}{1501.66}\right) = 77.6^\circ$$ **Ans.**

$$\beta = \cos^{-1}\left(\frac{-16.82}{1501.66}\right) = 90.6^\circ$$ **Ans.**

$$\gamma = \cos^{-1}\left(\frac{-1466.71}{1501.66}\right) = 168^\circ$$ **Ans.**

2–95.

At a given instant, the position of a plane at A and a train at B are measured relative to a radar antenna at O. Determine the distance d between A and B at this instant. To solve the problem, formulate a position vector, directed from A to B, and then determine its magnitude.



SOLUTION

Position Vector: The coordinates of points A and B are

$$A(-5\cos 60^\circ \cos 35^\circ, -5\cos 60^\circ \sin 35^\circ, 5\sin 60^\circ)\text{ km}$$

$$= A(-2.048, -1.434, 4.330)\text{ km}$$

$$B(2\cos 25^\circ \sin 40^\circ, 2\cos 25^\circ \cos 40^\circ, -2\sin 25^\circ)\text{ km}$$

$$= B(1.165, 1.389, -0.845)\text{ km}$$

The position vector $\mathbf{r}_{AB}$ can be established from the coordinates of points A and B.

$$\mathbf{r}_{AB} = \{[1.165 - (-2.048)]\mathbf{i} + [1.389 - (-1.434)]\mathbf{j} + (-0.845 - 4.330)\mathbf{k}\}\text{ km}$$

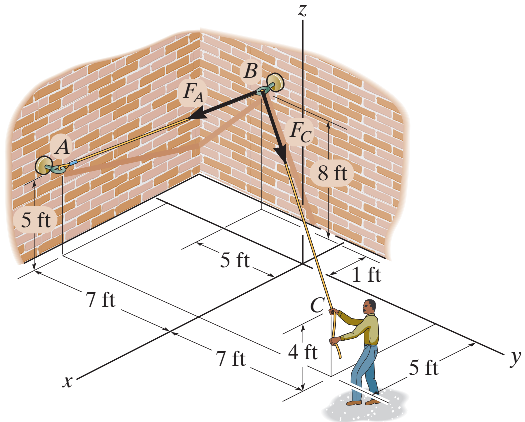
$$= \{3.213\mathbf{i} + 2.822\mathbf{j} - 5.175)\mathbf{k}\}\text{ km}$$

The distance between points A and B is

$$d = r_{AB} = \sqrt{3.213^2 + 2.822^2 + (-5.175)^2} = 6.71\text{ km}$$ **Ans.**

***2–96.**

The man pulls on the rope at C with a force of 70 lb which causes the forces $\mathbf{F}_A$ and $\mathbf{F}_C$ at B to have this same magnitude. Express each of these two forces as Cartesian vectors.



SOLUTION

Unit Vectors: The coordinate points A, B, and C are shown in Fig. a. Thus,

$$\mathbf{u}_A = \frac{\mathbf{r}_A}{r_A} = \frac{[5-(-1)]\mathbf{i} + [-7-(-5)]\mathbf{j} + (5-8)\mathbf{k}}{\sqrt{[5-(-1)]^2 + [-7-(-5)]^2 + (5-8)^2}}$$

$$= \frac{6}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} + \frac{3}{7}\mathbf{k}$$

$$\mathbf{u}_C = \frac{\mathbf{r}_C}{r_C} = \frac{[5-(-1)]\mathbf{i} + [-7(-5)]\mathbf{j} + (4-8)\mathbf{k}}{\sqrt{[5-(-1)]^2 + [-7(-5)]^2 + (4-8)^2}}$$

$$= \frac{3}{7}\mathbf{i} + \frac{6}{7}\mathbf{j} + \frac{2}{7}\mathbf{k}$$

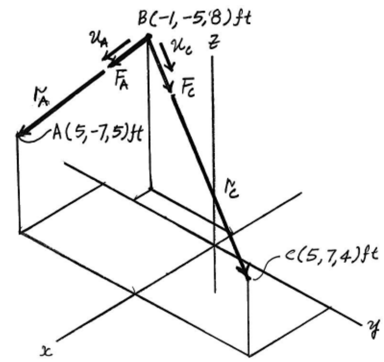
Force Vectors: Multiplying the magnitude of the force with its unit vector,

$$\mathbf{F}_A = F_A\mathbf{u}_A = 70\left(\frac{6}{7}\mathbf{i} - \frac{2}{7}\mathbf{j} + \frac{3}{7}\mathbf{k}\right)$$

$$= \{60\mathbf{i} - 20\mathbf{j} + 30\mathbf{k}\}\text{ lb}$$ **Ans.**

$$\mathbf{F}_C = F_C\mathbf{u}_C = 70\left(\frac{3}{7}\mathbf{i} + \frac{6}{7}\mathbf{j} + \frac{2}{7}\mathbf{k}\right)$$

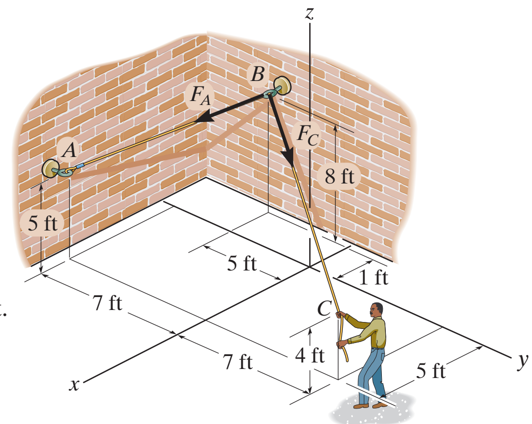
$$= \{30\mathbf{i} + 60\mathbf{j} + 20\mathbf{k}\}\text{ lb}$$ **Ans.**



(a)

2–97.

The man pulls on the rope at C with a force of 70 lb which causes the forces $\mathbf{F}_A$ and $\mathbf{F}_C$ at B to have this same magnitude. Determine the magnitude and coordinate direction angles of the resultant force acting at B.



SOLUTION

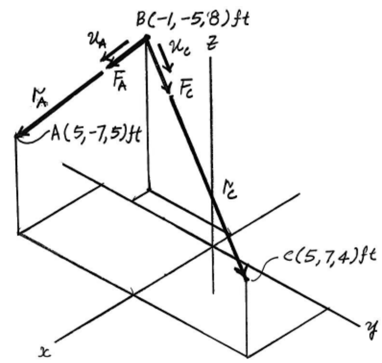
Force Vectors: The unit vectors $\mathbf{u}_B$ and $\mathbf{u}_C$ of $\mathbf{F}_B$ and $\mathbf{F}_C$ must be determined first. From Fig. *a*

$$\mathbf{u}_A = \frac{\mathbf{r}_A}{r_A} = \frac{[5-(-1)]\mathbf{i} + [-7(-5)]\mathbf{j} + (5-8)\mathbf{k}}{\sqrt{[5-(-1)]^2 + [-7(-5)]^2 + (5-8)^2}}$$

$$= \frac{6}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} + \frac{3}{7}\mathbf{k}$$

$$\mathbf{u}_C = \frac{\mathbf{r}_C}{r_C} = \frac{[5-(-1)]\mathbf{i} + [-7(-5)]\mathbf{j} + (4-8)\mathbf{k}}{\sqrt{[5-(-1)]^2 + [-7(-5)]^2 + (4-8)^2}}$$

$$= \frac{3}{7}\mathbf{i} + \frac{6}{7}\mathbf{j} + \frac{2}{7}\mathbf{k}$$



(a)

Thus, the force vectors $\mathbf{F}_B$ and $\mathbf{F}_C$ are given by

$$\mathbf{F}_A = F_A\mathbf{u}_A = 70\left(\frac{6}{7}\mathbf{i} - \frac{2}{7}\mathbf{j} + \frac{3}{7}\mathbf{k}\right) = \{60\mathbf{i} - 20\mathbf{j} + 30\mathbf{k}\}\text{ lb}$$

$$\mathbf{F}_C = F_C\mathbf{u}_C = 70\left(\frac{3}{7}\mathbf{i} + \frac{6}{7}\mathbf{j} + \frac{2}{7}\mathbf{k}\right) = \{30\mathbf{i} + 60\mathbf{j} + 20\mathbf{k}\}\text{ lb}$$

Resultant Force:

$$\mathbf{F}_R = \mathbf{F}_A + \mathbf{F}_C = (60\mathbf{i} - 20\mathbf{j} - 30\mathbf{k}) + (30\mathbf{i} + 60\mathbf{j} - 20\mathbf{k})$$

$$= \{90\mathbf{i} + 40\mathbf{j} - 50\mathbf{k}\}\text{ lb}$$

The magnitude of $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x{}^2 + (F_R)_y{}^2 + (F_R)_z{}^2}$$

$$= \sqrt{(90)^2 + (40)^2 + (-50)^2} = 110.45\text{ lb} = 110\text{ lb}$$ **Ans.**

The coordinate direction angles of $\mathbf{F}_R$ are

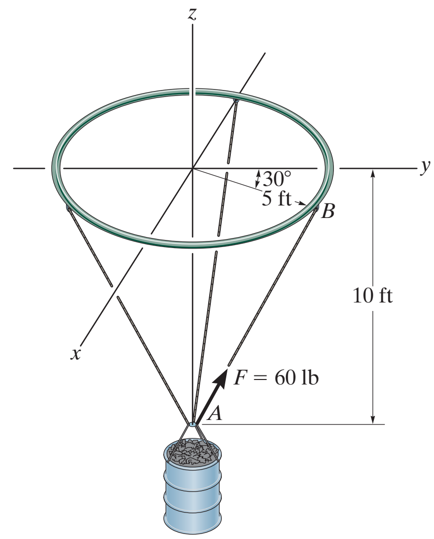
$$\alpha = \cos^{-1}\left[\frac{(F_R)_x}{F_R}\right] = \cos^{-1}\left(\frac{90}{110.45}\right) = 35.4°$$ **Ans.**

$$\beta = \cos^{-1}\left[\frac{(F_R)_y}{F_R}\right] = \cos^{-1}\left(\frac{40}{110.45}\right) = 68.8°$$ **Ans.**

$$\gamma = \cos^{-1}\left[\frac{(F_R)_z}{F_R}\right] = \cos^{-1}\left(\frac{-50}{110.45}\right) = 117°$$ **Ans.**

2–98.

The load at A creates a force of 60 lb in wire AB. Express this force as a Cartesian vector acting on A and directed toward B as shown.



SOLUTION

Unit Vector: First determine the position vector $\mathbf{r}_{AB}$. The coordinates of point B are

$B\,(5\sin 30°, 5\cos 30°, 0)\text{ ft} = B\,(2.50, 4.330, 0)\text{ ft}$

Then

$$\mathbf{r}_{AB} = \{(2.50 - 0)\mathbf{i} + (4.330 - 0)\mathbf{j} + [0 - (-10)]\mathbf{k}\}\text{ ft}$$

$$= \{2.50\mathbf{i} + 4.330\mathbf{j} + 10\mathbf{k}\}\text{ ft}$$

$$r_{AB} = \sqrt{2.50^2 + 4.330^2 + 10.0^2} = 11.180\text{ ft}$$

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{2.50\mathbf{i} + 4.330\mathbf{j} + 10\mathbf{k}}{11.180}$$

$$= 0.2236\mathbf{i} + 0.3873\mathbf{j} + 0.8944\mathbf{k}$$

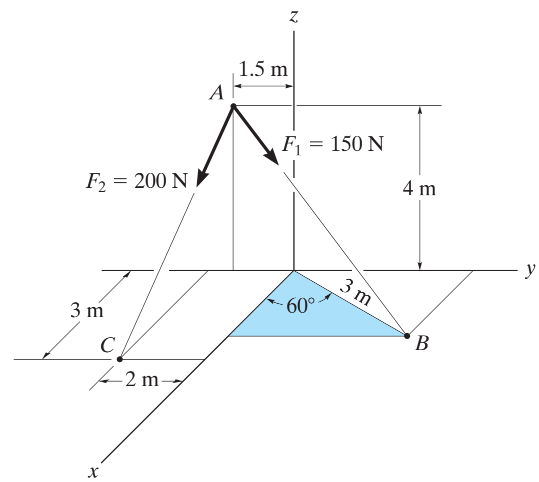
Force Vector:

$$\mathbf{F} = F\mathbf{u}_{AB} = 60\,\{0.2236\mathbf{i} + 0.3873\mathbf{j} + 0.8944\mathbf{k}\}\text{ lb}$$

$$= \{13.4\mathbf{i} + 23.2\mathbf{j} + 53.7\mathbf{k}\}\text{ lb}$$ **Ans.**

2–99.

Determine the magnitude and coordinate direction angles of the resultant force acting at point A.



SOLUTION

$$\mathbf{r}_{AC} = \{3\mathbf{i} - 0.5\mathbf{j} - 4\mathbf{k}\}\text{ m}$$

$$|\mathbf{r}_{AC}| = \sqrt{3^2 + (-0.5)^2 + (-4)^2} = \sqrt{25.25} = 5.02494$$

$$\mathbf{F}_2 = 200\left(\frac{3\mathbf{i} - 0.5\mathbf{j} - 4\mathbf{k}}{5.02494}\right) = (119.4044\mathbf{i} - 19.9007\mathbf{j} - 159.2059\mathbf{k})$$

$$\mathbf{r}_{AB} = (3\cos 60°\mathbf{i} + (1.5 + 3\sin 60°)\mathbf{j} - 4\mathbf{k})$$

$$\mathbf{r}_{AB} = (1.5\mathbf{i} + 4.0981\mathbf{j} + 4\mathbf{k})$$

$$|\mathbf{r}_{AB}| = \sqrt{(1.5)^2 + (4.0981)^2 + (-4)^2} = 5.9198$$

$$\mathbf{F}_1 = 150\left(\frac{1.5\mathbf{i} + 4.0981\mathbf{j} - 4\mathbf{k}}{5.9198}\right) = (38.0079\mathbf{i} + 103.8396\mathbf{j} - 101.3545\mathbf{k})$$

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 = (157.4124\mathbf{i} + 83.9389\mathbf{j} - 260.5607\mathbf{k})$$

$$F_R = \sqrt{(157.4124)^2 + (83.9389)^2 + (-260.5604)^2} = 315.7786 = 316\text{ N}$$ **Ans.**

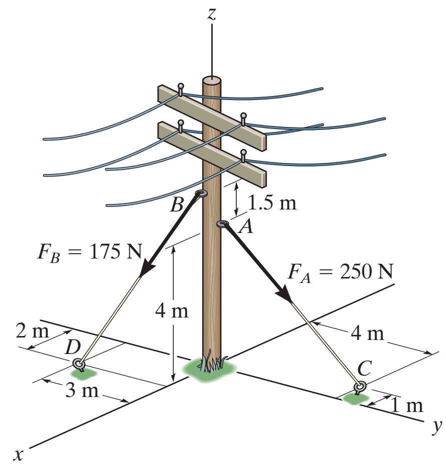
$$\alpha = \cos^{-1}\left(\frac{157.4124}{315.7786}\right) = 60.100° = 60.1°$$ **Ans.**

$$\beta = \cos^{-1}\left(\frac{83.9389}{315.7786}\right) = 74.585° = 74.6°$$ **Ans.**

$$\gamma = \cos^{-1}\left(\frac{-260.5607}{315.7786}\right) = 145.60° = 146°$$ **Ans.**

***2–100.**

The guy wires are used to support the telephone pole. Represent the force in each wire in Cartesian vector form. Neglect the diameter of the pole.



SOLUTION

Unit Vector:

$$\mathbf{r}_{AC} = \{(-1 - 0)\mathbf{i} + (4 - 0)\mathbf{j} + (0 - 4)\mathbf{k}\}\text{ m} = \{-1\mathbf{i} + 4\mathbf{j} - 4\mathbf{k}\}\text{ m}$$

$$r_{AC} = \sqrt{(-1)^2 + 4^2 + (-4)^2} = 5.745\text{ m}$$

$$\mathbf{u}_{AC} = \frac{\mathbf{r}_{AC}}{r_{AC}} = \frac{-1\mathbf{i} + 4\mathbf{j} - 4\mathbf{k}}{5.745} = -0.1741\mathbf{i} + 0.6963\mathbf{j} - 0.6963\mathbf{k}$$

$$\mathbf{r}_{BD} = \{(2 - 0)\,\mathbf{i} + (-3 - 0)\mathbf{j} + (0 - 5.5)\mathbf{k}\}\text{ m} = \{2\mathbf{i} - 3\mathbf{j} - 5.5\mathbf{k}\}\text{ m}$$

$$r_{BD} = \sqrt{2^2 + (-3)^2 + (-5.5)^2} = 6.576\text{ m}$$

$$\mathbf{u}_{BD} = \frac{\mathbf{r}_{BD}}{r_{BD}} = \frac{2\mathbf{i} - 3\mathbf{j} - 5.5\mathbf{k}}{6.576} = 0.3041\mathbf{i} - 0.4562\mathbf{j} - 0.8363\mathbf{k}$$

Force Vector:

$$\mathbf{F}_A = F_A\,\mathbf{u}_{AC} = 250\{-0.1741\mathbf{i} + 0.6963\mathbf{j} - 0.6963\mathbf{k}\}\text{ N}$$

$$= \{-43.52\mathbf{i} + 174.08\mathbf{j} - 174.08\mathbf{k}\}\text{ N}$$

$$= \{-43.5\mathbf{i} + 174\mathbf{j} - 174\mathbf{k}\}\text{ N}$$ **Ans.**

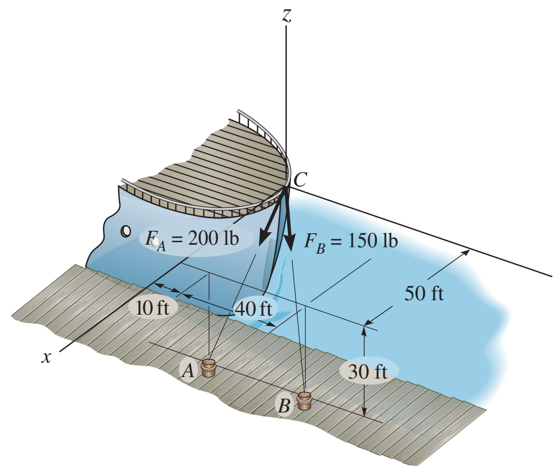
$$\mathbf{F}_B = F_B\mathbf{u}_{BD} = 175\{0.3041\mathbf{i} + 0.4562\mathbf{j} - 0.8363\mathbf{k}\}\text{ N}$$

$$= \{53.22\mathbf{i} - 79.83\mathbf{j} - 146.36\mathbf{k}\}\text{ N}$$

$$= \{53.2\mathbf{i} - 79.8\mathbf{j} - 146\mathbf{k}\}\text{ N}$$ **Ans.**

2–101.

The two mooring cables exert forces on the stern of a ship as shown. Represent each force as a Cartesian vector and determine the magnitude and coordinate direction angles of the resultant.



SOLUTION

Unit Vector:

$$\mathbf{r}_{CA} = \{(50-0)\mathbf{i} + (10-0)\mathbf{j} + (-30-0)\mathbf{k}\}\text{ ft} = \{50\mathbf{i} + 10\mathbf{j} - 30\mathbf{k}\}\text{ ft}$$

$$r_{CA} = \sqrt{50^2 + 10^2 + (-30)^2} = 59.16\text{ ft}$$

$$\mathbf{u}_{CA} = \frac{\mathbf{r}_{CA}}{r_{CA}} = \frac{50\mathbf{i} + 10\mathbf{j} - 30\mathbf{k}}{59.16} = 0.8452\mathbf{i} + 0.1690\mathbf{j} - 0.5071\mathbf{k}$$

$$\mathbf{r}_{CB} = \{(50-0)\mathbf{i} + (50-0)\mathbf{j} + (-30-0)\mathbf{k}\}\text{ ft} = \{50\mathbf{i} + 50\mathbf{j} - 30\mathbf{k}\}\text{ ft}$$

$$r_{CB} = \sqrt{50^2 + 50^2 + (-30)^2} = 76.81\text{ ft}$$

$$\mathbf{u}_{CB} = \frac{\mathbf{r}_{CA}}{r_{CA}} = \frac{50\mathbf{i} + 50\mathbf{j} - 30\mathbf{k}}{76.81} = 0.6509\mathbf{i} + 0.6509\mathbf{j} - 0.3906\mathbf{k}$$

Force Vector:

$$\mathbf{F}_A = F_A\mathbf{u}_{CA} = 200\{0.8452\mathbf{i} + 0.1690\mathbf{j} - 0.5071\mathbf{k}\}\text{ lb}$$

$$= \{169.03\mathbf{i} + 33.81\mathbf{j} - 101.42\mathbf{k}\}\text{ lb}$$

$$= \{169\mathbf{i} + 33.8\mathbf{j} - 101\mathbf{k}\}\text{ lb}$$ **Ans.**

$$\mathbf{F}_B = F_B\mathbf{u}_{CB} = 150\{0.6509\mathbf{i} + 0.6509\mathbf{j} - 0.3906\mathbf{k}\}\text{ lb}$$

$$= \{97.64\mathbf{i} + 97.64\mathbf{j} - 58.59\mathbf{k}\}\text{ lb}$$

$$= \{97.6\mathbf{i} + 97.6\mathbf{j} - 58.6\mathbf{k}\}\text{ lb}$$ **Ans.**

Resultant Force:

$$\mathbf{F}_R = \mathbf{F}_A + \mathbf{F}_B$$

$$= \{(169.03 + 97.64)\mathbf{i} + (33.81 + 97.64)\mathbf{j} + (-101.42 - 58.59)\mathbf{k}\}\text{ lb}$$

$$= \{266.67\mathbf{i} + 131.45\mathbf{j} - 160.00\mathbf{k}\}\text{ lb}$$

The magnitude of $\mathbf{F}_R$ is

$$F_R = \sqrt{266.67^2 + 131.45^2 + (-160.00)^2}$$

$$= 337.63\text{ lb} = 338\text{ lb}$$ **Ans.**

The coordinate direction angles of $\mathbf{F}_R$ are

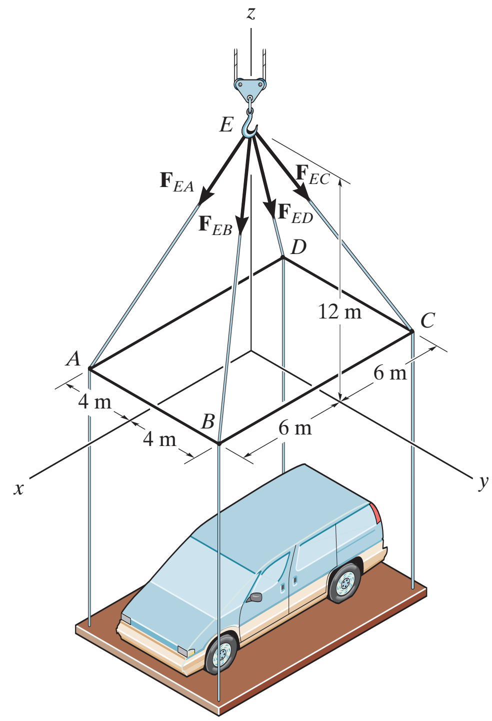
$$\cos\alpha = \frac{266.67}{337.63} \qquad \alpha = 37.8°$$ **Ans.**

$$\cos\beta = \frac{131.45}{337.63} \qquad \beta = 67.1°$$ **Ans.**

$$\cos\gamma = -\frac{160.00}{337.63} \qquad \gamma = 118°$$ **Ans.**

2–102.

Each of the four forces acting at E has a magnitude of 28 kN. Express each force as a Cartesian vector and determine the resultant force.



SOLUTION

$$F_{EA} = 28\left(\frac{6}{14}\mathbf{i} - \frac{4}{14}\mathbf{j} - \frac{12}{14}\mathbf{k}\right)$$

$$F_{EA} = \{12\mathbf{i} - 8\mathbf{j} - 24\mathbf{k}\}\ \text{kN}$$ **Ans.**

$$F_{EB} = 28\left(\frac{6}{14}\mathbf{i} + \frac{4}{14}\mathbf{j} - \frac{12}{14}\mathbf{k}\right)$$

$$F_{EB} = \{12\mathbf{i} + 8\mathbf{j} - 24\mathbf{k}\}\ \text{kN}$$ **Ans.**

$$F_{EC} = 28\left(\frac{-6}{14}\mathbf{i} + \frac{4}{14}\mathbf{j} - \frac{12}{14}\mathbf{k}\right)$$

$$F_{EC} = \{-12\mathbf{i} + 8\mathbf{j} - 24\mathbf{k}\}\ \text{kN}$$ **Ans.**

$$F_{ED} = 28\left(\frac{-6}{14}\mathbf{i} - \frac{4}{14}\mathbf{j} - \frac{12}{14}\mathbf{k}\right)$$

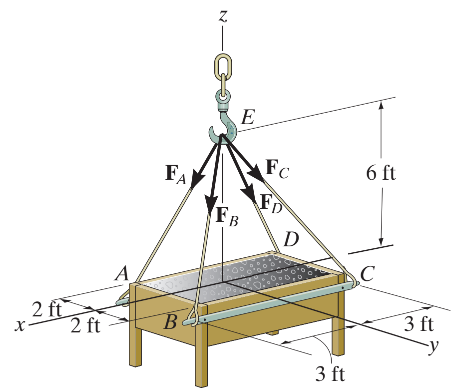
$$F_{ED} = \{-12\mathbf{i} - 8\mathbf{j} - 24\mathbf{k}\}\ \text{kN}$$ **Ans.**

$$\mathbf{F}_R = \mathbf{F}_{EA} + \mathbf{F}_{EB} + \mathbf{F}_{EC} + \mathbf{F}_{ED}$$

$$= \{-96\mathbf{k}\}\ \text{kN}$$ **Ans.**

2–103.

If the force in each cable tied to the bin is 70 lb, determine the magnitude and coordinate direction angles of the resultant force.



SOLUTION

Force Vectors: The unit vectors $\mathbf{u}_A$, $\mathbf{u}_B$, $\mathbf{u}_C$, and $\mathbf{u}_D$ of $\mathbf{F}_A$, $\mathbf{F}_B$, $\mathbf{F}_C$, and $\mathbf{F}_D$ must be determined first. From Fig. *a*,

$$\mathbf{u}_A = \frac{\mathbf{r}_A}{r_A} = \frac{(3-0)\mathbf{i} + (-2-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(3-0)^2 + (-2-0)^2 + (0-6)^2}} = \frac{3}{7}\mathbf{i} - \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$$

$$\mathbf{u}_B = \frac{\mathbf{r}_B}{r_B} = \frac{(3-0)\mathbf{i} + (2-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(3-0)^2 + (2-0)^2 + (0-6)^2}} = \frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$$

$$\mathbf{u}_C = \frac{\mathbf{r}_C}{r_C} = \frac{(-3-0)\mathbf{i} + (2-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(-3-0)^2 + (2-0)^2 + (0-6)^2}} = -\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$$

$$\mathbf{u}_D = \frac{\mathbf{r}_D}{r_D} = \frac{(-3-0)\mathbf{i} + (-2-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(-3-0)^2 + (-2-0)^2 + (0-6)^2}} = -\frac{3}{7}\mathbf{i} - \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$$

Thus, the force vectors $\mathbf{F}_A$, $\mathbf{F}_B$, $\mathbf{F}_C$, and $\mathbf{F}_D$ are given by

$$\mathbf{F}_A = F_A\mathbf{u}_A = 70\left(\frac{3}{7}\mathbf{i} - \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right) = [30\mathbf{i} - 20\mathbf{j} - 60\mathbf{k}]\text{ lb}$$

$$\mathbf{F}_B = F_B\mathbf{u}_B = 70\left(\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right) = [30\mathbf{i} + 20\mathbf{j} - 60\mathbf{k}]\text{ lb}$$

$$\mathbf{F}_C = F_C\mathbf{u}_C = 70\left(-\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right) = [-30\mathbf{i} + 20\mathbf{j} - 60\mathbf{k}]\text{ lb}$$

$$\mathbf{F}_D = F_D\mathbf{u}_D = 70\left(-\frac{3}{7}\mathbf{i} - \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right) = [-30\mathbf{i} - 20\mathbf{j} - 60\mathbf{k}]\text{ lb}$$

Resultant Force:

$$\mathbf{F}_R = \mathbf{F}_A + \mathbf{F}_B + \mathbf{F}_C + \mathbf{F}_D = (30\mathbf{i} - 20\mathbf{j} - 60\mathbf{k}) + (30\mathbf{i} + 20\mathbf{j} - 60\mathbf{k}) + (-30\mathbf{i} + 20\mathbf{j} - 60\mathbf{k}) + (-30\mathbf{i} - 20\mathbf{j} - 60\mathbf{k})$$
$$= \{-240\mathbf{k}\}\text{ N}$$

The magnitude of $\mathbf{F}_R$ is

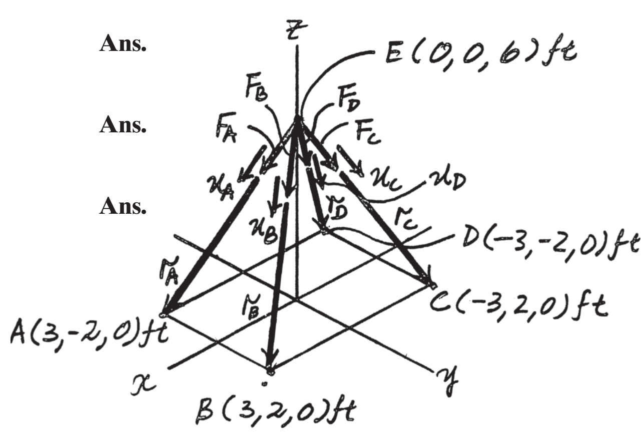
$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2 + (F_R)_z^2}$$
$$= \sqrt{0 + 0 + (-240)^2} = 240\text{ lb}$$ **Ans.**

The coordinate direction angles of $\mathbf{F}_R$ are

$$\alpha = \cos^{-1}\left[\frac{(F_R)_x}{F_R}\right] = \cos^{-1}\left(\frac{0}{240}\right) = 90^\circ$$ **Ans.**

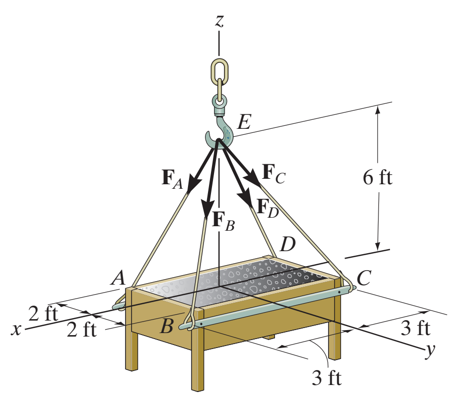
$$\beta = \cos^{-1}\left[\frac{(F_R)_y}{F_R}\right] = \cos^{-1}\left(\frac{0}{240}\right) = 90^\circ$$ **Ans.**

$$\gamma = \cos^{-1}\left[\frac{(F_R)_z}{F_R}\right] = \cos^{-1}\left(\frac{-240}{240}\right) = 180^\circ$$ **Ans.**



***2–104.**

If the resultant of the four forces is $\mathbf{F}_R = \{-360\mathbf{k}\}$ lb, determine the tension developed in each cable. Due to symmetry, the tension in the four cables is the same.



SOLUTION

Force Vectors: The unit vectors $\mathbf{u}_A$, $\mathbf{u}_B$, $\mathbf{u}_C$, and $\mathbf{u}_D$ of $\mathbf{F}_A$, $\mathbf{F}_B$, $\mathbf{F}_C$, and $\mathbf{F}_D$ must be determined first. From Fig. *a*,

$$\mathbf{u}_A = \frac{\mathbf{r}_A}{r_A} = \frac{(3-0)\mathbf{i} + (-2-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(3-0)^2 + (-2-0)^2 + (0-6)^2}} = \frac{3}{7}\mathbf{i} - \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$$

$$\mathbf{u}_B = \frac{\mathbf{r}_B}{r_B} = \frac{(3-0)\mathbf{i} + (2-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(3-0)^2 + (2-0)^2 + (0-6)^2}} = \frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$$

$$\mathbf{u}_C = \frac{\mathbf{r}_C}{r_C} = \frac{(-3-0)\mathbf{i} + (2-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(-3-0)^2 + (2-0)^2 + (0-6)^2}} = -\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$$

$$\mathbf{u}_D = \frac{\mathbf{r}_D}{r_D} = \frac{(-3-0)\mathbf{i} + (-2-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(-3-0)^2 + (-2-0)^2 + (0-6)^2}} = -\frac{3}{7}\mathbf{i} - \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$$

Since the magnitudes of $\mathbf{F}_A$, $\mathbf{F}_B$, $\mathbf{F}_C$, and $\mathbf{F}_D$ are the same and denoted as F, the four vectors or forces can be written as

$$\mathbf{F}_A = F_A\mathbf{u}_A = F\left(\frac{3}{7}\mathbf{i} - \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right)$$

$$\mathbf{F}_B = F_B\mathbf{u}_B = F\left(\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right)$$

$$\mathbf{F}_C = F_C\mathbf{u}_C = F\left(-\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right)$$

$$\mathbf{F}_D = F_D\mathbf{u}_D = F\left(-\frac{3}{7}\mathbf{i} - \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right)$$

Resultant Force: The vector addition of $\mathbf{F}_A$, $\mathbf{F}_B$, $\mathbf{F}_C$, and $\mathbf{F}_D$ is equal to $\mathbf{F}_R$. Thus,

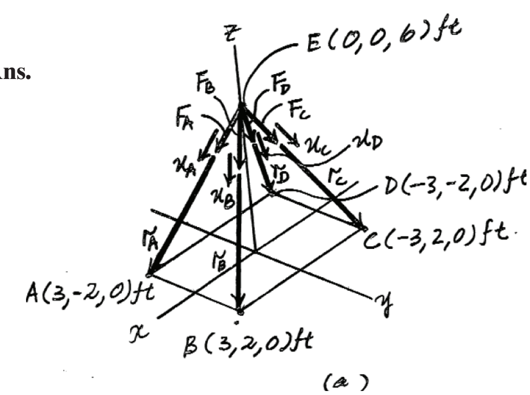
$$\mathbf{F}_R = \mathbf{F}_A + \mathbf{F}_B + \mathbf{F}_C + \mathbf{F}_D$$

$$\{-360\mathbf{k}\} = \left[F\left(\frac{3}{7}\mathbf{i} - \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right)\right] + \left[F\left(\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right)\right] + \left[F\left(-\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right) + \left[F\left(-\frac{3}{7}\mathbf{i} - \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right)\right]\right.$$

$$-360\mathbf{k} = -\frac{24}{7}\mathbf{k}$$

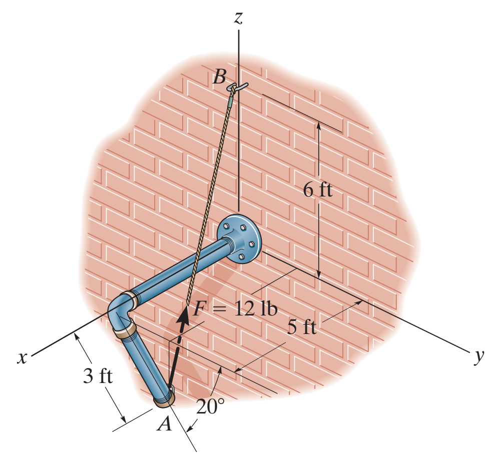
Thus,

$$360 = \frac{24}{7}F \qquad F = 105 \text{ lb}$$ **Ans.**



2–105.

The pipe is supported at its end by a cord AB. If the cord exerts a force of $F = 12$ lb on the pipe at A, express this force as a Cartesian vector.



SOLUTION

Unit Vector: The coordinates of point A are

$$A(5, 3\cos 20°, -3\sin 20°)\text{ ft} = A(5.00, 2.819, -1.206)\text{ ft}$$

Then

$$\mathbf{r}_{AB} = \{(0 - 5.00)\mathbf{i} + (0 - 2.819)\mathbf{j} + [6-(-1.206)]\mathbf{k}\}\text{ ft}$$

$$= \{-5.00\mathbf{i} - 2.819\mathbf{j} + 7.026\mathbf{k}\}\text{ ft}$$

$$r_{AB} = \sqrt{(-5.00)^2 + (-2.819)^2 + 7.026^2} = 9.073\text{ ft}$$

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{-5.00\mathbf{i} - 2.819\mathbf{j} + 7.026\mathbf{k}}{9.073}$$

$$= -0.5511\mathbf{i} - 0.3107\mathbf{j} + 0.7744\mathbf{k}$$

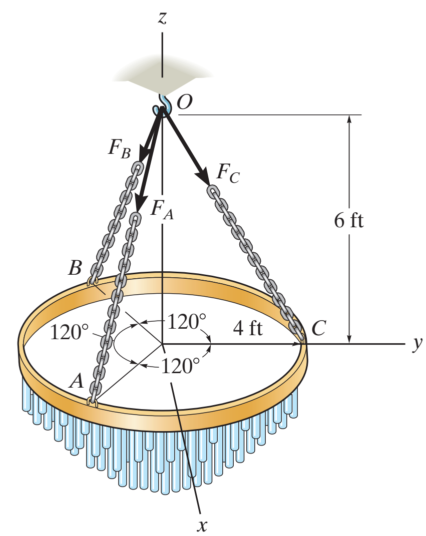
Force Vector:

$$\mathbf{F} = F\mathbf{u}_{AB} = 12\{-0.5511\mathbf{i} - 0.3107\mathbf{j} + 0.7744\mathbf{k}\}\text{ lb}$$

$$= \{-6.61\mathbf{i} - 3.73\mathbf{j} + 9.29\mathbf{k}\}\text{ lb}$$ **Ans.**

2–106.

The chandelier is supported by three chains which are concurrent at point O. If the force in each chain has a magnitude of 60 lb, express each force as a Cartesian vector and determine the magnitude and coordinate direction angles of the resultant force.



SOLUTION

$$\mathbf{F}_A = 60\frac{(4\cos 30°\,\mathbf{i} - 4\sin 30°\,\mathbf{j} - 6\,\mathbf{k})}{\sqrt{(4\cos 30°)^2 + (-4\sin 30°)^2 + (-6)^2}}$$

$$= \{28.8\,\mathbf{i} - 16.6\,\mathbf{j} - 49.9\,\mathbf{k}\}\text{ lb}$$ **Ans.**

$$\mathbf{F}_B = 60\frac{(-4\cos 30°\,\mathbf{i} - 4\sin 30°\,\mathbf{j} - 6\mathbf{k})}{\sqrt{(-4\cos 30°)^2 + (-4\sin 30°)^2 + (-6)^2}}$$

$$= \{-28.8\,\mathbf{i} - 16.6\,\mathbf{j} - 49.9\,\mathbf{k}\}\text{ lb}$$ **Ans.**

$$\mathbf{F}_C = 60\frac{(4\,\mathbf{j} - 6\,\mathbf{k})}{\sqrt{(4)^2 + (-6)^2}}$$

$$= \{33.3\,\mathbf{j} - 49.9\,\mathbf{k}\}\text{ lb}$$ **Ans.**

$$\mathbf{F}_R = \mathbf{F}_A + \mathbf{F}_B + \mathbf{F}_C = \{-149.8\,\mathbf{k}\}\text{ lb}$$

$$F_R = 150\text{ lb}$$ **Ans.**

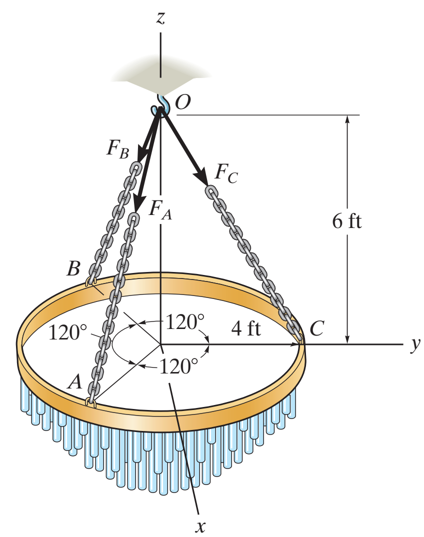
$$\alpha = 90°$$ **Ans.**

$$\beta = 90°$$ **Ans.**

$$\gamma = 180°$$ **Ans.**

2–107.

The chandelier is supported by three chains which are concurrent at point O. If the resultant force at O has a magnitude of 130 lb and is directed along the negative z axis, determine the force in each chain.



SOLUTION

$$\mathbf{F}_C = F\frac{(4\,\mathbf{j} - 6\,\mathbf{k})}{\sqrt{4^2 + (-6)^2}} = 0.5547\,F\mathbf{j} - 0.8321\,F\mathbf{k}$$

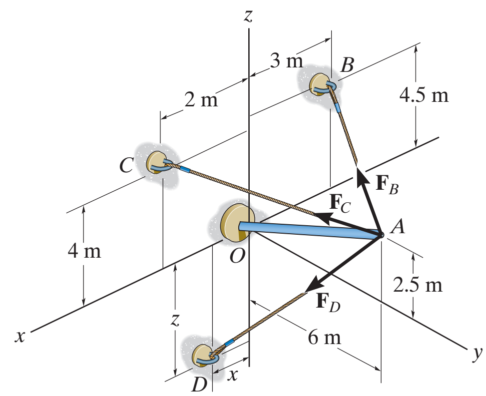
$$\mathbf{F}_A = \mathbf{F}_B = \mathbf{F}_C$$

$$F_{Rz} = \Sigma F_z; \qquad 130 = 3(0.8321F)$$

$$F = 52.1P$$ **Ans.**

***2–108.**

Determine the magnitude and coordinate direction angles of the resultant force. Set $F_B = 630\text{ N}$, $F_C = 520\text{ N}$ and $F_D = 750\text{ N}$, and $x = 3\text{ m}$ and $z = 3.5\text{ m}$.



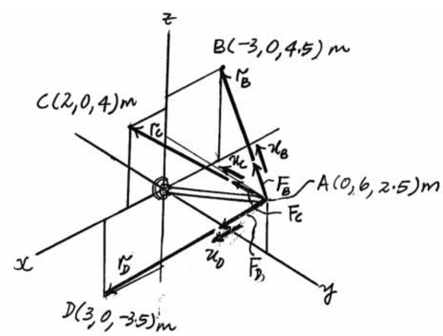
SOLUTION

Force Vectors: The unit vectors $\mathbf{u}_B$, $\mathbf{u}_C$, and $\mathbf{u}_D$ of $\mathbf{F}_B$, $\mathbf{F}_C$, and $\mathbf{F}_D$ must be determined first. From Fig. *a*,

$$\mathbf{u}_B = \frac{\mathbf{r}_B}{r_B} = \frac{(-3-0)\mathbf{i} + (0-6)\mathbf{j} + (4.5-2.5)\mathbf{k}}{\sqrt{(-3-0)^2 + (0-6)^2 + (4.5-2.5)^2}} = -\frac{3}{7}\mathbf{i} - \frac{6}{7}\mathbf{j} + \frac{2}{7}\mathbf{k}$$

$$\mathbf{u}_C = \frac{\mathbf{r}_C}{r_C} = \frac{(2-0)\mathbf{i} + (0-6)\mathbf{j} + (4-2.5)\mathbf{k}}{\sqrt{(2-0)^2 + (0-6)^2 + (4-2.5)^2}} = \frac{4}{13}\mathbf{i} - \frac{12}{13}\mathbf{j} + \frac{3}{13}\mathbf{k}$$

$$\mathbf{u}_D = \frac{\mathbf{r}_D}{r_D} = \frac{(3-0)\mathbf{i} + (0-6)\mathbf{j} + (-3.5-2.5)\mathbf{k}}{\sqrt{(0-3)^2 + (0-6)^2 + (-3.5-2.5)^2}} = \frac{1}{3}\mathbf{i} - \frac{2}{3}\mathbf{j} - \frac{2}{3}\mathbf{k}$$



(a)

Thus, the force vectors $\mathbf{F}_B$, $\mathbf{F}_C$, and $\mathbf{F}_D$ are given by

$$\mathbf{F}_B = F_B\mathbf{u}_B = 630\left(-\frac{3}{7}\mathbf{i} - \frac{6}{7}\mathbf{j} + \frac{2}{7}\mathbf{k}\right) = \{-270\mathbf{i} - 540\mathbf{j} + 180\mathbf{k}\}\text{ N}$$

$$\mathbf{F}_C = F_C\mathbf{u}_C = 520\left(\frac{4}{13}\mathbf{i} - \frac{12}{13}\mathbf{j} + \frac{3}{13}\mathbf{k}\right) = \{160\mathbf{i} - 480\mathbf{j} + 120\mathbf{k}\}\text{ N}$$

$$\mathbf{F}_D = F_D\mathbf{u}_D = 750\left(\frac{1}{3}\mathbf{i} - \frac{2}{3}\mathbf{j} - \frac{2}{3}\mathbf{k}\right) = \{250\mathbf{i} - 500\mathbf{j} - 500\mathbf{k}\}\text{ N}$$

Resultant Force:

$$\begin{aligned}\mathbf{F}_R &= \mathbf{F}_B + \mathbf{F}_C + \mathbf{F}_D = (-270\mathbf{i} - 540\mathbf{j} + 180\mathbf{k}) + (160\mathbf{i} - 480\mathbf{j} + 120\mathbf{k}) + (250\mathbf{i} - 500\mathbf{j} - 500\mathbf{k}) \\ &= [140\mathbf{i} - 1520\mathbf{j} - 200\mathbf{k}]\text{ N}\end{aligned}$$

The magnitude of $\mathbf{F}_R$ is

$$\begin{aligned}F_R &= \sqrt{(F_R)_x{}^2 + (F_R)_y{}^2 + (F_R)_z{}^2} \\ &= \sqrt{140^2 + (-1520)^2 + (-200)^2} = 1539.48\text{ N} = 1.54\text{ kN}\end{aligned}$$ **Ans.**

The coordinate direction angles of $\mathbf{F}_R$ are

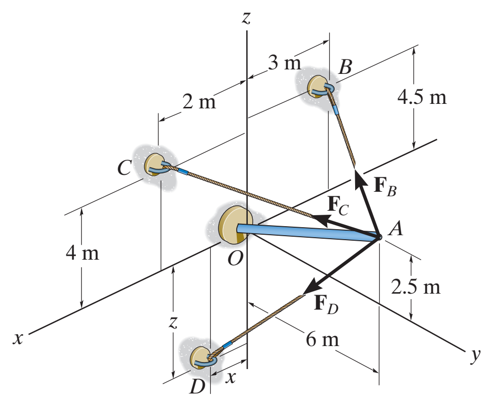
$$\alpha = \cos^{-1}\left[\frac{(F_R)_x}{F_R}\right] = \cos^{-1}\left(\frac{140}{1539.48}\right) = 84.8^\circ$$ **Ans.**

$$\beta = \cos^{-1}\left[\frac{(F_R)_y}{F_R}\right] = \cos^{-1}\left(\frac{-1520}{1539.48}\right) = 171^\circ$$ **Ans.**

$$\gamma = \cos^{-1}\left[\frac{(F_R)_z}{F_R}\right] = \cos^{-1}\left(\frac{-200}{1539.48}\right) = 97.5^\circ$$ **Ans.**

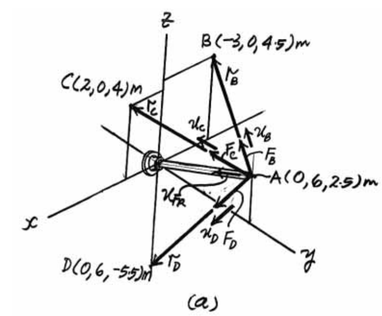
2–109.

If the magnitude of the resultant force is 1300 N and acts along the axis of the strut, directed from point A towards O, determine the magnitudes of the three forces acting on the strut. Set $x = 0$ and $z = 5.5$ m.



SOLUTION

Force Vectors: The unit vectors $\mathbf{u}_B$, $\mathbf{u}_C$, $\mathbf{u}_D$, and $\mathbf{u}_{F_R}$ of $\mathbf{F}_B$, $\mathbf{F}_C$, $\mathbf{F}_D$, and $\mathbf{F}_R$ must be determined first. From Fig. *a*,



$$\mathbf{u}_B = \frac{\mathbf{r}_B}{r_B} = \frac{(-3-0)\mathbf{i} + (0-6)\mathbf{j} + (4.5-2.5)\mathbf{k}}{\sqrt{(-3-0)^2 + (0-6)^2 + (4.5-2.5)^2}} = -\frac{3}{7}\mathbf{i} - \frac{6}{7}\mathbf{j} + \frac{2}{7}\mathbf{k}$$

$$\mathbf{u}_C = \frac{\mathbf{r}_C}{r_C} = \frac{(2-0)\mathbf{i} + (0-6)\mathbf{j} + (4-2.5)\mathbf{k}}{\sqrt{(2-0)^2 + (0-6)^2 + (4-2.5)^2}} = \frac{4}{13}\mathbf{i} - \frac{12}{13}\mathbf{j} + \frac{3}{13}\mathbf{k}$$

$$\mathbf{u}_D = \frac{\mathbf{r}_D}{r_D} = \frac{(0-0)\mathbf{i} + (0-6)\mathbf{j} + (-5.5-2.5)\mathbf{k}}{\sqrt{(0-0)^2 + (0-6)^2 + (-5.5-2.5)^2}} = -\frac{3}{5}\mathbf{j} + \frac{4}{5}\mathbf{k}$$

$$\mathbf{u}_{F_R} = \frac{\mathbf{r}_{AO}}{r_{AO}} = \frac{(0-0)\mathbf{i} + (0-6)\mathbf{j} + (0-2.5)\mathbf{k}}{\sqrt{(0-0)^2 + (0-6)^2 + (0-2.5)^2}} = -\frac{12}{13}\mathbf{j} + \frac{5}{13}\mathbf{k}$$

Thus, the force vectors $\mathbf{F}_B$, $\mathbf{F}_C$, $\mathbf{F}_D$, and $\mathbf{F}_R$ are given by

$$\mathbf{F}_B = F_B\mathbf{u}_B = -\frac{3}{7}F_B\mathbf{i} - \frac{6}{7}F_B\mathbf{j} + \frac{2}{7}F_B\mathbf{k}$$

$$\mathbf{F}_C = F_C\mathbf{u}_C = \frac{4}{13}F_C\mathbf{i} - \frac{12}{13}F_C\mathbf{j} + \frac{3}{13}F_C\mathbf{k}$$

$$\mathbf{F}_D = F_D\mathbf{u}_D = -\frac{3}{5}F_D\mathbf{j} - \frac{4}{5}F_D\mathbf{k}$$

$$\mathbf{F}_R = F_R\mathbf{u}_R = 1300\left(-\frac{12}{13}\mathbf{j} - \frac{5}{13}\mathbf{k}\right) = [-1200\mathbf{j} - 500\mathbf{k}]\text{ N}$$

Resultant Force:

$$\mathbf{F}_R = \mathbf{F}_B + \mathbf{F}_C + \mathbf{F}_D$$

$$-1200\mathbf{j} - 500\mathbf{k} = \left(-\frac{3}{7}F_B\mathbf{i} - \frac{6}{7}F_B\mathbf{j} + \frac{2}{7}F_B\mathbf{k}\right) + \left(\frac{4}{13}F_C\mathbf{i} - \frac{12}{13}F_C\mathbf{j} + \frac{3}{13}F_C\mathbf{k}\right) + \left(-\frac{3}{5}F_D\mathbf{j} - \frac{4}{5}F_D\mathbf{k}\right)$$

$$-1200\mathbf{j} - 500\mathbf{k} = \left(-\frac{3}{7}F_B + \frac{4}{13}F_C\right)\mathbf{i} + \left(-\frac{6}{7}F_B - \frac{12}{13}F_C - \frac{3}{5}F_D\mathbf{j}\right) + \left(\frac{2}{7}F_B + \frac{3}{13}F_C - \frac{4}{5}F_D\right)\mathbf{k}$$

Equating the **i**, **j**, and **k** components,

$$0 = -\frac{3}{7}F_B + \frac{4}{13}F_C \qquad (1)$$

$$-1200 = -\frac{6}{7}F_B - \frac{12}{13}F_C - \frac{3}{5}F_D\mathbf{j} \qquad (2)$$

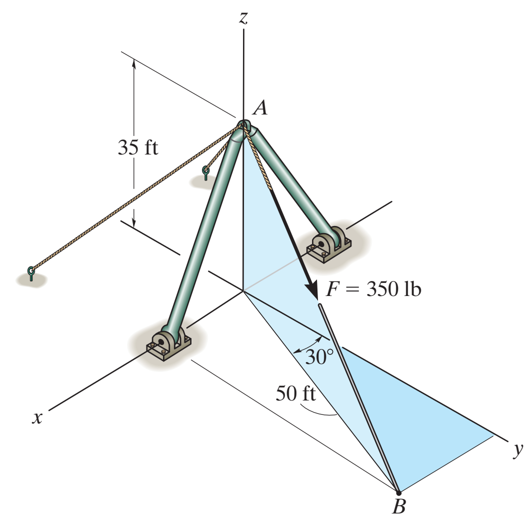
$$-500 = \frac{2}{7}F_B + \frac{3}{13}F_C - \frac{4}{5}F_D \qquad (3)$$

Solving Eqs. (1), (2), and (3), yields

$F_C = 442$ N $\qquad F_B = 318$ N $\qquad F_D = 866$ N **Ans.**

2–110.

The cable attached to the shear-leg derrick exerts a force on the derrick of $F = 350$ lb. Express this force as a Cartesian vector.



SOLUTION

Unit Vector: The coordinates of point B are

$$B(50 \sin 30°, 50 \cos 30°, 0) \text{ ft} = B(25.0, 43.301, 0) \text{ ft}$$

Then

$$\mathbf{r}_{AB} = \{(25.0 - 0)\mathbf{i} + (43.301 - 0)\mathbf{j} + (0 - 35)\mathbf{k}\} \text{ ft}$$

$$= \{25.0\mathbf{i} + 43.301\mathbf{j} - 35.0\mathbf{k}\} \text{ ft}$$

$$r_{AB} = \sqrt{25.0^2 + 43.301^2 + (-35.0)^2} = 61.033 \text{ ft}$$

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{25.0\mathbf{i} + 43.301\mathbf{j} - 35.0\mathbf{k}}{61.033}$$

$$= 0.4096\mathbf{i} + 0.7094\mathbf{j} - 0.5735\mathbf{k}$$

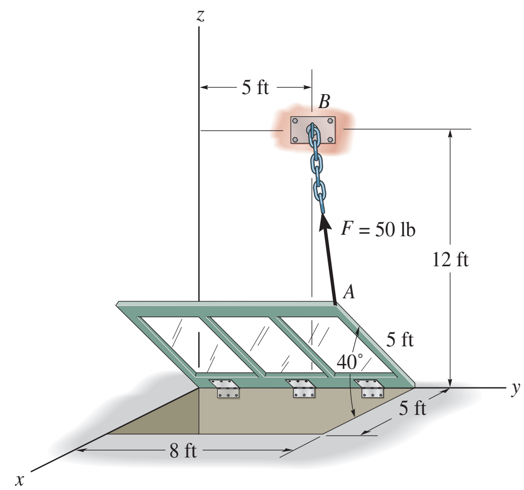
Force Vector:

$$\mathbf{F} = F\mathbf{u}_{AB} = 350\{0.4096\mathbf{i} + 0.7094\mathbf{j} - 0.5735\mathbf{k}\} \text{ lb}$$

$$= \{143\mathbf{i} + 248\mathbf{j} - 201\mathbf{k}\} \text{ lb}$$ **Ans.**

2–111.

The window is held open by chain AB. Determine the length of the chain, and express the 50-lb force acting at A along the chain as a Cartesian vector and determine its coordinate direction angles.



SOLUTION

Unit Vector: The coordinates of point A are

$$A(5\cos 40°, 8, 5\sin 40°)\text{ ft} = A(3.830, 8.00, 3.214)\text{ ft}$$

Then

$$\mathbf{r}_{AB} = \{(0 - 3.830)\mathbf{i} + (5 - 8.00)\mathbf{j} + (12 - 3.214)\mathbf{k}\}\text{ ft}$$

$$= \{-3.830\mathbf{i} - 3.00\mathbf{j} + 8.786\mathbf{k}\}\text{ ft}$$

$$r_{AB} = \sqrt{(-3.830)^2 + (-3.00)^2 + 8.786^2} = 10.043\text{ ft} = 10.0\text{ ft} \qquad \textbf{Ans.}$$

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{-3.830\mathbf{i} - 3.00\mathbf{j} + 8.786\mathbf{k}}{10.043}$$

$$= -0.3814\mathbf{i} - 0.2987\mathbf{j} + 0.8748\mathbf{k}$$

Force Vector:

$$\mathbf{F} = F\mathbf{u}_{AB} = 50\{-0.3814\mathbf{i} - 0.2987\mathbf{j} + 0.8748\mathbf{k}\}\text{ lb}$$

$$= \{-19.1\mathbf{i} - 14.9\mathbf{j} + 43.7\mathbf{k}\}\text{ lb} \qquad \textbf{Ans.}$$

Coordinate Direction Angles: From the unit vector $\mathbf{u}_{AB}$ obtained above, we have

$$\cos\alpha = -0.3814 \qquad \alpha = 112° \qquad \textbf{Ans.}$$

$$\cos\beta = -0.2987 \qquad \beta = 107° \qquad \textbf{Ans.}$$

$$\cos\gamma = 0.8748 \qquad \gamma = 29.0° \qquad \textbf{Ans.}$$

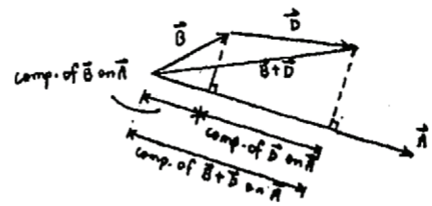
***2–112.**

Given the three vectors **A**, **B**, and **D**, show that $\mathbf{A} \cdot (\mathbf{B} + \mathbf{D}) = (\mathbf{A} \cdot \mathbf{B}) + (\mathbf{A} \cdot \mathbf{D})$.

SOLUTION

Since the component of $(\mathbf{B} + \mathbf{D})$ is equal to the sum of the components of **B** and **D**, then

$$\mathbf{A} \cdot (\mathbf{B} + \mathbf{D}) = \mathbf{A} \cdot \mathbf{B} + \mathbf{A} \cdot \mathbf{D} \qquad \textbf{(QED)}$$

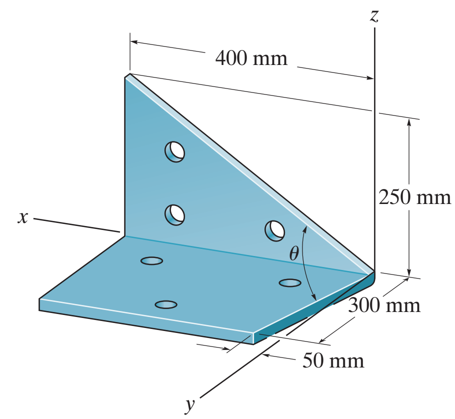


Also,

$$\begin{aligned} \mathbf{A} \cdot (\mathbf{B} + \mathbf{D}) &= (A_x \mathbf{i} + A_y \mathbf{j} + A_z \mathbf{k}) \cdot [(B_x + D_x)\mathbf{i} + (B_y + D_y)\mathbf{j} + (B_z + D_z)\mathbf{k}] \\ &= A_x (B_x + D_x) + A_y (B_y + D_y) + A_z (B_z + D_z) \\ &= (A_x B_x + A_y B_y + A_z B_z) + (A_x D_x + A_y D_y + A_z D_z) \\ &= (\mathbf{A} \cdot \mathbf{B}) + (\mathbf{A} \cdot \mathbf{D}) \qquad \textbf{(QED)} \end{aligned}$$

2–113.

Determine the angle θ between the edges of the sheet-metal bracket.



SOLUTION

$\mathbf{r}_1 = \{400\mathbf{i} + 250\mathbf{k}\}$ mm ; $\quad r_1 = 471.70$ mm

$\mathbf{r}_2 = \{50\mathbf{i} + 300\mathbf{j}\}$ mm ; $\quad r_2 = 304.14$ mm

$$\mathbf{r}_1 \cdot \mathbf{r}_2 = (400)(50) + 0(300) + 250(0) = 20\,000$$

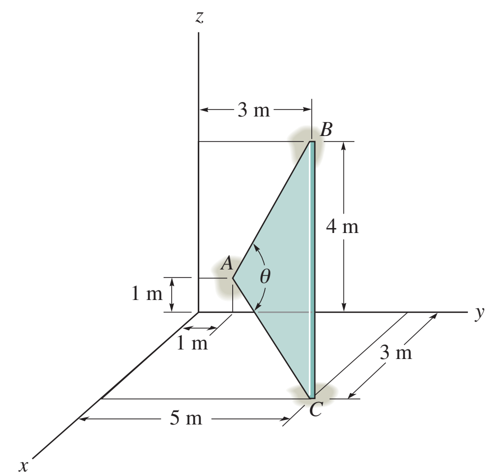
$$\theta = \cos^{-1}\left(\frac{\mathbf{r}_1 \cdot \mathbf{r}_2}{r_1 r_2}\right)$$

$$= \cos^{-1}\left(\frac{20\,000}{(471.70)(304.14)}\right) = 82.0^\circ$$

Ans.

2–114.

Determine the angle θ between the sides of the triangular plate.



SOLUTION

$\mathbf{r}_{AC} = \{3\,\mathbf{i} + 4\,\mathbf{j} - 1\,\mathbf{k}\}$ m

$r_{AC} = \sqrt{(3)^2 + (4)^2 + (-1)^2} = 5.0990$ m

$\mathbf{r}_{AB} = \{2\,\mathbf{j} + 3\,\mathbf{k}\}$ m

$r_{AB} = \sqrt{(2)^2 + (3)^2} = 3.6056$ m

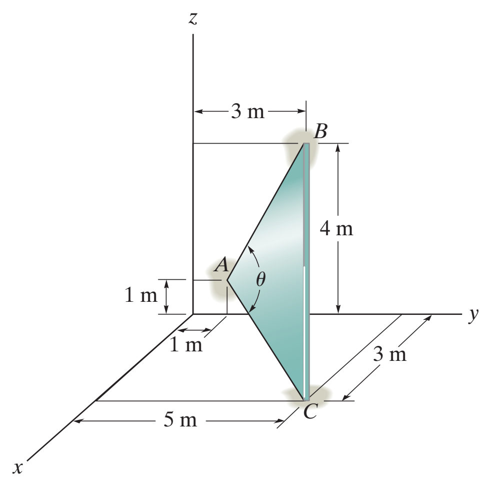
$\mathbf{r}_{AC} \cdot \mathbf{r}_{AB} = 0 + 4(2) + (-1)(3) = 5$

$$\theta = \cos^{-1}\left(\frac{\mathbf{r}_{AC} \cdot \mathbf{r}_{AB}}{r_{AC} r_{AB}}\right) = \cos^{-1}\frac{5}{(5.0990)(3.6056)}$$

$\theta = 74.219° = 74.2°$ **Ans.**

2–115.

Determine the length of side BC of the triangular plate. Solve the problem by finding the magnitude of $\mathbf{r}_{BC}$; then check the result by first finding θ, r_{AB}, and r_{AC} and then using the cosine law.



SOLUTION

$$\mathbf{r}_{BC} = \{3\,\mathbf{i} + 2\,\mathbf{j} - 4\,\mathbf{k}\}\text{ m}$$

$$r_{BC} = \sqrt{(3)^2 + (2)^2 + (-4)^2} = 5.39\text{ m} \qquad \textbf{Ans.}$$

Also,

$$\mathbf{r}_{AC} = \{3\,\mathbf{i} + 4\,\mathbf{j} - 1\,\mathbf{k}\}\text{ m}$$

$$r_{AC} = \sqrt{(3)^2 + (4)^2 + (-1)^2} = 5.0990\text{ m}$$

$$\mathbf{r}_{AB} = \{2\,\mathbf{j} + 3\,\mathbf{k}\}\text{ m}$$

$$r_{AB} = \sqrt{(2)^2 + (3)^2} = 3.6056\text{ m}$$

$$\mathbf{r}_{AC} \cdot \mathbf{r}_{AB} = 0 + 4(2) + (-1)(3) = 5$$

$$\theta = \cos^{-1}\left(\frac{\mathbf{r}_{AC} \cdot \mathbf{r}_{AB}}{r_{AC}\, r_{AB}}\right) = \cos^{-1}\frac{5}{(5.0990)(3.6056)}$$

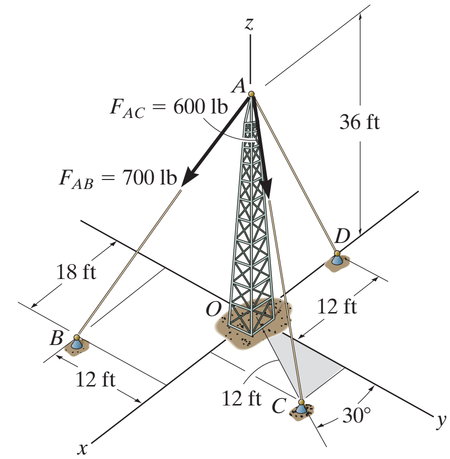
$$\theta = 74.219^\circ$$

$$r_{BC} = \sqrt{(5.0990)^2 + (3.6056)^2 - 2(5.0990)(3.6056)\cos 74.219^\circ}$$

$$r_{BC} = 5.39\text{ m} \qquad \textbf{Ans.}$$

***2–116.**

Determine the magnitude of the projected component of force $\mathbf{F}_{AB}$ acting along the z axis.



SOLUTION

Unit Vector: The unit vector $\mathbf{u}_{AB}$ must be determined first. From Fig. *a*,

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{(18-0)\mathbf{i} + (-12-0)\mathbf{j} + (0-36)\mathbf{k}}{\sqrt{(18-0)^2 + (-12-0)^2 + (0-36)^2}} = \frac{3}{7}\mathbf{i} - \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$$

Thus, the force vector $\mathbf{F}_{AB}$ is given by

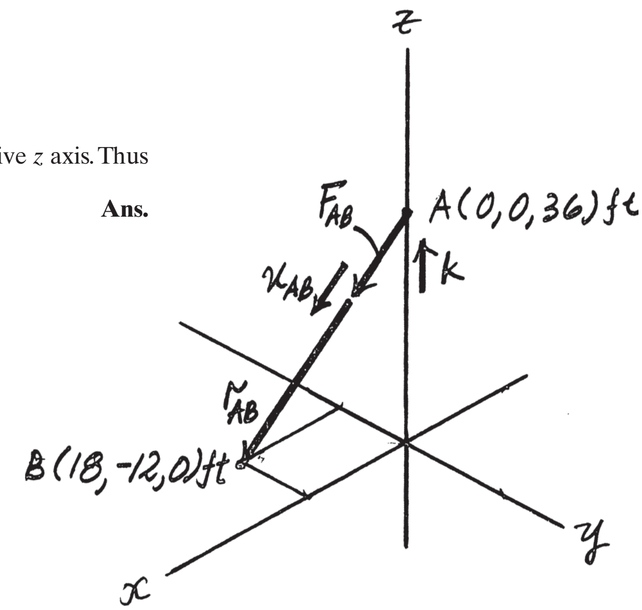
$$\mathbf{F}_{AB} = F_{AB}\,\mathbf{u}_{AB} = 700\left(\frac{3}{7}\mathbf{i} - \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right) = \{300\mathbf{i} - 200\mathbf{j} - 600\mathbf{k}\}\text{ lb}$$

Vector Dot Product: The projected component of $\mathbf{F}_{AB}$ along the z axis is

$$(F_{AB})_z = \mathbf{F}_{AB} \cdot \mathbf{k} = \left(300\mathbf{i} - 200\mathbf{j} - 600\mathbf{k}\right) \cdot \mathbf{k}$$
$$= -600\text{ lb}$$

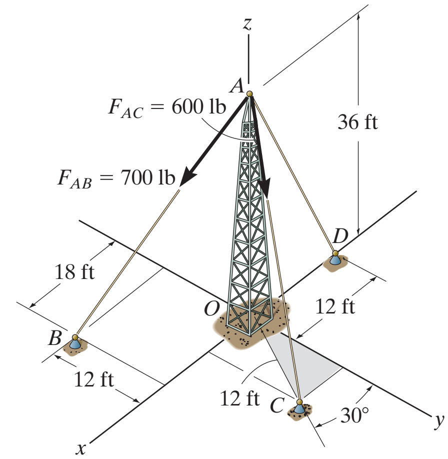
The negative sign indicates that ($\mathbf{F}_{AB}$)z is directed towards the negative z axis. Thus

$$(F_{AB})_z = 600\text{ lb}$$ **Ans.**



***2–117.**

Determine the magnitude of the projected component of force $\mathbf{F}_{AC}$ acting along the z axis.



SOLUTION

Unit Vector: The unit vector $\mathbf{u}_{AC}$ must be determined first. From Fig. a,

$$\mathbf{u}_{AC} = \frac{\mathbf{r}_{AC}}{r_{AC}} = \frac{(12 \sin 30° - 0)\mathbf{i} + (12 \cos 30° - 0)\mathbf{j} + (0 - 36)\mathbf{k}}{\sqrt{(12 \sin 30° - 0)^2 + (12 \cos 30° - 0)^2 + (0 - 36)^2}} = 0.1581\mathbf{i} + 0.2739\mathbf{j} - 0.9487\mathbf{k}$$

Thus, the force vector $\mathbf{F}_{AC}$ is given by

$$\mathbf{F}_{AC} = F_{AC}\mathbf{u}_{AC} = 600\left(0.1581\mathbf{i} + 0.2739\mathbf{j} - 0.9487\mathbf{k}\right) = \{94.87\mathbf{i} + 164.32\mathbf{j} - 569.21\mathbf{k}\}\text{ N}$$

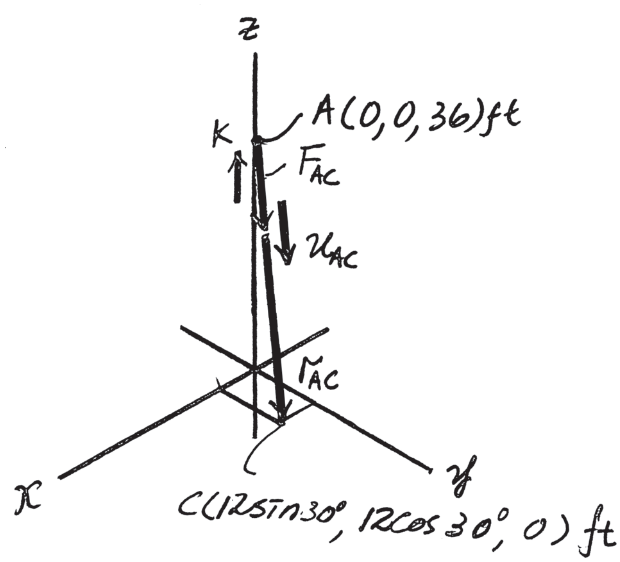
Vector Dot Product: The projected component of $\mathbf{F}_{AC}$ along the z axis is

$$(F_{AC})_z = \mathbf{F}_{AC} \cdot \mathbf{k} = \left(94.87\mathbf{i} + 164.32\mathbf{j} - 569.21\mathbf{k}\right) \cdot \mathbf{k}$$

$$= -569 \text{ lb}$$

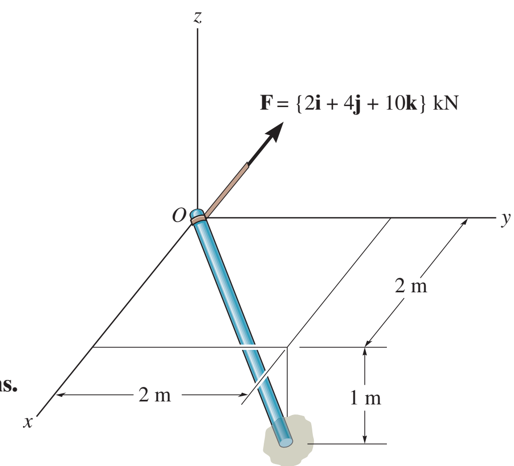
The negative sign indicates that $(\mathbf{F}_{AC})_z$ is directed towards the negative z axis. Thus

$$(F_{AC})_z = 569 \text{ lb} \qquad \textbf{Ans.}$$



2–118.

Determine the projection of the force **F** along the pole.



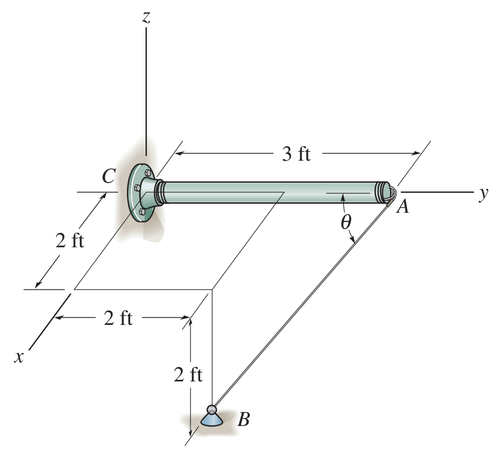
SOLUTION

$$\text{Proj } F = \mathbf{F}\cdot\mathbf{u}_a = (2\,\mathbf{i} + 4\,\mathbf{j} + 10\,\mathbf{k})\cdot\left(\frac{2}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} - \frac{1}{3}\mathbf{k}\right)$$

$$\text{Proj } F = 0.667 \text{ kN}$$ **Ans.**

2–119.

Determine the angle θ between the y axis of the pole and the wire AB.



SOLUTION

Position Vector:

$$\mathbf{r}_{AC} = \{-3\mathbf{j}\} \text{ ft}$$

$$\mathbf{r}_{AB} = \{(2-0)\mathbf{i} + (2-3)\mathbf{j} + (-2-0)\mathbf{k}\} \text{ ft}$$

$$= \{2\mathbf{i} - 1\mathbf{j} - 2\mathbf{k}\} \text{ ft}$$

The magnitudes of the position vectors are

$$r_{AC} = 3.00 \text{ ft} \qquad r_{AB} = \sqrt{2^2 + (-1)^2 + (-2)^2} = 3.00 \text{ ft}$$

The Angles Between Two Vectors θ: The dot product of two vectors must be determined first.

$$\mathbf{r}_{AC} \cdot \mathbf{r}_{AB} = (-3\mathbf{j}) \cdot (2\mathbf{i} - 1\mathbf{j} - 2\mathbf{k})$$

$$= 0(2) + (-3)(-1) + 0(-2)$$

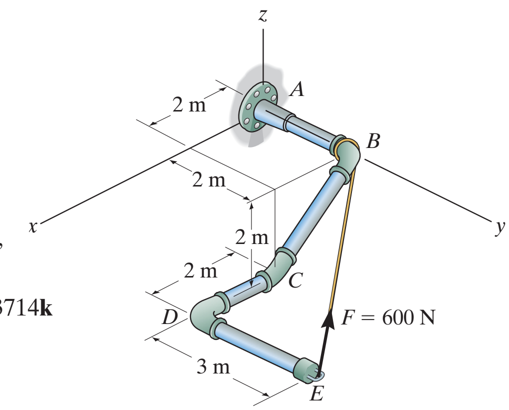
$$= 3$$

Then,

$$\theta = \cos^{-1}\left(\frac{\mathbf{r}_{AO} \cdot \mathbf{r}_{AB}}{r_{AO} r_{AB}}\right) = \cos^{-1}\left[\frac{3}{3.00(3.00)}\right] = 70.5^\circ$$ **Ans.**

***2–120.**

Determine the magnitudes of the components of $F = 600$ N acting along and perpendicular to segment DE of the pipe assembly.



SOLUTION

Unit Vectors: The unit vectors $\mathbf{u}_{EB}$ and $\mathbf{u}_{ED}$ must be determined first. From Fig. *a*,

$$\mathbf{u}_{EB} = \frac{\mathbf{r}_{EB}}{r_{EB}} = \frac{(0-4)\mathbf{i} + (2-5)\mathbf{j} + [0-(-2)]\mathbf{k}}{\sqrt{(0-4)^2 + (2-5)^2 + [0-(-2)]^2}} = -0.7428\mathbf{i} - 0.5571\mathbf{j} + 0.3714\mathbf{k}$$

$$\mathbf{u}_{ED} = -\mathbf{j}$$

Thus, the force vector **F** is given by

$$\mathbf{F} = F\mathbf{u}_{EB} = 600\left(-0.7428\mathbf{i} - 0.5571\mathbf{j} + 0.3714\mathbf{k}\right) = [-445.66\mathbf{i} - 334.25\mathbf{j} + 222.83\mathbf{k}]\text{ N}$$

Vector Dot Product: The magnitude of the component of **F** *p*arallel to segment DE of the pipe assembly is

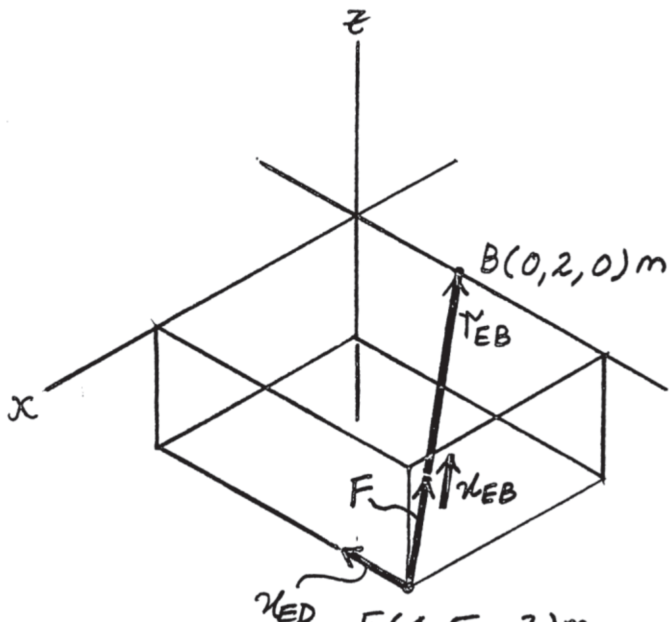
$$(F_{ED})_{\text{paral}} = \mathbf{F}\cdot\mathbf{u}_{ED} = \left(-445.66\mathbf{i} - 334.25\mathbf{j} + 222.83\mathbf{k}\right)\cdot\left(-\mathbf{j}\right)$$

$$= (-445.66)(0) + (-334.25)(-1) + (222.83)(0)$$

$$= 334.25 = 334\text{ N} \qquad \textbf{Ans.}$$

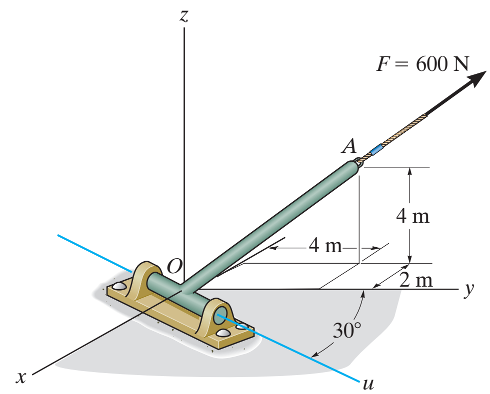
The component of **F** perpendicular to segment DE of the pipe assembly is

$$(F_{ED})_{\text{per}} = \sqrt{F^2 - (F_{ED})_{\text{paral}}{}^2} = \sqrt{600^2 - 334.25^2} = 498\text{ N} \qquad \textbf{Ans.}$$



2–121.

Determine the magnitude of the projection of force $F = 600$ N along the u axis.



SOLUTION

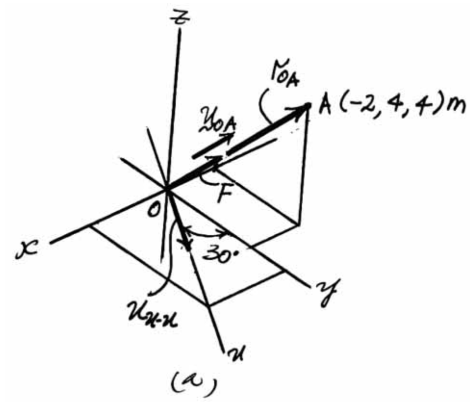
Unit Vectors: The unit vectors $\mathbf{u}_{OA}$ and $\mathbf{u}_u$ must be determined first. From Fig. a,

$$\mathbf{u}_{OA} = \frac{\mathbf{r}_{OA}}{r_{OA}} = \frac{(-2-0)\mathbf{i} + (4-0)\mathbf{j} + (4-0)\mathbf{k}}{\sqrt{(-2-0)^2 + (4-0)^2 + (4-0)^2}} = -\frac{1}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}$$

$$\mathbf{u}_u = \sin 30°\mathbf{i} + \cos 30°\mathbf{j}$$

Thus, the force vectors $\mathbf{F}$ is given by

$$\mathbf{F} = F\mathbf{u}_{OA} = 600\left(-\frac{1}{3}\mathbf{i} - \frac{2}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}\right) = \{-200\mathbf{i} + 400\mathbf{j} + 400\mathbf{k}\}\text{ N}$$



Vector Dot Product: The magnitude of the projected component of $\mathbf{F}$ along the u axis is

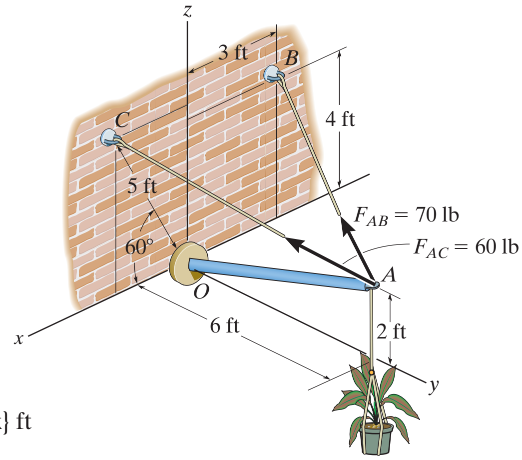
$$\mathbf{F}_u = F \cdot \mathbf{u}_u = (-200\mathbf{i} + 400\mathbf{j} + 400\mathbf{k}) \cdot (\sin 30°\mathbf{i} + \cos 30°\mathbf{j})$$

$$= (-200)(\sin 30°) + 400(\cos 30°) + 400(0)$$

$$= 246\text{ N} \qquad \textbf{Ans.}$$

2–122.

Determine the angle θ between cables AB and AC.



SOLUTION

Position Vectors: The position vectors $\mathbf{r}_{AB}$ and $\mathbf{r}_{AC}$ must be determined first. From Fig. *a*,

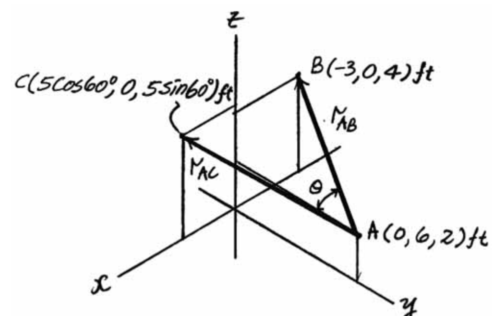
$$\mathbf{r}_{AB} = (-3 - 0)\mathbf{i} + (0 - 6)\mathbf{j} + (4 - 2)\mathbf{k} = \{-3\mathbf{i} - 6\mathbf{j} + 2\mathbf{k}\} \text{ ft}$$

$$\mathbf{r}_{AC} = (5\cos 60° - 0)\mathbf{i} + (0 - 6)\mathbf{j} + (5\sin 60° - 2)\mathbf{k} = \{2.5\mathbf{i} - 6\mathbf{j} + 2.330\mathbf{k}\} \text{ ft}$$

The magnitudes of $\mathbf{r}_{AB}$ and $\mathbf{r}_{AC}$ are

$$\mathbf{r}_{AB} = \sqrt{(-3)^2 + (-6)^2} = 7 \text{ ft}$$

$$\mathbf{r}_{AC} = \sqrt{2.5^2 + (-6)^2 + 2.330^2} = 6.905 \text{ ft}$$



(a)

Vector Dot Product:

$$\mathbf{r}_{AB} \cdot \mathbf{r}_{AC} = (-3\mathbf{i} - 6\mathbf{j} + 2\mathbf{k}) \cdot (2.5\mathbf{i} - 6\mathbf{j} + 2.330\mathbf{k})$$

$$= (-3)(2.5) + (-6)(-6) + (2)(2.330)$$

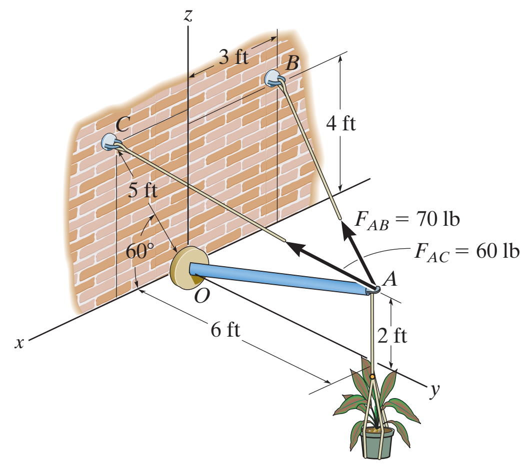
$$= 33.160 \text{ ft}^2$$

Thus,

$$\theta = \cos^{-1}\left(\frac{\mathbf{r}_{AB} \cdot \mathbf{r}_{AC}}{\mathbf{r}_{AB}\,\mathbf{r}_{AC}}\right) = \cos^{-1}\left[\frac{33.160}{7(6.905)}\right] = 46.7° \qquad \textbf{Ans.}$$

2–123.

Determine the angle ϕ between cable AC and strut AO.



SOLUTION

Position Vectors: The position vectors $\mathbf{r}_{AC}$ and $\mathbf{r}_{AO}$ must be determined first. From Fig. *a*,

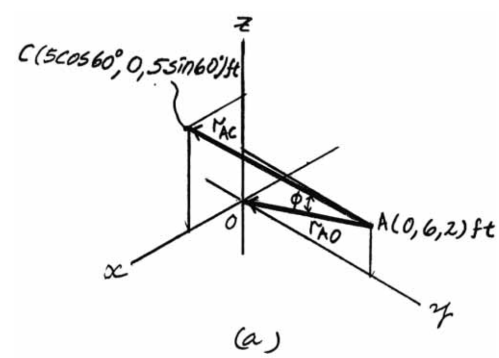
$$\mathbf{r}_{AC} = (5\cos 60° - 0)\mathbf{i} + (0 - 6)\mathbf{j} + (5\sin 60° - 2)\mathbf{k} = \{2.5\mathbf{i} - 6\mathbf{j} + 2.330\mathbf{k}\}\text{ ft}$$

$$\mathbf{r}_{AO} = (0 - 0)\mathbf{i} + (0 - 6)\mathbf{j} + (0 - 2)\mathbf{k} = \{-6\mathbf{j} - 2\mathbf{k}\}\text{ ft}$$

The magnitudes of $\mathbf{r}_{AC}$ and $\mathbf{r}_{AO}$ are

$$\mathbf{r}_{AC} = \sqrt{2.5^2 + (-6)^2 + 2.330^2} = 6.905\text{ ft}$$

$$\mathbf{r}_{AO} = \sqrt{(-6)^2 + (-2)^2} = \sqrt{40}\text{ ft}$$



Vector Dot Product:

$$\mathbf{r}_{AC}\cdot\mathbf{r}_{AO} = (2.5\mathbf{i} - 6\mathbf{j} + 2.330\mathbf{k})\cdot(-6\mathbf{j} - 2\mathbf{k})$$

$$= (2.5)(0) + (-6)(-6) + (2)(2.330)(-2)$$

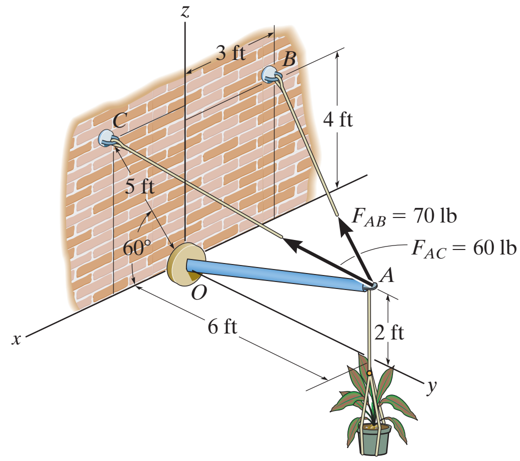
$$= 31.34\text{ ft}^2$$

Thus,

$$\phi = \cos^{-1}\left(\frac{\mathbf{r}_{AC}\cdot\mathbf{r}_{AO}}{\mathbf{r}_{AC}\,\mathbf{r}_{AO}}\right) = \cos^{-1}\left[\frac{31.34}{6.905\sqrt{40}}\right] = 44.1°$$ **Ans.**

***2–124.**

Determine the projected component of force $\mathbf{F}_{AB}$ along the axis of strut AO. Express the result as a Cartesian vector.

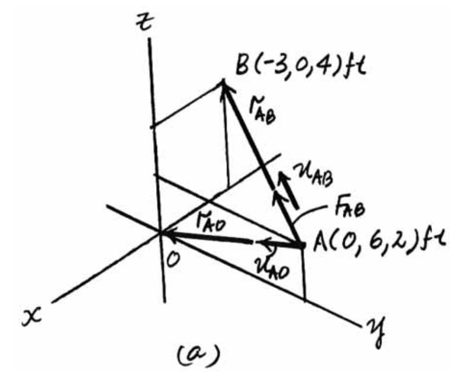


SOLUTION

Unit Vectors: The unit vectors $\mathbf{u}_{AB}$ and $\mathbf{u}_{AO}$ must be determined first. From Fig. *a*,

$$\mathbf{u}_{AB} = \frac{(-3-0)\mathbf{i} + (0-6)\mathbf{j} + (4-2)\mathbf{k}}{\sqrt{(-3-0)^2 + (0-6)^2 + (4-2)^2}} = -\frac{3}{7}\mathbf{i} - \frac{6}{7}\mathbf{j} + \frac{2}{7}\mathbf{k}$$

$$\mathbf{u}_{AO} = \frac{(0-0)\mathbf{i} + (0-6)\mathbf{j} + (0-2)\mathbf{k}}{\sqrt{(0-0)^2 + (0-6)^2 + (0-2)^2}} = -0.9487\mathbf{j} - 0.3162\,\mathbf{k}$$



Thus, the force vectors $\mathbf{F}_{AB}$ is

$$\mathbf{F}_{AB} = F_{AB}\mathbf{u}_{AB} = 70\left(-\frac{3}{7}\mathbf{i} - \frac{6}{7}\mathbf{j} + \frac{2}{7}\mathbf{k}\right) = \{-30\mathbf{i} - 60\mathbf{j} + 20\mathbf{k}\}\text{ lb}$$

Vector Dot Product: The magnitude of the projected component of $\mathbf{F}_{AB}$ along strut AO is

$$(F_{AB})_{AO} = \mathbf{F}_{AB}\cdot\mathbf{u}_{AO} = (-30\mathbf{i} - 60\mathbf{j} + 20\mathbf{k})\cdot(-0.9487\mathbf{j} - 0.3162\mathbf{k})$$

$$= (-30)(0) + (-60)(-0.9487) + (20)(-0.3162)$$

$$= 50.596\text{ lb}$$

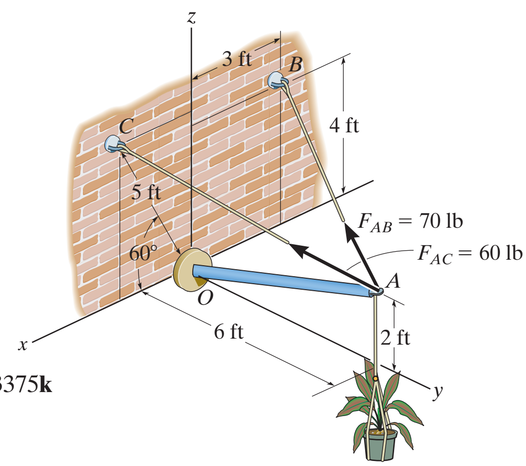
Thus, $(\mathbf{F}_{AB})_{AO}$ expressed in Cartesian vector form can be written as

$$(\mathbf{F}_{AB})_{AO} = (F_{AB})_{AO}\mathbf{u}_{AO} = 50.596(-0.9487\mathbf{j} - 0.3162\mathbf{k})$$

$$= \{-48\mathbf{j} - 16\mathbf{k}\}\text{ lb} \qquad \textbf{Ans.}$$

2–125.

Determine the projected component of force $\mathbf{F}_{AC}$ along the axis of strut AO. Express the result as a Cartesian vector.

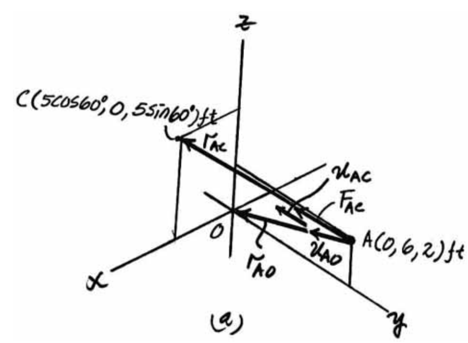


SOLUTION

Unit Vectors: The unit vectors $\mathbf{u}_{AC}$ and $\mathbf{u}_{AO}$ must be determined first. From Fig. *a*,

$$\mathbf{u}_{AC} = \frac{(5\cos 60° - 0)\mathbf{i} + (0 - 6)\mathbf{j} + (5\sin 60° - 2)\mathbf{k}}{\sqrt{(5\cos 60° - 0)^2 + (0-6)^2 + (0-2)^2}} = 0.3621\mathbf{i} - 0.8689\mathbf{j} + 0.3375\mathbf{k}$$

$$\mathbf{u}_{AO} = \frac{(0-0)\mathbf{i} + (0-6)\mathbf{j} + (0-2)\mathbf{k}}{\sqrt{(0-0)^2 + (0-6)^2 + (0-2)^2}} = -0.9487\mathbf{j} - 0.3162\,\mathbf{k}$$



Thus, the force vectors $\mathbf{F}_{AC}$ is given by

$$\mathbf{F}_{AC} = F_{AC}\mathbf{u}_{AC} = 60(0.3621\mathbf{i} - 0.8689\mathbf{j} + 0.3375\mathbf{k}) = \{21.72\mathbf{i} - 52.14\mathbf{j} + 20.25\mathbf{k}\}\text{ lb}$$

Vector Dot Product: The magnitude of the projected component of $\mathbf{F}_{AC}$ along strut AO is

$$\begin{aligned}(F_{AC})_{AO} = \mathbf{F}_{AC}\cdot\mathbf{u}_{AO} &= (21.72\mathbf{i} - 52.14\mathbf{j} + 20.25\mathbf{k})\cdot(-0.9487\mathbf{j} - 0.3162\mathbf{k})\\ &= (21.72)(0) + (-52.14)(-0.9487) + (20.25)(-0.3162)\\ &= 43.057\text{ lb}\end{aligned}$$

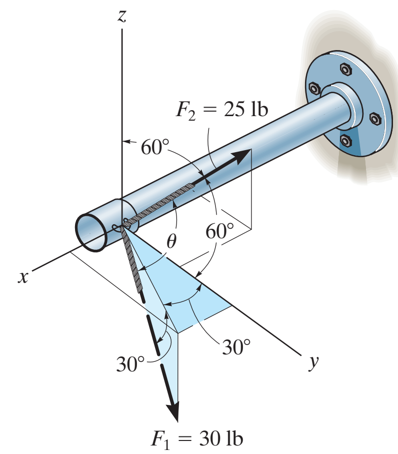
Thus, $(\mathbf{F}_{AC})_{AO}$ expressed in Cartesian vector form can be written as

$$\begin{aligned}(\mathbf{F}_{AC})_{AO} &= (F_{AC})_{AO}\mathbf{u}_{AO} = 43.057(-0.9487\mathbf{j} - 0.3162\mathbf{k})\\ &= \{-40.8\mathbf{j} - 13.6\mathbf{k}\}\text{ lb}\end{aligned}$$

Ans.

2–126.

Two cables exert forces on the pipe. Determine the magnitude of the projected component of $\mathbf{F}_1$ along the line of action of $\mathbf{F}_2$.



SOLUTION

Force Vector:

$$\mathbf{u}_{F_1} = \cos 30° \sin 30° \mathbf{i} + \cos 30° \cos 30° \mathbf{j} - \sin 30° \mathbf{k}$$

$$= 0.4330\mathbf{i} + 0.75\mathbf{j} - 0.5\mathbf{k}$$

$$\mathbf{F}_1 = F_R \mathbf{u}_{F_1} = 30(0.4330\mathbf{i} + 0.75\mathbf{j} - 0.5\mathbf{k}) \text{ lb}$$

$$= \{12.990\mathbf{i} + 22.5\mathbf{j} - 15.0\mathbf{k}\} \text{ lb}$$

Unit Vector: One can obtain the angle $\alpha = 135°$ for $\mathbf{F}_2$ using Eq. 2–8. $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$, with $\beta = 60°$ and $\gamma = 60°$. The unit vector along the line of action of $\mathbf{F}_2$ is

$$\mathbf{u}_{F_2} = \cos 135° \mathbf{i} + \cos 60° \mathbf{j} + \cos 60° \mathbf{k} = -0.7071\mathbf{i} + 0.5\mathbf{j} + 0.5\mathbf{k}$$

Projected Component of $\mathbf{F}_1$ Along the Line of Action of $\mathbf{F}_2$:

$$(F_1)_{F_2} = \mathbf{F}_1 \cdot \mathbf{u}_{F_2} = (12.990\mathbf{i} + 22.5\mathbf{j} - 15.0\mathbf{k}) \cdot (-0.7071\mathbf{i} + 0.5\mathbf{j} + 0.5\mathbf{k})$$

$$= (12.990)(-0.7071) + (22.5)(0.5) + (-15.0)(0.5)$$

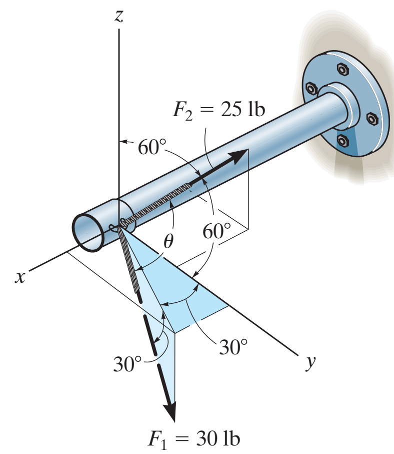
$$= -5.44 \text{ lb}$$

Negative sign indicates that the projected component of $(F_1)_{F_2}$ acts in the opposite sense of direction to that of $\mathbf{u}_{F_2}$.

The magnitude is $(F_1)_{F_2} = 5.44$ lb **Ans.**

2–127.

Determine the angle θ between the two cables attached to the pipe.



SOLUTION

Unit Vectors:

$$\mathbf{u}_{F_1} = \cos 30° \sin 30°\mathbf{i} + \cos 30° \cos 30°\mathbf{j} - \sin 30°\mathbf{k}$$

$$= 0.4330\mathbf{i} + 0.75\mathbf{j} - 0.5\mathbf{k}$$

$$\mathbf{u}_{F_2} = \cos 135°\mathbf{i} + \cos 60°\mathbf{j} + \cos 60°\mathbf{k}$$

$$= -0.7071\mathbf{i} + 0.5\mathbf{j} + 0.5\mathbf{k}$$

The Angles Between Two Vectors θ:

$$\mathbf{u}_{F_1} \cdot \mathbf{u}_{F_2} = (0.4330\mathbf{i} + 0.75\mathbf{j} - 0.5\mathbf{k}) \cdot (-0.7071\mathbf{i} + 0.5\mathbf{j} + 0.5\mathbf{k})$$

$$= 0.4330(-0.7071) + 0.75(0.5) + (-0.5)(0.5)$$

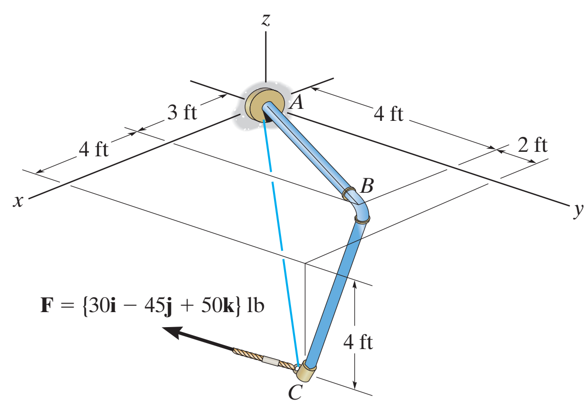
$$= -0.1812$$

Then,

$$\theta = \cos^{-1}\left(\mathbf{u}_{F_1} \cdot \mathbf{u}_{F_2}\right) = \cos^{-1}(-0.1812) = 100°$$ **Ans.**

***2–128.**

Determine the magnitudes of the components of **F** acting along and perpendicular to segment BC of the pipe assembly.



SOLUTION

Unit Vector: The unit vector $\mathbf{u}_{CB}$ must be determined first. From Fig. *a*

$$\mathbf{u}_{CB} = \frac{\mathbf{r}_{CB}}{r_{CB}} = \frac{(3-7)\mathbf{i} + (4-6)\mathbf{j} + [0-(-4)]\mathbf{k}}{\sqrt{(3-7)^2 + (4-6)^2 + [0-(-4)]^2}} = -\frac{2}{3}\mathbf{i} - \frac{1}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}$$

Vector Dot Product: The magnitude of the projected component of **F** parallel to segment BC of the pipe assembly is

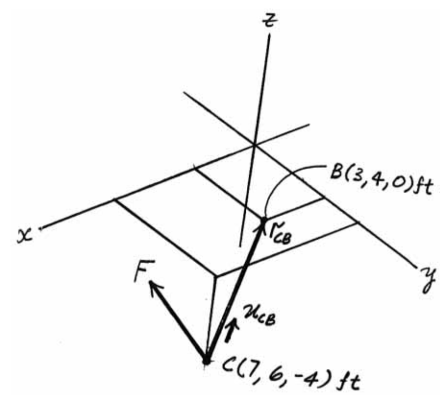
$$(F_{BC})_{pa} = \mathbf{F}\cdot\mathbf{u}_{CB} = (30\mathbf{i} - 45\mathbf{j} + 50\mathbf{k})\cdot\left(-\frac{2}{3}\mathbf{i} - \frac{1}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}\right)$$

$$= (30)\left(-\frac{2}{3}\right) + (-45)\left(-\frac{1}{3}\right) + 50\left(\frac{2}{3}\right)$$

$$= 28.33 \text{ lb} = 28.3 \text{ lb}$$ **Ans.**

The magnitude of **F** is $F = \sqrt{30^2 + (-45)^2 + 50^2} = \sqrt{5425}$ lb. Thus, the magnitude of the component of **F** perpendicular to segment BC of the pipe assembly can be determined from

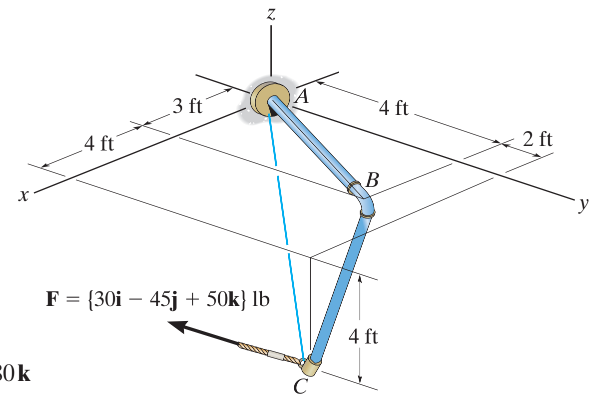
$$(F_{BC})_{pr} = \sqrt{F^2 - (F_{BC})_{pa}^{\,2}} = \sqrt{5425 - 28.33^2} = 68.0 \text{ lb}$$ **Ans.**



(a)

2–129.

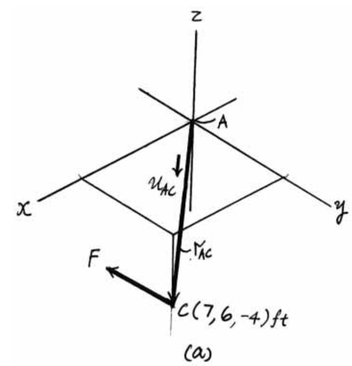
Determine the magnitude of the projected component of **F** along AC. Express this component as a Cartesian vector.



SOLUTION

Unit Vector: The unit vector $\mathbf{u}_{AC}$ must be determined first. From Fig. *a*

$$\mathbf{u}_{AC} = \frac{(7-0)\mathbf{i} + (6-0)\mathbf{j} + (-4-0)\mathbf{k}}{\sqrt{(7-0)^2 + (6-0)^2 + (-4-0)^2}} = 0.6965\,\mathbf{i} + 0.5970\,\mathbf{j} - 0.3980\,\mathbf{k}$$



Vector Dot Product: The magnitude of the projected component of **F** along line AC is

$$F_{AC} = \mathbf{F}\cdot\mathbf{u}_{AC} = (30\mathbf{i} - 45\mathbf{j} + 50\mathbf{k})\cdot(0.6965\mathbf{i} + 0.5970\mathbf{j} - 0.3980\mathbf{k})$$

$$= (30)(0.6965) + (-45)(0.5970) + 50(-0.3980)$$

$$= 25.87 \text{ lb}$$ **Ans.**

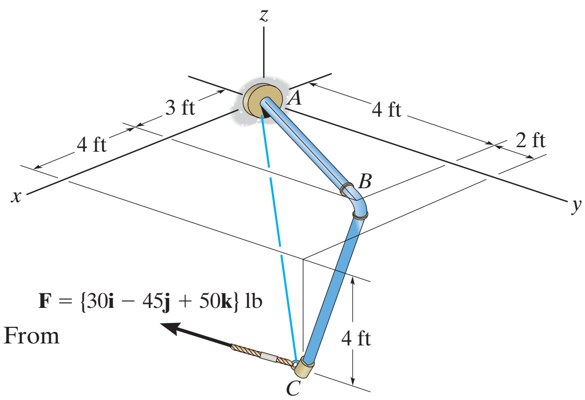
Thus, $\mathbf{F}_{AC}$ expressed in Cartesian vector form is

$$\mathbf{F}_{AC} = F_{AC}\,\mathbf{u}_{AC} = -25.87(0.6965\mathbf{i} + 0.5970\mathbf{j} - 0.3980\mathbf{k})$$

$$= \{-18.0\mathbf{i} - 15.4\mathbf{j} + 10.3\mathbf{k}\} \text{ lb}$$ **Ans.**

2–130.

Determine the angle θ between the pipe segments BA and BC.

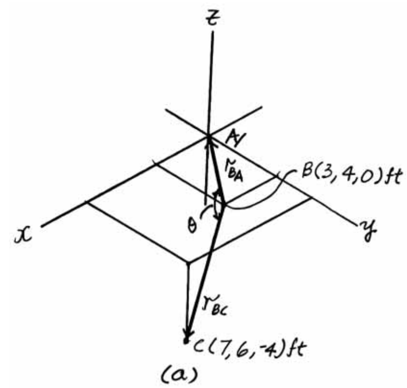


SOLUTION

Position Vectors: The position vectors $\mathbf{r}_{BA}$ and $\mathbf{r}_{BC}$ must be determined first. From Fig. *a*,

$$\mathbf{r}_{BA} = (0 - 3)\mathbf{i} + (0 - 4)\mathbf{j} + (0 - 0)\mathbf{k} = \{-3\mathbf{i} - 4\mathbf{j}\} \text{ ft}$$

$$\mathbf{r}_{BC} = (7 - 3)\mathbf{i} + (6 - 4)\mathbf{j} + (-4 - 0)\mathbf{k} = \{4\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}\} \text{ ft}$$



The magnitude of $\mathbf{r}_{BA}$ and $\mathbf{r}_{BC}$ are

$$r_{BA} = \sqrt{(-3)^2 + (-4)^2} = 5 \text{ ft}$$

$$r_{BC} = \sqrt{4^2 + 2^2 + (-4)^2} = 6 \text{ ft}$$

Vector Dot Product:

$$\mathbf{r}_{BA} \cdot \mathbf{r}_{BC} = (-3\mathbf{i} - 4\mathbf{j}) \cdot (4\mathbf{i} + 2\mathbf{j} - 4\mathbf{k})$$

$$= (-3)(4) + (-4)(2) + 0(-4)$$

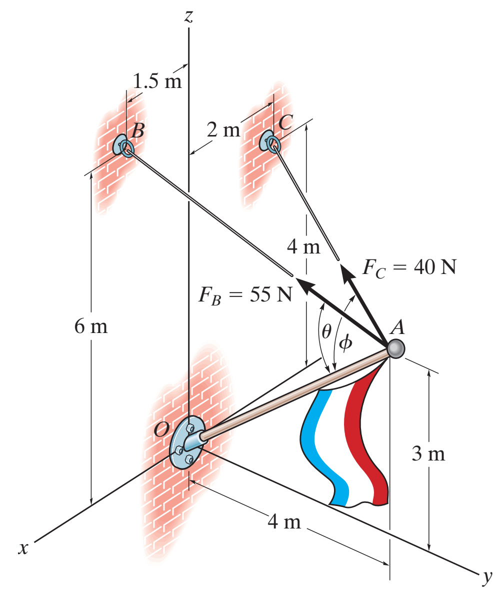
$$= -20 \text{ ft}^2$$

Thus,

$$\theta = \cos^{-1}\left(\frac{\mathbf{r}_{BA} \cdot \mathbf{r}_{BC}}{r_{BA}\, r_{BC}}\right) = \cos^{-1}\left[\frac{-20}{5(6)}\right] = 132^\circ$$ **Ans.**

2–131.

Determine the angles θ and ϕ made between the axes OA of the flag pole and AB and AC, respectively, of each cable.



SOLUTION

$\mathbf{r}_{AC} = \{-2\mathbf{i} - 4\mathbf{j} + 1\mathbf{k}\}$ m ; $\qquad r_{AC} = 4.58$ m

$\mathbf{r}_{AB} = \{1.5\mathbf{i} - 4\mathbf{j} + 3\mathbf{k}\}$ m; $\qquad r_{AB} = 5.22$ m

$\mathbf{r}_{AO} = \{-4\mathbf{j} - 3\mathbf{k}\}$ m; $\qquad r_{AO} = 5.00$ m

$$\mathbf{r}_{AB} \cdot \mathbf{r}_{AO} = (1.5)(0) + (-4)(-4) + (3)(-3) = 7$$

$$\theta = \cos^{-1}\left(\frac{\mathbf{r}_{AB} \cdot \mathbf{r}_{AO}}{r_{AB}\, r_{AO}}\right)$$

$$= \cos^{-1}\left(\frac{7}{5.22(5.00)}\right) = 74.4^\circ$$ **Ans.**

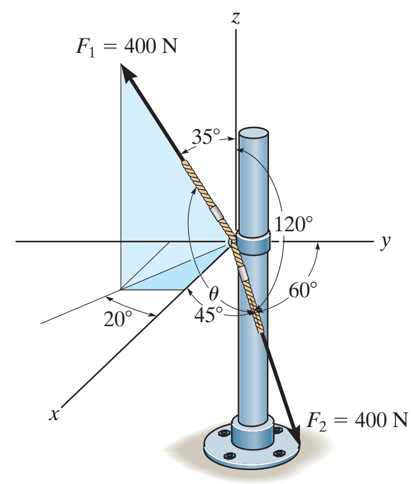
$$\mathbf{r}_{AC} \cdot \mathbf{r}_{AO} = (-2)(0) + (-4)(-4) + (1)(-3) = 13$$

$$\phi = \cos^{-1}\left(\frac{\mathbf{r}_{AC} \cdot \mathbf{r}_{AO}}{r_{AC}\, r_{AO}}\right)$$

$$= \cos^{-1}\left(\frac{13}{4.58(5.00)}\right) = 55.4^\circ$$ **Ans.**

***2–132.**

The cables each exert a force of 400 N on the post. Determine the magnitude of the projected component of $\mathbf{F}_1$ along the line of action of $\mathbf{F}_2$.



SOLUTION

Force Vector:

$$\mathbf{u}_{F_1} = \sin 35° \cos 20°\mathbf{i} - \sin 35° \sin 20°\mathbf{j} + \cos 35°\mathbf{k}$$

$$= 0.5390\mathbf{i} - 0.1962\mathbf{j} + 0.8192\mathbf{k}$$

$$\mathbf{F}_1 = F_1\mathbf{u}_{F_1} = 400(0.5390\mathbf{i} - 0.1962\mathbf{j} + 0.8192\mathbf{k})\text{ N}$$

$$= \{215.59\mathbf{i} - 78.47\mathbf{j} + 327.66\mathbf{k}\}\text{ N}$$

Unit Vector: The unit vector along the line of action of $\mathbf{F}_2$ is

$$\mathbf{u}_{F_2} = \cos 45°\mathbf{i} + \cos 60°\mathbf{j} + \cos 120°\mathbf{k}$$

$$= 0.7071\mathbf{i} + 0.5\mathbf{j} - 0.5\mathbf{k}$$

Projected Component of $\mathbf{F}_1$ ***Along Line of Action of*** $\mathbf{F}_2$:

$$(F_1)_{F_2} = \mathbf{F}_1 \cdot \mathbf{u}_{F_2} = (215.59\mathbf{i} - 78.47\mathbf{j} + 327.66\mathbf{k}) \cdot (0.7071\mathbf{i} + 0.5\mathbf{j} - 0.5\mathbf{k})$$

$$= (215.59)(0.7071) + (-78.47)(0.5) + (327.66)(-0.5)$$

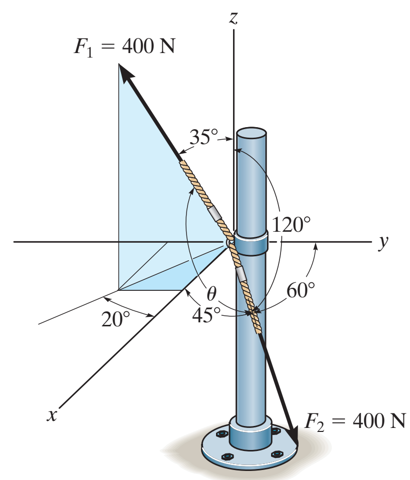
$$= -50.6\text{ N}$$

Negative sign indicates that the force component $(\mathbf{F}_1)F_2$ acts in the opposite sense of direction to that of $\mathbf{u}_{F2}$.

thus the magnitude is $(F_1)_{F_2} = 50.6$ N **Ans.**

2–133.

Determine the angle θ between the two cables attached to the post.



SOLUTION

Unit Vector:

$$\mathbf{u}_{F_1} = \sin 35° \cos 20°\mathbf{i} - \sin 35° \sin 20°\mathbf{j} + \cos 35°\mathbf{k}$$

$$= 0.5390\mathbf{i} - 0.1962\mathbf{j} + 0.8192\mathbf{k}$$

$$\mathbf{u}_{F_2} = \cos 45°\mathbf{i} + \cos 60°\mathbf{j} + \cos 120°\mathbf{k}$$

$$= 0.7071\mathbf{i} + 0.5\mathbf{j} - 0.5\mathbf{k}$$

The Angle Between Two Vectors θ: The dot product of two unit vectors must be determined first.

$$\mathbf{u}_{F_1} \cdot \mathbf{u}_{F_2} = (0.5390\mathbf{i} - 0.1962\mathbf{j} + 0.8192\mathbf{k}) \cdot (0.7071\mathbf{i} + 0.5\mathbf{j} - 0.5\mathbf{k})$$

$$= 0.5390(0.7071) + (-0.1962)(0.5) + 0.8192(-0.5)$$

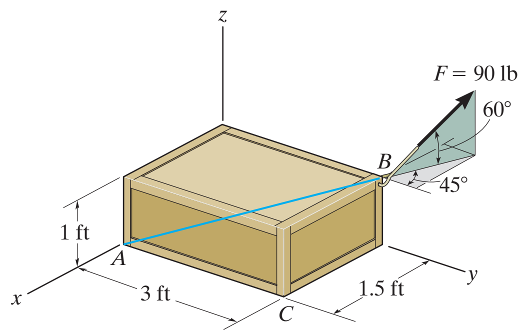
$$= -0.1265$$

Then,

$$\theta = \cos^{-1}\left(\mathbf{u}_{F_1} \cdot \mathbf{u}_{F_2}\right) = \cos^{-1}(-0.1265) = 97.3°$$ **Ans.**

2–134.

Determine the magnitudes of the components of force $F = 90$ lb acting parallel and perpendicular to diagonal AB of the crate.



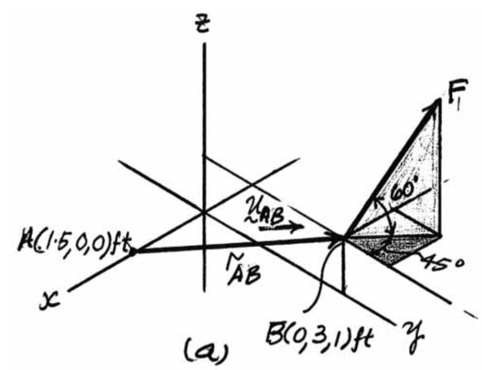
SOLUTION

Force and Unit Vector: The force vector **F** and unit vector $\mathbf{u}_{AB}$ must be determined first. From Fig. *a*

$$\mathbf{F} = 90(-\cos 60° \sin 45°\mathbf{i} + \cos 60° \cos 45°\mathbf{j} + \sin 60°\mathbf{k})$$

$$= \{-31.82\mathbf{i} + 31.82\mathbf{j} + 77.94\mathbf{k}\} \text{ lb}$$

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{(0 - 1.5)\mathbf{i} + (3 - 0)\mathbf{j} + (1 - 0)\mathbf{k}}{\sqrt{(0 - 1.5)^2 + (3 - 0)^2 + (1 - 0)^2}} = -\frac{3}{7}\mathbf{i} - \frac{6}{7}\mathbf{j} + \frac{2}{7}\mathbf{k}$$



Vector Dot Product: The magnitude of the projected component of **F** parallel to the diagonal AB is

$$[(F)_{AB}]_{pa} = \mathbf{F} \cdot \mathbf{u}_{AB} = (-31.82\mathbf{i} + 31.82\mathbf{j} + 77.94\mathbf{k}) \cdot \left(-\frac{3}{7}\mathbf{i} + \frac{6}{7}\mathbf{j} + \frac{2}{7}\mathbf{k}\right)$$

$$= (-31.82)\left(-\frac{3}{7}\right) + 31.82\left(\frac{6}{7}\right) + 77.94\left(\frac{2}{7}\right)$$

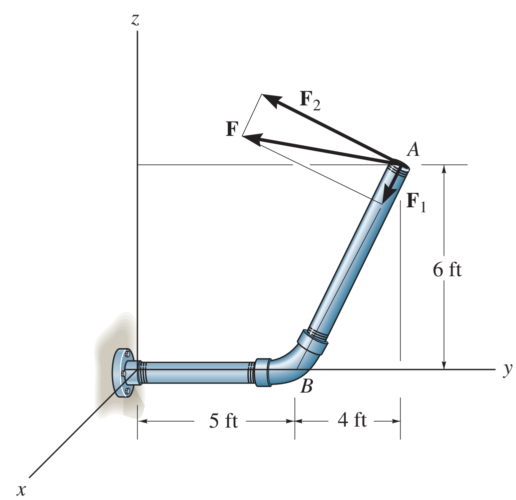
$$= 63.18 \text{ lb} = 63.2 \text{ lb}$$ **Ans.**

The magnitude of the component **F** perpendicular to the diagonal AB is

$$[(F)_{AB}]_{pr} = \sqrt{F^2 - [(F)_{AB}]_{pa}{}^2} = \sqrt{90^2 - 63.18^2} = 64.1 \text{ lb}$$ **Ans.**

2–135.

The force $\mathbf{F} = \{25\mathbf{i} - 50\mathbf{j} + 10\mathbf{k}\}$ lb acts at the end A of the pipe assembly. Determine the magnitude of the components $\mathbf{F}_1$ and $\mathbf{F}_2$ which act along the axis of AB and perpendicular to it.



SOLUTION

Unit Vector: The unit vector along AB axis is

$$\mathbf{u}_{AB} = \frac{(0-0)\mathbf{i} + (5-9)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(0-0)^2 + (5-9)^2 + (0-6)^2}} = -0.5547\mathbf{j} - 0.8321\mathbf{k}$$

Projected Component of* F *Along AB Axis:

$$F_1 = \mathbf{F}\cdot\mathbf{u}_{AB} = (25\mathbf{i} - 50\mathbf{j} + 10\mathbf{k})\cdot(-0.5547\mathbf{j} - 0.8321\mathbf{k})$$

$$= (25)(0) + (-50)(-0.5547) + (10)(-0.8321)$$

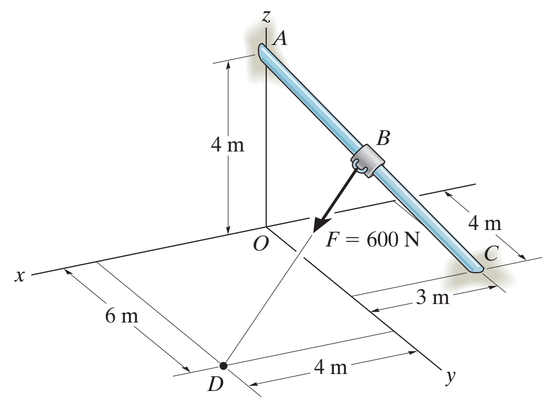
$$= 19.415 \text{ lb} = 19.4 \text{ lb}$$ **Ans.**

Component of* F *Perpendicular to AB Axis: The magnitude of force **F** is $F = \sqrt{25^2 + (-50)^2 + 10^2} = 56.789$ lb.

$$F_2 = \sqrt{F^2 - F_1^2} = \sqrt{56.789^2 - 19.414^2} = 53.4 \text{ lb}$$ **Ans.**

***2–136.**

Determine the components of **F** that act along rod *AC* and perpendicular to it. Point *B* is located at the midpoint of the rod.



SOLUTION

$\mathbf{r}_{AC} = (-3\mathbf{i} + 4\mathbf{j} - 4\mathbf{k}), \qquad r_{AC} = \sqrt{(-3)^2 + 4^2 + (-4)^2} = \sqrt{41}\text{ m}$

$$\mathbf{r}_{AB} = \frac{\mathbf{r}_{AC}}{2} = \frac{-3\mathbf{i} + 4\mathbf{j} + 4\mathbf{k}}{2} = -1.5\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$$

$\mathbf{r}_{AD} = \mathbf{r}_{AB} + \mathbf{r}_{BD}$

$\mathbf{r}_{BD} = \mathbf{r}_{AD} - \mathbf{r}_{AB}$

$= (4\mathbf{i} + 6\mathbf{j} - 4\mathbf{k}) - (-1.5\mathbf{i} + 2\mathbf{j} - 2\mathbf{k})$

$= \{5.5\mathbf{i} + 4\mathbf{j} - 2\mathbf{k}\}\text{ m}$

$r_{BD} = \sqrt{(5.5)^2 + (4)^2 + (-2)^2} = 7.0887\text{ m}$

$$\mathbf{F} = 600\left(\frac{\mathbf{r}_{BD}}{r_{BD}}\right) = 465.528\mathbf{i} + 338.5659\mathbf{j} - 169.2829\mathbf{k}$$

Component of **F** along $\mathbf{r}_{AC}$ is $\mathbf{F}_{||}$

$$F_{||} = \frac{\mathbf{F}\cdot\mathbf{r}_{AC}}{r_{AC}} = \frac{(465.528\mathbf{i} + 338.5659\mathbf{j} - 169.2829\mathbf{k})\cdot(-3\mathbf{i} + 4\mathbf{j} - 4\mathbf{k})}{\sqrt{41}}$$

$F_{||} = 99.1408 = 99.1\text{ N}$ **Ans.**

Component of F perpendicular to $\mathbf{r}_{AC}$ is $F_{\perp}$

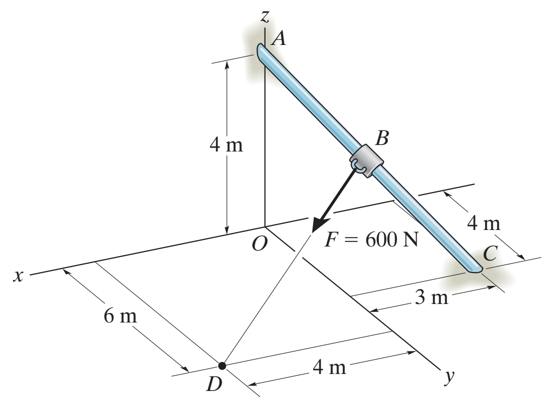
$F_{\perp}^2 + F_{||}^2 = F^2 = 600^2$

$F_{\perp}^2 = 600^2 - 99.1408^2$

$F_{\perp} = 591.75 = 592\text{ N}$ **Ans.**

2–137.

Determine the components of **F** that act along rod AC and perpendicular to it. Point B is located 3 m along the rod from end C.



SOLUTION

$\mathbf{r}_{CA} = 3\mathbf{i} - 4\mathbf{j} + 4\mathbf{k}$

$r_{CA} = 6.403124$

$$\mathbf{r}_{CB} = \frac{3}{6.403124}(\mathbf{r}_{CA}) = 1.40556\mathbf{i} - 1.874085\mathbf{j} + 1.874085\mathbf{k}$$

$\mathbf{r}_{OB} = \mathbf{r}_{OC} + \mathbf{r}_{CB}$

$\quad = -3\mathbf{i} + 4\mathbf{j} + \mathbf{r}_{CB}$

$\quad = -1.59444\mathbf{i} + 2.1259\mathbf{j} + 1.874085\mathbf{k}$

$\mathbf{r}_{OD} = \mathbf{r}_{OB} + \mathbf{r}_{BD}$

$\mathbf{r}_{BD} = \mathbf{r}_{OD} - \mathbf{r}_{OB} = (4\mathbf{i} + 6\mathbf{j}) - \mathbf{r}_{OB}$

$\quad = 5.5944\mathbf{i} + 3.8741\mathbf{j} - 1.874085\mathbf{k}$

$$r_{BD} = \sqrt{(5.5944)^2 + (3.8741)^2 + (-1.874085)^2} = 7.0582$$

$$\mathbf{F} = 600\left(\frac{\mathbf{r}_{BD}}{r_{BD}}\right) = 475.568\mathbf{i} + 329.326\mathbf{j} - 159.311\mathbf{k}$$

$\mathbf{r}_{AC} = (-3\mathbf{i} + 4\mathbf{j} - 4\mathbf{k}), \qquad r_{AC} = \sqrt{41}$

Component of **F** along $\mathbf{r}_{AC}$ is $\mathbf{F}_{||}$

$$F_{||} = \frac{\mathbf{F}\cdot\mathbf{r}_{AC}}{r_{AC}} = \frac{(475.568\mathbf{i} + 329.326\mathbf{j} - 159.311\mathbf{k})\cdot(-3\mathbf{i} + 4\mathbf{j} - 4\mathbf{k})}{\sqrt{41}}$$

$F_{||} = 82.4351 = 82.4\text{ N}$ **Ans.**

Component of **F** perpendicular to $\mathbf{r}_{AC}$ is $\mathbf{F}_{\perp}$

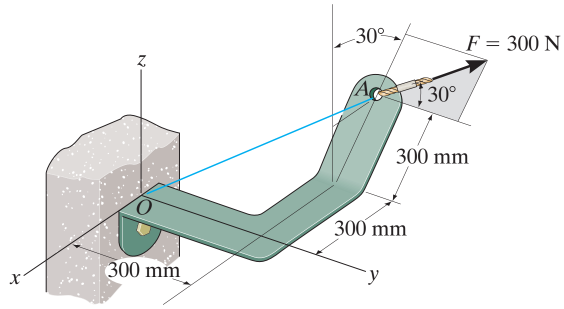
$F_{\perp}^2 + F_{||}^2 = F^2 = 600^2$

$F_{\perp}^2 = 600^2 - 82.4351^2$

$F_{\perp} = 594\text{ N}$ **Ans.**

2–138.

Determine the magnitudes of the projected components of the force $F = 300$ N acting along the x and y axes.



SOLUTION

Force Vector: The force vector **F** must be determined first. From Fig. *a*,

$$\mathbf{F} = -300 \sin 30° \sin 30° \mathbf{i} + 300 \cos 30° \mathbf{j} + 300 \sin 30° \cos 30° \mathbf{k}$$

$$= [-75\mathbf{i} + 259.81\mathbf{j} + 129.90\mathbf{k}] \text{ N}$$

Vector Dot Product: The magnitudes of the projected component of **F** along the x and y axes are

$$F_x = \mathbf{F} \cdot \mathbf{i} = \left(-75\mathbf{i} + 259.81\mathbf{j} + 129.90\mathbf{k}\right) \cdot \mathbf{i}$$

$$= -75(1) + 259.81(0) + 129.90(0)$$

$$= -75 \text{ N}$$

$$F_y = \mathbf{F} \cdot \mathbf{j} = \left(-75\mathbf{i} + 259.81\mathbf{j} + 129.90\mathbf{k}\right) \cdot \mathbf{j}$$

$$= -75(0) + 259.81(1) + 129.90(0)$$

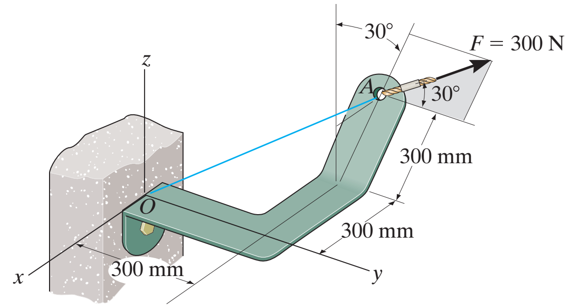
$$= 260 \text{ N}$$

The negative sign indicates that $\mathbf{F}_x$ is directed towards the negative x axis. Thus

$$F_x = 75 \text{ N}, \qquad F_y = 260 \text{ N}$$ **Ans.**

2–139.

Determine the magnitude of the projected component of the force $F = 300$ N acting along line OA.



SOLUTION

Force and Unit Vector: The force vector **F** and unit vector $\mathbf{u}_{OA}$ must be determined first. From Fig. a

$$\mathbf{F} = (-300 \sin 30° \sin 30°\mathbf{i} + 300 \cos 30°\mathbf{j} + 300 \sin 30° \cos 30°\mathbf{k})$$

$$= \{-75\mathbf{i} + 259.81\mathbf{j} + 129.90\mathbf{k}\} \text{ N}$$

$$\mathbf{u}_{OA} = \frac{\mathbf{r}_{OA}}{r_{OA}} = \frac{(-0.45 - 0)\mathbf{i} + (0.3 - 0)\mathbf{j} + (0.2598 - 0)\mathbf{k}}{\sqrt{(-0.45 - 0)^2 + (0.3 - 0)^2 + (0.2598 - 0)^2}} = -0.75\mathbf{i} + 0.5\mathbf{j} + 0.4330\mathbf{k}$$

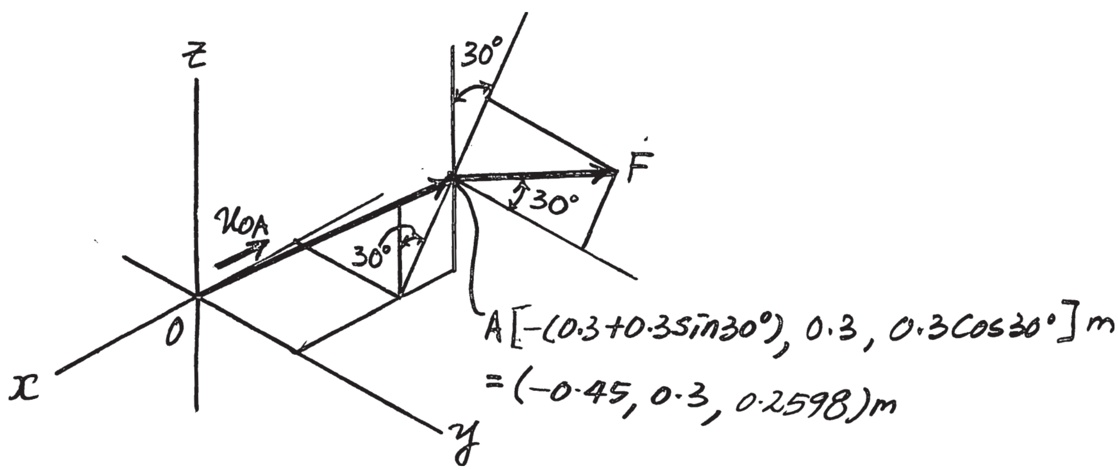
Vector Dot Product: The magnitude of the projected component of **F** along line OA is

$$F_{OA} = \mathbf{F} \cdot \mathbf{u}_{OA} = \left(-75\mathbf{i} + 259.81\mathbf{j} + 129.90\mathbf{k}\right) \cdot \left(-0.75\mathbf{i} + 0.5\mathbf{j} + 0.4330\mathbf{k}\right)$$

$$= (-75)(-0.75) + 259.81(0.5) + 129.90(0.4330)$$

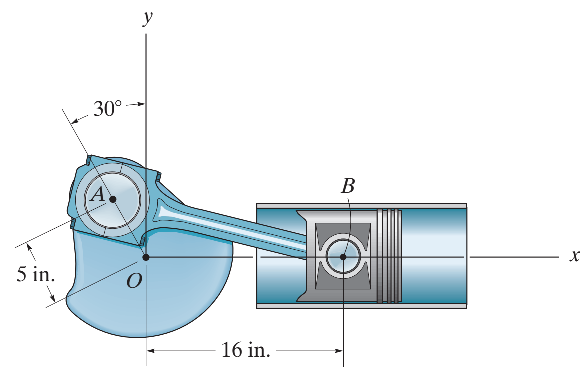
$$= 242 \text{ N}$$

Ans.



***2–140.**

Determine the length of the connecting rod AB by first formulating a Cartesian position vector from A to B and then determining its magnitude.



SOLUTION

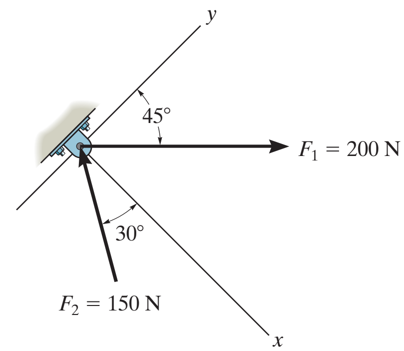
$$\mathbf{r}_{AB} = [16 - (-5 \sin 30°)]\mathbf{i} + (0 - 5 \cos 30°)\,\mathbf{j}$$

$$= \{18.5\,\mathbf{i} - 4.330\,\mathbf{j}\} \text{ in.}$$

$$r_{AB} = \sqrt{(18.5)^2 + (4.330)^2} = 19.0 \text{ in.}$$ **Ans.**

2–141.

Determine the x and y components of $\mathbf{F}_1$ and $\mathbf{F}_2$.



SOLUTION

$$F_{1x} = 200 \sin 45° = 141 \text{ N}$$ **Ans.**

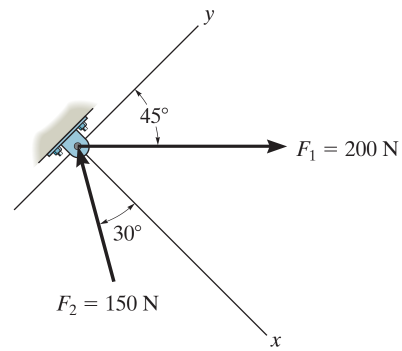
$$F_{1y} = 200 \cos 45° = 141 \text{ N}$$ **Ans.**

$$F_{2x} = -150 \cos 30° = -130 \text{ N}$$ **Ans.**

$$F_{2y} = 150 \sin 30° = 75 \text{ N}$$ **Ans.**

2–142.

Determine the magnitude of the resultant force and its direction, measured counterclockwise from the positive x axis.



SOLUTION

$+\searrow F_{Rx} = \Sigma F_x; \qquad F_{Rx} = -150 \cos 30° + 200 \sin 45° = 11.518 \text{ N}$

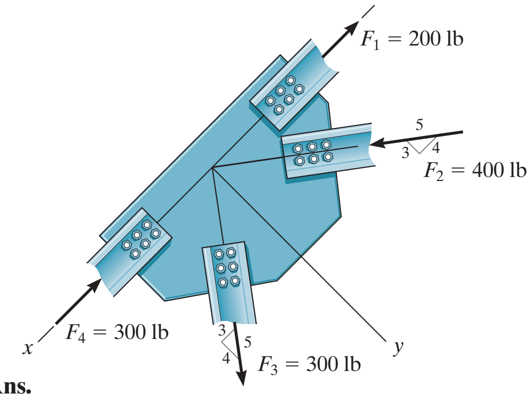
$\nearrow + F_{Ry} = \Sigma F_y; \qquad F_{Ry} = 150 \sin 30° + 200 \cos 45° = 216.421 \text{ N}$

$F_R = \sqrt{(11.518)^2 + (216.421)^2} = 217 \text{ N}$ **Ans.**

$\theta = \tan^{-1}\left(\frac{216.421}{11.518}\right) = 87.0°$ **Ans.**

2–143.

Determine the x and y components of each force acting on the *gusset plate* of the bridge truss. Show that the resultant force is zero.



SOLUTION

$F_{1x} = -200$ lb **Ans.**

$F_{1y} = 0$ **Ans.**

$F_{2x} = 400\left(\frac{4}{5}\right) = 320$ lb **Ans.**

$F_{2y} = -400\left(\frac{3}{5}\right) = -240$ lb **Ans.**

$F_{3x} = 300\left(\frac{3}{5}\right) = 180$ lb **Ans.**

$F_{3y} = 300\left(\frac{4}{5}\right) = 240$ lb **Ans.**

$F_{4x} = -300$ lb **Ans.**

$F_{4y} = 0$ **Ans.**

$F_{Rx} = F_{1x} + F_{2x} + F_{3x} + F_{4x}$

$F_{Rx} = -200 + 320 + 180 - 300 = 0$

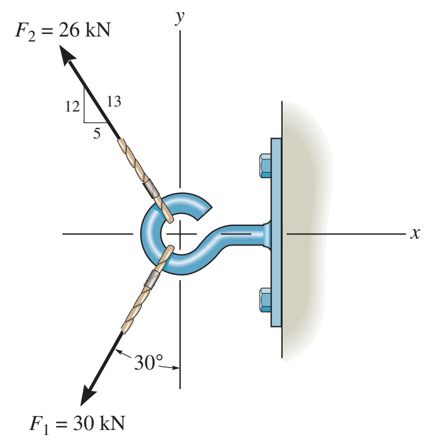
$F_{Ry} = F_{1y} + F_{2y} + F_{3y} + F_{4y}$

$F_{Ry} = 0 - 240 + 240 + 0 = 0$

Thus, $F_R = 0$

***2–144.**

Express $\mathbf{F}_1$ and $\mathbf{F}_2$ as Cartesian vectors.



SOLUTION

$$\mathbf{F}_1 = -30 \sin 30^\circ\, \mathbf{i} - 30 \cos 30^\circ\, \mathbf{j}$$

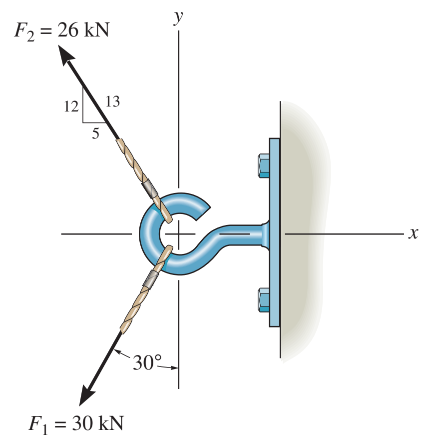
$$= \{-15.0\, \mathbf{i} - 26.0\, \mathbf{j}\} \text{ kN}$$ **Ans.**

$$\mathbf{F}_2 = -\frac{5}{13}(26)\, \mathbf{i} + \frac{12}{13}(26)\, \mathbf{j}$$

$$= \{-10.0\, \mathbf{i} + 24.0\, \mathbf{j}\} \text{ kN}$$ **Ans.**

2–145.

Determine the magnitude of the resultant force and its direction measured counterclockwise from the positive x axis.



SOLUTION

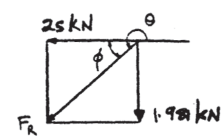
$\overset{+}{\rightarrow} F_{Rx} = \Sigma F_x;\quad F_{Rx} = -30 \sin 30° - \frac{5}{13}(26) = -25 \text{ kN}$

$+\uparrow F_{Ry} = \Sigma F_y;\quad F_{Ry} = -30 \cos 30° + \frac{12}{13}(26) = -1.981 \text{ kN}$

$$F_R = \sqrt{(-25)^2 + (-1.981)^2} = 25.1 \text{ kN}$$ **Ans.**

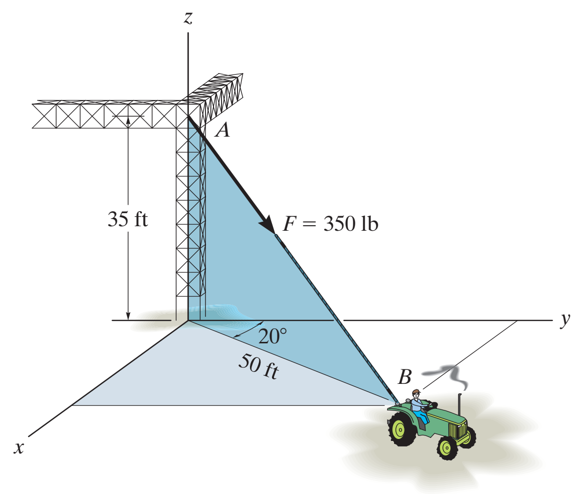
$$\phi = \tan^{-1}\left(\frac{1.981}{25}\right) = 4.53°$$

$$\theta = 180° + 4.53° = 185°$$ **Ans.**



2–146.

The cable attached to the tractor at B exerts a force of 350 lb on the framework. Express this force as a Cartesian vector.



SOLUTION

$$\mathbf{r} = 50 \sin 20°\mathbf{i} + 50 \cos 20°\mathbf{j} - 35\mathbf{k}$$

$$\mathbf{r} = \{17.10\mathbf{i} + 46.98\mathbf{j} - 35\mathbf{k}\} \text{ ft}$$

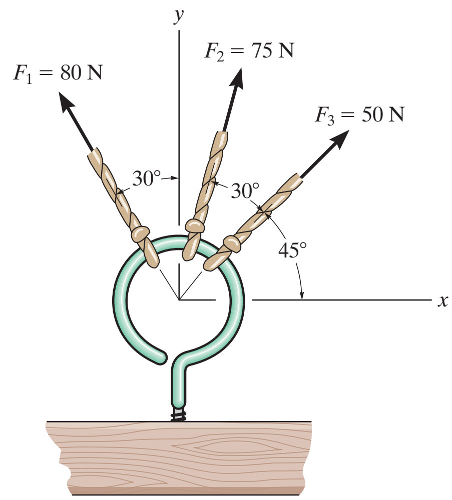
$$r = \sqrt{(17.10)^2 + (46.98)^2 + (-35)^2} = 61.03 \text{ ft}$$

$$\mathbf{u} = \frac{\mathbf{r}}{r} = (0.280\mathbf{i} + 0.770\mathbf{j} - 0.573\mathbf{k})$$

$$\mathbf{F} = F\mathbf{u} = \{98.1\mathbf{i} + 269\mathbf{j} - 201\mathbf{k}\} \text{ lb}$$ **Ans.**

2–147.

Determine the magnitude and direction of the resultant $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$ of the three forces by first finding the resultant $\mathbf{F}' = \mathbf{F}_1 + \mathbf{F}_3$ and then forming $\mathbf{F}_R = \mathbf{F}' + \mathbf{F}_2$. Specify its direction measured counterclockwise from the positive x axis.



SOLUTION

$$F' = \sqrt{(80)^2 + (50)^2 - 2(80)(50)\cos 105^\circ} = 104.7\ \text{N}$$

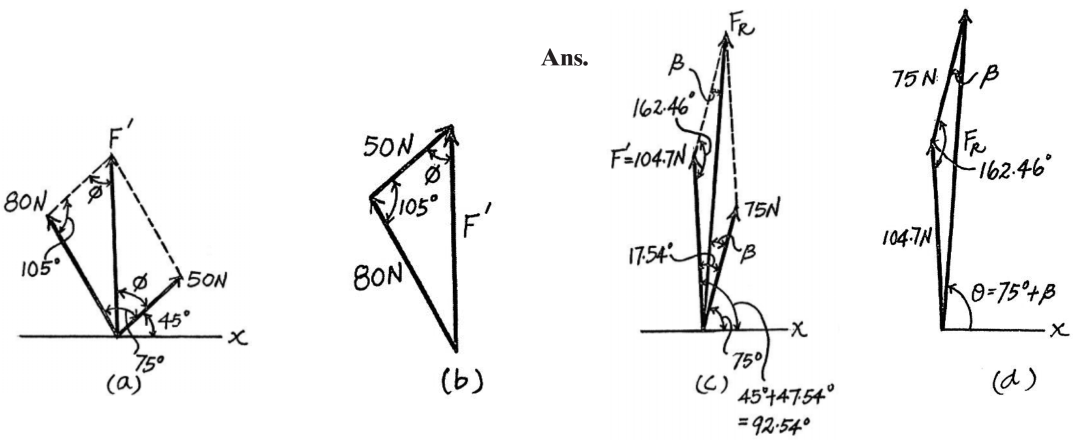
$$\frac{\sin\phi}{80} = \frac{\sin 105^\circ}{104.7}; \qquad \phi = 47.54^\circ$$

$$F_R = \sqrt{(104.7)^2 + (75)^2 - 2(104.7)(75)\cos 162.46^\circ}$$

$$F_R = 177.7 = 178\ \text{N}$$ **Ans.**

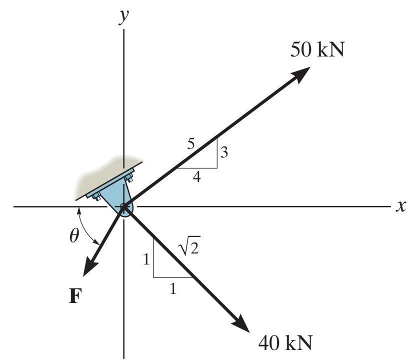
$$\frac{\sin\beta}{104.7} = \frac{\sin 162.46^\circ}{177.7}; \qquad \beta = 10.23^\circ$$

$$\theta = 75^\circ + 10.23^\circ = 85.2^\circ$$ **Ans.**



***2–148.**

If $\theta = 60°$ and $F = 20$ kN, determine the magnitude of the resultant force and its direction measured clockwise from the positive x axis.



SOLUTION

$\overset{+}{\rightarrow} F_{Rx} = \Sigma F_x; \qquad F_{Rx} = 50\left(\frac{4}{5}\right) + \frac{1}{\sqrt{2}}(40) - 20\cos 60° = 58.28 \text{ kN}$

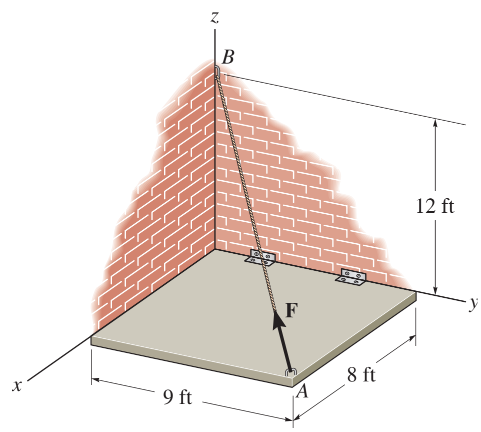
$+\uparrow F_{Ry} = \Sigma F_y; \qquad F_{Ry} = 50\left(\frac{3}{5}\right) - \frac{1}{\sqrt{2}}(40) - 20\sin 60° = -15.60 \text{ kN}$

$$F_R = \sqrt{(58.28)^2 + (-15.60)^2} = 60.3 \text{ kN}$$ **Ans.**

$$\phi = \tan^{-1}\left[\frac{15.60}{58.28}\right] = 15.0°$$ **Ans.**

2–149.

The hinged plate is supported by the cord AB. If the force in the cord is $F = 340$ lb, express this force, directed from A toward B, as a Cartesian vector. What is the length of the cord?



SOLUTION

Unit Vector:

$$\mathbf{r}_{AB} = \{(0 - 8)\mathbf{i} + (0 - 9)\mathbf{j} + (12 - 0)\mathbf{k}\}\ \text{ft}$$

$$= \{-8\mathbf{i} - 9\mathbf{j} + 12\mathbf{k}\}\ \text{ft}$$

$$r_{AB} = \sqrt{(-8)^2 + (-9)^2 + 12^2} = 17.0\ \text{ft} \qquad \textbf{Ans.}$$

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{-8\mathbf{i} - 9\mathbf{j} + 12\mathbf{k}}{17} = -\frac{8}{17}\mathbf{i} - \frac{9}{17}\mathbf{j} + \frac{12}{17}\mathbf{k}$$

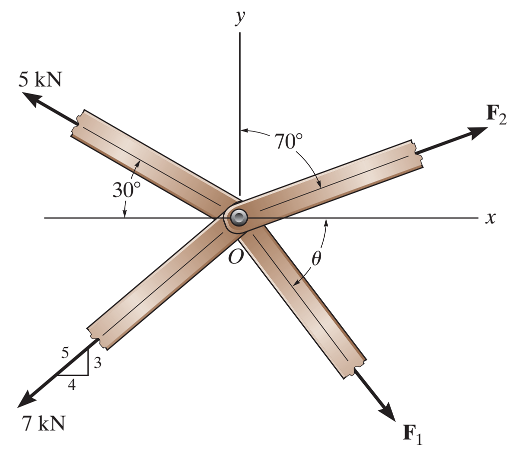
Force Vector:

$$\mathbf{F} = F\mathbf{u}_{AB} = 340\left\{-\frac{8}{17}\mathbf{i} - \frac{9}{17}\mathbf{j} + \frac{12}{17}\mathbf{k}\right\}\ \text{lb}$$

$$= \{-160\mathbf{i} - 180\mathbf{j} + 240\mathbf{k}\}\ \text{lb} \qquad \textbf{Ans.}$$

3–1.

The members of a truss are pin connected at joint O. Determine the magnitudes of $\mathbf{F}_1$ and $\mathbf{F}_2$ for equilibrium. Set $\theta = 60°$.



SOLUTION

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad F_2 \sin 70° + F_1 \cos 60° - 5 \cos 30° - \frac{4}{5}(7) = 0$

$0.9397F_2 + 0.5F_1 = 9.930$

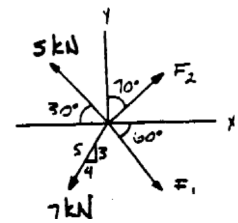
$+\uparrow \Sigma F_y = 0; \qquad F_2 \cos 70° + 5 \sin 30° - F_1 \sin 60° - \frac{3}{5}(7) = 0$

$0.3420F_2 - 0.8660F_1 = 1.7$

Solving:

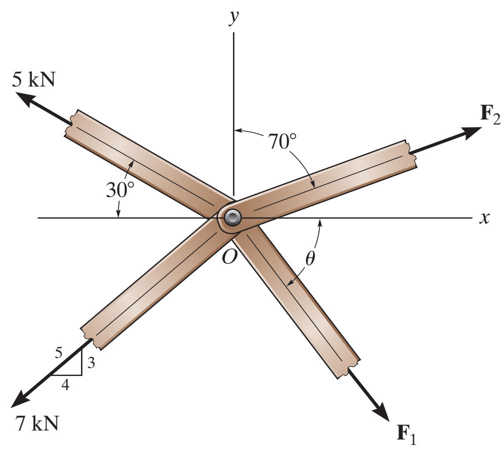
$F_2 = 9.60 \text{ kN}$ **Ans.**

$F_1 = 1.83 \text{ kN}$ **Ans.**



3–2.

The members of a truss are pin connected at joint O. Determine the magnitude of $\mathbf{F}_1$ and its angle θ for equilibrium. Set $F_2 = 6$ kN.



SOLUTION

$\xrightarrow{+} \Sigma F_x = 0;\quad 6 \sin 70^\circ + F_1 \cos\theta - 5\cos 30^\circ - \frac{4}{5}(7) = 0$

$F_1 \cos\theta = 4.2920$

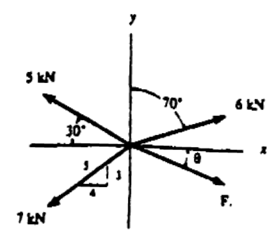
$+\uparrow \Sigma F_y = 0;\quad 6\cos 70^\circ + 5 \sin 30^\circ - F_1 \sin\theta - \frac{3}{5}(7) = 0$

$F_1 \sin\theta = 0.3521$

Solving:

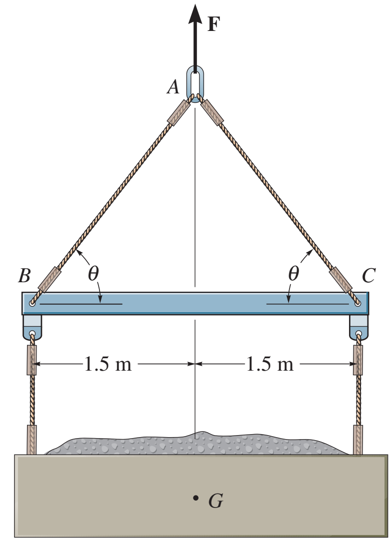
$\theta = 4.69^\circ$ **Ans.**

$F_1 = 4.31$ kN **Ans.**



3–3.

The lift sling is used to hoist a container having a mass of 500 kg. Determine the force in each of the cables AB and AC as a function of θ. If the maximum tension allowed in each cable is 5 kN, determine the shortest lengths of cables AB and AC that can be used for the lift. The center of gravity of the container is located at G.



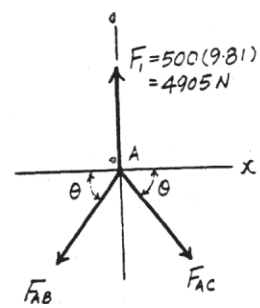
SOLUTION

Free-Body Diagram: By observation, the force $\mathbf{F}_1$ has to support the entire weight of the container. Thus, $F_1 = 500(9.81) = 4905$ N.

Equations of Equilibrium:

$\xrightarrow{+} \Sigma F_x = 0; \qquad F_{AC} \cos\theta - F_{AB} \cos\theta = 0 \quad F_{AC} = F_{AB} = F$

$+\uparrow \Sigma F_y = 0; \qquad 4905 - 2F \sin\theta = 0 \quad F = \{2452.5 \cos\theta\}$ N



Thus,

$$F_{AC} = F_{AB} = F = \{2.45 \cos\theta\} \text{ kN}$$ **Ans.**

If the maximum allowable tension in the cable is 5 kN, then

$$2452.5 \cos\theta = 5000$$

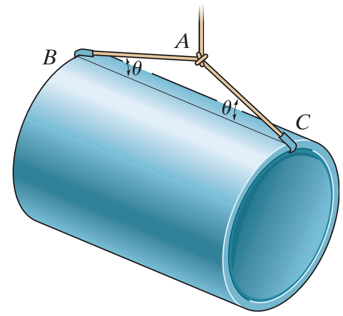
$$\theta = 29.37°$$

From the geometry, $l = \dfrac{1.5}{\cos\theta}$ and $\theta = 29.37°$. Therefore

$$l = \frac{1.5}{\cos 29.37°} = 1.72 \text{ m}$$ **Ans.**

***3–4.**

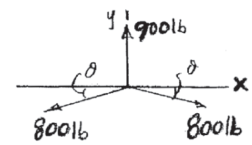
Cords AB and AC can each sustain a maximum tension of 800 lb. If the drum has a weight of 900 lb, determine the smallest angle θ at which they can be attached to the drum.



SOLUTION

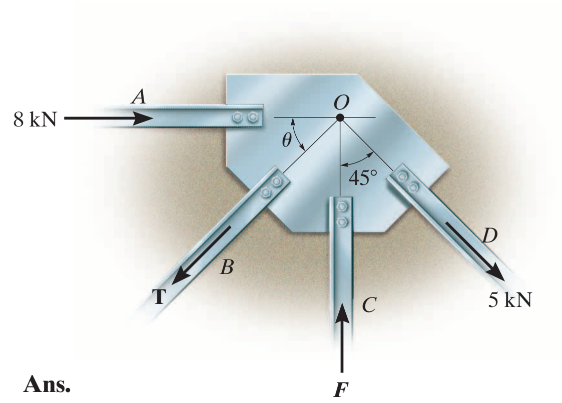
$+\uparrow \Sigma F_y = 0; \qquad 900 - 2(800)\sin\theta = 0$

$\theta = 34.2^\circ$ **Ans.**



3–5.

The members of a truss are connected to the gusset plate. If the forces are concurrent at point O, determine the magnitudes of **F** and **T** for equilibrium. Take $\theta = 30°$.



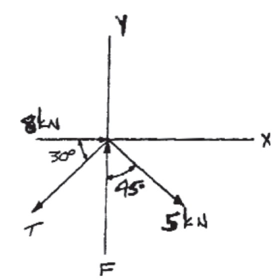
SOLUTION

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad -T \cos 30° + 8 + 5 \sin 45° = 0$

$T = 13.32 = 13.3 \text{ kN}$ **Ans.**

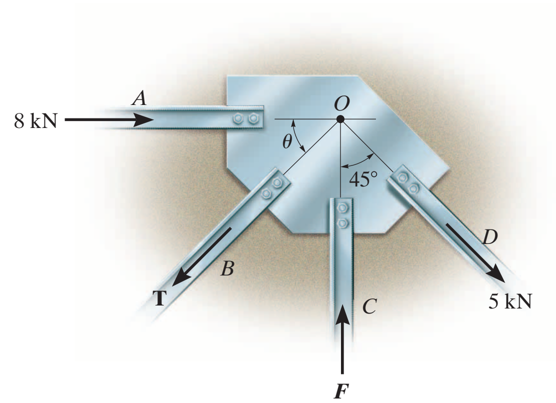
$+\uparrow \Sigma F_y = 0; \qquad F - 13.32 \sin 30° - 5 \cos 45° = 0$

$F = 10.2 \text{ kN}$ **Ans.**



3–6.

The gusset plate is subjected to the forces of four members. Determine the force in member B and its proper orientation θ for equilibrium. The forces are concurrent at point O. Take $F = 12$ kN.



SOLUTION

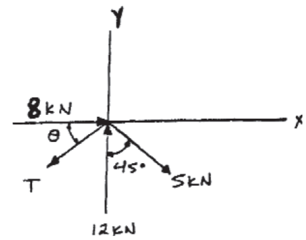
$\xrightarrow{+} \Sigma F_x = 0; \qquad 8 - T\cos\theta + 5\sin 45^\circ = 0$

$+\uparrow \Sigma F_y = 0; \qquad 12 - T\sin\theta - 5\cos 45^\circ = 0$

Solving,

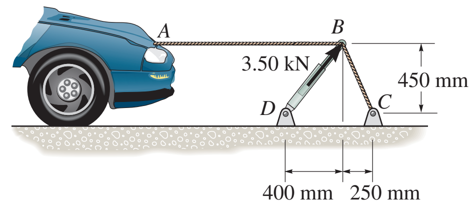
$$T = 14.3 \text{ kN} \qquad \textbf{Ans.}$$

$$\theta = 36.3^\circ \qquad \textbf{Ans.}$$



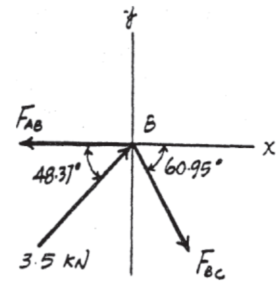
3–7.

The device shown is used to straighten the frames of wrecked autos. Determine the tension of each segment of the chain, i.e., AB and BC, if the force which the hydraulic cylinder DB exerts on point B is 3.50 kN, as shown.



SOLUTION

Equations of Equilibrium: A direct solution for F_{BC} can be obtained by summing forces along the y axis.



$+\uparrow \Sigma F_y = 0; \qquad 3.5 \sin 48.37° - F_{BC} \sin 60.95° = 0$

$$F_{BC} = 2.993 \text{ kN} = 2.99 \text{ kN}$$ **Ans.**

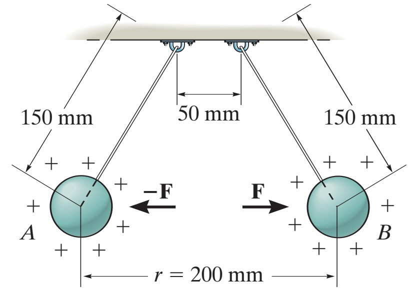
Using the result $F_{BC} = 2.993$ kN and summing forces along x axis, we have

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 3.5 \cos 48.37° + 2.993 \cos 60.95° - F_{AB} = 0$

$$F_{AB} = 3.78 \text{ kN}$$ **Ans.**

***3–8.**

Two electrically charged pith balls, each having a mass of 0.2 g, are suspended from light threads of equal length. Determine the resultant horizontal force of repulsion, F, acting on each ball if the measured distance between them is $r = 200$ mm.



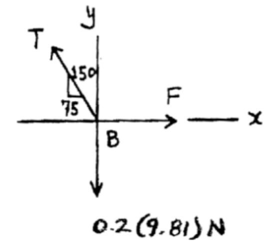
SOLUTION

$$\xrightarrow{+} \Sigma F_x = 0; \qquad F - T\left(\frac{75}{150}\right) = 0$$

$$+\uparrow \Sigma F_y = 0; \qquad T\left[\frac{\sqrt{150^2 - 75^2}}{150}\right] - 0.2(9.81)(10^{-3}) = 0$$

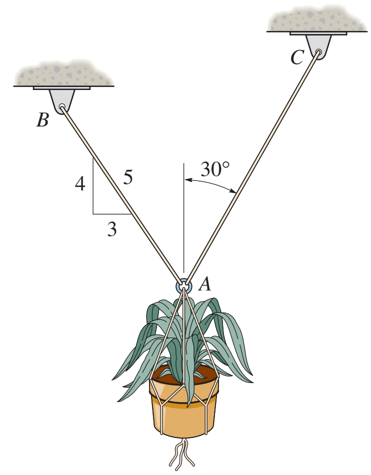
$$T = 2.266(10^{-3})\ \text{N}$$

$$F = 1.13\ \text{mN}$$ **Ans.**



3–9.

Determine the maximum weight of the flowerpot that can be supported without exceeding a cable tension of 50 lb in either cable AB or AC.



SOLUTION

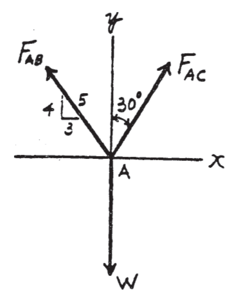
Equations of Equilibrium:

$\xrightarrow{+} \Sigma F_x = 0; \qquad F_{AC} \sin 30° - F_{AB}\left(\frac{3}{5}\right) = 0$

$$F_{AC} = 1.20 F_{AB} \qquad \textbf{(1)}$$

$+\uparrow \Sigma F_y = 0; \qquad F_{AC} \cos 30° + F_{AB}\left(\frac{4}{5}\right) - W = 0$

$$0.8660 F_{AC} + 0.8 F_{AB} = W \qquad \textbf{(2)}$$



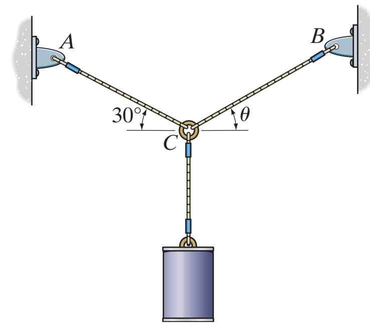
Since $F_{AC} > F_{AB}$, failure will occur first at cable AC with $F_{AC} = 50$ lb. Then solving Eqs. (1) and (2) yields

$$F_{AB} = 41.67 \text{ lb}$$

$$W = 76.6 \text{ lb} \qquad \textbf{Ans.}$$

3–10.

Determine the tension developed in wires CA and CB required for equilibrium of the 10-kg cylinder. Take $\theta = 40°$.



SOLUTION

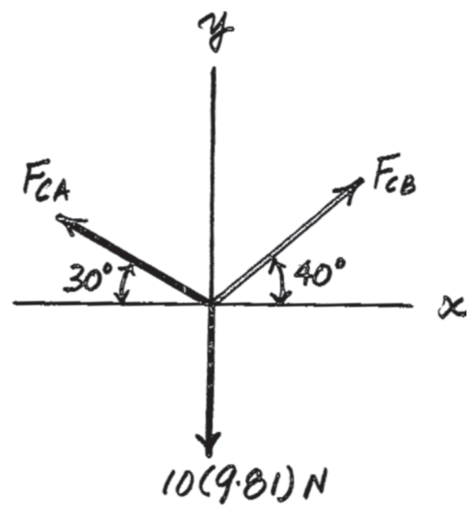
Equations of Equilibrium: Applying the equations of equilibrium along the x and y axes to the free-body diagram shown in Fig. a,

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad F_{CB} \cos 40° - F_{CA} \cos 30° = 0$ **(1)**

$+\uparrow \Sigma F_y = 0; \qquad F_{CB} \sin 40° + F_{CA} \sin 30° - 10(9.81) = 0$ **(2)**

Solving Eqs. (1) and (2), yields

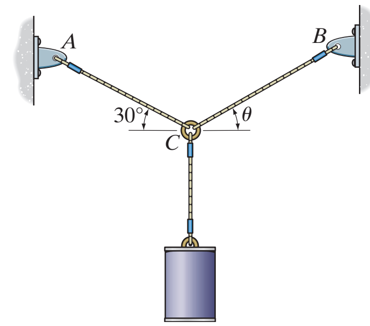
$F_{CA} = 80.0 \text{ N} \qquad F_{CB} = 90.4 \text{ N}$ **Ans.**



(a)

3–11.

If cable CB is subjected to a tension that is twice that of cable CA, determine the angle θ for equilibrium of the 10-kg cylinder. Also, what are the tensions in wires CA and CB?



SOLUTION

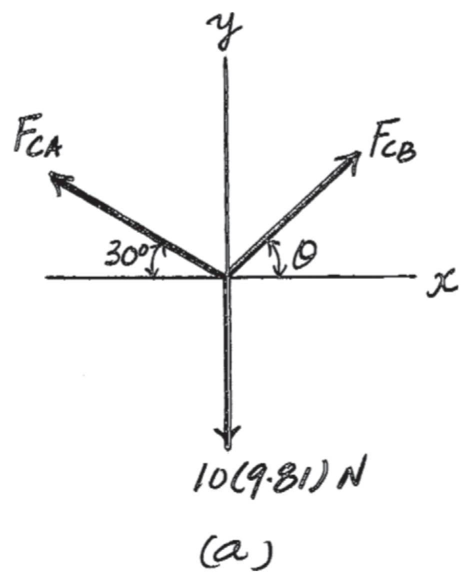
Equations of Equilibrium: Applying the equations of equilibrium along the x and y axes,

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\qquad F_{CB} \cos\theta - F_{CA} \cos 30^\circ = 0$ **(1)**

$+\uparrow \Sigma F_y = 0;$ $\qquad F_{CB} \sin\theta + F_{CA} \sin 30^\circ - 10(9.81) = 0$ **(2)**

However, it is required that

$$F_{CB} = 2F_{CA} \qquad \textbf{(3)}$$

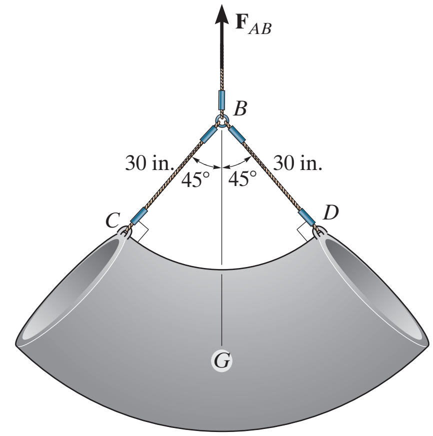


Solving Eqs. (1), (2), and (3), yields

$\theta = 64.3^\circ$ $\qquad F_{CB} = 85.2 \text{ N}$ $\qquad F_{CA} = 42.6 \text{ N}$ **Ans.**

***3–12.**

The concrete pipe elbow has a weight of 400 lb and the center of gravity is located at point G. Determine the force $\mathbf{F}_{AB}$ and the tension in cables BC and BD needed to support it.



SOLUTION

Free-Body Diagram: By observation, Force $\mathbf{F}_{AB}$ must equal the weight of the concrete pipe. Thus,

$$F_{AB} = 400 \text{ lb} \qquad \textbf{Ans.}$$

The tension force in cable CD is the same throughout the cable, that is $F_{BC} = F_{BD}$.

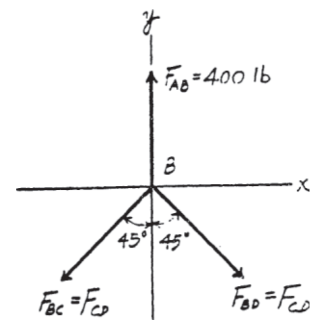
Equations of Equilibrium:

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad F_{BD} \sin 45° - F_{BC} \sin 45° = 0$

$F_{BC} = F_{BD} = F$

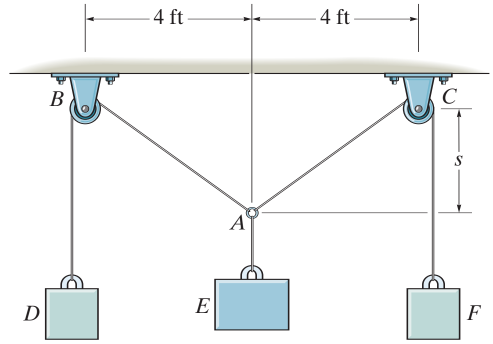
$+\uparrow \Sigma F_y = 0; \qquad 400 - 2F \cos 45° = 0$

$$F = F_{BD} = F_{BC} = 283 \text{ lb} \qquad \textbf{Ans.}$$



3–13.

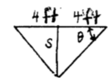
Blocks D and F weigh 5 lb each and block E weighs 8 lb. Determine the sag s for equilibrium. Neglect the size of the pulleys.



SOLUTION

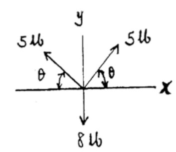
$+\uparrow \Sigma F_y = 0;$ $\quad 2(5)\sin\theta - 8 = 0$

$$\theta = \sin^{-1}(0.8) = 53.13°$$



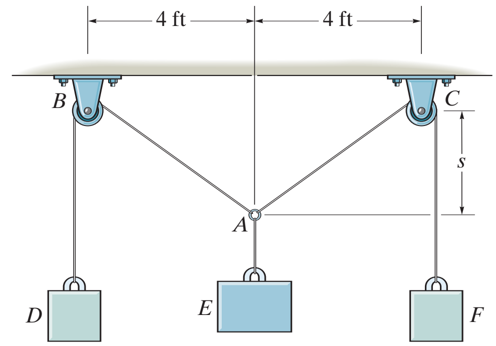
$$\tan\theta = \frac{s}{4}$$

$$s = 4\tan 53.13° = 5.33 \text{ ft}$$ **Ans.**



3–14.

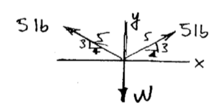
If blocks D and F weigh 5 lb each, determine the weight of block E if the sag $s = 3$ ft. Neglect the size of the pulleys.



SOLUTION

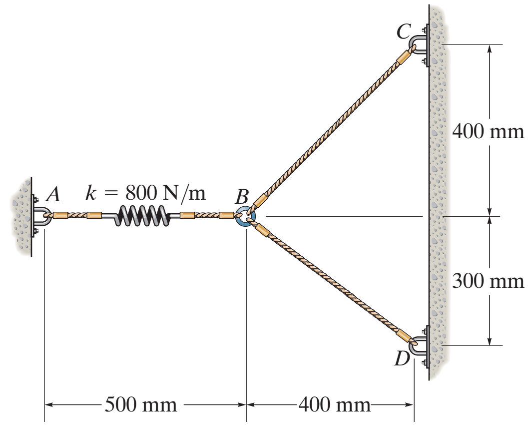
$$+\uparrow \Sigma F_y = 0; \qquad 2(5)\left(\frac{3}{5}\right) - W = 0$$

$$W = 6 \text{ lb}$$ **Ans.**



■3–15.

The spring has a stiffness of $k = 800$ N/m and an unstretched length of 200 mm. Determine the force in cables BC and BD when the spring is held in the position shown.



SOLUTION

The Force in The Spring: The spring stretches $s = l - l_0 = 0.5 - 0.2 = 0.3$ m. Applying Eq. 3–2, we have

$$F_{sp} = ks = 800(0.3) = 240 \text{ N}$$

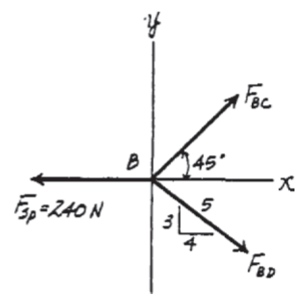
Equations of Equilibrium:

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad F_{BC} \cos 45° + F_{BD}\left(\frac{4}{5}\right) - 240 = 0$$

$$0.7071F_{BC} + 0.8F_{BD} = 240 \qquad \textbf{(1)}$$

$$+\uparrow \Sigma F_y = 0; \qquad F_{BC} \sin 45° - F_{BD}\left(\frac{3}{5}\right) = 0$$

$$F_{BC} = 0.8485F_{BD} \qquad \textbf{(2)}$$

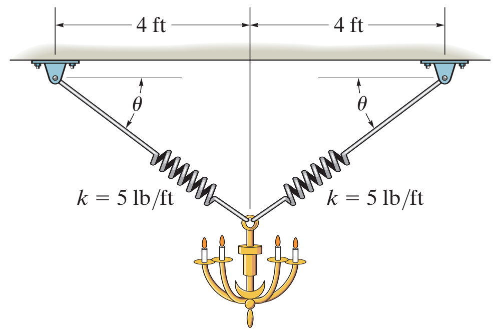


Solving Eqs. (1) and (2) yields,

$$F_{BD} = 171 \text{ N} \qquad F_{BC} = 145 \text{ N} \qquad \textbf{Ans.}$$

***■3–16.**

The 10-lb lamp fixture is suspended from two springs, each having an unstretched length of 4 ft and stiffness of $k = 5$ lb/ft. Determine the angle θ for equilibrium.



SOLUTION

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad T\cos\theta - T\cos\theta = 0$

$+\uparrow \Sigma F_y = 0;$ $\quad 2T\sin\theta - 10 = 0$

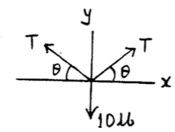
$$T\sin\theta = 5 \text{ lb}$$

$F = ks;$ $\quad T = 5\left(\dfrac{4}{\cos\theta} - 4\right)$

$$T = 20\left(\frac{1}{\cos\theta} - 1\right)$$

$$20\left(\frac{\sin\theta}{\cos\theta} - \sin\theta\right) = 5$$

$$\tan\theta - \sin\theta = 0.25$$

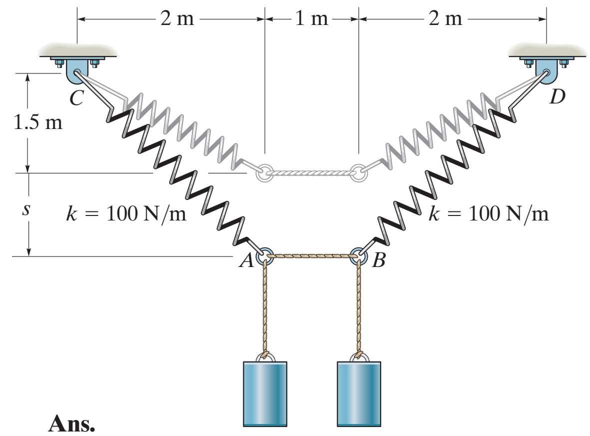


Solving by trial and error,

$$\theta = 43.0^\circ$$ **Ans.**

3–17.

Determine the mass of each of the two cylinders if they cause a sag of $s = 0.5$ m when suspended from the rings at A and B. Note that $s = 0$ when the cylinders are removed.

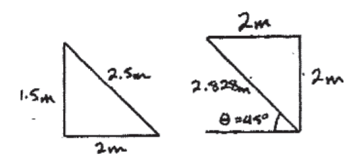


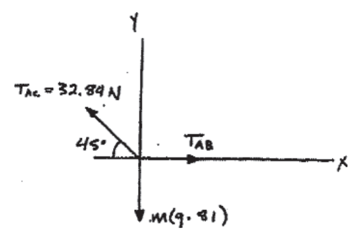
SOLUTION

$T_{AC} = 100\text{ N/m}\,(2.828 - 2.5) = 32.84\text{ N}$

$+\uparrow \Sigma F_y = 0; \qquad 32.84 \sin 45° - m(9.81) = 0$

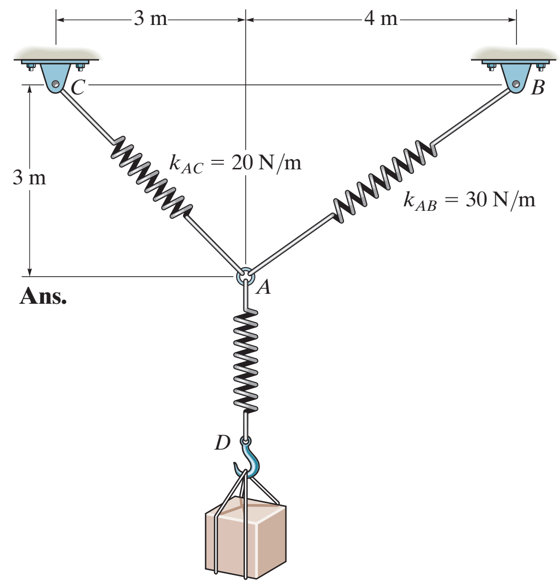
$m = 2.37$ kg **Ans.**





3–18.

Determine the stretch in springs AC and AB for equilibrium of the 2-kg block. The springs are shown in the equilibrium position.



SOLUTION

$F_{AD} = 2(9.81) = x_{AD}(40) \qquad x_{AD} = 0.4905 \text{ m}$ **Ans.**

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad F_{AB}\left(\frac{4}{5}\right) - F_{AC}\left(\frac{1}{\sqrt{2}}\right) = 0$

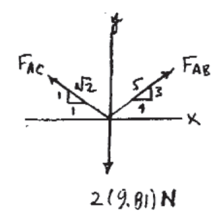
$+\uparrow \Sigma F_y = 0; \qquad F_{AC}\left(\frac{1}{\sqrt{2}}\right) + F_{AB}\left(\frac{3}{5}\right) - 2(9.81) = 0$

$F_{AC} = 15.86 \text{ N}$

$x_{AC} = \frac{15.86}{20} = 0.793 \text{ m}$ **Ans.**

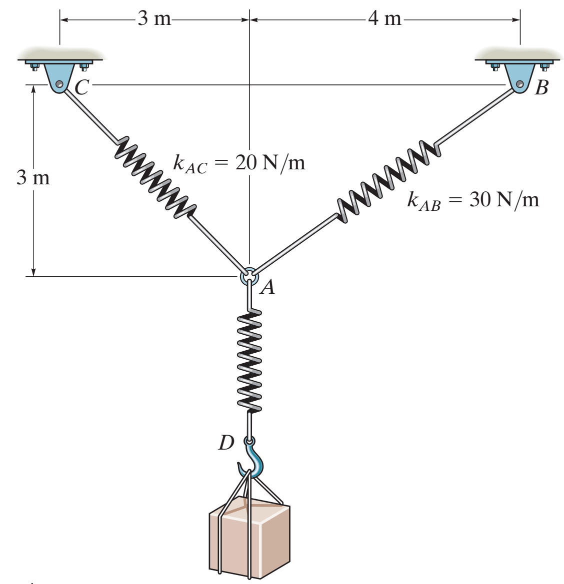
$F_{AB} = 14.01 \text{ N}$

$x_{AB} = \frac{14.01}{30} = 0.467 \text{ m}$ **Ans.**



3–19.

The unstretched length of spring AB is 3 m. If the block is held in the equilibrium position shown, determine the mass of the block at D.



SOLUTION

$$F = kx = 30(5 - 3) = 60 \text{ N}$$

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad T\cos 45° - 60\left(\frac{4}{5}\right) = 0$

$$T = 67.88 \text{ N}$$

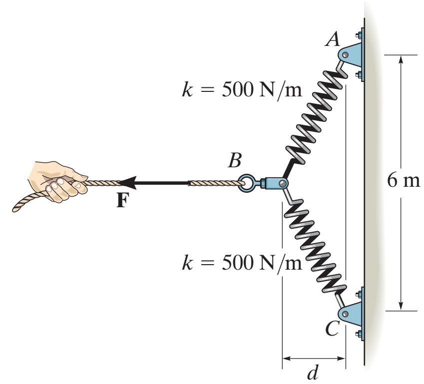
$+\uparrow \Sigma F_y = 0;$ $\quad -W + 67.88 \sin 45° + 60\left(\frac{3}{5}\right) = 0$

$$W = 84 \text{ N}$$

$$m = \frac{84}{9.81} = 8.56 \text{ kg}$$ **Ans.**

***3–20.**

The springs BA and BC each have a stiffness of 500 N/m and an unstretched length of 3 m. Determine the horizontal force **F** applied to the cord which is attached to the *small* ring B so that the displacement of the ring from the wall is $d = 1.5$ m.

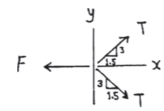


SOLUTION

$\xrightarrow{+} \Sigma F_x = 0; \qquad \dfrac{1.5}{\sqrt{11.25}}(T)(2) - F = 0$

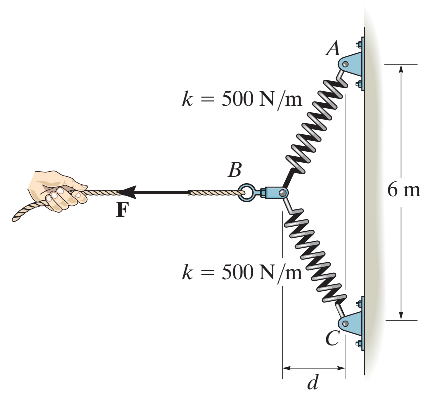
$T = ks = 500(\sqrt{3^2 + (1.5)^2} - 3) = 177.05 \text{ N}$

$F = 158 \text{ N}$ **Ans.**

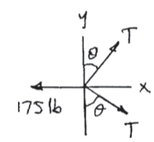


3–21.

The springs BA and BC each have a stiffness of 500 N/m and an unstretched length of 3 m. Determine the displacement d of the cord from the wall when a force $F = 175$ N is applied to the cord.



SOLUTION



$\xrightarrow{+} \Sigma F_x = 0; \qquad 175 = 2T \sin\theta$

$$T \sin\theta = 87.5$$

$$T\left[\frac{d}{\sqrt{3^2 + d^2}}\right] = 87.5$$

$$T = ks = 500(\sqrt{3^2 + d^2} - 3)$$

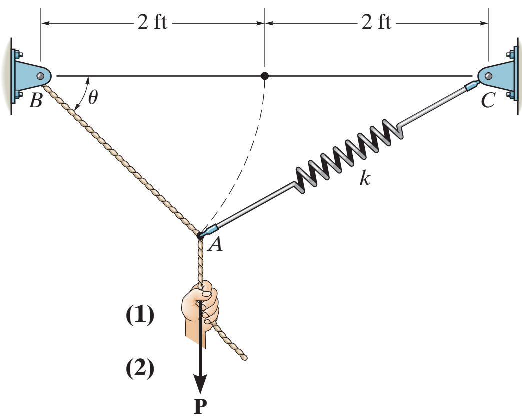
$$d\left(1 - \frac{3}{\sqrt{9 + d^2}}\right) = 0.175$$

By trial and error:

$$d = 1.56 \text{ m}$$ **Ans.**

■3–22.

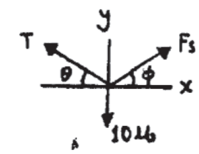
A vertical force $P = 10$ lb is applied to the ends of the 2-ft cord AB and spring AC. If the spring has an unstretched length of 2 ft, determine the angle θ for equilibrium. Take $k = 15$ lb/ft.



SOLUTION

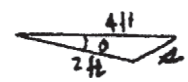
$\xrightarrow{+} \Sigma F_x = 0; \qquad F_s \cos\phi - T\cos\theta = 0 \qquad$ **(1)**

$+\uparrow \Sigma F_y = 0; \qquad T\sin\theta + F_s\sin\phi - 10 = 0 \qquad$ **(2)**



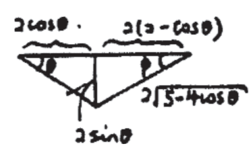
$$s = \sqrt{(4)^2 + (2)^2 - 2(4)(2)\cos\theta} - 2 = 2\sqrt{5 - 4\cos\theta} - 2$$

$$F_s = ks = 2k(\sqrt{5 - 4\cos\theta} - 1)$$



From Eq. (1): $T = F_s\left(\dfrac{\cos\phi}{\cos\theta}\right)$

$$T = 2k\left(\sqrt{5 - 4\cos\theta} - 1\right)\left(\frac{2 - \cos\theta}{\sqrt{5 - 4\cos\theta}}\right)\left(\frac{1}{\cos\theta}\right)$$



From Eq. (2):

$$\frac{2k\left(\sqrt{5-4\cos} - 1\right)(2-\cos\theta)}{\sqrt{5-4\cos\theta}}\tan\theta + \frac{2k\left(\sqrt{5-4\cos\theta} - 1\right)2\sin\theta}{2\sqrt{5-4\cos\theta}} = 10$$

$$\frac{\left(\sqrt{5-4\cos\theta} - 1\right)}{\sqrt{5-4\cos\theta}}(2\tan\theta - \sin\theta + \sin\theta) = \frac{10}{2k}$$

$$\frac{\tan\theta\left(\sqrt{5-4\cos\theta} - 1\right)}{\sqrt{5-4\cos\theta}} = \frac{10}{4k}$$

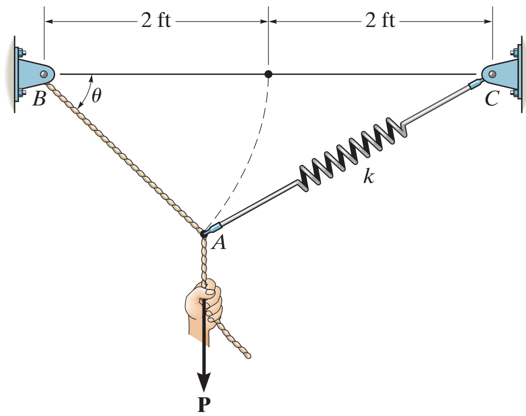
Set $k = 15$ lb/ft

Solving for θ by trial and error,

$$\theta = 35.0^\circ$$ **Ans.**

3–23.

Determine the unstretched length of spring AC if a force $P = 80$ lb causes the angle $\theta = 60°$ for equilibrium. Cord AB is 2 ft long. Take $k = 50$ lb/ft.



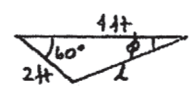
SOLUTION

$$l = \sqrt{4^2 + 2^2 - 2(2)(4)\cos 60°}$$

$$l = \sqrt{12}$$

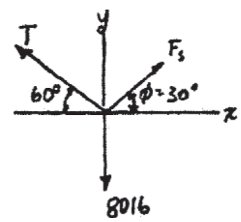
$$\frac{\sqrt{12}}{\sin 60°} = \frac{2}{\sin \phi}$$

$$\phi = \sin^{-1}\left(\frac{2 \sin 60°}{\sqrt{12}}\right) = 30°$$



$+\uparrow \Sigma F_y = 0; \qquad T \sin 60° + F_s \sin 30° - 80 = 0$

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad -T \cos 60° + F_s \cos 30° = 0$



Solving for F_s,

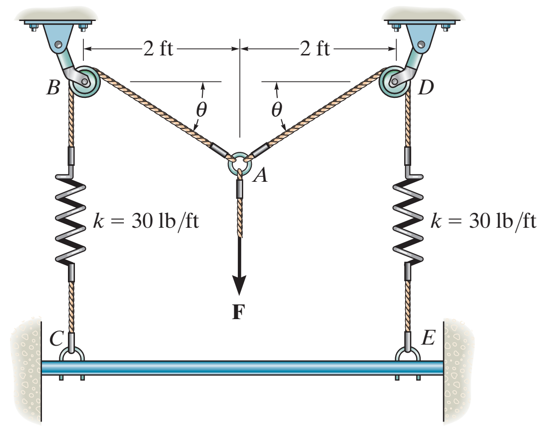
$F_s = 40$ lb

$F_s = kx$

$40 = 50(\sqrt{12} - l') \qquad l = \sqrt{12} - \dfrac{40}{50} = 2.66$ ft **Ans.**

***3–24.**

The springs on the rope assembly are originally unstretched when $\theta = 0°$. Determine the tension in each rope when $F = 90$ lb. Neglect the size of the pulleys at B and D.

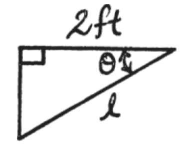


SOLUTION

$$l = \frac{2}{\cos\theta}$$

$$T = kx = k(l - l_0) = 30\left(\frac{2}{\cos\theta} - 2\right) = 60\left(\frac{1}{\cos\theta} - 1\right) \qquad \textbf{(1)}$$

$$+\uparrow \Sigma F_y = 0; \qquad 2T\sin\theta - 90 = 0 \qquad \textbf{(2)}$$



Substituting Eq. (1) into (2) yields:

$$120(\tan\theta - \sin\theta) - 90 = 0$$

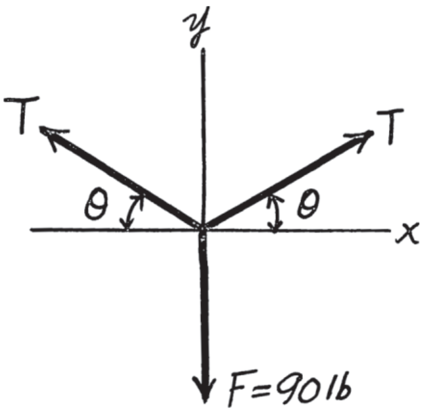
$$\tan\theta - \sin\theta = 0.75$$

By trial and error:

$$\theta = 57.957°$$

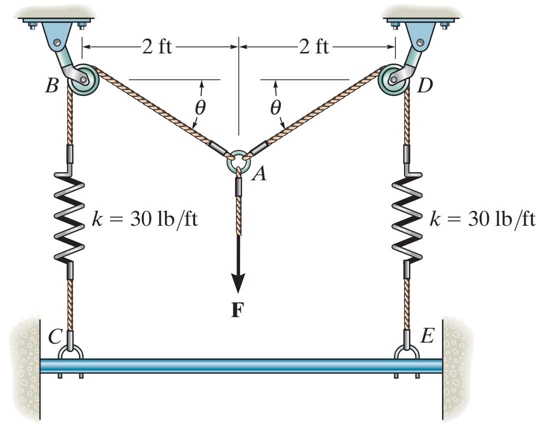
From Eq. (1),

$$T = 60\left(\frac{1}{\cos 57.957°} - 1\right) = 53.1 \text{ lb} \qquad \textbf{Ans.}$$



3–25.

The springs on the rope assembly are originally stretched 1 ft when $\theta = 0°$. Determine the vertical force F that must be applied so that $\theta = 30°$.



SOLUTION

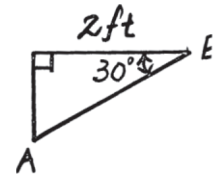
$$BA = \frac{2}{\cos 30°} = 2.3094 \text{ ft}$$

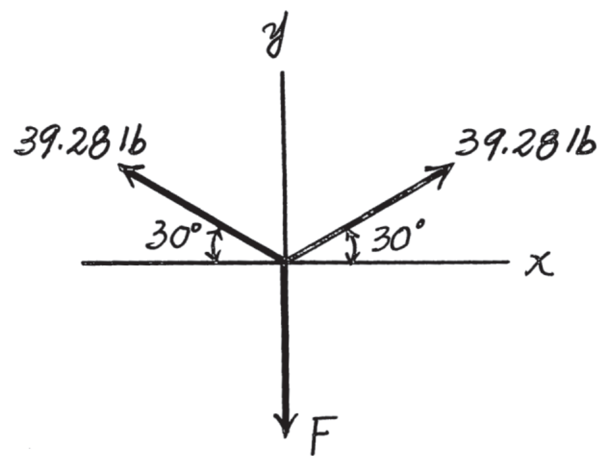
When $\theta = 30°$, the springs are stretched 1 ft + (2.3094 − 2) ft = 1.3094 ft

$F_s = kx = 30(1.3094) = 39.28$ lb

$+\uparrow \Sigma F_y = 0; \qquad 2(39.28) \sin 30° - F = 0$

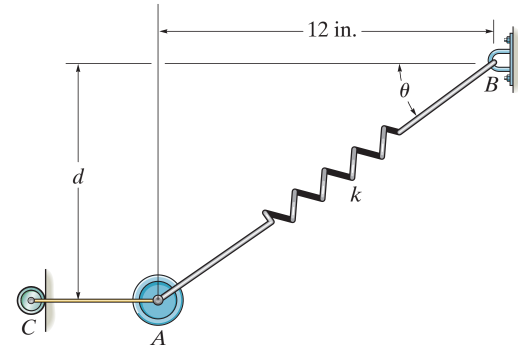
$F = 39.3$ lb **Ans.**





3–26.

The 10-lb weight A is supported by the cord AC and roller C, and by the spring that has a stiffness of $k = 10$ lb/in. If the unstretched length of the spring is 12 in. determine the distance d to where the weight is located when it is in equilibrium.



SOLUTION

$+\uparrow \Sigma F_y = 0; \qquad F_s \sin\theta - 10 = 0$

$F_s = kx; \qquad F_s = 10\left(\dfrac{12}{\cos\theta} - 12\right)$

$= 120(\sec\theta - 1)$

Thus,

$$120(\sec\theta - 1)\sin\theta = 10$$

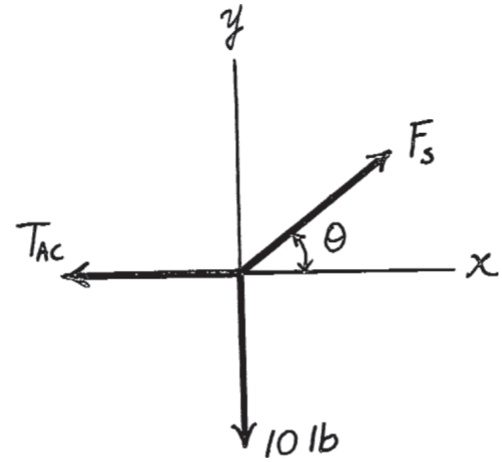
$$(\tan\theta - \sin\theta) = \frac{1}{12}$$

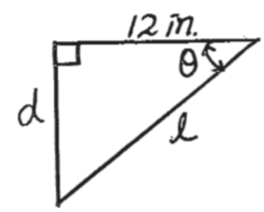
Solving,

$$\theta = 30.71^\circ$$

$$d = 12\tan 30.71^\circ = 7.13 \text{ in.}$$

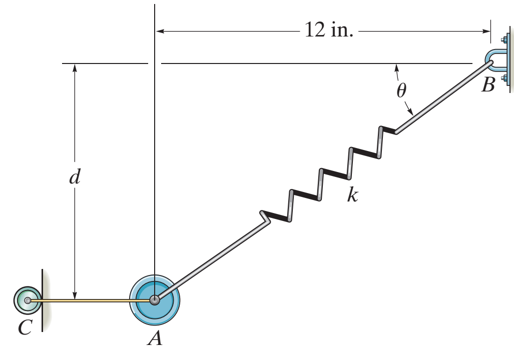
Ans.





3–27.

The 10-lb weight A is supported by the cord AC and roller C, and by spring AB. If the spring has an unstretched length of 8 in. and the weight is in equilibrium when $d = 4$ in., determine the stiffness k of the spring.



SOLUTION

$+\uparrow \Sigma F_y = 0; \qquad F_s \sin\theta - 10 = 0$

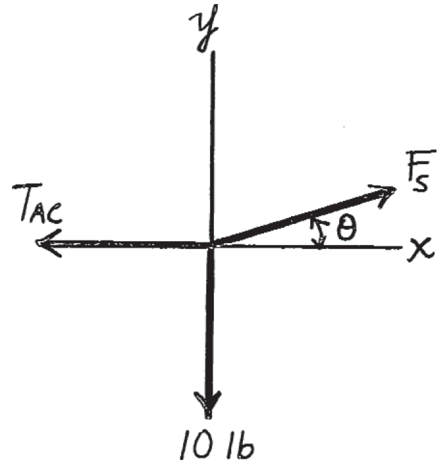
$F_s = kx; \qquad F_s = k\left(\dfrac{12}{\cos\theta} - 8\right)$

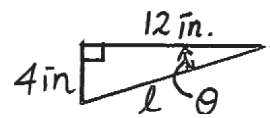
$\tan\theta = \dfrac{4}{12}; \qquad \theta = 18.435°$

Thus,

$$k\left(\frac{12}{\cos 18.435°} - 8\right)\sin 18.435° = 10$$

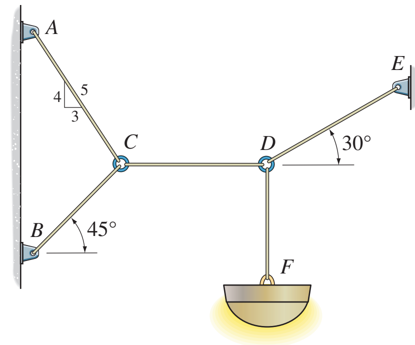
$$k = 6.80 \text{ lb/in.}$$ **Ans.**





***3–28.**

Determine the tension developed in each cord required for equilibrium of the 20-kg lamp.

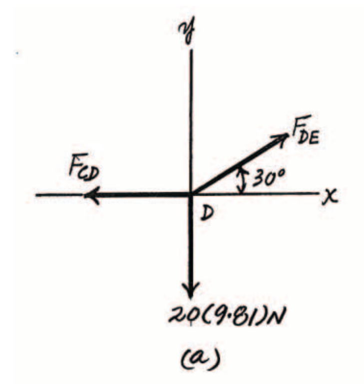


SOLUTION

Equations of Equilibrium: Applying the equations of equilibrium along the x and y axes to the free-body diagram of joint D shown in Fig. a, we have

$\overset{+}{\rightarrow}\Sigma F_x = 0;\quad F_{DE}\sin 30^\circ - 20(9.81) = 0 \qquad F_{DE} = 392.4\text{ N} = 392\text{ N}$ **Ans.**

$+\uparrow\Sigma F_y = 0;\quad 392.4\cos 30^\circ - F_{CD} = 0 \qquad F_{CD} = 339.83\text{ N} = 340\text{ N}$ **Ans.**



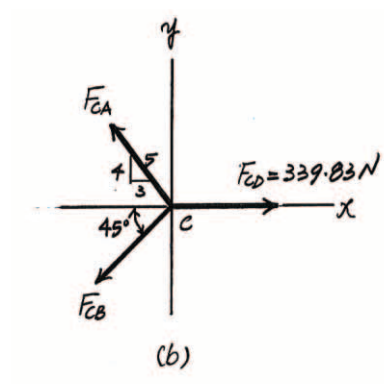
Using the result $F_{CD} = 339.83$ N and applying the equations of equilibrium along the x and y axes to the free-body diagram of joint D shown in Fig. b, we have

$$\overset{+}{\rightarrow}\Sigma F_x = 0;\qquad 339.83 - F_{CA}\left(\frac{3}{5}\right) - F_{CD}\cos 45^\circ = 0 \tag{1}$$

$$+\uparrow\Sigma F_y = 0;\qquad F_{CA}\left(\frac{4}{5}\right) - F_{CB}\sin 45^\circ = 0 \tag{2}$$

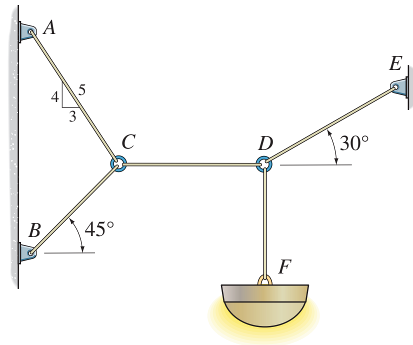
Solving Eqs. (1) and (2), yields

$F_{CB} = 275\text{ N} \qquad F_{CA} = 243\text{ N}$ **Ans.**



3–29.

Determine the maximum mass of the lamp that the cord system can support so that no single cord develops a tension exceeding 400 N.

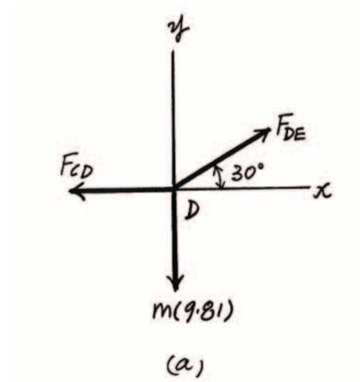


SOLUTION

Equations of Equilibrium: Applying the equations of equilibrium along the x and y axes to the free-body diagram of joint D shown in Fig. a, we have

$$+\uparrow \Sigma F_y = 0; \qquad F_{DE} \sin 30° - m(9.81) = 0 \qquad F_{DE} = 19.62m \qquad \textbf{Ans.}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 19.62m \cos 30° - F_{CD} = 0 \qquad F_{CD} = 16.99m \qquad \textbf{Ans.}$$



Using the result $F_{CD} = 16.99m$ and applying the equations of equilibrium along the x and y axes to the free-body diagram of joint D shown in Fig. b, we have

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 16.99m - F_{CA}\left(\frac{3}{5}\right) - F_{CD} \cos 45° = 0 \qquad \textbf{(1)}$$

$$+\uparrow \Sigma F_y = 0; \qquad F_{CA}\left(\frac{4}{5}\right) - F_{CB} \sin 45° = 0 \qquad \textbf{(2)}$$

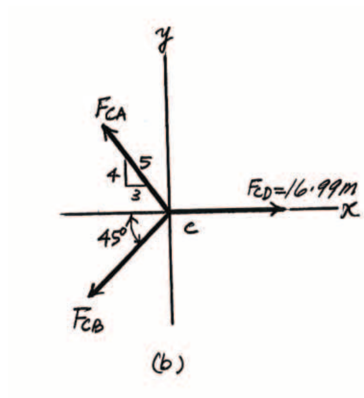
Solving Eqs. (1) and (2), yields

$F_{CB} = 13.73m \qquad F_{CA} = 12.14m$

Notice that cord DE is subjected to the greatest tensile force, and so it will achieve the maximum allowable tensile force first. Thus

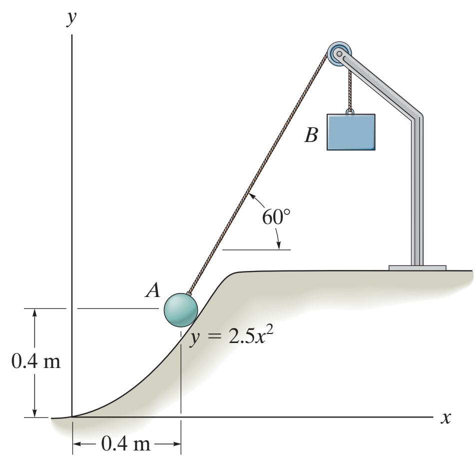
$$F_{DE} = 400 = 19.62m$$

$$m = 20.4 \text{ kg} \qquad \textbf{Ans.}$$



3–30.

A 4-kg sphere rests on the smooth parabolic surface. Determine the normal force it exerts on the surface and the mass m_B of block B needed to hold it in the equilibrium position shown.

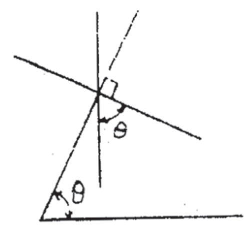


SOLUTION

Geometry: The angle θ which the surface makes with the horizontal is to be determined first.

$$\tan\theta\Big|_{x=0.4\text{ m}} = \frac{dy}{dx}\Big|_{x=0.4\text{ m}} = 5.0x\Big|_{x=0.4\text{ m}} = 2.00$$

$$\theta = 63.43^\circ$$



Free-Body Diagram: The tension in the cord is the same throughout the cord and is equal to the weight of block B, $W_B = m_B\,(9.81)$.

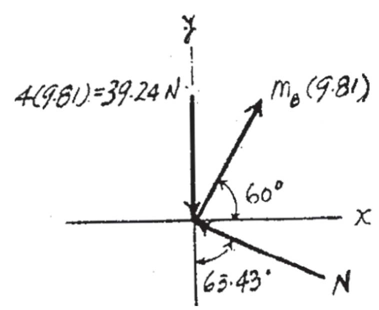
Equations of Equilibrium:

$\overset{+}{\rightarrow}\Sigma F_x = 0;\qquad m_B(9.81)\cos 60^\circ - N\sin 63.43^\circ = 0$

$$N = 5.4840 m_B \qquad \textbf{(1)}$$

$+\uparrow\Sigma F_y = 0;\qquad m_B(9.81)\sin 60^\circ + N\cos 63.43^\circ - 39.24 = 0$

$$8.4957 m_B + 0.4472N = 39.24 \qquad \textbf{(2)}$$

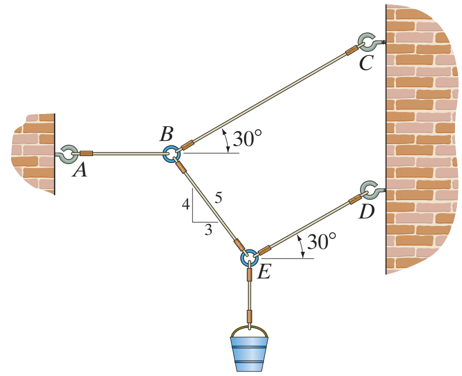


Solving Eqs. (1) and (2) yields

$$m_B = 3.58\text{ kg} \qquad N = 19.7\text{ N} \qquad \textbf{Ans.}$$

3–31.

If the bucket weighs 50 lb, determine the tension developed in each of the wires.



SOLUTION

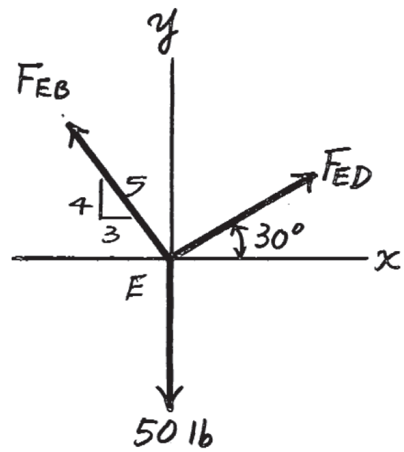
Equations of Equilibrium: First, we will apply the equations of equilibrium along the x and y axes to the free-body diagram of joint E shown in Fig. a.

$$\xrightarrow{+} \Sigma F_x = 0; \qquad F_{ED} \cos 30° - F_{EB}\left(\frac{3}{5}\right) = 0 \qquad \textbf{(1)}$$

$$+\uparrow \Sigma F_y = 0; \qquad F_{ED} \sin 30° + F_{EB}\left(\frac{4}{5}\right) - 50 = 0 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2), yields

$$F_{ED} = 30.2 \text{ lb} \qquad F_{EB} = 43.61 \text{ lb} = 43.6 \text{ lb} \qquad \textbf{Ans.}$$



(a)

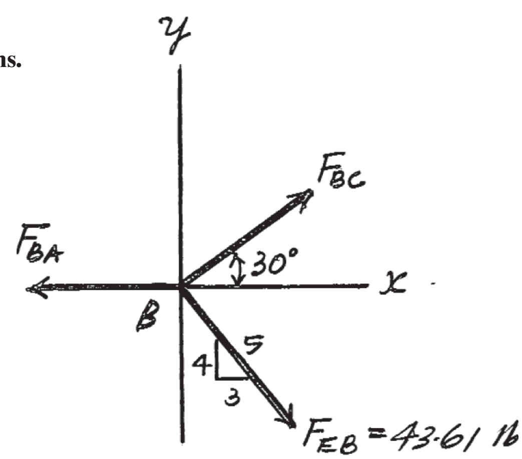
Using the result $F_{EB} = 43.61$ lb and applying the equations of equilibrium to the free-body diagram of joint B shown in Fig. b,

$$+\uparrow \Sigma F_y = 0; \qquad F_{BC} \sin 30° - 43.61\left(\frac{4}{5}\right) = 0$$

$$F_{BC} = 69.78 \text{ lb} = 69.8 \text{ lb} \qquad \textbf{Ans.}$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad 69.78 \cos 30° + 43.61\left(\frac{3}{5}\right) - F_{BA} = 0$$

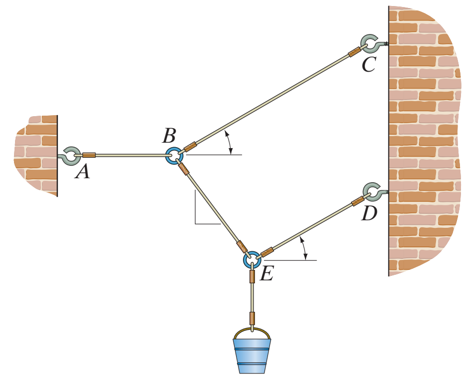
$$F_{BA} = 86.6 \text{ lb} \qquad \textbf{Ans.}$$



(b)

***3–32.**

Determine the maximum weight of the bucket that the wire system can support so that no single wire develops a tension exceeding 100 lb.

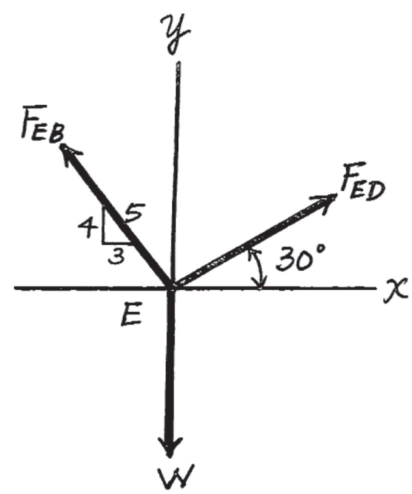


SOLUTION

Equations of Equilibrium: First,we will apply the equations of equilibrium along the x and y axes to the free-body diagram of joint E shown in Fig. a.

$$\xrightarrow{+} \Sigma F_x = 0; \qquad F_{ED}\cos 30^\circ - F_{EB}\left(\frac{3}{5}\right) = 0 \qquad \textbf{(1)}$$

$$+\uparrow \Sigma F_y = 0; \qquad F_{ED}\sin 30^\circ + F_{EB}\left(\frac{4}{5}\right) - W = 0 \qquad \textbf{(2)}$$



(a)

Solving,

$$F_{EB} = 0.8723W \qquad F_{ED} = 0.6043W$$

Using the result $F_{EB} = 0.8723W$ and applying the equations of equilibrium to the free-body diagram of joint B shown in Fig. b,

$$+\uparrow \Sigma F_y = 0; \qquad F_{BC}\sin 30^\circ - 0.8723W\left(\frac{4}{5}\right) = 0$$

$$F_{BC} = 1.3957W$$

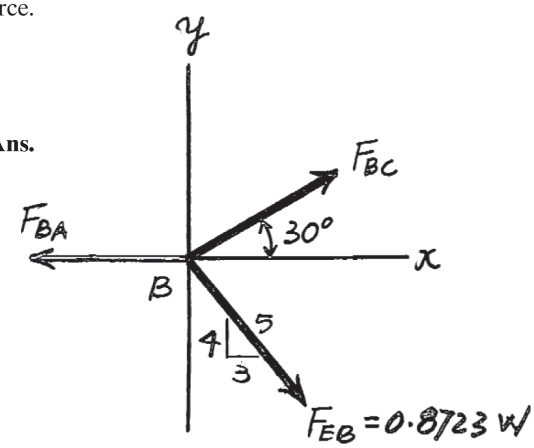
$$\xrightarrow{+} \Sigma F_x = 0; \qquad 1.3957W\cos 30^\circ + 0.8723W\left(\frac{3}{5}\right) - F_{BA} = 0$$

$$F_{BA} = 1.7320W$$

From these results, notice that wire BA is subjected to the greatest tensile force. Thus, it will achieve the maximum allowable tensile force first.

$$F_{BA} = 100 = 1.7320W$$

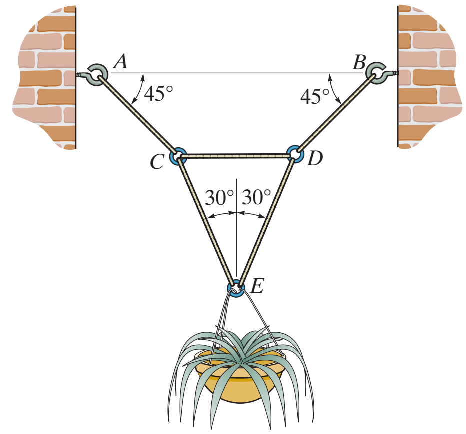
$$W = 57.7 \text{ lb} \qquad \textbf{Ans.}$$



(b)

3–33.

Determine the tension developed in each wire which is needed to support the 50-lb flowerpot.



SOLUTION

Equations of Equilibrium: First, we will apply the equations of equilibrium along the x and y axes to the free-body diagram of joint E shown in Fig. a.

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad F_{ED} \sin 30^\circ - F_{EC} \sin 30^\circ = 0 \qquad (1)$$

$$+\uparrow \Sigma F_y = 0; \qquad F_{ED} \cos 30^\circ + F_{EC} \cos 30^\circ - 50 = 0 \qquad (2)$$

Solving Eqs. (1) and (2), yields

$F_{ED} = F_{EC} = 28.87 \text{ lb} = 28.9 \text{ lb}$ **Ans.**

Using the result and applying the equations of equilibrium along the x and y axes to the free-body diagram of joint C shown in Fig. b, we have

$$+\uparrow \Sigma F_y = 0; \qquad F_{CA} \sin 45^\circ - 28.87 \cos 30^\circ = 0$$

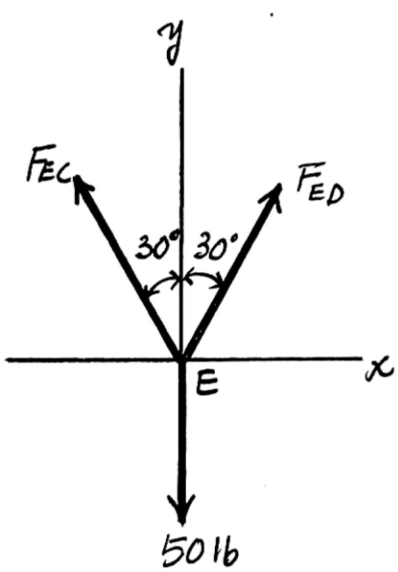
$F_{CA} = 35.36 \text{ lb} = 35.4 \text{ lb}$ **Ans.**

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad F_{CD} + 28.87 \sin 30^\circ - 35.36 \cos 45^\circ = 0$$

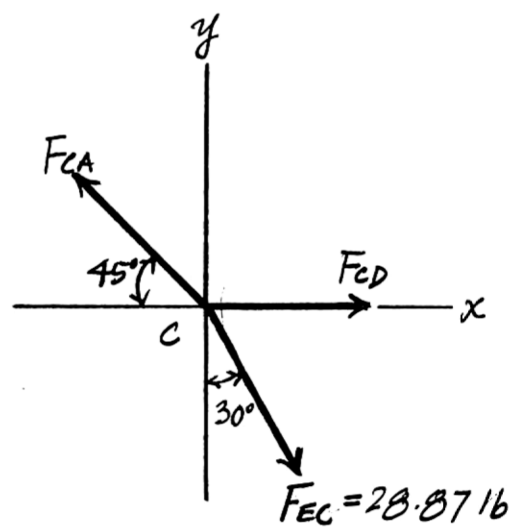
$F_{CD} = 10.6 \text{ lb}$ **Ans.**

Due to symmetry,

$F_{DB} = F_{CA} = 35.4 \text{ lb}$ **Ans.**



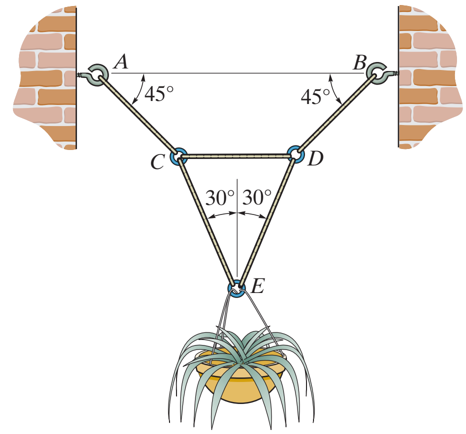
(a)



(b)

3–34.

If the tension developed in each of the wires is not allowed to exceed 40 lb, determine the maximum weight of the flowerpot that can be safely supported.



SOLUTION

Equations of Equilibrium: First, we will apply the equations of equilibrium along the x and y axes to the free-body diagram of joint E shown in Fig. a.

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad F_{ED}\sin 30^\circ - F_{EC}\sin 30^\circ = 0 \qquad \textbf{(1)}$$

$$+\uparrow\Sigma F_y = 0; \qquad F_{ED}\cos 30^\circ + F_{EC}\cos 30^\circ - W = 0 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2), yields

$$F_{ED} = F_{EC} = 0.5774W \qquad \textbf{Ans.}$$

Using the result $F_{EC} = 0.5774W$ and applying the equations of equilibrium along the x and y axes to the free-body diagram of joint C shown in Fig. b, we have

$$+\uparrow\Sigma F_y = 0; \qquad F_{CA}\sin 45^\circ - 0.5774W\cos 30^\circ = 0$$

$$F_{CA} = 0.7071W$$

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad F_{CD} + 0.5774W\sin 30^\circ - 0.7071W\cos 45^\circ = 0$$

$$F_{CD} = 0.2113W$$

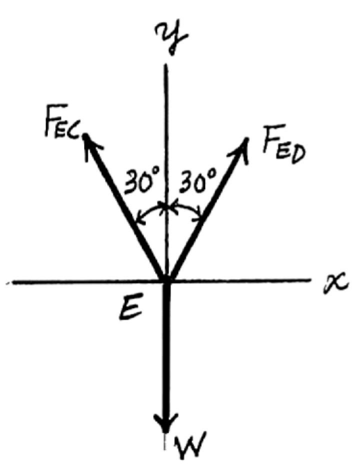
Due to symmetry,

$$F_{DB} = F_{CA} = 0.7071W$$

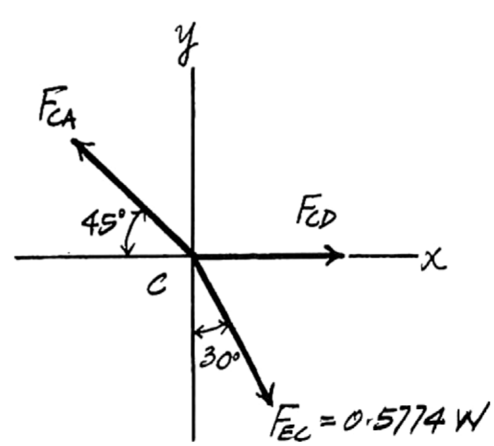
From this result, notice that cables DB and CA are subjected to the greater tensile forces. Thus, they will achieve the maximum allowable tensile force first.

$$F_{DB} = F_{CA} = 0.7071W$$

$$W = 56.6 \text{ lb} \qquad \textbf{Ans.}$$



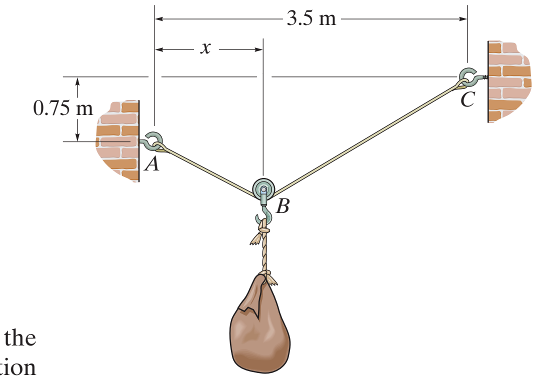
(a)



(b)

3–35.

Cable ABC has a length of 5 m. Determine the position x and the tension developed in ABC required for equilibrium of the 100-kg sack. Neglect the size of the pulley at B.



SOLUTION

Equations of Equilibrium: Since cable ABC passes over the smooth pulley at B, the tension in the cable is constant throughout its entire length. Applying the equation of equilibrium along the y axis to the free-body diagram in Fig. a, we have

$$+\uparrow \Sigma F_y = 0; \qquad 2T \sin \phi - 100(9.81) = 0 \qquad (1)$$

Geometry: Referring to Fig. b, we can write

$$\frac{3.5 - x}{\cos \phi} + \frac{x}{\cos \phi} = 5$$

$$\phi = \cos^{-1}\left(\frac{3.5}{5}\right) = 45.57°$$

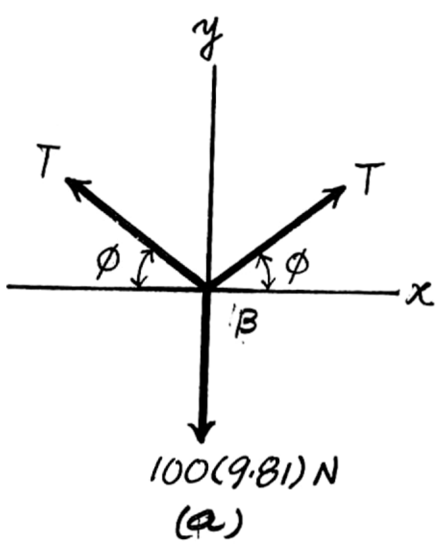
Also,

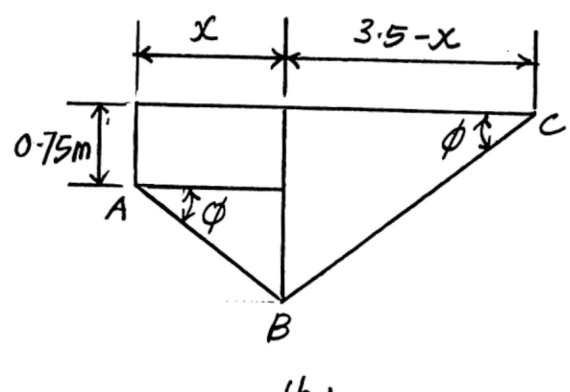
$$x \tan 45.57° + 0.75 = (3.5 - x) \tan 45.57°$$

$$x = 1.38 \text{ m}$$ **Ans.**

Substituting $\phi = 45.57°$ into Eq. (1), yields

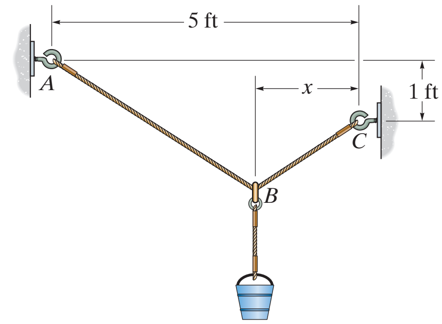
$$T = 687 \text{ N}$$ **Ans.**





***3–36.**

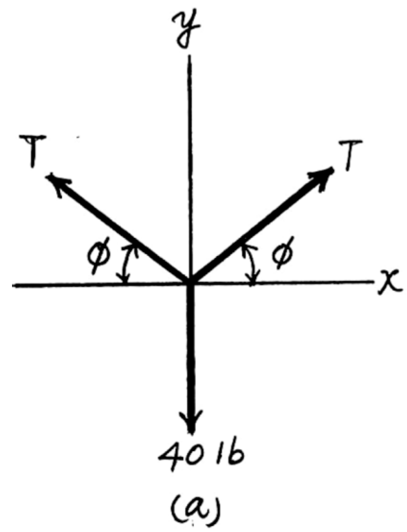
The single elastic cord ABC is used to support the 40-lb load. Determine the position x and the tension in the cord that is required for equilibrium. The cord passes through the smooth ring at B and has an unstretched length of 6 ft and stiffness of $k = 50$ lb/ft.



SOLUTION

Equations of Equilibrium: Since elastic cord ABC passes over the smooth ring at B, the tension in the cord is constant throughout its entire length. Applying the equation of equilibrium along the y axis to the free-body diagram in Fig. a, we have

$+\uparrow \Sigma F_y = 0;$ $$2T \sin\phi - 40 = 0 \qquad \textbf{(1)}$$



Geometry: Referring to Fig. (b), the stretched length of cord ABC is

$$l_{ABC} = \frac{x}{\cos\phi} + \frac{5-x}{\cos\phi} = \frac{5}{\cos\phi} \qquad \textbf{(2)}$$

Also,

$$x\tan\phi + 1 = (5-x)\tan\phi$$

$$x = \frac{5\tan\phi - 1}{2\tan\phi} \qquad \textbf{(3)}$$

Spring Force Formula: Applying the spring force formula using Eq. (2), we obtain

$$F_{sp} = k(l_{ABC} - l_0)$$

$$T = 50\left[\frac{5}{\cos\phi} - 6\right] \qquad \textbf{(4)}$$

Substituting Eq. (4) into Eq. (1) yields

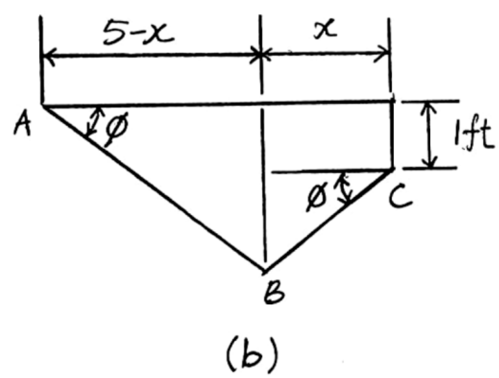
$$5\tan\phi - 6\sin\phi = 0.4$$

Solving the above equation by trial and error

$$\phi = 40.86^\circ$$

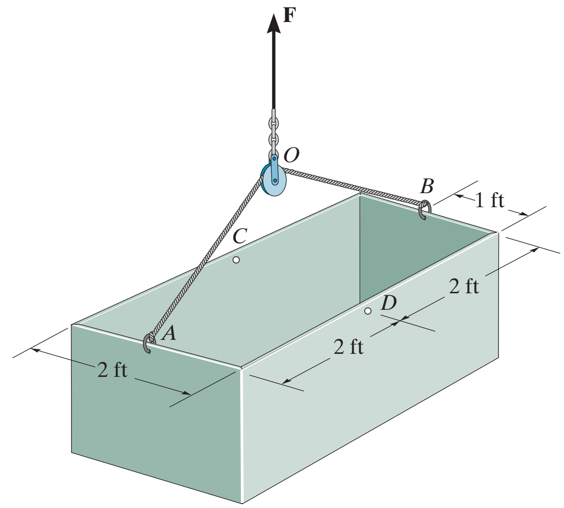
Substituting $\phi = 40.86^\circ$ into Eqs. (1) and (3) yields

$$T = 30.6 \text{ lb} \qquad x = 1.92 \text{ ft} \qquad \textbf{Ans.}$$



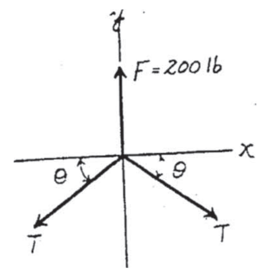
3–37.

The 200-lb uniform tank is suspended by means of a 6-ft-long cable, which is attached to the sides of the tank and passes over the small pulley located at O. If the cable can be attached at either points A and B, or C and D, determine which attachment produces the least amount of tension in the cable. What is this tension?



SOLUTION

Free-Body Diagram: By observation, the force **F** has to support the entire weight of the tank. Thus, $F = 200$ lb. The tension in cable AOB or COD is the same throughout the cable.

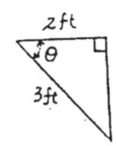


Equations of Equilibrium:

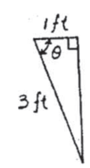
$\xrightarrow{+} \Sigma F_x = 0; \qquad T\cos\theta - T\cos\theta = 0 \qquad$ ***(Satisfied!)***

$+\uparrow \Sigma F_y = 0; \qquad 200 - 2T\sin\theta = 0 \qquad T = \dfrac{100}{\sin\theta}$ **(1)**

From the function obtained above, one realizes that in order to produce the least amount of tension in the cable, $\sin\theta$ hence θ must be as great as possible. Since the attachment of the cable to point C and D produces a greater θ $\left(\theta = \cos^{-1}\frac{1}{3} = 70.53°\right)$ as compared to the attachment of the cable to points A and B $\left(\theta = \cos^{-1}\frac{2}{3} = 48.19°\right)$,



the attachment of the cable to point C and D will produce the least amount of tension in the cable. **Ans.**

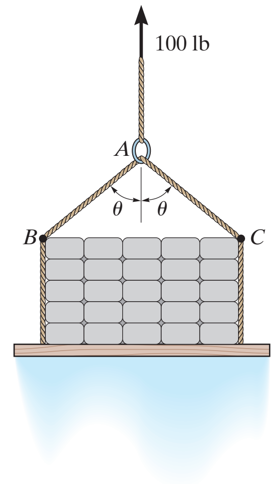


Thus,

$$T = \frac{100}{\sin 70.53°} = 106 \text{ lb}$$ **Ans.**

3–38.

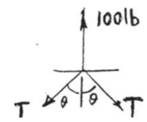
The sling *BAC* is used to lift the 100-lb load with constant velocity. Determine the force in the sling and plot its value T (ordinate) as a function of its orientation θ, where $0 \leq \theta \leq 90°$.

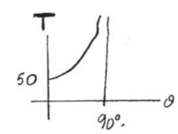


SOLUTION

$$+\uparrow \Sigma F_{xy} = 0; \qquad 100 - 2T\cos\theta = 0$$

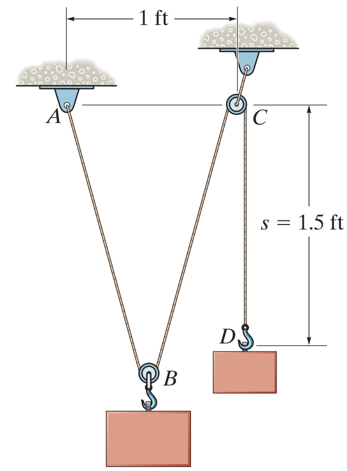
$$T = \{50 \sec\theta\} \text{ lb} \qquad \textbf{Ans.}$$





■3–39.

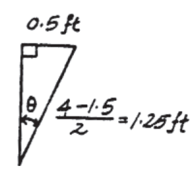
A "scale" is constructed with a 4-ft-long cord and the 10-lb block D. The cord is fixed to a pin at A and passes over two *small* pulleys at B and C. Determine the weight of the suspended block at B if the system is in equilibrium when $s = 1.5$ ft.



SOLUTION

Free-Body Diagram: The tension force in the cord is the same throughout the cord, that is, 10 lb. From the geometry,

$$\theta = \sin^{-1}\left(\frac{0.5}{1.25}\right) = 23.58°$$

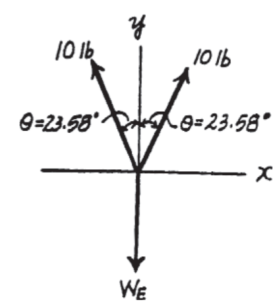


Equations of Equilibrium:

$$\xrightarrow{+} \Sigma F_x = 0; \qquad 10 \sin 23.58° - 10 \sin 23.58° = 0 \qquad \textbf{(\textit{Satisfied!})}$$

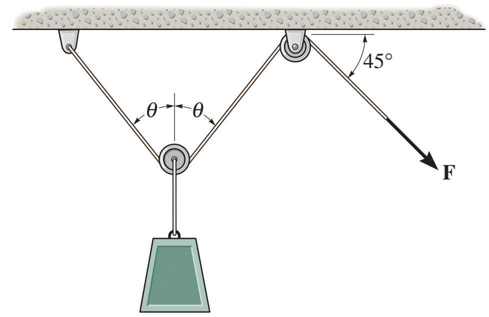
$$+\uparrow \Sigma F_y = 0; \qquad 2(10) \cos 23.58° - W_B = 0$$

$$W_B = 18.3 \text{ lb} \qquad \textbf{Ans.}$$



***3–40.**

The load has a mass of 15 kg and is lifted by the pulley system shown. Determine the force **F** in the cord as a function of the angle θ. Plot the function of force F versus the angle θ for $0 \leq \theta \leq 90°$.



SOLUTION

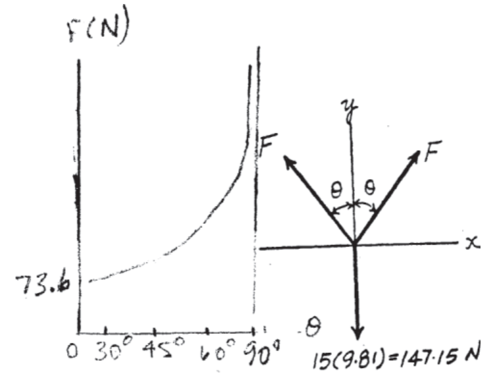
Free-Body Diagram: The tension force is the same throughout the cord.

Equations of Equilibrium:

$\xrightarrow{+} \Sigma F_x = 0;$ $\quad F \sin \theta - F \sin \theta = 0$ **(*Satisfied!*)**

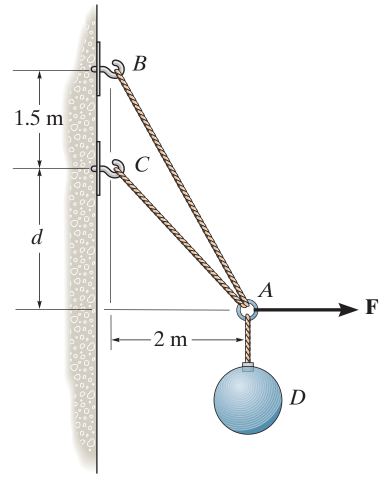
$+\uparrow \Sigma F_y = 0;$ $\quad 2F \cos \theta - 147.15 = 0$

$$F = \{73.6 \sec \theta\} \text{ N}$$ **Ans.**



3–41.

Determine the forces in cables AC and AB needed to hold the 20-kg ball D in equilibrium. Take $F = 300$ N and $d = 1$ m.



SOLUTION

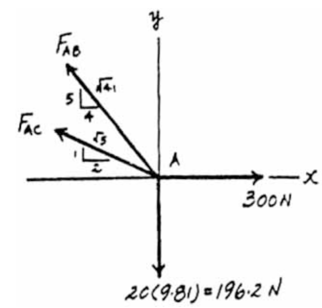
Equations of Equilibrium:

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad 300 - F_{AB}\left(\frac{4}{\sqrt{41}}\right) - F_{AC}\left(\frac{2}{\sqrt{5}}\right) = 0$$

$$06247F_{AB} + 0.8944F_{AC} = 300 \qquad \textbf{(1)}$$

$$+\uparrow\Sigma F_y = 0; \qquad F_{AB}\left(\frac{5}{\sqrt{41}}\right) + F_{AC}\left(\frac{1}{\sqrt{5}}\right) - 196.2 = 0$$

$$0.7809F_{AB} + 0.4472F_{AC} = 196.2 \qquad \textbf{(2)}$$

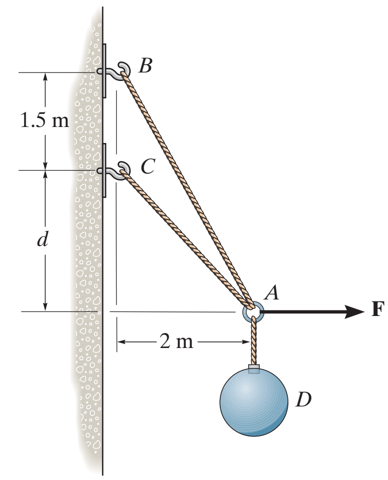


Solving Eqs. (1) and (2) yields

$F_{AB} = 98.6$ N $\qquad F_{AC} = 267$ N **Ans.**

3–42.

The ball D has a mass of 20 kg. If a force of $F = 100$ N is applied horizontally to the ring at A, determine the largest dimension d so that the force in cable AC is zero.



SOLUTION

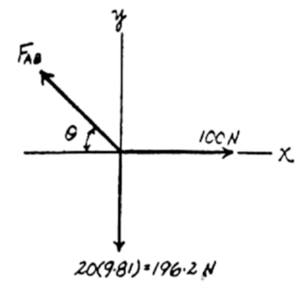
Equations of Equilibrium:

$\xrightarrow{+} \Sigma F_x = 0;\quad 100 - F_{AB}\cos\theta = 0 \quad F_{AB}\cos\theta = 100$ **(1)**

$+\uparrow \Sigma F_y = 0;\quad F_{AB}\sin\theta - 196.2 = 0 \quad F_{AB}\sin\theta = 196.2$ **(2)**

Solving Eqs. (1) and (2) yields

$$\theta = 62.99^\circ \qquad F_{AB} = 220.21\text{ N}$$



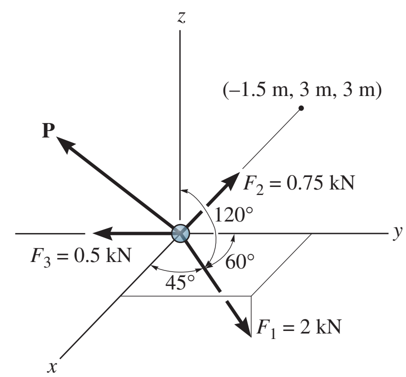
From the geometry,

$$d + 1.5 = 2\tan 62.99^\circ$$

$$d = 2.42\text{ m}$$ **Ans.**

3–43.

Determine the magnitude and direction of the force **P** required to keep the concurrent force system in equilibrium.



SOLUTION

Cartesian Vector Notation:

$$\mathbf{F}_1 = 2\{\cos 45°\mathbf{i} + \cos 60°\mathbf{j} + \cos 120°\mathbf{k}\}\text{ kN} = \{1.414\mathbf{i} + 1.00\mathbf{j} - 1.00\mathbf{k}\}\text{ kN}$$

$$\mathbf{F}_2 = 0.75\left(\frac{-1.5\mathbf{i} + 3\mathbf{j} + 3\mathbf{k}}{\sqrt{(-1.5)^2 + 3^2 + 3^2}}\right) = \{-0.250\mathbf{i} + 0.50\mathbf{j} + 0.50\mathbf{k}\}\text{ kN}$$

$$\mathbf{F}_3 = \{-0.50\mathbf{j}\}\text{ kN}$$

$$\mathbf{P} = P_x\mathbf{i} + P_y\mathbf{j} + P_z\mathbf{k}$$

Equations of Equilibrium:

$$\Sigma\mathbf{F} = \mathbf{0}; \qquad \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \mathbf{P} = \mathbf{0}$$

$$(P_x + 1.414 - 0.250)\,\mathbf{i} + (P_y + 1.00 + 0.50 - 0.50)\,\mathbf{j} + (P_z - 1.00 + 0.50)\,\mathbf{k} = \mathbf{0}$$

Equating **i**, **j**, and **k** components, we have

$$P_x + 1.414 - 0.250 = 0 \qquad P_x = -1.164\text{ kN}$$

$$P_y + 1.00 + 0.50 - 0.50 = 0 \qquad P_y = -1.00\text{ kN}$$

$$P_z - 1.00 + 0.50 = 0 \qquad P_z = 0.500\text{ kN}$$

The magnitude of **P** is

$$P = \sqrt{P_x^2 + P_y^2 + P_z^2}$$

$$= \sqrt{(-1.164)^2 + (-1.00)^2 + (0.500)^2}$$

$$= 1.614\text{ kN} = 1.61\text{ kN} \qquad \textbf{Ans.}$$

The coordinate direction angles are

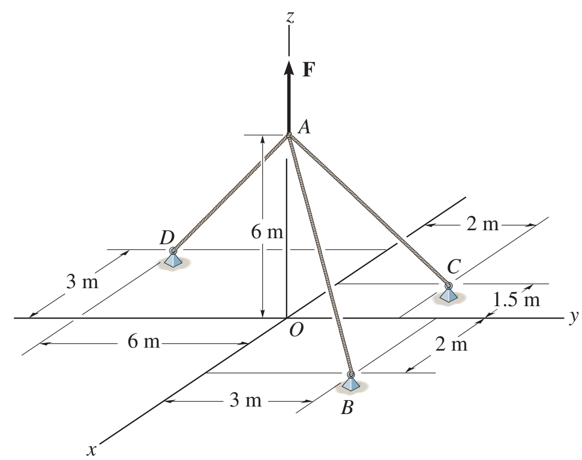
$$\alpha = \cos^{-1}\left(\frac{P_x}{P}\right) = \cos^{-1}\left(\frac{-1.164}{1.614}\right) = 136° \qquad \textbf{Ans.}$$

$$\beta = \cos^{-1}\left(\frac{P_y}{P}\right) = \cos^{-1}\left(\frac{-1.00}{1.614}\right) = 128° \qquad \textbf{Ans.}$$

$$\gamma = \cos^{-1}\left(\frac{P_z}{P}\right) = \cos^{-1}\left(\frac{0.500}{1.614}\right) = 72.0° \qquad \textbf{Ans.}$$

***3–44.**

If cable AB is subjected to a tension of 700 N, determine the tension in cables AC and AD and the magnitude of the vertical force $\mathbf{F}$.



SOLUTION

Cartesian Vector Notation:

$$\mathbf{F}_{AB} = 700\left(\frac{2\mathbf{i} + 3\mathbf{j} - 6\mathbf{k}}{\sqrt{2^2 + 3^2 + (-6)^2}}\right) = \{200\mathbf{i} + 300\mathbf{j} - 600\mathbf{k}\}\ \text{N}$$

$$\mathbf{F}_{AC} = F_{AC}\left(\frac{-1.5\mathbf{i} + 2\mathbf{j} - 6\mathbf{k}}{\sqrt{(-1.5)^2 + 2^2 + (-6)^2}}\right) = -0.2308F_{AC}\mathbf{i} + 0.3077F_{AC}\mathbf{j} - 0.9231F_{AC}\mathbf{k}$$

$$\mathbf{F}_{AD} = F_{AD}\left(\frac{-3\mathbf{i} - 6\mathbf{j} - 6\mathbf{k}}{\sqrt{(-3)^2 + (-6)^2 + (-6)^2}}\right) = -0.3333F_{AD}\mathbf{i} - 0.6667F_{AD}\mathbf{j} - 0.6667F_{AD}\mathbf{k}$$

$$\mathbf{F} = F\mathbf{k}$$

Equations of Equilibrium:

$$\Sigma\mathbf{F} = \mathbf{0}; \qquad \mathbf{F}_{AB} + \mathbf{F}_{AC} + \mathbf{F}_{AD} + \mathbf{F} = \mathbf{0}$$

$$(200 - 0.2308F_{AC} - 0.3333F_{AD})\mathbf{i} + (300 + 0.3077F_{AC} - 0.6667F_{AD})\mathbf{j}$$

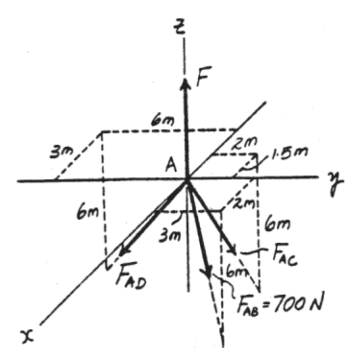
$$+ (-600 - 0.9231F_{AC} - 0.6667F_{AD} + F)\mathbf{k} = \mathbf{0}$$

Equating **i**, **j**, and **k** components, we have

$$200 - 0.2308F_{AC} - 0.3333F_{AD} = 0 \qquad \textbf{(1)}$$

$$300 + 0.3077F_{AC} - 0.6667F_{AD} = 0 \qquad \textbf{(2)}$$

$$-600 - 0.9231F_{AC} - 0.6667F_{AD} + F = 0 \qquad \textbf{(3)}$$

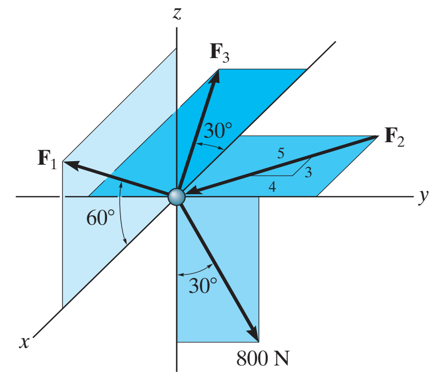


Solving Eqs. (1), (2) and (3) yields

$F_{AC} = 130$ N $\quad F_{AD} = 510$ N $\quad F = 1060$ N $= 1.06$ kN **Ans.**

3–45.

Determine the magnitudes of $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ for equilibrium of the particle.



SOLUTION

$$\mathbf{F}_1 = F_1\{\cos 60°\mathbf{i} + \sin 60°\mathbf{k}\}$$

$$= \{0.5F_1\mathbf{i} + 0.8660F_1\mathbf{k}\}\,\text{N}$$

$$\mathbf{F}_2 = F_2\{\frac{3}{5}\,\mathbf{i} - \frac{4}{5}\,\mathbf{j}\}$$

$$= \{\,0.6F_2\,\mathbf{i} - 0.8F_2\mathbf{j}\}\,\text{N}$$

$$\mathbf{F}_3 = F_3\,\{-\cos 30°\mathbf{i} - \sin 30°\mathbf{j}\}$$

$$\{-0.8660F_3\mathbf{i} - 0.5F_3\mathbf{j}\,\}\,\text{N}$$

$\Sigma F_x = 0;\qquad 0.5F_1 + 0.6F_2 - 0.8660F_3 = 0$

$\Sigma F_y = 0;\qquad -0.8F_2 - 0.5F_3 + 800 \sin 30° = 0$

$\Sigma F_z = 0;\qquad 0.8660F_1 - 800\,\cos 30° = 0$

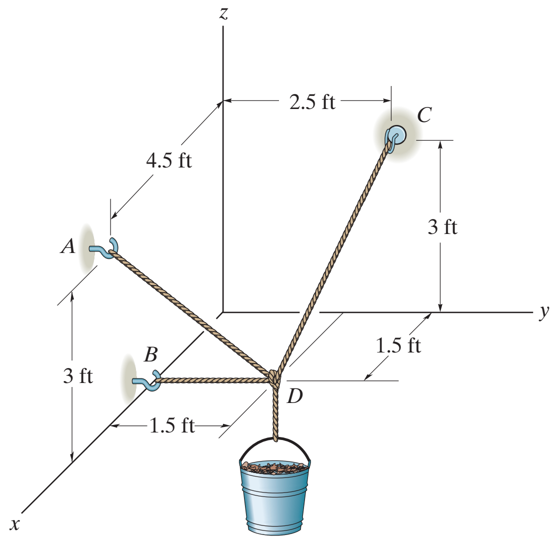
$F_1 = 800\text{ N}$ **Ans.**

$F_2 = 147\text{ N}$ **Ans.**

$F_3 = 564\text{ N}$ **Ans.**

3–46.

If the bucket and its contents have a total weight of 20 lb, determine the force in the supporting cables *DA*, *DB*, and *DC*.



SOLUTION

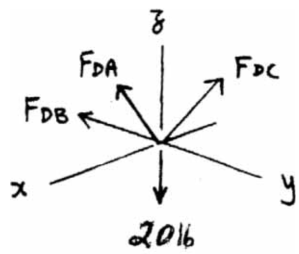
$$\mathbf{u}_{DA} = \{\frac{3}{4.5}\mathbf{i} - \frac{1.5}{4.5}\mathbf{j} + \frac{3}{4.5}\mathbf{k}\}$$

$$\mathbf{u}_{DC} = \{-\frac{1.5}{3.5}\mathbf{i} + \frac{1}{3.5}\mathbf{j} + \frac{3}{3.5}\mathbf{k}\}$$

$$\Sigma F_x = 0; \qquad \frac{3}{4.5}F_{DA} - \frac{1.5}{3.5}F_{DC} = 0$$

$$\Sigma F_y = 0; \qquad -\frac{1.5}{4.5}F_{DA} - F_{DB} + \frac{1}{3.5}F_{DC} = 0$$

$$\Sigma F_z = 0; \qquad \frac{3}{4.5}F_{DA} + \frac{3}{3.5}F_{DC} - 20 = 0$$



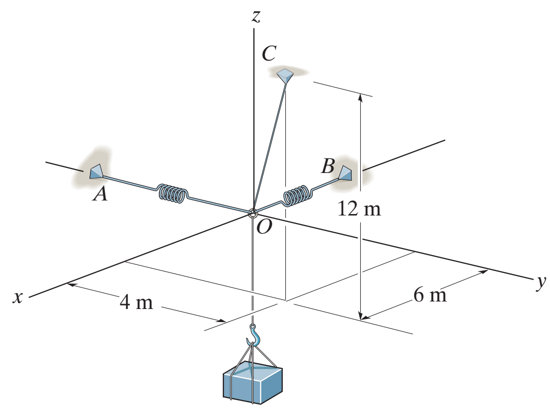
$F_{DA} = 10.0$ lb **Ans.**

$F_{DB} = 1.11$ lb **Ans.**

$F_{DC} = 15.6$ lb **Ans.**

3–47.

Determine the stretch in each of the two springs required to hold the 20-kg crate in the equilibrium position shown. Each spring has an unstretched length of 2 m and a stiffness of $k = 300$ N/m.



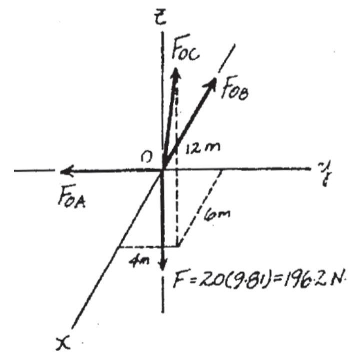
SOLUTION

Cartesian Vector Notation:

$$\mathbf{F}_{OC} = F_{OC}\left(\frac{6\mathbf{i} + 4\mathbf{j} + 12\mathbf{k}}{\sqrt{6^2 + 4^2 + 12^2}}\right) = \frac{3}{7}F_{OC}\mathbf{i} + \frac{2}{7}F_{OC}\mathbf{j} + \frac{6}{7}F_{OC}\mathbf{k}$$

$$\mathbf{F}_{OA} = -F_{OA}\mathbf{j} \qquad \mathbf{F}_{OB} = -F_{OB}\mathbf{i}$$

$$\mathbf{F} = \{-196.2\mathbf{k}\}\text{ N}$$



Equations of Equilibrium:

$$\Sigma\mathbf{F} = \mathbf{0}; \qquad \mathbf{F}_{OC} + \mathbf{F}_{OA} + \mathbf{F}_{OB} + \mathbf{F} = \mathbf{0}$$

$$\left(\frac{3}{7}F_{OC} - F_{OB}\right)\mathbf{i} + \left(\frac{2}{7}F_{OC} - F_{OA}\right)\mathbf{j} + \left(\frac{6}{7}F_{OC} - 196.2\right)\mathbf{k} = \mathbf{0}$$

Equating **i**, **j**, and **k** components, we have

$$\frac{3}{7}F_{OC} - F_{OB} = 0 \qquad \textbf{(1)}$$

$$\frac{2}{7}F_{OC} - F_{OA} = 0 \qquad \textbf{(2)}$$

$$\frac{6}{7}F_{OC} - 196.2 = 0 \qquad \textbf{(3)}$$

Solving Eqs. (1),(2) and (3) yields

$$F_{OC} = 228.9\text{ N} \qquad F_{OB} = 98.1\text{ N} \qquad F_{OA} = 65.4\text{ N}$$

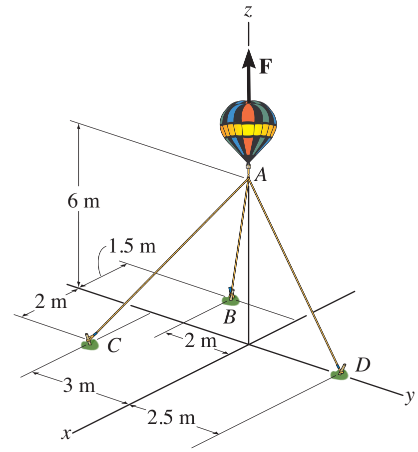
Spring Elongation: Using spring formula, Eq. 3–2, the spring elongation is $s = \frac{F}{k}$.

$$s_{OB} = \frac{98.1}{300} = 0.327\text{ m} = 327\text{ mm} \qquad \textbf{Ans.}$$

$$s_{OA} = \frac{65.4}{300} = 0.218\text{ m} = 218\text{ mm} \qquad \textbf{Ans.}$$

***3–48.**

If the balloon is subjected to a net uplift force of $F = 800$ N, determine the tension developed in ropes AB, AC, AD.



SOLUTION

Force Vectors: We can express each of the forces on the free-body diagram shown in Fig. (*a*) in Cartesian vector form as

$$\mathbf{F}_{AB} = F_{AB}\left[\frac{(-1.5-0)\mathbf{i} + (-2-0)\mathbf{j} + (-6-0)\mathbf{k}}{\sqrt{(-1.5-0)^2 + (-2-0)^2 + (-6-0)^2}}\right] = -\frac{3}{13}F_{AB}\,\mathbf{i} - \frac{4}{13}F_{AB}\,\mathbf{j} - \frac{12}{13}F_{AB}\,\mathbf{k}$$

$$\mathbf{F}_{AC} = F_{AC}\left[\frac{(2-0)\mathbf{i} + (-3-0)\mathbf{j} + (-6-0)\mathbf{k}}{\sqrt{(2-0)^2 + (-3-0)^2 + (-6-0)^2}}\right] = \frac{2}{7}F_{AC}\,\mathbf{i} - \frac{3}{7}F_{AC}\,\mathbf{j} - \frac{6}{7}F_{AC}\,\mathbf{k}$$

$$\mathbf{F}_{AD} = F_{AD}\left[\frac{(0-0)\mathbf{i} + (2.5-0)\mathbf{j} + (-6-0)\mathbf{k}}{\sqrt{(0-0)^2 + (2.5-0)^2 + (-6-0)^2}}\right] = \frac{5}{13}F_{AD}\,\mathbf{j} - \frac{12}{13}F_{AD}\,\mathbf{k}$$

$$\mathbf{W} = \{800\mathbf{k}\}\text{N}$$

Equations of Equilibrium: Equilibrium requires

$$\Sigma\mathbf{F} = \mathbf{0}; \qquad \mathbf{F}_{AB} + \mathbf{F}_{AC} + \mathbf{F}_{AD} + \mathbf{W} = \mathbf{0}$$

$$\left(-\frac{3}{13}F_{AB}\,\mathbf{i} - \frac{4}{13}F_{AB}\,\mathbf{j} - \frac{12}{13}F_{AB}\,\mathbf{k}\right) + \left(\frac{2}{7}F_{AC}\,\mathbf{i} - \frac{3}{7}F_{AC}\,\mathbf{j} - \frac{6}{7}F_{AC}\,\mathbf{k}\right) + \left(\frac{5}{13}F_{AD}\,\mathbf{j} - \frac{12}{13}F_{AD}\,\mathbf{k}\right) + 800\mathbf{k} = 0$$

$$\left(-\frac{3}{13}F_{AB} + \frac{2}{7}F_{AC}\right)\mathbf{i} + \left(-\frac{4}{13}F_{AB} - \frac{3}{7}F_{AC} - \frac{5}{13}F_{AD}\right)\mathbf{j} + \left(-\frac{12}{13}F_{AB} - \frac{6}{7}F_{AC} - \frac{12}{13}F_{AD} + 800\right)\mathbf{k} = 0$$

Equating the **i**, **j**, and **k** components yields

$$-\frac{3}{13}F_{AB} + \frac{2}{7}F_{AC} = 0 \qquad \textbf{(1)}$$

$$-\frac{4}{13}F_{AB} - \frac{3}{7}F_{AC} + \frac{5}{13}F_{AD} = 0 \qquad \textbf{(2)}$$

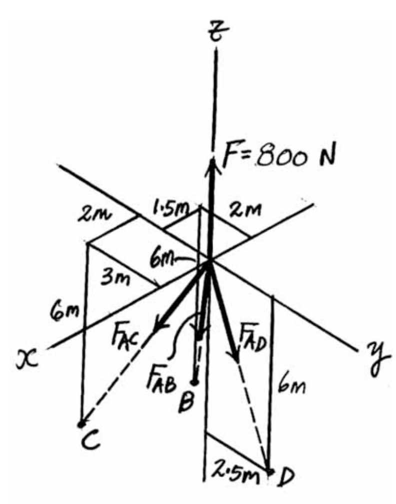
$$-\frac{12}{13}F_{AB} - \frac{6}{7}F_{AC} - \frac{12}{13}F_{AD} + 800 = 0 \qquad \textbf{(3)}$$

Solving Eqs. (1) through (3) yields

$\mathbf{F}_{AC} = 203$ N **Ans.**

$\mathbf{F}_{AB} = 251$ N **Ans.**

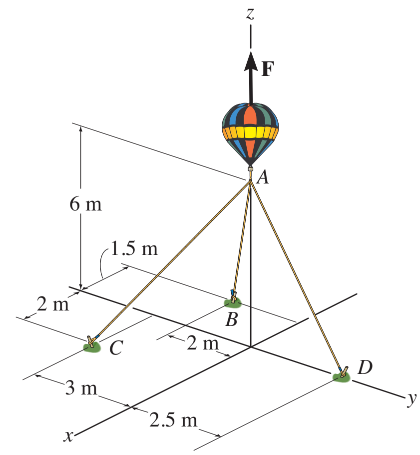
$\mathbf{F}_{AD} = 427$ N **Ans.**



(a)

3–49.

If each one of the ropes will break when it is subjected to a tensile force of 450 N, determine the maximum uplift force **F** the balloon can have before one of the ropes breaks.



SOLUTION

Force Vectors: We can express each of the forces on the free-body diagram shown in Fig. *a* in Cartesian vector form as

$$\mathbf{F}_{AB} = F_{AB}\left[\frac{(-1.5-0)\mathbf{i} + (-2-0)\mathbf{j} + (-6-0)\mathbf{k}}{\sqrt{(-1.5-0)^2 + (-2-0)^2 + (-6-0)^2}}\right] = -\frac{3}{13}F_{AB}\,\mathbf{i} - \frac{4}{13}F_{AB}\,\mathbf{j} - \frac{12}{13}F_{AB}\,\mathbf{k}$$

$$\mathbf{F}_{AC} = F_{AC}\left[\frac{(2-0)\mathbf{i} + (-3-0)\mathbf{j} + (-6-0)\mathbf{k}}{\sqrt{(2-0)^2 + (-3-0)^2 + (-6-0)^2}}\right] = \frac{2}{7}F_{AC}\,\mathbf{i} - \frac{3}{7}F_{AC}\,\mathbf{j} - \frac{6}{7}F_{AC}\,\mathbf{k}$$

$$\mathbf{F}_{AD} = F_{AD}\left[\frac{(0-0)\mathbf{i} + (2.5-0)\mathbf{j} + (-6-0)\mathbf{k}}{\sqrt{(0-0)^2 + (2.5-0)^2 + (-6-0)^2}}\right] = \frac{5}{13}F_{AD}\,\mathbf{j} - \frac{12}{13}F_{AD}\,\mathbf{k}$$

$$\mathbf{F} = F\,\mathbf{k}$$

Equations of Equilibrium: Equilibrium requires

$$\Sigma\mathbf{F} = \mathbf{0}; \qquad \mathbf{F}_{AB} + \mathbf{F}_{AC} + \mathbf{F}_{AD} + \mathbf{F} = \mathbf{0}$$

$$\left(-\frac{3}{13}F_{AB}\,\mathbf{i} - \frac{4}{13}F_{AB}\,\mathbf{j} - \frac{12}{13}F_{AB}\,\mathbf{k}\right) + \left(\frac{2}{7}F_{AC}\,\mathbf{i} - \frac{3}{7}F_{AC}\,\mathbf{j} - \frac{6}{7}F_{AC}\,\mathbf{k}\right) + \left(\frac{5}{13}F_{AD}\,\mathbf{j} - \frac{12}{13}F_{AD}\,\mathbf{k}\right) + F\mathbf{k} = 0$$

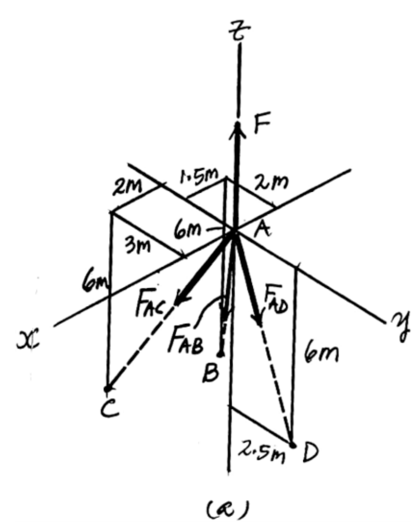
$$\left(-\frac{3}{13}F_{AB} + \frac{2}{7}F_{AC}\right)\mathbf{i} + \left(-\frac{4}{13}F_{AB} - \frac{3}{7}F_{AC} + \frac{5}{13}F_{AD}\right)\mathbf{j} + \left(-\frac{12}{13}F_{AB} - \frac{6}{7}F_{AC} - \frac{12}{13}F_{AD} + F\right)\mathbf{k} = 0$$

Equating the **i**, **j**, and **k** components yields

$$-\frac{3}{13}F_{AB} + \frac{2}{7}F_{AC} = 0 \qquad \textbf{(1)}$$

$$-\frac{4}{13}F_{AB} - \frac{3}{7}F_{AC} + \frac{5}{13}F_{AD} = 0 \qquad \textbf{(2)}$$

$$-\frac{12}{13}F_{AB} - \frac{6}{7}F_{AC} - \frac{12}{13}F_{AD} + F = 0 \qquad \textbf{(3)}$$



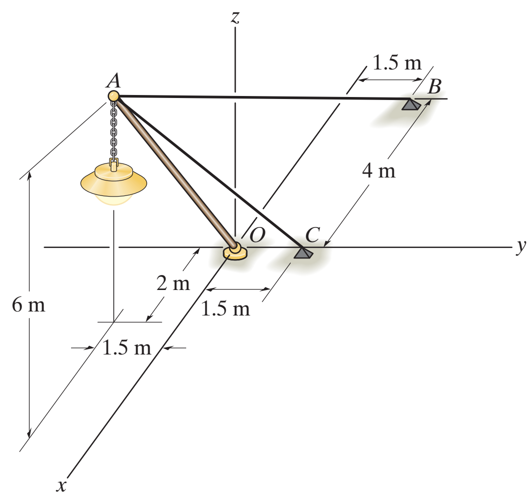
Assume that cord AD will break first. Substituting $F_{AD} = 450$ N into Eqs. (2) and (3) and solving Eqs. (1) through (3), yields

$F_{AB} = 264.71$ N
$F_{AC} = 213.8$ N
$F = 842.99$ N $= 843$ N **Ans.**

Since $F_{AC} = 213.8\text{ N} < 450\text{ N}$ and $F_{AB} = 264.71\text{ N} < 450\text{ N}$, our assumption is correct.

■**3–50.**

The lamp has a mass of 15 kg and is supported by a pole *AO* and cables *AB* and *AC*. If the force in the pole acts along its axis, determine the forces in *AO*, *AB*, and *AC* for equilibrium.



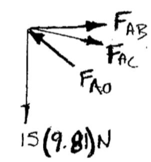
SOLUTION

$$\mathbf{F}_{AO} = F_{AO}\{\frac{2}{6.5}\mathbf{i} - \frac{1.5}{6.5}\mathbf{j} + \frac{6}{6.5}\mathbf{k}\}\ \text{N}$$

$$\mathbf{F}_{AB} = F_{AB}\{-\frac{6}{9}\mathbf{i} + \frac{3}{9}\mathbf{j} - \frac{6}{9}\mathbf{k}\}\ \text{N}$$

$$\mathbf{F}_{AC} = F_{AC}\{-\frac{2}{7}\mathbf{i} + \frac{3}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\}\ \text{N}$$

$$\mathbf{W} = 15(9.81)\mathbf{k} = \{-147.15\mathbf{k}\}\text{N}$$



$\Sigma F_x = 0;\qquad 0.3077F_{AO} - 0.6667F_{AB} - 0.2857F_{AC} = 0$

$\Sigma F_y = 0;\qquad -0.2308F_{AO} + 0.3333F_{AB} + 0.4286F_{AC} = 0$

$\Sigma F_z = 0;\qquad 0.9231F_{AO} - 0.667F_{AB} - 0.8571F_{AC} - 147.15 = 0$

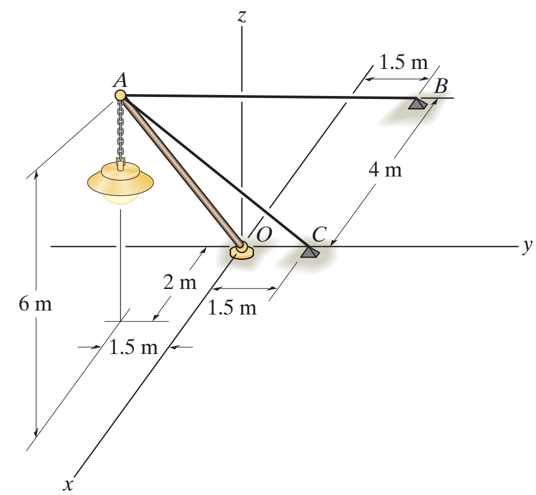
$F_{AO} = 319\ \text{N}$ **Ans.**

$F_{AB} = 110\ \text{N}$ **Ans.**

$F_{AC} = 85.8\ \text{N}$ **Ans.**

3–51.

Cables AB and AC can sustain a maximum tension of 500 N, and the pole can support a maximum compression of 300 N. Determine the maximum weight of the lamp that can be supported in the position shown. The force in the pole acts along the axis of the pole.



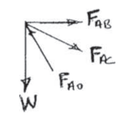
SOLUTION

$$\mathbf{F}_{AO} = F_{AO}\{\frac{2}{6.5}\mathbf{i} - \frac{1.5}{6.5}\mathbf{j} + \frac{6}{6.5}\mathbf{k}\}\text{ N}$$

$$\mathbf{F}_{AB} = F_{AB}\{-\frac{6}{9}\mathbf{i} + \frac{3}{9}\mathbf{j} - \frac{6}{9}\mathbf{k}\}\text{ N}$$

$$\mathbf{F}_{AC} = F_{AC}\{-\frac{2}{7}\mathbf{i} + \frac{3}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\}\text{ N}$$

$$\mathbf{W} = \{W\mathbf{k}\}\text{ N}$$



$$\Sigma F_x = 0; \qquad \frac{2}{6.5}F_{AO} - \frac{6}{9}F_{AB} - \frac{2}{7}F_{AC} = 0$$

$$\Sigma F_y = 0; \qquad -\frac{1.5}{6.5}F_{AO} + \frac{3}{9}F_{AB} + \frac{3}{7}F_{AC} = 0$$

$$\Sigma F_z = 0; \qquad \frac{6}{6.5}F_{AO} - \frac{6}{9}F_{AB} - \frac{6}{7}F_{AC} - W = 0$$

1) Assume $F_{AB} = 500$ N

$$\frac{2}{6.5}F_{AO} - \frac{6}{9}(500) - \frac{2}{7}F_{AC} = 0$$

$$-\frac{1.5}{6.5}F_{AO} + \frac{3}{9}(500) + \frac{3}{7}F_{AC} = 0$$

$$\frac{6}{6.5}F_{AO} - \frac{6}{9}(500) - \frac{6}{7}F_{AC} - W = 0$$

Solving,

$$F_{AO} = 1444.444\text{ N} > 300\text{ N } (\mathbf{N.G!})$$

$$F_{AC} = 388.889\text{ N}$$

$$W = 666.667\text{ N}$$

2) Assume $F_{AC} = 500$ N

$$\frac{2}{6.5}F_{AO} - \frac{6}{9}F_{AB} - \frac{2}{7}(500) = 0$$

$$-\frac{1.5}{6.5}F_{AO} + \frac{3}{9}F_{AB} + \frac{3}{7}(500) = 0$$

$$\frac{6}{6.5}F_{AO} - \frac{6}{9}F_{AB} - \frac{6}{7}(500) - W = 0$$

Solving,

$$F_{AO} = 1857.143\text{ N} > 300\text{ N } (\mathbf{N.G!})$$

$$F_{AB} = 642.857\text{ N} > 500\text{ N } (\mathbf{N.G!})$$

3) Assume $F_{AO} = 300$ N

$$\frac{2}{6.5}(300) - \frac{6}{9}F_{AB} - \frac{2}{7}F_{AC} = 0$$

$$-\frac{1.5}{6.5}(300) + \frac{3}{9}F_{AB} + \frac{3}{7}F_{AC} = 0$$

$$\frac{6}{6.5}(300) - \frac{6}{9}F_{AB} - \frac{6}{7}F_{AC} - W = 0$$

Solving,

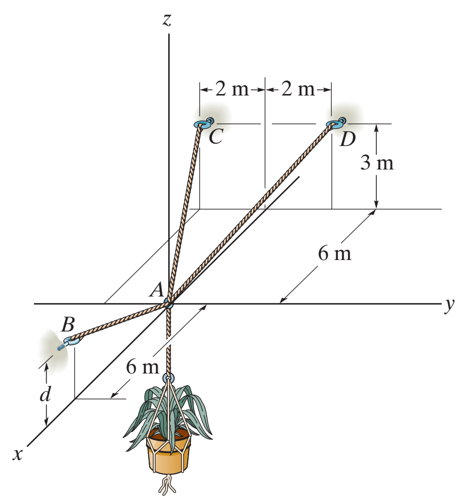
$$F_{AC} = 80.8\text{ N}$$

$$F_{AB} = 104\text{ N}$$

$$W = 138\text{ N} \qquad \textbf{Ans.}$$

***3–52.**

The 50-kg pot is supported from A by the three cables. Determine the force acting in each cable for equilibrium. Take $d = 2.5$ m.



SOLUTION

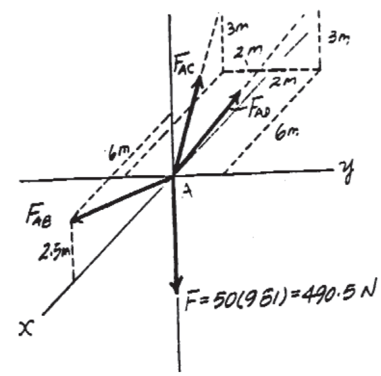
Cartesian Vector Notation:

$$\mathbf{F}_{AB} = F_{AB}\left(\frac{6\mathbf{i}+2.5\mathbf{k}}{\sqrt{6^2+2.5^2}}\right) = \frac{12}{13}F_{AB}\,\mathbf{i}+\frac{5}{13}F_{AB}\,\mathbf{k}$$

$$\mathbf{F}_{AC} = F_{AC}\left(\frac{-6\mathbf{i}-2\mathbf{j}+3\mathbf{k}}{\sqrt{(-6)^2+(-2)^2+3^2}}\right) = -\frac{6}{7}F_{AC}\,\mathbf{i}-\frac{2}{7}F_{AC}\,\mathbf{j}+\frac{3}{7}F_{AC}\,\mathbf{k}$$

$$\mathbf{F}_{AD} = F_{AD}\left(\frac{-6\mathbf{i}+2\mathbf{j}+3\mathbf{k}}{\sqrt{(-6)^2+2^2+3^2}}\right) = -\frac{6}{7}F_{AD}\,\mathbf{i}+\frac{2}{7}F_{AD}\,\mathbf{j}+\frac{3}{7}F_{AD}\,\mathbf{k}$$

$$\mathbf{F} = \{-490.5\mathbf{k}\}\text{ N}$$



Equations of Equilibrium:

$$\Sigma\mathbf{F} = \mathbf{0};\qquad \mathbf{F}_{AB}+\mathbf{F}_{AC}+\mathbf{F}_{AD}+\mathbf{F} = \mathbf{0}$$

$$\left(\frac{12}{13}F_{AB}-\frac{6}{7}F_{AC}-\frac{6}{7}F_{AD}\right)\mathbf{i}+\left(-\frac{2}{7}F_{AC}+\frac{2}{7}F_{AD}\right)\mathbf{j}$$

$$+\left(\frac{5}{13}F_{AB}+\frac{3}{7}F_{AC}+\frac{3}{7}F_{AD}-490.5\right)\mathbf{k} = \mathbf{0}$$

Equating **i**, **j**, and **k** components, we have

$$\frac{12}{13}F_{AB}-\frac{6}{7}F_{AC}-\frac{6}{7}F_{AD} = 0 \qquad \textbf{(1)}$$

$$-\frac{2}{7}F_{AC}+\frac{2}{7}F_{AD} = 0 \qquad \textbf{(2)}$$

$$\frac{5}{13}F_{AB}+\frac{3}{7}F_{AC}+\frac{3}{7}F_{AD}-490.5 = 0 \qquad \textbf{(3)}$$

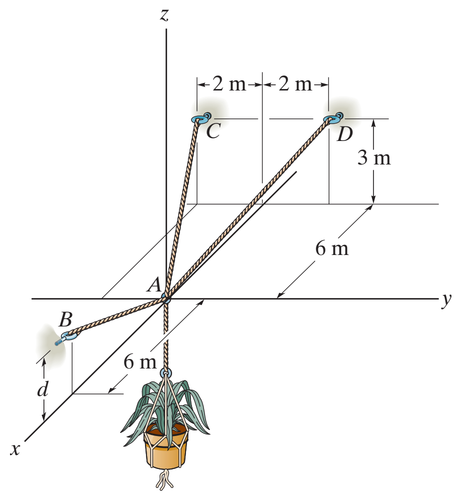
Solving Eqs. (1), (2) and (3) yields

$$F_{AC} = F_{AD} = 312\text{ N}$$

$$F_{AB} = 580\text{ N} \qquad \textbf{Ans.}$$

3–53.

Determine the height d of cable AB so that the force in cables AD and AC is one-half as great as the force in cable AB. What is the force in each cable for this case? The flower pot has a mass of 50 kg.



SOLUTION

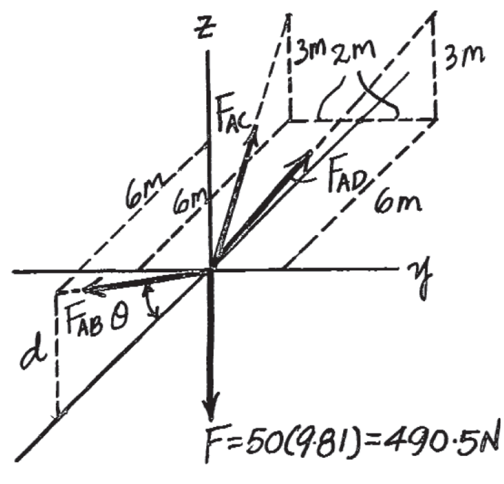
Cartesian Vector Notation:

$$\mathbf{F}_{AB} = (F_{AB})_x\,\mathbf{i} + (F_{AB})_y\,\mathbf{k}$$

$$\mathbf{F}_{AC} = \frac{F_{AB}}{2}\left(\frac{-6\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}}{\sqrt{(-6)^2 + (-2)^2 + 3^2}}\right) = -\frac{3}{7}F_{AB}\mathbf{i} - \frac{1}{7}F_{AB}\,\mathbf{j} + \frac{3}{14}F_{AB}\,\mathbf{k}$$

$$\mathbf{F}_{AD} = \frac{F_{AB}}{2}\left(\frac{-6\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}}{\sqrt{(-6)^2 + 2^2 + 3^2}}\right) = -\frac{3}{7}F_{AB}\mathbf{i} + \frac{1}{7}F_{AB}\,\mathbf{j} + \frac{3}{14}F_{AB}\,\mathbf{k}$$

$$\mathbf{F} = \{-490.5\mathbf{k}\}\text{ N}$$



Equations of Equilibrium:

$$\Sigma\mathbf{F} = \mathbf{0}; \qquad \mathbf{F}_{AB} + \mathbf{F}_{AC} + \mathbf{F}_{AD} + \mathbf{F} = \mathbf{0}$$

$$\left((F_{AB})_x - \frac{3}{7}F_{AB} - \frac{3}{7}F_{AB}\right)\mathbf{i} + \left(-\frac{1}{7}F_{AB} + \frac{1}{7}F_{AB}\right)\mathbf{j} + \left((F_{AB})_z + \frac{3}{14}F_{AB} + \frac{3}{14}F_{AB} - 490.5\right)\mathbf{k} = \mathbf{0}$$

Equating **i**, **j**, and **k** components, we have

$$(F_{AB})_x - \frac{3}{7}F_{AB} - \frac{3}{7}F_{AB} = 0 \qquad (F_{AB})_x = \frac{6}{7}F_{AB} \qquad \textbf{(1)}$$

$$-\frac{1}{7}F_{AB} + \frac{1}{7}F_{AB} = 0 \qquad \textit{\textbf{(Satisfied!)}}$$

$$(F_{AB})_z + \frac{3}{14}F_{AB} + \frac{3}{14}F_{AB} - 490.5 = 0 \qquad (F_{AB})_z = 490.5 - \frac{3}{7}F_{AB} \qquad \textbf{(2)}$$

However, $F_{AB}^2 = (F_{AB})_x^2 + (F_{AB})_z^2$, then substitute Eqs. (1) and (2) into this expression yields

$$F_{AB}^2 = \left(\frac{6}{7}F_{AB}\right)^2 + \left(490.5 - \frac{3}{7}F_{AB}\right)^2$$

Solving for positive root, we have

$$F_{AB} = 519.79\text{ N} = 520\text{ N} \qquad \textbf{Ans.}$$

Thus, $$F_{AC} = F_{AD} = \frac{1}{2}(519.79) = 260\text{ N} \qquad \textbf{Ans.}$$

Also,

$$(F_{AB})_x = \frac{6}{7}(519.79) = 445.53\text{ N}$$

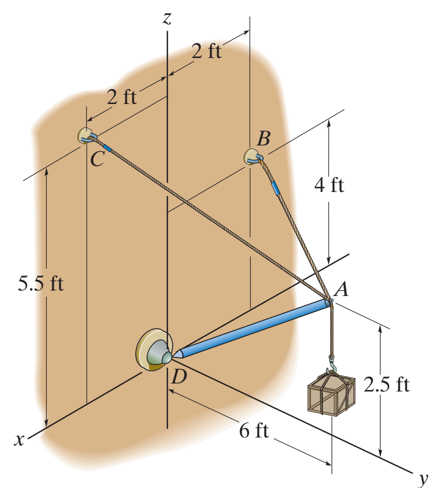
$$(F_{AB})_z = 490.5 - \frac{3}{7}(519.79) = 267.73\text{ N}$$

then, $$\theta = \tan^{-1}\left[\frac{(F_{AB})_z}{(F_{AB})_x}\right] = \tan^{-1}\left(\frac{267.73}{445.53}\right) = 31.00°$$

$$d = 6\tan\theta = 6\tan 31.00° = 3.61\text{ m} \qquad \textbf{Ans.}$$

3–54.

Determine the tension developed in cables AB and AC and the force developed along strut AD for equilibrium of the 400-lb crate.



SOLUTION

Force Vectors: We can express each of the forces on the free-body diagram shown in Fig. *a* in Cartesian vector form as

$$\mathbf{F}_{AB} = F_{AB}\left[\frac{(-2-0)\mathbf{i} + (-6-0)\mathbf{j} + (1.5-0)\mathbf{k}}{\sqrt{(-2-0)^2 + (-6-0)^2 + (1.5-0)^2}}\right] = -\frac{4}{13}F_{AB}\,\mathbf{i} - \frac{12}{13}F_{AB}\,\mathbf{j} + \frac{3}{13}F_{AB}\,\mathbf{k}$$

$$\mathbf{F}_{AC} = F_{AC}\left[\frac{(2-0)\mathbf{i} + (-6-0)\mathbf{j} + (3-0)\mathbf{k}}{\sqrt{(2-0)^2 + (-6-0)^2 + (3-0)^2}}\right] = \frac{2}{7}F_{AC}\,\mathbf{i} - \frac{6}{7}F_{AC}\,\mathbf{j} + \frac{3}{7}F_{AC}\,\mathbf{k}$$

$$\mathbf{F}_{AD} = F_{AD}\left[\frac{(0-0)\mathbf{i} + [0-(-6)]\mathbf{j} + [0-(-2.5)]\mathbf{k}}{\sqrt{(0-0)^2 + [0-(-6)]^2 + (0-(-2.5))]^2}}\right] = \frac{12}{13}F_{AD}\,\mathbf{j} + \frac{5}{13}F_{AD}\,\mathbf{k}$$

$$\mathbf{W} = \{-400\mathbf{k}\}\text{ lb}$$

Equations of Equilibrium: Equilibrium requires

$$\Sigma\mathbf{F} = \mathbf{0}; \qquad \mathbf{F}_{AB} + \mathbf{F}_{AC} + \mathbf{F}_{AD} + \mathbf{W} = \mathbf{0}$$

$$\left(-\frac{4}{13}F_{AB}\,\mathbf{i} - \frac{12}{13}F_{AB}\,\mathbf{j} + \frac{3}{13}F_{AB}\,\mathbf{k}\right) + \left(\frac{2}{7}F_{AC}\,\mathbf{i} - \frac{6}{7}F_{AC}\,\mathbf{j} + \frac{3}{7}F_{AC}\,\mathbf{k}\right) + \left(\frac{12}{13}F_{AD}\,\mathbf{j} + \frac{5}{13}F_{AD}\,\mathbf{k}\right) + (-400\mathbf{k}) = 0$$

$$\left(-\frac{4}{13}F_{AB} + \frac{2}{7}F_{AC}\right)\mathbf{i} + \left(-\frac{12}{13}F_{AB} - \frac{6}{7}F_{AC} + \frac{12}{13}F_{AD}\right)\mathbf{j} + \left(\frac{3}{13}F_{AB} + \frac{3}{7}F_{AC} + \frac{5}{13}F_{AD} - 400\right)\mathbf{k} = 0$$

Equating the **i**, **j**, and **k** components yields

$$-\frac{4}{13}F_{AB} + \frac{2}{7}F_{AC} = 0 \qquad \textbf{(1)}$$

$$-\frac{12}{13}F_{AB} - \frac{6}{7}F_{AC} + \frac{12}{13}F_{AD} = 0 \qquad \textbf{(2)}$$

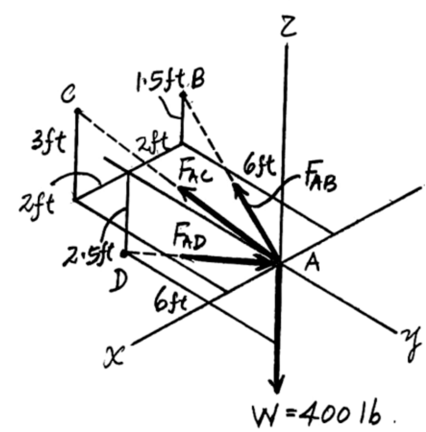
$$\frac{3}{13}F_{AB} + \frac{3}{7}F_{AC} + \frac{5}{13}F_{AD} - 400 = 0 \qquad \textbf{(3)}$$

Solving Eqs. (1) through (3) yields

$F_{AB} = 274$ lb **Ans.**

$F_{AC} = 295$ lb **Ans.**

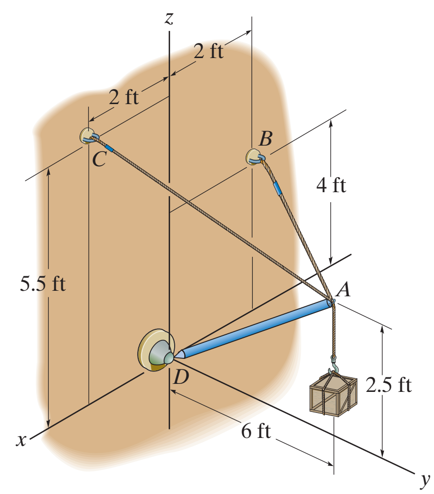
$F_{AD} = 547$ lb **Ans.**



(a)

3–55.

If the tension developed in each of the cables cannot exceed 300 lb, determine the largest weight of the crate that can be supported. Also, what is the force developed along strut AD?



SOLUTION

Force Vectors: We can express each of the forces on the free-body diagram shown in Fig. *a* in Cartesian vector form as

$$\mathbf{F}_{AB} = F_{AB}\left[\frac{(-2-0)\mathbf{i} + (-6-0)\mathbf{j} + (1.5-0)\mathbf{k}}{\sqrt{(-2-0)^2 + (-6-0)^2 + (1.5-0)^2}}\right] = -\frac{4}{13}F_{AB}\,\mathbf{i} - \frac{12}{13}F_{AB}\,\mathbf{j} + \frac{3}{13}F_{AB}\,\mathbf{k}$$

$$\mathbf{F}_{AC} = F_{AC}\left[\frac{(2-0)\mathbf{i} + (-6-0)\mathbf{j} + (3-0)\mathbf{k}}{\sqrt{(2-0)^2 + (-6-0)^2 + (3-0)^2}}\right] = \frac{2}{7}F_{AC}\,\mathbf{i} - \frac{6}{7}F_{AC}\,\mathbf{j} + \frac{3}{7}F_{AC}\,\mathbf{k}$$

$$\mathbf{F}_{AD} = F_{AD}\left[\frac{(0-0)\mathbf{i} + [0-(-6)]\mathbf{j} + [0-(-2.5)]\mathbf{k}}{\sqrt{(0-0)^2 + [0-(-6)]^2 + [0-(-2.5)]^2}}\right] = \frac{12}{13}F_{AD}\,\mathbf{j} + \frac{5}{13}F_{AD}\,\mathbf{k}$$

$$\mathbf{W} = -W\mathbf{k}$$

Equations of Equilibrium: Equilibrium requires

$$\Sigma\mathbf{F} = \mathbf{0}; \qquad \mathbf{F}_{AB} + \mathbf{F}_{AC} + \mathbf{F}_{AD} + \mathbf{W} = \mathbf{0}$$

$$\left(-\frac{4}{13}F_{AB}\,\mathbf{i} - \frac{12}{13}F_{AB}\,\mathbf{j} + \frac{3}{13}F_{AB}\,\mathbf{k}\right) + \left(\frac{2}{7}F_{AC}\,\mathbf{i} - \frac{6}{7}F_{AC}\,\mathbf{j} + \frac{3}{7}F_{AC}\,\mathbf{k}\right) + \left(\frac{12}{13}F_{AD}\,\mathbf{j} + \frac{5}{13}F_{AD}\,\mathbf{k}\right) + (-W\mathbf{k}) = 0$$

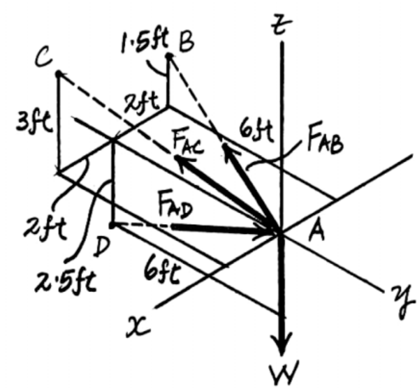
$$\left(-\frac{4}{13}F_{AB} + \frac{2}{7}F_{AC}\right)\mathbf{i} + \left(-\frac{12}{13}F_{AB} - \frac{6}{7}F_{AC} + \frac{12}{13}F_{AD}\right)\mathbf{j} + \left(\frac{3}{13}F_{AB} + \frac{3}{7}F_{AC} + \frac{5}{13}F_{AD} - W\right)\mathbf{k} = 0$$

Equating the **i**, **j**, and **k** components yields

$$-\frac{4}{13}F_{AB} + \frac{2}{7}F_{AC} = 0 \qquad \textbf{(1)}$$

$$-\frac{12}{13}F_{AB} - \frac{6}{7}F_{AC} + \frac{12}{13}F_{AD} = 0 \qquad \textbf{(2)}$$

$$\frac{3}{13}F_{AB} + \frac{3}{7}F_{AC} + \frac{5}{13}F_{AD} - W = 0 \qquad \textbf{(3)}$$



(a)

Let us assume that cable AC achieves maximum tension first. Substituting $F_{AC} = 300$ lb into Eqs. (1) through (3) and solving, yields

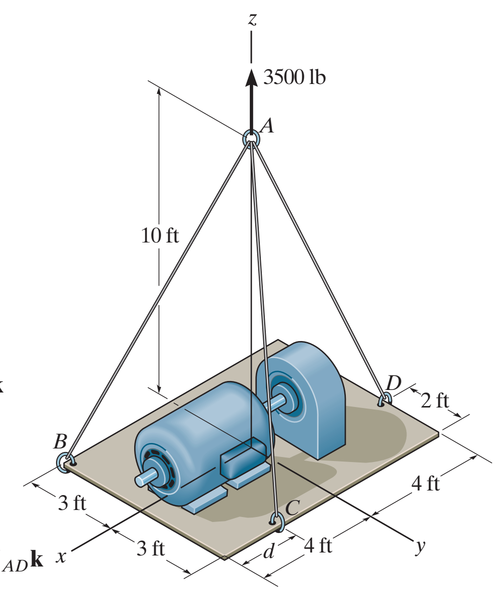
$F_{AB} = 278.57$ lb

$F_{AD} = 557$ lb $\qquad W = 407$ lb $\qquad$ **Ans.**

Since $F_{AB} = 278.57 \text{ lb} < 300 \text{ lb}$, our assumption is correct.

***3–56.**

Determine the force in each cable needed to support the 3500-lb platform. Set $d = 2$ ft.



SOLUTION

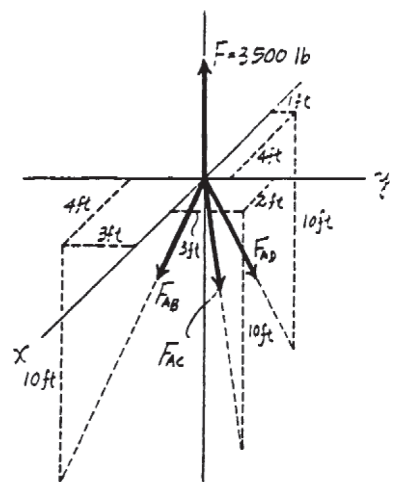
Cartesian Vector Notation:

$$\mathbf{F}_{AB} = F_{AB}\left(\frac{4\mathbf{i} - 3\mathbf{j} - 10\mathbf{k}}{\sqrt{4^2 + (-3)^2 + (-10)^2}}\right) = 0.3578F_{AB}\mathbf{i} - 0.2683F_{AB}\mathbf{j} - 0.8944F_{AB}\mathbf{k}$$

$$\mathbf{F}_{AC} = F_{AC}\left(\frac{2\mathbf{i} + 3\mathbf{j} - 10\mathbf{k}}{\sqrt{2^2 + 3^2 + (-10)^2}}\right) = 0.1881F_{AC}\mathbf{i} + 0.2822F_{AC}\mathbf{j} - 0.9407F_{AC}\mathbf{k}$$

$$\mathbf{F}_{AD} = F_{AD}\left(\frac{-4\mathbf{i} + 1\mathbf{j} - 10\mathbf{k}}{\sqrt{(-4)^2 + 1^2 + (-10)^2}}\right) = -0.3698F_{AD}\mathbf{i} + 0.09245F_{AD}\mathbf{j} - 0.9245F_{AD}\mathbf{k}$$

$\mathbf{F} = \{3500\mathbf{k}\}$ lb



Equations of Equilibrium:

$$\Sigma\mathbf{F} = \mathbf{0}; \qquad \mathbf{F}_{AB} + \mathbf{F}_{AC} + \mathbf{F}_{AD} + \mathbf{F} = \mathbf{0}$$

$$(0.3578F_{AB} + 0.1881F_{AC} - 0.3698F_{AD})\,\mathbf{i} + (-0.2683F_{AB} + 0.2822F_{AC} + 0.09245F_{AD})\mathbf{j} + (-0.8944F_{AB} - 0.9407F_{AC} - 0.9245F_{AD} + 3500)\mathbf{k} = 0$$

Equating **i**, **j**, and **k** components, we have

$$0.3578F_{AB} + 0.1881F_{AC} - 0.3698F_{AD} = 0 \qquad \textbf{(1)}$$

$$-0.2683F_{AB} + 0.2822F_{AC} + 0.09245F_{AD} = 0 \qquad \textbf{(2)}$$

$$-0.8944F_{AB} - 0.9407F_{AC} - 0.9245F_{AD} + 3500 = 0 \qquad \textbf{(3)}$$

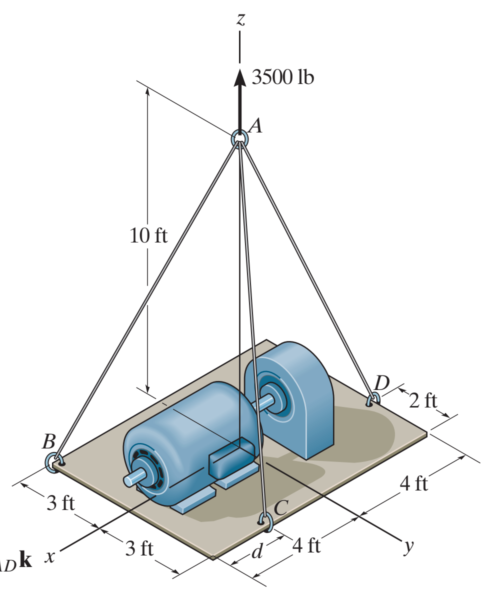
Solving Eqs. (1), (2) and (3) yields

$F_{AB} = 1369.59 \text{ lb} = 1.37 \text{ kip}$ $\qquad F_{AC} = 744.11 \text{ lb} = 0.744 \text{ kip}$ **Ans.**

$F_{AD} = 1703.62 \text{ lb} = 1.70 \text{ kip}$ **Ans.**

3–57.

Determine the force in each cable needed to support the 3500-lb platform. Set $d = 4$ ft.



SOLUTION

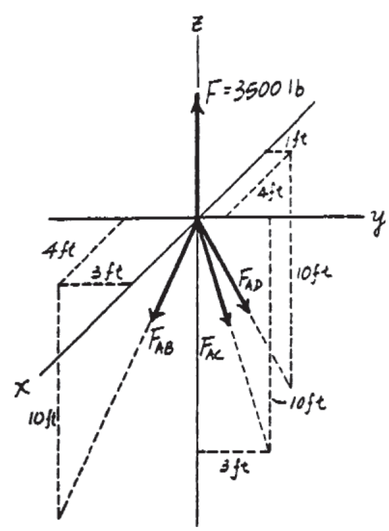
Cartesian Vector Notation:

$$\mathbf{F}_{AB} = F_{AB}\left(\frac{4\mathbf{i} - 3\mathbf{j} - 10\mathbf{k}}{\sqrt{4^2 + (-3)^2 + (-10)^2}}\right) = 0.3578F_{AB}\mathbf{i} - 0.2683F_{AB}\mathbf{j} - 0.8944F_{AB}\,\mathbf{k}$$

$$\mathbf{F}_{AC} = F_{AC}\left(\frac{3\mathbf{j} - 10\mathbf{k}}{\sqrt{3^2 + (-10)^2}}\right) = 0.2873F_{AC}\mathbf{j} - 0.9578F_{AC}\,\mathbf{k}$$

$$\mathbf{F}_{AD} = F_{AD}\left(\frac{-4\mathbf{i} + 1\mathbf{j} - 10\mathbf{k}}{\sqrt{(-4)^2 + 1^2 + (-10)^2}}\right) = -0.3698F_{AD}\mathbf{i} + 0.09245F_{AD}\mathbf{j} - 0.9245F_{AD}\mathbf{k}$$

$$\mathbf{F} = \{3500\mathbf{k}\}\text{ lb}$$



Equations of Equilibrium:

$$\Sigma\mathbf{F} = \mathbf{0}; \qquad \mathbf{F}_{AB} + \mathbf{F}_{AC} + \mathbf{F}_{AD} + \mathbf{F} = \mathbf{0}$$

$$(0.3578F_{AB} - 0.3698F_{AD})\mathbf{i} + (-0.2683F_{AB} + 0.2873F_{AC} + 0.09245F_{AD})\mathbf{j}$$
$$+ (-0.8944F_{AB} - 0.9578F_{AC} - 0.9245F_{AD} + 3500)\mathbf{k} = 0$$

Equating **i**, **j**, and **k** components, we have

$$0.3578F_{AB} - 0.3698F_{AD} = 0 \qquad \textbf{(1)}$$

$$-0.2683F_{AB} + 0.2873F_{AC} + 0.09245F_{AD} = 0 \qquad \textbf{(2)}$$

$$-0.8944F_{AB} - 0.9578F_{AC} - 0.9245F_{AD} + 3500 = 0 \qquad \textbf{(3)}$$

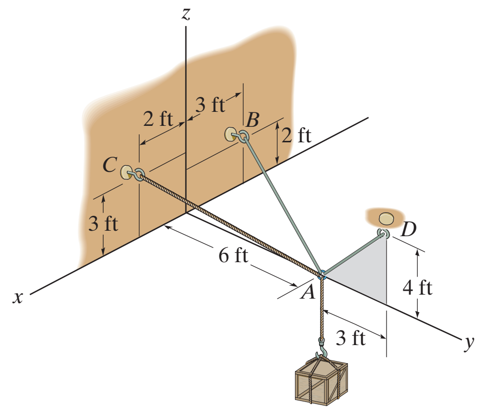
Solving Eqs. (1), (2) and (3) yields

$F_{AB} = 1467.42\text{ lb} = 1.47\text{ kip}$ $\qquad F_{AC} = 913.53\text{ lb} = 0.914\text{ kip}$ **Ans.**

$F_{AD} = 1419.69\text{ lb} = 1.42\text{ kip}$ **Ans.**

3–58.

Determine the tension developed in each cable for equilibrium of the 300-lb crate.



SOLUTION

Force Vectors: We can express each of the forces shown in Fig. *a* in Cartesian vector form as

$$\mathbf{F}_{AB} = F_{AB}\left[\frac{(-3-0)\mathbf{i} + (-6-0)\mathbf{j} + (2-0)\mathbf{k}}{\sqrt{(-3-0)^2 + (-6-0)^2 + (2-0)^2}}\right] = -\frac{3}{7}F_{AB}\,\mathbf{i} - \frac{6}{7}F_{AB}\,\mathbf{j} + \frac{2}{7}F_{AB}\,\mathbf{k}$$

$$\mathbf{F}_{AC} = F_{AC}\left[\frac{(2-0)\mathbf{i} + (-6-0)\mathbf{j} + (3-0)\mathbf{k}}{\sqrt{(2-0)^2 + (-6-0)^2 + (3-0)^2}}\right] = \frac{2}{7}F_{AC}\,\mathbf{i} - \frac{6}{7}F_{AC}\,\mathbf{j} + \frac{3}{7}F_{AC}\,\mathbf{k}$$

$$\mathbf{F}_{AD} = F_{AD}\left[\frac{(0-0)\mathbf{i} + (3-0)\mathbf{j} + (4-0)\mathbf{k}}{\sqrt{(0-0)^2 + (3-0)^2 + (4-0)^2}}\right] = \frac{3}{5}F_{AD}\,\mathbf{j} + \frac{4}{5}F_{AD}\,\mathbf{k}$$

$$\mathbf{W} = \{-300\mathbf{k}\}\ \text{lb}$$

Equations of Equilibrium: Equilibrium requires

$$\Sigma\mathbf{F} = \mathbf{0}; \qquad \mathbf{F}_{AB} + \mathbf{F}_{AC} + \mathbf{F}_{AD} + \mathbf{W} = \mathbf{0}$$

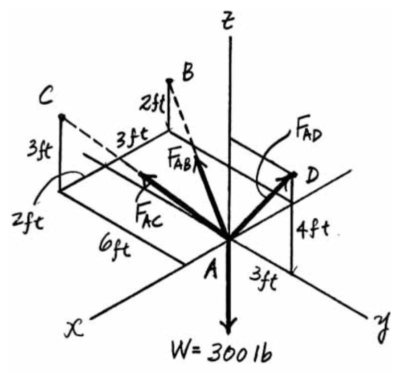
$$\left(-\frac{3}{7}F_{AB}\,\mathbf{i} - \frac{6}{7}F_{AB}\,\mathbf{j} + \frac{2}{7}F_{AB}\,\mathbf{k}\right) + \left(\frac{2}{7}F_{AC}\,\mathbf{i} - \frac{6}{7}F_{AC}\,\mathbf{j} + \frac{3}{7}F_{AC}\,\mathbf{k}\right) + \left(\frac{3}{5}F_{AD}\,\mathbf{j} + \frac{4}{5}F_{AD}\,\mathbf{k}\right) + (-300\mathbf{k}) = 0$$

Equating the **i**, **j**, and **k** components yields

$$-\frac{3}{7}F_{AB} + \frac{2}{7}F_{AC} = 0 \qquad \textbf{(1)}$$

$$-\frac{6}{7}F_{AB} - \frac{6}{7}F_{AC} + \frac{3}{5}F_{AD} = 0 \qquad \textbf{(2)}$$

$$\frac{2}{7}F_{AB} + \frac{3}{7}F_{AC} + \frac{4}{5}F_{AD} - 300 = 0 \qquad \textbf{(3)}$$



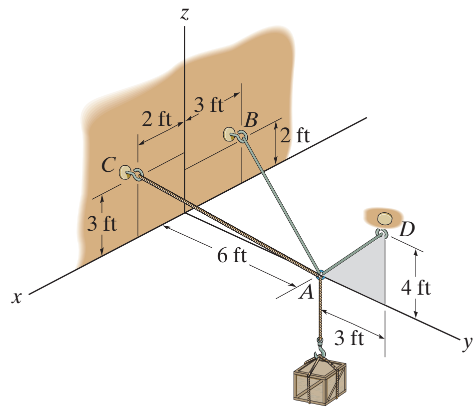
(a)

Solving Eqs. (1) through (3) yields

$F_{AB} = 79.2$ lb $\qquad F_{AC} = 119$ lb $\qquad F_{AD} = 283$ lb **Ans.**

3–59.

Determine the maximum weight of the crate that can be suspended from cables AB, AC, and AD so that the tension developed in any one of the cables does not exceed 250 lb.



SOLUTION

Force Vectors: We can express each of the forces shown in Fig. *a* in Cartesian vector form as

$$\mathbf{F}_{AB} = F_{AB}\left[\frac{(-3-0)\mathbf{i} + (-6-0)\mathbf{j} + (2-0)\mathbf{k}}{\sqrt{(-3-0)^2 + (-6-0)^2 + (2-0)^2}}\right] = -\frac{3}{7}F_{AB}\,\mathbf{i} - \frac{6}{7}F_{AB}\,\mathbf{j} + \frac{2}{7}F_{AB}\,\mathbf{k}$$

$$\mathbf{F}_{AC} = F_{AC}\left[\frac{(2-0)\mathbf{i} + (-6-0)\mathbf{j} + (3-0)\mathbf{k}}{\sqrt{(2-0)^2 + (-6-0)^2 + (3-0)^2}}\right] = \frac{2}{7}F_{AC}\,\mathbf{i} - \frac{6}{7}F_{AC}\,\mathbf{j} + \frac{3}{7}F_{AC}\,\mathbf{k}$$

$$\mathbf{F}_{AD} = F_{AD}\left[\frac{(0-0)\mathbf{i} + (3-0)\mathbf{j} + (4-0)\mathbf{k}}{\sqrt{(0-0)^2 + (3-0)^2 + (4-0)^2}}\right] = \frac{3}{5}F_{AD}\,\mathbf{j} + \frac{4}{5}F_{AD}\,\mathbf{k}$$

$$\mathbf{W} = -W_C\,\mathbf{k}$$

Equations of Equilibrium: Equilibrium requires

$$\Sigma\mathbf{F} = \mathbf{0}; \qquad \mathbf{F}_{AB} + \mathbf{F}_{AC} + \mathbf{F}_{AD} + \mathbf{W} = \mathbf{0}$$

$$\left(-\frac{3}{7}F_{AB}\,\mathbf{i} - \frac{6}{7}F_{AB}\,\mathbf{j} + \frac{2}{7}F_{AB}\,\mathbf{k}\right) + \left(\frac{2}{7}F_{AC}\,\mathbf{i} - \frac{6}{7}F_{AC}\,\mathbf{j} + \frac{3}{7}F_{AC}\,\mathbf{k}\right) + \left(\frac{3}{5}F_{AD}\,\mathbf{j} + \frac{4}{5}F_{AD}\,\mathbf{k}\right) + (-W_C\,\mathbf{k}) = 0$$

$$\left(-\frac{3}{7}F_{AB} + \frac{2}{7}F_{AC}\right)\mathbf{i} + \left(-\frac{6}{7}F_{AB} - \frac{6}{7}F_{AC} + \frac{3}{5}F_{AD}\right)\mathbf{j} + \left(\frac{2}{7}F_{AB} + \frac{3}{7}F_{AC} + \frac{4}{5}F_{AD} - W_C\right)\mathbf{k} = 0$$

Equating the **i**, **j**, and **k** components yields

$$-\frac{3}{7}F_{AB} + \frac{2}{7}F_{AC} = 0 \qquad \textbf{(1)}$$

$$-\frac{6}{7}F_{AB} - \frac{6}{7}F_{AC} + \frac{3}{5}F_{AD} = 0 \qquad \textbf{(2)}$$

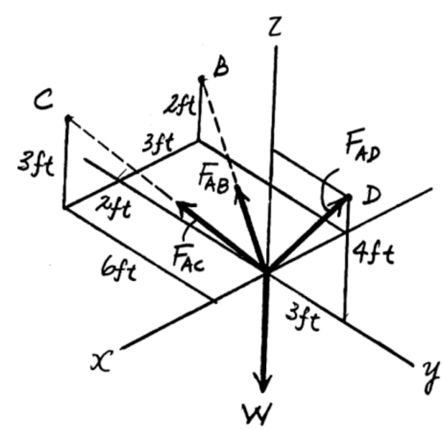
$$\frac{2}{7}F_{AB} + \frac{3}{7}F_{AC} + \frac{4}{5}F_{AD} - W_C = 0 \qquad \textbf{(3)}$$

Assuming that cable AD achieves maximum tension first, substituting $F_{AD} = 250$ lb into Eqs. (2) and (3), and solving Eqs. (1) through (3) yields

$F_{AB} = 70$ lb $\qquad F_{AC} = 105$ lb

$W_C = 265$ lb **Ans.**

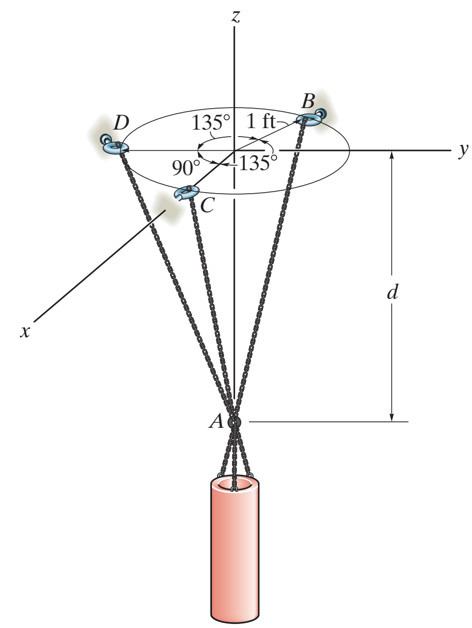
Since $F_{AB} = 70 \text{ lb} < 250$ lb and $F_{AC} = 105$ lb, the above assumption is correct.



(a)

***3–60.**

The 800-lb cylinder is supported by three chains as shown. Determine the force in each chain for equilibrium. Take $d = 1$ ft.



SOLUTION

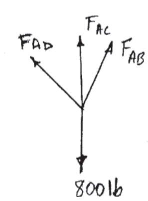
$$\mathbf{F}_{AD} = F_{AD}\left(\frac{-1\mathbf{j} + 1\mathbf{k}}{\sqrt{(-1)^2 + 1^2}}\right) = -0.7071F_{AD}\mathbf{j} + 0.7071F_{AD}\mathbf{k}$$

$$\mathbf{F}_{AC} = F_{AC}\left(\frac{1\mathbf{i} + 1\mathbf{k}}{\sqrt{1^2 + 1^2}}\right) = 0.7071F_{AC}\mathbf{i} + 0.7071F_{AC}\mathbf{k}$$

$$\mathbf{F}_{AB} = F_{AB}\left(\frac{-0.7071\mathbf{i} + 0.7071\mathbf{j} + 1\mathbf{k}}{\sqrt{(-0.7071)^2 + 0.7071^2 + 1^2}}\right)$$

$$= -0.5F_{AB}\mathbf{i} + 0.5F_{AB}\mathbf{j} + 0.7071F_{AB}\mathbf{k}$$

$$\mathbf{F} = \{-800\mathbf{k}\}\text{ lb}$$



$$\Sigma\mathbf{F} = \mathbf{0}; \qquad \mathbf{F}_{AD} + \mathbf{F}_{AC} + \mathbf{F}_{AB} + \mathbf{F} = \mathbf{0}$$

$$(-0.7071F_{AD}\mathbf{j} + 0.7071F_{AD}\mathbf{k}) + (0.7071F_{AC}\mathbf{i} + 0.7071F_{AC}\mathbf{k})$$
$$+ (-0.5F_{AB}\mathbf{i} + 0.5F_{AB}\mathbf{j} + 0.7071F_{AB}\mathbf{k}) + (-800\mathbf{k}) = \mathbf{0}$$

$$(0.7071F_{AC} - 0.5F_{AB})\mathbf{i} + (-0.7071F_{AD} + 0.5F_{AB})\mathbf{j}$$
$$+ (0.7071F_{AD} + 0.7071F_{AC} + 0.7071F_{AB} - 800)\mathbf{k} = \mathbf{0}$$

$$\Sigma F_x = 0; \qquad 0.7071F_{AC} - 0.5F_{AB} = 0 \qquad \textbf{(1)}$$

$$\Sigma F_y = 0; \qquad -0.7071F_{AD} + 0.5F_{AB} = 0 \qquad \textbf{(2)}$$

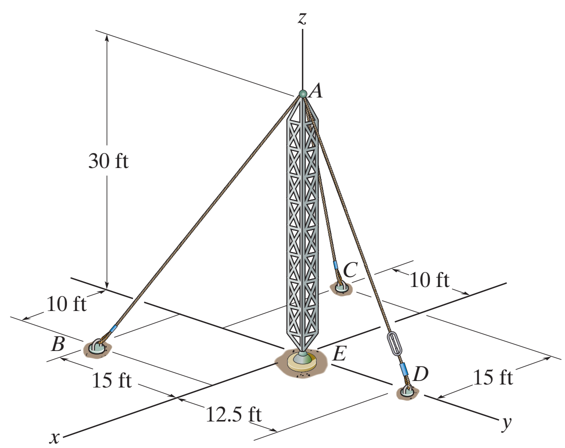
$$\Sigma F_z = 0; \qquad 0.7071F_{AD} + 0.7071F_{AC} + 0.7071F_{AB} - 800 = 0 \qquad \textbf{(3)}$$

Solving Eqs. (1), (2), and (3) yields:

$F_{AB} = 469$ lb $\qquad F_{AC} = F_{AD} = 331$ lb $\qquad$ **Ans.**

3–61.

If cable AD is tightened by a turnbuckle and develops a tension of 1300 lb, determine the tension developed in cables AB and AC and the force developed along the antenna tower AE at point A.



SOLUTION

Force Vectors: We can express each of the forces on the free-body diagram shown in Fig. *a* in Cartesian vector form as

$$\mathbf{F}_{AB} = F_{AB}\left[\frac{(10-0)\mathbf{i} + (-15-0)\mathbf{j} + (-30-0)\mathbf{k}}{\sqrt{(10-0)^2 + (-15-0)^2 + (-30-0)^2}}\right] = \frac{2}{7}F_{AB}\,\mathbf{i} - \frac{3}{7}F_{AB}\,\mathbf{j} - \frac{6}{7}F_{AB}\,\mathbf{k}$$

$$\mathbf{F}_{AC} = F_{AC}\left[\frac{(-15-0)\mathbf{i} + (-10-0)\mathbf{j} + (-30-0)\mathbf{k}}{\sqrt{(-15-0)^2 + (-10-0)^2 + (-30-0)^2}}\right] = -\frac{3}{7}F_{AC}\,\mathbf{i} - \frac{2}{7}F_{AC}\,\mathbf{j} - \frac{6}{7}F_{AC}\,\mathbf{k}$$

$$\mathbf{F}_{AD} = F_{AD}\left[\frac{(0-0)\mathbf{i} + (12.5-0)\mathbf{j} + (-30-0)\mathbf{k}}{\sqrt{(0-0)^2 + (12.5-0)^2 + (-30-0)^2}}\right] = \{500\mathbf{j} - 1200\mathbf{k}\}\text{ lb}$$

$$\mathbf{F}_{AE} = F_{AE}\,\mathbf{k}$$

Equations of Equilibrium: Equilibrium requires

$$\Sigma\mathbf{F} = \mathbf{0}; \qquad \mathbf{F}_{AB} + \mathbf{F}_{AC} + \mathbf{F}_{AD} + \mathbf{F}_{AE} = \mathbf{0}$$

$$\left(\frac{2}{7}F_{AB}\,\mathbf{i} - \frac{3}{7}F_{AB}\,\mathbf{j} - \frac{6}{7}F_{AB}\,\mathbf{k}\right) + \left(-\frac{3}{7}F_{AC}\,\mathbf{i} - \frac{2}{7}F_{AC}\,\mathbf{j} - \frac{6}{7}F_{AC}\,\mathbf{k}\right) + (500\mathbf{j} - 1200\mathbf{k}) + F_{AE}\,\mathbf{k} = 0$$

$$\left(\frac{2}{7}F_{AB} - \frac{3}{7}F_{AC}\right)\mathbf{i} + \left(-\frac{3}{7}F_{AB} - \frac{2}{7}F_{AC} + 500\right)\mathbf{j} + \left(-\frac{6}{7}F_{AB} - \frac{6}{7}F_{AC} + F_{AE} - 1200\right)\mathbf{k} = 0$$

Equating the **i**, **j**, and **k** components yields

$$\frac{2}{7}F_{AB} - \frac{3}{7}F_{AC} = 0 \qquad \textbf{(1)}$$

$$-\frac{3}{7}F_{AB} - \frac{2}{7}F_{AC} + 500 = 0 \qquad \textbf{(2)}$$

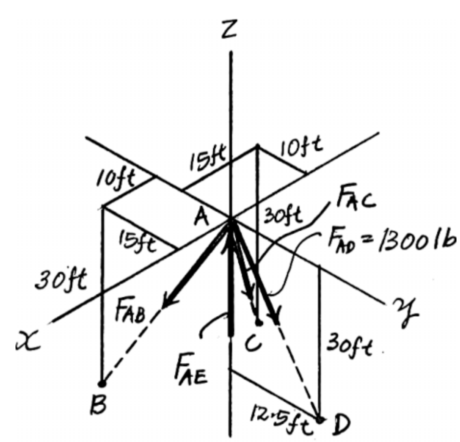
$$-\frac{6}{7}F_{AB} - \frac{6}{7}F_{AC} + F_{AE} - 1200 = 0 \qquad \textbf{(3)}$$

Solving Eqs. (1) through (3) yields

$F_{AB} = 808$ lb **Ans.**

$F_{AC} = 538$ lb **Ans.**

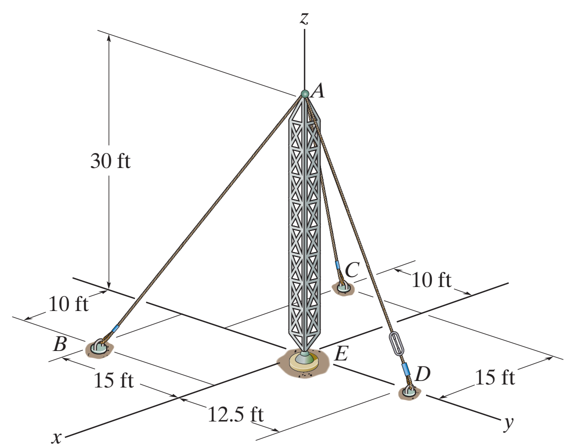
$F_{AE} = 2354$ lb $= 2.35$ kip **Ans.**



(a)

3–62.

If the tension developed in either cable AB or AC cannot exceed 1000 lb, determine the maximum tension that can be developed in cable AD when it is tightened by the turnbuckle. Also, what is the force developed along the antenna tower at point A?



SOLUTION

Force Vectors: We can express each of the forces on the free-body diagram shown in Fig. *a* in Cartesian vector form as

$$\mathbf{F}_{AB} = F_{AB}\left[\frac{(10-0)\mathbf{i} + (-15-0)\mathbf{j} + (-30-0)\mathbf{k}}{\sqrt{(10-0)^2 + (-15-0)^2 + (-30-0)^2}}\right] = \frac{2}{7}F_{AB}\,\mathbf{i} - \frac{3}{7}F_{AB}\,\mathbf{j} - \frac{6}{7}F_{AB}\,\mathbf{k}$$

$$\mathbf{F}_{AC} = F_{AC}\left[\frac{(-15-0)\mathbf{i} + (-10-0)\mathbf{j} + (-30-0)\mathbf{k}}{\sqrt{(-15-0)^2 + (-10-0)^2 + (-30-0)^2}}\right] = -\frac{3}{7}F_{AC}\,\mathbf{i} - \frac{2}{7}F_{AC}\,\mathbf{j} - \frac{6}{7}F_{AC}\,\mathbf{k}$$

$$\mathbf{F}_{AD} = F\left[\frac{(0-0)\mathbf{i} + (12.5-0)\mathbf{j} + (-30-0)\mathbf{k}}{\sqrt{(0-0)^2 + (12.5-0)^2 + (-30-0)^2}}\right] = \frac{5}{13}F\mathbf{j} - \frac{12}{13}F\,\mathbf{k}$$

$$\mathbf{F}_{AE} = F_{AE}\,\mathbf{k}$$

Equations of Equilibrium: Equilibrium requires

$$\Sigma\mathbf{F} = \mathbf{0}; \qquad \mathbf{F}_{AB} + \mathbf{F}_{AC} + \mathbf{F}_{AD} + \mathbf{F}_{AE} = \mathbf{0}$$

$$\left(\frac{2}{7}F_{AB}\,\mathbf{i} - \frac{3}{7}F_{AB}\,\mathbf{j} - \frac{6}{7}F_{AB}\,\mathbf{k}\right) + \left(-\frac{3}{7}F_{AC}\,\mathbf{i} - \frac{2}{7}F_{AC}\,\mathbf{j} - \frac{6}{7}F_{AC}\,\mathbf{k}\right) + \left(\frac{5}{13}F\mathbf{j} - \frac{12}{13}F\,\mathbf{k}\right) + F_{AE}\mathbf{k} = 0$$

$$\left(\frac{2}{7}F_{AB} - \frac{3}{7}F_{AC}\right)\mathbf{i} + \left(-\frac{3}{7}F_{AB} - \frac{2}{7}F_{AC} + \frac{5}{13}F\right)\mathbf{j} + \left(-\frac{6}{7}F_{AB} - \frac{6}{7}F_{AC} - \frac{12}{13}F + F_{AE}\right)\mathbf{k} = 0$$

Equating the **i**, **j**, and **k** components yields

$$\frac{2}{7}F_{AB} - \frac{3}{7}F_{AC} = 0 \qquad \textbf{(1)}$$

$$-\frac{3}{7}F_{AB} - \frac{2}{7}F_{AC} + \frac{5}{13}F = 0 \qquad \textbf{(2)}$$

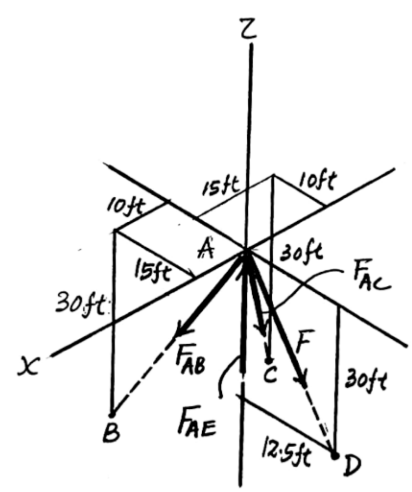
$$-\frac{6}{7}F_{AB} - \frac{6}{7}F_{AC} - \frac{12}{13}F + F_{AE} = 0 \qquad \textbf{(3)}$$

Let us assume that cable AB achieves maximum tension first. Substituting $F_{AB} = 1000$ lb into Eqs. (1) through (3) and solving yields

$F_{AC} = 666.67$ lb

$F_{AE} = 2914 \text{ lb} = 2.91 \text{ kip} \qquad F = 1610 \text{ lb} = 1.61 \text{ kip}$ **Ans.**

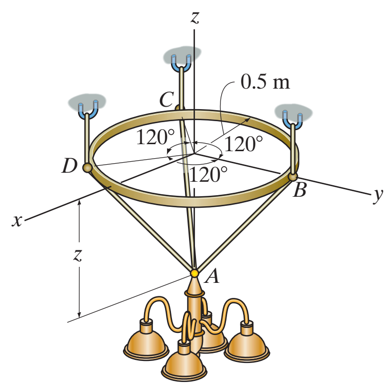
Since $F_{AC} = 666.67 \text{ lb} < 1000$ lb, our assumption is correct.



(a)

3–63.

The thin ring can be adjusted vertically between three equally long cables from which the 100-kg chandelier is suspended. If the ring remains in the horizontal plane and $z = 600$ mm, determine the tension in each cable.



SOLUTION

Geometry: Referring to the geometry of the free-body diagram shown in Fig. *a*, the lengths of cables AB, AC, and AD are all $l = \sqrt{0.5^2 + 0.6^2} = \sqrt{0.61}$ m

Equations of Equilibrium: Equilibrium requires

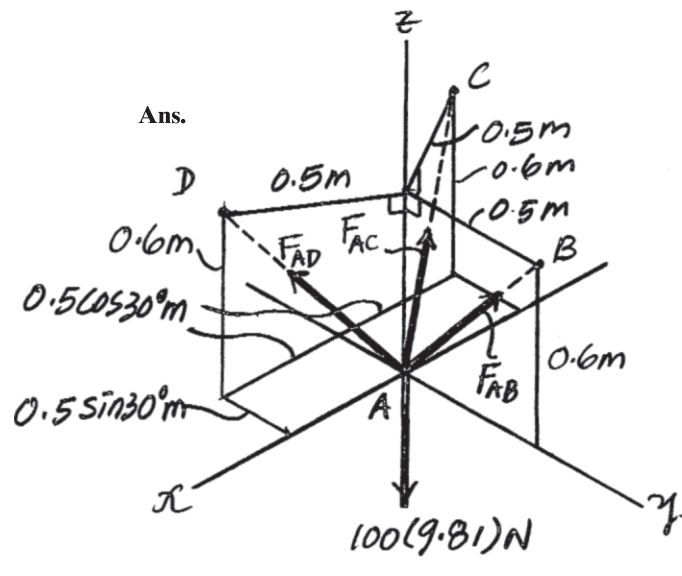
$$\Sigma F_x = 0; \quad F_{AD}\left(\frac{0.5\cos 30°}{\sqrt{0.61}}\right) - F_{AC}\left(\frac{0.5\cos 30°}{\sqrt{0.61}}\right) = 0 \quad F_{AD} = F_{AC} = F$$

$$\Sigma F_y = 0; \quad F_{AB}\left(\frac{0.5}{\sqrt{0.61}}\right) - 2\left[F\left(\frac{0.5\sin 30°}{\sqrt{0.61}}\right)\right] = 0 \qquad F_{AB} = F$$

Thus, cables AB, AC, and AD all develop the same tension.

$$\Sigma F_z = 0; \quad 3F\left(\frac{0.6}{\sqrt{0.61}}\right) - 100(9.81) = 0$$

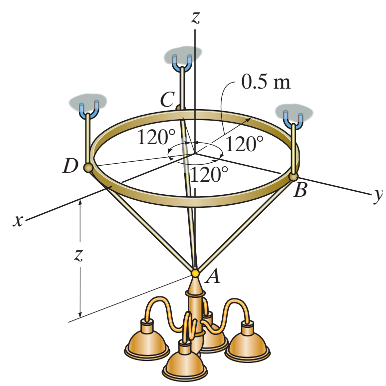
$$F_{AB} = F_{AC} = F_{AD} = 426 \text{ N}$$ **Ans.**



(a)

***3–64.**

The thin ring can be adjusted vertically between three equally long cables from which the 100-kg chandelier is suspended. If the ring remains in the horizontal plane and the tension in each cable is not allowed to exceed 1 kN, determine the smallest allowable distance z required for equilibrium.



SOLUTION

Geometry: Referring to the geometry of the free-body diagram shown in Fig. a, the lengths of cables AB, AC, and AD are all $l = \sqrt{0.5^2 + z^2}$.

Equations of Equilibrium: Equilibrium requires

$$\Sigma F_x = 0; \quad F_{AD}\left(\frac{0.5\cos 30^\circ}{\sqrt{0.5^2 + z^2}}\right) - F_{AC}\left(\frac{0.5\cos 30^\circ}{\sqrt{0.5^2 + z^2}}\right) = 0 \qquad F_{AD} = F_{AC} = F$$

$$\Sigma F_y = 0; \quad F_{AB}\left(\frac{0.5}{\sqrt{0.5^2 + z^2}}\right) - 2\left[F\left(\frac{0.5\sin 30^\circ}{\sqrt{0.5^2 + z^2}}\right)\right] = 0 \qquad F_{AB} = F$$

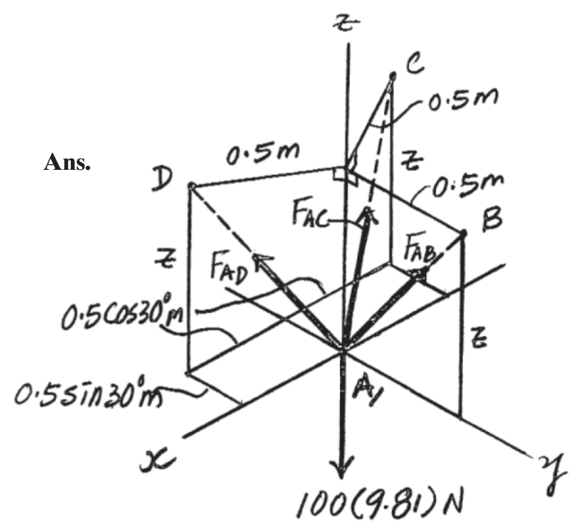
Thus, cables AB, AC, and AD all develop the same tension.

$$\Sigma F_z = 0; \quad 3F\left(\frac{z}{\sqrt{0.5^2 + z^2}}\right) - 100(9.81) = 0$$

Cables AB, AC, and AD will also achieve maximum tension simultaneously. Substituting $F = 1000$ N, we obtain

$$3(1000)\left(\frac{z}{\sqrt{0.5^2 + z^2}}\right) - 100(9.81) = 0$$

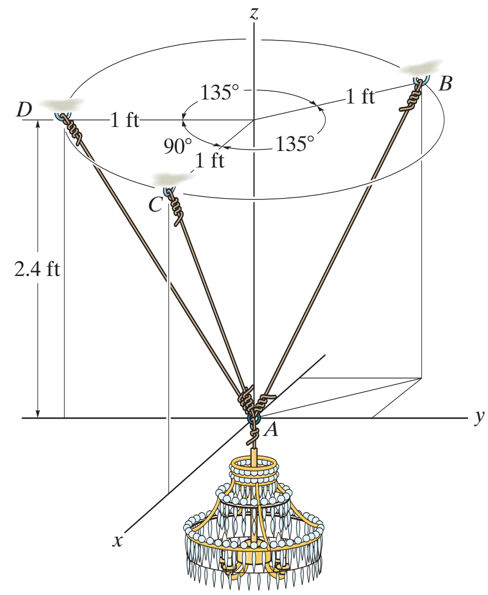
$$z = 0.1730 \text{ m} = 173 \text{ mm}$$ **Ans.**



(a)

3–65.

The 80-lb chandelier is supported by three wires as shown. Determine the force in each wire for equilibrium.



SOLUTION

$$\Sigma F_x = 0; \qquad \frac{1}{2.6}F_{AC} - \frac{1}{2.6}F_{AB}\cos 45° = 0$$

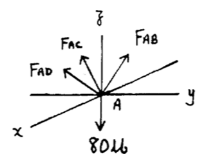
$$\Sigma F_y = 0; \qquad -\frac{1}{2.6}F_{AD} + \frac{1}{2.6}F_{AB}\sin 45° = 0$$

$$\Sigma F_z = 0; \qquad \frac{2.4}{2.6}F_{AC} + \frac{2.4}{2.6}F_{AD} + \frac{2.4}{2.6}F_{AB} - 80 = 0$$

Solving,

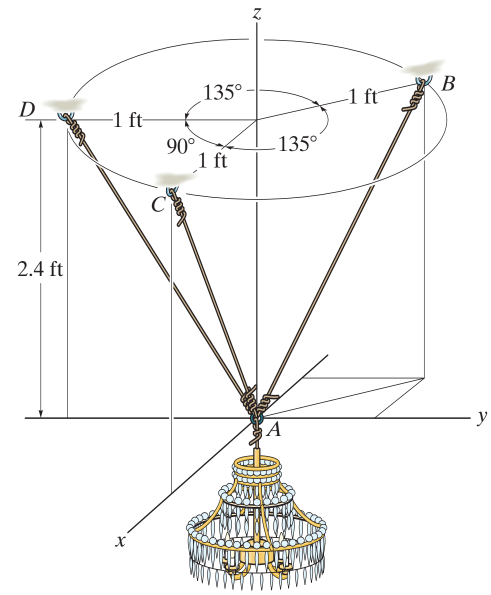
$$F_{AB} = 35.9 \text{ lb} \qquad \textbf{Ans.}$$

$$F_{AC} = F_{AD} = 25.4 \text{ lb} \qquad \textbf{Ans.}$$



3–66.

If each wire can sustain a maximum tension of 120 lb before it fails, determine the greatest weight of the chandelier the wires will support in the position shown.

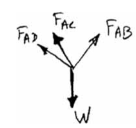


SOLUTION

$$\Sigma F_x = 0; \qquad \frac{1}{2.6}F_{AC} - \frac{1}{2.6}F_{AB}\cos 45° = 0 \qquad \textbf{(1)}$$

$$\Sigma F_y = 0; \qquad -\frac{1}{2.6}F_{AD} + \frac{1}{2.6}F_{AB}\sin 45° = 0 \qquad \textbf{(2)}$$

$$\Sigma F_z = 0; \qquad \frac{2.4}{2.6}F_{AC} + \frac{2.4}{2.6}F_{AD} + \frac{2.4}{2.6}F_{AB} - W = 0 \qquad \textbf{(3)}$$



Assume $F_{AC} = 120$ lb. From Eq. (1)

$$\frac{1}{2.6}(120) - \frac{1}{2.6}F_{AB}\cos 45° = 0$$

$$F_{AB} = 169.71 > 120 \text{ lb } (\textbf{N.G!})$$

Assume $F_{AB} = 120$ lb. From Eqs. (1) and (2)

$$\frac{1}{2.6}F_{AC} - \frac{1}{2.6}(120)(\cos 45°) = 0$$

$$F_{AC} = 84.853 \text{ lb} < 120 \text{ lb } (\textbf{O.K!})$$

$$-\frac{1}{2.6}F_{AD} + \frac{1}{2.6}(120)\sin 45° = 0$$

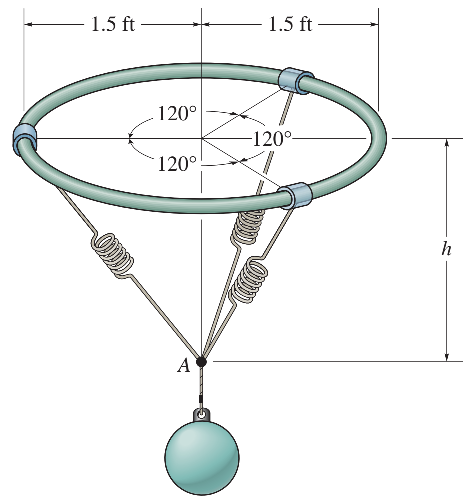
$$F_{AD} = 84.853 \text{ lb} < 120 \text{ lb } (\textbf{O.K!})$$

Thus,

$$W = \frac{2.4}{2.6}(F_{AC} + F_{AD} + F_{AB}) = 267.42 = 267 \text{ lb} \qquad \textbf{Ans.}$$

■3–67.

The 80-lb ball is suspended from the horizontal ring using three springs each having an unstretched length of 1.5 ft and stiffness of 50 lb/ft. Determine the vertical distance h from the ring to point A for equilibrium.



SOLUTION

Equation of Equilibrium: This problem can be easily solved if one realizes that due to symmetry all springs are subjected to a same tensile force of F_{sp}. Summing forces along z axis yields

$$\Sigma F_z = 0; \qquad 3F_{sp} \cos \gamma - 80 = 0 \qquad \textbf{(1)}$$

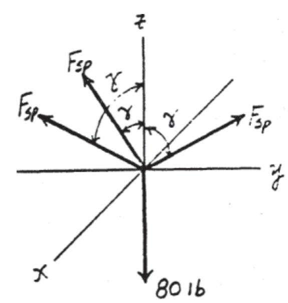
Spring Force: Applying Eq. 3–2, we have

$$F_{sp} = ks = k(l - l_0) = 50\left(\frac{1.5}{\sin \gamma} - 1.5\right) = \frac{75}{\sin \gamma} - 75 \qquad \textbf{(2)}$$

Substituting Eq. (2) into (1) yields

$$3\left(\frac{75}{\sin \gamma} - 75\right)\cos \gamma - 80 = 0$$

$$\tan \gamma = \frac{45}{16}(1 - \sin \gamma)$$



Solving by trial and error, we have

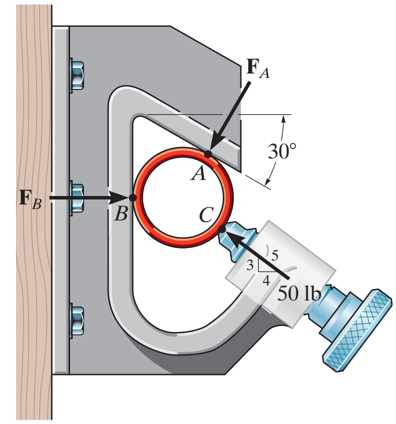
$$\gamma = 42.4425°$$

Geometry:

$$h = \frac{1.5}{\tan \gamma} = \frac{1.5}{\tan 42.4425°} = 1.64 \text{ ft} \qquad \textbf{Ans.}$$

***3–68.**

The pipe is held in place by the vise. If the bolt exerts a force of 50 lb on the pipe in the direction shown, determine the forces F_A and F_B that the smooth contacts at A and B exert on the pipe.



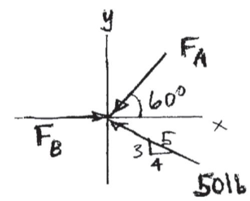
SOLUTION

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad F_B - F_A \cos 60^\circ - 50\left(\frac{4}{5}\right) = 0$$

$$+\uparrow \Sigma F_y = 0; \qquad -F_A \sin 60^\circ + 50\left(\frac{3}{5}\right) = 0$$

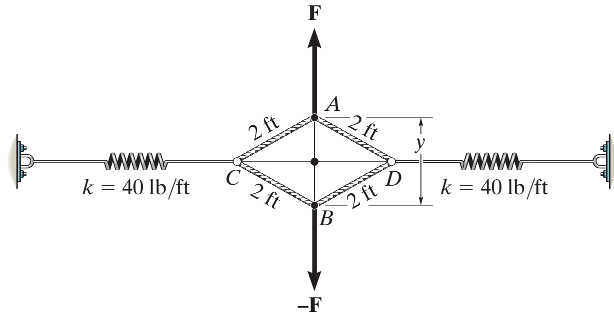
$$F_A = 34.6 \text{ lb} \qquad \textbf{Ans.}$$

$$F_B = 57.3 \text{ lb} \qquad \textbf{Ans.}$$



3–69.

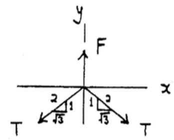
When y is zero, the springs sustain a force of 60 lb. Determine the magnitude of the applied vertical forces **F** and $-\mathbf{F}$ required to pull point A away from point B a distance of $y = 2$ ft. The ends of cords CAD and CBD are attached to rings at C and D.



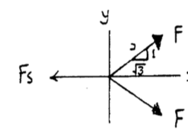
SOLUTION

Initial spring stretch:

$$s_1 = \frac{60}{40} = 1.5 \text{ ft}$$



$+\uparrow \Sigma F_y = 0; \qquad F - 2\left(\frac{1}{2}T\right) = 0; \qquad F = T$



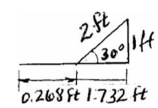
$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad -F_s + 2\left(\frac{\sqrt{3}}{2}\right)F = 0$

$$F_s = 1.732F$$

Final stretch is $1.5 + 0.268 = 1.768$ ft

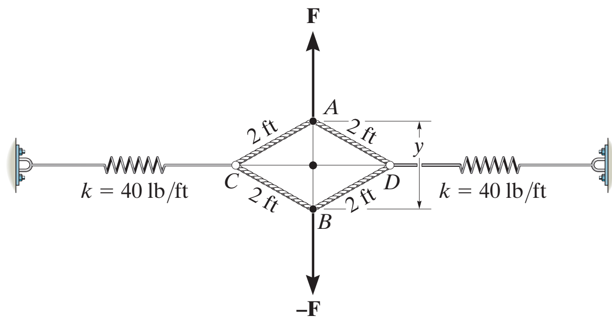
$$40(1.768) = 1.732F$$

$$F = 40.8 \text{ lb}$$ **Ans.**



3–70.

When y is zero, the springs are each stretched 1.5 ft. Determine the distance y if a force of $F = 60$ lb is applied to points A and B as shown. The ends of cords CAD and CBD are attached to rings at C and D.



SOLUTION

$+\uparrow \Sigma F_y = 0;$ $\quad 2\,T \sin\theta = 60$

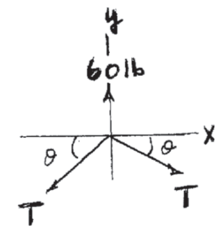
$$T \sin\theta = 30$$

$\xrightarrow{+} \Sigma F_x = 0;$ $\quad 2T\cos\theta = F_{sp}$

$$F_{sp}\tan\theta = 60 \qquad \textbf{(1)}$$

$$F_{sp} = kx$$

$$F_{sp} = 40(1.5 + 2 - 2\cos\theta)$$



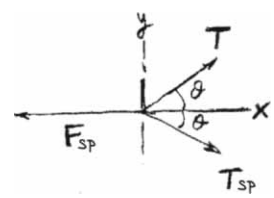
Substitute F in Eq. 1

$$40(1.5 + 2 - 2\cos\theta)\tan\theta = 60$$

$$(3.5 - 2\cos\theta)\tan\theta = 1.5$$

$$3.5\tan\theta - 2\sin\theta = 1.5$$

$$1.75\tan\theta - \sin\theta = 0.75$$

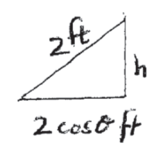


By trial and error:

$$\theta = 37.96^\circ$$

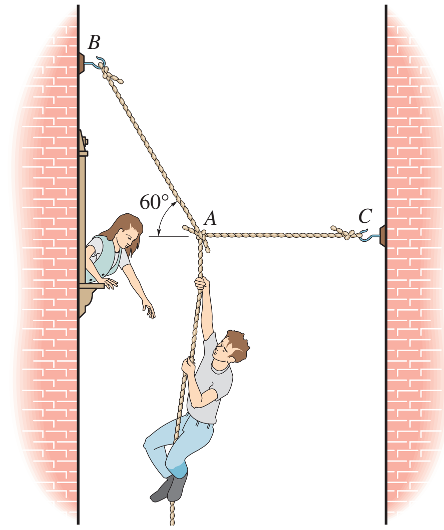
$$\frac{y}{2} = 2\sin 37.96^\circ$$

$$y = 2.46 \text{ ft} \qquad \textbf{Ans.}$$



3–71.

Romeo tries to reach Juliet by climbing with constant velocity up a rope which is knotted at point A. Any of the three segments of the rope can sustain a maximum force of 2 kN before it breaks. Determine if Romeo, who has a mass of 65 kg, can climb the rope, and if so, can he along with Juliet, who has a mass of 60 kg, climb down with constant velocity?



SOLUTION

$+\uparrow \Sigma F_y = 0; \qquad T_{AB} \sin 60° - 65(9.81) = 0$

$T_{AB} = 736.29 \text{ N} < 2000 \text{ N}$

$\xrightarrow{+} \Sigma F_x = 0; \qquad T_{AC} - 736.29 \cos 60° = 0$

$T_{AC} = 368.15 \text{ N} < 2000 \text{ N}$

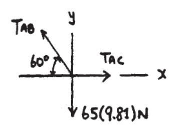
Yes, Romeo can climb up the rope. **Ans.**

$+\uparrow \Sigma F_y = 0; \qquad T_{AB} \sin 60° - 125(9.81) = 0$

$T_{AB} = 1415.95 \text{ N} < 2000 \text{ N}$

$\xrightarrow{+} \Sigma F_x = 0; \qquad T_{AC} - 1415.95 \cos 60° = 0$

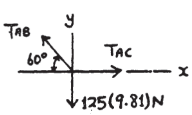
$T_{AC} = 708 \text{ N} < 2000 \text{ N}$



Also, for the vertical segment,

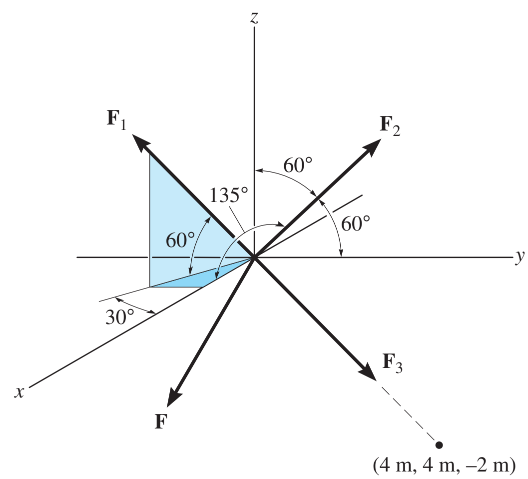
$T = 125(9.81) = 1226 \text{ N} < 2000 \text{ N}$

Yes, Romeo and Juliet can climb down. **Ans.**



***■3–72.**

Determine the magnitudes of forces $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ necessary to hold the force $\mathbf{F} = \{-9\mathbf{i} - 8\mathbf{j} - 5\mathbf{k}\}$ kN in equilibrium.



SOLUTION

$\Sigma F_x = 0; \qquad F_1 \cos 60° \cos 30° + F_2 \cos 135° + \frac{4}{6}F_3 - 9 = 0$

$\Sigma F_y = 0; \qquad -F_1 \cos 60° \sin 30° + F_2 \cos 60° + \frac{4}{6}F_3 - 8 = 0$

$\Sigma F_z = 0; \qquad F_1 \sin 60° + F_2 \cos 60° - \frac{2}{6}F_3 - 5 = 0$

$0.433F_1 - 0.707F_2 + 0.667F_3 = 9$

$-0.250F_1 + 0.500F_2 + 0.667F_3 = 8$

$0.866F_1 + 0.500F_2 - 0.333F_3 = 5$

Solving,

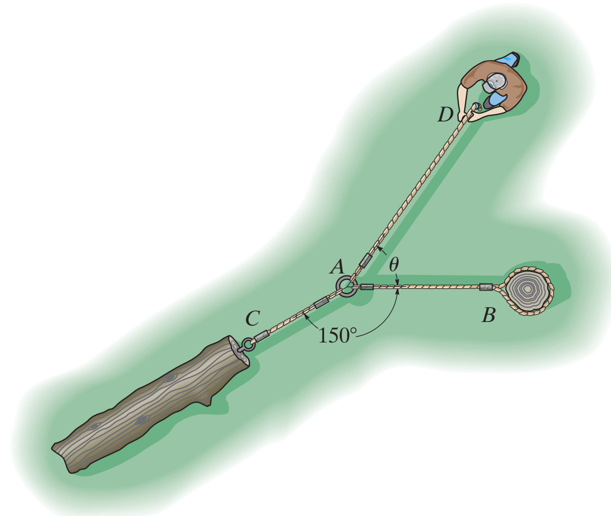
$F_1 = 8.26 \text{ kN}$ **Ans.**

$F_2 = 3.84 \text{ kN}$ **Ans.**

$F_3 = 12.2 \text{ kN}$ **Ans.**

3–73.

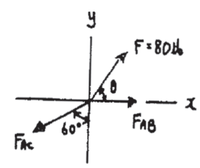
The man attempts to pull the log at C by using the three ropes. Determine the direction θ in which he should pull on his rope with a force of 80 lb, so that he exerts a maximum force on the log. What is the force on the log for this case? Also, determine the direction in which he should pull in order to maximize the force in the rope attached to B. What is this maximum force?



SOLUTION

$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad F_{AB} + 80\cos\theta - F_{AC}\sin 60^\circ = 0$ **(1)**

$+\uparrow\Sigma F_y = 0; \qquad 80\sin\theta - F_{AC}\cos 60^\circ = 0$ **(2)**



$$F_{AC} = 160\sin\theta$$

$$\frac{dF_{AC}}{d\theta} = 160\cos\theta = 0$$

$\theta = 90^\circ$ **Ans.**

$F_{AC} = 160\text{ lb}$ **Ans.**

From Eq. (1),

$$F_{AC}\sin 60^\circ = F_{AB} + 80\cos\theta$$

Substitute into Eq. (2),

$$80\sin\theta\sin 60^\circ = (F_{AB} + 80\cos\theta)\cos 60^\circ$$

$$F_{AB} = 138.6\sin\theta - 80\cos\theta$$

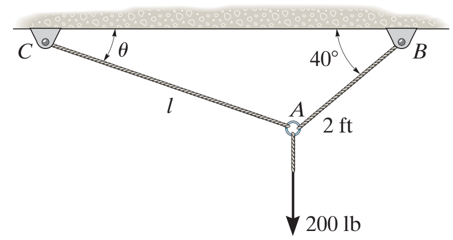
$$\frac{dF_{AB}}{d\theta} = 138.6\cos\theta + 80\sin\theta = 0$$

$\theta = \tan^{-1}\left[\frac{138.6}{-80}\right] = 120^\circ$ **Ans.**

$F_{AB} = 138.6\sin 120^\circ - 80\cos 120^\circ = 160\text{ lb}$ **Ans.**

■3–74.

The ring of negligible size is subjected to a vertical force of 200 lb. Determine the longest length l of cord AC such that the tension acting in AC is 160 lb. Also, what is the force acting in cord AB? *Hint:* Use the equilibrium condition to determine the required angle θ for attachment, then determine l using trigonometry applied to ΔABC.

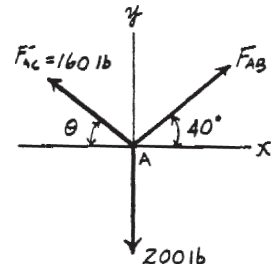


SOLUTION

Equations of Equilibrium:

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad F_{AB}\cos 40° - 160\cos\theta = 0 \qquad \textbf{(1)}$$

$$+\uparrow \Sigma F_y = 0; \qquad F_{AB}\sin 40° + 160\sin\theta - 200 = 0 \qquad \textbf{(2)}$$



Solving Eqs. (1) and (2) and choosing the smallest value of θ, yields

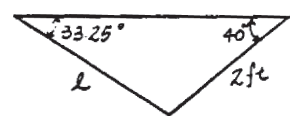
$$\theta = 33.25°$$

$$F_{AB} = 175 \text{ lb} \qquad \textbf{Ans.}$$

Geometry: Applying law of sines, we have

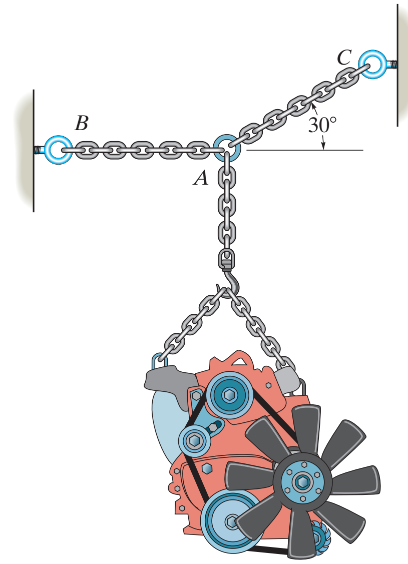
$$\frac{l}{\sin 40°} = \frac{2}{\sin 33.25°}$$

$$l = 2.34 \text{ ft} \qquad \textbf{Ans.}$$



3–75.

Determine the maximum weight of the engine that can be supported without exceeding a tension of 450 lb in chain AB and 480 lb in chain AC.



SOLUTION

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad F_{AC} \cos 30° - F_{AB} = 0$ **(1)**

$+\uparrow \Sigma F_y = 0; \qquad F_{AC} \sin 30° - W = 0$ **(2)**

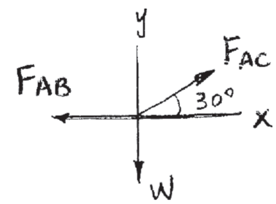
Assuming cable AB reaches the maximum tension $F_{AB} = 450$ lb.

From Eq. (1) $F_{AC} \cos 30° - 450 = 0 \qquad F_{AC} = 519.6 \text{ lb} > 480 \text{ lb}$ (No Good)

Assuming cable AC reaches the maximum tension $F_{AC} = 480$ lb.

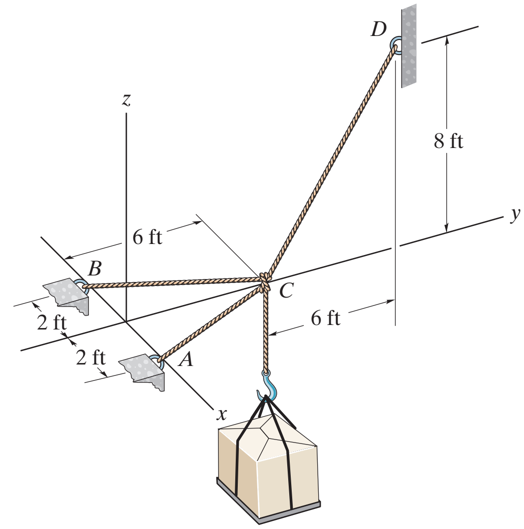
From Eq. (1) $480 \cos 30° - F_{AB} = 0 \qquad F_{AB} = 415.7 \text{ lb} < 450 \text{ lb}$ (OK)

From Eq. (2) $480 \sin 30° - W = 0 \qquad W = 240 \text{ lb}$ **Ans.**



***3–76.**

Determine the force in each cable needed to support the 500-lb load.



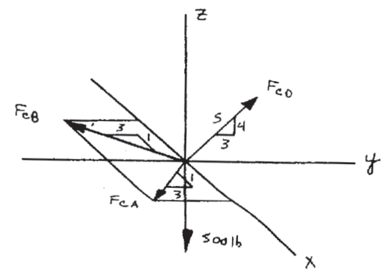
SOLUTION

At C:

$$\Sigma F_x = 0; \qquad F_{CA}\left(\frac{1}{\sqrt{10}}\right) - F_{CB}\left(\frac{1}{\sqrt{10}}\right) = 0$$

$$\Sigma F_y = 0; \qquad -F_{CA}\left(\frac{3}{\sqrt{10}}\right) - F_{CB}\left(\frac{3}{\sqrt{10}}\right) + F_{CD}\left(\frac{3}{5}\right) = 0$$

$$\Sigma F_z = 0; \qquad -500 + F_{CD}\left(\frac{4}{5}\right) = 0$$



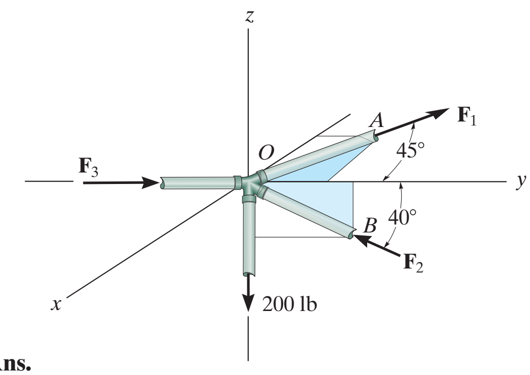
Solving:

$$F_{CD} = 625 \text{ lb}$$ **Ans.**

$$F_{CA} = F_{CB} = 198 \text{ lb}$$ **Ans.**

3–77.

The joint of a space frame is subjected to four member forces. Member OA lies in the x–y plane and member OB lies in the y–z plane. Determine the forces acting in each of the members required for equilibrium of the joint.



SOLUTION

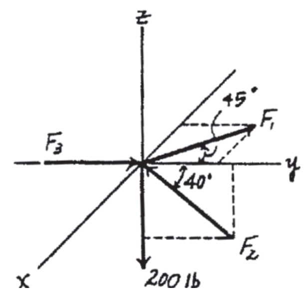
Equation of Equilibrium:

$\Sigma F_x = 0;\quad -F_1 \sin 45° = 0 \qquad F_1 = 0$ **Ans.**

$\Sigma F_z = 0;\quad F_2 \sin 40° - 200 = 0 \qquad F_2 = 311.14 \text{ lb} = 311 \text{ lb}$ **Ans.**

Using the results $F_1 = 0$ and $F_2 = 311.14$ lb and then summing forces along the y axis, we have

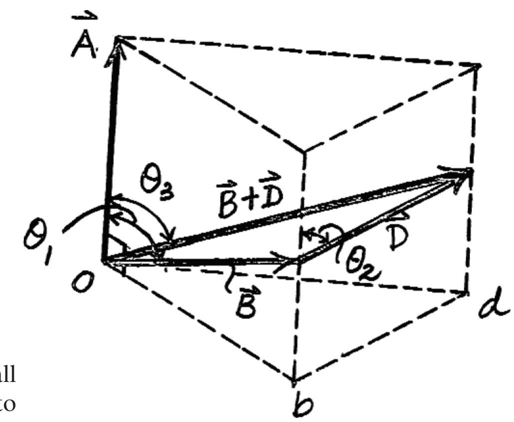
$\Sigma F_y = 0;\quad F_3 - 311.14 \cos 40° = 0 \qquad F_3 = 238 \text{ lb}$ **Ans.**



4–1.

If **A**, **B**, and **D** are given vectors, prove the distributive law for the vector cross product, i.e., $\mathbf{A} \times (\mathbf{B} + \mathbf{D}) = (\mathbf{A} \times \mathbf{B}) + (\mathbf{A} \times \mathbf{D})$.

SOLUTION



Consider the three vectors; with **A** vertical.

Note *obd* is perpendicular to **A**.

$$od = |\mathbf{A} \times (\mathbf{B} + \mathbf{D})| = |\mathbf{A}||\mathbf{B} + \mathbf{D}| \sin \theta_3$$

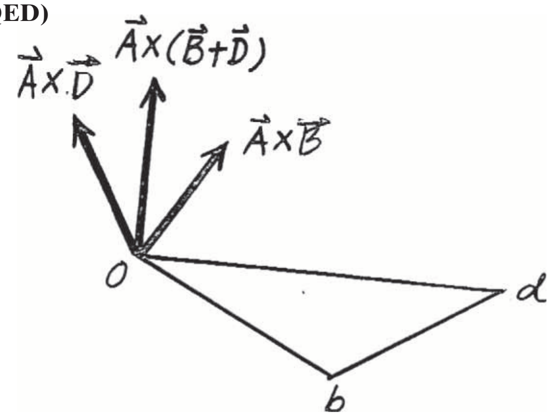
$$ob = |\mathbf{A} \times \mathbf{B}| = |\mathbf{A}||\mathbf{B}| \sin \theta_1$$

$$bd = |\mathbf{A} \times \mathbf{D}| = |\mathbf{A}||\mathbf{D}| \sin \theta_2$$

Also, these three cross products all lie in the plane *obd* since they are all perpendicular to **A**. As noted the magnitude of each cross product is proportional to the length of each side of the triangle.

The three vector cross products also form a closed triangle $o'b'd'$ which is similar to triangle *obd*. Thus from the figure,

$$\mathbf{A} \times (\mathbf{B} + \mathbf{D}) = (\mathbf{A} \times \mathbf{B}) + (\mathbf{A} \times \mathbf{D}) \qquad \textbf{(QED)}$$



Note also,

$$\mathbf{A} = A_x\,\mathbf{i} + A_y\,\mathbf{j} + A_z\,\mathbf{k}$$

$$\mathbf{B} = B_x\,\mathbf{i} + B_y\,\mathbf{j} + B_z\,\mathbf{k}$$

$$\mathbf{D} = D_x\,\mathbf{i} + D_y\,\mathbf{j} + D_z\,\mathbf{k}$$

$$\mathbf{A} \times (\mathbf{B} + \mathbf{D}) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x + D_x & B_y + D_y & B_z + D_z \end{vmatrix}$$

$$= [A_y(B_z + D_z) - A_z(B_y + D_y)]\mathbf{i}$$

$$- [A_x(B_z + D_z) - A_z(B_x + D_x)]\mathbf{j}$$

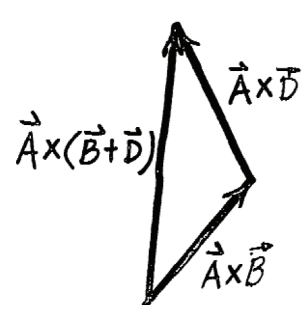
$$+ [A_x(B_y + D_y) - A_y(B_x + D_x)]\mathbf{k}$$

$$= [(A_y B_z - A_z B_y)\mathbf{i} - (A_x B_z - A_z B_x)]\mathbf{j} + (A_x B_y - A_y B_x)\mathbf{k}$$

$$+ [(A_y D_z - A_z D_y)\mathbf{i} - (A_x D_z - A_z D_x)\mathbf{j} + (A_x D_y - A_y D_x)\mathbf{k}$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ D_x & D_y & D_z \end{vmatrix}$$

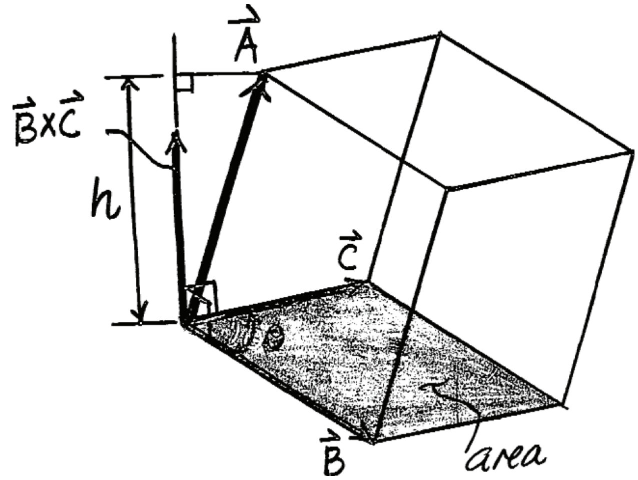
$$= (\mathbf{A} \times \mathbf{B}) + (\mathbf{A} \times \mathbf{D}) \qquad \textbf{(QED)}$$



4–2.

Prove the triple scalar product identity $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = (\mathbf{A} \times \mathbf{B}) \cdot \mathbf{C}$.

SOLUTION



As shown in the figure

$$\text{Area} = B(C \sin \theta) = |\mathbf{B} \times \mathbf{C}|$$

Thus,

$$\text{Volume of parallelepiped is } |\mathbf{B} \times \mathbf{C}||h|$$

But,

$$|h| = |\mathbf{A} \cdot \mathbf{u}_{(\mathbf{B}\times\mathbf{C})}| = \left|\mathbf{A} \cdot \left(\frac{\mathbf{B} \times \mathbf{C}}{|\mathbf{B} \times \mathbf{C}|}\right)\right|$$

Thus,

$$\text{Volume} = |\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C})|$$

Since $|(\mathbf{A} \times \mathbf{B}) \cdot \mathbf{C}|$ represents this same volume then

$$\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = (\mathbf{A} \times \mathbf{B}) \cdot \mathbf{C} \qquad \textbf{(QED)}$$

Also,

$$LHS = \mathbf{A} \cdot (\mathbf{B} \times \mathbf{C})$$

$$= (A_x \mathbf{i} + A_y \mathbf{j} + A_z \mathbf{k}) \cdot \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix}$$

$$= A_x (B_y C_z - B_z C_y) - A_y (B_x C_z - B_z C_x) + A_z (B_x C_y - B_y C_x)$$

$$= A_x B_y C_z - A_x B_z C_y - A_y B_x C_z + A_y B_z C_x + A_z B_x C_y - A_z B_y C_x$$

$$RHS = (\mathbf{A} \times \mathbf{B}) \cdot \mathbf{C}$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} \cdot (C_x \mathbf{i} + C_y \mathbf{j} + C_z \mathbf{k})$$

$$= C_x(A_y B_z - A_z B_y) - C_y(A_x B_z - A_z B_x) + C_z(A_x B_y - A_y B_x)$$

$$= A_x B_y C_z - A_x B_z C_y - A_y B_x C_z + A_y B_z C_x + A_z B_x C_y - A_z B_y C_x$$

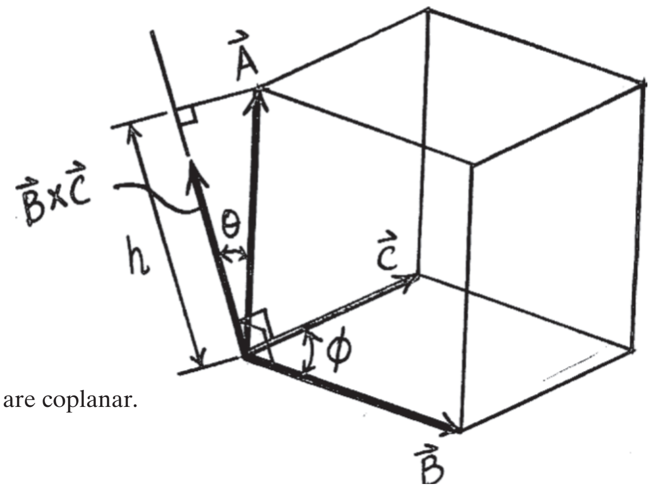
Thus, $LHS = RHS$

$$\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = (\mathbf{A} \times \mathbf{B}) \cdot \mathbf{C} \qquad \textbf{(QED)}$$

4–3.

Given the three nonzero vectors **A**, **B**, and **C**, show that if $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = 0$, the three vectors *must* lie in the same plane.

SOLUTION



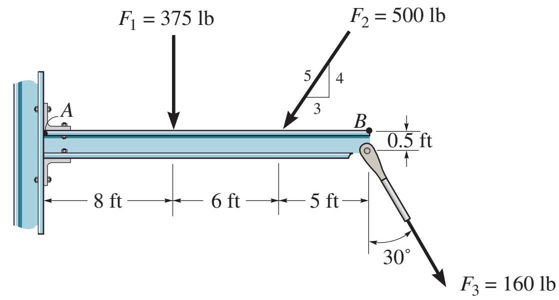
Consider,

$$
\begin{aligned}
|\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C})| &= |\mathbf{A}|\,|\mathbf{B} \times \mathbf{C}\,| \cos \theta \\
&= (|\mathbf{A}| \cos \theta)|\mathbf{B} \times \mathbf{C}| \\
&= |h|\,|\mathbf{B} \times \mathbf{C}| \\
&= BC\,|h| \sin \phi \\
&= \text{volume of parallelepiped.}
\end{aligned}
$$

If $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = 0$, then the volume equals zero, so that **A, B,** and **C** are coplanar.

***4–4.**

Determine the moment about point A of each of the three forces acting on the beam.



SOLUTION

$$\zeta + (M_{F_1})_A = -375(8)$$

$$= -3000 \text{ lb} \cdot \text{ft} = 3.00 \text{ kip} \cdot \text{ft} \quad \textit{(Clockwise)}$$ **Ans.**

$$\zeta + (M_{F_2})_A = -500\left(\frac{4}{5}\right)(14)$$

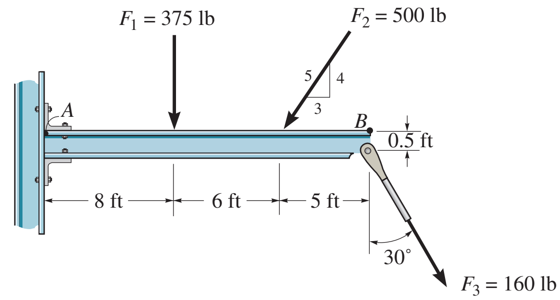
$$= -5600 \text{ lb} \cdot \text{ft} = 5.60 \text{ kip} \cdot \text{ft} \quad \textit{(Clockwise)}$$ **Ans.**

$$\zeta + (M_{F_3})_A = -160(\cos 30°)(19) + 160 \sin 30°(0.5)$$

$$= -2593 \text{ lb} \cdot \text{ft} = 2.59 \text{ kip} \cdot \text{ft} \quad \textit{(Clockwise)}$$ **Ans.**

4–5.

Determine the moment about point B of each of the three forces acting on the beam.



SOLUTION

$$\circlearrowleft +(M_{F_1})_B = 375(11)$$

$$= 4125 \text{ lb} \cdot \text{ft} = 4.125 \text{ kip} \cdot \text{ft} \quad \textbf{\textit{(Counterclockwise)}}$$ **Ans.**

$$\circlearrowleft +(M_{F_2})_B = 500\left(\frac{4}{5}\right)(5)$$

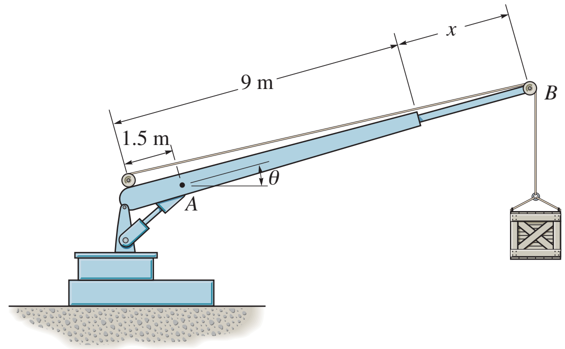
$$= 2000 \text{ lb} \cdot \text{ft} = 2.00 \text{ kip} \cdot \text{ft} \quad \textbf{\textit{(Counterclockwise)}}$$ **Ans.**

$$\circlearrowleft +(M_{F_3})_B = 160 \sin 30^\circ(0.5) - 160 \cos 30^\circ(0)$$

$$= 40.0 \text{ lb} \cdot \text{ft} \quad \textbf{\textit{(Counterclockwise)}}$$ **Ans.**

4–6.

The crane can be adjusted for any angle $0° \leq \theta \leq 90°$ and any extension $0 \leq x \leq 5$ m. For a suspended mass of 120 kg, determine the moment developed at A as a function of x and θ. What values of both x and θ develop the maximum possible moment at A? Compute this moment. Neglect the size of the pulley at B.



SOLUTION

$$\circlearrowleft + M_A = -120(9.81)(7.5 + x)\cos\theta$$

$$= \{-1177.2 \cos\theta(7.5 + x)\} \text{ N}\cdot\text{m}$$

$$= \{1.18 \cos\theta(7.5 + x)\} \text{ kN}\cdot\text{m} \quad \textbf{\textit{(Clockwise)}}$$ **Ans.**

The maximum moment at A occurs when $\theta = 0°$ and $x = 5$ m. **Ans.**

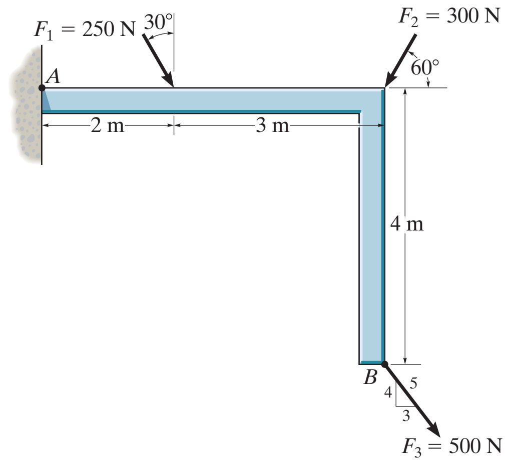
$$\circlearrowleft + (M_A)_{\max} = \{-1177.2 \cos 0°(7.5 + 5)\} \text{ N}\cdot\text{m}$$

$$= -14\,715 \text{ N}\cdot\text{m}$$

$$= 14.7 \text{ kN}\cdot\text{m} \quad \textbf{\textit{(Clockwise)}}$$ **Ans.**

4–7.

Determine the moment of each of the three forces about point A.



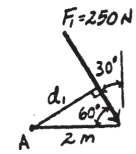
SOLUTION

The moment arm measured perpendicular to each force from point A is

$$d_1 = 2 \sin 60° = 1.732 \text{ m}$$

$$d_2 = 5 \sin 60° = 4.330 \text{ m}$$

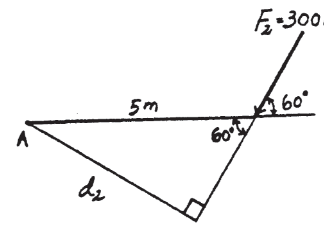
$$d_3 = 2 \sin 53.13° = 1.60 \text{ m}$$



Using each force where $M_A = Fd$, we have

$$\circlearrowleft + (M_{F_1})_A = -250(1.732)$$

$$= -433 \text{ N} \cdot \text{m} = 433 \text{ N} \cdot \text{m} \quad \textit{\textbf{(Clockwise)}}$$ **Ans.**

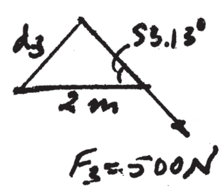


$$\circlearrowleft + (M_{F_2})_A = -300(4.330)$$

$$= -1299 \text{ N} \cdot \text{m} = 1.30 \text{ kN} \cdot \text{m} \quad \textit{\textbf{(Clockwise)}}$$ **Ans.**

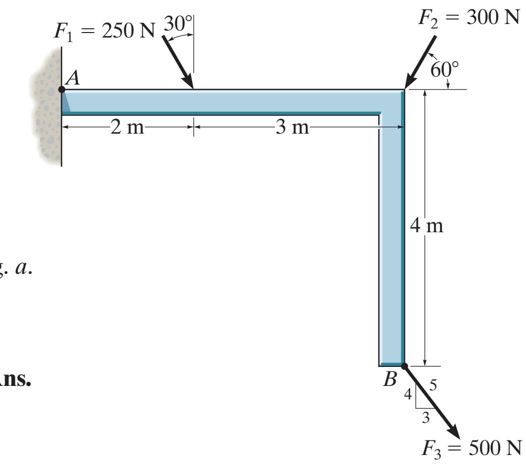
$$\circlearrowleft + (M_{F_3})_A = -500(1.60)$$

$$= -800 \text{ N} \cdot \text{m} = 800 \text{ N} \cdot \text{m} \quad \textit{\textbf{(Clockwise)}}$$ **Ans.**



***4–8.**

Determine the moment of each of the three forces about point B.



SOLUTION

The forces are resolved into horizontal and vertical component as shown in Fig. *a*. For $\mathbf{F}_1$,

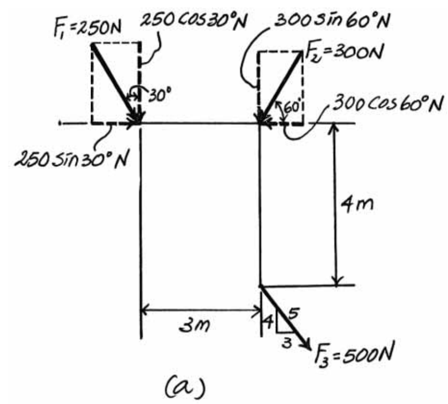
$$\circlearrowleft + M_B = 250 \cos 30°(3) - 250 \sin 30°(4)$$
$$= 149.51 \text{ N}\cdot\text{m} = 150 \text{ N}\cdot\text{m} \circlearrowleft \qquad \textbf{Ans.}$$

For $\mathbf{F}_2$,

$$\circlearrowleft + M_B = 300 \sin 60°(0) + 300 \cos 60°(4)$$
$$= 600 \text{ N}\cdot\text{m} \circlearrowleft \qquad \textbf{Ans.}$$

Since the line of action of $\mathbf{F}_3$ passes through B, its moment arm about point B is zero. Thus

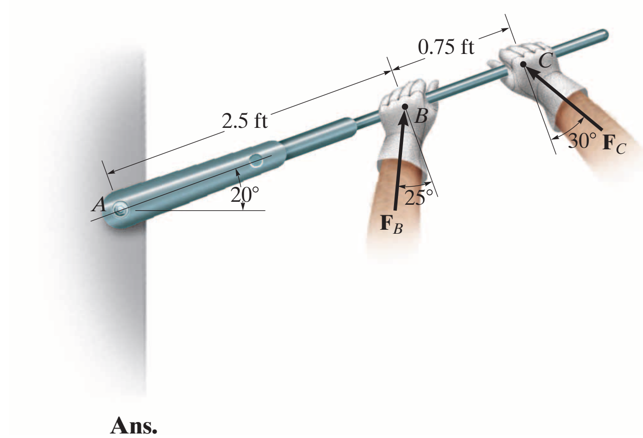
$$M_B = 0 \qquad \textbf{Ans.}$$



(a)

4–9.

Determine the moment of each force about the bolt located at A. Take $F_B = 40$ lb, $F_C = 50$ lb.



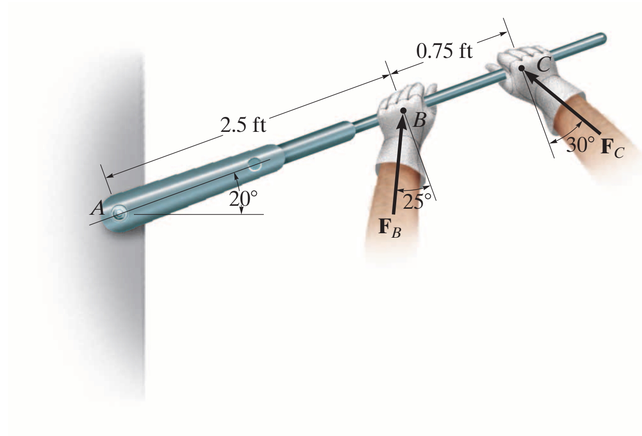
SOLUTION

$$\circlearrowleft + M_B = 40 \cos 25°(2.5) = 90.6 \text{ lb} \cdot \text{ft} \circlearrowleft$$ **Ans.**

$$\circlearrowleft + M_C = 50 \cos 30°(3.25) = 141 \text{ lb} \cdot \text{ft} \circlearrowleft$$ **Ans.**

4–10.

If $F_B = 30$ lb and $F_C = 45$ lb, determine the resultant moment about the bolt located at A.



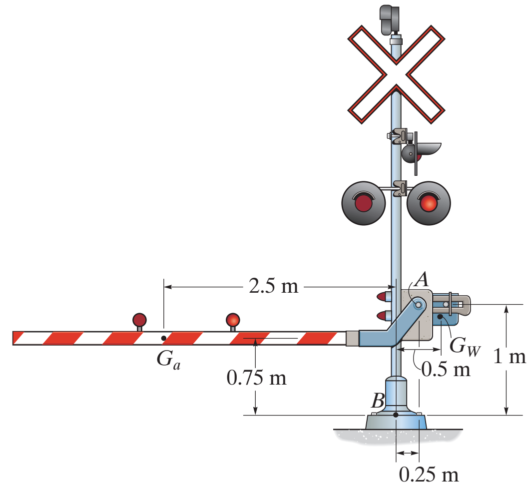
SOLUTION

$$\circlearrowleft + M_A = 30 \cos 25°(2.5) + 45 \cos 30°(3.25)$$

$$= 195 \text{ lb} \cdot \text{ft} \circlearrowleft$$

4–11.

The railway crossing gate consists of the 100-kg gate arm having a center of mass at G_a and the 250-kg counterweight having a center of mass at G_W. Determine the magnitude and directional sense of the resultant moment produced by the weights about point A.



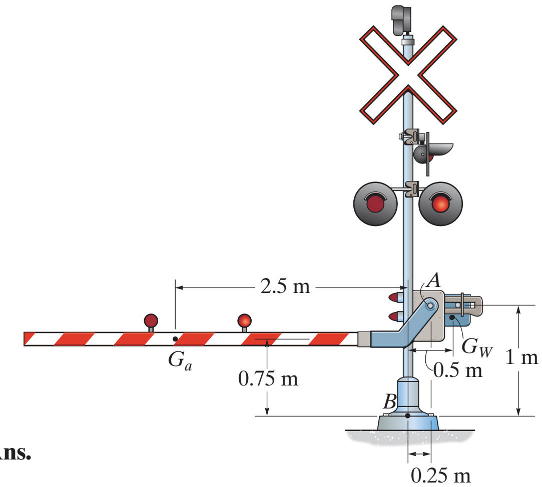
SOLUTION

$$+(M_R)_A = \Sigma Fd; \quad (M_R)_A = 100(9.81)(2.5 + 0.25) - 250(9.81)(0.5 - 0.25)$$

$$= 2084.625 \text{ N} \cdot \text{m} = 2.08 \text{ kN} \cdot \text{m} \; \textbf{(\textit{Counterclockwise})}$$ **Ans.**

***4–12.**

The railway crossing gate consists of the 100-kg gate arm having a center of mass at G_a and the 250-kg counterweight having a center of mass at G_W. Determine the magnitude and directional sense of the resultant moment produced by the weights about point B.



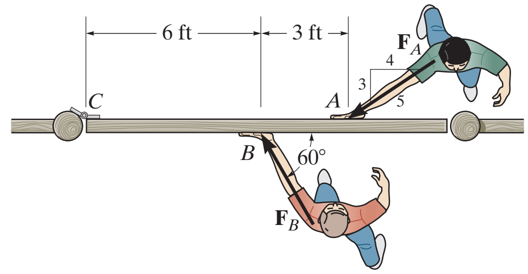
SOLUTION

$$\circlearrowleft +(M_R)_B = \Sigma Fd; \quad (M_R)_B = 100(9.81)(2.5) - 250(9.81)(0.5)$$

$$= 1226.25\ \text{N} \cdot \text{m} = 1.23\ \text{kN} \cdot \text{m}\ \textbf{(\textit{Counterclockwise})}$$ **Ans.**

***4–13.**

The two boys push on the gate with forces of $F_A = 30$ lb, and $F_B = 50$ lb, as shown. Determine the moment of each force about C. Which way will the gate rotate, clockwise or counterclockwise? Neglect the thickness of the gate.



SOLUTION

$$\circlearrowleft + (M_{F_A})_C = -30\left(\frac{3}{5}\right)(9)$$

$$= -162 \text{ lb} \cdot \text{ft} = 162 \text{ lb} \cdot \text{ft} \textbf{ (\textit{Clockwise})}$$ **Ans.**

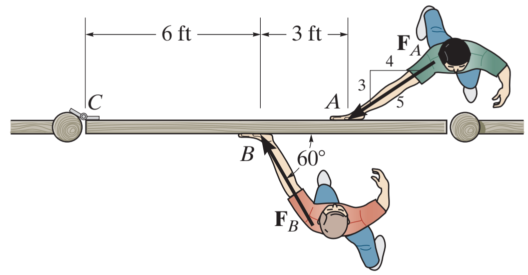
$$\circlearrowleft + (M_{F_B})_C = 50(\sin 60°)(6)$$

$$= 260 \text{ lb} \cdot \text{ft} \textbf{ (\textit{Counterclockwise})}$$ **Ans.**

Since $(M_{F_B})_C > (M_{F_A})_C$, the gate will rotate ***Counterclockwise***. **Ans.**

4–14.

Two boys push on the gate as shown. If the boy at B exerts a force of $F_B = 30$ lb, determine the magnitude of the force F_A the boy at A must exert in order to prevent the gate from turning. Neglect the thickness of the gate.



SOLUTION

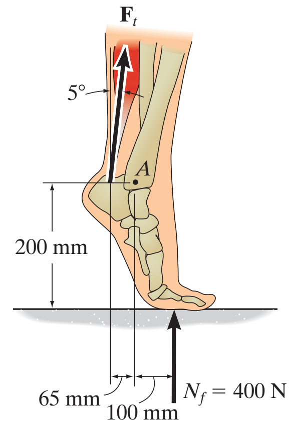
In order to prevent the gate from turning, the resultant moment about point C must be equal to zero.

$$+M_{R_C} = \Sigma Fd; \qquad M_{R_C} = 0 = 30 \sin 60°(6) - F_A\left(\frac{3}{5}\right)(9)$$

$$F_A = 28.9 \text{ lb} \qquad \textbf{Ans.}$$

4–15.

The Achilles tendon force of $F_t = 650$ N is mobilized when the man tries to stand on his toes. As this is done, each of his feet is subjected to a reactive force of $N_f = 400$ N. Determine the resultant moment of $\mathbf{F}_t$ and $\mathbf{N}_f$ about the ankle joint A.



SOLUTION

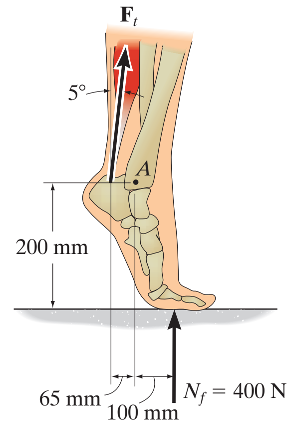
Referring to Fig. a,

$$\zeta+(M_R)_A = \Sigma Fd; \qquad (M_R)_A = 400(0.1) - 650(0.065)\cos 5^\circ$$

$$= -2.09 \text{ N}\cdot\text{m} = 2.09 \text{ N}\cdot\text{m} \textbf{ (\textit{Clockwise})} \quad \textbf{Ans.}$$

***4–16.**

The Achilles tendon force $\mathbf{F}_t$ is mobilized when the man tries to stand on his toes. As this is done, each of his feet is subjected to a reactive force of $N_t = 400$ N. If the resultant moment produced by forces $\mathbf{F}_t$ and $\mathbf{N}_f$ about the ankle joint A is required to be zero, determine the magnitude of $\mathbf{F}_f$.

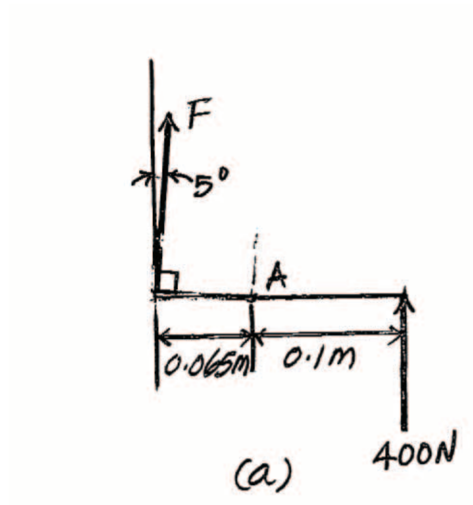


SOLUTION

Referring to Fig. *a*,

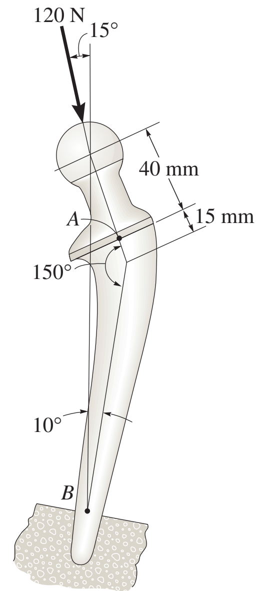
$$\circlearrowleft +(M_R)_A = \Sigma Fd; \qquad 0 = 400(0.1) - F\cos 5°(0.065)$$

$$F = 618 \text{ N} \qquad \textbf{Ans.}$$



4–17.

The total hip replacement is subjected to a force of $F = 120$ N. Determine the moment of this force about the neck at A and the stem at B.



SOLUTION

Moment About Point A: The angle between the line of action of the load and the neck axis is $20° - 15° = 5°$.

$$\circlearrowleft + M_A = 120 \sin 5°(0.04)$$

$$= 0.418 \text{ N} \cdot \text{m} \quad \textit{(Counterclockwise)} \qquad \textbf{Ans.}$$

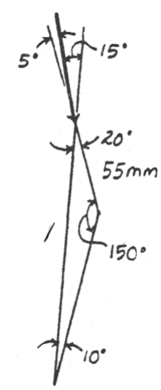
Moment About Point B: The dimension l can be determined using the law of sines.

$$\frac{l}{\sin 150°} = \frac{55}{\sin 10°} \qquad l = 158.4 \text{ mm} = 0.1584 \text{ m}$$

Then,

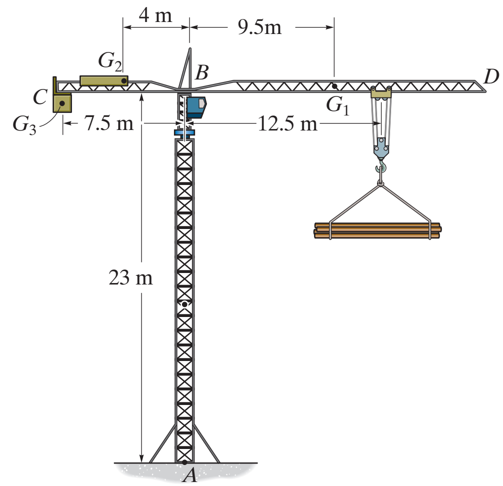
$$\circlearrowleft + \quad M_B = -120 \sin 15°(0.1584)$$

$$= -4.92 \text{ N} \cdot \text{m} = 4.92 \text{ N} \cdot \text{m} \quad \textit{(Clockwise)} \qquad \textbf{Ans.}$$



4–18.

The tower crane is used to hoist the 2-Mg load upward at constant velocity. The 1.5-Mg jib BD, 0.5-Mg jib BC, and 6-Mg counterweight C have centers of mass at G_1, G_2, and G_3, respectively. Determine the resultant moment produced by the load and the weights of the tower crane jibs about point A and about point B.



SOLUTION

Since the moment arms of the weights and the load measured to points A and B are the same, the resultant moments produced by the load and the weight about points A and B are the same.

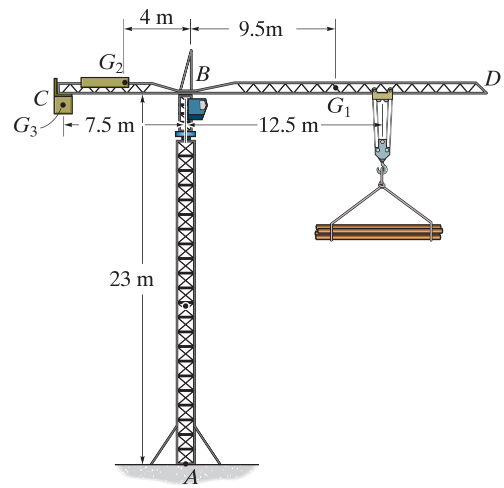
$\zeta + (M_R)_A = (M_R)_B = \Sigma Fd; \qquad (M_R)_A = (M_R)_B = 6000(9.81)(7.5) + 500(9.81)(4) - 1500(9.81)(9.5)$

$-2000(9.81)(12.5) = 76\,027.5 \text{ N}\cdot\text{m} = 76.0 \text{ kN}\cdot\text{m}$ **(*Counterclockwise*)**

Ans.

4–19.

The tower crane is used to hoist a 2-Mg load upward at constant velocity. The 1.5-Mg jib BD and 0.5-Mg jib BC have centers of mass at G_1 and G_2, respectively. Determine the required mass of the counterweight C so that the resultant moment produced by the load and the weight of the tower crane jibs about point A is zero. The center of mass for the counterweight is located at G_3.



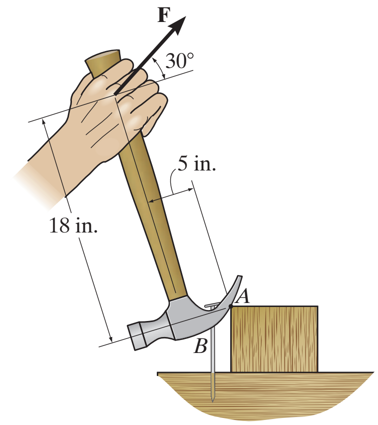
SOLUTION

$\zeta+ (M_R)_A = \Sigma Fd; \qquad 0 = M_C(9.81)(7.5) + 500(9.81)(4) - 1500(9.81)(9.5) - 2000(9.81)(12.5)$

$M_C = 4966.67 \text{ kg} = 4.97 \text{ Mg}$ **Ans.**

***4–20.**

The handle of the hammer is subjected to the force of $F = 20$ lb. Determine the moment of this force about the point A.

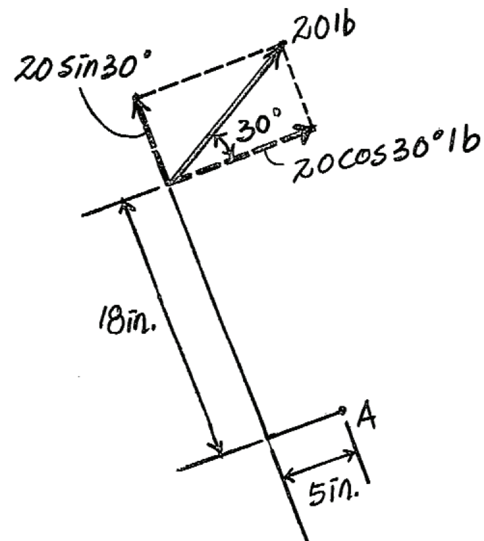


SOLUTION

Resolving the 20-lb force into components parallel and perpendicular to the hammer, Fig. *a*, and applying the principle of moments,

$$\circlearrowleft + M_A = -20 \cos 30°(18) - 20 \sin 30°(5)$$

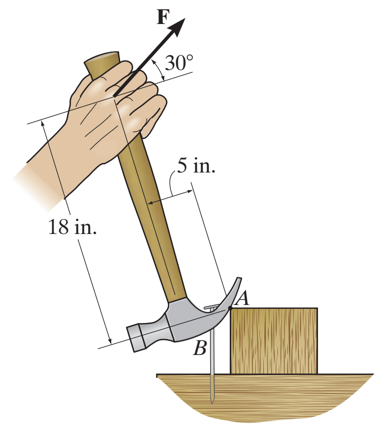
$$= -361.77 \text{ lb} \cdot \text{in} = 362 \text{ lb} \cdot \text{in} \ \ \textbf{(\textit{Clockwise})} \qquad \textbf{Ans.}$$



(a)

4–21.

In order to pull out the nail at B, the force **F** exerted on the handle of the hammer must produce a clockwise moment of 500 lb. in. about point A. Determine the required magnitude of force **F**.

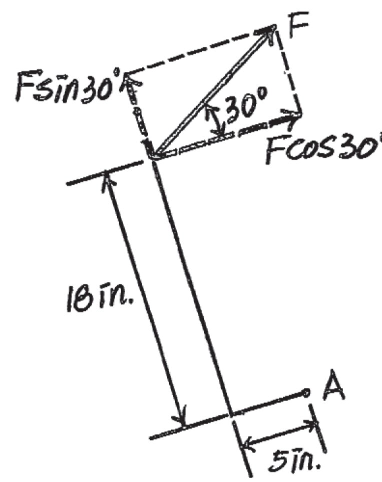


SOLUTION

Resolving force **F** into components parallel and perpendicular to the hammer, Fig. a, and applying the principle of moments,

$$\circlearrowleft +M_A = -500 = -F\cos 30°(18) - F\sin 30°(5)$$

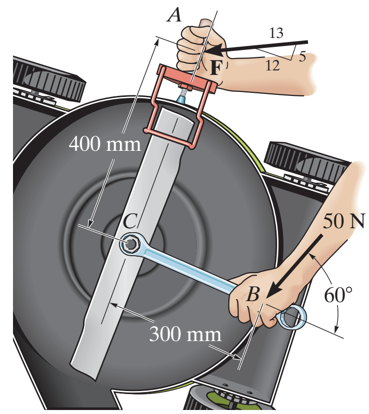
$$F = 27.6 \text{ lb} \qquad \textbf{Ans.}$$



(a)

4–22.

The tool at A is used to hold a power lawnmower blade stationary while the nut is being loosened with the wrench. If a force of 50 N is applied to the wrench at B in the direction shown, determine the moment it creates about the nut at C. What is the magnitude of force **F** at A so that it creates the opposite moment about C?



SOLUTION

$$\curvearrowleft + M_A = 50 \sin 60°(0.3)$$

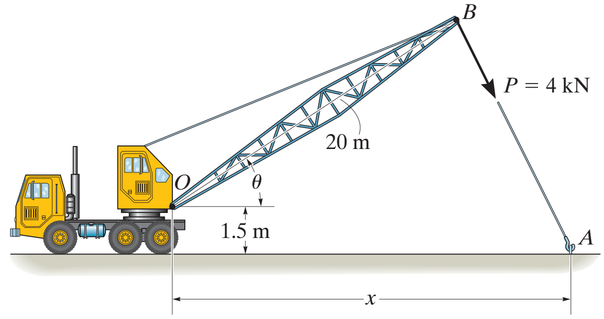
$$M_A = 12.99 = 13.0 \text{ N} \cdot \text{m}$$ **Ans.**

$$\curvearrowright + M_A = 0; \quad -12.99 + F\left(\frac{12}{13}\right)(0.4) = 0$$

$$F = 35.2 \text{ N}$$ **Ans.**

4–23.

The towline exerts a force of $P = 4$ kN at the end of the 20-m-long crane boom. If $\theta = 30°$, determine the placement x of the hook at A so that this force creates a maximum moment about point O. What is this moment?



SOLUTION

Maximum moment, $OB \perp BA$

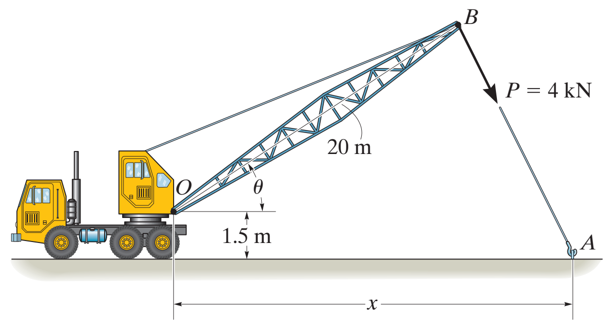
$$\circlearrowleft + (M_O)_{max} = -4\text{kN}(20) = 80 \text{ kN} \cdot \text{m} \circlearrowright$$ **Ans.**

$$4 \text{ kN} \sin 60°(x) - 4 \text{ kN} \cos 60°(1.5) = 80 \text{ kN} \cdot \text{m}$$

$$x = 24.0 \text{ m}$$ **Ans.**

***4–24.**

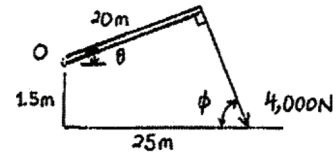
The towline exerts a force of $P = 4$ k N at the end of the 20-m-long crane boom. If $x = 25$ m, determine the position θ of the boom so that this force creates a maximum moment about point O. What is this moment?



SOLUTION

Maximum moment, $OB \perp BA$

$$\circlearrowleft + (M_O)_{max} = 4000(20) = 80\,000 \text{ N} \cdot \text{m} = 80.0 \text{ kN} \cdot \text{m} \qquad \textbf{Ans.}$$



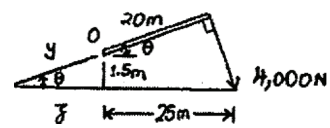
$$4000 \sin \phi(25) - 4000 \cos \phi(1.5) = 80\,000$$

$$25 \sin \phi - 1.5 \cos \phi = 20$$

$$\phi = 56.43^\circ$$

$$\theta = 90^\circ - 56.43^\circ = 33.6^\circ \qquad \textbf{Ans.}$$

Also,



$$(1.5)^2 + z^2 = y^2$$

$$2.25 + z^2 = y^2$$

Similar triangles

$$\frac{20 + y}{z} = \frac{25 + z}{y}$$

$$20y + y^2 = 25z + z^2$$

$$20(\sqrt{2.25 + z^2}) + 2.25 + z^2 = 25z + z^2$$

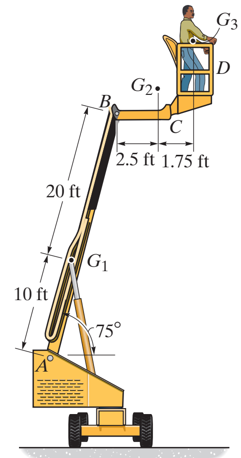
$$z = 2.260 \text{ m}$$

$$y = 2.712 \text{ m}$$

$$\theta = \cos^{-1}\left(\frac{2.260}{2.712}\right) = 33.6^\circ \qquad \textbf{Ans.}$$

4–25.

If the 1500-lb boom AB, the 200-lb cage BCD, and the 175-lb man have centers of gravity located at points G_1, G_2 and G_3, respectively, determine the resultant moment produced by each weight about point A.



SOLUTION

Moment of the weight of boom AB about point A:

$$\circlearrowleft + M_A = -1500(10\cos 75°) = -3882.29 \text{ lb} \cdot \text{ft} = 3.88 \text{ kip} \cdot \text{ft} \ (\textbf{\textit{Clockwise}}) \qquad \textbf{Ans.}$$

Moment of the weight of cage BCD about point A:

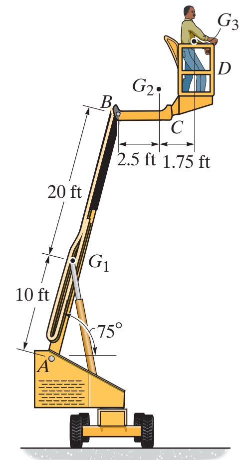
$$\circlearrowleft + M_A = -200(30\cos 75° + 2.5) = -2052.91 \text{ lb} \cdot \text{ft} = 2.05 \text{ kip} \cdot \text{ft} \ (\textbf{\textit{Clockwise}}) \qquad \textbf{Ans.}$$

Moment of the weight of the man about point A:

$$\circlearrowleft + M_A = -175(30\cos 75° + 4.25) = -2102.55 \text{ lb} \cdot \text{ft} = 2.10 \text{ kip} \cdot \text{ft} \ (\textbf{\textit{Clockwise}}) \qquad \textbf{Ans.}$$

4–26.

If the 1500-lb boom AB, the 200-lb cage BCD, and the 175-lb man have centers of gravity located at points G_1, G_2 and G_3, respectively, determine the resultant moment produced by all the weights about point A.

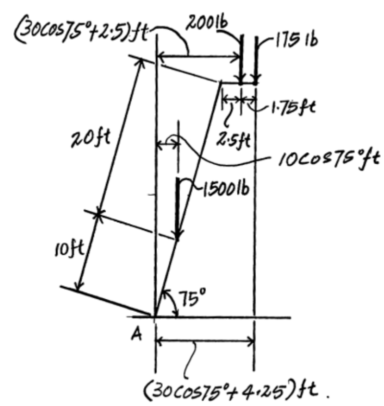


SOLUTION

Referring to Fig. *a*, the resultant moment of the weight about point A is given by

$$\circlearrowleft +(M_R)_A = \Sigma Fd; \qquad (M_R)_A = -1500(10\cos 75°) - 200(30\cos 75° + 2.5) - 175(30\cos 75° + 4.25)$$

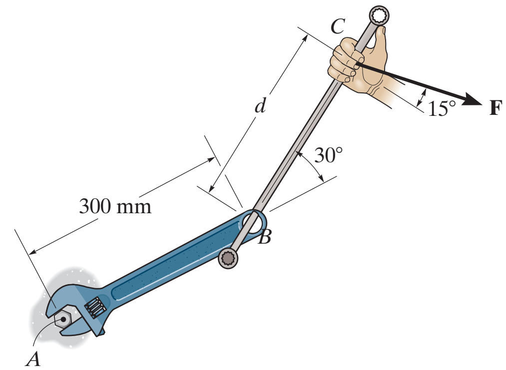
$$= -8037.75\ \text{lb}\cdot\text{ft} = 8.04\ \text{kip}\cdot\text{ft}\ \textbf{(\textit{Clockwise})} \qquad \textbf{Ans.}$$



(a)

4–27.

The connected bar BC is used to increase the lever arm of the crescent wrench as shown. If the applied force is $F = 200$ N and $d = 300$ mm, determine the moment produced by this force about the bolt at A.

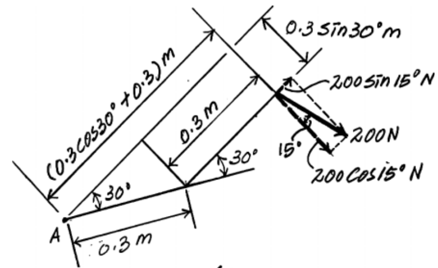


SOLUTION

By resolving the 200-N force into components parallel and perpendicular to the box wrench BC, Fig. a, the moment can be obtained by adding algebraically the moments of these two components about point A in accordance with the principle of moments.

$$\circlearrowleft +(M_R)_A = \Sigma Fd; \qquad M_A = 200 \sin 15°(0.3 \sin 30°) - 200 \cos 15°(0.3 \cos 30° + 0.3)$$

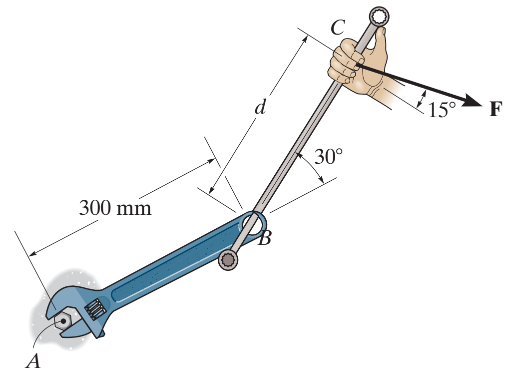
$$= -100.38 \text{ N} \cdot \text{m} = 100 \text{ N} \cdot \text{m} \textbf{ (\textit{Clockwise})} \qquad \textbf{Ans.}$$



(a)

***4–28.**

The connected bar BC is used to increase the lever arm of the crescent wrench as shown. If a clockwise moment of $M_A = 120\,\text{N}\cdot\text{m}$ is needed to tighten the bolt at A and the force $F = 200$ N, determine the required extension d in order to develop this moment.

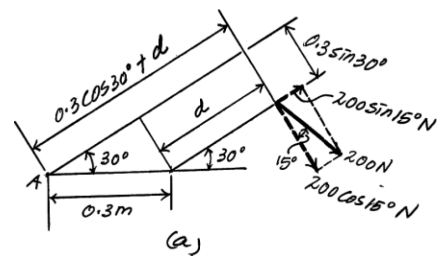


SOLUTION

By resolving the 200-N force into components parallel and perpendicular to the box wrench BC, Fig. a, the moment can be obtained by adding algebraically the moments of these two components about point A in accordance with the principle of moments.

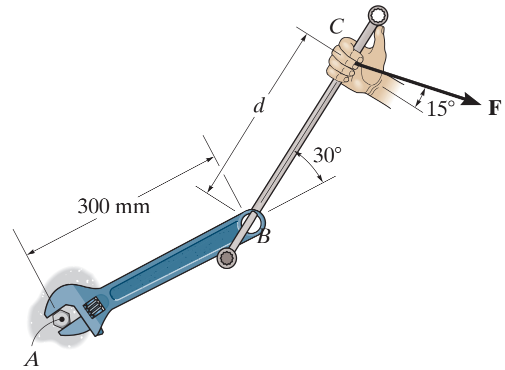
$$\circlearrowleft +(M_R)_A = \Sigma Fd; \quad -120 = 200 \sin 15^\circ(0.3 \sin 30^\circ) - 200 \cos 15^\circ(0.3 \cos 30^\circ + d)$$

$$d = 0.4016\text{ m} = 402\text{ mm} \qquad \textbf{Ans.}$$



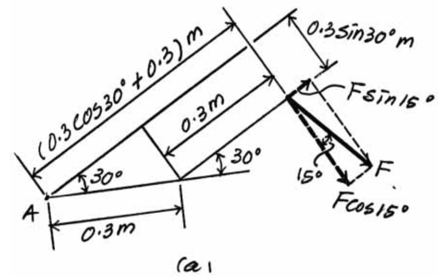
4–29.

The connected bar BC is used to increase the lever arm of the crescent wrench as shown. If a clockwise moment of $M_A = 120\,\text{N}\cdot\text{m}$ is needed to tighten the nut at A and the extension $d = 300$ mm, determine the required force **F** in order to develop this moment.



SOLUTION

By resolving force **F** into components parallel and perpendicular to the box wrench BC, Fig. a, the moment of **F** can be obtained by adding algebraically the moments of these two components about point A in accordance with the principle of moments.

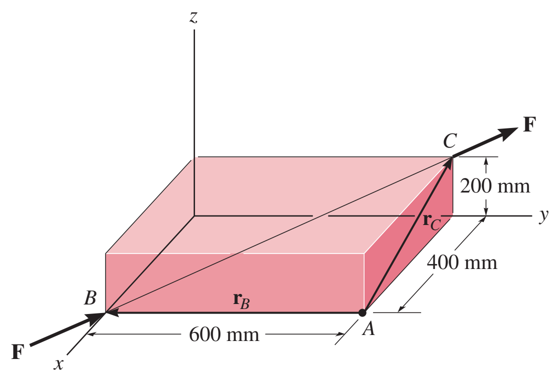


$$\circlearrowleft +(M_R)_A = \Sigma Fd; \qquad -120 = F\sin 15°(0.3\sin 30°) - F\cos 15°(0.3\cos 30° + 0.3)$$

$$F = 239\text{ N} \qquad \textbf{Ans.}$$

4–30.

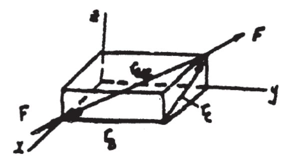
A force **F** having a magnitude of $F = 100$ N acts along the diagonal of the parallelepiped. Determine the moment of **F** about point A, using $\mathbf{M}_A = \mathbf{r}_B \times \mathbf{F}$ and $\mathbf{M}_A = \mathbf{r}_C \times \mathbf{F}$.



SOLUTION

$$\mathbf{F} = 100\left(\frac{-0.4\,\mathbf{i} + 0.6\,\mathbf{j} + 0.2\,\mathbf{k}}{0.7483}\right)$$

$$\mathbf{F} = \{-53.5\,\mathbf{i} + 80.2\,\mathbf{j} + 26.7\,\mathbf{k}\}\text{ N}$$



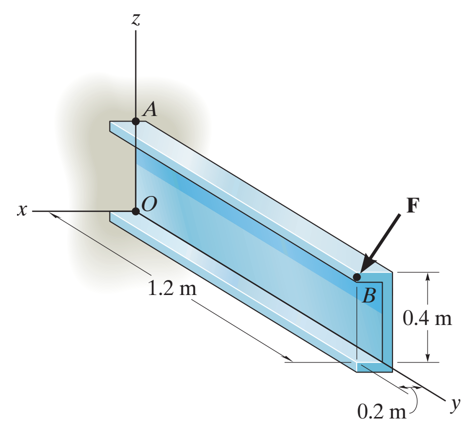
$$\mathbf{M}_A = \mathbf{r}_B \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -0.6 & 0 \\ -53.5 & 80.2 & 26.7 \end{vmatrix} = \{-16.0\,\mathbf{i} - 32.1\,\mathbf{k}\}\text{ N}\cdot\text{m}$$ **Ans.**

Also,

$$\mathbf{M}_A = \mathbf{r}_C \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.4 & 0 & 0.2 \\ -53.5 & 80.2 & 26.7 \end{vmatrix} = \{-16.0\,\mathbf{i} - 32.1\,\mathbf{k}\}\text{ N}\cdot\text{m}$$ **Ans.**

4–31.

The force $\mathbf{F} = \{600\mathbf{i} + 300\mathbf{j} - 600\mathbf{k}\}$ N acts at the end of the beam. Determine the moment of the force about point A.



SOLUTION

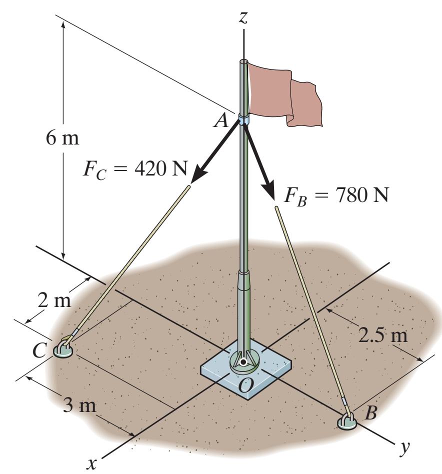
$$\mathbf{r} = \{0.2\mathbf{i} + 1.2\mathbf{j}\} \text{ m}$$

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.2 & 1.2 & 0 \\ 600 & 300 & -600 \end{vmatrix}$$

$$\mathbf{M}_O = \{-720\mathbf{i} + 120\mathbf{j} - 660\mathbf{k}\} \text{ N} \cdot \text{m} \qquad \textbf{Ans.}$$

***4–32.**

Determine the moment produced by force $\mathbf{F}_B$ about point O. Express the result as a Cartesian vector.



SOLUTION

Position Vector and Force Vectors: Either position vector $\mathbf{r}_{OA}$ or $\mathbf{r}_{OB}$ can be used to determine the moment of $\mathbf{F}_B$ about point O.

$$\mathbf{r}_{OA} = [6\mathbf{k}] \text{ m} \qquad \mathbf{r}_{OB} = [2.5\mathbf{j}] \text{ m}$$

The force vector $\mathbf{F}_B$ is given by

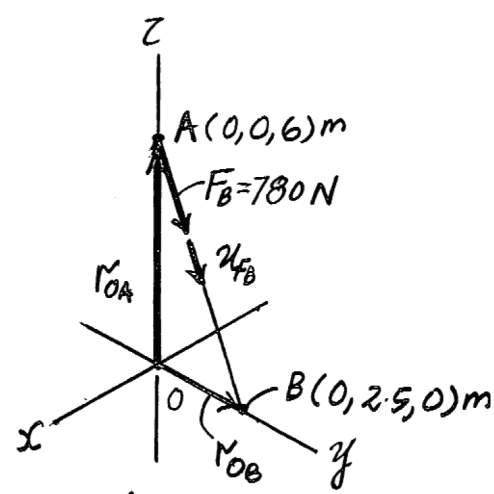
$$\mathbf{F}_B = F_B \mathbf{u}_{FB} = 780\left[\frac{(0-0)\mathbf{i} + (2.5-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(0-0)^2 + (2.5-0)^2 + (0-6)^2}}\right] = [300\mathbf{j} - 720\mathbf{k}] \text{ N}$$

Vector Cross Product: The moment of $\mathbf{F}_B$ about point O is given by

$$\mathbf{M}_O = \mathbf{r}_{OA} \times \mathbf{F}_B = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 6 \\ 0 & 300 & -720 \end{vmatrix} = [-1800\mathbf{i}] \text{ N}\cdot\text{m} = [-1.80\mathbf{i}] \text{ kN}\cdot\text{m} \qquad \textbf{Ans.}$$

or

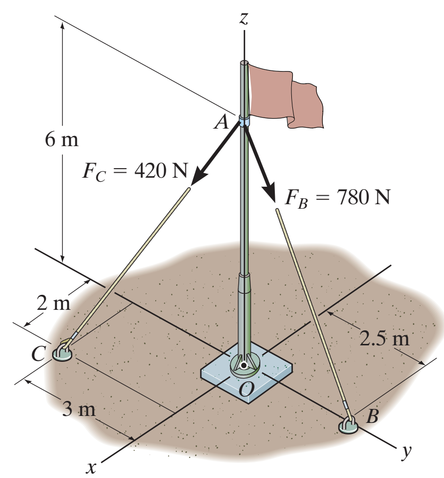
$$\mathbf{M}_O = \mathbf{r}_{OB} \times \mathbf{F}_B = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 2.5 & 0 \\ 0 & 300 & -720 \end{vmatrix} = [-1800\mathbf{i}] \text{ N}\cdot\text{m} = [-1.80\mathbf{i}] \text{ kN}\cdot\text{m} \qquad \textbf{Ans.}$$



(a)

4–33.

Determine the moment produced by force $\mathbf{F}_C$ about point O. Express the result as a Cartesian vector



SOLUTION

Position Vector and Force Vectors: Either position vector $\mathbf{r}_{OA}$ or $\mathbf{r}_{OC}$ can be used to determine the moment of F_C about point O.

$$\mathbf{r}_{OA} = \{6\mathbf{k}\}\text{ m}$$

$$\mathbf{r}_{OC} = (2-0)\mathbf{i} + (-3-0)\mathbf{j} + (0-0)\mathbf{k} = [2\mathbf{i} - 3\mathbf{j}]\text{ m}$$

The force vector F_C is given by

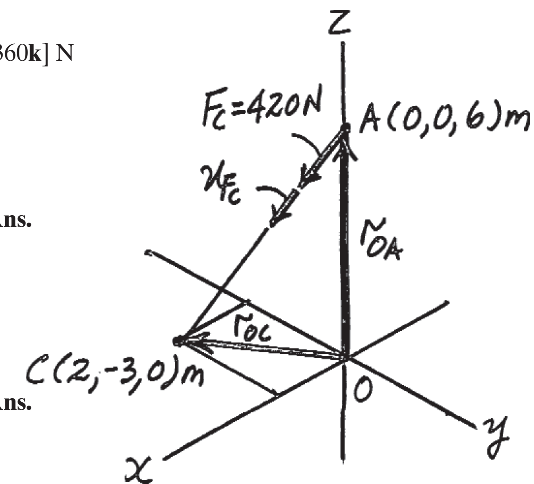
$$\mathbf{F}_C = F_C\mathbf{u}_{FC} = 420B\frac{(2-0)\mathbf{i} + (-3-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(2-0)^2 + (-3-0)^2 + (0-6)^2}}\Bigg] = [120\mathbf{i} - 180\mathbf{j} - 360\mathbf{k}]\text{ N}$$

Vector Cross Product: The moment of F_C about point O is given by

$$\mathbf{M}_O = \mathbf{r}_{OA} \times \mathbf{F}_C = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 6 \\ 120 & -180 & -360 \end{vmatrix} = [1080\mathbf{i} + 720\mathbf{j}]\text{ N}\cdot\text{m} \qquad \textbf{Ans.}$$

or

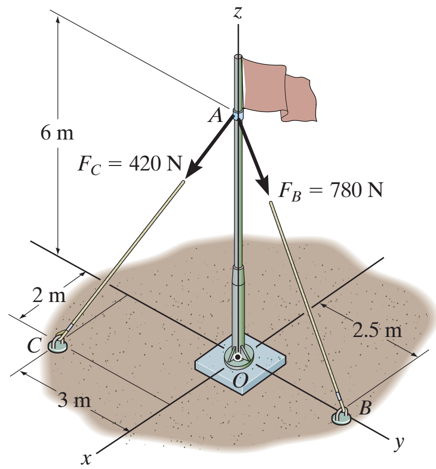
$$\mathbf{M}_O = \mathbf{r}_{OC} \times \mathbf{F}_C = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -3 & 0 \\ 120 & -180 & -360 \end{vmatrix} = [1080\mathbf{i} + 720\mathbf{j}]\text{ N}\cdot\text{m} \qquad \textbf{Ans.}$$



(a)

4–34.

Determine the resultant moment produced by forces $\mathbf{F}_B$ and $\mathbf{F}_C$ about point O. Express the result as a Cartesian vector.



SOLUTION

Position Vector and Force Vectors: The position vector $\mathbf{r}_{OA}$ and force vectors $\mathbf{F}_B$ and $\mathbf{F}_C$, Fig. *a*, must be determined first.

$$\mathbf{r}_{OA} = \{6\mathbf{k}\}\text{ m}$$

$$\mathbf{F}_B = F_B\,\mathbf{u}_{FB} = 780\left[\frac{(0-0)\mathbf{i} + (2.5-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(0-0)^2 + (2.5-0)^2 + (0-6)^2}}\right] = [300\mathbf{j} - 720\mathbf{k}]\text{ N}$$

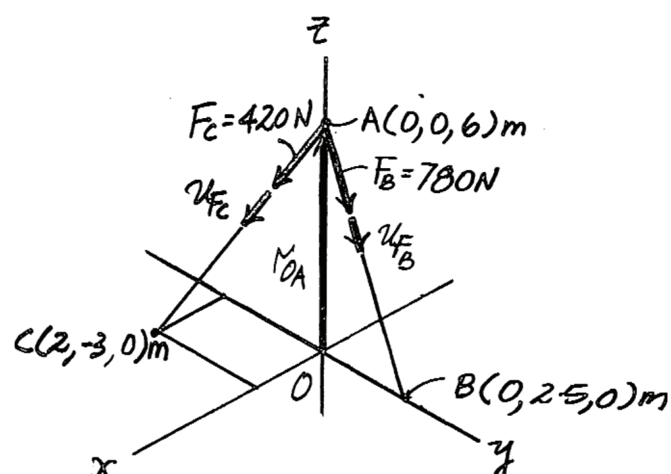
$$\mathbf{F}_C = F_C\mathbf{u}_{FC} = 420\left[\frac{(2-0)\mathbf{i} + (-3-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(2-0)^2 + (-3-0)^2 + (0-6)^2}}\right] = [120\mathbf{i} - 180\mathbf{j} - 360\mathbf{k}]\text{ N}$$

Resultant Moment: The resultant moment of $\mathbf{F}_B$ and $\mathbf{F}_C$ about point O is given by

$$\mathbf{M}_O = \mathbf{r}_{OA} \times \mathbf{F}_B + \mathbf{r}_{OA} \times \mathbf{F}_C$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 6 \\ 0 & 300 & -720 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 6 \\ 120 & -180 & -360 \end{vmatrix}$$

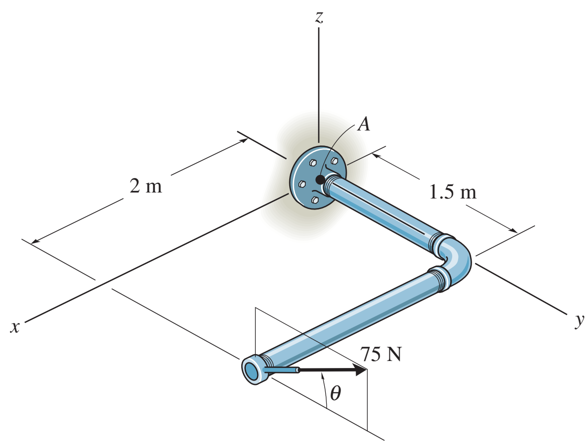
$$= [-720\mathbf{i} + 720\mathbf{j}]\text{ N}\cdot\text{m}$$ **Ans.**



(a)

■4–35.

Using a ring collar the 75-N force can act in the vertical plane at various angles θ. Determine the magnitude of the moment it produces about point A, plot the result of M (ordinate) versus θ (abscissa) for $0° \leq \theta \leq 180°$, and specify the angles that give the maximum and minimum moment.



SOLUTION

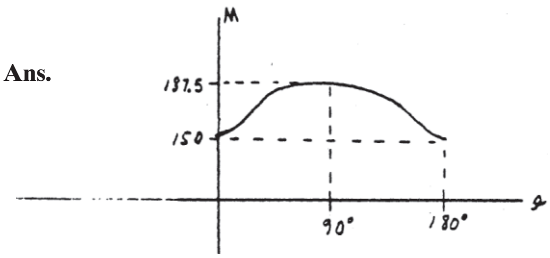
$$\mathbf{M}_A = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 1.5 & 0 \\ 0 & 75\cos\theta & 75\sin\theta \end{vmatrix}$$

$$= 112.5\sin\theta\,\mathbf{i} - 150\sin\theta\,\mathbf{j} + 150\cos\theta\,\mathbf{k}$$

$$M_A = \sqrt{(112.5\sin\theta)^2 + (-150\sin\theta)^2 + (150\cos\theta)^2} = \sqrt{12\,656.25\sin^2\theta + 22\,500}$$

$$\frac{dM_A}{d\theta} = \frac{1}{2}(12\,656.25\sin^2\theta + 22\,500)^{-\frac{1}{2}}(12\,656.25)(2\sin\theta\cos\theta) = 0$$

$\sin\theta\cos\theta = 0;\quad \theta = 0°, 90°, 180°$ **Ans.**

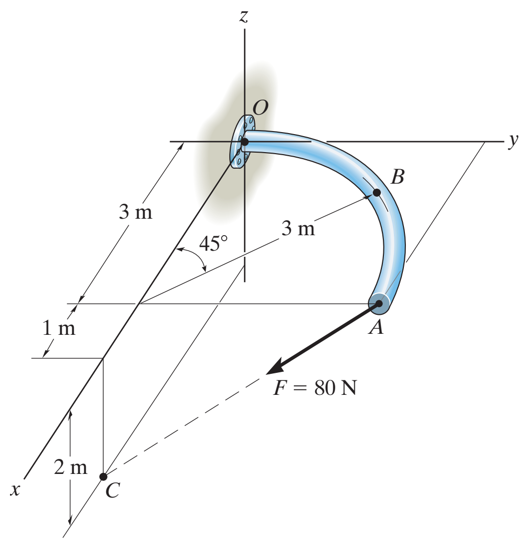


$M_{max} = 187.5\text{ N}\cdot\text{m at }\theta = 90°$

$M_{min} = 150\text{ N}\cdot\text{m at }\theta = 0°, 180°$

***4–36.**

The curved rod lies in the x–y plane and has a radius of 3 m. If a force of $F = 80$ N acts at its end as shown, determine the moment of this force about point O.



SOLUTION

$$\mathbf{r}_{AC} = \{1\mathbf{i} - 3\mathbf{j} - 2\mathbf{k}\}\text{ m}$$

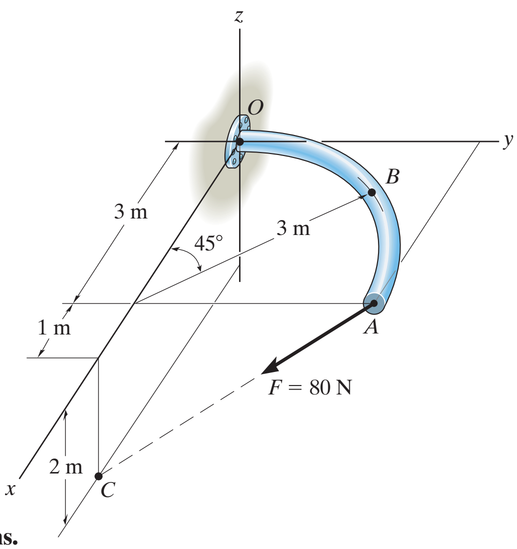
$$r_{AC} = \sqrt{(1)^2 + (-3)^2 + (-2)^2} = 3.742\text{ m}$$

$$\mathbf{M}_O = \mathbf{r}_{OC} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 0 & -2 \\ \frac{1}{3.742}(80) & -\frac{3}{3.742}(80) & -\frac{2}{3.742}(80) \end{vmatrix}$$

$$\mathbf{M}_O = \{-128\mathbf{i} + 128\mathbf{j} - 257\mathbf{k}\}\text{ N}\cdot\text{m}$$ **Ans.**

4–37.

The curved rod lies in the x–y plane and has a radius of 3 m. If a force of $F = 80$ N acts at its end as shown, determine the moment of this force about point B.



SOLUTION

$$\mathbf{r}_{AC} = \{1\mathbf{i} - 3\mathbf{j} - 2\mathbf{k}\}\text{ m}$$

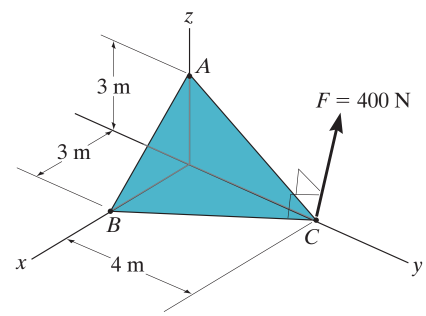
$$r_{AC} = \sqrt{(1)^2 + (-3)^2 + (-2)^2} = 3.742\text{ m}$$

$$\mathbf{M}_B = \mathbf{r}_{BA} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3\cos 45^\circ & (3 - 3\sin 45^\circ) & 0 \\ \frac{1}{3.742}(80) & -\frac{3}{3.742}(80) & -\frac{2}{3.742}(80) \end{vmatrix}$$

$$\mathbf{M}_B = \{-37.6\mathbf{i} + 90.7\mathbf{j} - 155\mathbf{k}\}\text{ N}\cdot\text{m}$$ **Ans.**

4–38.

Force **F** acts perpendicular to the inclined plane. Determine the moment produced by **F** about point A. Express the result as a Cartesian vector.

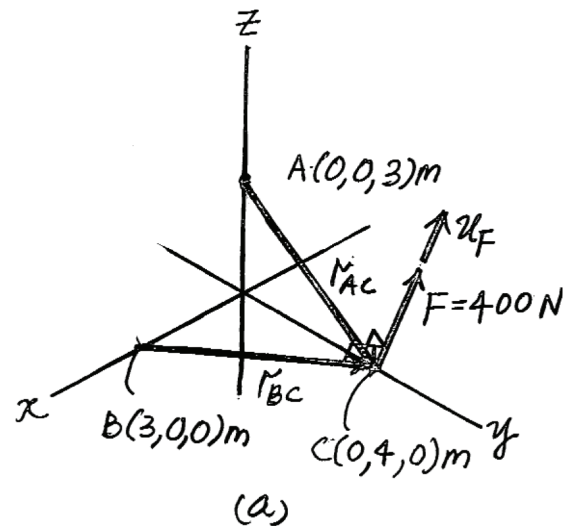


SOLUTION

Force Vector: Since force **F** is perpendicular to the inclined plane, its unit vector $\mathbf{u}_F$ is equal to the unit vector of the cross product, $\mathbf{b} = \mathbf{r}_{AC} \times \mathbf{r}_{BC}$, Fig. *a*. Here

$$\mathbf{r}_{AC} = (0-0)\mathbf{i} + (4-0)\mathbf{j} + (0-3)\mathbf{k} = [4\mathbf{j} - 3\mathbf{k}]\text{ m}$$

$$\mathbf{r}_{BC} = (0-3)\mathbf{i} + (4-0)\mathbf{j} + (0-0)\mathbf{k} = [-3\mathbf{i} + 4\mathbf{j}]\text{ m}$$



(a)

Thus,

$$\mathbf{b} = \mathbf{r}_{CA} \times \mathbf{r}_{CB} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 4 & -3 \\ -3 & 4 & 0 \end{vmatrix}$$

$$= [12\mathbf{i} + 9\mathbf{j} + 12\mathbf{k}]\text{ m}^2$$

Then,

$$\mathbf{u}_F = \frac{\mathbf{b}}{b} = \frac{12\mathbf{i} + 9\mathbf{j} + 12\mathbf{k}}{\sqrt{12^2 + 9^2 + 12^2}} = 0.6247\mathbf{i} + 0.4685\mathbf{j} + 0.6247\mathbf{k}$$

And finally

$$\mathbf{F} = F\mathbf{u}_F = 400(0.6247\mathbf{i} + 0.4685\mathbf{j} + 0.6247\mathbf{k})$$

$$= [249.88\mathbf{i} + 187.41\mathbf{j} + 249.88\mathbf{k}]\text{ N}$$

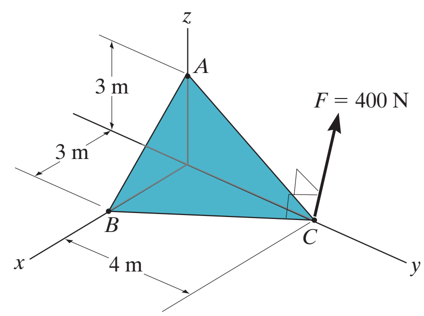
Vector Cross Product: The moment of **F** about point A is

$$\mathbf{M}_A = \mathbf{r}_{AC} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 4 & -3 \\ 249.88 & 187.41 & 249.88 \end{vmatrix}$$

$$= [1.56\mathbf{i} - 0.750\mathbf{j} - 1.00\mathbf{k}]\text{ kN}\cdot\text{m}$$ **Ans.**

4–39.

Force **F** acts perpendicular to the inclined plane. Determine the moment produced by **F** about point B. Express the result as a Cartesian vector.



SOLUTION

Force Vector: Since force **F** is perpendicular to the inclined plane, its unit vector $\mathbf{u}_F$ is equal to the unit vector of the cross product, $\mathbf{b} = \mathbf{r}_{AC} \times \mathbf{r}_{BC}$, Fig. *a*. Here

$$\mathbf{r}_{AC} = (0 - 0)\mathbf{i} + (4 - 0)\mathbf{j} + (0 - 3)\mathbf{k} = [4\mathbf{j} - 3\mathbf{k}] \text{ m}$$

$$\mathbf{r}_{BC} = (0 - 3)\mathbf{i} + (4 - 0)\mathbf{j} + (0 - 0)\mathbf{k} = [-3\mathbf{k} + 4\mathbf{j}] \text{ m}$$

Thus,

$$\mathbf{b} = \mathbf{r}_{CA} \times \mathbf{r}_{CB} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 4 & -3 \\ -3 & 4 & 0 \end{vmatrix} = [12\mathbf{i} + 9\mathbf{j} + 12\mathbf{k}] \text{ m}^2$$

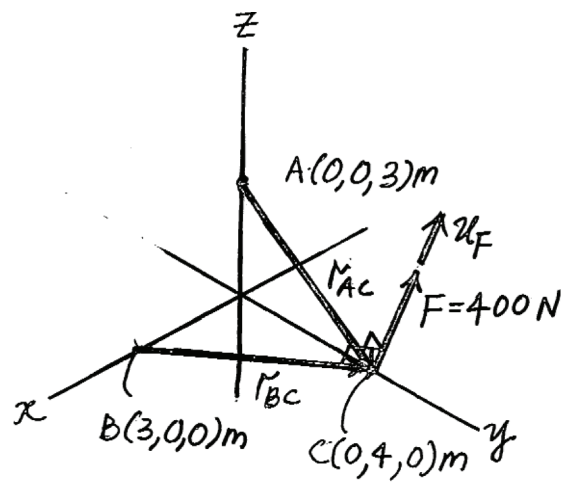
Then,

$$\mathbf{u}_F = \frac{\mathbf{b}}{b} = \frac{12\mathbf{i} + 9\mathbf{j} + 12\mathbf{k}}{\sqrt{12^2 + 9^2 + 12^2}} = 0.6247\mathbf{i} + 0.4685\mathbf{j} + 0.6247\mathbf{k}$$

And finally

$$\mathbf{F} = F\mathbf{u}_F = 400(0.6247\mathbf{i} + 0.4685\mathbf{j} + 0.6247\mathbf{k})$$

$$= [249.88\mathbf{i} + 187.41\mathbf{j} + 249.88\mathbf{k}] \text{ N}$$



(a)

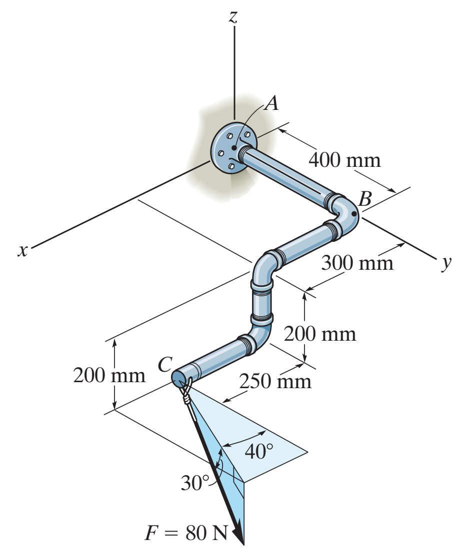
Vector Cross Product: The moment of **F** about point B is

$$\mathbf{M}_B = \mathbf{r}_{BC} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -3 & 4 & 0 \\ 249.88 & 187.41 & 249.88 \end{vmatrix}$$

$$= [1.00\mathbf{i} + 0.750\mathbf{j} - 1.56\mathbf{k}] \text{ kN} \cdot \text{m}$$ **Ans.**

***4–40.**

The pipe assembly is subjected to the 80-N force. Determine the moment of this force about point A.



SOLUTION

Position Vector And Force Vector:

$$\mathbf{r}_{AC} = \{(0.55 - 0)\mathbf{i} + (0.4 - 0)\mathbf{j} + (-0.2 - 0)\mathbf{k}\}\text{ m}$$

$$= \{0.55\mathbf{i} + 0.4\mathbf{j} - 0.2\mathbf{k}\}\text{ m}$$

$$\mathbf{F} = 80(\cos 30^\circ \sin 40^\circ\mathbf{i} + \cos 30^\circ \cos 40^\circ\mathbf{j} - \sin 30^\circ\mathbf{k})\text{ N}$$

$$= (44.53\mathbf{i} + 53.07\mathbf{j} - 40.0\mathbf{k}\}\text{ N}$$

Moment of Force* F *About Point A: Applying Eq. 4–7, we have

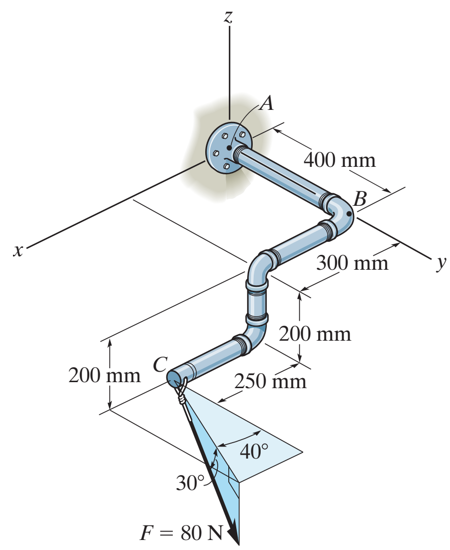
$$\mathbf{M}_A = \mathbf{r}_{AC} \times \mathbf{F}$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.55 & 0.4 & -0.2 \\ 44.53 & 53.07 & -40.0 \end{vmatrix}$$

$$= \{-5.39\mathbf{i} + 13.1\mathbf{j} + 11.4\mathbf{k}\}\text{ N}\cdot\text{m}$$ **Ans.**

4–41.

The pipe assembly is subjected to the 80-N force. Determine the moment of this force about point B.



SOLUTION

Position Vector And Force Vector:

$$\mathbf{r}_{BC} = \{(0.55 - 0)\mathbf{i} + (0.4 - 0.4)\mathbf{j} + (-0.2 - 0)\mathbf{k}\}\text{ m}$$

$$= \{0.55\mathbf{i} - 0.2\mathbf{k}\}\text{ m}$$

$$\mathbf{F} = 80\,(\cos 30^\circ \sin 40^\circ \mathbf{i} + \cos 30^\circ \cos 40^\circ \mathbf{j} - \sin 30^\circ \mathbf{k})\text{ N}$$

$$= (44.53\mathbf{i} + 53.07\mathbf{j} - 40.0\mathbf{k}\}\text{ N}$$

Moment of Force* F *About Point B: Applying Eq. 4–7, we have

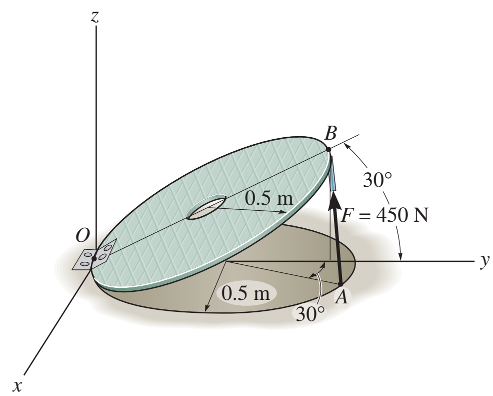
$$\mathbf{M}_B = \mathbf{r}_{BC} \times \mathbf{F}$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.55 & 0 & -0.2 \\ 44.53 & 53.07 & -40.0 \end{vmatrix}$$

$$= \{10.6\mathbf{i} + 13.1\mathbf{j} + 29.2\mathbf{k}\}\text{ N}\cdot\text{m} \qquad \textbf{Ans.}$$

4–42.

Strut AB of the 1-m-diameter hatch door exerts a force of 450 N on point B. Determine the moment of this force about point O.



SOLUTION

Position Vector And Force Vector:

$$\mathbf{r}_{OB} = \{(0 - 0)\mathbf{i} + (1\cos 30° - 0)\mathbf{j} + (1\sin 30° - 0)\mathbf{k}\}\text{ m}$$

$$= \{0.8660\mathbf{j} + 0.5\mathbf{k}\}\text{ m}$$

$$\mathbf{r}_{OA} = \{(0.5\sin 30° - 0)\mathbf{i} + (0.5 + 0.5\cos 30° - 0)\mathbf{j} + (0 - 0)\mathbf{k}\}\text{ m}$$

$$= \{0.250\mathbf{i} + 0.9330\mathbf{j}\}\text{ m}$$

$$\mathbf{F} = 450\left(\frac{(0 - 0.5\sin 30°)\mathbf{i} + [1\cos 30° - (0.5 + 0.5\cos 30°)]\mathbf{j} + (1\sin 30° - 0)\mathbf{k}}{\sqrt{(0 - 0.5\sin 30°)^2 + [1\cos 30° - (0.5 + 0.5\cos 30°)]^2 + (1\sin 30° - 0)^2}}\right)\text{N}$$

$$= \{-199.82\mathbf{i} - 53.54\mathbf{j} + 399.63\mathbf{k}\}\text{ N}$$

Moment of Force* F *About Point O: Applying Eq. 4–7, we have

$$\mathbf{M}_O = \mathbf{r}_{OB} \times \mathbf{F}$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0.8660 & 0.5 \\ -199.82 & -53.54 & 399.63 \end{vmatrix}$$

$$= \{373\mathbf{i} - 99.9\mathbf{j} + 173\mathbf{k}\}\text{ N}\cdot\text{m}$$ **Ans.**

Or

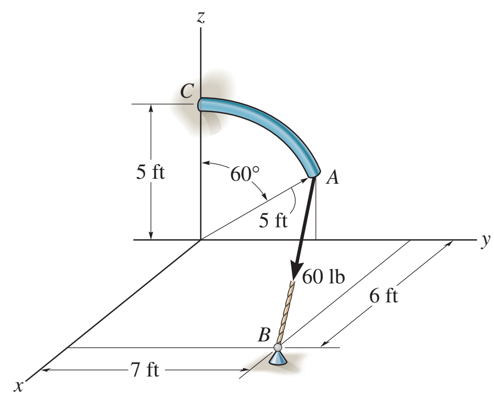
$$\mathbf{M}_O = \mathbf{r}_{OA} \times \mathbf{F}$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.250 & 0.9330 & 0 \\ -199.82 & -53.54 & 399.63 \end{vmatrix}$$

$$= \{373\mathbf{i} - 99.9\mathbf{j} + 173\mathbf{k}\}\text{ N}\cdot\text{m}$$ **Ans.**

4–43.

The curved rod has a radius of 5 ft. If a force of 60 lb acts at its end as shown, determine the moment of this force about point C.



SOLUTION

Position Vector and Force Vector:

$$\mathbf{r}_{CA} = \{(5\sin 60° - 0)\mathbf{j} + (5\cos 60° - 5)\mathbf{k}\}\text{ m}$$

$$= \{4.330\mathbf{j} - 2.50\mathbf{k}\}\text{ m}$$

$$\mathbf{F}_{AB} = 60\left(\frac{(6-0)\mathbf{i} + (7 - 5\sin 60°)\mathbf{j} + (0 - 5\cos 60°)\mathbf{k}}{\sqrt{(6-0)^2 + (7 - 5\sin 60°)^2 + (0 - 5\cos 60°)^2}}\right)\text{ lb}$$

$$= \{51.231\mathbf{i} + 22.797\mathbf{j} - 21.346\mathbf{k}\}\text{ lb}$$

Moment of Force $\mathbf{F}_{AB}$ About Point C: Applying Eq. 4–7, we have

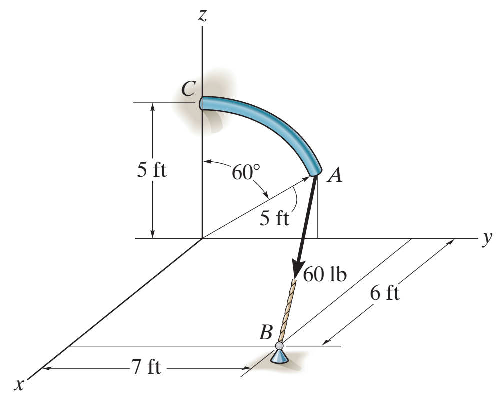
$$\mathbf{M}_C = \mathbf{r}_{CA} \times \mathbf{F}_{AB}$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 4.330 & -2.50 \\ 51.231 & 22.797 & -21.346 \end{vmatrix}$$

$$= \{-35.4\mathbf{i} - 128\mathbf{j} - 222\mathbf{k}\}\text{ lb}\cdot\text{ft}$$ **Ans.**

***4–44.**

Determine the smallest force F that must be applied along the rope in order to cause the curved rod, which has a radius of 5 ft, to fail at the support C. This requires a moment of $M = 80\ \text{lb}\cdot\text{ft}$ to be developed at C.



SOLUTION

Position Vector and Force Vector:

$$\mathbf{r}_{CA} = \{(5 \sin 60° - 0)\mathbf{j} + (5 \cos 60° - 5)\mathbf{k}\}\ \text{m}$$
$$= \{4.330\mathbf{j} - 2.50\,\mathbf{k}\}\ \text{m}$$

$$\mathbf{F}_{AB} = F\left(\frac{(6-0)\mathbf{i} + (7 - 5\sin 60°)\mathbf{j} + (0 - 5\cos 60°)\mathbf{k}}{\sqrt{(6-0)^2 + (7 - 5\sin 60°)^2 + (0 - 5\cos 60°)^2}}\right)\ \text{lb}$$
$$= 0.8539F_{\mathbf{i}} + 0.3799F_{\mathbf{j}} - 0.3558F_{\mathbf{k}}$$

Moment of Force $\mathbf{F_{AB}}$ About Point C:

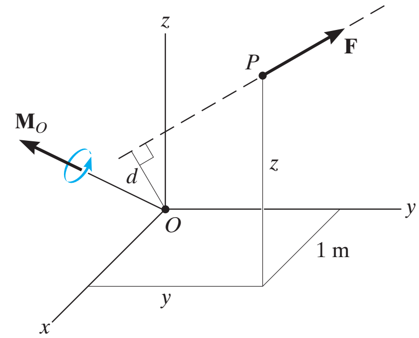
$$\mathbf{M}_C = \mathbf{r}_{CA} \times \mathbf{F}_{AB}$$
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 4.330 & -2.50 \\ 0.8539F & 0.3799F & -0.3558F \end{vmatrix}$$
$$= -0.5909F_{\mathbf{i}} - 2.135\mathbf{j} - 3.697\mathbf{k}$$

Require

$$80 = \sqrt{(0.5909)^2 + (-2.135)^2 + (-3.697)^2}\,F$$
$$F = 18.6\ \text{lb.} \qquad \textbf{Ans.}$$

4–45.

A force of $\mathbf{F} = \{6\mathbf{i} - 2\mathbf{j} + 1\mathbf{k}\}$ kN produces a moment of $\mathbf{M}_O = \{4\mathbf{i} + 5\mathbf{j} - 14\mathbf{k}\}$ kN·m about the origin of coordinates, point O. If the force acts at a point having an x coordinate of $x = 1$ m, determine the y and z coordinates.



SOLUTION

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F}$$

$$4\mathbf{i} + 5\mathbf{j} - 14\mathbf{k} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & y & z \\ 6 & -2 & 1 \end{vmatrix}$$

$$4 = y + 2z$$

$$5 = -1 + 6z$$

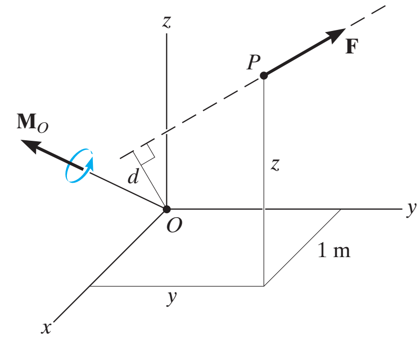
$$-14 = -2 - 6y$$

$y = 2$ m **Ans.**

$z = 1$ m **Ans.**

4–46.

The force $\mathbf{F} = \{6\mathbf{i} + 8\mathbf{j} + 10\mathbf{k}\}$ N creates a moment about point O of $\mathbf{M}_O = \{-14\mathbf{i} + 8\mathbf{j} + 2\mathbf{k}\}$ N·m. If the force passes through a point having an x coordinate of 1 m, determine the y and z coordinates of the point. Also, realizing that $M_O = Fd$, determine the perpendicular distance d from point O to the line of action of $\mathbf{F}$.



SOLUTION

$$-14\mathbf{i} + 8\mathbf{j} + 2\mathbf{k} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & y & z \\ 6 & 8 & 10 \end{vmatrix}$$

$$-14 = 10y - 8z$$

$$8 = -10 + 6z$$

$$2 = 8 - 6y$$

$$y = 1 \text{ m} \qquad \textbf{Ans.}$$

$$z = 3 \text{ m} \qquad \textbf{Ans.}$$

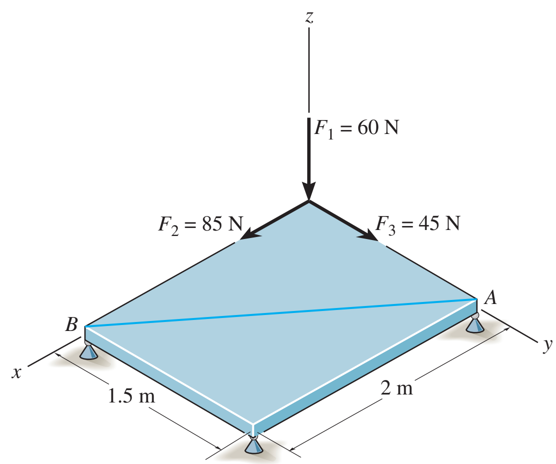
$$M_O = \sqrt{(-14)^2 + (8)^2 + (2)^2} = 16.25 \text{ N}\cdot\text{m}$$

$$F = \sqrt{(6)^2 + (8)^2 + (10)^2} = 14.14 \text{ N}$$

$$d = \frac{16.25}{14.14} = 1.15 \text{ m} \qquad \textbf{Ans.}$$

4–47.

Determine the magnitude of the moment of each of the three forces about the axis AB. Solve the problem (a) using a Cartesian vector approach and (b) using a scalar approach.



SOLUTION

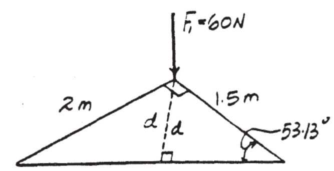
a) *Vector Analysis*

Position Vector and Force Vector:

$$\mathbf{r}_1 = \{-1.5\mathbf{j}\}\text{ m} \qquad \mathbf{r}_2 = \mathbf{r}_3 = \mathbf{0}$$

$$\mathbf{F}_1 = \{-60\mathbf{k}\}\text{ N} \qquad \mathbf{F}_2 = \{85\mathbf{i}\}\text{ N} \quad \mathbf{F}_3 = \{45\mathbf{j}\}\text{ N}$$

Unit Vector Along AB Axis:



$$\mathbf{u}_{AB} = \frac{(2-0)\mathbf{i} + (0-1.5)\mathbf{j}}{\sqrt{(2-0)^2 + (0-1.5)^2}} = 0.8\mathbf{i} - 0.6\mathbf{j}$$

Moment of Each Force About AB Axis: Applying Eq. 4–11, we have

$$(M_{AB})_1 = \mathbf{u}_{AB} \cdot (\mathbf{r}_1 \times \mathbf{F}_1)$$

$$= \begin{vmatrix} 0.8 & -0.6 & 0 \\ 0 & -1.5 & 0 \\ 0 & 0 & -60 \end{vmatrix}$$

$$= 0.8[(-1.5)(-60) - 0] - 0 + 0 = 72.0\text{ N}\cdot\text{m}$$ **Ans.**

$$(M_{AB})_2 = \mathbf{u}_{AB} \cdot (\mathbf{r}_2 \times \mathbf{F}_2)$$

$$= \begin{vmatrix} 0.8 & -0.6 & 0 \\ 0 & 0 & 0 \\ 85 & 0 & 0 \end{vmatrix} = 0$$ **Ans.**

$$(M_{AB})_3 = \mathbf{u}_{AB} \cdot (\mathbf{r}_3 \times \mathbf{F}_3)$$

$$= \begin{vmatrix} 0.8 & -0.6 & 0 \\ 0 & 0 & 0 \\ 0 & 45 & 0 \end{vmatrix} = 0$$ **Ans.**

b) *Scalar Analysis:* Since moment arm from force $\mathbf{F}_2$ and $\mathbf{F}_3$ is equal to zero,

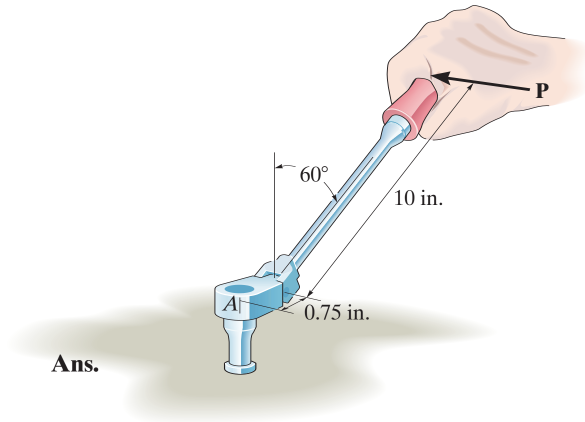
$$(M_{AB})_2 = (M_{AB})_3 = 0$$ **Ans.**

Moment arm d from force $\mathbf{F}_1$ to axis AB is $d = 1.5 \sin 53.13° = 1.20$ m,

$$(M_{AB})_1 = F_1 d = 60(1.20) = 72.0\text{ N}\cdot\text{m}$$ **Ans.**

***4–48.**

The flex-headed ratchet wrench is subjected to a force of $P = 16$ lb, applied perpendicular to the handle as shown. Determine the moment or torque this imparts along the vertical axis of the bolt at A.



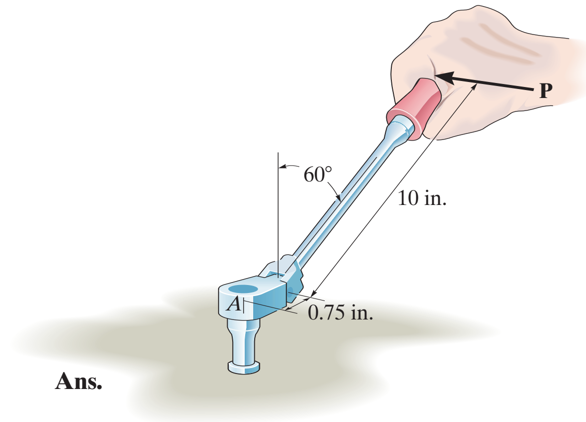
SOLUTION

$$M = 16(0.75 + 10 \sin 60°)$$

$$M = 151 \text{ lb} \cdot \text{in.}$$ **Ans.**

4–49.

If a torque or moment of 80 lb · in. is required to loosen the bolt at A, determine the force P that must be applied perpendicular to the handle of the flex-headed ratchet wrench.



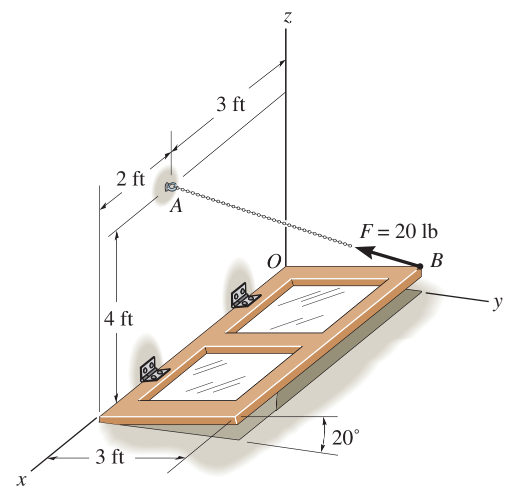
SOLUTION

$$80 = P(0.75 + 10 \sin 60°)$$

$$P = \frac{80}{9.41} = 8.50 \text{ lb}$$ **Ans.**

4–50.

The chain AB exerts a force of 20 lb on the door at B. Determine the magnitude of the moment of this force along the hinged axis x of the door.



SOLUTION

Position Vector and Force Vector:

$$\mathbf{r}_{OA} = \{(3-0)\mathbf{i} + (4-0)\mathbf{k}\}\text{ ft} = \{3\mathbf{i} + 4\mathbf{k}\}\text{ ft}$$

$$\mathbf{r}_{OB} = \{(0-0)\mathbf{i} + (3\cos 20^\circ - 0)\mathbf{j} + (3\sin 20^\circ - 0)\mathbf{k}\}\text{ ft}$$

$$= \{2.8191\mathbf{j} + 1.0261\mathbf{k}\}\text{ ft}$$

$$\mathbf{F} = 20\left(\frac{(3-0)\mathbf{i} + (0-3\cos 20^\circ)\mathbf{j} + (4-3\sin 20^\circ)\mathbf{k}}{\sqrt{(3-0)^2 + (0-3\cos 20^\circ)^2 + (4-3\sin 20^\circ)^2}}\right)\text{lb}$$

$$= \{11.814\mathbf{i} - 11.102\mathbf{j} + 11.712\mathbf{k}\}\text{ lb}$$

Moment of Force* F *About x Axis: The unit vector along the x axis is **i**. Applying Eq. 4–11, we have

$$M_x = \mathbf{i}\cdot(\mathbf{r}_{OA}\times\mathbf{F})$$

$$= \begin{vmatrix} 1 & 0 & 0 \\ 3 & 0 & 4 \\ 11.814 & -11.102 & 11.712 \end{vmatrix}$$

$$= 1[0(11.712) - (-11.102)(4)] - 0 + 0$$

$$= 44.4\text{ lb}\cdot\text{ft}$$ **Ans.**

Or

$$M_x = \mathbf{i}\cdot(\mathbf{r}_{OB}\times\mathbf{F})$$

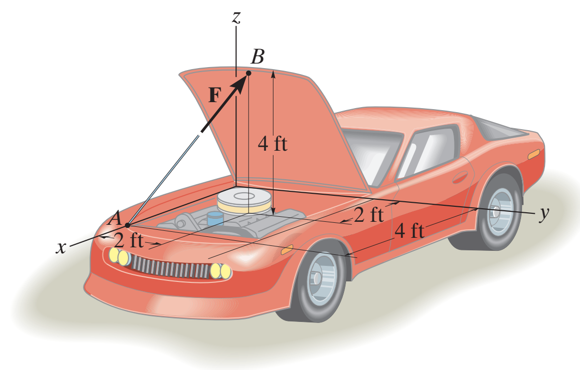
$$= \begin{vmatrix} 1 & 0 & 0 \\ 0 & 2.8191 & 1.0261 \\ 11.814 & -11.102 & 11.712 \end{vmatrix}$$

$$= 1[2.8191(11.712) - (-11.102)(1.0261)] - 0 + 0$$

$$= 44.4\text{ lb}\cdot\text{ft}$$ **Ans.**

4–51.

The hood of the automobile is supported by the strut AB, which exerts a force of $F = 24$ lb on the hood. Determine the moment of this force about the hinged axis y.



SOLUTION

$\mathbf{r} = \{4\mathbf{i}\}$ m

$$\mathbf{F} = 24\left(\frac{-2\mathbf{i} + 2\mathbf{j} + 4\mathbf{k}}{\sqrt{(-2)^2 + (2)^2 + (4)^2}}\right)$$

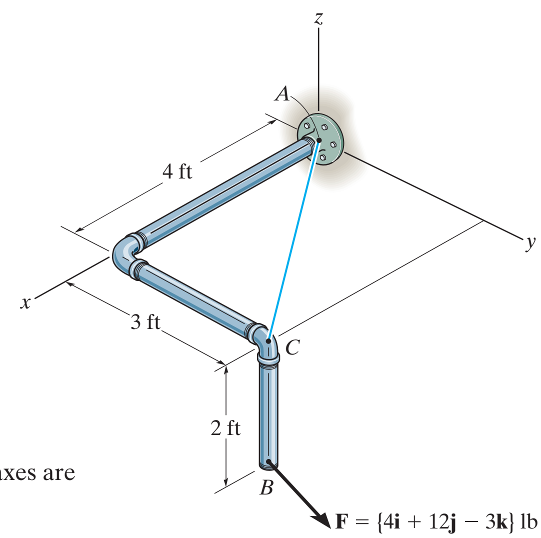
$$= \{-9.80\mathbf{i} + 9.80\mathbf{j} + 19.60\mathbf{k}\} \text{ lb}$$

$$M_y = \begin{vmatrix} 0 & 1 & 0 \\ 4 & 0 & 0 \\ -9.80 & 9.80 & 19.60 \end{vmatrix} = -78.4 \text{ lb} \cdot \text{ft}$$

$\mathbf{M}_y = \{-78.4\mathbf{j}\}$ lb · ft **Ans.**

***4–52.**

Determine the magnitude of the moments of the force **F** about the x, y, and z axes. Solve the problem (a) using a Cartesian vector approach and (b) using a scalar approach.



SOLUTION

a) ***Vector Analysis***

Position Vector:

$$\mathbf{r}_{AB} = \{(4-0)\,\mathbf{i} + (3-0)\mathbf{j} + (-2-0)\mathbf{k}\}\text{ ft} = \{4\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}\}\text{ ft}$$

Moment of Force* F *About x, y, and z Axes: The unit vectors along x, y, and z axes are **i**, **j**, and **k** respectively. Applying Eq. 4–11, we have

$$M_x = \mathbf{i}\cdot(\mathbf{r}_{AB}\times\mathbf{F})$$

$$= \begin{vmatrix} 1 & 0 & 0 \\ 4 & 3 & -2 \\ 4 & 12 & -3 \end{vmatrix}$$

$$= 1[3(-3) - (12)(-2)] - 0 + 0 = 15.0\text{ lb}\cdot\text{ft}$$ **Ans.**

$$M_y = \mathbf{j}\cdot(\mathbf{r}_{AB}\times\mathbf{F})$$

$$= \begin{vmatrix} 0 & 1 & 0 \\ 4 & 3 & -2 \\ 4 & 12 & -3 \end{vmatrix}$$

$$= 0 - 1[4(-3) - (4)(-2)] + 0 = 4.00\text{ lb}\cdot\text{ft}$$ **Ans.**

$$M_z = \mathbf{k}\cdot(\mathbf{r}_{AB}\times\mathbf{F})$$

$$= \begin{vmatrix} 0 & 0 & 1 \\ 4 & 3 & -2 \\ 4 & 12 & -3 \end{vmatrix}$$

$$= 0 - 0 + 1[4(12) - (4)(3)] = 36.0\text{ lb}\cdot\text{ft}$$ **Ans.**

b) ***Scalar Analysis***

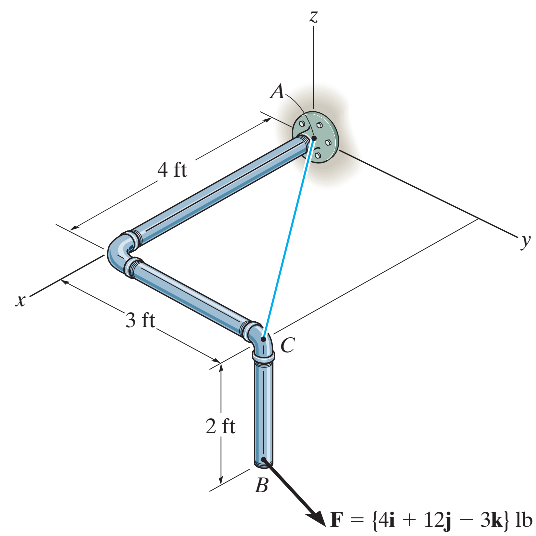
$M_x = \Sigma M_x;\qquad M_x = 12(2) - 3(3) = 15.0\text{ lb}\cdot\text{ft}$ **Ans.**

$M_y = \Sigma M_y;\qquad M_y = -4(2) + 3(4) = 4.00\text{ lb}\cdot\text{ft}$ **Ans.**

$M_z = \Sigma M_z;\qquad M_z = -4(3) + 12(4) = 36.0\text{ lb}\cdot\text{ft}$ **Ans.**

4–53.

Determine the moment of the force **F** about an axis extending between *A* and *C*. Express the result as a Cartesian vector.



SOLUTION

Position Vector:

$$\mathbf{r}_{CB} = \{-2\mathbf{k}\}\text{ ft}$$

$$\mathbf{r}_{AB} = \{(4-0)\mathbf{i} + (3-0)\mathbf{j} + (-2-0)\mathbf{k}\}\text{ ft} = \{4\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}\}\text{ ft}$$

Unit Vector Along AC Axis:

$$\mathbf{u}_{AC} = \frac{(4-0)\mathbf{i} + (3-0)\mathbf{j}}{\sqrt{(4-0)^2 + (3-0)^2}} = 0.8\mathbf{i} + 0.6\mathbf{j}$$

Moment of Force* F *About AC Axis: With $\mathbf{F} = \{4\mathbf{i} + 12\mathbf{j} - 3\mathbf{k}\}$ lb, applying Eq. 4–7, we have

$$M_{AC} = \mathbf{u}_{AC} \cdot (\mathbf{r}_{CB} \times \mathbf{F})$$

$$= \begin{vmatrix} 0.8 & 0.6 & 0 \\ 0 & 0 & -2 \\ 4 & 12 & -3 \end{vmatrix}$$

$$= 0.8[(0)(-3) - 12(-2)] - 0.6[0(-3) - 4(-2)] + 0$$

$$= 14.4\text{ lb}\cdot\text{ft}$$

Or

$$M_{AC} = \mathbf{u}_{AC} \cdot (\mathbf{r}_{AB} \times \mathbf{F})$$

$$= \begin{vmatrix} 0.8 & 0.6 & 0 \\ 4 & 3 & -2 \\ 4 & 12 & -3 \end{vmatrix}$$

$$= 0.8[(3)(-3) - 12(-2)] - 0.6[4(-3) - 4(-2)] + 0$$

$$= 14.4\text{ lb}\cdot\text{ft}$$

Expressing $\mathbf{M}_{AC}$ as a Cartesian vector yields

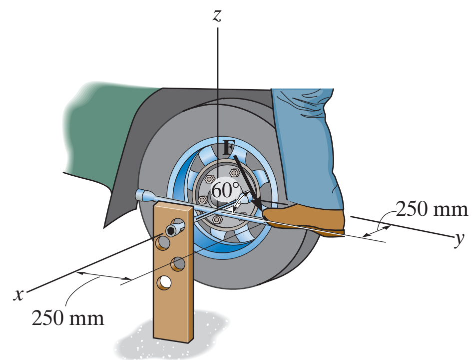
$$\mathbf{M}_{AC} = M_{AC}\,\mathbf{u}_{AC}$$

$$= 14.4(0.8\mathbf{i} + 0.6\mathbf{j})$$

$$= \{11.5\mathbf{i} + 8.64\mathbf{j}\}\text{ lb}\cdot\text{ft}$$ **Ans.**

4–54.

The board is used to hold the end of a four-way lug wrench in the position shown when the man applies a force of $F = 100$ N. Determine the magnitude of the moment produced by this force about the x axis. Force **F** lies in a vertical plane.



SOLUTION

Vector Analysis

Moment About the x Axis: The position vector $\mathbf{r}_{AB}$, Fig. *a*, will be used to determine the moment of **F** about the x axis.

$$\mathbf{r}_{AB} = (0.25 - 0.25)\mathbf{i} + (0.25 - 0)\mathbf{j} + (0 - 0)\mathbf{k} = \{0.25\mathbf{j}\}\text{ m}$$

The force vector **F**, Fig. *a*, can be written as

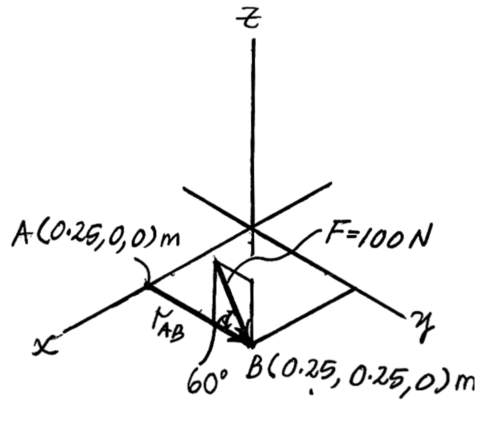
$$\mathbf{F} = 100(\cos 60°\mathbf{j} - \sin 60°\mathbf{k}) = \{50\mathbf{j} - 86.60\mathbf{k}\}\text{ N}$$

Knowing that the unit vector of the x axis is **i**, the magnitude of the moment of **F** about the x axis is given by

$$M_x = \mathbf{i}\cdot\mathbf{r}_{AB}\times\mathbf{F} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 0.25 & 0 \\ 0 & 50 & -86.60 \end{vmatrix}$$

$$= 1[0.25(-86.60) - 50(0)] + 0 + 0$$

$$= -21.7\text{ N}\cdot\text{m} \qquad \textbf{Ans.}$$



(a)

The negative sign indicates that M_x is directed towards the negative x axis.

Scalar Analysis

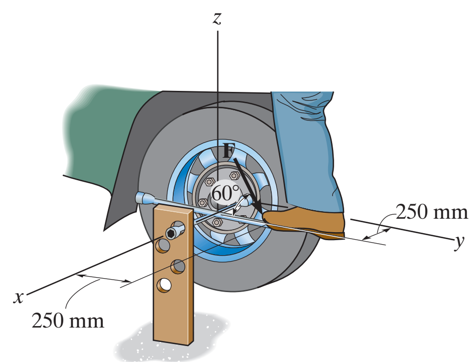
This problem can be solved by summing the moment about the x axis

$$M_x = \Sigma M_x; \qquad M_x = -100\ \sin 60°(0.25) + 100\ \cos 60°(0)$$

$$= -21.7\text{ N}\cdot\text{m} \quad \textbf{Ans.}$$

4–55.

The board is used to hold the end of a four-way lug wrench in position. If a torque of 30 N·m about the x axis is required to tighten the nut, determine the required magnitude of the force **F** that the man's foot must apply on the end of the wrench in order to turn it. Force **F** lies in a vertical plane.



SOLUTION

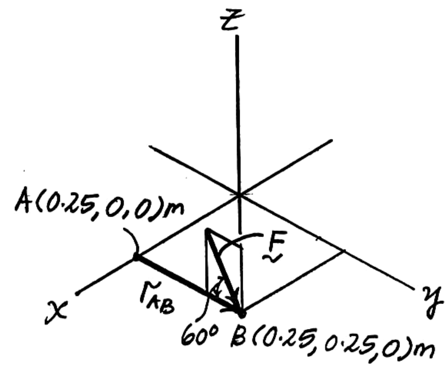
Vector Analysis

Moment About the x Axis: The position vector $\mathbf{r}_{AB}$, Fig. a, will be used to determine the moment of **F** about the x axis.

$$\mathbf{r}_{AB} = (0.25 - 0.25)\mathbf{i} + (0.25 - 0)\mathbf{j} + (0 - 0)\mathbf{k} = \{0.25\mathbf{j}\}\text{ m}$$

The force vector **F**, Fig. a, can be written as

$$\mathbf{F} = F(\cos 60°\mathbf{j} - \sin 60°\mathbf{k}) = 0.5F\mathbf{j} - 0.8660F\mathbf{k}$$



(a)

Knowing that the unit vector of the x axis is **i**, the magnitude of the moment of **F** about the x axis is given by

$$M_x = \mathbf{i} \cdot \mathbf{r}_{AB} \times \mathbf{F} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 0.25 & 0 \\ 0 & 0.5F & -0.8660F \end{vmatrix}$$

$$= 1[0.25(-0.8660F) - 0.5F(0)] + 0 + 0$$

$$= -0.2165F \qquad \textbf{Ans.}$$

The negative sign indicates that M_x is directed towards the negative x axis. The magnitude of **F** required to produce $M_x = 30$ N·m can be determined from

$$30 = 0.2165F$$

$$F = 139\text{ N} \qquad \textbf{Ans.}$$

Scalar Analysis

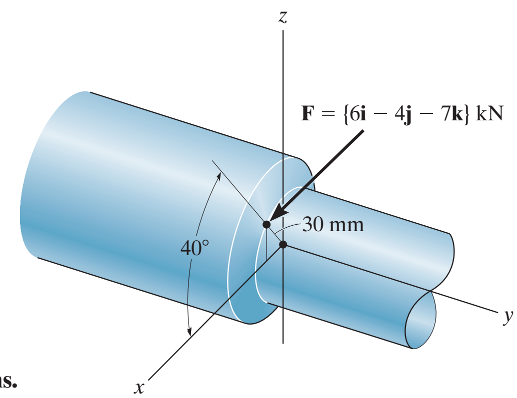
This problem can be solved by summing the moment about the x axis

$$M_x = \Sigma M_x; \qquad -30 = -F\sin 60°(0.25) + F\cos 60°(0)$$

$$F = 139\text{ N} \qquad \textbf{Ans.}$$

***4–56.**

The cutting tool on the lathe exerts a force **F** on the shaft as shown. Determine the moment of this force about the *y* axis of the shaft.



SOLUTION

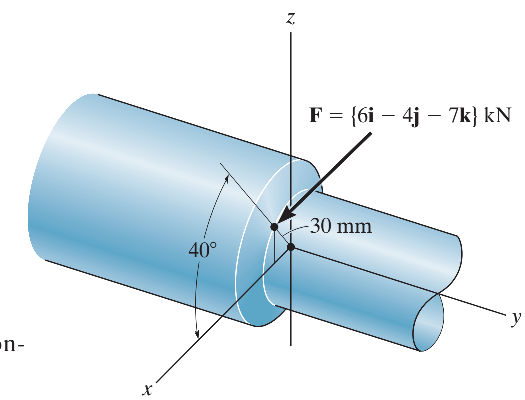
$$M_y = \mathbf{u}_y \cdot (\mathbf{r} \times \mathbf{F})$$

$$= \begin{vmatrix} 0 & 1 & 0 \\ 30 \cos 40^\circ & 0 & 30 \sin 40^\circ \\ 6 & -4 & -7 \end{vmatrix}$$

$$M_y = 276.57 \text{ N} \cdot \text{mm} = 0.277 \text{ N} \cdot \text{m}$$ **Ans.**

4–57.

The cutting tool on the lathe exerts a force **F** on the shaft as shown. Determine the moment of this force about the x and z axes.

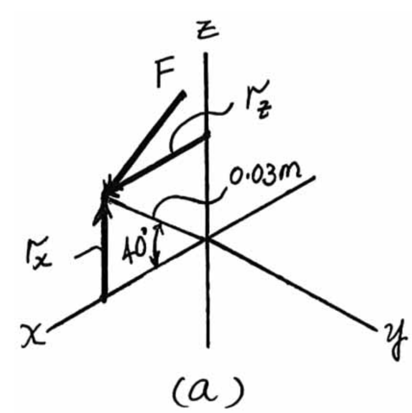


SOLUTION

Moment About x and y Axes: Position vectors $\mathbf{r}_x$ and $\mathbf{r}_z$ shown in Fig. *a* can be conveniently used in computing the moment of **F** about x and z axes respectively.

$$\mathbf{r}_x = \{0.03 \sin 40° \mathbf{k}\} \text{ m} \qquad \mathbf{r}_z = \{0.03 \cos 40° \mathbf{i}\} \text{ m}$$

Knowing that the unit vectors for x and z axes are **i** and **k** respectively. Thus, the magnitudes of moment of **F** about x and z axes are given by



$$M_x = \mathbf{i} \cdot \mathbf{r}_x \times \mathbf{F} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 0 & 0.03 \sin 40° \\ 6 & -4 & -7 \end{vmatrix}$$

$$= 1[0(-07) - (-4)(0.03 \sin 40°)] - 0 + 0$$

$$= 0.07713 \text{ kN} \cdot \text{m} = 77.1 \text{ N} \cdot \text{m}$$

$$M_z = \mathbf{k} \cdot \mathbf{r}_z \times \mathbf{F} = \begin{vmatrix} 0 & 0 & 1 \\ 0.03 \cos 40° & 0 & 0 \\ 6 & -4 & 7 \end{vmatrix}$$

$$= 0 - 0 + 1[0.03 \cos 40°(-4) - 6(0)]$$

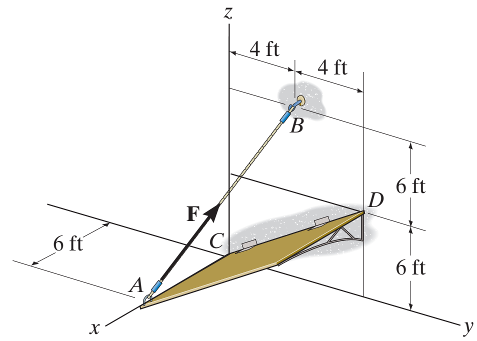
$$= -\ 0.09193 \text{ kN} \cdot \text{m} = -91.9 \text{ N} \cdot \text{m}$$

Thus,

$$M_x = M_x\mathbf{i} = \{77.1\mathbf{i}\} \text{ N} \cdot \text{m} \qquad M_z = M_z\mathbf{k} = \{-91.9\ \mathbf{k}\} \text{ N} \cdot \text{m}$$ **Ans.**

4–58.

If the tension in the cable is $F = 140$ lb, determine the magnitude of the moment produced by this force about the hinged axis, CD, of the panel.



SOLUTION

Moment About the *CD* Axis: Either position vector $\mathbf{r}_{CA}$ or $\mathbf{r}_{DB}$, Fig. *a*, can be used to determine the moment of **F** about the *CD* axis.

$$\mathbf{r}_{CA} = (6 - 0)\mathbf{i} + (0 - 0)\mathbf{j} + (0 - 0)\mathbf{k} = [6\mathbf{i}] \text{ ft}$$

$$\mathbf{r}_{DB} = (0 - 0)\mathbf{i} + (4 - 8)\mathbf{j} + (12 - 6)\mathbf{k} = [-4\mathbf{j} + 6\mathbf{k}] \text{ ft}$$

Referring to Fig. *a*, the force vector **F** can be written as

$$\mathbf{F} = F\mathbf{u}_{AB} = 140\left[\frac{(0 - 6)\mathbf{i} + (4 - 0)\mathbf{j} + (12 - 0)\mathbf{k}}{\sqrt{(0 - 6)^2 + (4 - 0)^2 + (12 - 0)^2}}\right] = [-60\mathbf{i} + 40\mathbf{j} + 120\mathbf{k}] \text{ lb}$$

The unit vector $\mathbf{u}_{CD}$, Fig. *a*, that specifies the direction of the *CD* axis is given by

$$\mathbf{u}_{CD} = \frac{(0 - 0)\mathbf{i} + (8 - 0)\mathbf{j} + (6 - 0)\mathbf{k}}{\sqrt{(0 - 0)^2 + (8 - 0)^2 + (6 - 0)^2}} = \frac{4}{5}\mathbf{j} + \frac{3}{5}\mathbf{k}$$

Thus, the magnitude of the moment of **F** about the *CD* axis is given by

$$M_{CD} = \mathbf{u}_{CD} \cdot \mathbf{r}_{CA} \times \mathbf{F} = \begin{vmatrix} 0 & \frac{4}{5} & \frac{3}{5} \\ 6 & 0 & 0 \\ -60 & 40 & 120 \end{vmatrix}$$

$$= 0 - \frac{4}{5}[6(120) - (-60)(0)] + \frac{3}{5}[6(40) - (-60)(0)]$$

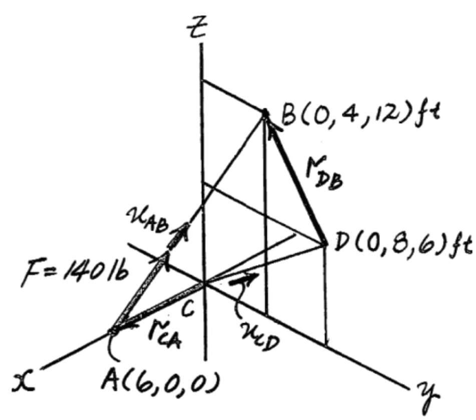
$$= -432 \text{ lb} \cdot \text{ft} \qquad \textbf{Ans.}$$

or

$$M_{CD} = \mathbf{u}_{CD} \cdot \mathbf{r}_{DB} \times \mathbf{F} = \begin{vmatrix} 0 & \frac{4}{5} & \frac{3}{5} \\ 0 & -4 & 6 \\ -60 & 40 & 120 \end{vmatrix}$$

$$= 0 - \frac{4}{5}[0(120) - (-60)(6)] + \frac{3}{5}[0(40) - (-60)(-4)]$$

$$= -432 \text{ lb} \cdot \text{ft}$$



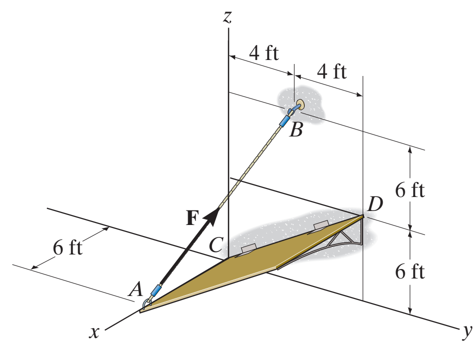
(a)

The negative sign indicates that $\mathbf{M}_{CD}$ acts in the opposite sense to that of $\mathbf{u}_{CD}$. Thus,

$$M_{CD} = 432 \text{ lb} \cdot \text{ft} \qquad \textbf{Ans.}$$

4–59.

Determine the magnitude of force **F** in cable AB in order to produce a moment of 500 lb·ft about the hinged axis CD, which is needed to hold the panel in the position shown.



SOLUTION

Moment About the CD Axis: Either position vector $\mathbf{r}_{CA}$ or $\mathbf{r}_{CB}$, Fig. *a*, can be used to determine the moment of **F** about the CD axis.

$$\mathbf{r}_{CA} = (6-0)\mathbf{i} + (0-0)\mathbf{j} + (0-0)\mathbf{k} = [6\mathbf{i}]\text{ft}$$

$$\mathbf{r}_{CB} = (0-0)\mathbf{i} + (4-0)\mathbf{j} + (12-0)\mathbf{k} = [4\mathbf{j} + 12\mathbf{k}]\text{ft}$$

Referring to Fig. *a*, the force vector **F** can be written as

$$\mathbf{F} = F\mathbf{u}_{AB} = F\left[\frac{(0-6)\mathbf{i} + (4-0)\mathbf{j} + (12-0)\mathbf{k}}{\sqrt{(0-6)^2 + (4-0)^2 + (12-0)^2}}\right] = -\frac{3}{7}F\mathbf{i} + \frac{2}{7}F\mathbf{j} + \frac{6}{7}F\mathbf{k}$$

The unit vector $\mathbf{u}_{CD}$, Fig. *a*, that specifies the direction of the CD axis is given by

$$\mathbf{u}_{CD} = \frac{(0-0)\mathbf{i} + (8-0)\mathbf{j} + (6-0)\mathbf{k}}{\sqrt{(0-0)^2 + (8-0)^2 + (6-0)^2}} = \frac{4}{5}\mathbf{j} + \frac{3}{5}\mathbf{k}$$

Thus, the magnitude of the moment of **F** about the CD axis is required to be $\mathbf{M}_{CD} = |500|$ lb·ft. Thus,

$$M_{CD} = \mathbf{u}_{CD} \cdot \mathbf{r}_{CA} \times \mathbf{F}$$

$$|500| = \begin{vmatrix} 0 & \frac{4}{5} & \frac{3}{5} \\ 6 & 0 & 0 \\ -\frac{3}{7}F & \frac{2}{7}F & \frac{6}{7}F \end{vmatrix}$$

$$-500 = 0 - \frac{4}{5}\left[6\left(\frac{6}{7}F\right) - \left(-\frac{3}{7}F\right)(0)\right] + \frac{3}{5}\left[6\left(\frac{2}{7}F\right) - \left(-\frac{3}{7}F\right)(0)\right]$$

$$F = 162 \text{ lb}$$ **Ans.**

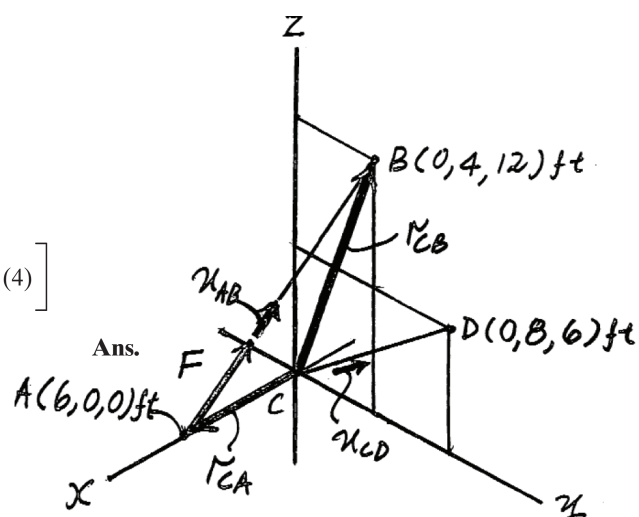
or

$$M_{CD} = \mathbf{u}_{CD} \cdot \mathbf{r}_{CB} \times \mathbf{F}$$

$$|500| = \begin{vmatrix} 0 & \frac{4}{5} & \frac{3}{5} \\ 0 & 4 & 12 \\ -\frac{3}{7}F & \frac{2}{7}F & \frac{6}{7}F \end{vmatrix}$$

$$-500 = 0 - \frac{4}{5}\left[0\left(\frac{6}{7}F\right) - \left(-\frac{3}{7}F\right)(12)\right] + \frac{3}{5}\left[0\left(\frac{2}{7}F\right) - \left(-\frac{3}{7}F\right)(4)\right]$$

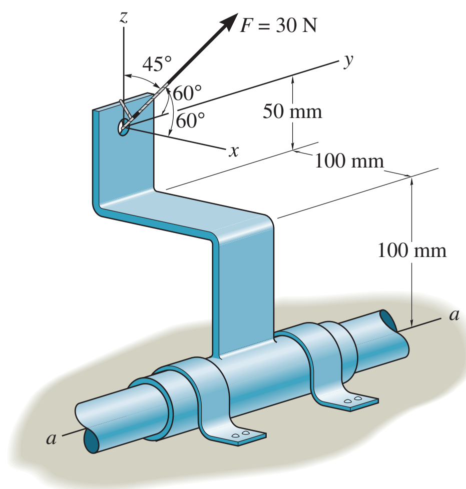
$$F = 162 \text{ lb}$$ **Ans.**



(a)

***4–60.**

The force of $F = 30$ N acts on the bracket as shown. Determine the moment of the force about the $a-a$ axis of the pipe. Also, determine the coordinate direction angles of F in order to produce the maximum moment about the $a-a$ axis. What is this moment?



SOLUTION

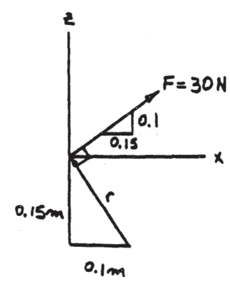
$$\mathbf{F} = 30\,(\cos 60°\,\mathbf{i} + \cos 60°\,\mathbf{j} + \cos 45°\,\mathbf{k})$$

$$= \{15\,\mathbf{i} + 15\,\mathbf{j} + 21.21\,\mathbf{k}\}\text{ N}$$

$$\mathbf{r} = \{-0.1\,\mathbf{i} + 0.15\,\mathbf{k}\}\text{ m}$$

$$\mathbf{u} = \mathbf{j}$$

$$M_a = \begin{vmatrix} 0 & 1 & 0 \\ -0.1 & 0 & 0.15 \\ 15 & 15 & 21.21 \end{vmatrix} = 4.37\text{ N}\cdot\text{m} \qquad \textbf{Ans.}$$



F must be perpendicular to **u** and **r**.

$$\mathbf{u}_F = \frac{0.15}{0.1803}\mathbf{i} + \frac{0.1}{0.1803}\mathbf{k}$$

$$= 0.8321\mathbf{i} + 0.5547\mathbf{k}$$

$$\alpha = \cos^{-1} 0.8321 = 33.7° \qquad \textbf{Ans.}$$

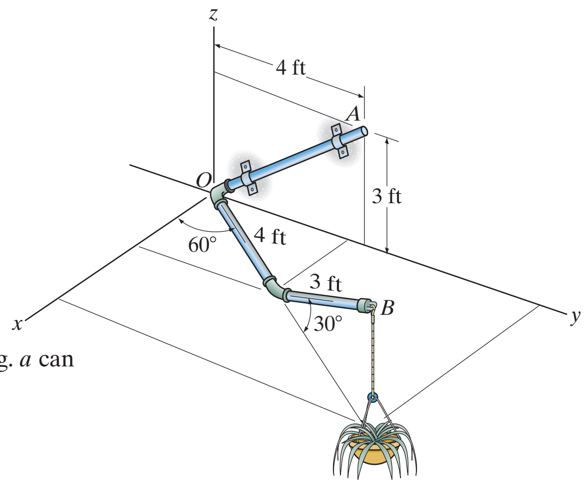
$$\beta = \cos^{-1} 0 = 90° \qquad \textbf{Ans.}$$

$$\gamma = \cos^{-1} 0.5547 = 56.3° \qquad \textbf{Ans.}$$

$$M = 30\,(0.1803) = 5.41\text{ N}\cdot\text{m} \qquad \textbf{Ans.}$$

4–61.

The pipe assembly is secured on the wall by the two brackets. If the flower pot has a weight of 50 lb, determine the magnitude of the moment produced by the weight about the x, y, and z axes.



SOLUTION

Moment About x, y, and z Axes: Position vectors $\mathbf{r}_x$, $\mathbf{r}_y$, and $\mathbf{r}_z$ shown in Fig. *a* can be conveniently used in computing the moment of **W** about x, y, and z axes.

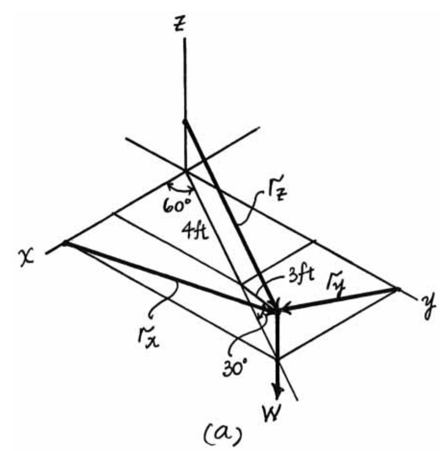
$$\mathbf{r}_x = \{(4 + 3\cos 30°)\sin 60°\mathbf{j} + 3\sin 30°\mathbf{k}\} \text{ ft}$$
$$= \{5.7141\mathbf{j} + 1.5\mathbf{k}\} \text{ ft}$$

$$\mathbf{r}_y = \{(4 + 3\cos 30°)\cos 60°\mathbf{i} + 3\sin 30°\mathbf{k}\} \text{ ft}$$
$$= \{3.2990\mathbf{i} + 1.5\mathbf{k}\} \text{ ft}$$

$$\mathbf{r}_z = \{(4 + 3\cos 30°)\cos 60°\mathbf{i} + (4 + 3\cos 30°)\sin 60°\mathbf{j}\} \text{ ft}$$
$$= \{3.2990\mathbf{i} + 5.7141\mathbf{j}\} \text{ ft}$$

The Force vector is given by

$$\mathbf{W} = W(-\mathbf{k}) = \{-50\,\mathbf{k}\} \text{ lb}$$



Knowing that the unit vectors for x, y, and z axes are **i**, **j**, and **k** respectively. Thus, the magnitudes of the moment of **W** about x, y, and z axes are given by

$$M_x = \mathbf{i}\cdot\mathbf{r}_x \times \mathbf{W} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 5.7141 & 1.5 \\ 0 & 0 & -50 \end{vmatrix}$$
$$= 1[5.7141(-50) - 0(1.5)] - 0 + 0$$
$$= -285.70 \text{ lb}\cdot\text{ft} = -286 \text{ lb}\cdot\text{ft} \qquad \textbf{Ans.}$$

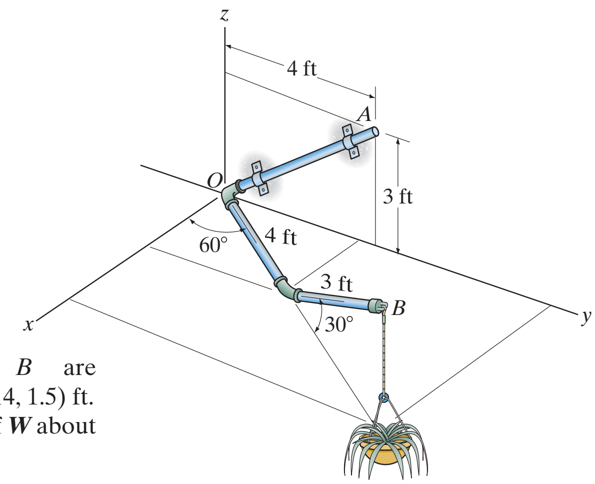
$$M_y = \mathbf{j}\cdot\mathbf{r}_y \times \mathbf{W} = \begin{vmatrix} 0 & 1 & 0 \\ 3.2990 & 0 & 1.5 \\ 0 & 0 & -50 \end{vmatrix}$$
$$= 0 - 1[3.2990(-50) - 0(1.5)] + 0$$
$$= 164.95 \text{ lb}\cdot\text{ft} = 165 \text{ lb}\cdot\text{ft} \qquad \textbf{Ans.}$$

$$M_z = \mathbf{k}\cdot\mathbf{r}_z \times \mathbf{W} = \begin{vmatrix} 0 & 0 & 1 \\ 3.2990 & 5.7141 & 0 \\ 0 & 0 & -5 \end{vmatrix}$$
$$= 0 - 0 + 1[3.2990(0) - 0(5.7141)]$$
$$= 0 \textbf{Ans.}$$

The negative sign indicates that $\mathbf{M}_x$ is directed towards the negative x axis.

4–62.

The pipe assembly is secured on the wall by the two brackets. If the flower pot has a weight of 50 lb, determine the magnitude of the moment produced by the weight about the *OA* axis.



SOLUTION

Moment About the OA Axis: The coordinates of point B are $[(4 + 3\cos 30°)\cos 60°, (4 + 3\cos 30°)\sin 60°, 3\sin 30°]$ ft $= (3.299, 5.714, 1.5)$ ft. Either position vector $\mathbf{r}_{OB}$ or $\mathbf{r}_{AB}$ can be used to determine the moment of $\boldsymbol{W}$ about the *OA* axis.

$$\mathbf{r}_{OB} = (3.299 - 0)\mathbf{i} + (5.714 - 0)\mathbf{j} + (1.5 - 0)\mathbf{k} = [3.299\mathbf{i} + 5.714\mathbf{j} + 1.5\mathbf{k}] \text{ ft}$$

$$\mathbf{r}_{AB} = (3.299 - 0)\mathbf{i} + (5.714 - 4)\mathbf{j} + (1.5 - 3)\mathbf{k} = [3.299\mathbf{i} + 1.714\mathbf{j} - 1.5\mathbf{k}] \text{ ft}$$

Since $\mathbf{W}$ is directed towards the negative z axis, we can write $\mathbf{W} = [-50\mathbf{k}]$ lb

The unit vector $\mathbf{u}_{OA}$, Fig. *a*, that specifies the direction of the *OA* axis is given by

$$\mathbf{u}_{OA} = \frac{(0-0)\mathbf{i} + (4-0)\mathbf{j} + (3-0)\mathbf{k}}{\sqrt{(0-0)^2 + (4-0)^2 + (3-0)^2}} = \frac{4}{5}\mathbf{j} + \frac{3}{5}\mathbf{k}$$

The magnitude of the moment of $\mathbf{W}$ about the *OA* axis is given by

$$M_{OA} = \mathbf{u}_{OA} \cdot \mathbf{r}_{OB} \times \mathbf{W} = \begin{vmatrix} 0 & \frac{4}{5} & \frac{3}{5} \\ 3.299 & 5.714 & 1.5 \\ 0 & 0 & -50 \end{vmatrix}$$

$$= 0 - \frac{4}{5}[3.299(-50) - 0(1.5)] + \frac{3}{5}[3.299(0) - 0(5.714)]$$

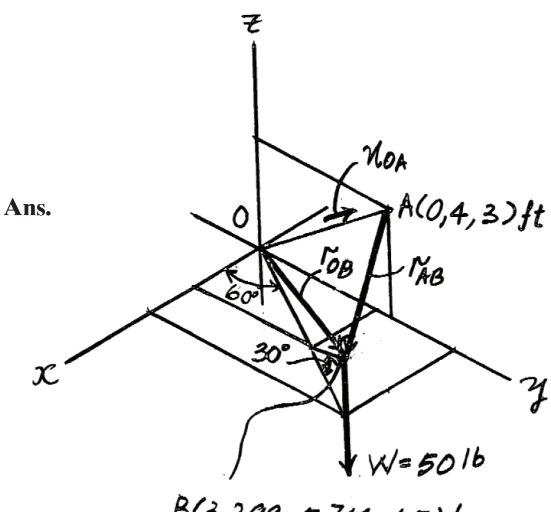
$$= 132 \text{ lb} \cdot \text{ft}$$ **Ans.**

or

$$M_{OA} = \mathbf{u}_{OA} \cdot \mathbf{r}_{AB} \times \mathbf{W} = \begin{vmatrix} 0 & \frac{4}{5} & \frac{3}{5} \\ 3.299 & 1.714 & -1.5 \\ 0 & 0 & -50 \end{vmatrix}$$

$$= 0 - \frac{4}{5}[3.299(-50) - 0(-1.5)] + \frac{3}{5}[3.299(0) - 0(1.714)]$$

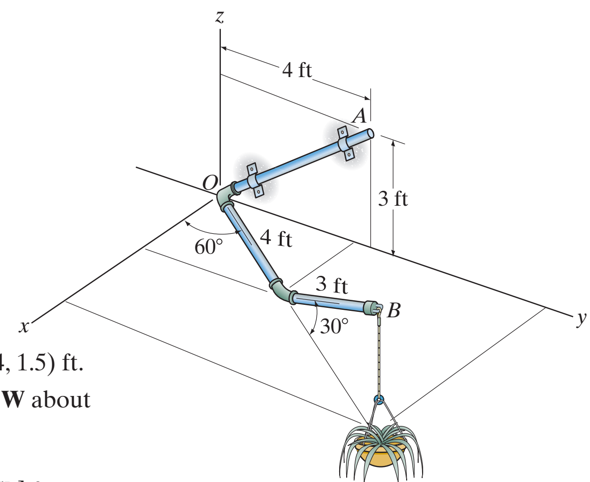
$$= 132 \text{ lb} \cdot \text{ft}$$ **Ans.**



(a)

4–63.

The pipe assembly is secured on the wall by the two brackets. If the frictional force of both brackets can resist a maximum moment of 150 lb·ft, determine the largest weight of the flower pot that can be supported by the assembly without causing it to rotate about the *OA* axis.



SOLUTION

Moment About the OA Axis: The coordinates of point *B* are $[(4 + 3\cos 30°)\cos 60°, (4 + 3\cos 30°)\sin 60°, 3\sin 30°]\text{ft} = (3.299, 5.174, 1.5)$ ft. Either position vector $\mathbf{r}_{OB}$ or $\mathbf{r}_{OC}$ can be used to determine the moment of **W** about the *OA* axis.

$$\mathbf{r}_{OA} = (3.299 - 0)\mathbf{i} + (5.714 - 0)\mathbf{j} + (1.5 - 0)\mathbf{k} = [3.299\mathbf{i} + 5.714\mathbf{j} + 1.5\mathbf{k}]\text{ ft}$$

$$\mathbf{r}_{AB} = (3.299 - 0)\mathbf{i} + (5.714 - 4)\mathbf{j} + (1.5 - 3)\mathbf{k} = [3.299\mathbf{i} + 1.714\mathbf{j} - 1.5\mathbf{k}]\text{ ft}$$

Since **W** is directed towards the negative *z* axis, we can write $\mathbf{W} = -W\mathbf{k}$

The unit vector $\mathbf{u}_{OA}$, Fig. *a*, that specifies the direction of the *OA* axis is given by

$$\mathbf{u}_{OA} = \frac{(0-0)\mathbf{i} + (4-0)\mathbf{j} + (3-0)\mathbf{k}}{\sqrt{(0-0)^2 + (4-0)^2 + (3-0)^2}} = \frac{4}{5}\mathbf{j} + \frac{3}{5}\mathbf{k}$$

Since it is required that the magnitude of the moment of **W** about the *OA* axis not exceed 150 ft·lb, we can write

$$M_{OA} = \mathbf{u}_{OA} \cdot \mathbf{r}_{OB} \times \mathbf{W}$$

$$|150| = \begin{vmatrix} 0 & \frac{4}{5} & \frac{3}{5} \\ 3.299 & 5.714 & 1.5 \\ 0 & 0 & -W \end{vmatrix}$$

$$150 = 0 - \frac{4}{5}[3.299(-W) - 0(1.5)] + \frac{3}{5}[3.299(0) - 0(5.714)]$$

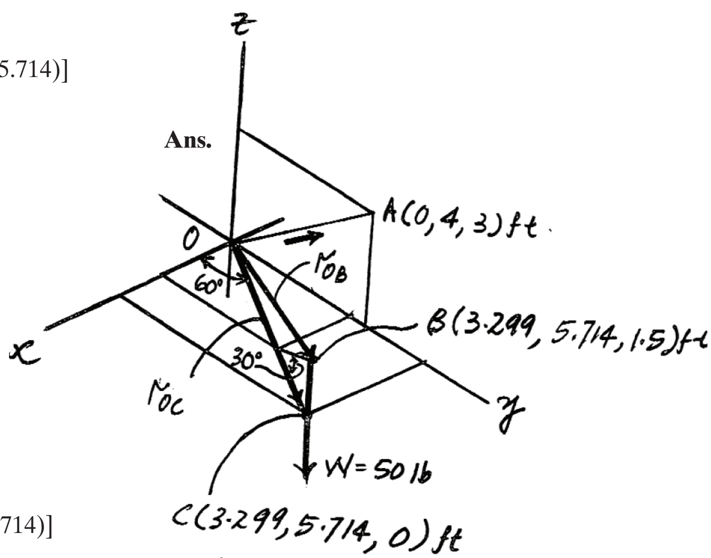
$$W = 56.8\text{ lb}$$ **Ans.**

or

$$M_{OA} = \mathbf{u}_{OA} \cdot \mathbf{r}_{AB} \times \mathbf{W}$$

$$|150| = \begin{vmatrix} 0 & \frac{4}{5} & \frac{3}{5} \\ 3.299 & 5.714 & 0 \\ 0 & 0 & -W \end{vmatrix}$$

$$150 = 0 - \frac{4}{5}[3.299(-W) - 0(0)] + \frac{3}{5}[3.299(0) - 0(5.714)]$$

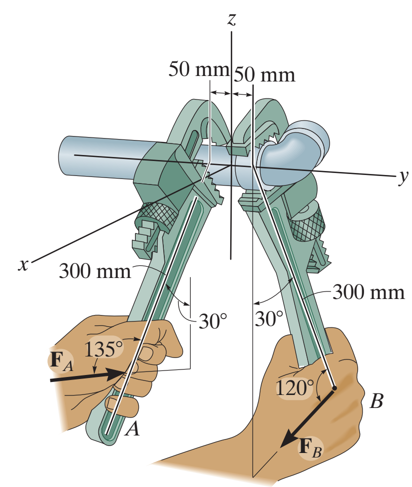


(a)

$$W = 56.8\text{ lb}$$ **Ans.**

***4–64**

The wrench A is used to hold the pipe in a stationary position while wrench B is used to tighten the elbow fitting. If $F_B = 150$ N, determine the magnitude of the moment produced by this force about the y axis. Also, what is the magnitude of force $\mathbf{F}_A$ in order to counteract this moment?



SOLUTION

Vector Analysis

Moment of F_B About the y Axis: The position vector $\mathbf{r}_{CB}$, Fig. a, will be used to determine the moment of $\mathbf{F}_B$ about the y axis.

$$\mathbf{r}_{CB} = (-0.15 - 0)\mathbf{j} + (0.05 - 0.05)\mathbf{j} + (-0.2598 - 0)\mathbf{k} = \{-0.15\mathbf{i} - 0.2598\mathbf{k}\}\text{ m}$$

Referring to Fig. a, the force vector $\mathbf{F}_B$ can be written as

$$\mathbf{F}_B = 150(\cos 60°\mathbf{i} - \sin 60°\mathbf{k}) = \{75\mathbf{i} - 129.90\mathbf{k}\}\text{ N}$$

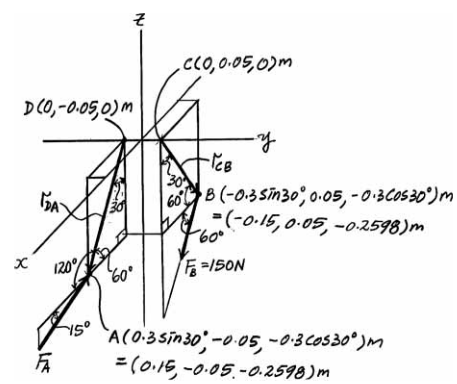
Knowing that the unit vector of the y axis is $\mathbf{j}$, the magnitude of the moment of $\mathbf{F}_B$ about the y axis is given by

$$M_y = \mathbf{j}\cdot\mathbf{r}_{CB}\times\mathbf{F}_B = \begin{vmatrix} 0 & 1 & 0 \\ -0.15 & 0 & -0.2598 \\ 75 & 0 & -129.90 \end{vmatrix}$$

$$= 0 - 1[-0.15(-129.90) - 75(-0.2598)] + 0$$

$$= -38.97\text{ N}\cdot\text{m} = 39.0\text{ N}\cdot\text{m} \qquad \textbf{Ans.}$$

The negative sign indicates that M_y is directed towards the negative y axis.



Moment of F_A About the y Axis: The position vector $\mathbf{r}_{DA}$, Fig. a, will be used to determine the moment of $\mathbf{F}_A$ about the y axis.

$$\mathbf{r}_{DA} = (0.15 - 0)\mathbf{i} + [-0.05 - (-0.05)]\mathbf{j} + (-0.2598 - 0)\mathbf{k} = \{0.15\mathbf{i} - 0.2598\mathbf{k}\}\text{ m}$$

Referring to Fig. a, the force vector $\mathbf{F}_A$ can be written as

$$\mathbf{F}_A = F_A(-\cos 15°\mathbf{i} + \sin 15°\mathbf{k}) = -0.9659F_A\mathbf{i} + 0.2588F_A\mathbf{k}$$

Since the moment of $\mathbf{F}_A$ about the y axis is required to counter that of $\mathbf{F}_B$ about the same axis, $\mathbf{F}_A$ must produce a moment of equal magnitude but in the opposite sense to that of $\mathbf{F}_A$.

$$M_x = \mathbf{j}\cdot\mathbf{r}_{DA}\times\mathbf{F}_B$$

$$+0.38.97 = \begin{vmatrix} 0 & 1 & 0 \\ 0.15 & 0 & -0.2598 \\ -0.9659F_A & 0 & 0.2588F_A \end{vmatrix}$$

$$+0.38.97 = 0 - 1[0.15(0.2588F_A) - (-0.9659F_A)(-0.2598)] + 0$$

$$F_A = 184\text{ N} \qquad \textbf{Ans.}$$

Scalar Analysis

This problem can be solved by first taking the moments of $\mathbf{F}_B$ and then $\mathbf{F}_A$ about the y axis. For $\mathbf{F}_B$ we can write

$$M_y = \Sigma M_y; \qquad M_y = -150\cos 60°(0.3\cos 30°) - 150\sin 60°(0.3\sin 30°)$$

$$= -38.97\text{ N}\cdot\text{m} \qquad \textbf{Ans.}$$

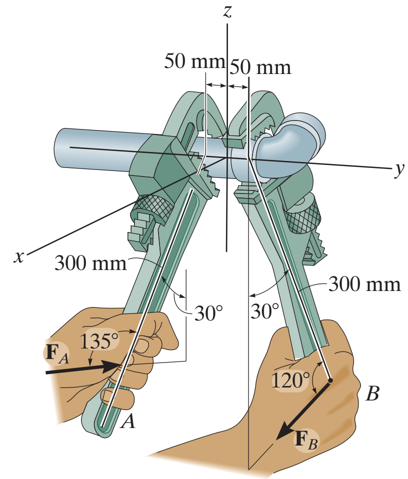
The moment of $\mathbf{F}_A$, about the y axis also must be equal in magnitude but opposite in sense to that of $\mathbf{F}_B$ about the same axis

$$M_y = \Sigma M_y; \qquad 38.97 = F_A\cos 15°(0.3\cos 30°) - F_A\sin 15°(0.3\sin 30°)$$

$$F_A = 184\text{ N} \qquad \textbf{Ans.}$$

4–65.

The wrench A is used to hold the pipe in a stationary position while wrench B is used to tighten the elbow fitting. Determine the magnitude of force F_B in order to develop a torque of 50 N · m about the y axis. Also, what is the required magnitude of force $\mathbf{F}_A$ in order to counteract this moment?



SOLUTION

Vector Analysis

Moment of $\mathbf{F}_B$ About the y Axis: The position vector $\mathbf{r}_{CB}$, Fig. *a*, will be used to determine the moment of $\mathbf{F}_B$ about the y axis.

$$\mathbf{r}_{CB} = (-0.15 - 0)\mathbf{i} + (0.05 - 0.05)\mathbf{j} + (-0.2598 - 0)\mathbf{k} = \{-0.15\mathbf{i} - 0.2598\mathbf{k}\}\text{ m}$$

Referring to Fig. *a*, the force vector $\mathbf{F}_B$ can be written as

$$\mathbf{F}_B = F_B(\cos 60°\mathbf{i} - \sin 60°\mathbf{k}) = 0.5F_B\mathbf{i} - 0.8660F_B\mathbf{k}$$

Knowing that the unit vector of the y axis is $\mathbf{j}$, the moment of $\mathbf{F}_B$ about the y axis is required to be equal to -50 N · m, which is given by

$$M_y = \mathbf{j} \cdot \mathbf{r}_{CB} \times \mathbf{F}_B$$

$$-50\mathbf{i} = \begin{vmatrix} 0 & 1 & 0 \\ -0.15 & 0 & -0.2598 \\ 0.5F_B & 0 & -0.8660F_B \end{vmatrix}$$

$$-50 = 0 - 1[-0.15(-0.8660F_B) - 0.5F_B(-0.2598)] + 0$$

$$F_B = 192\text{ N} \qquad \textbf{Ans.}$$

Moment of $\mathbf{F}_A$ About the y Axis: The position vector $\mathbf{r}_{DA}$, Fig. a, will be used to determine the moment of $\mathbf{F}_A$ about the y axis.

$$\mathbf{r}_{DA} = (0.15 - 0)\mathbf{i} + [-0.05 - (-0.05)]\mathbf{j} + (-0.2598 - 0)\mathbf{k} = \{-0.15\mathbf{i} - 0.2598\mathbf{k}\}\text{ m}$$

Referring to Fig. *a*, the force vector $\mathbf{F}_A$ can be written as

$$\mathbf{F}_A = F_A(-\cos 15°\mathbf{i} + \sin 15°\mathbf{k}) = -0.9659F_A\mathbf{i} + 0.2588F_A\mathbf{k}$$

Since the moment of $\mathbf{F}_A$ about the y axis is required to produce a countermoment of 50 N · m about the y axis, we can write

$$M_y = \mathbf{j} \cdot \mathbf{r}_{DA} \times \mathbf{F}_A$$

$$50 = \begin{vmatrix} 0 & 1 & 0 \\ 0.15 & 0 & -0.2598 \\ -0.9659F_A & 0 & 0.2588F_A \end{vmatrix}$$

$$50 = 0 - 1[0.15(0.2588F_A) - (-0.9659F_A)(-0.2598)] + 0$$

$$F_A = 236\text{ N} \cdot \text{m} \qquad \textbf{Ans.}$$

Scalar Analysis

This problem can be solved by first taking the moments of $\mathbf{F}_B$ and then $\mathbf{F}_A$ about the y axis. For $\mathbf{F}_B$ we can write

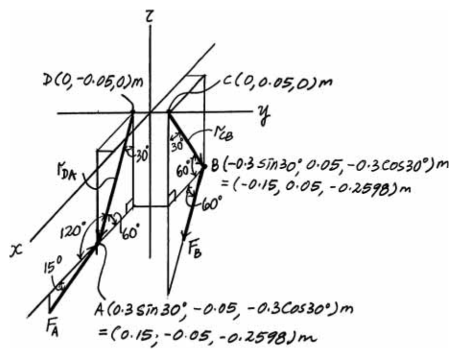
$$M_y = \Sigma M_y; \qquad -50 = -F_B \cos 60°(0.3 \cos 30°) - F_B \sin 60°(0.3 \sin 30°)$$

$$F_B = 192\text{ N} \qquad \textbf{Ans.}$$

For $\mathbf{F}_A$, we can write

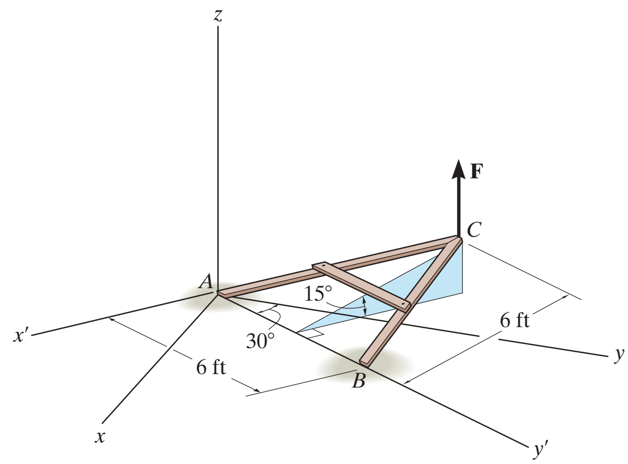
$$M_y = \Sigma M_y; \qquad 50 = F_A \cos 15°(0.3 \cos 30°) - F_A \sin 15°(0.3 \sin 30°)$$

$$F_A = 236\text{ N} \qquad \textbf{Ans.}$$



4–66.

The A-frame is being hoisted into an upright position by the vertical force of $F = 80$ lb. Determine the moment of this force about the y axis when the frame is in the position shown.



SOLUTION

Using x', y', z:

$$\mathbf{u}_y = -\sin 30^\circ\,\mathbf{i}' + \cos 30^\circ\,\mathbf{j}'$$

$$\mathbf{r}_{AC} = -6\cos 15^\circ\mathbf{i}' + 3\,\mathbf{j}' + 6\sin 15^\circ\,\mathbf{k}$$

$$\mathbf{F} = 80\,\mathbf{k}$$

$$M_y = \begin{vmatrix} -\sin 30^\circ & \cos 30^\circ & 0 \\ -6\cos 15^\circ & 3 & 6\sin 15^\circ \\ 0 & 0 & 80 \end{vmatrix} = -120 + 401.53 + 0$$

$M_y = 282\ \text{lb}\cdot\text{ft}$ **Ans.**

Also, using x, y, z:

Coordinates of point C:

$$x = 3\sin 30^\circ - 6\cos 15^\circ\cos 30^\circ = -3.52\ \text{ft}$$

$$y = 3\cos 30^\circ + 6\cos 15^\circ\sin 30^\circ = 5.50\ \text{ft}$$

$$z = 6\sin 15^\circ = 1.55\ \text{ft}$$

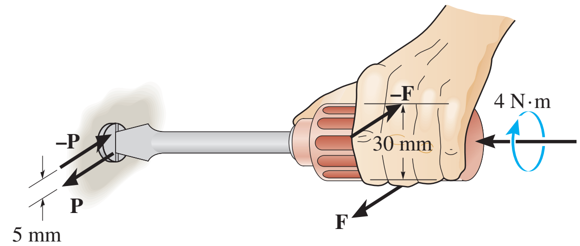
$$\mathbf{r}_{AC} = -3.52\,\mathbf{i} + 5.50\,\mathbf{j} + 1.55\,\mathbf{k}$$

$$\mathbf{F} = 80\,\mathbf{k}$$

$$M_y = \begin{vmatrix} 0 & 1 & 0 \\ -3.52 & 5.50 & 1.55 \\ 0 & 0 & 80 \end{vmatrix} = 282\ \text{lb}\cdot\text{ft}$$ **Ans.**

4–67.

A twist of 4 N · m is applied to the handle of the screwdriver. Resolve this couple moment into a pair of couple forces **F** exerted on the handle and **P** exerted on the blade.



SOLUTION

For the handle

$M_C = \Sigma M_x;$ $\qquad F(0.03) = 4$

$$F = 133 \text{ N}$$ **Ans.**

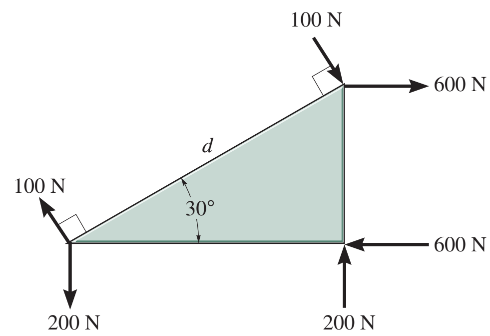
For the blade,

$M_C = \Sigma M_x;$ $\qquad P(0.005) = 4$

$$P = 800 \text{ N}$$ **Ans.**

***4–68.**

The ends of the triangular plate are subjected to three couples. Determine the plate dimension d so that the resultant couple is 350 N · m clockwise.



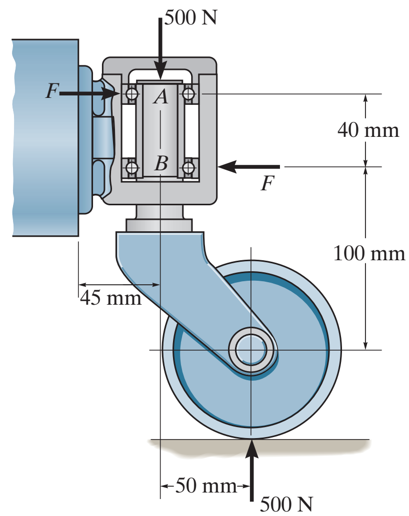
SOLUTION

$$\circlearrowleft + M_R = \Sigma M_A; \qquad -350 = 200(d \cos 30^\circ) - 600(d \sin 30^\circ) - 100d$$

$$d = 1.54 \text{ m}$$ **Ans.**

4–69.

The caster wheel is subjected to the two couples. Determine the forces **F** that the bearings exert on the shaft so that the resultant couple moment on the caster is zero.



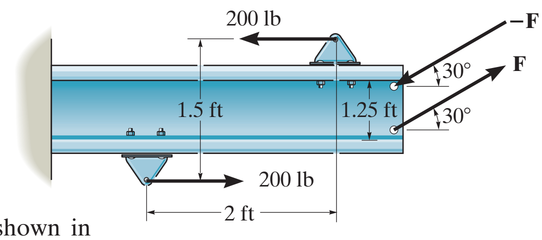
SOLUTION

$$\circlearrowleft +\Sigma M_A = 0; \qquad 500(50) - F(40) = 0$$

$$F = 625 \text{ N}$$ **Ans.**

4–70.

Two couples act on the beam. If $F = 125$ lb, determine the resultant couple moment.

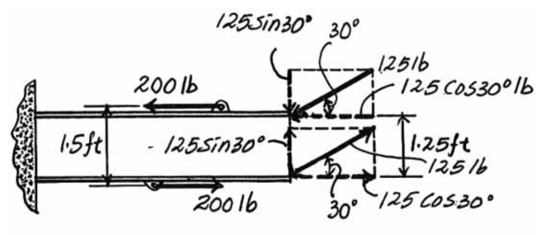


SOLUTION

125 lb couple is resolved in to their horizontal and vertical components as shown in Fig. *a*.

$$\circlearrowleft + (M_R)_C = 200(1.5) + 125 \cos 30°(1.25)$$

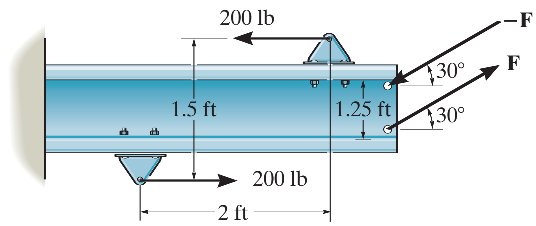
$$= 435.32 \text{ lb}\cdot\text{ft} = 435 \text{ lb}\cdot\text{ft} \circlearrowleft$$ **Ans.**



(a)

4–71.

Two couples act on the beam. Determine the magnitude of **F** so that the resultant couple moment is 450 lb · ft, counterclockwise. Where on the beam does the resultant couple moment act?



SOLUTION

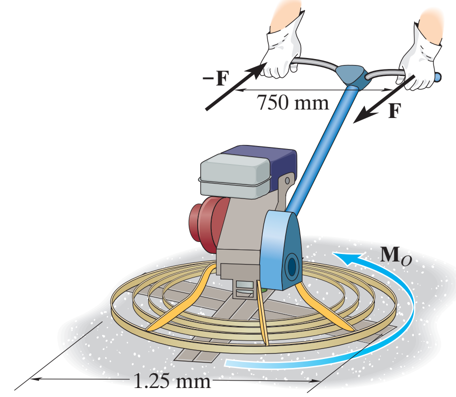
$$\circlearrowleft + M_R = \Sigma M; \qquad 450 = 200(1.5) + F\cos 30°(1.25)$$

$$F = 139 \text{ lb} \qquad \textbf{Ans.}$$

The resultant couple moment is a free vector. It can act at any point on the beam.

***4–72.**

Friction on the concrete surface creates a couple moment of $M_O = 100\ \text{N}\cdot\text{m}$ on the blades of the trowel. Determine the magnitude of the couple forces so that the resultant couple moment on the trowel is zero. The forces lie in the horizontal plane and act perpendicular to the handle of the trowel.



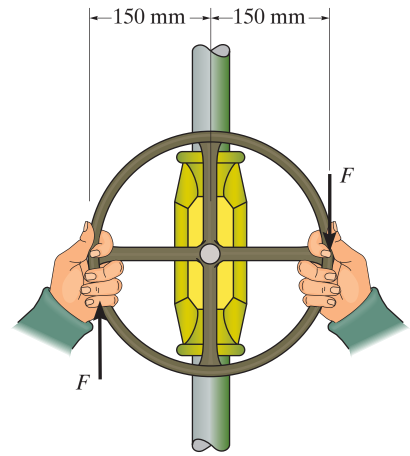
SOLUTION

Couple Moment: The couple moment of **F** about the vertical axis is $M_C = F(0.75) = 0.75F$. Since the resultant couple moment about the vertical axis is required to be zero, we can write

$(M_c)_R = \Sigma M_{z};\qquad 0 = 100 - 0.75F \qquad F = 133\ \text{N}$ **Ans.**

4–73.

The man tries to open the valve by applying the couple forces of $F = 75$ N to the wheel. Determine the couple moment produced.



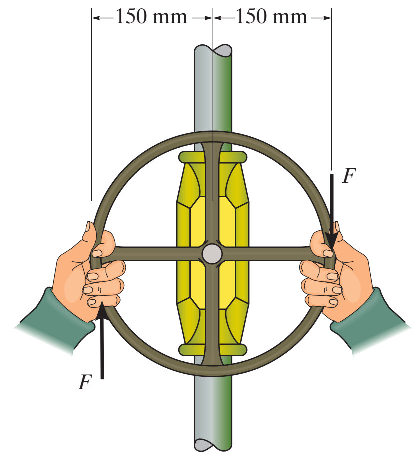
SOLUTION

$\circlearrowleft + M_c = \Sigma M;$ $\quad M_c = -75(0.15 + 0.15)$

$= -22.5 \text{ N}\cdot\text{m} = 22.5 \text{ N}\cdot\text{m} \circlearrowright$ **Ans.**

4–74.

If the valve can be opened with a couple moment of $25\ \text{N}\cdot\text{m}$, determine the required magnitude of each couple force which must be applied to the wheel.



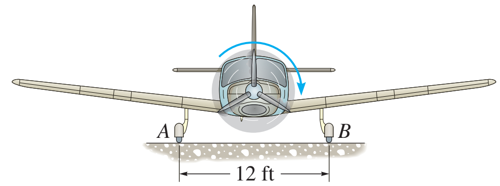
SOLUTION

$\curvearrowleft + M_c = \Sigma M;\qquad -25 = -F(0.15 + 0.15)$

$F = 83.3\ \text{N}$ **Ans.**

4–75.

When the engine of the plane is running, the vertical reaction that the ground exerts on the wheel at A is measured as 650 lb. When the engine is turned off, however, the vertical reactions at A and B are 575 lb each. The difference in readings at A is caused by a couple acting on the propeller when the engine is running. This couple tends to overturn the plane counterclockwise, which is opposite to the propeller's clockwise rotation. Determine the magnitude of this couple and the magnitude of the reaction force exerted at B when the engine is running.



SOLUTION

When the engine of the plane is turned on, the resulting couple moment exerts an additional force of $F = 650 - 575 = 75.0$ lb on wheel A and a lesser the reactive force on wheel B of $F = 75.0$ lb as well. Hence,

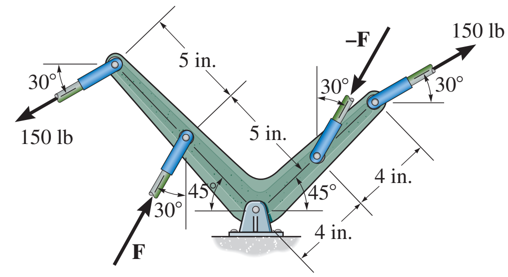
$$M = 75.0(12) = 900 \text{ lb} \cdot \text{ft} \qquad \textbf{Ans.}$$

The reactive force at wheel B is

$$R_B = 575 - 75.0 = 500 \text{ lb} \qquad \textbf{Ans.}$$

***4–76.**

Determine the magnitude of the couple force **F** so that the resultant couple moment on the crank is zero.



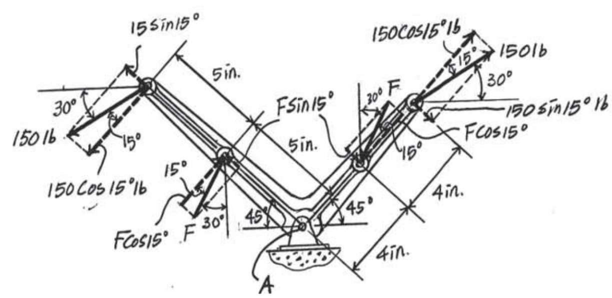
SOLUTION

By resolving **F** and the 150-lb couple into components parallel and perpendicular to the lever arm of the crank, Fig. *a*, and summing the moment of these two force components about point *A*, we have

$$\curvearrowleft +(M_C)_R = \Sigma M_A; \qquad 0 = 150 \cos 15°(10) - F \cos 15°(5) - F \sin 15°(4) - 150 \sin 15°(8)$$

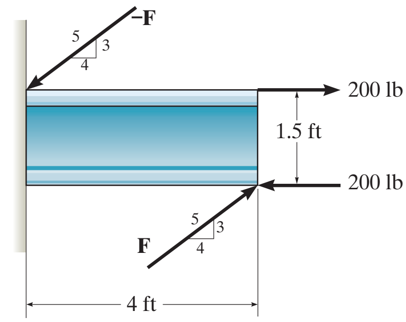
$$F = 194 \text{ lb} \qquad \textbf{Ans.}$$

Note: Since the line of action of the force component parallel to the lever arm of the crank passes through point *A*, no moment is produced about this point.



4–77.

Two couples act on the beam as shown. If $F = 150$ lb, determine the resultant couple moment.

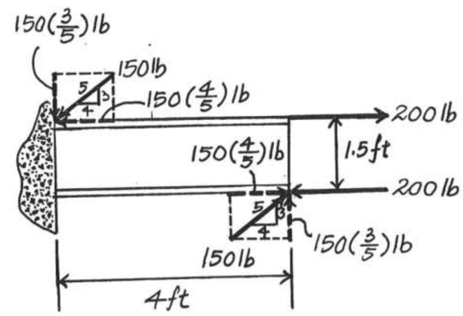


SOLUTION

150 lb couple is resolved into their horizontal and vertical components as shown in Fig. *a*

$$\circlearrowleft + (M_R)_c = 150\left(\frac{4}{5}\right)(1.5) + 150\left(\frac{3}{5}\right)(4) - 200(1.5)$$

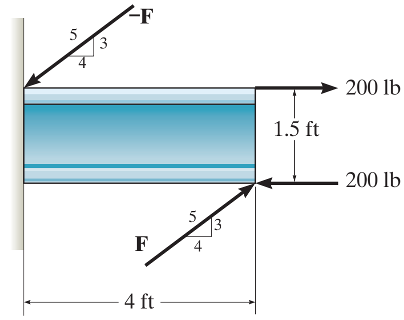
$$= 240 \text{ lb} \cdot \text{ft} \circlearrowleft$$ **Ans.**



(a)

4–78.

Two couples act on the beam as shown. Determine the magnitude of **F** so that the resultant couple moment is 300 lb · ft counterclockwise. Where on the beam does the resultant couple act?



SOLUTION

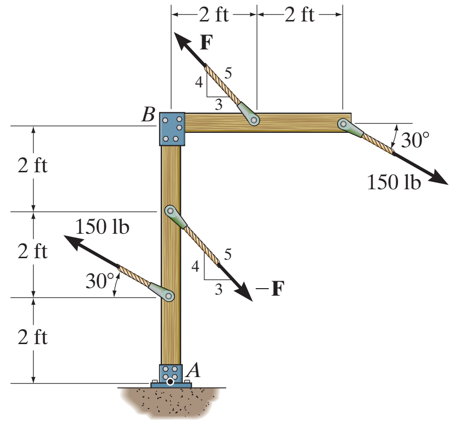
$$\curvearrowleft + (M_C)_R = \frac{3}{5}F(4) + \frac{4}{5}F(1.5) - 200(1.5) = 300$$

$$F = 167 \text{ lb}$$ **Ans.**

Resultant couple can act anywhere. **Ans.**

4–79.

If $F = 200$ lb, determine the resultant couple moment.



SOLUTION

a) By resolving the 150-lb and 200-lb couples into their x and y components, Fig. a, the couple moments $(MC)_1$ and $(MC)_2$ produced by the 150-lb and 200-lb couples, respectively, are given by

$$\circlearrowleft +(M_C)_1 = -150 \cos 30°(4) - 150 \sin 30°(4) = -819.62 \text{ lb}\cdot\text{ft} = 819.62 \text{ lb}\cdot\text{ft} \circlearrowright$$

$$\circlearrowleft +(M_C)_2 = 200\left(\frac{4}{5}\right)(2) + 200\left(\frac{3}{5}\right)(2) = 560 \text{ lb}\cdot\text{ft}$$

Thus, the resultant couple moment can be determined from

$$\circlearrowleft +(M_C)_R = (M_C)_1 + (M_C)_2$$

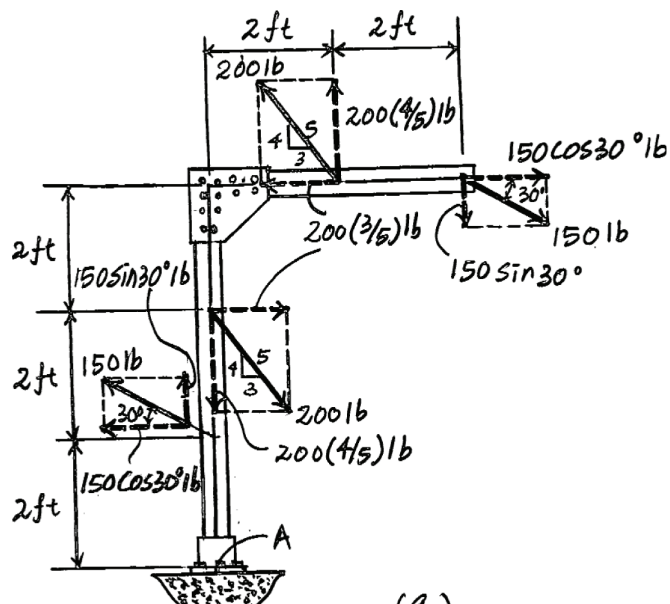
$$= -819.62 + 560 = -259.62 \text{ lb}\cdot\text{ft} = 260 \text{ lb}\cdot\text{ft} \textbf{ (\textit{Clockwise})} \qquad \textbf{Ans.}$$

b) By resolving the 150-lb and 200-lb couples into their x and y components, Fig. a, and summing the moments of these force components algebraically about point A,

$$\circlearrowleft +(M_C)_R = \Sigma M_A\,;\, (M_C)_R = -150 \sin 30°(4) - 150 \cos 30°(6) + 200\left(\frac{4}{5}\right)(2) + 200\left(\frac{3}{5}\right)(6)$$

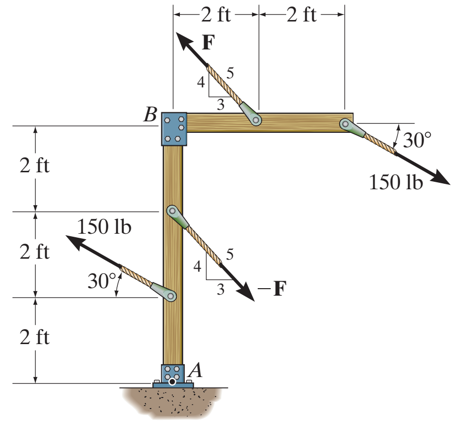
$$- 200\left(\frac{3}{5}\right)(4) + 200\left(\frac{4}{5}\right)(0) + 150 \cos 30°(2) + 150 \sin 30°(0)$$

$$= -259.62 \text{ lb}\cdot\text{ft} = 260 \text{ lb}\cdot\text{ft} \textbf{ (\textit{Clockwise})} \qquad \textbf{Ans.}$$



***4–80.**

Determine the required magnitude of force **F** if the resultant couple moment on the frame is 200 lb · ft, clockwise.



SOLUTION

By resolving **F** and the 150-lb couple into their x and y components, Fig. a, the couple moments $(M_C)_1$ and $(M_C)_2$ produced by **F** and the 150-lb couple, respectively, are given by

$$\circlearrowleft +(M_C)_1 = F\left(\frac{4}{5}\right)(2) + F\left(\frac{3}{5}\right)(2) = 2.8F$$

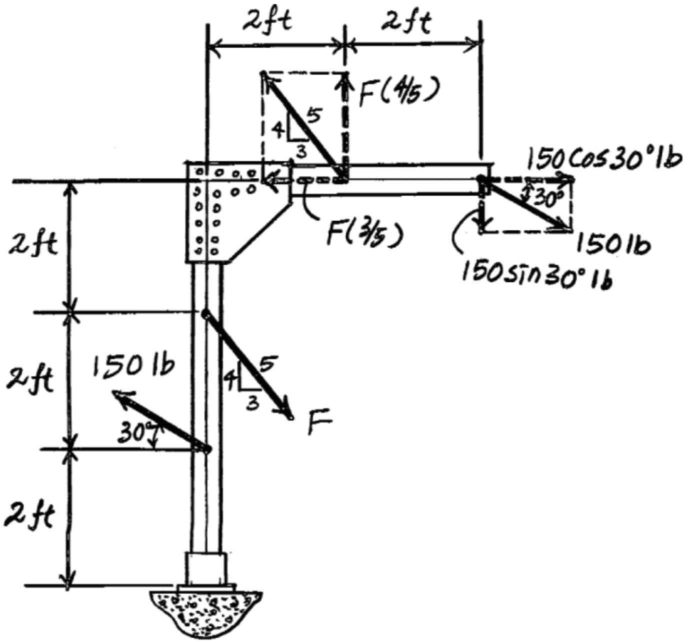
$$\circlearrowleft +(M_C)_2 = -150 \cos 30°(4) - 150 \sin 30°(4) = -819.62 \text{ lb}\cdot\text{ft} = 819.62 \text{ lb}\cdot\text{ft} \circlearrowright$$

The resultant couple moment acting on the beam is required to be 200 lb · ft, clockwise. Thus,

$$\circlearrowleft +(M_C)_R = (M_C)_1 + (M_C)_2$$

$$-200 = 2.8F - 819.62$$

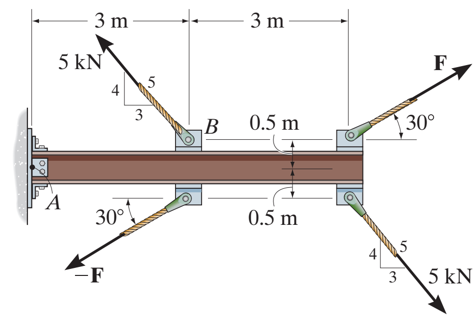
$$F = 221 \text{ lb}$$ **Ans.**



(a)

4–81.

Two couples act on the cantilever beam. If $F = 6\text{ kN}$, determine the resultant couple moment.



SOLUTION

a) By resolving the 6-kN and 5-kN couples into their x and y components, Fig. a, the couple moments $(M_c)_1$ and $(M_c)_2$ produced by the 6-kN and 5-kN couples, respectively, are given by

$$\circlearrowleft +(M_C)_1 = 6 \sin 30°(3) - 6 \cos 30°(0.5 + 0.5) = 3.804 \text{ kN} \cdot \text{m}$$

$$\circlearrowleft +(M_C)_2 = 5\left(\frac{3}{5}\right)(0.5 + 0.5) - 5\left(\frac{4}{5}\right)(3) = -9 \text{ kN} \cdot \text{m}$$

Thus, the resultant couple moment can be determined from

$$(M_C)_R = (M_C)_1 + (M_C)_2$$

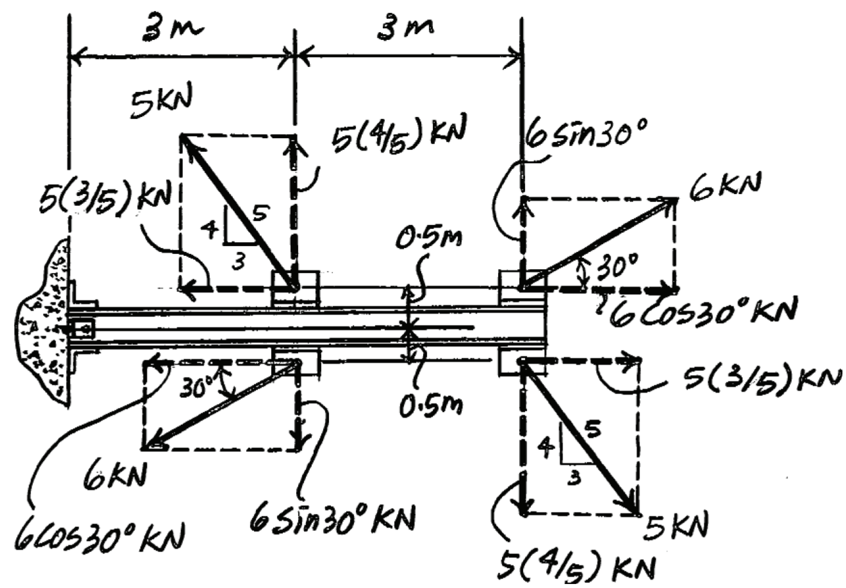
$$= 3.804 - 9 = -5.196 \text{ kN} \cdot \text{m} = 5.20 \text{ kN} \cdot \text{m} \textbf{ (\textit{Clockwise})} \qquad \textbf{Ans.}$$

b) By resolving the 6-kN and 5-kN couples into their x and y components, Fig. a, and summing the moments of these force components about point A, we can write

$$\circlearrowleft +(M_C)_R = \Sigma M_A; \qquad (M_C)_R = 5\left(\frac{3}{5}\right)(0.5) + 5\left(\frac{4}{5}\right)(3) - 6 \cos 30°(0.5) - 6 \sin 30°(3)$$

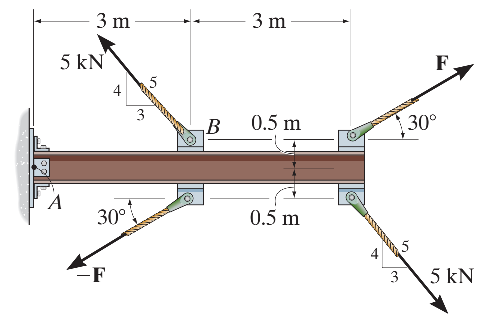
$$+ 6 \sin 30°(6) - 6 \cos 30°(0.5) + 5\left(\frac{3}{5}\right)(0.5) - 5\left(\frac{4}{5}\right)(6)$$

$$= -5.196 \text{ kN} \cdot \text{m} = 5.20 \text{ kN} \cdot \text{m} \textbf{ (\textit{Clockwise})} \qquad \textbf{Ans.}$$



4–82.

Determine the required magnitude of force **F**, if the resultant couple moment on the beam is to be zero.



SOLUTION

By resolving **F** and the 5-kN couple into their x and y components, Fig. a, the couple moments $(M_c)_1$ and $(M_c)_2$ produced by **F** and the 5-kN couple, respectively, are given by

$$\circlearrowleft +(M_C)_1 = F \sin 30°(3) - F \cos 30°(1) = 0.6340F$$

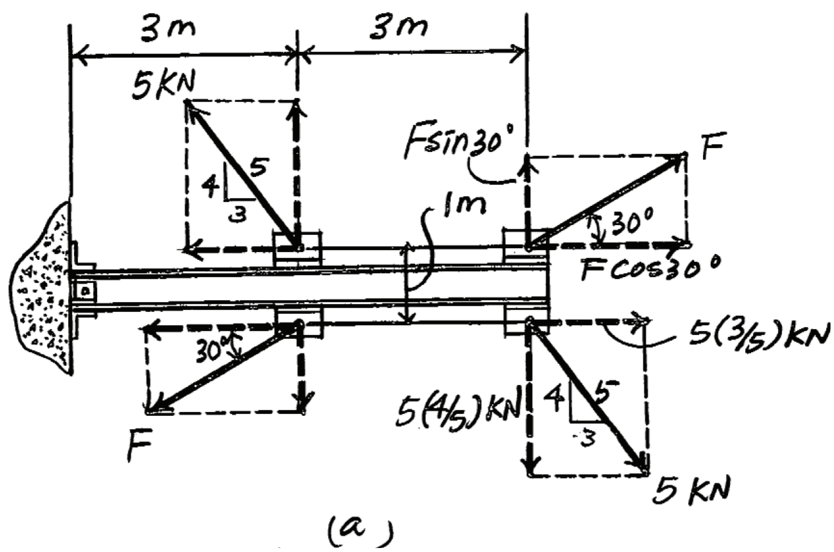
$$\circlearrowleft +(M_C)_2 = 5\left(\frac{3}{5}\right)(1) - 5\left(\frac{4}{5}\right)(3) = -9\text{ kN}\cdot\text{m}$$

The resultant couple moment acting on the beam is required to be zero. Thus,

$$(M_C)_R = (M_C)_1 + (M_C)_2$$

$$0 = 0.6340F - 9$$

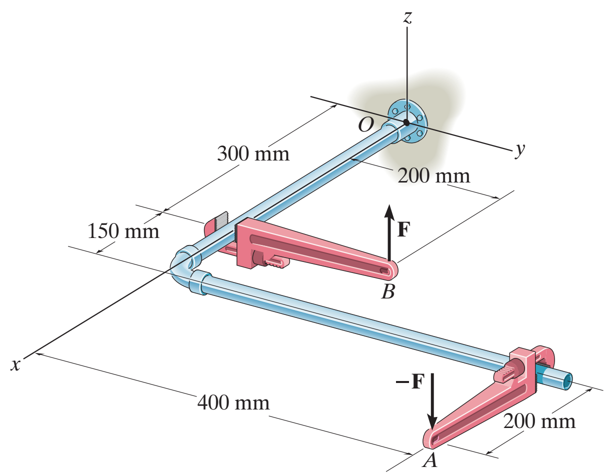
$$F = 14.2\text{ kN}\cdot\text{m}$$ **Ans.**



(a)

4–83.

Express the moment of the couple acting on the pipe assembly in Cartesian vector form. Solve the problem (a) using Eq. 4–13, and (b) summing the moment of each force about point O. Take $\mathbf{F} = \{25\mathbf{k}\}$ N.



SOLUTION

(a) $\mathbf{M}_C = \mathbf{r}_{AB} \times (25\mathbf{k})$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.35 & -0.2 & 0 \\ 0 & 0 & 25 \end{vmatrix}$$

$\mathbf{M}_C = \{-5\mathbf{i} + 8.75\mathbf{j}\}\ \text{N}\cdot\text{m}$ **Ans.**

(b) $\mathbf{M}_C = \mathbf{r}_{OB} \times (25\mathbf{k}) + \mathbf{r}_{OA} \times (-25\mathbf{k})$

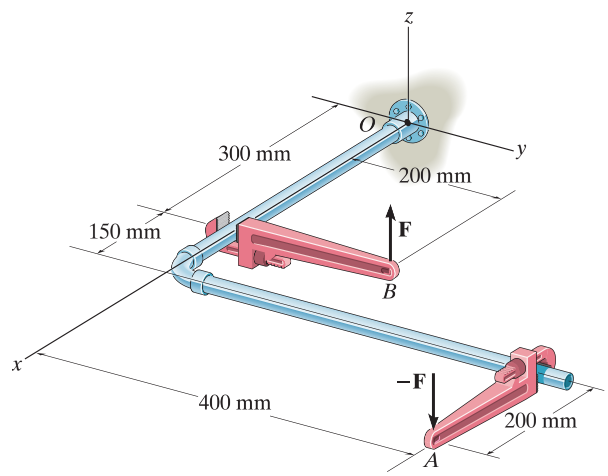
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.3 & 0.2 & 0 \\ 0 & 0 & 25 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.65 & 0.4 & 0 \\ 0 & 0 & -25 \end{vmatrix}$$

$\mathbf{M}_C = (5 - 10)\mathbf{i} + (-7.5 + 16.25)\mathbf{j}$

$\mathbf{M}_C = \{-5\mathbf{i} + 8.75\mathbf{j}\}\ \text{N}\cdot\text{m}$ **Ans.**

***4–84.**

If the couple moment acting on the pipe has a magnitude of 400 N · m, determine the magnitude F of the vertical force applied to each wrench.



SOLUTION

$$\mathbf{M}_C = \mathbf{r}_{AB} \times (F\mathbf{k})$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.35 & -0.2 & 0 \\ 0 & 0 & F \end{vmatrix}$$

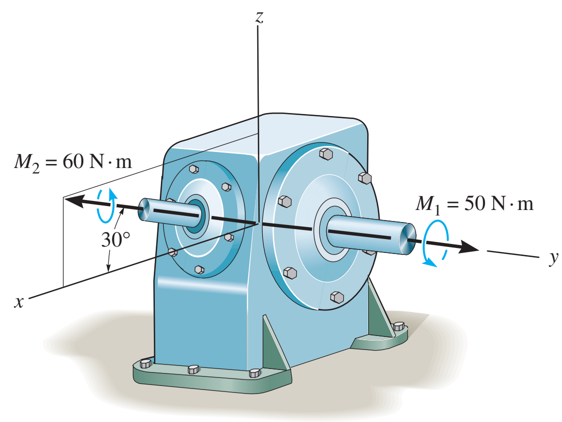
$$\mathbf{M}_C = \{-0.2F\mathbf{i} + 0.35F\mathbf{j}\}\ \text{N}\cdot\text{m}$$

$$M_C = \sqrt{(-0.2F)^2 + (0.35F)^2} = 400$$

$$F = \frac{400}{\sqrt{(-0.2)^2 + (0.35)^2}} = 992\ \text{N}$$ **Ans.**

4–85.

The gear reducer is subjected to the couple moments shown. Determine the resultant couple moment and specify its magnitude and coordinate direction angles.



SOLUTION

Express Each Couple Moment as a Cartesian Vector:

$$\mathbf{M}_1 = \{50\mathbf{j}\}\ \text{N}\cdot\text{m}$$

$$\mathbf{M}_2 = 60(\cos 30°\mathbf{i} + \sin 30°\mathbf{k})\ \text{N}\cdot\text{m} = \{51.96\mathbf{i} + 30.0\mathbf{k}\}\ \text{N}\cdot\text{m}$$

Resultant Couple Moment:

$$\mathbf{M}_R = \Sigma\mathbf{M}; \qquad \mathbf{M}_R = \mathbf{M}_1 + \mathbf{M}_2$$

$$= \{51.96\mathbf{i} + 50.0\mathbf{j} + 30.0\mathbf{k}\}\ \text{N}\cdot\text{m}$$

$$= \{52.0\mathbf{i} + 50\mathbf{j} + 30\mathbf{k}\}\ \text{N}\cdot\text{m}$$ **Ans.**

The magnitude of the resultant couple moment is

$$M_R = \sqrt{51.96^2 + 50.0^2 + 30.0^2}$$

$$= 78.102\ \text{N}\cdot\text{m} = 78.1\ \text{N}\cdot\text{m}$$ **Ans.**

The coordinate direction angles are

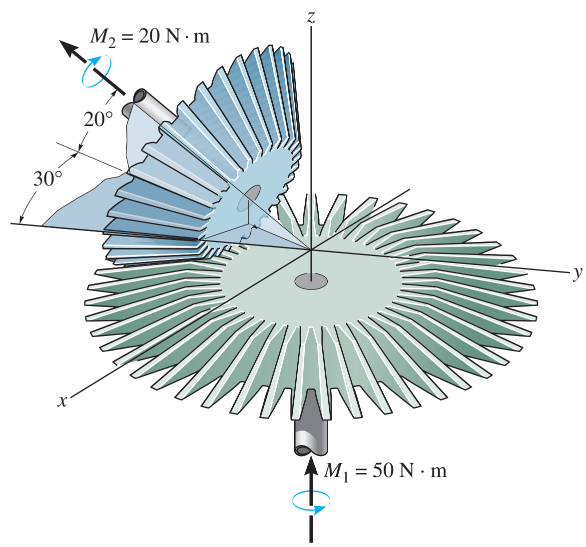
$$\alpha = \cos^{-1}\left(\frac{51.96}{78.102}\right) = 48.3°$$ **Ans.**

$$\beta = \cos^{-1}\left(\frac{50.0}{78.102}\right) = 50.2°$$ **Ans.**

$$\gamma = \cos^{-1}\left(\frac{30.0}{78.102}\right) = 67.4°$$ **Ans.**

4–86.

The meshed gears are subjected to the couple moments shown. Determine the magnitude of the resultant couple moment and specify its coordinate direction angles.



SOLUTION

$$\mathbf{M}_1 = \{50\mathbf{k}\}\ \text{N}\cdot\text{m}$$

$$\mathbf{M}_2 = 20(-\cos 20^\circ \sin 30^\circ \mathbf{i} - \cos 20^\circ \cos 30^\circ \mathbf{j} + \sin 20^\circ \mathbf{k})\ \text{N}\cdot\text{m}$$

$$= \{-9.397\mathbf{i} - 16.276\mathbf{j} + 6.840\mathbf{k}\}\ \text{N}\cdot\text{m}$$

Resultant Couple Moment:

$$\mathbf{M}_R = \Sigma\mathbf{M}; \qquad \mathbf{M}_R = \mathbf{M}_1 + \mathbf{M}_2$$

$$= \{-9.397\mathbf{i} - 16.276\mathbf{j} + (50 + 6.840)\mathbf{k}\}\ \text{N}\cdot\text{m}$$

$$= \{-9.397\mathbf{i} - 16.276\mathbf{j} + 56.840\mathbf{k}\}\ \text{N}\cdot\text{m}$$

The magnitude of the resultant couple moment is

$$M_R = \sqrt{(-9.397)^2 + (-16.276)^2 + (56.840)^2}$$

$$= 59.867\ \text{N}\cdot\text{m} = 59.9\ \text{N}\cdot\text{m} \qquad \textbf{Ans.}$$

The coordinate direction angles are

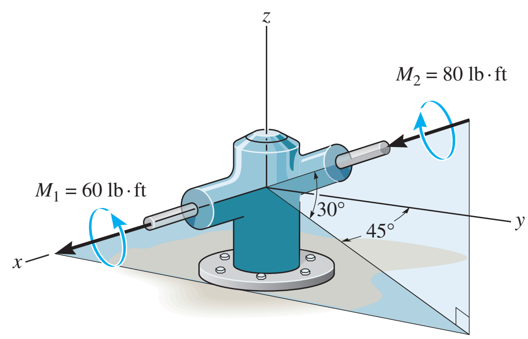
$$\alpha = \cos^{-1}\left(\frac{-9.397}{59.867}\right) = 99.0^\circ \qquad \textbf{Ans.}$$

$$\beta = \cos^{-1}\left(\frac{-16.276}{59.867}\right) = 106^\circ \qquad \textbf{Ans.}$$

$$\gamma = \cos^{-1}\left(\frac{56.840}{59.867}\right) = 18.3^\circ \qquad \textbf{Ans.}$$

4–87.

The gear reducer is subject to the couple moments shown. Determine the resultant couple moment and specify its magnitude and coordinate direction angles.



SOLUTION

Express Each:

$$\mathbf{M}_1 = \{60\mathbf{i}\}\ \text{lb}\cdot\text{ft}$$

$$\mathbf{M}_2 = 80(-\cos 30^\circ \sin 45^\circ \mathbf{i} - \cos 30^\circ \cos 45^\circ \mathbf{j} - \sin 30^\circ \mathbf{k})\ \text{lb}\cdot\text{ft}$$

$$= \{-48.99\mathbf{i} - 48.99\mathbf{j} - 40.0\mathbf{k}\}\ \text{lb}\cdot\text{ft}$$

Resultant Couple Moment:

$$\mathbf{M}_R = \Sigma\mathbf{M}; \qquad \mathbf{M}_R = \mathbf{M}_1 + \mathbf{M}_2$$

$$= \{(60 - 48.99)\mathbf{i} - 48.99\mathbf{j} - 40.0\mathbf{k}\}\ \text{lb}\cdot\text{ft}$$

$$= \{11.01\mathbf{i} - 48.99\mathbf{j} - 40.0\mathbf{k}\}\ \text{lb}\cdot\text{ft}$$

$$= \{11.0\mathbf{i} - 49.0\mathbf{j} - 40.0\mathbf{k}\}\ \text{lb}\cdot\text{ft}$$ **Ans.**

The magnitude of the resultant couple moment is

$$M_R = \sqrt{11.01^2 + (-48.99)^2 + (-40.0)^2}$$

$$= 64.20\ \text{lb}\cdot\text{ft} = 64.2\ \text{lb}\cdot\text{ft}$$ **Ans.**

The coordinate direction angles are

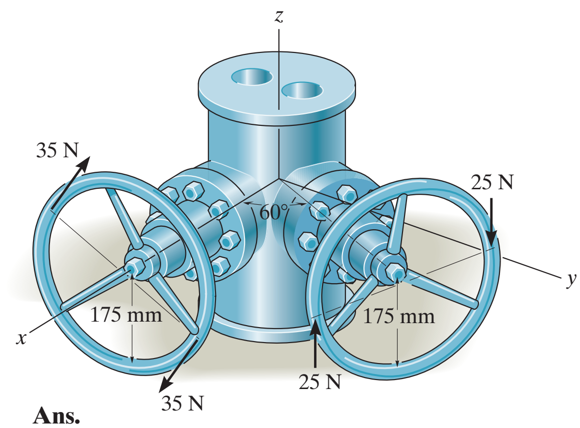
$$\alpha = \cos^{-1}\left(\frac{11.01}{64.20}\right) = 80.1^\circ$$ **Ans.**

$$\beta = \cos^{-1}\left(\frac{-48.99}{64.20}\right) = 140^\circ$$ **Ans.**

$$\gamma = \cos^{-1}\left(\frac{-40.0}{64.20}\right) = 129^\circ$$ **Ans.**

***4–88.**

A couple acts on each of the handles of the minidual valve. Determine the magnitude and coordinate direction angles of the resultant couple moment.



SOLUTION

$M_x = -35(0.35) - 25(0.35)\cos 60° = -16.625$

$M_y = -25(0.35)\sin 60° = -7.5777\ \text{N}\cdot\text{m}$

$|M| = \sqrt{(-16.625)^2 + (-7.5777)^2} = 18.2705 = 18.3\ \text{N}\cdot\text{m}$ **Ans.**

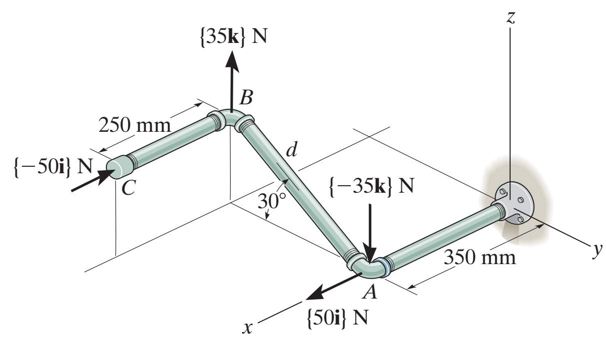
$\alpha = \cos^{-1}\left(\dfrac{-16.625}{18.2705}\right) = 155°$ **Ans.**

$\beta = \cos^{-1}\left(\dfrac{-7.5777}{18.2705}\right) = 115°$ **Ans.**

$\gamma = \cos^{-1}\left(\dfrac{0}{18.2705}\right) = 90°$ **Ans.**

4–89.

Determine the resultant couple moment of the two couples that act on the pipe assembly. The distance from A to B is $d = 400$ mm. Express the result as a Cartesian vector.



SOLUTION

Vector Analysis

Position Vector:

$$\mathbf{r}_{AB} = \{(0.35 - 0.35)\mathbf{i} + (-0.4\cos 30^\circ - 0)\mathbf{j} + (0.4\sin 30^\circ - 0)\mathbf{k}\}\ \text{m}$$

$$= \{-0.3464\mathbf{j} + 0.20\mathbf{k}\}\ \text{m}$$

Couple Moments: With $\mathbf{F}_1 = \{35\mathbf{k}\}$ N and $\mathbf{F}_2 = \{-50\mathbf{i}\}$ N, applying Eq. 4–15, we have

$$(\mathbf{M}_C)_1 = \mathbf{r}_{AB} \times \mathbf{F}_1$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -0.3464 & 0.20 \\ 0 & 0 & 35 \end{vmatrix} = \{-12.12\mathbf{i}\}\ \text{N}\cdot\text{m}$$

$$(\mathbf{M}_C)_2 = \mathbf{r}_{AB} \times \mathbf{F}_2$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -0.3464 & 0.20 \\ -50 & 0 & 0 \end{vmatrix} = \{-10.0\mathbf{j} - 17.32\mathbf{k}\}\ \text{N}\cdot\text{m}$$

Resultant Couple Moment:

$$\mathbf{M}_R = \Sigma\mathbf{M}; \qquad \mathbf{M}_R = (\mathbf{M}_C)_1 + (\mathbf{M}_C)_2$$

$$= \{-12.1\mathbf{i} - 10.0\mathbf{j} - 17.3\mathbf{k}\}\text{N}\cdot\text{m} \qquad \textbf{Ans.}$$

Scalar Analysis: Summing moments about x, y, and z axes, we have

$$(M_R)_x = \Sigma M_x; \qquad (M_R)_x = -35(0.4\cos 30^\circ) = -12.12\ \text{N}\cdot\text{m}$$

$$(M_R)_y = \Sigma M_y; \qquad (M_R)_y = -50(0.4\sin 30^\circ) = -10.0\ \text{N}\cdot\text{m}$$

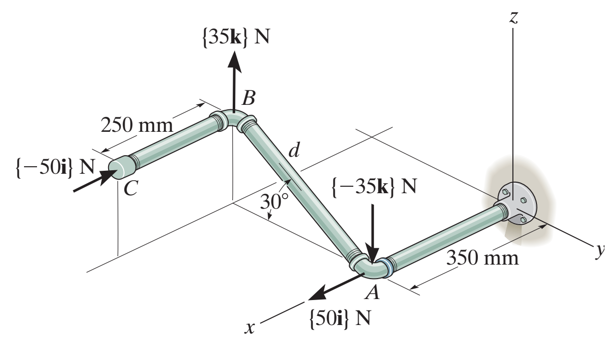
$$(M_R)_z = \Sigma M_z; \qquad (M_R)_z = -50(0.4\cos 30^\circ) = -17.32\ \text{N}\cdot\text{m}$$

Express $\mathbf{M}_R$ as a Cartesian vector, we have

$$\mathbf{M}_R = \{-12.1\mathbf{i} - 10.0\mathbf{j} - 17.3\mathbf{k}\}\ \text{N}\cdot\text{m}$$

4–90.

Determine the distance d between A and B so that the resultant couple moment has a magnitude of $M_R = 20\ \text{N}\cdot\text{m}$.



SOLUTION

Position Vector:

$$\mathbf{r}_{AB} = \{(0.35 - 0.35)\mathbf{i} + (-d\cos 30° - 0)\mathbf{j} + (d\sin 30° - 0)\mathbf{k}\}\ \text{m}$$

$$= \{-0.8660d\,\mathbf{j} + 0.50d\,\mathbf{k}\}\ \text{m}$$

Couple Moments: With $\mathbf{F}_1 = \{35\mathbf{k}\}$ N and $\mathbf{F}_2 = \{-50\mathbf{i}\}$ N, applying Eq. 4–15, we have

$$(\mathbf{M}_C)_1 = \mathbf{r}_{AB} \times \mathbf{F}_1$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -0.8660d & 0.50d \\ 0 & 0 & 35 \end{vmatrix} = \{-30.31d\,\mathbf{i}\}\ \text{N}\cdot\text{m}$$

$$(\mathbf{M}_C)_2 = \mathbf{r}_{AB} \times \mathbf{F}_2$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -0.8660d & 0.50d \\ -50 & 0 & 0 \end{vmatrix} = \{-25.0d\,\mathbf{j} - 43.30d\,\mathbf{k}\}\ \text{N}\cdot\text{m}$$

Resultant Couple Moment:

$$\mathbf{M}_R = \Sigma\mathbf{M}; \qquad \mathbf{M}_R = (\mathbf{M}_C)_1 + (\mathbf{M}_C)_2$$

$$= \{-30.31d\,\mathbf{i} - 25.0d\,\mathbf{j} - 43.30d\,\mathbf{k}\}\ \text{N}\cdot\text{m}$$

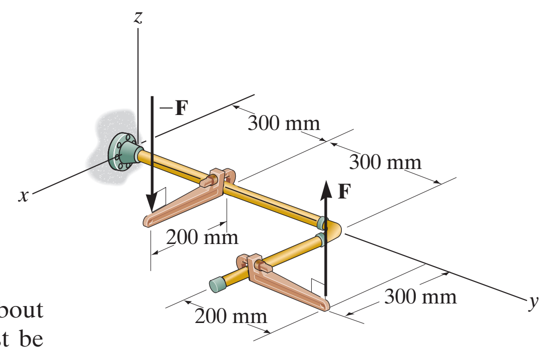
The magnitude of $\mathbf{M}_R$ is $20\ \text{N}\cdot\text{m}$, thus

$$20 = \sqrt{(-30.31d)^2 + (-25.0d)^2 + (43.30d)^2}$$

$$d = 0.3421\ \text{m} = 342\ \text{mm}$$ **Ans.**

4–91.

If $F = 80$ N, determine the magnitude and coordinate direction angles of the couple moment. The pipe assembly lies in the x–y plane.



SOLUTION

It is easiest to find the couple moment of **F** by taking the moment of **F** or –**F** about point A or B, respectively, Fig. a. Here the position vectors $\mathbf{r}_{AB}$ and $\mathbf{r}_{BA}$ must be determined first.

$$\mathbf{r}_{AB} = (0.3 - 0.2)\mathbf{i} + (0.8 - 0.3)\mathbf{j} + (0 - 0)\mathbf{k} = [0.1\mathbf{i} + 0.5\mathbf{j}]\text{ m}$$

$$\mathbf{r}_{BA} = (0.2 - 0.3)\mathbf{i} + (0.3 - 0.8)\mathbf{j} + (0 - 0)\mathbf{k} = [-0.1\mathbf{i} - 0.5\mathbf{j}]\text{ m}$$

The force vectors **F** and –**F** can be written as

$$\mathbf{F} = \{80\,\mathbf{k}\}\text{ N and } -\mathbf{F} = [-80\,\mathbf{k}]\text{ N}$$

Thus, the couple moment of **F** can be determined from

$$\mathbf{M}_c = \mathbf{r}_{AB} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.1 & 0.5 & 0 \\ 0 & 0 & 80 \end{vmatrix} = [40\mathbf{i} - 8\mathbf{j}]\text{ N}\cdot\text{m}$$

or

$$\mathbf{M}_c = \mathbf{r}_{BA} \times -\mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.1 & -0.5 & 0 \\ 0 & 0 & -80 \end{vmatrix} = [40\mathbf{i} - 8\mathbf{j}]\text{ N}\cdot\text{m}$$

The magnitude of M_c is given by

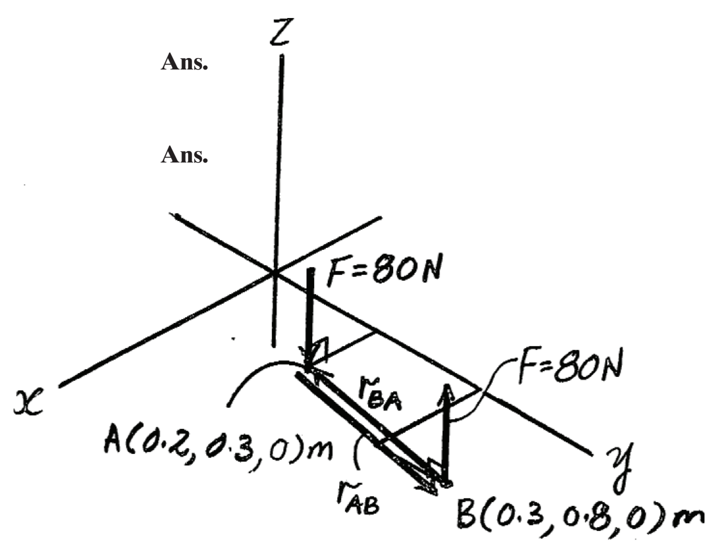
$$M_c = \sqrt{M_x{}^2 + M_y{}^2 + M_z{}^2} = \sqrt{40^2 + (-8)^2 + 0^2} = 40.79\text{ N}\cdot\text{m} = 40.8\text{ N}\cdot\text{m} \qquad \textbf{Ans.}$$

The coordinate angles of $\mathbf{M}_c$ are

$$\alpha = \cos^{-1}\left(\frac{M_x}{M}\right) = \cos\left(\frac{40}{40.79}\right) = 11.3^\circ \qquad \textbf{Ans.}$$

$$\beta = \cos^{-1}\left(\frac{M_y}{M}\right) = \cos\left(\frac{-8}{40.79}\right) = 101^\circ \qquad \textbf{Ans.}$$

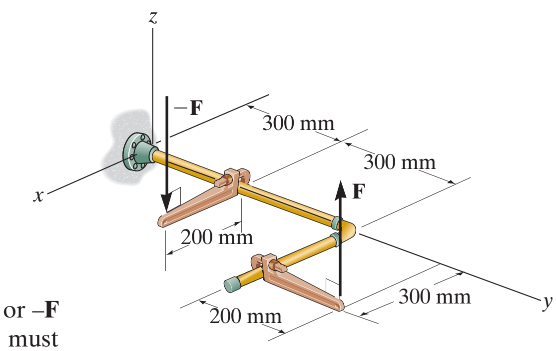
$$\gamma = \cos^{-1}\left(\frac{M_z}{M}\right) = \cos\left(\frac{0}{40.79}\right) = 90^\circ \qquad \textbf{Ans.}$$



(a)

***4–92.**

If the magnitude of the couple moment acting on the pipe assembly is 50 N · m, determine the magnitude of the couple forces applied to each wrench. The pipe assembly lies in the x–y plane.



SOLUTION

It is easiest to find the couple moment of **F** by taking the moment of either **F** or –**F** about point A or B, respectively, Fig. a. Here the position vectors $\mathbf{r}_{AB}$ and $\mathbf{r}_{BA}$ must be determined first.

$$\mathbf{r}_{AB} = (0.3 - 0.2)\mathbf{i} + (0.8 - 0.3)\mathbf{j} + (0 - 0)\mathbf{k} = [0.1\mathbf{i} + 0.5\mathbf{j}]\text{ m}$$

$$\mathbf{r}_{BA} = (0.2 - 0.3)\mathbf{i} + (0.3 - 0.8)\mathbf{j} + (0 - 0)\mathbf{k} = [-0.1\mathbf{i} - 0.5\mathbf{j}]\text{ m}$$

The force vectors **F** and –**F** can be written as
$\mathbf{F} = \{F\mathbf{k}\}$ N and $-\mathbf{F} = [-F\mathbf{k}]$N

Thus, the couple moment of **F** can be determined from

$$\mathbf{M}_c = \mathbf{r}_{AB} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.1 & 0.5 & 0 \\ 0 & 0 & F \end{vmatrix} = 0.5F\mathbf{i} - 0.1F\mathbf{j}$$

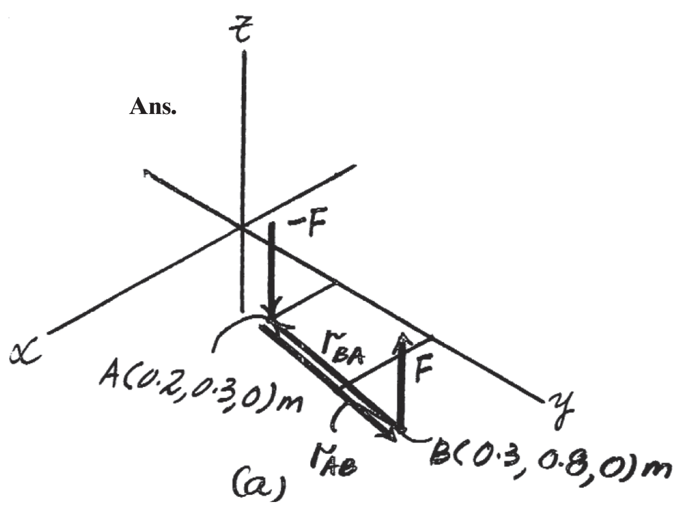
The magnitude of M_c is given by

$$M_c = \sqrt{M_x{}^2 + M_y{}^2 + M_z{}^2} = \sqrt{(0.5F)^2 + (0.1F)^2 + 0^2} = 0.5099F$$

Since M_c is required to equal 50 N · m,

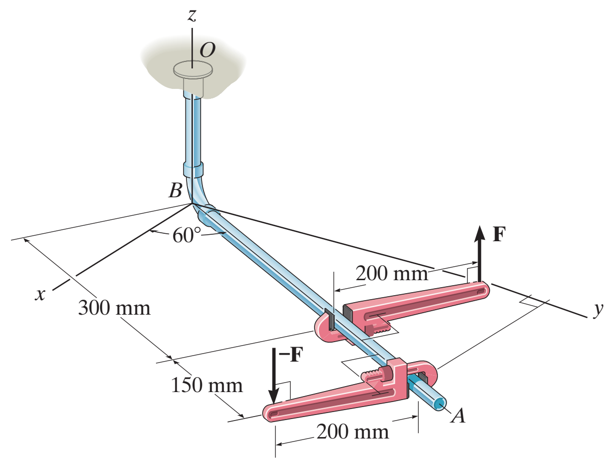
$$50 = 0.5099F$$

$$F = 98.1\text{ N}$$ **Ans.**



4–93.

If $\mathbf{F} = \{100\mathbf{k}\}$ N, determine the couple moment that acts on the assembly. Express the result as a Cartesian vector. Member BA lies in the x–y plane.

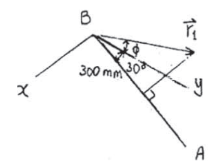


SOLUTION

$$\phi = \tan^{-1}\left(\frac{2}{3}\right) - 30° = 3.69°$$

$$\mathbf{r}_1 = \{-360.6 \sin 3.69°\mathbf{i} + 360.6 \cos 3.69°\mathbf{j}\}$$

$$= \{-23.21\mathbf{i} + 359.8\mathbf{j}\} \text{ mm}$$

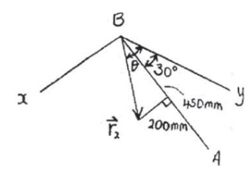


$$\theta = \tan^{-1}\left(\frac{2}{4.5}\right) + 30° = 53.96°$$

$$\mathbf{r}_2 = \{492.4 \sin 53.96°\mathbf{i} + 492.4 \cos 53.96°\mathbf{j}\}$$

$$= \{398.2\mathbf{i} + 289.7\mathbf{j}\} \text{ mm}$$

$$\mathbf{M}_c = (\mathbf{r}_1 - \mathbf{r}_2) \times \mathbf{F}$$

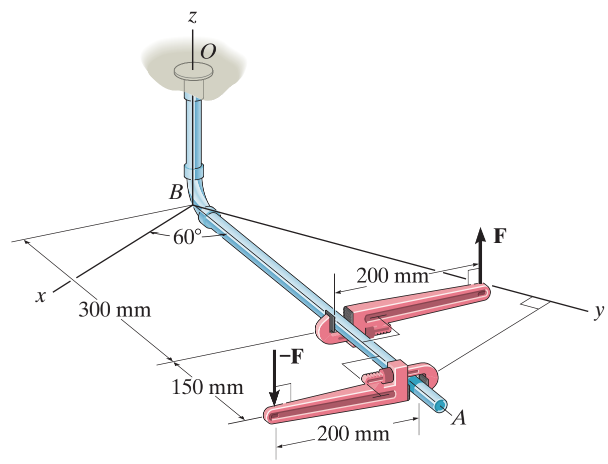


$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -421.4 & 70.10 & 0 \\ 0 & 0 & 100 \end{vmatrix}$$

$$\mathbf{M}_c = \{7.01\mathbf{i} + 42.1\mathbf{j}\} \text{ N} \cdot \text{m}$$ **Ans.**

4–94.

If the magnitude of the resultant couple moment is 15 N · m, determine the magnitude F of the forces applied to the wrenches.



SOLUTION

$$\phi = \tan^{-1}\left(\frac{2}{3}\right) - 30^\circ = 3.69^\circ$$

$$\mathbf{r}_1 = \{-360.6 \sin 3.69^\circ \mathbf{i} + 360.6 \cos 3.69^\circ \mathbf{j}\}$$

$$= \{-23.21\mathbf{i} + 359.8\mathbf{j}\} \text{ mm}$$

$$\theta = \tan^{-1}\left(\frac{2}{4.5}\right) + 30^\circ = 53.96^\circ$$

$$\mathbf{r}_2 = \{492.4 \sin 53.96^\circ \mathbf{i} + 492.4 \cos 53.96^\circ \mathbf{j}\}$$

$$= \{398.2\mathbf{i} + 289.7\mathbf{j}\} \text{ mm}$$

$$\mathbf{M}_c = (\mathbf{r}_1 - \mathbf{r}_2) \times \mathbf{F}$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -421.4 & 70.10 & 0 \\ 0 & 0 & F \end{vmatrix}$$

$$\mathbf{M}_c = \{0.0701F\mathbf{i} + 0.421F\mathbf{j}\} \text{ N} \cdot \text{m}$$

$$M_c = \sqrt{(0.0701F)^2 + (0.421F)^2} = 15$$

$$F = \frac{15}{\sqrt{(0.0701)^2 + (0.421)^2}} = 35.1 \text{ N}$$ **Ans.**

Also, align y' axis along BA.

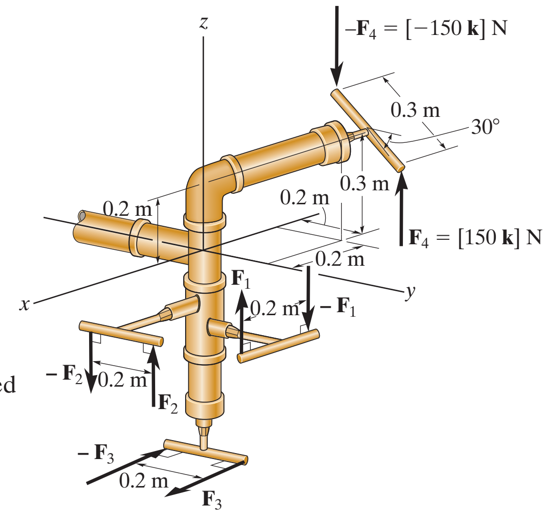
$$\mathbf{M}_c = -F(0.15)\mathbf{i}' + F(0.4)\mathbf{j}'$$

$$15 = \sqrt{(F(-0.15))^2 + (F(0.4))^2}$$

$$F = 35.1 \text{ N}$$ **Ans.**

4–95.

If $F_1 = 100\,\text{N}$, $F_2 = 120\,\text{N}$ and $F_3 = 80\,\text{N}$, determine the magnitude and coordinate direction angles of the resultant couple moment.



SOLUTION

Couple Moment: The position vectors $\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3$, and $\mathbf{r}_4$, Fig. *a*, must be determined first.

$\mathbf{r}_1 = \{0.2\mathbf{i}\}$ m $\qquad \mathbf{r}_2 = \{0.2\mathbf{j}\}$ m $\qquad \mathbf{r}_3 = \{0.2\mathbf{j}\}$ m

From the geometry of Figs. *b* and *c*, we obtain

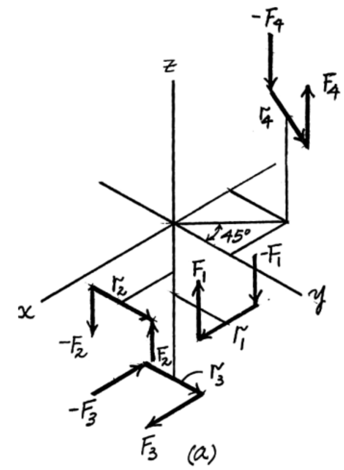
$$\mathbf{r}_4 = 0.3\cos 30^\circ \cos 45^\circ \mathbf{i} + 0.3 \cos 30^\circ \sin 45^\circ \mathbf{j} - 0.3 \sin 30^\circ \mathbf{k}$$
$$= \{0.1837\mathbf{i} + 0.1837\mathbf{j} - 0.15\mathbf{k}\}\ \text{m}$$

The force vectors $\mathbf{F}_1, \mathbf{F}_2$, and $\mathbf{F}_3$ are given by

$\mathbf{F}_1 = \{100\mathbf{k}\}$ N $\qquad \mathbf{F}_2 = \{120\mathbf{k}\}$ N $\qquad \mathbf{F}_3 = \{80\mathbf{i}\}$ N

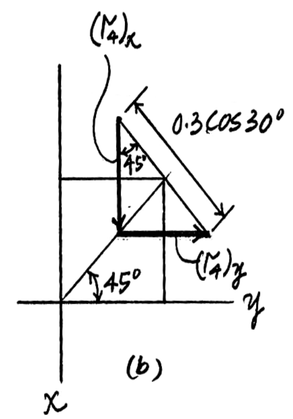
Thus,

$$\mathbf{M}_1 = \mathbf{r}_1 \times \mathbf{F}_1 = (0.2\mathbf{i}) \times (100\mathbf{k}) = \{-20\mathbf{j}\}\ \text{N}\cdot\text{m}$$
$$\mathbf{M}_2 = \mathbf{r}_2 \times \mathbf{F}_2 = (0.2\mathbf{j}) \times (120\mathbf{k}) = \{24\mathbf{i}\}\ \text{N}\cdot\text{m}$$
$$\mathbf{M}_3 = \mathbf{r}_3 \times \mathbf{F}_3 = (0.2\mathbf{j}) \times (80\mathbf{i}) = \{-16\mathbf{k}\}\ \text{N}\cdot\text{m}$$
$$\mathbf{M}_4 = \mathbf{r}_4 \times \mathbf{F}_4 = (0.1837\mathbf{i} + 0.1837\mathbf{j} - 0.15\mathbf{k}) \times (150\mathbf{k}) = \{27.56\mathbf{i} - 27.56\mathbf{j}\}\ \text{N}\cdot\text{m}$$



Resultant Moment: The resultant couple moment is given by

$$(\mathbf{M}_c)_R = \Sigma \mathbf{M}_c; \qquad (\mathbf{M}_c)_R = \mathbf{M}_1 + \mathbf{M}_2 + \mathbf{M}_3 + \mathbf{M}_4$$
$$= (-20\mathbf{j}) + (24\mathbf{i}) + (-16\mathbf{k}) + (27.56\mathbf{i} - 27.56\mathbf{j})$$
$$= \{51.56\mathbf{i} - 47.56\mathbf{j} - 16\mathbf{k}\}\ \text{N}\cdot\text{m}$$



The magnitude of the couple moment is

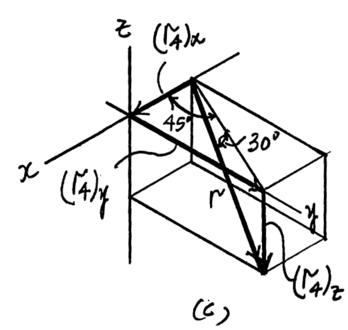
$$(\mathbf{M}_c)_R = \sqrt{[(\mathbf{M}_c)_R]_x^{\,2} + [(\mathbf{M}_c)_R]_y^{\,2} + [(\mathbf{M}_c)_R]_z^{\,2}}$$
$$= \sqrt{(51.56)^2 + (-47.56)^2 + (-16)^2}$$
$$= 71.94\ \text{N}\cdot\text{m} = 71.9\ \text{N}\cdot\text{m} \qquad \textbf{Ans.}$$

The coordinate angles of $(\mathbf{M}_c)_R$ are

$$\alpha = \cos^{-1}\left(\frac{[(M_c)_R]_x}{(M_c)_R}\right) = \cos\left(\frac{51.56}{71.94}\right) = 44.2^\circ \qquad \textbf{Ans.}$$

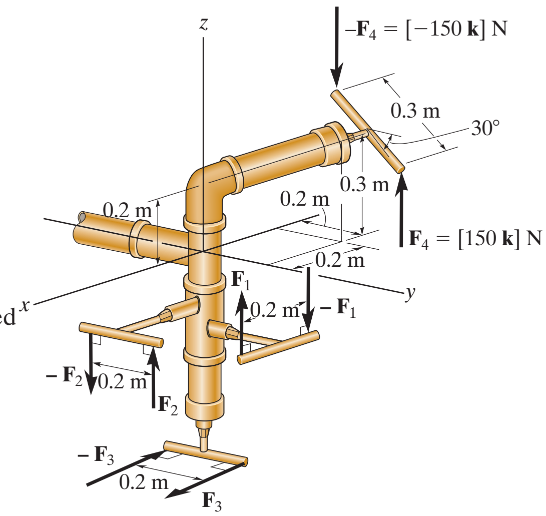
$$\beta = \cos^{-1}\left(\frac{[(M_c)_R]_y}{(M_c)_R}\right) = \cos\left(\frac{-47.56}{71.94}\right) = 131^\circ \qquad \textbf{Ans.}$$

$$\gamma = \cos^{-1}\left(\frac{[(M_c)_R]_z}{(M_c)_R}\right) = \cos\left(\frac{-16}{71.94}\right) = 103^\circ \qquad \textbf{Ans.}$$



***4–96.**

Determine the required magnitude of $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ so that the resultant couple moment is $(M_c)_R = [50\,\mathbf{i} - 45\,\mathbf{j} - 20\,\mathbf{k}]$ N·m.



SOLUTION

Couple Moment: The position vectors $\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3$, and $\mathbf{r}_4$, Fig. *a*, must be determined first.

$\mathbf{r}_1 = \{0.2\mathbf{i}\}$ m $\qquad \mathbf{r}_2 = \{0.2\mathbf{j}\}$ m $\qquad \mathbf{r}_3 = \{0.2\mathbf{j}\}$ m

From the geometry of Figs. *b* and *c*, we obtain

$$\mathbf{r}_4 = 0.3\cos 30°\cos 45°\mathbf{i} + 0.3\cos 30°\sin 45°\mathbf{j} - 0.3\sin 30°\mathbf{k}$$
$$= \{0.1837\mathbf{i} + 0.1837\mathbf{j} - 0.15\mathbf{k}\}\text{ m}$$

The force vectors $\mathbf{F}_1, \mathbf{F}_2$, and $\mathbf{F}_3$ are given by

$\mathbf{F}_1 = F_1\mathbf{k} \qquad \mathbf{F}_2 = F_2\mathbf{k} \qquad \mathbf{F}_3 = F_3\mathbf{i}$

Thus,

$$\mathbf{M}_1 = \mathbf{r}_1 \times \mathbf{F}_1 = (0.2\mathbf{i}) \times (F_1\mathbf{k}) = -0.2\,F_1\mathbf{j}$$
$$\mathbf{M}_2 = \mathbf{r}_2 \times \mathbf{F}_2 = (0.2\mathbf{j}) \times (F_2\mathbf{k}) = 0.2\,F_2\mathbf{i}$$
$$\mathbf{M}_3 = \mathbf{r}_3 \times \mathbf{F}_3 = (0.2\mathbf{j}) \times (F_3\mathbf{i}) = -0.2\,F_3\mathbf{k}$$
$$\mathbf{M}_4 = \mathbf{r}_4 \times \mathbf{F}_4 = (0.1837\mathbf{i} + 0.1837\mathbf{j} - 0.15\mathbf{k}) \times (150\mathbf{k}) = \{27.56\mathbf{i} - 27.56\mathbf{j}\}\text{ N}\cdot\text{m}$$

Resultant Moment: The resultant couple moment required to equal $(\mathbf{M}_c)_R = \{50\mathbf{i} - 45\mathbf{j} - 20\mathbf{k}\}$ N·m. Thus,

$(\mathbf{M}_c)_R = \Sigma\mathbf{M}_c; \qquad (\mathbf{M}_c)_R = \mathbf{M}_1 + \mathbf{M}_2 + \mathbf{M}_3 + \mathbf{M}_4$

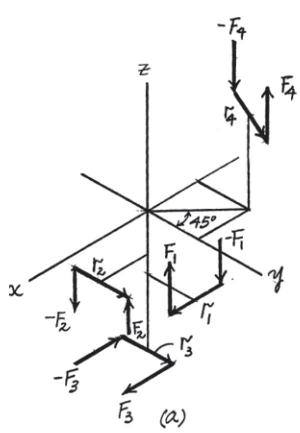
$$50\mathbf{i} - 45\mathbf{j} - 20\mathbf{k} = (-0.2F_1\mathbf{j}) + (0.2F_2\mathbf{i}) + (-0.2F_3\mathbf{k}) + (27.56\mathbf{i} - 27.56\mathbf{j})$$
$$50\mathbf{i} - 45\mathbf{j} - 20\mathbf{k} = (0.2F_2 + 27.56)\mathbf{i} + (-0.2F_1 - 27.56)\mathbf{j} - 0.2F_3\mathbf{k}$$

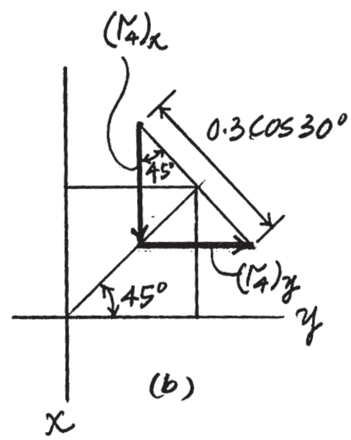
Equating the **i**, **j**, and **k** components yields

$50 = 0.2F_2 + 27.56 \qquad F_2 = 112\text{ N}$ **Ans.**

$-45 = -0.2F_1 - 27.56 \qquad F_1 = 87.2\text{ N}$ **Ans.**

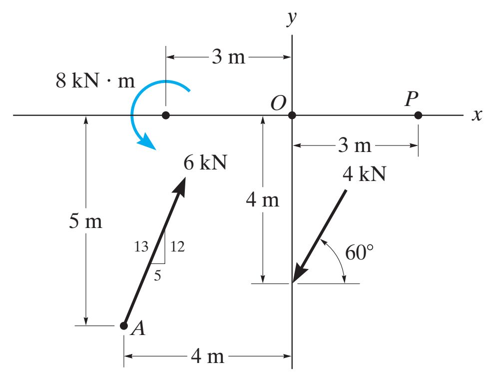
$-20 = -0.2F_3 \qquad F_3 = 100\text{ N}$ **Ans.**





4–97.

Replace the force and couple system by an equivalent force and couple moment at point O.



SOLUTION

$$\xrightarrow{+} \Sigma F_{Rx} = \Sigma F_x; \qquad F_{Rx} = 6\left(\frac{5}{13}\right) - 4\cos 60^\circ$$

$$= 0.30769 \text{ kN}$$

$$+\uparrow \Sigma F_{Ry} = \Sigma F_y; \qquad F_{Ry} = 6\left(\frac{12}{13}\right) - 4\sin 60^\circ$$

$$= 2.0744 \text{ kN}$$

$$F_R = \sqrt{(0.30769)^2 + (2.0744)^2} = 2.10 \text{ kN}$$ **Ans.**

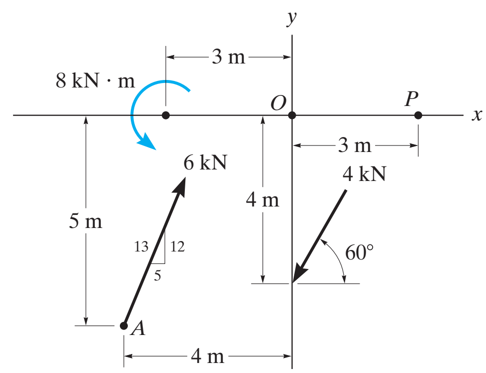
$$\theta = \tan^{-1}\left[\frac{2.0744}{0.30769}\right] = 81.6^\circ \measuredangle$$ **Ans.**

$$\circlearrowleft + M_O = \Sigma M_O; \qquad M_O = 8 - 6\left(\frac{12}{13}\right)(4) + 6\left(\frac{5}{13}\right)(5) - 4\cos 60^\circ(4)$$

$$M_O = -10.62 \text{ kN}\cdot\text{m} = 10.6 \text{ kN}\cdot\text{m} \circlearrowright$$ **Ans.**

4–98.

Replace the force and couple system by an equivalent force and couple moment at point P.



SOLUTION

$$\overset{+}{\rightarrow} \Sigma F_{Rx} = \Sigma F_x; \qquad F_{Rx} = 6\left(\frac{5}{13}\right) - 4\cos 60^\circ$$

$$= 0.30769 \text{ kN}$$

$$+\uparrow \Sigma F_{Ry} = \Sigma F_y; \qquad F_{Ry} = 6\left(\frac{12}{13}\right) - 4\sin 60^\circ$$

$$= 2.0744 \text{ kN}$$

$$F_R = \sqrt{(0.30769)^2 + (2.0744)^2} = 2.10 \text{ kN} \qquad \textbf{Ans.}$$

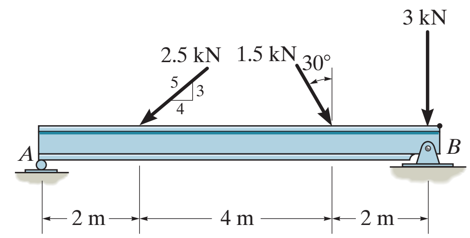
$$\theta = \tan^{-1}\left[\frac{2.0744}{0.30769}\right] = 81.6^\circ \measuredangle \qquad \textbf{Ans.}$$

$$\circlearrowleft + M_P = \Sigma M_P; \qquad M_P = 8 - 6\left(\frac{12}{13}\right)(7) + 6\left(\frac{5}{13}\right)(5) - 4\cos 60^\circ(4) + 4\sin 60^\circ(3)$$

$$M_P = -16.8 \text{ kN}\cdot\text{m} = 16.8 \text{ kN}\cdot\text{m} \circlearrowright \qquad \textbf{Ans.}$$

4–99.

Replace the force system acting on the beam by an equivalent force and couple moment at point A.



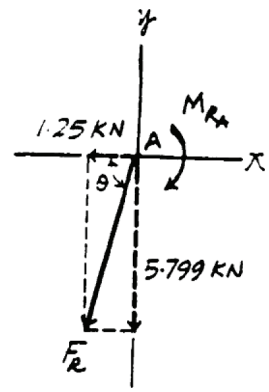
SOLUTION

$$\overset{+}{\rightarrow} F_{R_x} = \Sigma F_x; \qquad F_{R_x} = 1.5 \sin 30° - 2.5\left(\frac{4}{5}\right)$$

$$= -1.25 \text{ kN} = 1.25 \text{ kN} \leftarrow$$

$$+\uparrow F_{R_y} = \Sigma F_y; \qquad F_{R_y} = -1.5 \cos 30° - 2.5\left(\frac{3}{5}\right) - 3$$

$$= -5.799 \text{ kN} = 5.799 \text{ kN} \downarrow$$



Thus,

$$F_R = \sqrt{F_{R_x}^2 + F_{R_y}^2} = \sqrt{1.25^2 + 5.799^2} = 5.93 \text{ kN}$$ **Ans.**

and

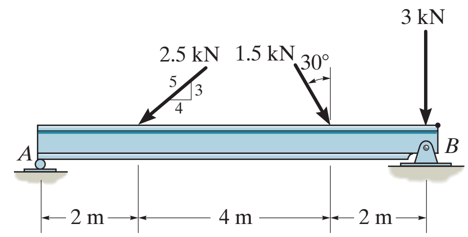
$$\theta = \tan^{-1}\left(\frac{F_{R_y}}{F_{R_x}}\right) = \tan^{-1}\left(\frac{5.799}{1.25}\right) = 77.8° \measuredangle$$ **Ans.**

$$\circlearrowleft + M_{R_A} = \Sigma M_A; \qquad M_{R_A} = -2.5\left(\frac{3}{5}\right)(2) - 1.5 \cos 30°(6) - 3(8)$$

$$= -34.8 \text{ kN}\cdot\text{m} = 34.8 \text{ kN}\cdot\text{m} \quad \textbf{(\textit{Clockwise})}$$ **Ans.**

***4–100.**

Replace the force system acting on the beam by an equivalent force and couple moment at point B.



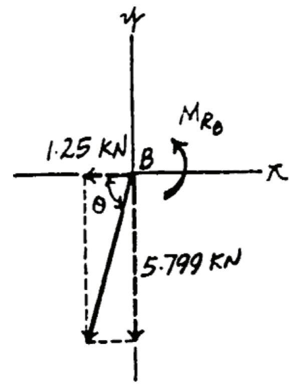
SOLUTION

$$\xrightarrow{+} F_{R_x} = \Sigma F_x; \qquad F_{R_x} = 1.5 \sin 30° - 2.5\left(\frac{4}{5}\right)$$

$$= -1.25 \text{ kN} = 1.25 \text{ kN} \leftarrow$$

$$+\uparrow F_{R_y} = \Sigma F_y; \qquad F_{R_y} = -1.5 \cos 30° - 2.5\left(\frac{3}{5}\right) - 3$$

$$= -5.799 \text{ kN} = 5.799 \text{ kN} \downarrow$$



Thus,

$$F_R = \sqrt{F_{R_x}^2 + F_{R_y}^2} = \sqrt{1.25^2 + 5.799^2} = 5.93 \text{ kN}$$ **Ans.**

and

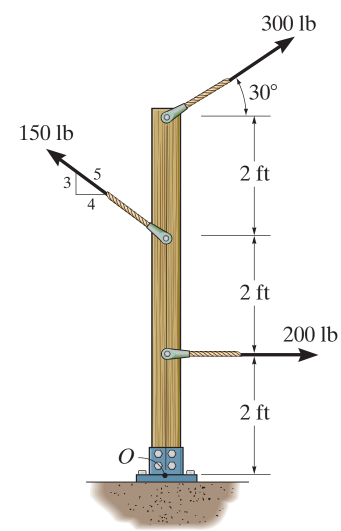
$$\theta = \tan^{-1}\left(\frac{F_{R_y}}{F_{R_x}}\right) = \tan^{-1}\left(\frac{5.799}{1.25}\right) = 77.8° \ \measuredangle$$ **Ans.**

$$\circlearrowleft + M_{R_B} = \Sigma M_{R_B}; \qquad M_B = 1.5\cos 30°(2) + 2.5\left(\frac{3}{5}\right)(6)$$

$$= 11.6 \text{ kN} \cdot \text{m} \quad (\textit{Counterclockwise})$$ **Ans.**

4–101.

Replace the force system acting on the post by a resultant force and couple moment at point O.



SOLUTION

Equivalent Resultant Force: Forces $\mathbf{F}_1$ and $\mathbf{F}_2$ are resolved into their x and y components, Fig. a. Summing these force components algebraically along the x and y axes, we have

$$\overset{+}{\rightarrow} \Sigma(F_R)_x = \Sigma F_x; \qquad (F_R)_x = 300\cos 30° - 150\left(\frac{4}{5}\right) + 200 = 339.81 \text{ lb} \rightarrow$$

$$+\uparrow (F_R)_y = \Sigma F_y; \qquad (F_R)_y = 300\sin 30° + 150\left(\frac{3}{5}\right) = 240 \text{ lb} \uparrow$$

The magnitude of the resultant force $\mathbf{F}_R$ is given by

$$F_R = \sqrt{(F_R)_x^{\,2} + (F_R)_y^{\,2}} = \sqrt{339.81^2 + 240^2} = 416.02 \text{ lb} = 416 \text{ lb} \qquad \textbf{Ans.}$$

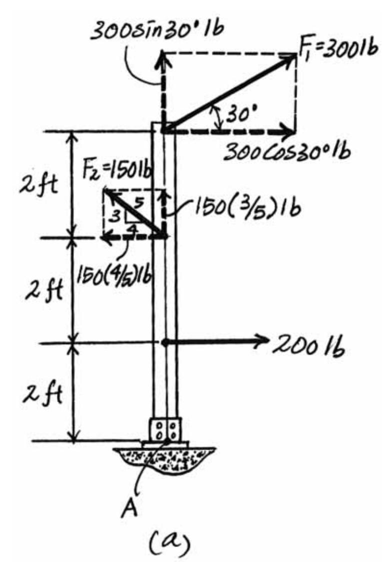
The angle θ of $\mathbf{F}_R$ is

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left[\frac{240}{339.81}\right] = 35.23° = 35.2° \;\measuredangle \qquad \textbf{Ans.}$$

Equivalent Resultant Couple Moment: Applying the principle of moments, Figs. a and b, and summing the moments of the force components algebraically about point A, we can write

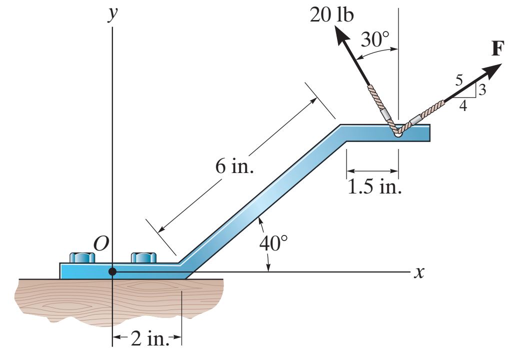
$$\circlearrowleft + (M_R)_A = \Sigma M_A; \qquad (M_R)_A = 150\left(\frac{4}{5}\right)(4) - 200(2) - 300\cos 30°(6)$$

$$= -1478.85 \text{ lb}\cdot\text{ft} = 1.48 \text{ kip}\cdot\text{ft} \;\textbf{(\textit{Clockwise}) Ans.}$$



4–102.

Replace the two forces by an equivalent resultant force and couple moment at point O. Set $F = 20$ lb.



SOLUTION

$\xrightarrow{+} F_{Rx} = \Sigma F_x;$ $\quad F_{Rx} = \frac{4}{5}(20) - 20 \sin 30° = 6 \text{ lb}$

$+\uparrow F_{Ry} = \Sigma F_y;$ $\quad F_{Ry} = 20 \cos 30° + \frac{3}{5}(20) = 29.32 \text{ lb}$

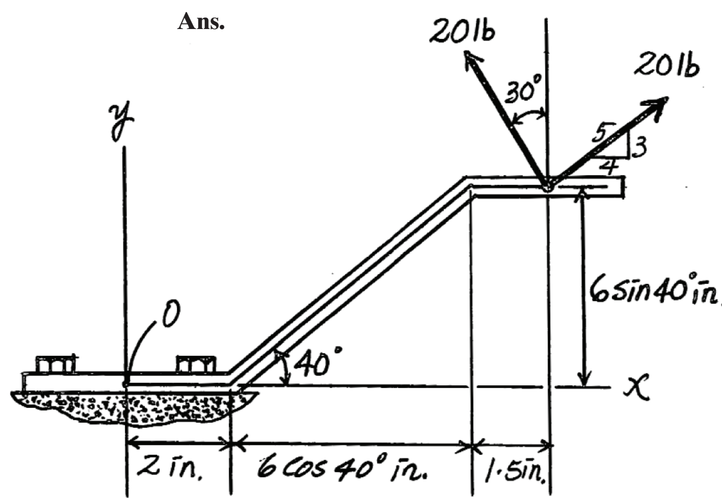
$$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2} = \sqrt{6^2 + (29.32)^2} = 29.9 \text{ lb} \qquad \textbf{Ans.}$$

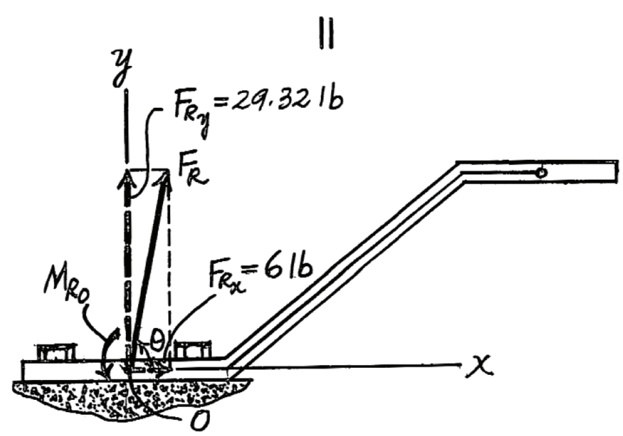
$$\theta = \tan^{-1}\frac{F_{Ry}}{F_{Rx}} = \tan^{-1}\left(\frac{29.32}{6}\right) = 78.4° \ \measuredangle \qquad \textbf{Ans.}$$

$\circlearrowleft +M_{R_O} = \Sigma M_O;$ $\quad M_{R_O} = 20 \sin 30°(6 \sin 40°) + 20 \cos 30°(3.5 + 6 \cos 40°)$

$$-\frac{4}{5}(20)(6 \sin 40°) + \frac{3}{5}(20)(3.5 + 6 \cos 40°)$$

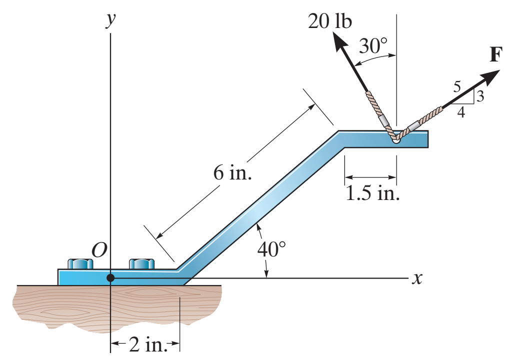
$$= 214 \text{ lb} \cdot \text{in.} \circlearrowright \qquad \textbf{Ans.}$$





4–103.

Replace the two forces by an equivalent resultant force and couple moment at point O. Set $F = 15$ lb.



SOLUTION

$\overset{+}{\rightarrow} F_{Rx} = \Sigma F_x;\qquad F_{Rx} = \frac{4}{5}(15) - 20 \sin 30° = 2 \text{ lb}$

$+\uparrow F_{Ry} = \Sigma F_y;\qquad F_{Ry} = 20 \cos 30° + \frac{3}{5}(15) = 26.32 \text{ lb}$

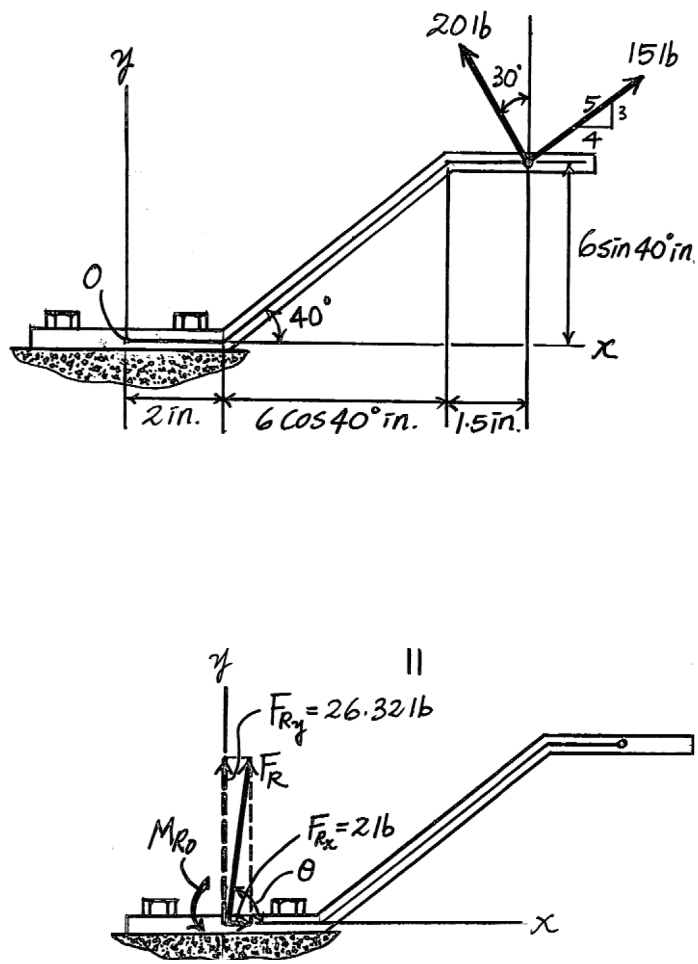
$$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2} = \sqrt{2^2 + 26.32^2} = 26.4 \text{ lb} \qquad \textbf{Ans.}$$

$$\theta = \tan^{-1}\frac{F_{Ry}}{F_{Rx}} = \tan^{-1}\left(\frac{26.32}{2}\right) = 85.7° \measuredangle \qquad \textbf{Ans.}$$

$\circlearrowleft + M_{R_O} = \Sigma M_O;\qquad M_{R_O} = 20 \sin 30°(6 \sin 40°) + 20 \cos 30°(3.5 + 6 \cos 40°)$

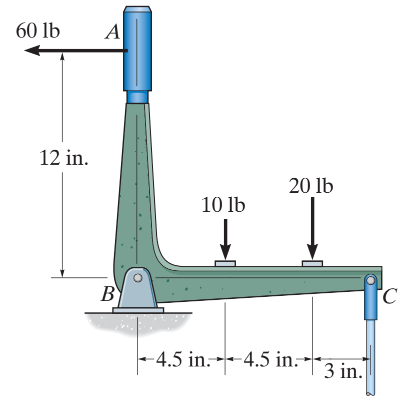
$$-\frac{4}{5}(15)(6 \sin 40°) + \frac{3}{5}(15)(3.5 + 6 \cos 40°)$$

$$= 205 \text{ lb} \cdot \text{in.} \circlearrowleft \qquad \textbf{Ans.}$$



***4–104.**

Replace the force system acting on the crank by a resultant force, and specify where its line of action intersects *BA* measured from the pin at *B*.



SOLUTION

Equivalent Resultant Force: Summing the forces, Fig. *a*, algebraically along the *x* and *y* axes, we have

$$\overset{+}{\rightarrow} \Sigma(F_R)_x = \Sigma F_x; \qquad (F_R)_x = -60 \text{ lb} = 60 \text{ lb} \leftarrow$$

$$+\uparrow (F_R)_y = \Sigma F_y; \qquad (F_R)_y = -10 - 20 = -30 \text{ lb} = 30 \text{ lb} \downarrow$$

The magnitude of the resultant force $\mathbf{F}_R$ is given by

$$F_R = \sqrt{(F_R)_x^{\,2} + (F_R)_y^{\,2}} = \sqrt{60^2 + 30^2} = 67.08 \text{ lb} = 67.1 \text{ lb} \qquad \textbf{Ans.}$$

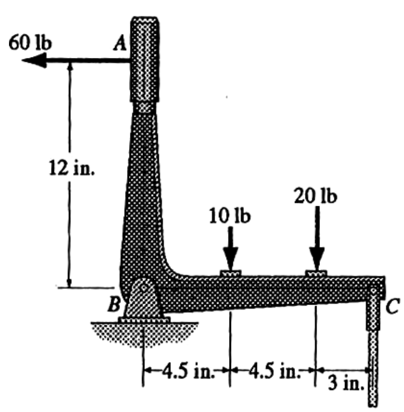
The angle θ of $\mathbf{F}_R$ is

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left[\frac{30}{60}\right] = 26.57^\circ = 26.6^\circ \qquad \textbf{Ans.}$$

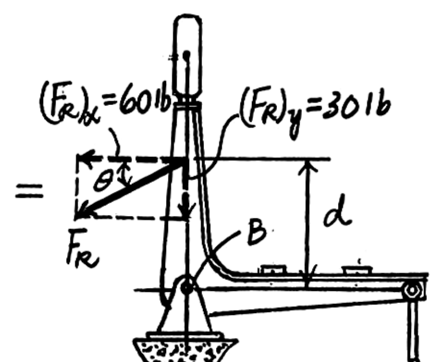
Location of Resultant Force: Summing the moments of the forces shown in Fig. *a* and the force components shown in Fig. *b* algebraically about point *B*, we can write

$$\circlearrowleft + (M_R)_B = \Sigma M_B; \qquad 60(d) = 60(12) - 10(4.5) - 20(9)$$

$$d = 8.25 \text{ in.} \qquad \textbf{Ans.}$$



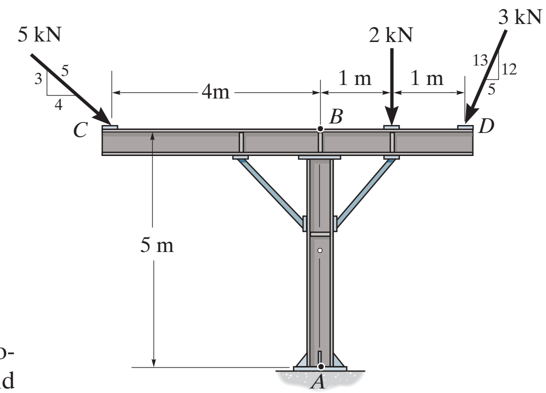
(a)



(b)

4–105.

Replace the force system acting on the frame by a resultant force and couple moment at point A.



SOLUTION

Equivalent Resultant Force: Resolving $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ into their x and y components, Fig. a, and summing these force components algebraically along the x and y axes, we have

$$\overset{+}{\rightarrow} \Sigma(F_R)_x = \Sigma F_x; \qquad (F_R)_x = 5\left(\frac{4}{5}\right) - 3\left(\frac{5}{13}\right) = 2.846 \text{ kN} \rightarrow$$

$$+\uparrow (F_R)_y = \Sigma F_y; \qquad (F_R)_y = -5\left(\frac{3}{5}\right) - 2 - 3\left(\frac{12}{13}\right) = -7.769 \text{ kN} = 7.769 \text{ kN} \downarrow$$

The magnitude of the resultant force $\mathbf{F}_R$ is given by

$$F_R = \sqrt{(F_R)_x^{\,2} + (F_R)_y^{\,2}} = \sqrt{2.846^2 + 7.769^2} = 8.274 \text{ kN} = 8.27 \text{ kN} \qquad \textbf{Ans.}$$

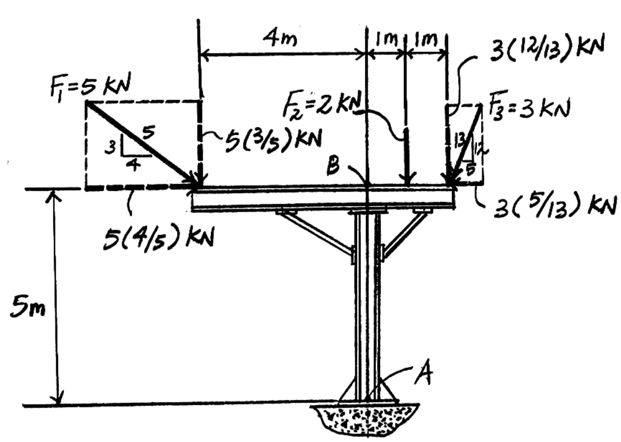
The angle θ of $\mathbf{F}_R$ is

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left[\frac{7.769}{2.846}\right] = 69.88^\circ = 69.9^\circ \qquad \textbf{Ans.}$$

Equivalent Couple Moment: Applying the principle of moments and summing the moments of the force components algebraically about point A, we can write

$$\circlearrowleft + (M_R)_A = \Sigma M_A; \qquad (M_R)_A = 5\left(\frac{3}{5}\right)(4) - 5\left(\frac{4}{5}\right)(5) - 2(1) - 3\left(\frac{12}{13}\right)(2) + 3\left(\frac{5}{13}\right)(5)$$

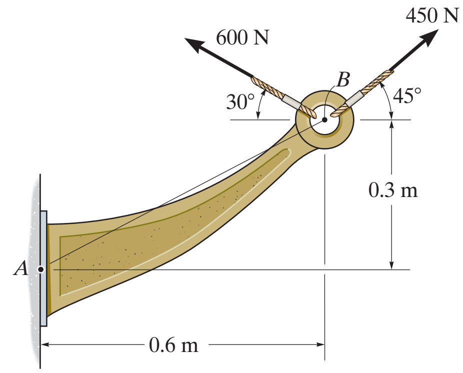
$$= -9.768 \text{ kN}\cdot\text{m} = 9.77 \text{ kN}\cdot\text{m } \textbf{(\textit{Clockwise})} \qquad \textbf{Ans.}$$



(a)

4–106.

Replace the force system acting on the bracket by a resultant force and couple moment at point A.



SOLUTION

Equivalent Resultant Force: Forces $\mathbf{F}_1$ and $\mathbf{F}_2$ are resolved into their x and y components, Fig. *a*. Summing these force components algebraically along the x and y axes, we have

$$\overset{+}{\rightarrow} \Sigma(F_R)_x = \Sigma F_x; \quad (F_R)_x = 450 \cos 45° - 600 \cos 30° = -201.42 \text{ N} = 201.42 \text{ N} \quad \leftarrow$$

$$+\uparrow (F_R)_y = \Sigma F_y; \quad (F_R)_y = 450 \sin 45° + 600 \sin 30° = 618.20 \text{ N} \quad \uparrow$$

The magnitude of the resultant force $\mathbf{F}_R$ is given by

$$F_R = \sqrt{(F_R)_x{}^2 + (F_R)_y{}^2} = \sqrt{201.4^2 + 618.20^2} = 650.18 \text{ kN} = 650 \text{ N} \qquad \textbf{Ans.}$$

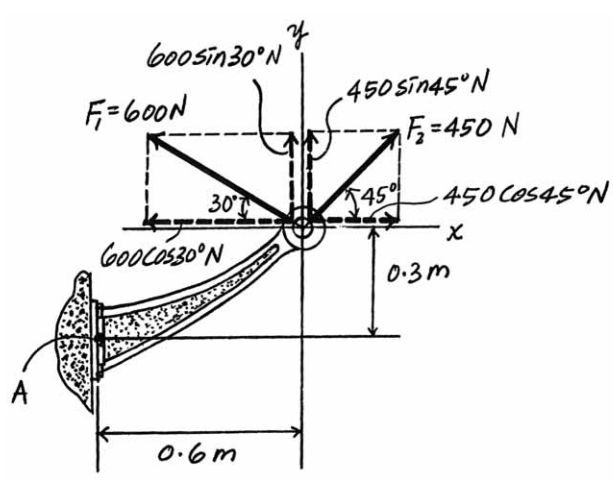
The angle θ of $\mathbf{F}_R$ is

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left[\frac{618.20}{201.4}\right] = 71.95° = 72.0° \qquad \textbf{Ans.}$$

Equivalent Resultant Couple Moment: Applying the principle of moments, Figs. *a* and *b*, and summing the moments of the force components algebraically about point A, we can write

$$\circlearrowleft + (M_R)_A = \Sigma M_A; \quad (M_R)_A = 600 \sin 30°(0.6) + 600 \cos 30°(0.3) + 450 \sin 45°(0.6) - 450 \cos 45°(0.3)$$

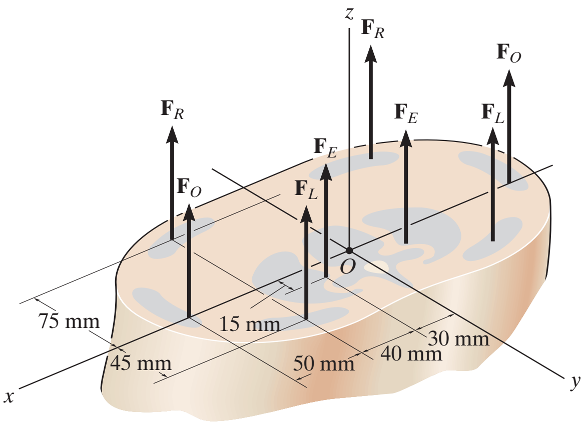
$$= 431.36 \text{ N} \cdot \text{m} = 431 \text{ N} \cdot \text{m} \textbf{ (\textit{Counterclockwise})} \qquad \textbf{Ans.}$$



(a)

4–107.

A biomechanical model of the lumbar region of the human trunk is shown. The forces acting in the four muscle groups consist of $F_R = 35$ N for the rectus, $F_O = 45$ N for the oblique, $F_L = 23$ N for the lumbar latissimus dorsi, and $F_E = 32$ N for the erector spinae. These loadings are symmetric with respect to the y–z plane. Replace this system of parallel forces by an equivalent force and couple moment acting at the spine, point O. Express the results in Cartesian vector form.



SOLUTION

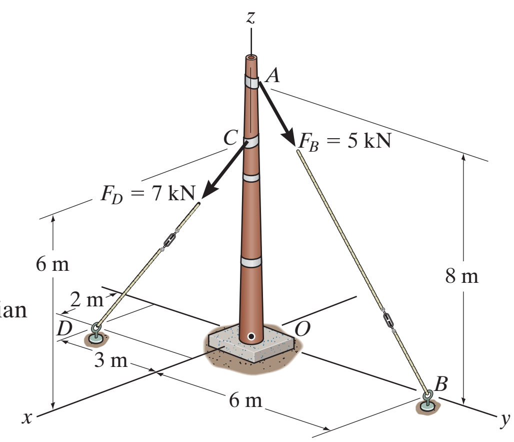
$\mathbf{F}_R = \Sigma \mathbf{F}_z;$ $\quad \mathbf{F}_R = \{2(35 + 45 + 23 + 32)\mathbf{k}\} = \{270\mathbf{k}\}$ N **Ans.**

$\mathbf{M}_{RO_x} = \Sigma \mathbf{M}_{O_x};$ $\quad \mathbf{M}_{RO} = [-2(35)(0.075) + 2(32)(0.015) + 2(23)(0.045)]\mathbf{i}$

$\mathbf{M}_{RO} = \{-2.22\mathbf{i}\}\ \text{N}\cdot\text{m}$ **Ans.**

***4–108.**

Replace the two forces acting on the post by a resultant force and couple moment at point O. Express the results in Cartesian vector form.



SOLUTION

Equivalent Resultant Force: The forces $\mathbf{F}_B$ and $\mathbf{F}_D$, Fig. *a*, expressed in Cartesian vector form can be written as

$$\mathbf{F}_B = F_B\mathbf{u}_{AB} = 5\left[\frac{(0-0)\mathbf{i} + (6-0)\mathbf{j} + (0-8)\mathbf{k}}{\sqrt{(0-0)^2 + (6-0)^2 + (0-8)^2}}\right] = [3\mathbf{j} - 4\mathbf{k}]\ \text{kN}$$

$$\mathbf{F}_D = F_D\mathbf{u}_{CD} = 7\left[\frac{(2-0)\mathbf{i} + (-3-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(2-0)^2 + (-3-0)^2 + (0-6)^2}}\right] = [2\mathbf{i} - 3\mathbf{j} - 6\mathbf{k}]\ \text{kN}$$

The resultant force $\mathbf{F}_R$ is given by

$$\mathbf{F}_R = \Sigma\mathbf{F};\quad \mathbf{F}_R = \mathbf{F}_B + \mathbf{F}_D$$

$$= (3\mathbf{j} - 4\mathbf{k}) + (2\mathbf{i} - 3\mathbf{j} - 6\mathbf{k})$$

$$= [2\mathbf{i} - 10\mathbf{k}]\ \text{kN}$$ **Ans.**

Equivalent Resultant Force: The position vectors $\mathbf{r}_{OB}$ and $\mathbf{r}_{OC}$ are

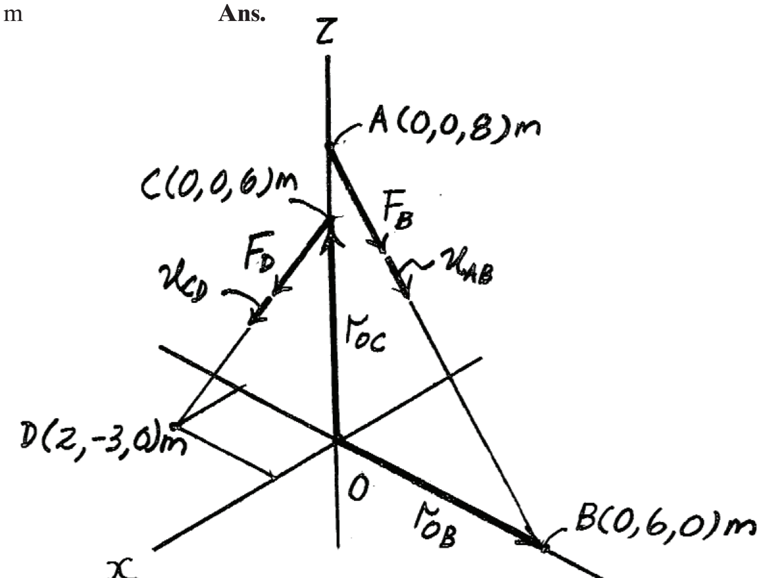
$$\mathbf{r}_{OB} = \{6\mathbf{j}\}\ \text{m} \qquad \mathbf{r}_{OC} = [6\mathbf{k}]\ \text{m}$$

Thus, the resultant couple moment about point O is given by

$$(\mathbf{M}_R)_O = \Sigma\mathbf{M}_O;\qquad (\mathbf{M}_R)_O = \mathbf{r}_{OB} \times \mathbf{F}_B + \mathbf{r}_{OC} \times \mathbf{F}_D$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 6 & 0 \\ 0 & 3 & -4 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 6 \\ 2 & -3 & -6 \end{vmatrix}$$

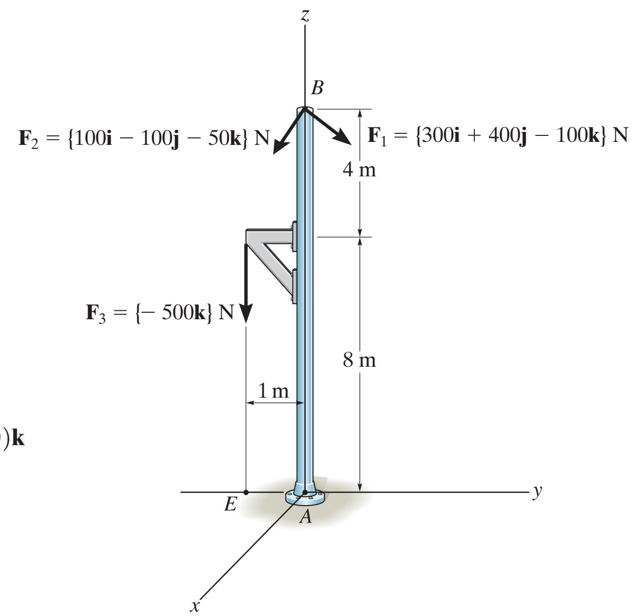
$$= [-6\mathbf{i} + 12\mathbf{j}]\ \text{kN}\cdot\text{m}$$ **Ans.**



(a)

4–109.

Replace the force system by an equivalent force and couple moment at point A.



SOLUTION

$\mathbf{F}_R = \Sigma\mathbf{F};\quad \mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$

$$= (300 + 100)\mathbf{i} + (400 - 100)\mathbf{j} + (-100 - 50 - 500)\mathbf{k}$$

$$= \{400\mathbf{i} + 300\mathbf{j} - 650\mathbf{k}\}\text{ N}$$

The position vectors are $\mathbf{r}_{AB} = \{12\mathbf{k}\}$ m and $\mathbf{r}_{AE} = \{-1\mathbf{j}\}$ m.

$\mathbf{M}_{R_A} = \Sigma\mathbf{M}_A;\quad \mathbf{M}_{R_A} = \mathbf{r}_{AB} \times \mathbf{F}_1 + \mathbf{r}_{AB} \times \mathbf{F}_2 + \mathbf{r}_{AE} \times \mathbf{F}_3$

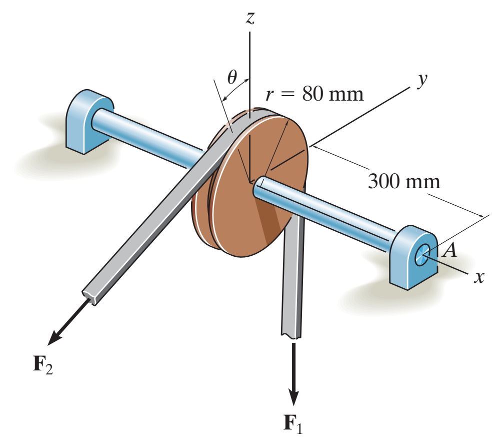
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 12 \\ 300 & 400 & -100 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 12 \\ 100 & -100 & -50 \end{vmatrix}$$

$$+ \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -1 & 0 \\ 0 & 0 & -500 \end{vmatrix}$$

$$= \{-3100\mathbf{i} + 4800\mathbf{j}\}\text{ N}\cdot\text{m}$$ **Ans.**

4–110.

The belt passing over the pulley is subjected to forces $\mathbf{F}_1$ and $\mathbf{F}_2$, each having a magnitude of 40 N. $\mathbf{F}_1$ acts in the $-\mathbf{k}$ direction. Replace these forces by an equivalent force and couple moment at point A. Express the result in Cartesian vector form. Set $\theta = 0°$ so that $\mathbf{F}_2$ acts in the $-\mathbf{j}$ direction.



SOLUTION

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$

$$\mathbf{F}_R = \{-40\mathbf{j} - 40\mathbf{k}\}\text{ N}$$ **Ans.**

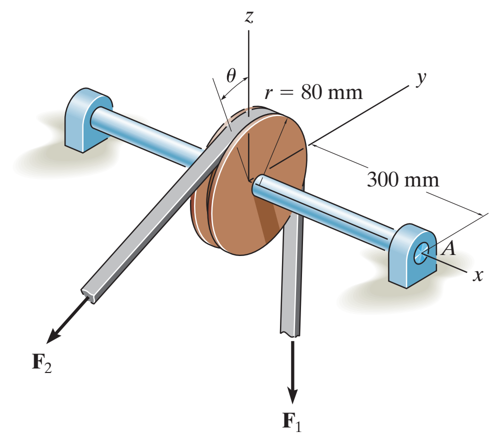
$$\mathbf{M}_{RA} = \Sigma(\mathbf{r} \times \mathbf{F})$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.3 & 0 & 0.08 \\ 0 & -40 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.3 & 0.08 & 0 \\ 0 & 0 & -40 \end{vmatrix}$$

$$\mathbf{M}_{RA} = \{-12\mathbf{j} + 12\mathbf{k}\}\text{ N}\cdot\text{m}$$ **Ans.**

4–111.

The belt passing over the pulley is subjected to two forces $\mathbf{F}_1$ and $\mathbf{F}_2$, each having a magnitude of 40 N. $\mathbf{F}_1$ acts in the $-\mathbf{k}$ direction. Replace these forces by an equivalent force and couple moment at point A. Express the result in Cartesian vector form. Take $\theta = 45°$.



SOLUTION

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$

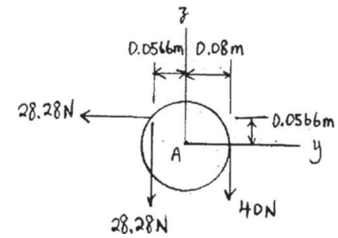
$$= -40 \cos 45°\mathbf{j} + (-40 - 40 \sin 45°)\mathbf{k}$$

$$\mathbf{F}_R = \{-28.3\mathbf{j} - 68.3\mathbf{k}\} \text{ N}$$ **Ans.**

$$\mathbf{r}_{AF1} = \{-0.3\mathbf{i} + 0.08\mathbf{j}\} \text{ m}$$

$$\mathbf{r}_{AF2} = -0.3\mathbf{i} - 0.08 \sin 45°\mathbf{j} + 0.08 \cos 45°\mathbf{k}$$

$$= \{-0.3\mathbf{i} - 0.0566\mathbf{j} + 0.0566\mathbf{k}\} \text{ m}$$



$$\mathbf{M}_{RA} = (\mathbf{r}_{AF1} \times \mathbf{F}_1) + (\mathbf{r}_{AF2} \times \mathbf{F}_2)$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.3 & 0.08 & 0 \\ 0 & 0 & -40 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.3 & -0.0566 & 0.0566 \\ 0 & -40 \cos 45° & -40 \sin 45° \end{vmatrix}$$

$$\mathbf{M}_{RA} = \{-20.5\mathbf{j} + 8.49\mathbf{k}\} \text{ N}\cdot\text{m}$$ **Ans.**

Also,

$$M_{RA_x} = \Sigma M_{A_x}$$

$$M_{RA_x} = 28.28(0.0566) + 28.28(0.0566) - 40(0.08)$$

$$M_{RA_x} = 0$$

$$M_{RA_y} = \Sigma M_{A_y}$$

$$M_{RA_y} = -28.28(0.3) - 40(0.3)$$

$$M_{RA_y} = -20.5 \text{ N}\cdot\text{m}$$

$$M_{RA_z} = \Sigma M_{A_z}$$

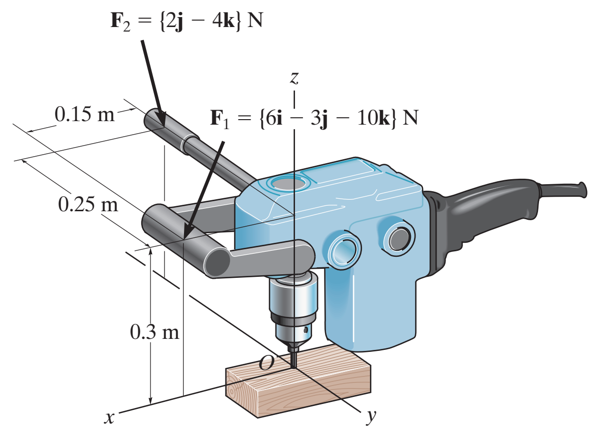
$$M_{RA_z} = 28.28(0.3)$$

$$M_{RA_z} = 8.49 \text{ N}\cdot\text{m}$$

$$\mathbf{M}_{RA} = \{-20.5\mathbf{j} + 8.49\mathbf{k}\} \text{ N}\cdot\text{m}$$ **Ans.**

***4–112.**

Handle forces $\mathbf{F}_1$ and $\mathbf{F}_2$ are applied to the electric drill. Replace this force system by an equivalent resultant force and couple moment acting at point O. Express the results in Cartesian vector form.



SOLUTION

$\mathbf{F}_R = \Sigma\mathbf{F};\quad \mathbf{F}_R = 6\mathbf{i} - 3\mathbf{j} - 10\mathbf{k} + 2\mathbf{j} - 4\mathbf{k}$

$= \{6\mathbf{i} - 1\mathbf{j} - 14\mathbf{k}\}\text{ N}$ **Ans.**

$\mathbf{M}_{RO} = \Sigma\mathbf{M}_O;$

$$\mathbf{M}_{RO} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.15 & 0 & 0.3 \\ 6 & -3 & -10 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -0.25 & 0.3 \\ 0 & 2 & -4 \end{vmatrix}$$

$= 0.9\mathbf{i} + 3.30\mathbf{j} - 0.450\mathbf{k} + 0.4\mathbf{i}$

$= \{1.30\mathbf{i} + 3.30\mathbf{j} - 0.450\mathbf{k}\}\text{ N}\cdot\text{m}$ **Ans.**

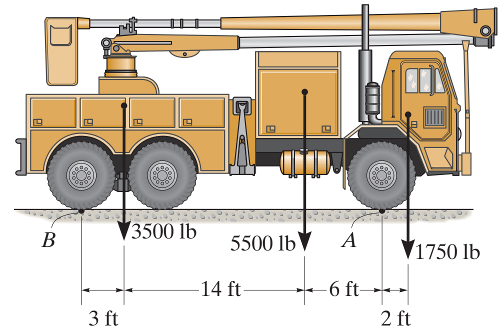
Note that $F_{Rz} = -14$ N pushes the drill bit down into the stock.

$(M_{RO})_x = 1.30\text{ N}\cdot\text{m}$ and $(M_{RO})_y = 3.30\text{ N}\cdot\text{m}$ cause the drill bit to bend.

$(M_{RO})_z = -0.450\text{ N}\cdot\text{m}$ causes the drill case and the spinning drill bit to rotate about the z-axis.

4–113.

The weights of the various components of the truck are shown. Replace this system of forces by an equivalent resultant force and specify its location measured from B.



SOLUTION

$$+\uparrow F_R = \Sigma F_y; \qquad F_R = -1750 - 5500 - 3500$$

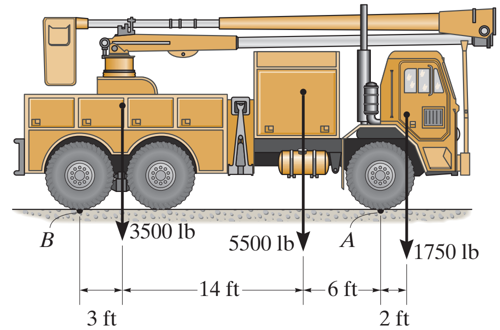
$$= -10\,750 \text{ lb} = 10.75 \text{ kip}\downarrow \qquad \textbf{Ans.}$$

$$\circlearrowleft + M_{R_A} = \Sigma M_A; \quad -10\,750d = -3500(3) - 5500(17) - 1750(25)$$

$$d = 13.7 \text{ ft} \qquad \textbf{Ans.}$$

4–114.

The weights of the various components of the truck are shown. Replace this system of forces by an equivalent resultant force and specify its location measured from point A.

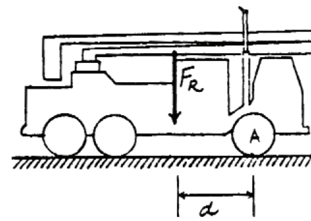


SOLUTION

Equivalent Force:

$$+\uparrow F_R = \Sigma F_y; \qquad F_R = -1750 - 5500 - 3500$$

$$= -10\,750 \text{ lb} = 10.75 \text{ kip}\downarrow \qquad \textbf{Ans.}$$



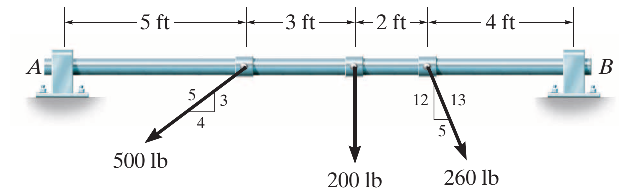
Location of Resultant Force From Point A:

$$\circlearrowleft + M_{R_A} = \Sigma M_A; \qquad 10\,750(d) = 3500(20) + 5500(6) - 1750(2)$$

$$d = 9.26 \text{ ft} \qquad \textbf{Ans.}$$

4–115.

Replace the three forces acting on the shaft by a single resultant force. Specify where the force acts, measured from end A.



SOLUTION

$$\xrightarrow{+} F_{Rx} = \Sigma F_x; \qquad F_{Rx} = -500\left(\frac{4}{5}\right) + 260\left(\frac{5}{13}\right) = -300 \text{ lb} = 300 \text{ lb} \leftarrow$$

$$+\uparrow F_{Ry} = \Sigma F_y; \qquad F_{Ry} = -500\left(\frac{3}{5}\right) - 200 - 260\left(\frac{12}{13}\right) = -740 \text{ lb} = 740 \text{ lb} \downarrow$$

$$F = \sqrt{(-300)^2 + (-740)^2} = 798 \text{ lb}$$ **Ans.**

$$\theta = \tan^{-1}\left(\frac{740}{300}\right) = 67.9^\circ$$ **Ans.**

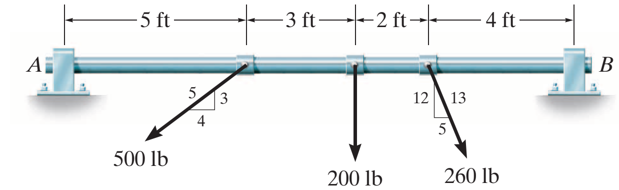
$$\circlearrowleft + M_{RA} = \Sigma M_A; \qquad 740(x) = 500\left(\frac{3}{5}\right)(5) + 200(8) + 260\left(\frac{12}{13}\right)(10)$$

$$740(x) = 5500$$

$$x = 7.43 \text{ ft}$$ **Ans.**

***4–116.**

Replace the three forces acting on the shaft by a single resultant force. Specify where the force acts, measured from end *B*.



SOLUTION

$$\overset{+}{\rightarrow} F_{Rx} = \Sigma F_x; \qquad F_{Rx} = -500\left(\frac{4}{5}\right) + 260\left(\frac{5}{13}\right) = -300 \text{ lb} = 300 \text{ lb} \leftarrow$$

$$+\uparrow F_{Ry} = \Sigma F_y; \qquad F_{Ry} = -500\left(\frac{3}{5}\right) - 200 - 260\left(\frac{12}{13}\right) = -740 \text{ lb} = 740 \text{ lb} \downarrow$$

$$F = \sqrt{(-300)^2 + (-740)^2} = 798 \text{ lb}$$ **Ans.**

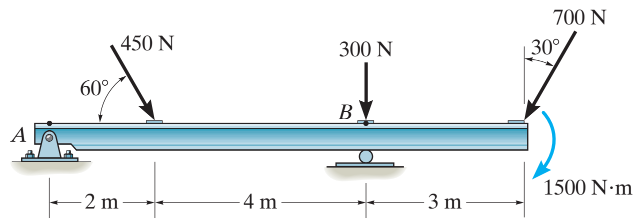
$$\theta = \tan^{-1}\left(\frac{740}{300}\right) = 67.9^\circ$$ **Ans.**

$$\circlearrowleft + M_{RB} = \Sigma M_B; \qquad 740(x) = 500\left(\frac{3}{5}\right)(9) + 200(6) + 260\left(\frac{12}{13}\right)(4)$$

$$x = 6.57 \text{ ft}$$ **Ans.**

4–117.

Replace the loading acting on the beam by a single resultant force. Specify where the force acts, measured from end A.



SOLUTION

$\overset{+}{\rightarrow} F_{Rx} = \Sigma F_x; \qquad F_{Rx} = 450 \cos 60° - 700 \sin 30° = -125 \text{ N} = 125 \text{ N} \leftarrow$

$+\uparrow F_{Ry} = \Sigma F_y; \qquad F_{Ry} = -450 \sin 60° - 700 \cos 30° - 300 = -1296 \text{ N} = 1296 \text{ N} \downarrow$

$F = \sqrt{(-125)^2 + (-1296)^2} = 1302 \text{ N}$ **Ans.**

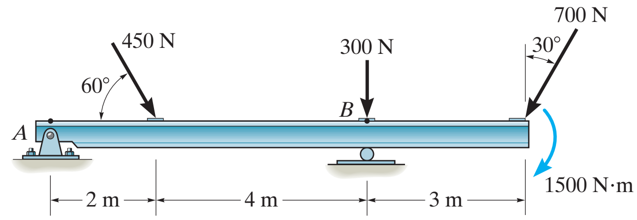
$\theta = \tan^{-1}\left(\frac{1296}{125}\right) = 84.5°$ ↙ **Ans.**

$\circlearrowleft + M_{RA} = \Sigma M_A; \qquad 1296(x) = 450 \sin 60°(2) + 300(6) + 700 \cos 30°(9) + 1500$

$x = 7.36 \text{ m}$ **Ans.**

4–118.

Replace the loading acting on the beam by a single resultant force. Specify where the force acts, measured from B.



SOLUTION

$\xrightarrow{+} F_{Rx} = \Sigma F_x;\quad F_{Rx} = 450 \cos 60° - 700 \sin 30° = -125 \text{ N} = 125 \text{ N} \leftarrow$

$+\uparrow F_{Ry} = \Sigma F_y;\quad F_{Ry} = -450 \sin 60° - 700 \cos 30° - 300 = -1296 \text{ N} = 1296 \text{ N} \downarrow$

$F = \sqrt{(-125)^2 + (-1296)^2} = 1302 \text{ N}$ **Ans.**

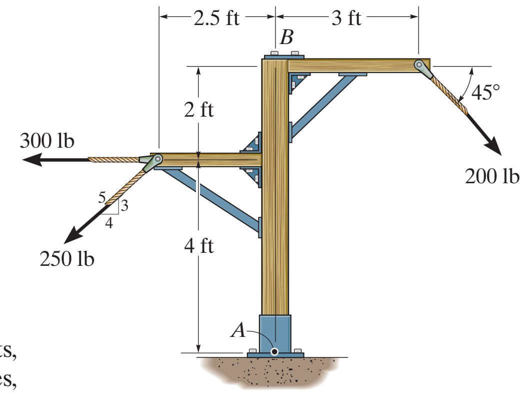
$\theta = \tan^{-1}\left(\frac{1296}{125}\right) = 84.5°$ **Ans.**

$\circlearrowleft + M_{RB} = \Sigma M_B;\quad 1296(x) = -450 \sin 60°(4) + 700 \cos 30°(3) + 1500$

$x = 1.36 \text{ m (to the right)}$ **Ans.**

4–119.

Replace the force system acting on the frame by a resultant force, and specify where its line of action intersects member AB, measured from point A.



SOLUTION

Equivalent Resultant Force: Resolving $\mathbf{F}_1$ and $\mathbf{F}_3$ into their x and y components, Fig. a, and summing these force components algebraically along the x and y axes, we have

$$\overset{+}{\rightarrow} \Sigma(F_R)_x = \Sigma F_x; \qquad (F_R)_x = 200\cos 45° - 250\left(\frac{4}{5}\right) - 300 = -358.58 \text{ lb} = 358.58 \text{ lb} \leftarrow$$

$$+\uparrow (F_R)_y = \Sigma F_y; \qquad (F_R)_y = -200\sin 45° - 250\left(\frac{3}{5}\right) = -291.42 \text{ lb} = 291.42 \text{ lb}\downarrow$$

The magnitude of the resultant force $\mathbf{F}_R$ is given by

$$F_R = \sqrt{(F_R)_x{}^2 + (F_R)_y{}^2} = \sqrt{358.58^2 + 291.42^2} = 462.07 \text{ lb} = 462 \text{ lb} \qquad \textbf{Ans.}$$

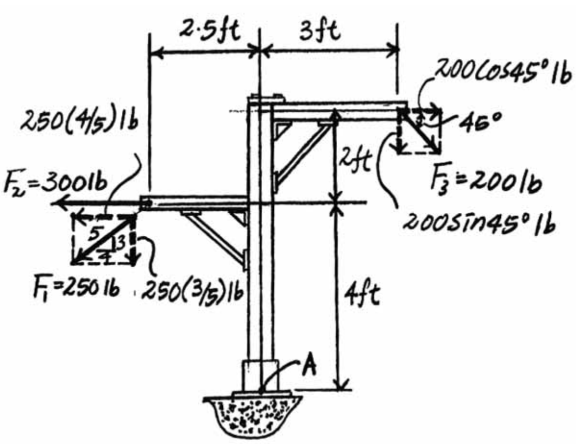
The angle θ of $\mathbf{F}_R$ is

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left[\frac{291.42}{358.58}\right] = 39.1° \qquad \textbf{Ans.}$$

Location of Resultant Force: Applying the principle of moments to Figs. a and b, and summing the moments of the force components algebraically about point A, we can write

$$\circlearrowleft +(M_R)_A = \Sigma M_A; \qquad 358.58(d) = 250\left(\frac{3}{5}\right)(2.5) + 250\left(\frac{4}{5}\right)(4) + 300(4) - 200\cos 45°(6) - 200\sin 45°(3)$$

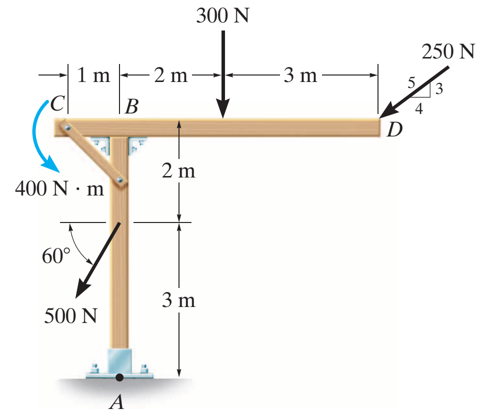
$$d = 3.07 \text{ ft} \qquad \textbf{Ans.}$$



(a)

***4–120.**

Replace the loading on the frame by a single resultant force. Specify where its line of action intersects member AB, measured from A.



SOLUTION

$$\xrightarrow{+} \Sigma F_x = F_{Rx}; \qquad F_{Rx} = -250\left(\frac{4}{5}\right) - 500(\cos 60^\circ) = -450 \text{ N} = 450 \text{ N} \leftarrow$$

$$+\uparrow \Sigma F_y = \Sigma F_y; \qquad F_{Ry} = -300 - 250\left(\frac{3}{5}\right) - 500 \sin 60^\circ = -883.0127 \text{ N} = 883.0127 \text{ N} \downarrow$$

$$F_R = \sqrt{(-450)^2 + (-883.0127)^2} = 991 \text{ N}$$ **Ans.**

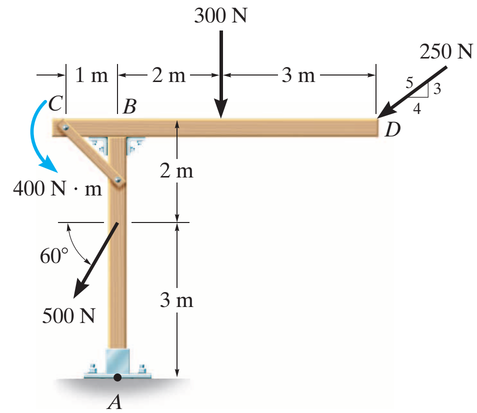
$$\theta = \tan^{-1}\left(\frac{883.0127}{450}\right) = 63.0^\circ \; \measuredangle$$

$$\circlearrowleft + M_{RA} = \Sigma M_A; \qquad 450y = 400 + (500 \cos 60^\circ)(3) + 250\left(\frac{4}{5}\right)(5) - 300(2) - 250\left(\frac{3}{5}\right)(5)$$

$$y = \frac{800}{450} = 1.78 \text{ m}$$ **Ans.**

4–121.

Replace the loading on the frame by a single resultant force. Specify where its line of action intersects member CD, measured from end C.



SOLUTION

$$\xrightarrow{+} \Sigma F_x = F_{Rx}; \qquad F_{Rx} = -250\left(\frac{4}{5}\right) - 500(\cos 60^\circ) = -450 \text{ N} = 450 \text{ N} \leftarrow$$

$$+\uparrow \Sigma F_y = \Sigma F_y; \qquad F_{Ry} = -300 - 250\left(\frac{3}{5}\right) - 500 \sin 60^\circ = -883.0127 \text{ N} = 883.0127 \text{ N} \downarrow$$

$$F_R = \sqrt{(-450)^2 + (-883.0127)^2} = 991 \text{ N}$$ **Ans.**

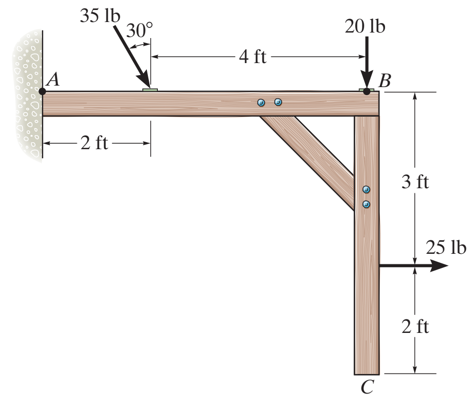
$$\theta = \tan^{-1}\left(\frac{883.0127}{450}\right) = 63.0^\circ \measuredangle$$

$$\circlearrowleft + M_{RA} = \Sigma M_C; \qquad 883.0127x = -400 + 300(3) + 250\left(\frac{3}{5}\right)(6) + 500 \cos 60^\circ(2) + (500 \sin 60^\circ)(1)$$

$$x = \frac{2333}{883.0127} = 2.64 \text{ m}$$ **Ans.**

4–122.

Replace the force system acting on the frame by an equivalent resultant force and specify where the resultant's line of action intersects member AB, measured from point A.



SOLUTION

$$\xrightarrow{+} F_{Rx} = \Sigma F_x; \qquad F_{Rx} = 35 \sin 30° + 25 = 42.5 \text{ lb}$$

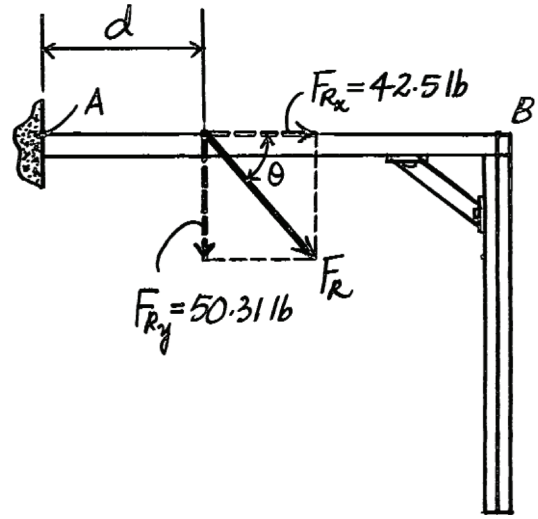
$$+\downarrow F_{Ry} = \Sigma F_y; \qquad F_{Ry} = 35 \cos 30° + 20 = 50.31 \text{ lb}$$

$$F_R = \sqrt{(42.5)^2 + (50.31)^2} = 65.9 \text{ lb}$$ **Ans.**

$$\theta = \tan^{-1}\left(\frac{50.31}{42.5}\right) = 49.8° \;\; \measuredangle$$ **Ans.**

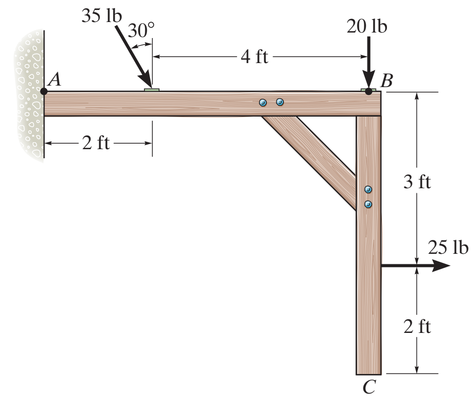
$$\circlearrowleft + M_{RA} = \Sigma M_A; \qquad 50.31\,(d) = 35 \cos 30°\,(2) + 20\,(6) - 25\,(3)$$

$$d = 2.10 \text{ ft}$$ **Ans.**



4–123.

Replace the force system acting on the frame by an equivalent resultant force and specify where the resultant's line of action intersects member BC, measured from point B.



SOLUTION

$\overset{+}{\rightarrow} F_{Rx} = \Sigma F_x; \qquad F_{Rx} = 35 \sin 30° + 25 = 42.5 \text{ lb}$

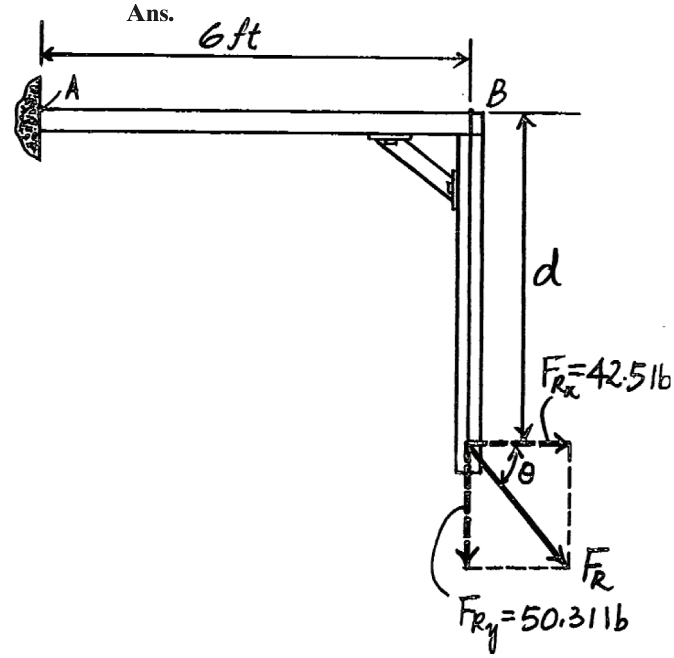
$+\downarrow F_{Ry} = \Sigma F_y; \qquad F_{Ry} = 35 \cos 30° + 20 = 50.31 \text{ lb}$

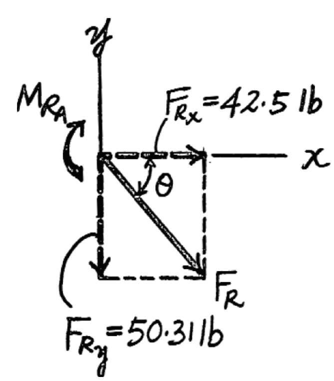
$$F_R = \sqrt{(42.5)^2 + (50.31)^2} = 65.9 \text{ lb}$$ **Ans.**

$$\theta = \tan^{-1}\left(\frac{50.31}{42.5}\right) = 49.8° \;\measuredangle$$ **Ans.**

$\circlearrowleft + M_{RA} = \Sigma M_A; \qquad 50.31(6) - 42.5(d) = 35 \cos 30° (2) + 20(6) - 25(3)$

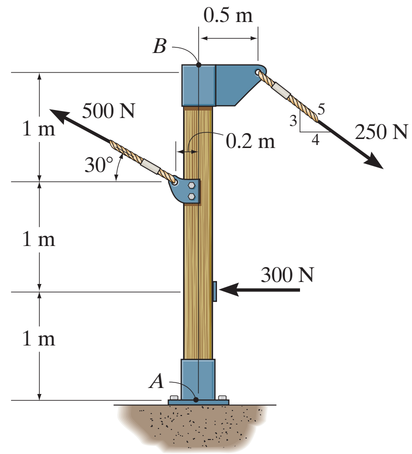
$$d = 4.62 \text{ ft}$$ **Ans.**





***4–124.**

Replace the force system acting on the post by a resultant force, and specify where its line of action intersects the post AB measured from point A.



SOLUTION

Equivalent Resultant Force: Forces $\mathbf{F}_1$ and $\mathbf{F}_2$ are resolved into their x and y components, Fig. a. Summing these force components algebraically along the x and y axes,

$$\overset{+}{\rightarrow}(F_R)_x = \Sigma F_x; \qquad (F_R)_x = 250\left(\frac{4}{5}\right) - 500\cos 30^\circ - 300 = -533.01\text{ N} = 533.01\text{ N} \leftarrow$$

$$+\uparrow(F_R)_y = \Sigma F_y; \qquad (F_R)_y = 500\sin 30^\circ - 250\left(\frac{3}{5}\right) = 100\text{ N}\uparrow$$

The magnitude of the resultant force $\mathbf{F}_R$ is given by

$$F_R = \sqrt{(F_R)_x{}^2 + (F_R)_y{}^2} = \sqrt{533.01^2 + 100^2} = 542.31\text{ N} = 542\text{ N} \qquad \textbf{Ans.}$$

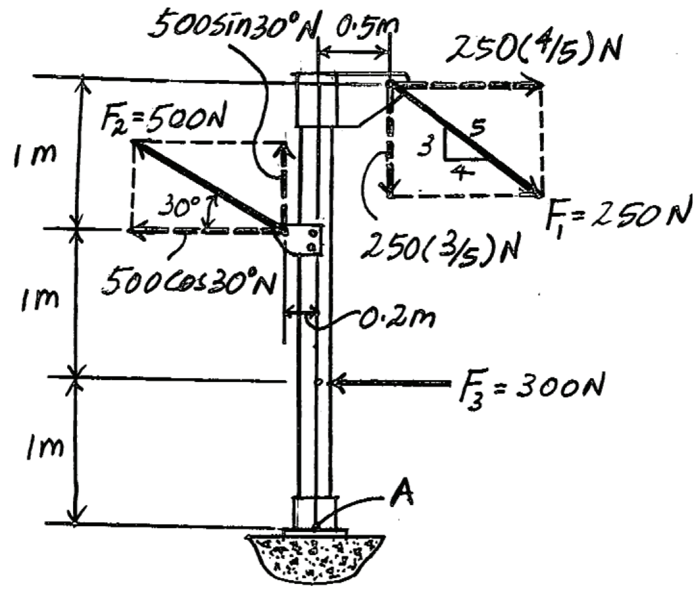
The angle θ of $\mathbf{F}_R$ is

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left[\frac{100}{533.01}\right] = 10.63^\circ = 10.6^\circ \qquad \textbf{Ans.}$$

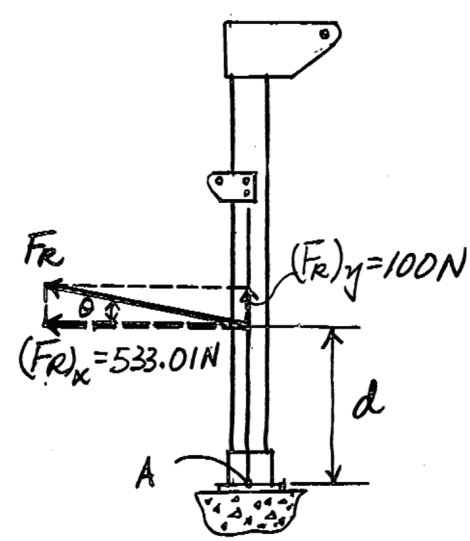
Location of the Resultant Force: Applying the principle of moments, Figs. a and b, and summing the moments of the force components algebraically about point A,

$$\circlearrowleft +(M_R)_A = \Sigma M_A; \qquad 533.01(d) = 500\cos 30^\circ(2) - 500\sin 30^\circ(0.2) - 250\left(\frac{3}{5}\right)(0.5) - 250\left(\frac{4}{5}\right)(3) + 300(1)$$

$$d = 0.8274\text{ mm} = 827\text{ mm} \qquad \textbf{Ans.}$$



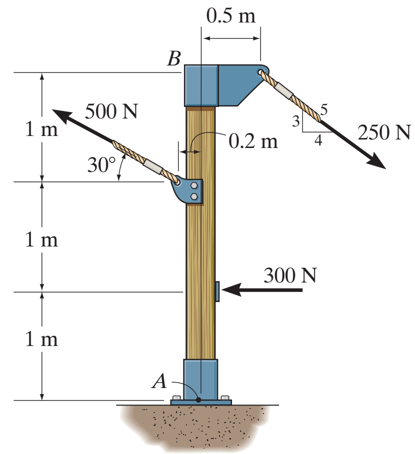
(a)



(b)

4–125.

Replace the force system acting on the post by a resultant force, and specify where its line of action intersects the post AB measured from point B.



SOLUTION

Equivalent Resultant Force: Forces $\mathbf{F}_1$ and $\mathbf{F}_2$ are resolved into their x and y components, Fig. a. Summing these force components algebraically along the x and y axes,

$$\overset{+}{\rightarrow} \Sigma(F_R)_x = \Sigma F_x; \qquad (F_R)_x = 250\left(\frac{4}{5}\right) - 500\cos 30° - 300 = -533.01\text{N} = 533.01 \text{ N} \leftarrow$$

$$+\uparrow (F_R)_y = \Sigma F_y; \qquad (F_R)_y = 500 \sin 30° - 250\left(\frac{3}{5}\right) = 100 \text{ N}\uparrow$$

The magnitude of the resultant force $\mathbf{F}_R$ is given by

$$F_R = \sqrt{(F_R)_x{}^2 + (F_R)_y{}^2} = \sqrt{533.01^2 + 100^2} = 542.31 \text{ N} = 542 \text{ N} \qquad \textbf{Ans.}$$

The angle θ of $\mathbf{F}_R$ is

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left[\frac{100}{533.01}\right] = 10.63° = 10.6° \qquad \textbf{Ans.}$$

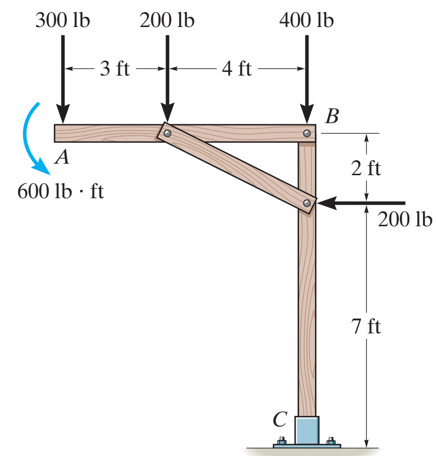
Location of the Resultant Force: Applying the principle of moments, Figs. a and b, and summing the moments of the force components algebraically about point B,

$$\circlearrowleft +(M_R)_B = \Sigma M_b; \qquad -533.01(d) = -500\cos 30°(1) - 500\sin 30°(0.2) - 250\left(\frac{3}{5}\right)(0.5) - 300(2)$$

$$d = 2.17 \text{ m} \qquad \textbf{Ans.}$$

4–126.

Replace the loading on the frame by a single resultant force. Specify where its line of action intersects member AB, measured from A.



SOLUTION

$\overset{+}{\rightarrow} F_{Rx} = \Sigma F_x; \qquad F_{Rx} = -200 \text{ lb} = 200\text{lb} \quad \leftarrow$

$+\uparrow F_{Ry} = \Sigma F_y; \qquad F_{Ry} = -300 - 200 - 400 = -900 \text{ lb} = 900 \text{ lb} \quad \downarrow$

$F = \sqrt{(-200)^2 + (-900)^2} = 922 \text{ lb}$ **Ans.**

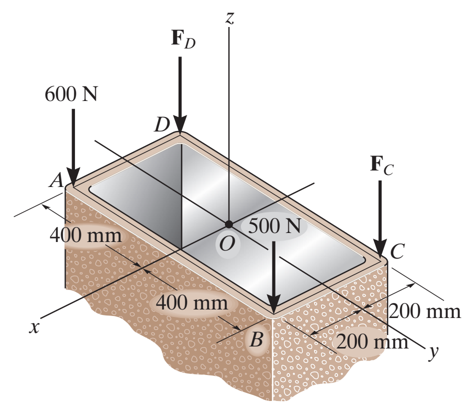
$\theta = \tan^{-1}\left(\frac{900}{200}\right) = 77.5°$ ⦣ **Ans.**

$\circlearrowleft + M_{RA} = \Sigma M_A; \qquad 900(x) = 200(3) + 400(7) + 200(2) - 600 = 0$

$x = \frac{3200}{900} = 3.56 \text{ ft}$ **Ans.**

4–127.

The tube supports the four parallel forces. Determine the magnitudes of forces $\mathbf{F}_C$ and $\mathbf{F}_D$ acting at C and D so that the equivalent resultant force of the force system acts through the midpoint O of the tube.



SOLUTION

Since the resultant force passes through point O, the resultant moment components about x and y axes are both zero.

$$\Sigma M_x = 0; \qquad F_D(0.4) + 600(0.4) - F_C(0.4) - 500(0.4) = 0$$

$$F_C - F_D = 100 \qquad \textbf{(1)}$$

$$\Sigma M_y = 0; \qquad 500(0.2) + 600(0.2) - F_C(0.2) - F_D(0.2) = 0$$

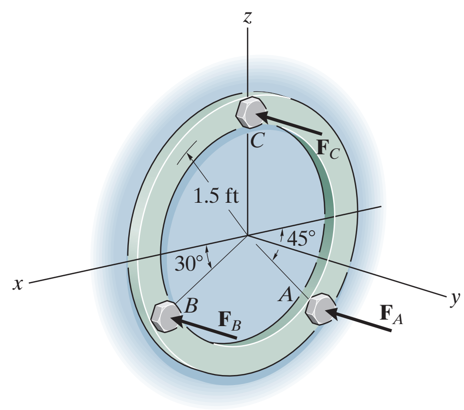
$$F_C + F_D = 1100 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields:

$$F_C = 600\text{ N} \qquad F_D = 500\text{ N} \qquad \textbf{Ans.}$$

***4–128.**

Three parallel bolting forces act on the circular plate. Determine the resultant force, and specify its location (x, z) on the plate. $F_A = 200$ lb, $F_B = 100$ lb, and $F_C = 400$ lb.



SOLUTION

Equivalent Force:

$$F_R = \Sigma F_y; \qquad -F_R = -400 - 200 - 100$$

$$F_R = 700 \text{ lb} \qquad \textbf{Ans.}$$

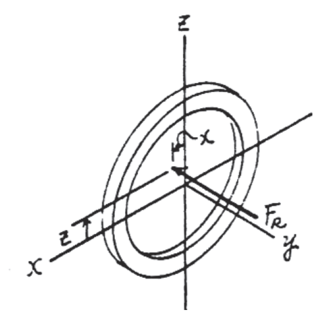
Location of Resultant Force:

$$M_{R_x} = \Sigma M_x; \qquad 700(z) = 400(1.5) - 200(1.5 \sin 45°) - 100(1.5 \sin 30°)$$

$$z = 0.447 \text{ ft} \qquad \textbf{Ans.}$$

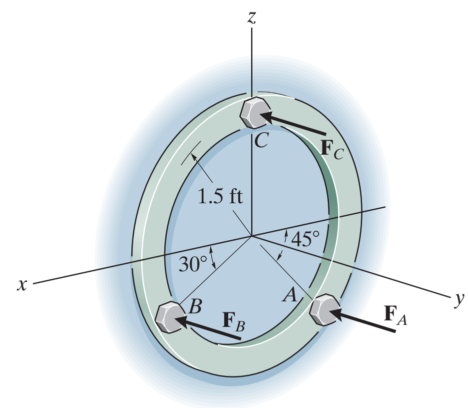
$$M_{R_z} = \Sigma M_z; \qquad -700(x) = 200(1.5 \cos 45°) - 100(1.5 \cos 30°)$$

$$x = -0.117 \text{ ft} \qquad \textbf{Ans.}$$



4–129.

The three parallel bolting forces act on the circular plate. If the force at A has a magnitude of $F_A = 200$ lb, determine the magnitudes of $\mathbf{F}_B$ and $\mathbf{F}_C$ so that the resultant force $\mathbf{F}_R$ of the system has a line of action that coincides with the y axis. *Hint:* This requires $\Sigma M_x = 0$ and $\Sigma M_z = 0$.



SOLUTION

Since $\mathbf{F}_R$ coincides with y axis, $M_{R_x} = M_{R_y} = 0$.

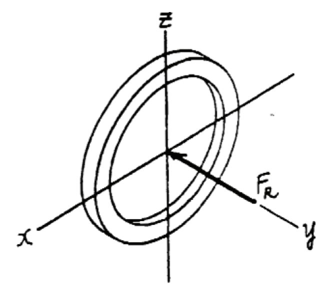
$$M_{R_z} = \Sigma M_z; \qquad 0 = 200(1.5 \cos 45°) - F_B(1.5 \cos 30°)$$

$$F_B = 163.30 \text{ lb} = 163 \text{ lb} \qquad \textbf{Ans.}$$

Using the result $F_B = 163.30$ lb,

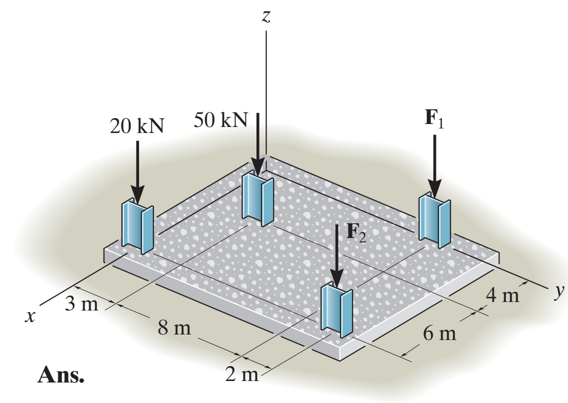
$$M_{R_x} = \Sigma M_x; \qquad 0 = F_C(1.5) - 200(1.5 \sin 45°) - 163.30(1.5 \sin 30°)$$

$$F_C = 223 \text{ lb} \qquad \textbf{Ans.}$$



4–130.

The building slab is subjected to four parallel column loadings. Determine the equivalent resultant force and specify its location (x, y) on the slab. Take $F_1 = 30$ kN, $F_2 = 40$ kN.



SOLUTION

$+\uparrow F_R = \Sigma F_z; \qquad F_R = -20 - 50 - 30 - 40 = -140\,\text{kN} = 140\,\text{kN}\downarrow$ **Ans.**

$(M_R)_x = \Sigma M_x; \qquad -140y = -50(3) - 30(11) - 40(13)$

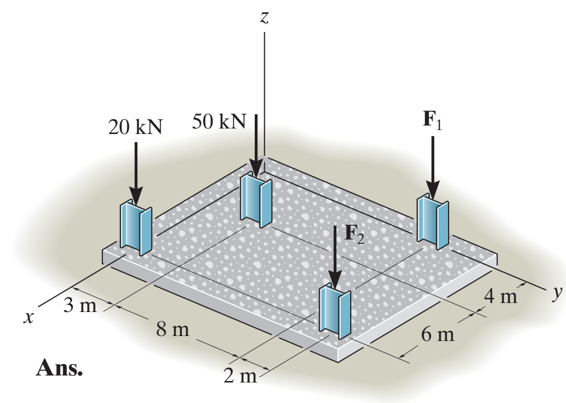
$y = 7.14\text{ m}$ **Ans.**

$(M_R)_y = \Sigma M_y; \qquad 140x = 50(4) + 20(10) + 40(10)$

$x = 5.71\text{ m}$ **Ans.**

4–131.

The building slab is subjected to four parallel column loadings. Determine the equivalent resultant force and specify its location (x, y) on the slab. Take $F_1 = 20$ kN, $F_2 = 50$ kN.



SOLUTION

$+\downarrow F_R = \Sigma F_z; \qquad F_R = 20 + 50 + 20 + 50 = 140 \text{ kN}$ **Ans.**

$M_{R\,y} = \Sigma M_y; \qquad 140(x) = (50)(4) + 20(10) + 50(10)$

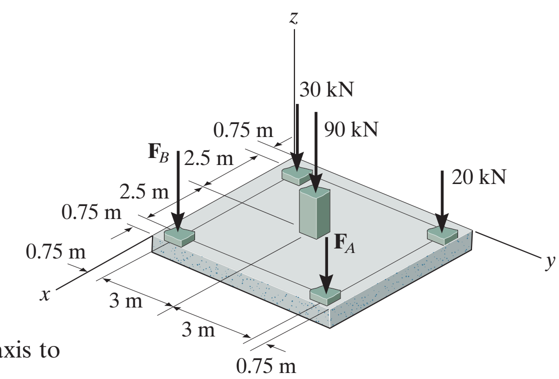
$x = 6.43 \text{ m}$ **Ans.**

$M_{R\,x} = \Sigma M_x; \qquad -140(y) = -(50)(3) - 20(11) - 50(13)$

$y = 7.29 \text{ m}$ **Ans.**

***4–132.**

If $F_A = 40$ kN and $F_B = 35$ kN, determine the magnitude of the resultant force and specify the location of its point of application (x, y) on the slab.



SOLUTION

Equivalent Resultant Force: By equating the sum of the forces along the z axis to the resultant force $\mathbf{F}_R$, Fig. b,

$$+\uparrow F_R = \Sigma F_z; \qquad -F_R = -30 - 20 - 90 - 35 - 40$$

$$F_R = 215 \text{ kN} \qquad \textbf{Ans.}$$

Point of Application: By equating the moment of the forces and $\mathbf{F}_R$, about the x and y axes,

$$(M_R)_x = \Sigma M_x; \qquad -215(y) = -35(0.75) - 30(0.75) - 90(3.75) - 20(6.75) - 40(6.75)$$

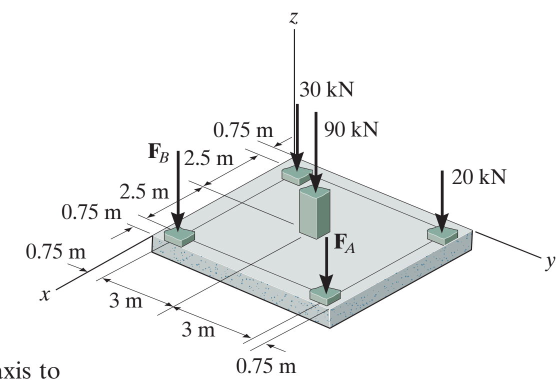
$$y = 3.68 \text{ m} \qquad \textbf{Ans.}$$

$$(M_R)_y = \Sigma M_y; \qquad 215(x) = 30(0.75) + 20(0.75) + 90(3.25) + 35(5.75) + 40(5.75)$$

$$x = 3.54 \text{ m} \qquad \textbf{Ans.}$$

4–133.

If the resultant force is required to act at the center of the slab, determine the magnitude of the column loadings $\mathbf{F}_A$ and $\mathbf{F}_B$ and the magnitude of the resultant force.



SOLUTION

Equivalent Resultant Force: By equating the sum of the forces along the z axis to the resultant force $\mathbf{F}_R$,

$+\uparrow F_R = \Sigma F_z; \qquad -F_R = -30 - 20 - 90 - F_A - F_B$

$$F_R = 140 + F_A + F_B \qquad \textbf{(1)}$$

Point of Application: By equating the moment of the forces and $\mathbf{F}_R$, about the x and y axes,

$(M_R)_x = \Sigma M_x; \qquad -F_R(3.75) = -F_B(0.75) - 30(0.75) - 90(3.75) - 20(6.75) - F_A(6.75)$

$$F_R = 0.2F_B + 1.8F_A + 132 \qquad \textbf{(2)}$$

$(M_R)_y = \Sigma M_y; \qquad F_R(3.25) = 30(0.75) + 20(0.75) + 90(3.25) + F_A(5.75) + F_B(5.75)$

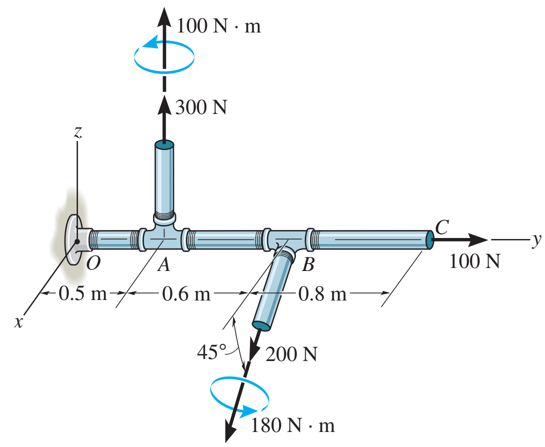
$$F_R = 1.769F_A + 1.769F_B + 101.54 \qquad \textbf{(3)}$$

Solving Eqs.(1) through (3) yields

$F_A = 30\,\text{kN} \qquad F_B = 20\text{ kN} \qquad F_R = 190\,\text{kN}$ **Ans.**

4–134.

Replace the two wrenches and the force, acting on the pipe assembly, by an equivalent resultant force and couple moment at point O.



SOLUTION

Force And Moment Vectors:

$$\mathbf{F}_1 = \{300\mathbf{k}\}\text{ N} \qquad \mathbf{F}_3 = \{100\mathbf{j}\}\text{ N}$$

$$\mathbf{F}_2 = 200\{\cos 45°\mathbf{i} - \sin 45°\mathbf{k}\}\text{ N}$$

$$= \{141.42\mathbf{i} - 141.42\mathbf{k}\}\text{ N}$$

$$\mathbf{M}_1 = \{100\mathbf{k}\}\text{ N}\cdot\text{m}$$

$$\mathbf{M}_2 = 180\{\cos 45°\mathbf{i} - \sin 45°\mathbf{k}\}\text{ N}\cdot\text{m}$$

$$= \{127.28\mathbf{i} - 127.28\mathbf{k}\}\text{ N}\cdot\text{m}$$

Equivalent Force and Couple Moment At Point O:

$\mathbf{F}_R = \Sigma\mathbf{F};\qquad \mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$

$$= 141.42\mathbf{i} + 100.0\mathbf{j} + (300 - 141.42)\mathbf{k}$$

$$= \{141\mathbf{i} + 100\mathbf{j} + 159\mathbf{k}\}\text{ N} \qquad \textbf{Ans.}$$

The position vectors are $\mathbf{r}_1 = \{0.5\mathbf{j}\}$ m and $\mathbf{r}_2 = \{1.1\mathbf{j}\}$ m.

$\mathbf{M}_{R_O} = \Sigma\mathbf{M}_O;\qquad \mathbf{M}_{R_O} = \mathbf{r}_1 \times \mathbf{F}_1 + \mathbf{r}_2 \times \mathbf{F}_2 + \mathbf{M}_1 + \mathbf{M}_2$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0.5 & 0 \\ 0 & 0 & 300 \end{vmatrix}$$

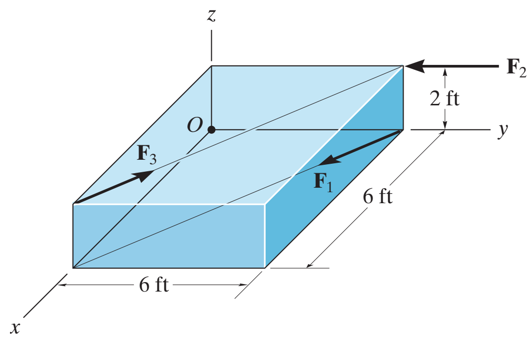
$$+ \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 1.1 & 0 \\ 141.42 & 0 & -141.42 \end{vmatrix}$$

$$+ 100\mathbf{k} + 127.28\mathbf{i} - 127.28\mathbf{k}$$

$$= \{122\mathbf{i} - 183\mathbf{k}\}\text{ N}\cdot\text{m} \qquad \textbf{Ans.}$$

4–135.

The three forces acting on the block each have a magnitude of 10 lb. Replace this system by a wrench and specify the point where the wrench intersects the z axis, measured from point O.



SOLUTION

$$\mathbf{F}_R = \{-10\mathbf{j}\}\text{ lb}$$

$$\mathbf{M}_O = (6\mathbf{j} + 2\mathbf{k}) \times (-10\mathbf{j}) + 2(10)(-0.707\mathbf{i} - 0.707\mathbf{j})$$

$$= \{5.858\mathbf{i} - 14.14\mathbf{j}\}\text{ lb}\cdot\text{ft}$$

Require

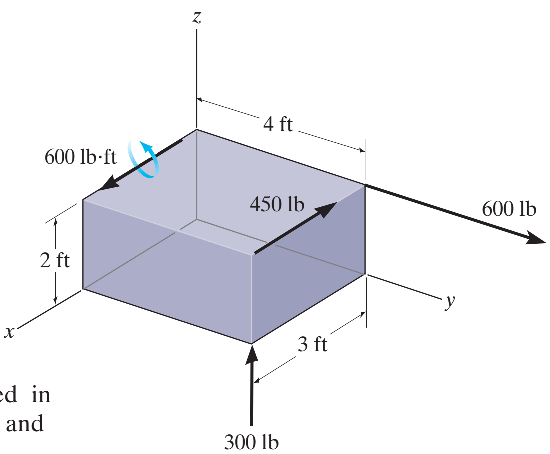
$$z = \frac{5.858}{10} = 0.586\text{ ft}$$ **Ans.**

$$\mathbf{F}_W = \{-10\mathbf{j}\}\text{ lb}$$ **Ans.**

$$\mathbf{M}_W = \{-14.1\mathbf{j}\}\text{ lb}\cdot\text{ft}$$ **Ans.**

***4–136.**

Replace the force and couple moment system acting on the rectangular block by a wrench. Specify the magnitude of the force and couple moment of the wrench and where its line of action intersects the x–y plane.



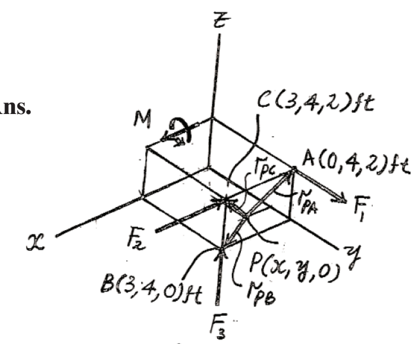
SOLUTION

Equivalent Resultant Force: The resultant forces $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ expressed in Cartesian vector form can be written as $\mathbf{F}_1 = [600\mathbf{j}]$ lb, $\mathbf{F}_2 = [-450\mathbf{i}]$ lb, and $\mathbf{F}_3 = [300\mathbf{k}]$ lb. The force of the wrench can be determined from

$$\mathbf{F}_R = \Sigma\mathbf{F}; \quad \mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = 600\mathbf{j} - 450\mathbf{i} + 300\mathbf{k} = [-450\mathbf{i} + 600\mathbf{j} + 300\mathbf{k}]\text{ lb}$$

Thus, the magnitude of the wrench force is given by

$$\mathbf{F}_R = \sqrt{(F_R)_x{}^2 + (F_R)_y{}^2 + (F_R)_z{}^2} = \sqrt{(-450)^2 + 600^2 + 300^2} = 807.77\text{ lb} = 808\text{ lb} \qquad \textbf{Ans.}$$



(a)

Equivalent Couple Moment: Here, we will assume that the axis of the wrench passes through point P, Figs. a and b. Since $\mathbf{M}_W$ is collinear with $\mathbf{F}_R$,

$$\mathbf{M}_W = M_W\mathbf{u}_{F_R} = M_W\left[\frac{-450\mathbf{i} + 600\mathbf{j} + 300\mathbf{k}}{\sqrt{(-450)^2 + 600^2 + 300^2}}\right]$$

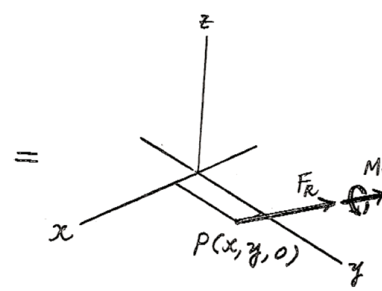
$$= -0.5571M_w\mathbf{i} + 0.7428M_w\mathbf{j} + 0.3714M_w\mathbf{k}$$

The position vectors $\mathbf{r}_{PA}$, $\mathbf{r}_{PB}$, and $\mathbf{r}_{PC}$ are

$$\mathbf{r}_{PA} = (0 - x)\mathbf{i} + (4 - y)\mathbf{j} + (2 - 0)\mathbf{k} = -x\mathbf{i} + (4 - y)\mathbf{j} + 2\mathbf{k}$$

$$\mathbf{r}_{PB} = (3 - x)\mathbf{i} + (4 - y)\mathbf{j} + (0 - 0)\mathbf{k} = (3 - x)\mathbf{i} + (4 - y)\mathbf{j}$$

$$\mathbf{r}_{PC} = (3 - x)\mathbf{i} + (4 - y)\mathbf{j} + (2 - 0)\mathbf{k} = (3 - x)\mathbf{i} + (4 - y)\mathbf{j} + 2\mathbf{k}$$



(b)

The couple moment $\mathbf{M}$ expressed in Cartesian vector form is written as $\mathbf{M} = [600\mathbf{i}]$ lb·ft. Summing the moments of $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ about point P and including $\mathbf{M}$,

$$\mathbf{M}_W = \Sigma\mathbf{M}_P; \qquad \mathbf{M}_W = \mathbf{r}_{PA} \times \mathbf{F}_1 + \mathbf{r}_{PC} \times \mathbf{F}_2 + \mathbf{r}_{PB} \times \mathbf{F}_3 + \mathbf{M}$$

$$-0.5571M_w\mathbf{i} + 0.7428M_w\mathbf{j} + 0.3714M_w\mathbf{k} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -x & (4 - y) & 2 \\ 0 & 600 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ (3 - x) & (4 - y) & 2 \\ -450 & 0 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ (3 - x) & (4 - y) & 0 \\ 0 & 0 & 300 \end{vmatrix} + 600\mathbf{i}$$

$$-0.5571M_w\mathbf{i} + 0.7428M_w\mathbf{j} + 0.3714M_w\mathbf{k} = (600 - 300y)\mathbf{i} + (300x - 1800)\mathbf{j} + (1800 - 600x - 450y)\mathbf{k}$$

Equating the $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ components,

$$-0.5571M_w = 600 - 300y \qquad \textbf{(1)}$$

$$0.7428M_w = 300x - 1800 \qquad \textbf{(2)}$$

$$0.3714M_w = 1800 - 600x - 450y \qquad \textbf{(3)}$$

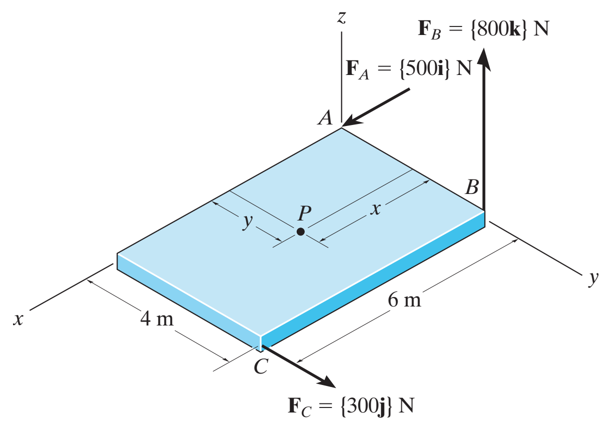
Solving Eqs. (1),(2),and (3) yields

$$x = 3.52\text{ ft} \qquad y = 0.138\text{ ft} \qquad M_W = -1003\text{ lb}\cdot\text{ft} \qquad \textbf{Ans.}$$

The negative sign indicates that $\mathbf{M}_W$ acts in the opposite sense to that of $\mathbf{F}_R$.

4–137.

Replace the three forces acting on the plate by a wrench. Specify the magnitude of the force and couple moment for the wrench and the point $P(x, y)$ where its line of action intersects the plate.



SOLUTION

$$\mathbf{F}_R = \{500\mathbf{i} + 300\mathbf{j} + 800\mathbf{k}\}\text{ N}$$

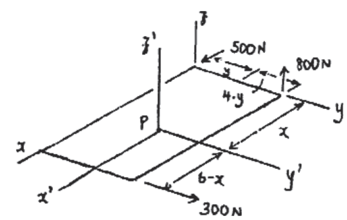
$$F_R = \sqrt{(500)^2 + (300)^2 + (800)^2} = 990\text{ N} \qquad \textbf{Ans.}$$

$$\mathbf{u}_{FR} = \{0.5051\mathbf{i} + 0.3030\mathbf{j} + 0.8081\mathbf{k}\}$$

$$M_{R_{x'}} = \Sigma M_{x'}; \qquad M_{R_{x'}} = 800(4 - y)$$

$$M_{R_{y'}} = \Sigma M_{y'}; \qquad M_{R_{y'}} = 800x$$

$$M_{R_{z'}} = \Sigma M_{z'}; \qquad M_{R_{z'}} = 500y + 300(6 - x)$$



Since $\mathbf{M}_R$ also acts in the direction of $\mathbf{u}_{FR}$,

$$M_R(0.5051) = 800(4 - y)$$

$$M_R(0.3030) = 800x$$

$$M_R(0.8081) = 500y + 300(6 - x)$$

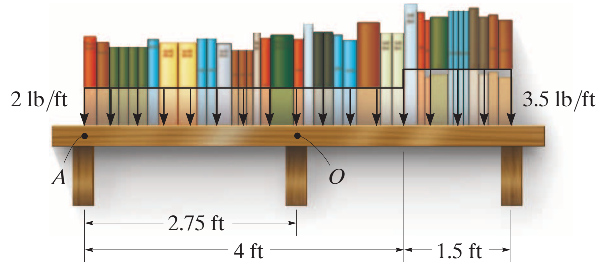
$$M_R = 3.07\text{ kN}\cdot\text{m} \qquad \textbf{Ans.}$$

$$x = 1.16\text{ m} \qquad \textbf{Ans.}$$

$$y = 2.06\text{ m} \qquad \textbf{Ans.}$$

4–138.

The loading on the bookshelf is distributed as shown. Determine the magnitude of the equivalent resultant location, measured from point O.

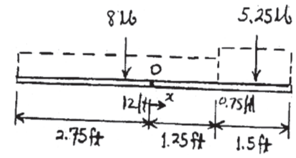


SOLUTION

$+\downarrow F_{RO} = \Sigma F;$ $\quad F_{RO} = 8 + 5.25 = 13.25 = 13.2 \text{ lb}\downarrow$ **Ans.**

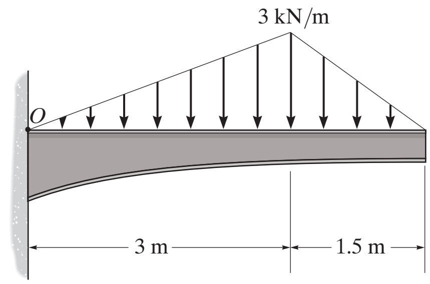
$\circlearrowleft + M_{RO} = \Sigma M_O;$ $\quad 13.25x = 5.25(0.75 + 1.25) - 8(2 - 1.25)$

$x = 0.340 \text{ ft}$ **Ans.**



4–139.

Replace the distributed loading with an equivalent resultant force, and specify its location on the beam measured from point O.



SOLUTION

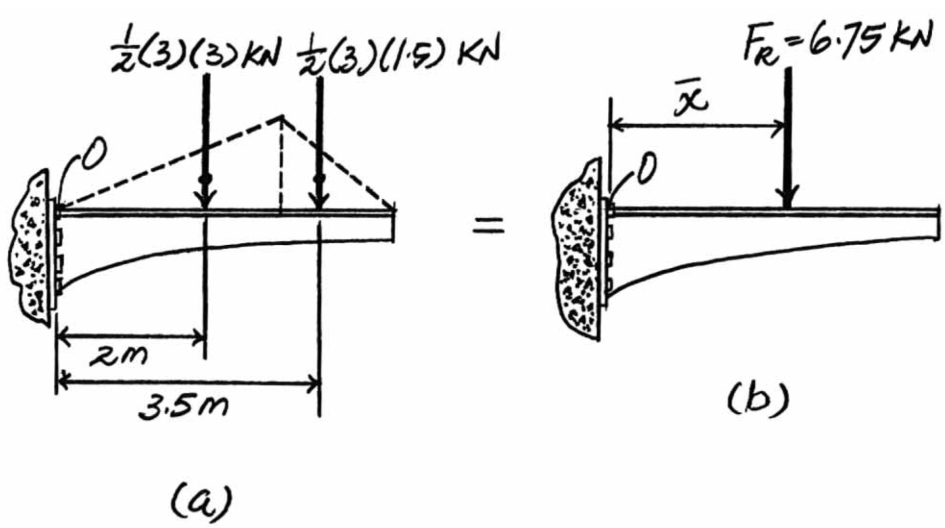
Loading: The distributed loading can be divided into two parts as shown in Fig. *a*.
Equations of Equilibrium: Equating the forces along the y axis of Figs. *a* and *b*, we have

$$+\downarrow F_R = \Sigma F; \qquad F_R = \frac{1}{2}(3)(3) + \frac{1}{2}(3)(1.5) = 6.75 \text{ kN} \downarrow$$ **Ans.**

If we equate the moment of F_R, Fig. *b*, to the sum of the moment of the forces in Fig. *a* about point O, we have

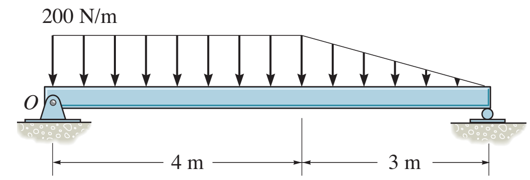
$$\circlearrowleft + (M_R)_O = \Sigma M_O; \qquad -6.75(\bar{x}) = -\frac{1}{2}(3)(3)(2) - \frac{1}{2}(3)(1.5)(3.5)$$

$$\bar{x} = 2.5 \text{ m}$$ **Ans.**



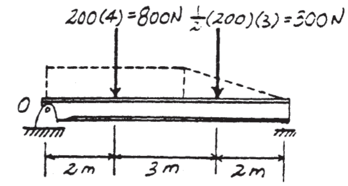
***4–140.**

Replace the loading by an equivalent force and couple moment acting at point O.



SOLUTION

Equivalent Force and Couple Moment At Point O:



$+\uparrow F_R = \Sigma F_y;\qquad F_R = -800 - 300$

$= -1100\text{ N} = 1.10\text{ kN}\downarrow$ **Ans.**

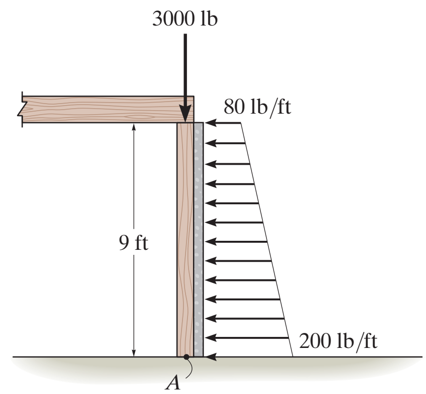
$\circlearrowleft + M_{R_O} = \Sigma M_O;\qquad M_{R_O} = -800(2) - 300(5)$

$= -3100\text{ N}\cdot\text{m}$

$= 3.10\text{ kN}\cdot\text{m}$ ***(Clockwise)*** **Ans.**

4–141.

The column is used to support the floor which exerts a force of 3000 lb on the top of the column. The effect of soil pressure along its side is distributed as shown. Replace this loading by an equivalent resultant force and specify where it acts along the column, measured from its base A.



SOLUTION

$$\overset{+}{\rightarrow} \Sigma F_{Rx} = \Sigma F_x; \qquad F_{Rx} = 720 + 540 = 1260 \text{ lb}$$

$$+\downarrow F_{Ry} = \Sigma F_y; \qquad F_{Ry} = 3000 \text{ lb}$$

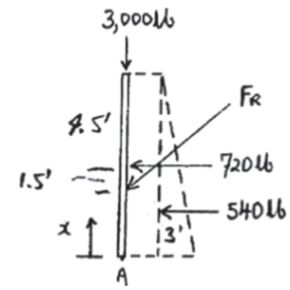
$$F_R = \sqrt{(1260)^2 + (3000)^2} = 3254 \text{ lb}$$

$$F_R = 3.25 \text{ kip} \qquad \textbf{Ans.}$$

$$\theta = \tan^{-1}\left[\frac{3000}{1260}\right] = 67.2^\circ \qquad \textbf{Ans.}$$

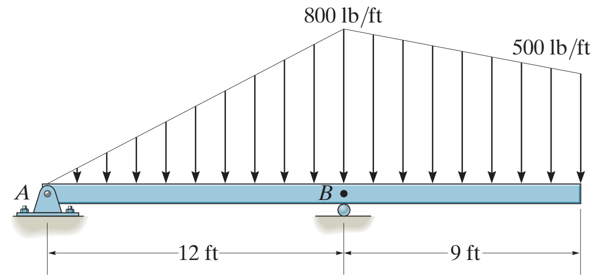
$$\circlearrowleft + M_{RA} = \Sigma M_A; \qquad 1260x = 540(3) + 720(4.5)$$

$$x = 3.86 \text{ ft} \qquad \textbf{Ans.}$$



4–142.

Replace the loading by an equivalent resultant force and specify its location on the beam, measured from point *B*.



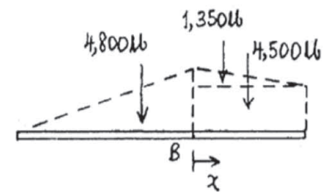
SOLUTION

$+\downarrow F_R = \Sigma F;$ $\quad F_R = 4800 + 1350 + 4500 = 10\,650 \text{ lb}$

$F_R = 10.6 \text{ kip} \downarrow$ **Ans.**

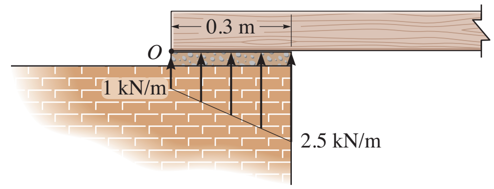
$\circlearrowleft + M_{RB} = \Sigma M_B;$ $\quad 10\,650x = -4800(4) + 1350(3) + 4500(4.5)$

$x = 0.479 \text{ ft}$ **Ans.**



4–143.

The masonry support creates the loading distribution acting on the end of the beam. Simplify this load to a single resultant force and specify its location measured from point O.



SOLUTION

Equivalent Resultant Force:

$+\uparrow F_R = \Sigma F_y; \qquad F_R = 1(0.3) + \frac{1}{2}(2.5 - 1)(0.3) = 0.525 \text{ kN} \uparrow$ **Ans.**

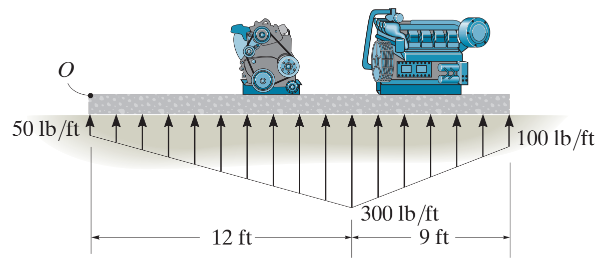
Location of Equivalent Resultant Force:

$\zeta + (M_R)_O = \Sigma M_O; \qquad 0.525(d) = 0.300(0.15) + 0.225(0.2)$

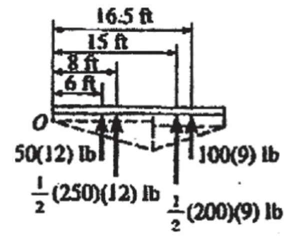
$d = 0.171 \text{ m}$ **Ans.**

***4–144.**

The distribution of soil loading on the bottom of a building slab is shown. Replace this loading by an equivalent resultant force and specify its location, measured from point O.



SOLUTION



$$+\uparrow F_R = \Sigma F_y; \quad F_R = 50(12) + \tfrac{1}{2}(250)(12) + \tfrac{1}{2}(200)(9) + 100(9)$$

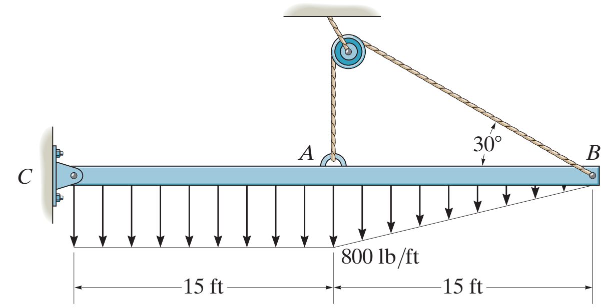
$$= 3900 \text{ lb} = 3.90 \text{ kip} \uparrow$$ **Ans.**

$$\circlearrowleft +M_{Ro} = \Sigma M_O; \quad 3900(d) = 50(12)(6) + \tfrac{1}{2}(250)(12)(8) + \tfrac{1}{2}(200)(9)(15) + 100(9)(16.5)$$

$$d = 11.3 \text{ ft}$$ **Ans.**

4–145.

Replace the distributed loading by an equivalent resultant force, and specify its location on the beam, measured from the pin at *C*.



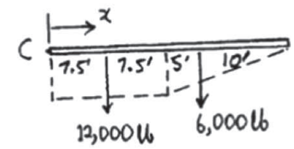
SOLUTION

$+\downarrow F_R = \Sigma F;$ $\quad F_R = 12\,000 + 6000 = 18\,000 \text{ lb}$

$F_R = 18.0 \text{ kip} \downarrow$ **Ans.**

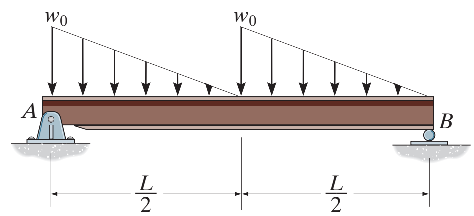
$\circlearrowleft + M_{RC} = \Sigma M_C;$ $\quad 18\,000x = 12\,000(7.5) + 6000(20)$

$x = 11.7 \text{ ft}$ **Ans.**



4–146.

Replace the distributed loading with an equivalent resultant force, and specify its location on the beam measured from point A.



SOLUTION

Loading: The distributed loading can be divided into two parts as shown in Fig. *a*. The magnitude and location of the resultant force of each part acting on the beam are also shown in Fig. *a*.

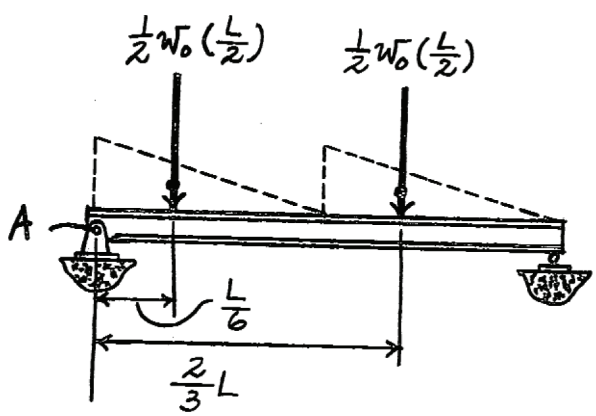
Resultants: Equating the sum of the forces along the y axis of Figs. *a* and *b*,

$$+\downarrow F_R = \Sigma F; \qquad F_R = \frac{1}{2}w_0\left(\frac{L}{2}\right) + \frac{1}{2}w_0\left(\frac{L}{2}\right) = \frac{1}{2}w_0 L \downarrow \qquad \textbf{Ans.}$$

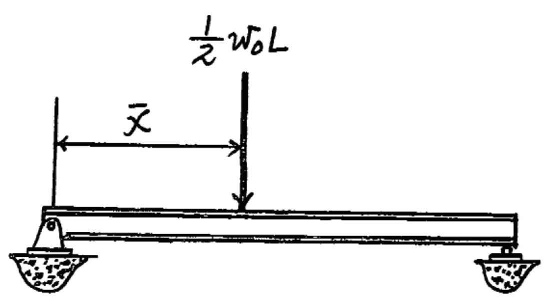
If we equate the moments of $\mathbf{F}_R$, Fig. *b*, to the sum of the moment of the forces in Fig. *a* about point A,

$$\circlearrowleft +(M_R)_A = \Sigma M_A; \qquad -\frac{1}{2}w_0 L(\bar{x}) = -\frac{1}{2}w_0\left(\frac{L}{2}\right)\left(\frac{L}{6}\right) - \frac{1}{2}w_0\left(\frac{L}{2}\right)\left(\frac{2}{3}L\right)$$

$$\bar{x} = \frac{5}{12}L \qquad \textbf{Ans.}$$



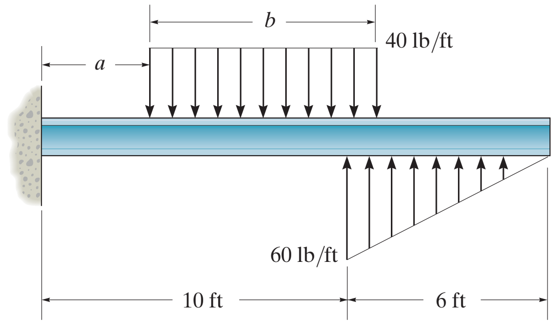
(a)



(b)

4–147.

The beam is subjected to the distributed loading. Determine the length b of the uniform load and its position a on the beam such that the resultant force and couple moment acting on the beam are zero.

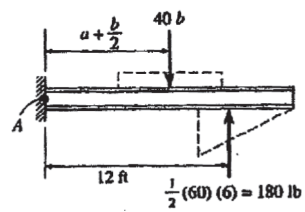


SOLUTION

Require $F_R = 0$.

$$+\uparrow F_R = \Sigma F_y; \quad 0 = 180 - 40b$$

$$b = 4.50 \text{ ft}$$ **Ans.**



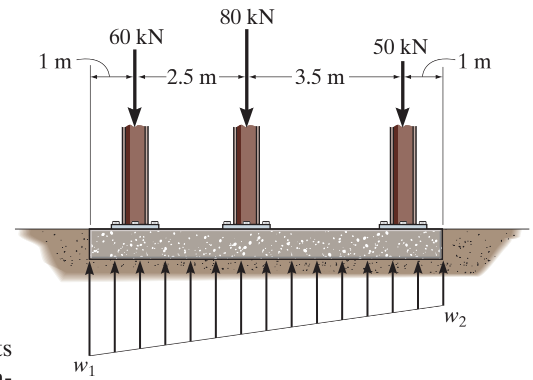
Require $M_{R_A} = 0$. Using the result $b = 4.50$ ft, we have

$$\zeta + M_{R_A} = \Sigma M_A; \quad 0 = 180(12) - 40(4.50)\left(a + \frac{4.50}{2}\right)$$

$$a = 9.75 \text{ ft}$$ **Ans.**

***4–148.**

If the soil exerts a trapezoidal distribution of load on the bottom of the footing, determine the intensities w_1 and w_2 of this distribution needed to support the column loadings.



SOLUTION

Loading: The trapezoidal reactive distributed load can be divided into two parts as shown on the free-body diagram of the footing, Fig. *a*. The magnitude and location measured from point A of the resultant force of each part are also indicated in Fig. *a*.

Equations of Equilibrium: Writing the moment equation of equilibrium about point B, we have

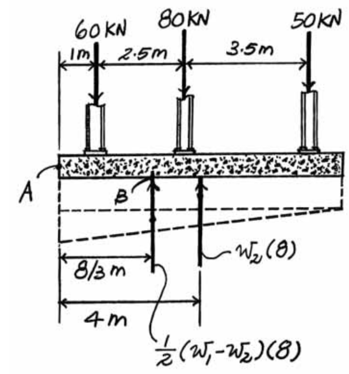
$$\circlearrowleft + \Sigma M_B = 0; \quad w_2(8)\left(4 - \frac{8}{3}\right) + 60\left(\frac{8}{3} - 1\right) - 80\left(3.5 - \frac{8}{3}\right) - 50\left(7 - \frac{8}{3}\right) = 0$$

$$w_2 = 17.1875 \text{ kN/m} = 17.2 \text{ kN/m} \qquad \textbf{Ans.}$$

Using the result of w_2 and writing the force equation of equilibrium along the y axis, we obtain

$$+\uparrow \Sigma F_y = 0; \quad \frac{1}{2}(w_1 - 17.1875)8 + 17.1875(8) - 60 - 80 - 50 = 0$$

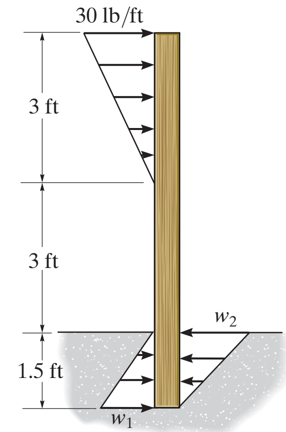
$$w_1 = 30.3125 \text{ kN/m} = 30.3 \text{ kN/m} \qquad \textbf{Ans.}$$



(a)

4–149.

The post is embedded into a concrete footing so that it is fixed supported. If the reaction of the concrete on the post can be approximated by the distributed loading shown, determine the intensity of w_1 and w_2 so that the resultant force and couple moment on the post due to the loadings are both zero.



SOLUTION

Loading: The magnitude and location of the resultant forces of each triangular distributed load are indicated in Fig. *a*.

Resultants: The resultant force $\mathbf{F}_R$ of the triangular distributed load is required to be zero. Referring to Fig. *a* and summing the forces along the *x* axis, we have

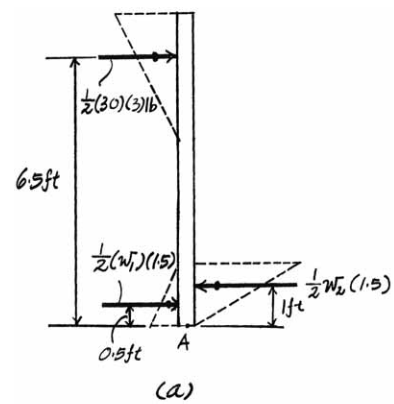
$$\overset{+}{\rightarrow} F_R = 0 = \Sigma F_x; \qquad 0 = \frac{1}{2}(30)(3) + \frac{1}{2}(w_1)(1.5) - \frac{1}{2}(w_2)(1.5)$$

$$0.75w_2 - 0.75w_1 = 45 \qquad \textbf{(1)}$$

Also, the resultant couple moment $\mathbf{M}_R$ of the triangular distributed load is required to be zero. Here, the moment will be summed about point A, as in Fig. *a*.

$$\circlearrowleft +(M_R)_A = 0 = \Sigma M_A; \qquad 0 = \frac{1}{2}(w_2)(1.5)(1) - \frac{1}{2}(w_1)(1.5)(0.5) - \frac{1}{2}(30)(3)(6.5)$$

$$0.75w_2 - 0.375w_1 = 292.5 \qquad \textbf{(2)}$$

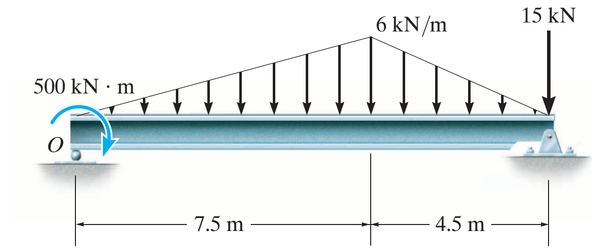


Solving Eqs. (1) and (2), yields

$$w_1 = 660 \text{ lb/ft} \qquad w_2 = 720 \text{ lb/ft} \qquad \textbf{Ans.}$$

4–150.

Replace the loading by an equivalent force and couple moment acting at point *O*.



SOLUTION

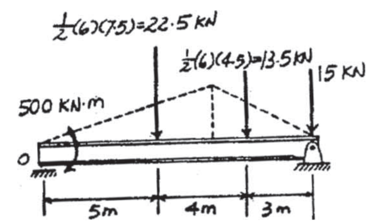
$+\uparrow F_R = \Sigma F_y;\qquad F_R = -22.5 - 13.5 - 15.0$

$= -51.0 \text{ kN} = 51.0 \text{ kN}\downarrow$ **Ans.**

$\circlearrowleft + M_{R_o} = \Sigma M_o;\qquad M_{R_o} = -500 - 22.5(5) - 13.5(9) - 15(12)$

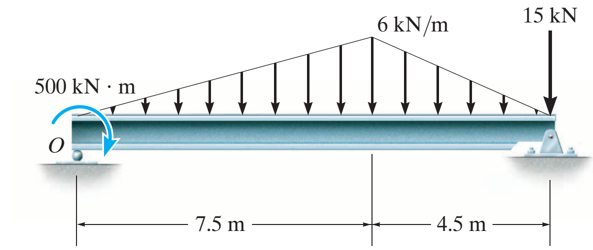
$= -914 \text{ kN}\cdot\text{m}$

$= 914 \text{ kN}\cdot\text{m}$ **(*Clockwise*)** **Ans.**



4–151.

Replace the loading by a single resultant force, and specify the location of the force measured from point *O*.



SOLUTION

Equivalent Resultant Force:

$+\uparrow F_R = \Sigma F_y;$ $\quad -F_R = -22.5 - 13.5 - 15$

$F_R = 51.0 \text{ kN} \downarrow$ **Ans.**

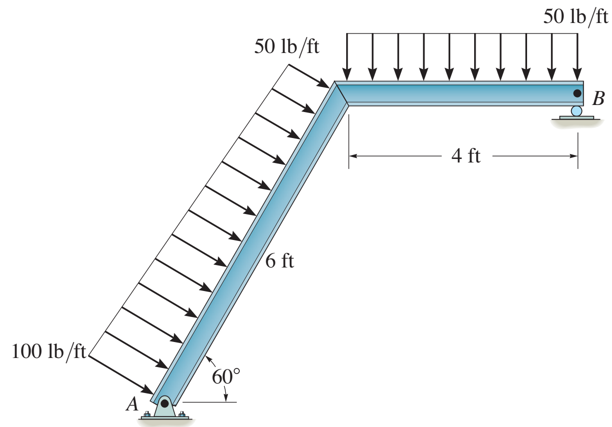
Location of Equivalent Resultant Force:

$\circlearrowleft + (M_R)_O = \Sigma M_O;$ $\quad -51.0(d) = -500 - 22.5(5) - 13.5(9) - 15(12)$

$d = 17.9 \text{ m}$ **Ans.**

***4–152.**

Replace the loading by an equivalent resultant force and couple moment at point A.



SOLUTION

$$F_1 = \frac{1}{2}(6)(50) = 150 \text{ lb}$$

$$F_2 = (6)(50) = 300 \text{ lb}$$

$$F_3 = (4)(50) = 200 \text{ lb}$$

$\xrightarrow{+} F_{Rx} = \Sigma F_x;\qquad F_{Rx} = 150 \sin 60° + 300 \sin 60° = 389.71 \text{ lb}$

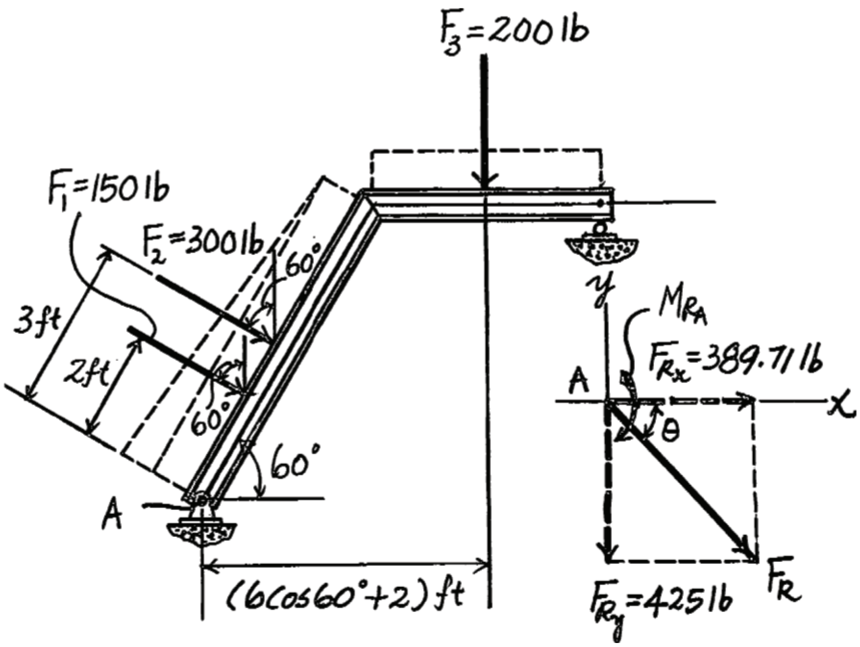
$+\downarrow F_{Ry} = \Sigma F_y;\qquad F_{Ry} = 150 \cos 60° + 300 \cos 60° + 200 = 425 \text{ lb}$

$$F_R = \sqrt{(389.71)^2 + (425)^2} = 577 \text{ lb}$$ **Ans.**

$$\theta = \tan^{-1}\left(\frac{425}{389.71}\right) = 47.5° \;\measuredangle$$ **Ans.**

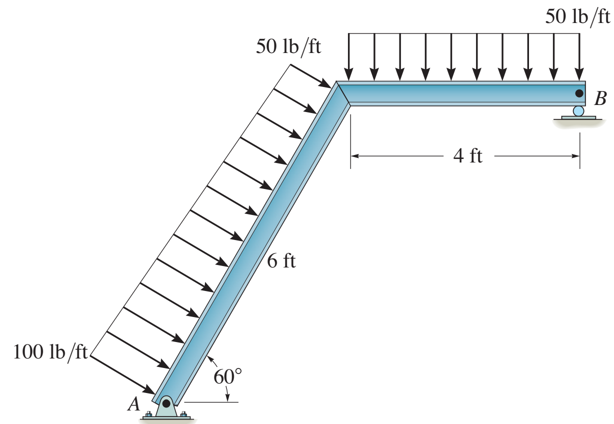
$\circlearrowleft + M_{RA} = \Sigma M_A;\qquad M_{RA} = 150\,(2) + 300\,(3) + 200\,(6 \cos 60° + 2)$

$$= 2200 \text{ lb}\cdot\text{ft} = 2.20 \text{ kip}\cdot\text{ft} \;\circlearrowright$$ **Ans.**



4–153.

Replace the loading by an equivalent resultant force and couple moment acting at point B.



SOLUTION

$$F_1 = \frac{1}{2}(6)(50) = 150 \text{ lb}$$

$$F_2 = (6)(50) = 300 \text{ lb}$$

$$F_2 = (4)(50) = 200 \text{ lb}$$

$\xrightarrow{+} F_{Rx} = \Sigma F_x;\qquad F_{Rx} = 150 \sin 60° + 300 \sin 60° = 389.71 \text{ lb}$

$+\downarrow F_{Ry} = \Sigma F_y;\qquad F_{Ry} = 150 \cos 60° + 300 \cos 60° + 200 = 425 \text{ lb}$

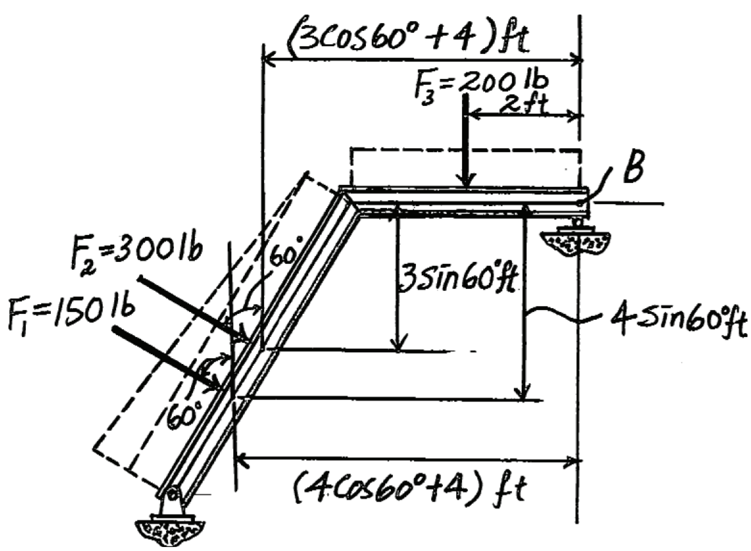
$$F_R = \sqrt{(389.71)^2 + (425)^2} = 577 \text{ lb}$$ **Ans.**

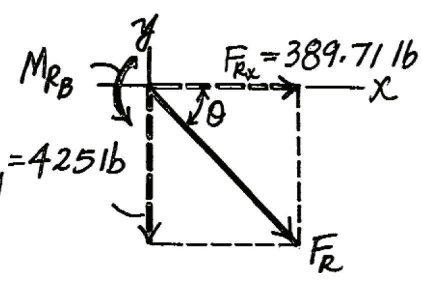
$$\theta = \tan^{-1}\left(\frac{425}{389.71}\right) = 47.5°$$ **Ans.**

$\circlearrowleft + M_{RB} = \Sigma M_B;\qquad M_{RB} = 150 \cos 60° (4 \cos 60° + 4) + 150 \sin 60° (4 \sin 60°)$

$+ 300 \cos 60° (3 \cos 60° + 4) + 300 \sin 60° (3 \sin 60°) + 200 (2)$

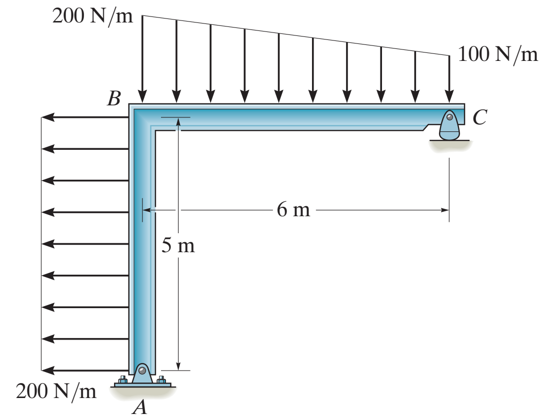
$$M_{RB} = 2800 \text{ lb}\cdot\text{ft} = 2.80 \text{ kip}\cdot\text{ft} \circlearrowright$$ **Ans.**





4–154.

Replace the distributed loading by an equivalent resultant force and specify where its line of action intersects member *AB*, measured from *A*.



SOLUTION

$\stackrel{+}{\leftarrow} \Sigma F_{Rx} = \Sigma F_x; \qquad F_{Rx} = 1000 \text{ N}$

$+\downarrow F_{Ry} = \Sigma F_y; \qquad F_{Ry} = 900 \text{ N}$

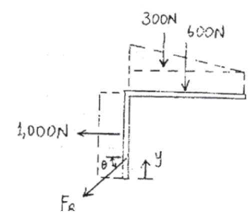
$F_R = \sqrt{(1000)^2 + (900)^2} = 1345 \text{ N}$

$F_R = 1.35 \text{ kN}$ **Ans.**

$\theta = \tan^{-1}\left[\frac{900}{1000}\right] = 42.0^\circ$ **Ans.**

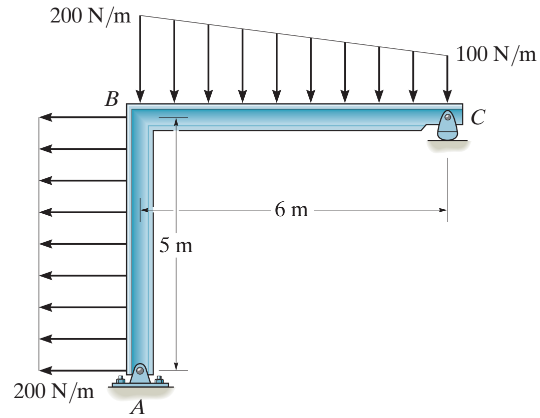
$\circlearrowleft + M_{RA} = \Sigma M_A; \qquad 1000y = 1000(2.5) - 300(2) - 600(3)$

$y = 0.1 \text{ m}$ **Ans.**



4–155.

Replace the distributed loading by an equivalent resultant force and specify where its line of action intersects member *BC*, measured from *C*.



SOLUTION

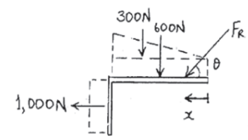
$\overset{+}{\leftarrow}\ \Sigma F_{Rx} = \Sigma F_x;\qquad F_{Rx} = 1000\ \text{N}$

$+\downarrow F_{Ry} = \Sigma F_y;\qquad F_{Ry} = 900\ \text{N}$

$F_R = \sqrt{(1000)^2 + (900)^2} = 1345\ \text{N}$

$F_R = 1.35\ \text{kN}$ **Ans.**

$\theta = \tan^{-1}\left[\dfrac{900}{1000}\right] = 42.0°$ **Ans.**

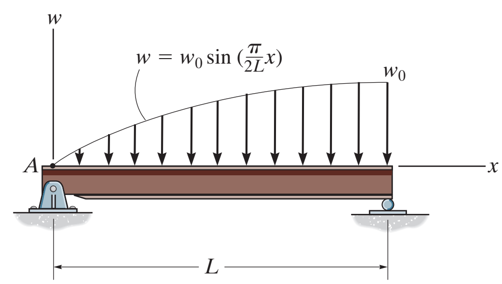


$\circlearrowleft + M_{RC} = \Sigma M_C;\qquad 900x = 600(3) + 300(4) - 1000(2.5)$

$x = 0.556\ \text{m}$ **Ans.**

***4–156.**

Replace the distributed loading with an equivalent resultant force, and specify its location on the beam measured from point A.



SOLUTION

Resultant: The magnitude of the differential force $d\mathbf{F}_R$ is equal to the area of the element shown shaded in Fig. *a*. Thus,

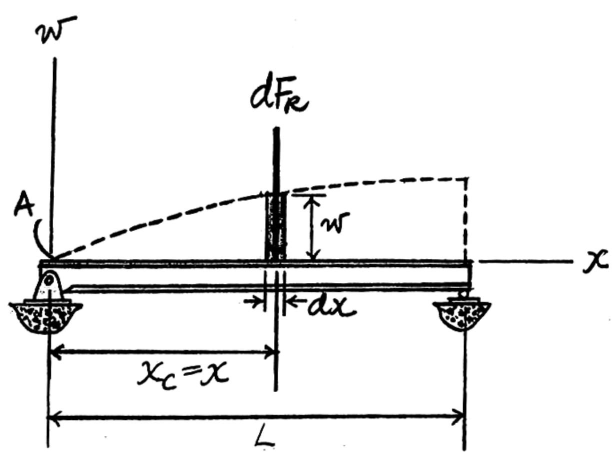
$$dF_R = w\,dx = \left(w_0 \sin \frac{\pi}{2L}x\right)dx$$

Integrating $d\mathbf{F}_R$ over the entire length of the beam gives the resultant force $\mathbf{F}_R$.

$$+\downarrow \qquad F_R = \int_L dF_R = \int_0^L \left(w_0 \sin \frac{\pi}{2L}x\right)dx = \left(-\frac{2w_0L}{\pi}\cos\frac{\pi}{2L}x\right)\Big|_0^L = \frac{2w_0L}{\pi}\downarrow \qquad \textbf{Ans.}$$

Location: The location of $d\mathbf{F}_R$ on the beam is $x_c = x$ measured from point A. Thus, the location $\bar{x}$ of $\mathbf{F}_R$ measured from point A is given by

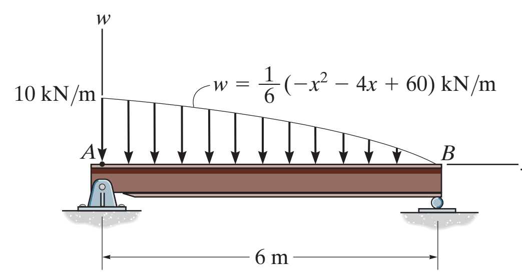
$$\bar{x} = \frac{\int_L x_c dF_R}{\int_L dF_R} = \frac{\int_0^L x\left(w_0 \sin \frac{\pi}{2L}x\right)dx}{\frac{2w_0L}{\pi}} = \frac{\frac{4w_0L^2}{\pi^2}}{\frac{2w_0L}{\pi}} = \frac{2L}{\pi} \qquad \textbf{Ans.}$$



(a)

4–157.

Replace the distributed loading with an equivalent resultant force, and specify its location on the beam measured from point A.



SOLUTION

Resultant: The magnitude of the differential force $d\mathbf{F}_R$ is equal to the area of the element shown shaded in Fig. *a*. Thus,

$$dF_R = w\,dx = \frac{1}{6}(-x^2 - 4x + 60)dx$$

Integrating $d\mathbf{F}_R$ over the entire length of the beam gives the resultant force $\mathbf{F}_R$.

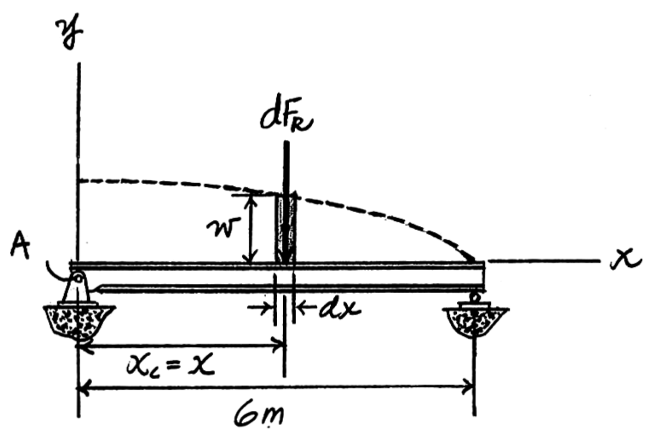
$$+\downarrow \qquad F_R = \int_L dF_R = \int_0^{6\text{ m}} \frac{1}{6}(-x^2 - 4x + 60)dx = \frac{1}{6}\left[-\frac{x^3}{3} - 2x^2 + 60x\right]\Bigg|_0^{6\text{ m}}$$

$$= 36 \text{ kN} \downarrow \qquad \textbf{Ans.}$$

Location: The location of $d\mathbf{F}_R$ on the beam is $x_c = x$, measured from point A. Thus the location $\overline{x}$ of $\mathbf{F}_R$ measured from point A is

$$\overline{x} = \frac{\int_L x_c dF_R}{\int_L dF_R} = \frac{\int_0^{6\text{ m}} x\left[\frac{1}{6}(-x^2 - 4x + 60)\right]dx}{36} = \frac{\int_0^{6\text{m}} \frac{1}{6}(-x^3 - 4x^2 + 60x)dx}{36} = \frac{\frac{1}{6}\left(-\frac{x^4}{4} - \frac{4x^3}{3} + 30x^2\right)\Bigg|_0^{6\text{m}}}{36}$$

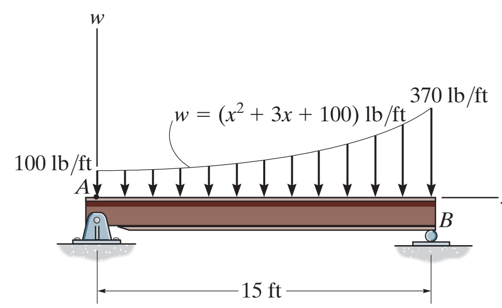
$$= 2.17 \text{ m} \qquad \textbf{Ans.}$$



(a)

4–158.

Replace the distributed loading with an equivalent resultant force, and specify its location on the beam measured from point A.



SOLUTION

Resultant: The magnitude of the differential force $d\mathbf{F}_R$ is equal to the area of the element shown shaded in Fig. *a*. Thus,

$$dF_R = w\,dx = \left(x^2 + 3x + 100\right)dx$$

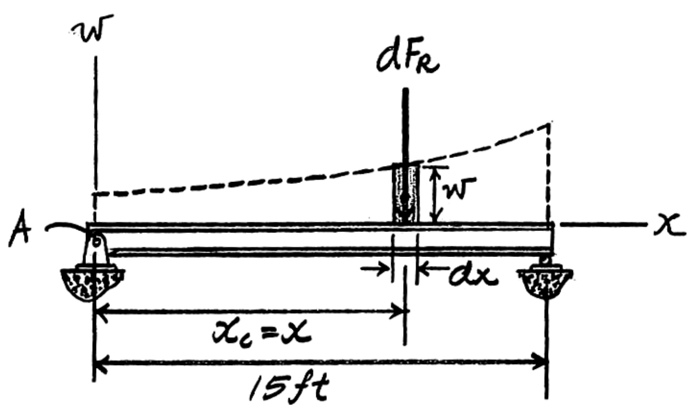
Integrating $d\mathbf{F}_R$ over the entire length of the beam gives the resultant force $\mathbf{F}_R$.

$$+\downarrow \qquad F_R = \int_L dF_R = \int_0^L \left(x^2 + 3x + 100\right)dx = \left(\frac{x^3}{3} + \frac{3x^2}{2} + 100x\right)\Bigg|_0^{15\text{ ft}}$$

$$= 2962.5\text{ lb} = 2.96\text{ kip}$$ **Ans.**

Location: The location of $d\mathbf{F}_R$ on the beam is $x_c = x$ measured from point A. Thus, the location $\bar{x}$ of $\mathbf{F}_R$ measured from point A is given by

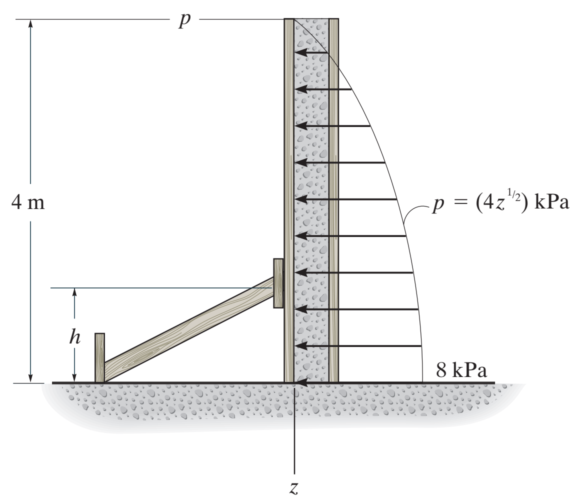
$$\bar{x} = \frac{\int_L x_c dF_R}{\int_L dF_R} = \frac{\int_0^{15\text{ ft}} x\left(x^2 + 3x + 100\right)dx}{2962.5} = \frac{\left(\frac{x^4}{4} + x^3 + 50x^2\right)\Big|_0^{15\text{ ft}}}{2962.5} = 9.21\text{ ft}$$ **Ans.**



(a)

4–159.

Wet concrete exerts a pressure distribution along the wall of the form. Determine the resultant force of this distribution and specify the height h where the bracing strut should be placed so that it lies through the line of action of the resultant force. The wall has a width of 5 m.



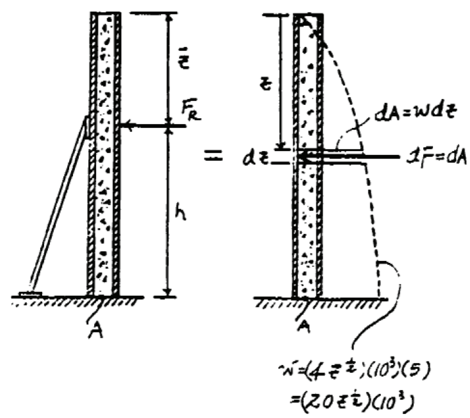
SOLUTION

Equivalent Resultant Force:

$$\stackrel{+}{\rightarrow} F_R = \Sigma F_x; \qquad -F_R = -\int dA = -\int_0^z w\,dz$$

$$F_R = \int_0^{4\text{ m}} \left(20z^{\frac{1}{2}}\right)\left(10^3\right) dz$$

$$= 106.67\left(10^3\right)\text{ N} = 107\text{ kN} \leftarrow$$ **Ans.**



Location of Equivalent Resultant Force:

$$\bar{z} = \frac{\int_A z\,dA}{\int_A dA} = \frac{\int_0^z zw\,dz}{\int_0^z w\,dz}$$

$$= \frac{\int_0^{4\text{ m}} z\left[\left(20z^{\frac{1}{2}}\right)(10^3)\right]dz}{\int_0^{4\text{ m}} \left(20z^{\frac{1}{2}}\right)(10^3)dz}$$

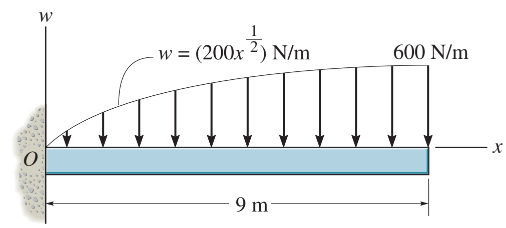
$$= \frac{\int_0^{4\text{ m}} \left[\left(20z^{\frac{3}{2}}\right)(10^3)\right]dz}{\int_0^{4\text{ m}} \left(20z^{\frac{1}{2}}\right)(10^3)dz}$$

$$= 2.40\text{ m}$$

Thus, $h = 4 - \bar{z} = 4 - 2.40 = 1.60\text{ m}$ **Ans.**

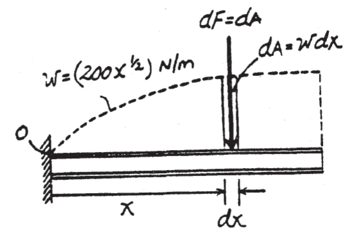
***4–160.**

Replace the loading by an equivalent force and couple moment acting at point O.



SOLUTION

Equivalent Resultant Force And Moment At Point O:



$$+\uparrow F_R = \Sigma F_y; \qquad F_R = -\int_A dA = -\int_0^x w\,dx$$

$$F_R = -\int_0^{9\text{ m}} \left(200x^{\frac{1}{2}}\right)dx$$

$$= -3600 \text{ N} = 3.60 \text{ kN} \downarrow$$ **Ans.**

$$\circlearrowleft + M_{R_O} = \Sigma M_O; \quad M_{R_O} = -\int_0^x xw\,dx$$

$$= -\int_0^{9\text{ m}} x\left(200x^{\frac{1}{2}}\right)dx$$

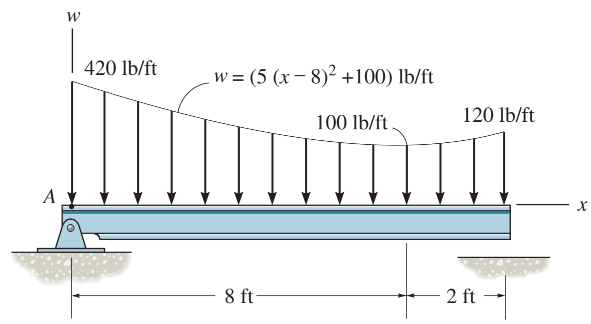
$$= -\int_0^{9\text{ m}} \left(200x^{\frac{3}{2}}\right)dx$$

$$= -19\,440 \text{ N}\cdot\text{m}$$

$$= 19.4 \text{ kN}\cdot\text{m}$$ ***(Clockwise)*** **Ans.**

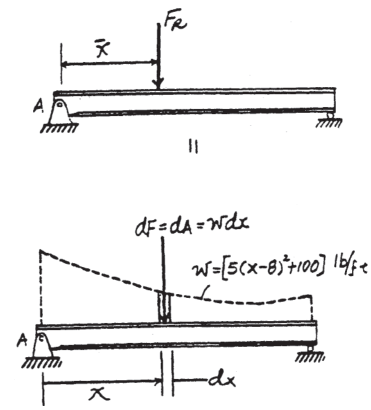
4–161.

Determine the magnitude of the equivalent resultant force of the distributed load and specify its location on the beam measured from point A.



SOLUTION

Equivalent Resultant Force:



$$+\uparrow F_R = \Sigma F_y; \qquad -F_R = -\int_A dA = -\int_0^x w\,dx$$

$$F_R = \int_0^{10\text{ ft}} [5(x-8)^2 + 100]dx$$

$$= 1866.67 \text{ lb} = 1.87 \text{ kip} \downarrow$$ **Ans.**

Location of Equivalent Resultant Force:

$$\tilde{x} = \frac{\int_A x\,dA}{\int_A dA} = \frac{\int_0^x xw\,dx}{\int_0^x w\,dx}$$

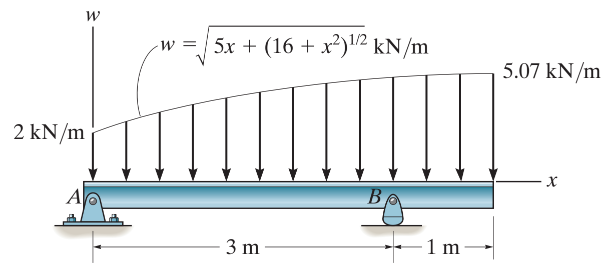
$$= \frac{\int_0^{10\text{ ft}} x[5(x-8)^2 + 100]dx}{\int_0^{10\text{ ft}} [5(x-8)^2 + 100]dx}$$

$$= \frac{\int_0^{10\text{ ft}} (5x^3 - 80x^2 + 420x)dx}{\int_0^{10\text{ ft}} [5(x-8)^2 + 100]dx}$$

$$= 3.66 \text{ ft}$$ **Ans.**

■**4–162.**

Determine the equivalent resultant force of the distributed loading and its location, measured from point A. Evaluate the integral using Simpson's rule.



SOLUTION

$$F_R = \int w dx = \int_0^4 \sqrt{5x + (16 + x^2)^{\frac{1}{2}}}\, dx$$

$$F_R = 14.9 \text{ kN}$$ **Ans.**

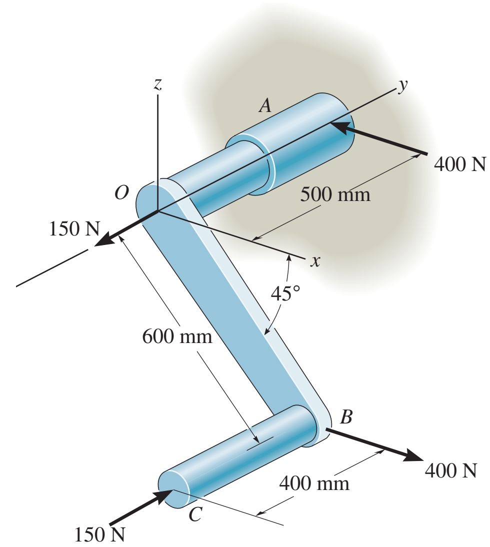
$$\int_0^4 \bar{x}\, dF = \int_0^4 (x)\sqrt{5x + (16x + x^2)^{\frac{1}{2}}}\, dx$$

$$= 33.74 \text{ kN} \cdot \text{m}$$

$$\bar{x} = \frac{33.74}{14.9} = 2.27 \text{ m}$$ **Ans.**

4–163.

Determine the resultant couple moment of the two couples that act on the assembly. Member OB lies in the x-z plane.



SOLUTION

For the 400-N forces:

$$\mathbf{M}_{C1} = \mathbf{r}_{AB} \times (400\mathbf{i})$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.6 \cos 45° & -0.5 & -0.6 \sin 45° \\ 400 & 0 & 0 \end{vmatrix}$$

$$= -169.7\mathbf{j} + 200\mathbf{k}$$

For the 150-N forces:

$$\mathbf{M}_{C2} = \mathbf{r}_{OB} \times (150\mathbf{j})$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.6 \cos 45° & 0 & -0.6 \sin 45° \\ 0 & 150 & 0 \end{vmatrix}$$

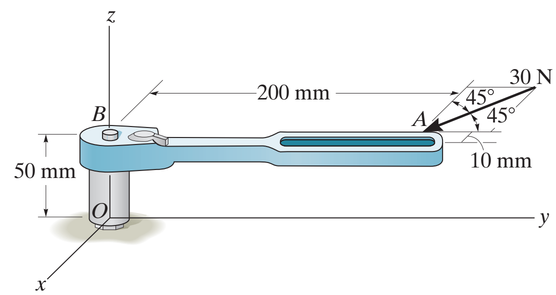
$$= 63.6\mathbf{i} + 63.6\mathbf{k}$$

$$\mathbf{M}_{CR} = \mathbf{M}_{C1} + \mathbf{M}_{C2}$$

$$\mathbf{M}_{CR} = \{63.6\mathbf{i} - 170\mathbf{j} + 264\mathbf{k}\}\ \text{N} \cdot \text{m}$$ **Ans.**

***4–164.**

The horizontal 30-N force acts on the handle of the wrench. What is the magnitude of the moment of this force about the z axis?



SOLUTION

Position Vector And Force Vectors:

$$\mathbf{r}_{BA} = \{-0.01\mathbf{i} + 0.2\mathbf{j}\}\text{ m}$$

$$\mathbf{r}_{OA} = [(-0.01 - 0)\mathbf{i} + (0.2 - 0)\mathbf{j} + (0.05 - 0)\mathbf{k}\}\text{ m}$$

$$= \{-0.01\mathbf{i} + 0.2\mathbf{j} + 0.05\mathbf{k}\}\text{ m}$$

$$\mathbf{F} = 30(\sin 45°\mathbf{i} - \cos 45°\mathbf{j})\text{ N}$$

$$= [21.213\mathbf{i} - 21.213\mathbf{j}]\text{ N}$$

Moment of Force* F *About z Axis: The unit vector along the z axis is $\mathbf{k}$. Applying Eq. 4–11, we have

$$M_z = \mathbf{k} \cdot (\mathbf{r}_{BA} \times \mathbf{F})$$

$$= \begin{vmatrix} 0 & 0 & 1 \\ -0.01 & 0.2 & 0 \\ 21.213 & -21.213 & 0 \end{vmatrix}$$

$$= 0 - 0 + 1[(-0.01)(-21.213) - 21.213(0.2)]$$

$$= -4.03\text{ N}\cdot\text{m}$$ **Ans.**

Or

$$M_z = \mathbf{k} \cdot (\mathbf{r}_{OA} \times \mathbf{F})$$

$$= \begin{vmatrix} 0 & 0 & 1 \\ -0.01 & 0.2 & 0.05 \\ 21.213 & -21.213 & 0 \end{vmatrix}$$

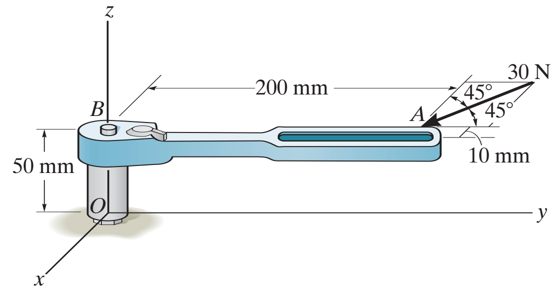
$$= 0 - 0 + 1[(-0.01)(-21.213) - 21.213(0.2)]$$

$$= -4.03\text{ N}\cdot\text{m}$$ **Ans.**

The negative sign indicates that $\mathbf{M}_z$, is directed along the negative z axis.

4–165.

The horizontal 30-N force acts on the handle of the wrench. Determine the moment of this force about point O. Specify the coordinate direction angles α, β, γ of the moment axis.



SOLUTION

Position Vector And Force Vectors:

$$\mathbf{r}_{OA} = \{(-0.01 - 0)\mathbf{i} + (0.2 - 0)\mathbf{j} + (0.05 - 0)\mathbf{k}\}\text{ m}$$

$$= \{-0.01\mathbf{i} + 0.2\mathbf{j} + 0.05\mathbf{k}\}\text{ m}$$

$$\mathbf{F} = 30(\sin 45°\mathbf{i} - \cos 45°\mathbf{j})\text{ N}$$

$$= \{21.213\mathbf{i} - 21.213\mathbf{j}\}\text{ N}$$

Moment of Force* F *About Point O: Applying Eq. 4–7, we have

$$\mathbf{M}_O = \mathbf{r}_{OA} \times \mathbf{F}$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.01 & 0.2 & 0.05 \\ 21.213 & -21.213 & 0 \end{vmatrix}$$

$$= \{1.061\mathbf{i} + 1.061\mathbf{j} - 4.031\mathbf{k}\}\text{ N}\cdot\text{m}$$

$$= \{1.06\mathbf{i} + 1.06\mathbf{j} - 4.03\mathbf{k}\}\text{ N}\cdot\text{m}$$ **Ans.**

The magnitude of $\mathbf{M}_O$ is

$$M_O = \sqrt{1.061^2 + 1.061^2 + (-4.031)^2} = 4.301\text{ N}\cdot\text{m}$$

The coordinate direction angles for $\mathbf{M}_O$ are

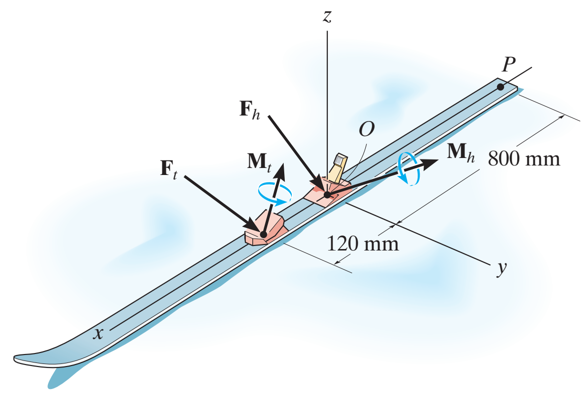
$$\alpha = \cos^{-1}\left(\frac{1.061}{4.301}\right) = 75.7°$$ **Ans.**

$$\beta = \cos^{-1}\left(\frac{1.061}{4.301}\right) = 75.7°$$ **Ans.**

$$\gamma = \cos^{-1}\left(\frac{-4.301}{4.301}\right) = 160°$$ **Ans.**

4–166.

The forces and couple moments that are exerted on the toe and heel plates of a snow ski are $\mathbf{F}_t = \{-50\mathbf{i} + 80\mathbf{j} - 158\mathbf{k}\}$ N, $\mathbf{M}_t = \{-6\mathbf{i} + 4\mathbf{j} + 2\mathbf{k}\}$ N · m, and $\mathbf{F}_h = \{-20\mathbf{i} + 60\mathbf{j} - 250\mathbf{k}\}$ N, $\mathbf{M}_h = \{-20\mathbf{i} + 8\mathbf{j} + 3\mathbf{k}\}$ N · m, respectively. Replace this system by an equivalent force and couple moment acting at point *P*. Express the results in Cartesian vector form.



SOLUTION

$$\mathbf{F}_R = \mathbf{F}_t + \mathbf{F}_h = \{-70\mathbf{i} + 140\mathbf{j} - 408\mathbf{k}\} \quad \text{N}$$ **Ans.**

$$\mathbf{M}_{RP} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.8 & 0 & 0 \\ -20 & 60 & -250 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.92 & 0 & 0 \\ -50 & 80 & -158 \end{vmatrix} + (-6\mathbf{i} + 4\mathbf{j} + 2\mathbf{k}) + (-20\mathbf{i} + 8\mathbf{j} + 3\mathbf{k})$$

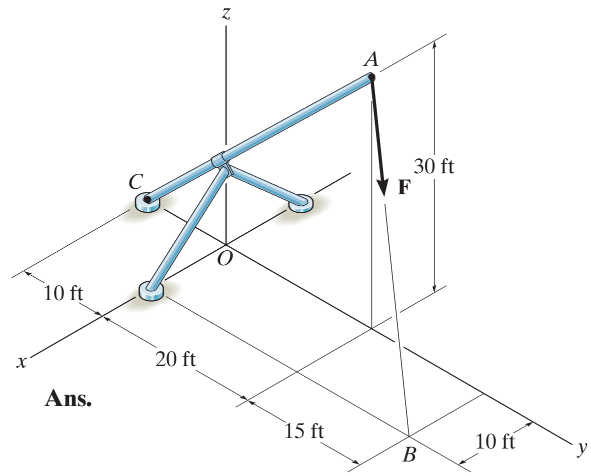
$$\mathbf{M}_{RP} = (200\mathbf{j} + 48\mathbf{k}) + (145.36\mathbf{j} + 73.6\mathbf{k}) + (-6\mathbf{i} + 4\mathbf{j} + 2\mathbf{k}) + (-20\mathbf{i} + 8\mathbf{j} + 3\mathbf{k})$$

$$\mathbf{M}_{RP} = \{-26\mathbf{i} + 357.36\mathbf{j} + 126.6\mathbf{k}\} \text{ N} \cdot \text{m}$$

$$\mathbf{M}_{RP} = \{-26\mathbf{i} + 357\mathbf{j} + 127\mathbf{k}\} \text{ N} \cdot \text{m}$$ **Ans.**

4–167.

Replace the force **F** having a magnitude of $F = 50$ lb and acting at point A by an equivalent force and couple moment at point C.



SOLUTION

$$\mathbf{F}_R = 50\left[\frac{(10\mathbf{i} + 15\mathbf{j} - 30\mathbf{k})}{\sqrt{(10)^2 + (15)^2 + (-30)^2}}\right]$$

$$\mathbf{F}_R = \{14.3\mathbf{i} + 21.4\mathbf{j} - 42.9\mathbf{k}\}\ \text{lb}$$ **Ans.**

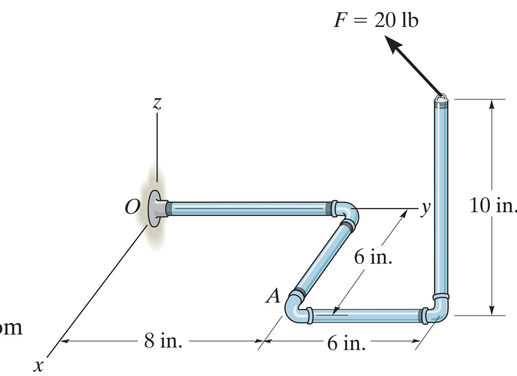
$$\mathbf{M}_{RC} = \mathbf{r}_{CB} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 10 & 45 & 0 \\ 14.29 & 21.43 & -42.86 \end{vmatrix}$$

$$= \{-1929\mathbf{i} + 428.6\mathbf{j} - 428.6\mathbf{k}\}\ \text{lb}\cdot\text{ft}$$

$$\mathbf{M}_A = \{-1.93\mathbf{i} + 0.429\mathbf{j} - 0.429\mathbf{k}\}\ \text{kip}\cdot\text{ft}$$ **Ans.**

***4–168.**

Determine the coordinate direction angles α, β, γ of **F**, which is applied to the end A of the pipe assembly, so that the moment of **F** about O is zero.



SOLUTION

Require $\mathbf{M}_O = \mathbf{0}$. This happens when force **F** is directed along line OA either from point O to A or from point A to O. The unit vectors $\mathbf{u}_{OA}$ and $\mathbf{u}_{AO}$ are

$$\mathbf{u}_{OA} = \frac{(6-0)\mathbf{i} + (14-0)\mathbf{j} + (10-0)\mathbf{k}}{\sqrt{(6-0)^2 + (14-0)^2 + (10-0)^2}}$$

$$= 0.3293\mathbf{i} + 0.7683\mathbf{j} + 0.5488\mathbf{k}$$

Thus,

$$\alpha = \cos^{-1} 0.3293 = 70.8^\circ \qquad \textbf{Ans.}$$

$$\beta = \cos^{-1} 0.7683 = 39.8^\circ \qquad \textbf{Ans.}$$

$$\gamma = \cos^{-1} 0.5488 = 56.7^\circ \qquad \textbf{Ans.}$$

or

$$\mathbf{u}_{AO} = \frac{(0-6)\mathbf{i} + (0-14)\mathbf{j} + (0-10)\mathbf{k}}{\sqrt{(0-6)^2 + (0-14)^2 + (0-10)^2}}$$

$$= -0.3293\mathbf{i} - 0.7683\mathbf{j} - 0.5488\mathbf{k}$$

Thus,

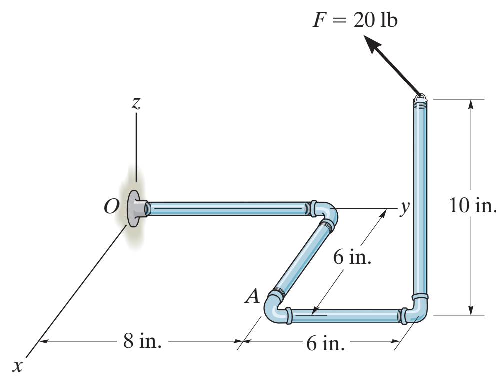
$$\alpha = \cos^{-1}(-0.3293) = 109^\circ \qquad \textbf{Ans.}$$

$$\beta = \cos^{-1}(-0.7683) = 140^\circ \qquad \textbf{Ans.}$$

$$\gamma = \cos^{-1}(-0.5488) = 123^\circ \qquad \textbf{Ans.}$$

4–169.

Determine the moment of the force **F** about point O. The force has coordinate direction angles of $\alpha = 60°$, $\beta = 120°$, $\gamma = 45°$. Express the result as a Cartesian vector.



SOLUTION

Position Vector And Force Vectors:

$$\mathbf{r}_{OA} = \{(6-0)\mathbf{i} + (14-0)\mathbf{j} + (10-0)\mathbf{k}\}\ \text{in.}$$

$$= \{6\mathbf{i} + 14\mathbf{j} + 10\mathbf{k}\}\ \text{in.}$$

$$\mathbf{F} = 20(\cos 60°\mathbf{i} + \cos 120°\mathbf{j} + \cos 45°\mathbf{k})\ \text{lb}$$

$$= (10.0\mathbf{i} - 10.0\mathbf{j} + 14.142\mathbf{k}\}\ \text{lb}$$

Moment of Force* F *About Point O: Applying Eq. 4–7, we have

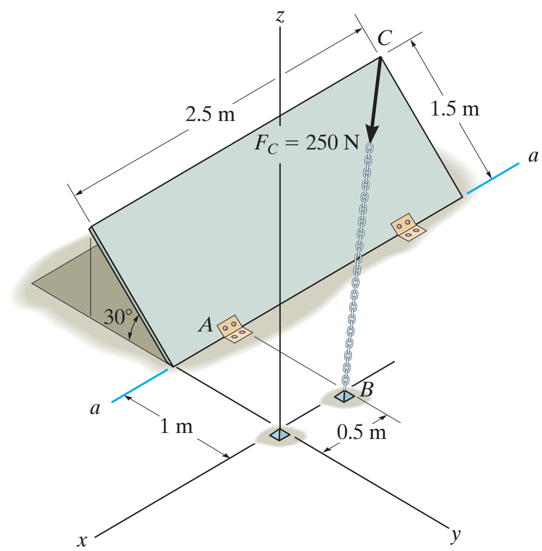
$$\mathbf{M}_O = \mathbf{r}_{OA} \times \mathbf{F}$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 6 & 14 & 10 \\ 10.0 & -10.0 & 14.142 \end{vmatrix}$$

$$= \{298\mathbf{i} + 15.1\mathbf{j} - 200\mathbf{k}\}\ \text{lb}\cdot\text{in}$$ **Ans.**

4–170.

Determine the moment of the force $\mathbf{F}_c$ about the door hinge at A. Express the result as a Cartesian vector.



SOLUTION

Position Vector And Force Vector:

$$\mathbf{r}_{AB} = \{[-0.5 - (-0.5)]\mathbf{i} + [0 - (-1)]\mathbf{j} + (0 - 0)\mathbf{k}\} \text{ m} = \{1\mathbf{j}\} \text{ m}$$

$$\mathbf{F}_C = 250\left(\frac{[-0.5 - (-2.5)]\mathbf{i} + \{0 - [-(1 + 1.5\cos 30°)]\}\mathbf{j} + (0 - 1.5\sin 30°)\mathbf{k}}{\sqrt{[-0.5 - (-2.5)]^2 + \{0 - [-(1 + 1.5\cos 30°)]\}^2 + (0 - 1.5\sin 30°)^2}}\right)\text{N}$$

$$= [159.33\mathbf{i} + 183.15\mathbf{j} - 59.75\mathbf{k}]\text{N}$$

Moment of Force $\mathbf{F}_c$ About Point A: Applying Eq. 4–7, we have

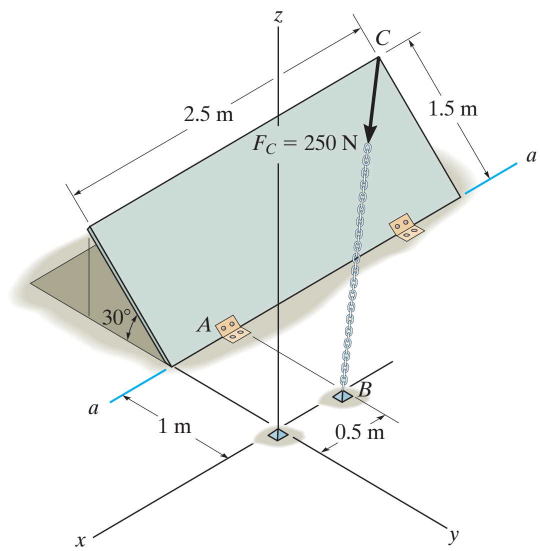
$$\mathbf{M}_A = \mathbf{r}_{AB} \times \mathbf{F}$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 1 & 0 \\ 159.33 & 183.15 & -59.75 \end{vmatrix}$$

$$= \{-59.7\mathbf{i} - 159\mathbf{k}\}\text{N} \cdot \text{m}$$ **Ans.**

4–171.

Determine the magnitude of the moment of the force $\mathbf{F}_c$ about the hinged axis *aa* of the door.



SOLUTION

Position Vector And Force Vectors:

$$\mathbf{r}_{AB} = \{[-0.5 - (-0.5)]\mathbf{i} + [0 - (-1)]\mathbf{j} + (0 - 0)\mathbf{k}\}\ \text{m} = \{1\mathbf{j}\}\ \text{m}$$

$$\mathbf{F}_C = 250\left(\frac{\{-0.5 - (-2.5)\}\mathbf{i} + \{0 - [-(1 + 1.5\cos 30°)]\}\mathbf{j} + (0 - 1.5\sin 30°)\mathbf{k}}{\sqrt{[-0.5 - (-2.5)]^2 + \{0 - [-(1 + 1.5\cos 30°)]\}^2 + (0 - 1.5\sin 30°)^2}}\right)N$$

$$= [159.33\mathbf{i} + 183.15\mathbf{j} - 59.75\mathbf{k}]\ \text{N}$$

Moment of Force $\mathbf{F}_c$ ***About a - a Axis:*** The unit vector along the $a - a$ axis is $\mathbf{i}$. Applying Eq. 4–11, we have

$$\mathbf{M}_{a-a} = \mathbf{i}\cdot(\mathbf{r}_{AB} \times \mathbf{F}_C)$$

$$= \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 159.33 & 183.15 & -59.75 \end{vmatrix}$$

$$= 1[1(-59.75) - (183.15)(0)] - 0 + 0$$

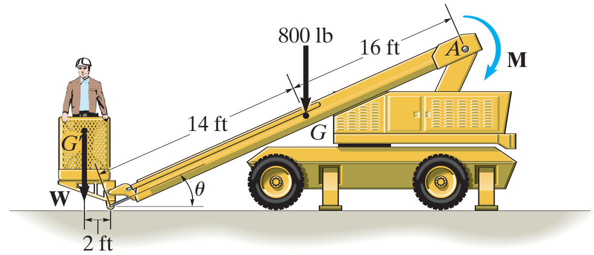
$$= -59.7\ \text{N}\cdot\text{m}$$

The negative sign indicates that M_{a-a} is directed toward the negative x axis.

$$M_{a-a} = 59.7\ \text{N}\cdot\text{m}$$ **Ans.**

***4–172.**

The boom has a length of 30 ft, a weight of 800 lb, and mass center at G. If the maximum moment that can be developed by the motor at A is $M = 20(10^3)$ lb·ft, determine the maximum load W, having a mass center at G', that can be lifted. Take $\theta = 30°$.



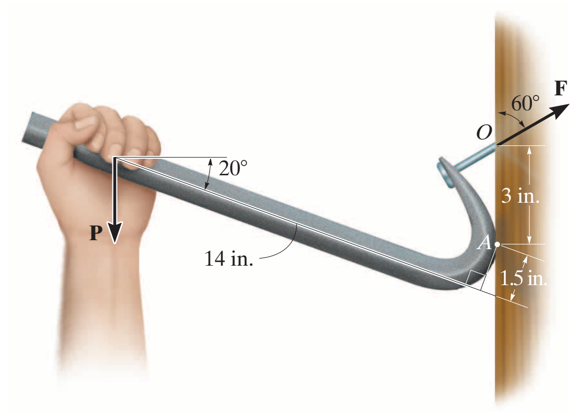
SOLUTION

$$20(10^3) = 800(16 \cos 30°) + W(30 \cos 30° + 2)$$

$$W = 319 \text{ lb}$$ **Ans.**

4–173.

If it takes a force of $F = 125$ lb to pull the nail out, determine the smallest vertical force **P** that must be applied to the handle of the crowbar. *Hint:* This requires the moment of **F** about point A to be equal to the moment of **P** about A. Why?



SOLUTION

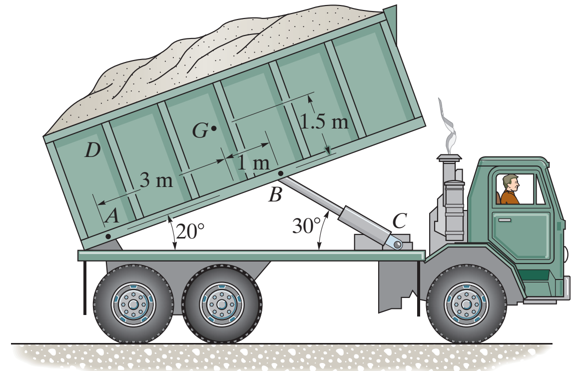
$$\circlearrowleft + M_F = 125(\sin 60°)(3) = 324.7595 \text{ lb} \cdot \text{in.}$$

$$\circlearrowleft + M_p = P(14 \cos 20° + 1.5 \sin 20°) = M_F = 324.7595 \text{ lb} \cdot \text{in.}$$

$$P = 23.8 \text{ lb}$$ **Ans.**

5–1.

Draw the free-body diagram of the dumpster D of the truck, which has a weight of 5000 lb and a center of gravity at G. It is supported by a pin at A and a pin-connected hydraulic cylinder BC (short link). Explain the significance of each force on the diagram. (See Fig. 5–7b.)



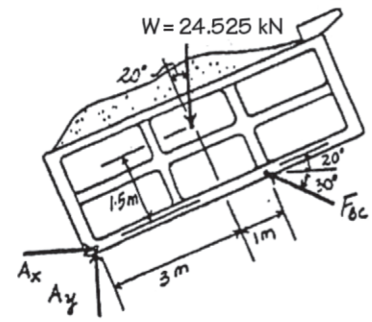
SOLUTION

The Significance of Each Force:

W is the effect of gravity (weight) on the dumpster.

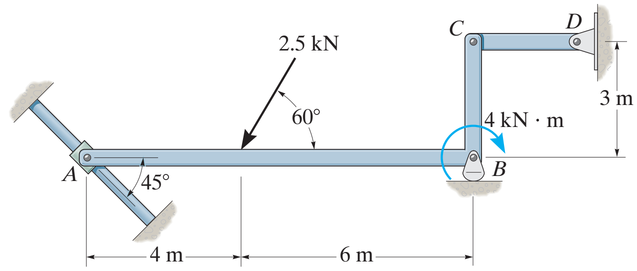
A_y and A_x are the pin A reactions on the dumpster.

F_{BC} is the hydraulic cylinder BC reaction on the dumpster.



5–2.

Draw the free-body diagram of member ABC which is supported by a smooth collar at A, rocker at B, and short link CD. Explain the significance of each force acting on the diagram. (See Fig. 5–7b.)



SOLUTION

The Significance of Each Force:

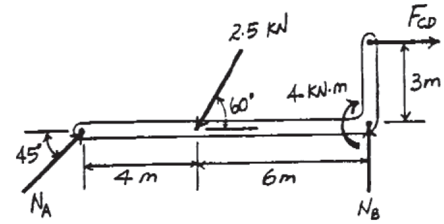
N_A is the smooth collar reaction on member ABC.

N_B is the rocker support B reaction on member ABC.

F_{CD} is the short link reaction on member ABC.

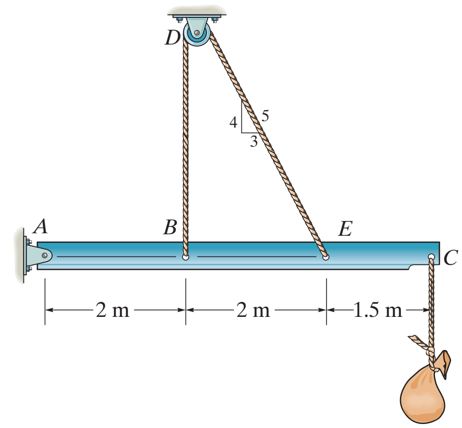
2.5 kN is the effect of external applied force on member ABC.

4 kN · m is the effect of external applied couple moment on member ABC.



5–3.

Draw the free-body diagram of the beam which supports the 80-kg load and is supported by the pin at A and a cable which wraps around the pulley at D. Explain the significance of each force on the diagram. (See Fig. 5–7b.)

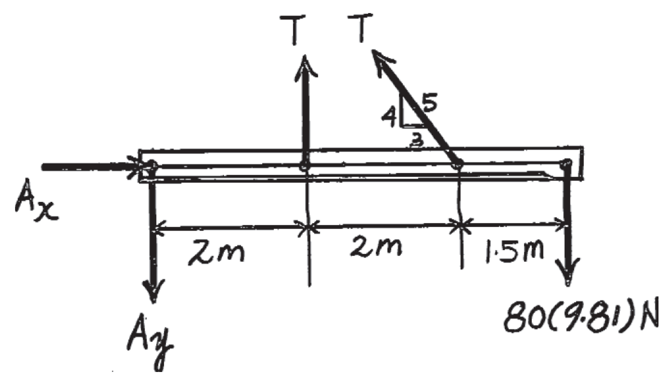


SOLUTION

T force of cable on beam.

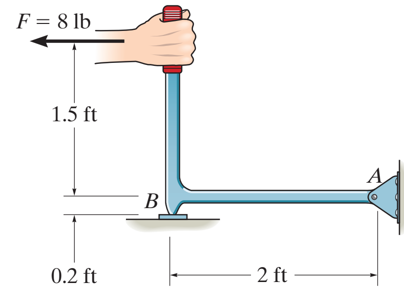
A_x, A_y force of pin on beam.

80(9.81)N force of load on beam.

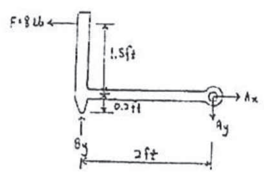


***5–4.**

Draw the free-body diagram of the hand punch, which is pinned at *A* and bears down on the smooth surface at *B*.

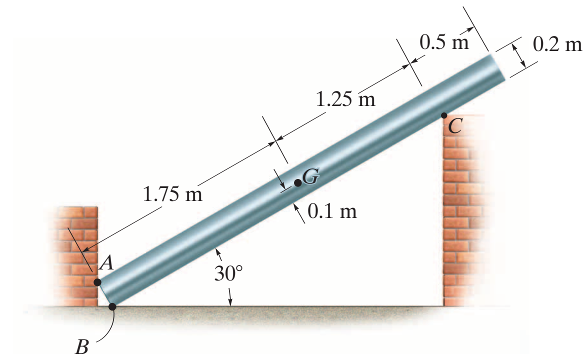


SOLUTION

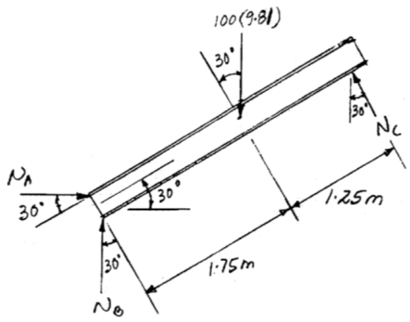


5–5.

Draw the free-body diagram of the uniform bar, which has a mass of 100 kg and a center of mass at *G*. The supports *A*, *B*, and *C* are smooth.

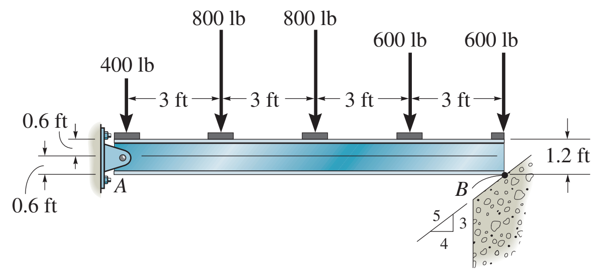


SOLUTION

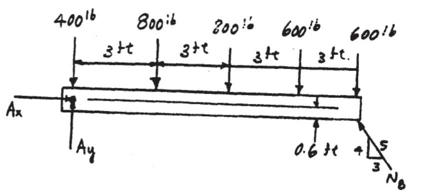


5–6.

Draw the free-body diagram of the beam, which is pin supported at *A* and rests on the smooth incline at *B*.

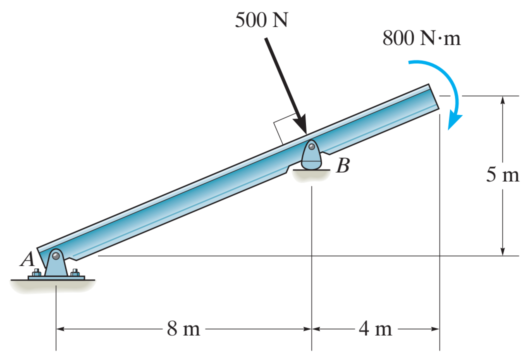


SOLUTION

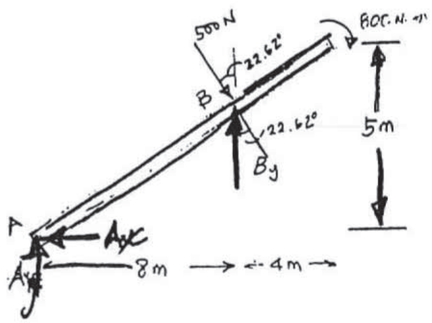


5–7.

Draw the free-body diagram of the beam, which is pin connected at A and rocker-supported at B.

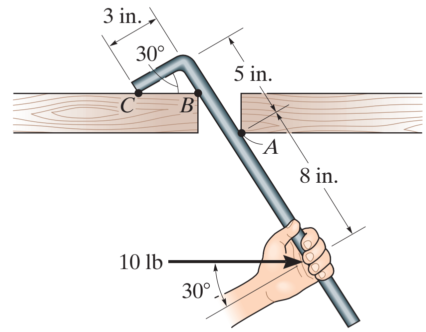


SOLUTION



***5–8.**

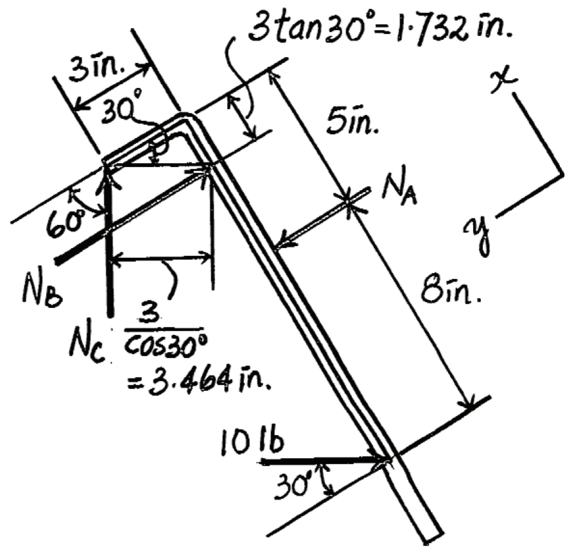
Draw the free-body diagram of the bar, which has a negligible thickness and smooth points of contact at A, B, and C. Explain the significance of each force on the diagram. (See Fig. 5–7*b*.)



SOLUTION

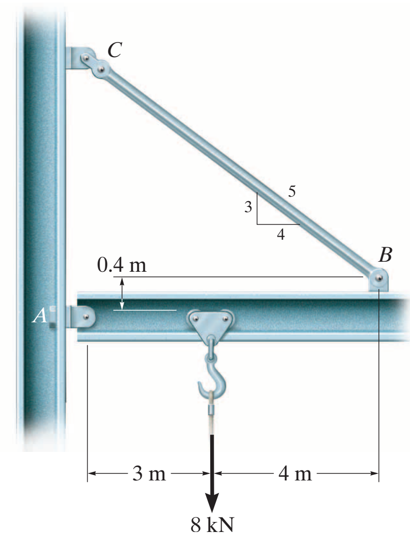
N_A, N_B, N_C force of wood on bar.

10 lb force of hand on bar.

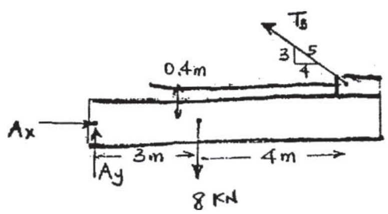


5–9.

Draw the free-body diagram of the jib crane AB, which is pin connected at A and supported by member (link) BC.

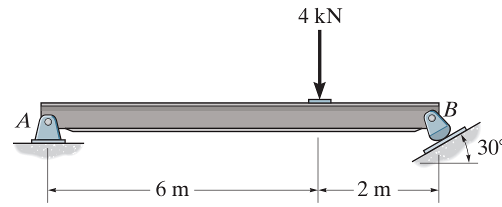


SOLUTION



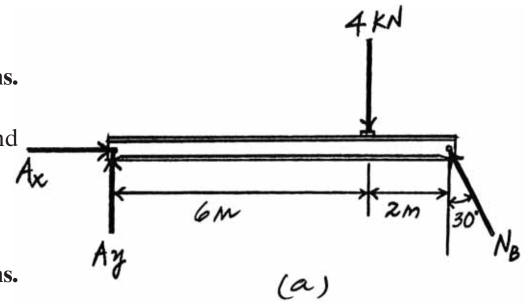
5–10.

Determine the horizontal and vertical components of reaction at the pin A and the reaction of the rocker B on the beam.



SOLUTION

Equations of Equilibrium: From the free-body diagram of the beam, Fig. a, N_B can be obtained by writing the moment equation of equilibrium about point A.



$$\circlearrowleft + \Sigma M_A = 0; \qquad N_B \cos 30°(8) - 4(6) = 0$$

$$N_B = 3.464 \text{ kN} = 3.46 \text{ kN} \qquad \textbf{Ans.}$$

Using this result and writing the force equations of equilibrium along the x and y axes, we have

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad A_x - 3.464 \sin 30° = 0$$

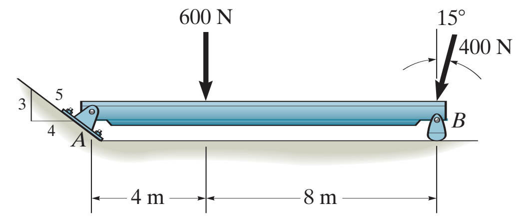
$$A_x = 1.73 \text{ kN} \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad A_y + 3.464 \cos 30° - 4 = 0$$

$$A_y = 1.00 \text{ kN} \qquad \textbf{Ans.}$$

5–11.

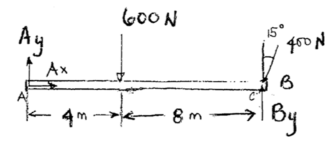
Determine the magnitude of the reactions on the beam at A and B. Neglect the thickness of the beam.



SOLUTION

$\circlearrowleft + \Sigma M_A = 0;$ $\quad B_y(12) - (400\cos 15°)(12) - 600(4) = 0$

$B_y = 586.37 = 586 \text{ N}$ **Ans.**



$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad A_x - 400 \sin 15° = 0$

$A_x = 103.528 \text{ N}$

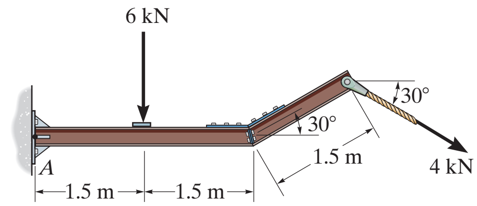
$+\uparrow \Sigma F_y = 0;$ $\quad A_y - 600 - 400 \cos 15° + 586.37 = 0$

$A_y = 400 \text{ N}$

$F_A = \sqrt{(103.528)^2 + (400)^2} = 413 \text{ N}$ **Ans.**

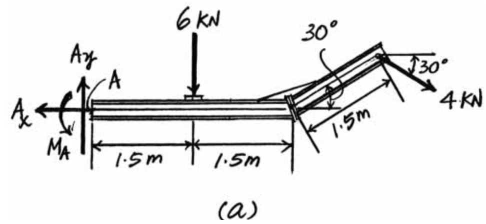
***5–12.**

Determine the components of the support reactions at the fixed support A on the cantilevered beam.



SOLUTION

Equations of Equilibrium: From the free-body diagram of the cantilever beam, Fig. *a*, A_x, A_y, and M_A can be obtained by writing the moment equation of equilibrium about point A.



$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 4 \cos 30° - A_x = 0$

$A_x = 3.46 \text{ kN}$ **Ans.**

$+\uparrow \Sigma F_y = 0; \qquad A_y - 6 - 4 \sin 30° = 0$

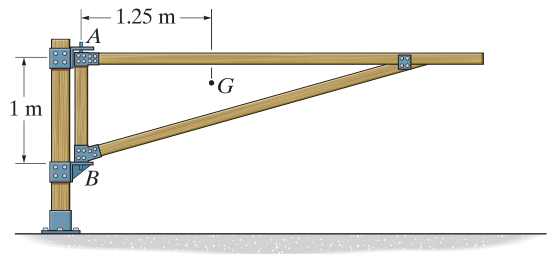
$A_y = 8 \text{ kN}$ **Ans.**

$\circlearrowleft + \Sigma M_A = 0; M_A - 6(1.5) - 4 \cos 30° \,(1.5 \sin 30°) - 4 \sin 30°(3 + 1.5 \cos 30°) = 0$

$M_A = 20.2 \text{ kN} \cdot \text{m}$ **Ans.**

5–13.

The 75-kg gate has a center of mass located at G. If A supports only a horizontal force and B can be assumed as a pin, determine the components of reaction at these supports.



SOLUTION

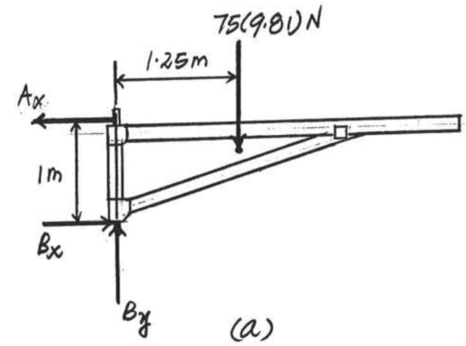
Equations of Equilibrium: From the free-body diagram of the gate, Fig. a, B_y and A_x can be obtained by writing the force equation of equilibrium along the y axis and the moment equation of equilibrium about point B.

$+\uparrow \Sigma F_y = 0; \qquad B_y - 75(9.81) = 0$

$B_y = 735.75 \text{ N} = 736 \text{ N}$ **Ans.**

$\circlearrowleft + \Sigma M_B = 0; \qquad A_x(1) - 75(9.81)(1.25) = 0$

$A_x = 919.69 \text{ N} = 920 \text{ N}$ **Ans.**



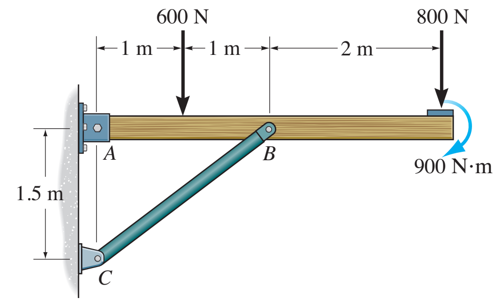
Using the result $A_x = 919.69$ N and writing the force equation of equilibrium along the x axis, we have

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad B_x - 919.69 = 0$

$B_x = 919.69 \text{ N} = 920 \text{ N}$ **Ans.**

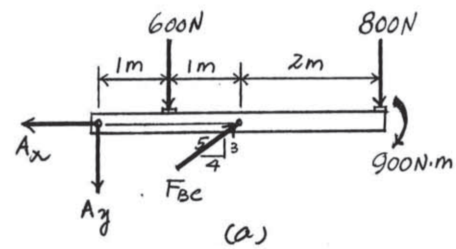
5–14.

The overhanging beam is supported by a pin at A and the two-force strut BC. Determine the horizontal and vertical components of reaction at A and the reaction at B on the beam.



SOLUTION

Equations of Equilibrium: Since line BC is a two-force member, it will exert a force $\mathbf{F}_{BC}$ directed along its axis on the beam as shown on the free-body diagram, Fig. a. From the free-body diagram, F_{BC} can be obtained by writing the moment equation of equilibrium about point A.



$$\circlearrowleft + \Sigma M_A = 0; \qquad F_{BC}\left(\frac{3}{5}\right)(2) - 600(1) - 800(4) - 900 = 0$$

$$F_{BC} = 3916.67 \text{ N} = 3.92 \text{ kN} \qquad \textbf{Ans.}$$

Using this result and writing the force equations of equilibrium along the x and y axes, we have

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 3916.67\left(\frac{4}{5}\right) - A_x = 0$$

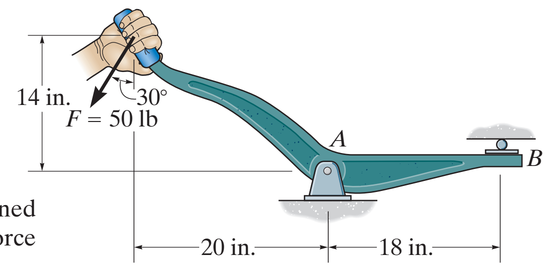
$$A_x = 3133.33 \text{ N} = 3.13 \text{ kN} \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad -A_y - 600 - 800 + 3916.67\left(\frac{3}{5}\right) = 0$$

$$A_y = 950 \text{ N} \qquad \textbf{Ans.}$$

5–15.

Determine the horizontal and vertical components of reaction at the pin at A and the reaction of the roller at B on the lever.



SOLUTION

Equations of Equilibrium: From the free-body diagram, F_B and A_x can be obtained by writing the moment equation of equilibrium about point A and the force equation of equilibrium along the x axis, respectively.

$\circlearrowleft + \Sigma M_A = 0;$ $\quad 50 \cos 30°(20) + 50 \sin 30°(14) - F_B(18) = 0$

$F_B = 67.56 \text{ lb} = 67.6 \text{ lb}$ **Ans.**

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad A_x - 50 \sin 30° = 0$

$A_x = 25 \text{ lb}$ **Ans.**

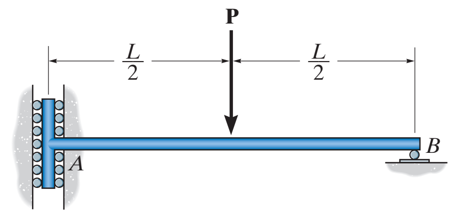
Using the result $F_B = 67.56$ lb and writing the force equation of equilibrium along the y axis, we have

$+\uparrow \Sigma F_y = 0;$ $\quad A_y - 50 \cos 30° - 67.56 = 0$

$A_y = 110.86 \text{ lb} = 111 \text{ lb}$ **Ans.**

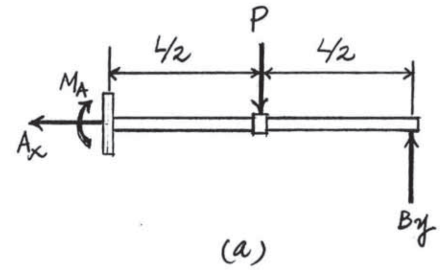
***5–16.**

Determine the components of reaction at the supports A and B on the rod.



SOLUTION

Equations of Equilibrium: Since the roller at A offers no resistance to vertical movement, the vertical component of reaction at support A is equal to zero. From the free-body diagram, A_x, B_y, and M_A can be obtained by writing the force equations of equilibrium along the x and y axes and the moment equation of equilibrium about point B, respectively.



$\overset{+}{\rightarrow}\Sigma F_x = 0;$ $\quad A_x = 0$ **Ans.**

$+\uparrow\Sigma F_y = 0;$ $\quad B_y - P = 0$

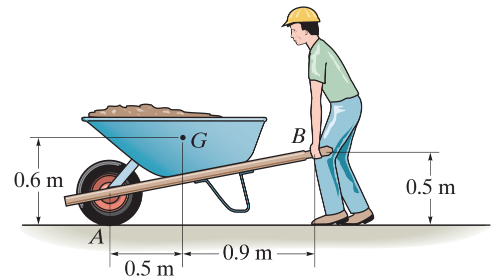
$B_y = P$ **Ans.**

$\circlearrowleft +\Sigma M_B = 0;$ $\quad P\left(\frac{L}{2}\right) - M_A = 0$

$M_A = \frac{PL}{2}$ **Ans.**

5–17.

If the wheelbarrow and its contents have a mass of 60 kg and center of mass at G, determine the magnitude of the resultant force which the man must exert on *each* of the two handles in order to hold the wheelbarrow in equilibrium.



SOLUTION

$\zeta + \Sigma M_B = 0;$ $\quad -A_y\,(1.4) + 60(9.81)(0.9) = 0$

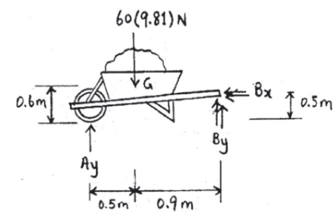
$A_y = 378.39 \text{ N}$

$+\uparrow \Sigma F_y = 0;$ $\quad 378.39 - 60(9.81) + 2B_y = 0$

$B_y = 105.11 \text{ N}$

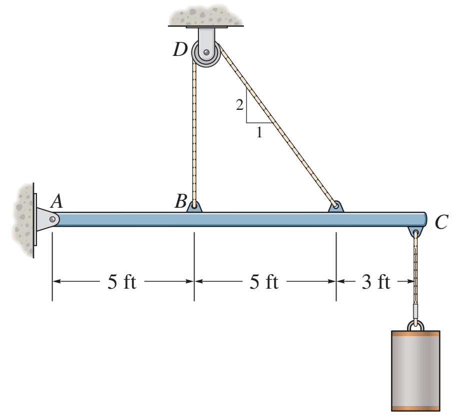
$\xrightarrow{+} \Sigma F_x = 0;$ $\quad B_x = 0$

$F_B = 105 \text{ N}$ **Ans.**



5–18.

Determine the tension in the cable and the horizontal and vertical components of reaction of the pin A. The pulley at D is frictionless and the cylinder weighs 80 lb.

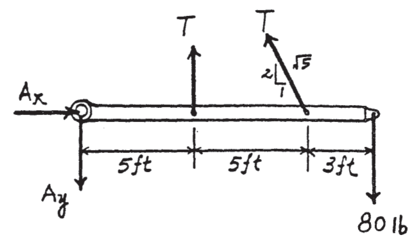


SOLUTION

Equations of Equilibrium: The tension force developed in the cable is the same throughout the whole cable. The force in the cable can be obtained directly by summing moments about point A.

$$\circlearrowleft + \Sigma M_A = 0; \quad T(5) + T\left(\frac{2}{\sqrt{5}}\right)(10) - 80(13) = 0$$

$$T = 74.583 \text{ lb} = 74.6 \text{ lb}$$ **Ans.**



$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad A_x - 74.583\left(\frac{1}{\sqrt{5}}\right) = 0$$

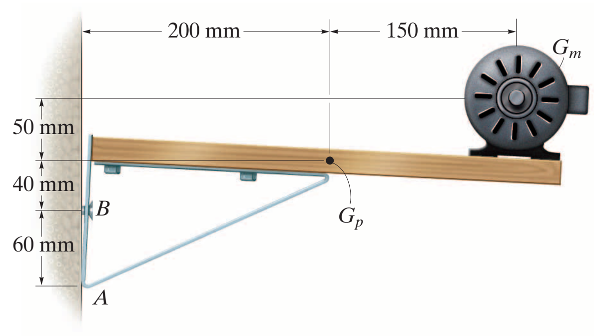
$$A_x = 33.4 \text{ lb}$$ **Ans.**

$$+\uparrow \Sigma F_y = 0; \quad 74.583 + 74.583\left(\frac{2}{\sqrt{5}}\right) - 80 - B_y = 0$$

$$A_y = 61.3 \text{ lb}$$ **Ans.**

5–19.

The shelf supports the electric motor which has a mass of 15 kg and mass center at G_m. The platform upon which it rests has a mass of 4 kg and mass center at G_p. Assuming that a single bolt B holds the shelf up and the bracket bears against the smooth wall at A, determine this normal force at A and the horizontal and vertical components of reaction of the bolt on the bracket.



SOLUTION

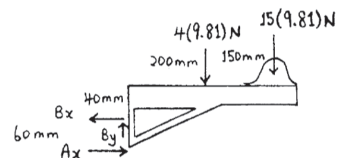
$\zeta + \Sigma M_A = 0;$ $\quad B_x(60) - 4(9.81)(200) - 15(9.81)(350) = 0$

$B_x = 989.18 = 989 \text{ N}$ **Ans.**

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad A_x = 989.18 = 989 \text{ N}$ **Ans.**

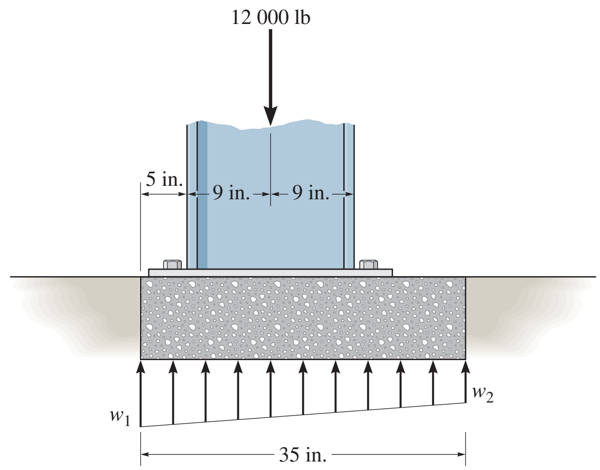
$+\uparrow \Sigma F_y = 0;$ $\quad B_y = 4(9.81) + 15(9.81)$

$B_y = 186.39 = 186 \text{ N}$ **Ans.**



***5–20.**

The pad footing is used to support the load of 12 000 lb. Determine the intensities w_1 and w_2 of the distributed loading acting on the base of the footing for the equilibrium.



SOLUTION

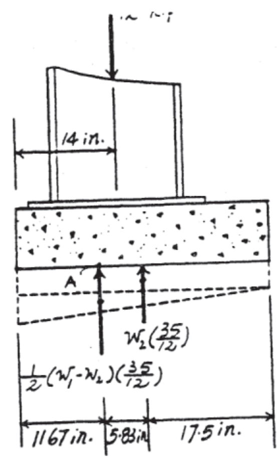
Equations of Equilibrium: The load intensity w_2 can be determined directly by summing moments about point A.

$$\circlearrowleft + \Sigma M_A = 0; \qquad w_2\left(\frac{35}{12}\right)(17.5 - 11.67) - 12(14 - 11.67) = 0$$

$$w_2 = 1.646 \text{ kip/ft} = 1.65 \text{ kip/ft} \qquad \textbf{Ans.}$$

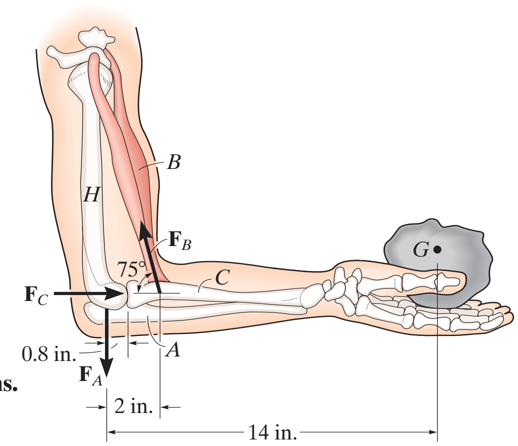
$$+\uparrow \Sigma F_y = 0; \qquad \frac{1}{2}(w_1 - 1.646)\left(\frac{35}{12}\right) + 1.646\left(\frac{35}{12}\right) - 12 = 0$$

$$w_1 = 6.58 \text{ kip/ft} \qquad \textbf{Ans.}$$



5–21.

When holding the 5-lb stone in equilibrium, the humerus H, assumed to be smooth, exerts normal forces $\mathbf{F}_C$ and $\mathbf{F}_A$ on the radius C and ulna A as shown. Determine these forces and the force $\mathbf{F}_B$ that the biceps B exerts on the radius for equilibrium. The stone has a center of mass at G. Neglect the weight of the arm.



SOLUTION

$\zeta + \Sigma M_B = 0; \qquad -5(12) + F_A(2) = 0$

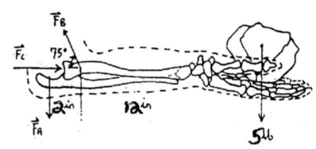
$F_A = 30 \text{ lb}$ **Ans.**

$+\uparrow \Sigma F_y = 0; \qquad F_B \sin 75° - 5 - 30 = 0$

$F_B = 36.2 \text{ lb}$ **Ans.**

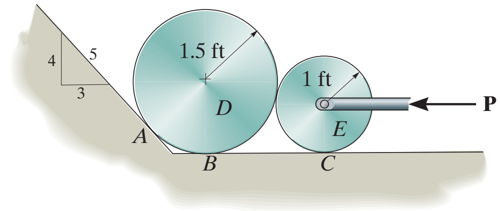
$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad F_C - 36.2 \cos 75° = 0$

$F_C = 9.38 \text{ lb}$ **Ans.**



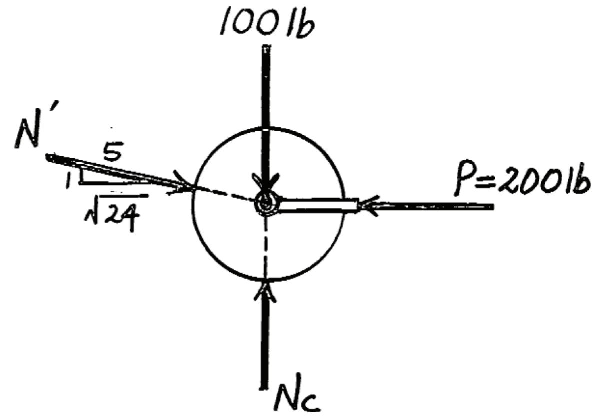
5–22.

The smooth disks D and E have a weight of 200 lb and 100 lb, respectively. If a horizontal force of $P = 200$ lb is applied to the center of disk E, determine the normal reactions at the points of contact with the ground at A, B, and C.



SOLUTION

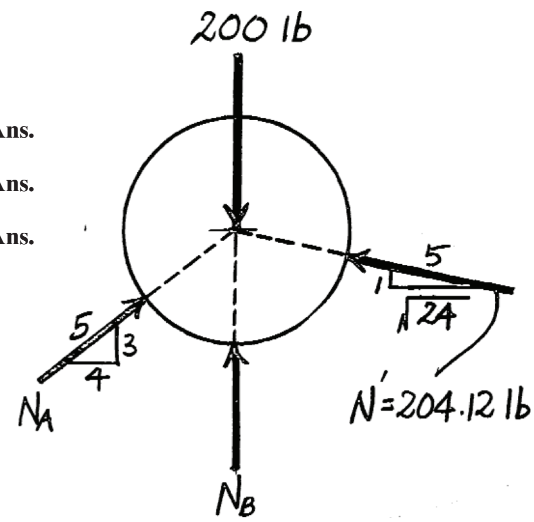
For disk E:



$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad -P + N'\left(\frac{\sqrt{24}}{5}\right) = 0$$

$$+\uparrow \Sigma F_y = 0; \qquad N_C - 100 - N'\left(\frac{1}{5}\right) = 0$$

For disk D:



$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad N_A\left(\frac{4}{5}\right) - N'\left(\frac{\sqrt{24}}{5}\right) = 0$$

$$+\uparrow \Sigma F_y = 0; \qquad N_A\left(\frac{3}{5}\right) + N_B - 200 + N'\left(\frac{1}{5}\right) = 0$$

Set $P = 200$ lb and solve:

$N' = 204.12$ lb

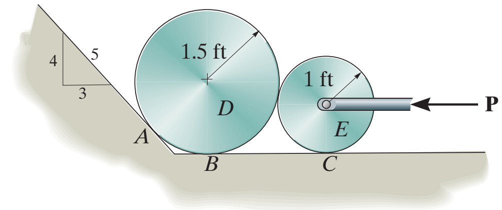
$N_A = 250$ lb **Ans.**

$N_B = 9.18$ lb **Ans.**

$N_C = 141$ lb **Ans.**

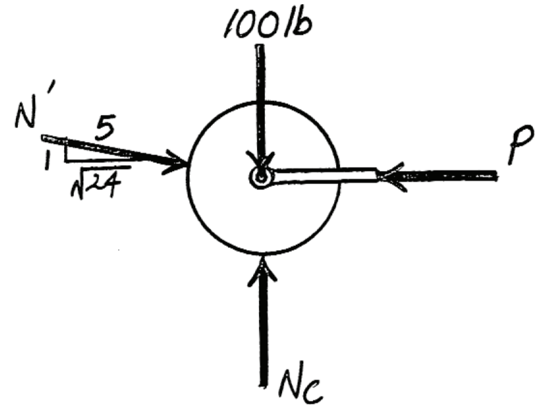
5–23.

The smooth disks D and E have a weight of 200 lb and 100 lb, respectively. Determine the largest horizontal force P that can be applied to the center of disk E without causing the disk D to move up the incline.



SOLUTION

For disk E:



$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad -P + N'\left(\frac{\sqrt{24}}{5}\right) = 0$$

$$+\uparrow \Sigma F_y = 0; \qquad N_C - 100 - N'\left(\frac{1}{5}\right) = 0$$

For disk D:

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad N_A\left(\frac{4}{5}\right) - N'\left(\frac{\sqrt{24}}{5}\right) = 0$$

$$+\uparrow \Sigma F_y = 0; \qquad N_A\left(\frac{3}{5}\right) + N_B - 200 + N'\left(\frac{1}{5}\right) = 0$$

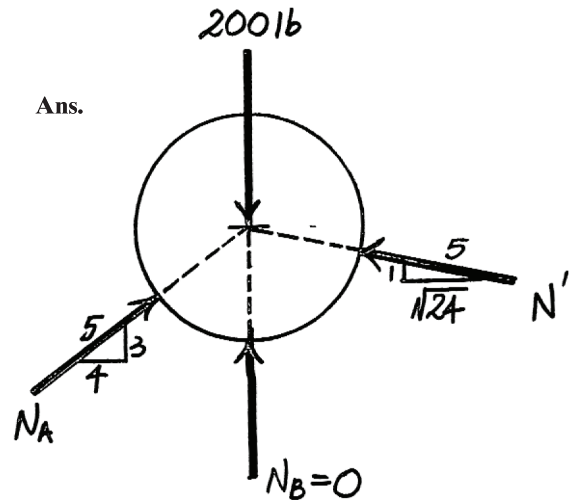
Require $N_B = 0$ for P_{max}. Solving,

$$N' = 214 \text{ lb}$$

$$P_{max} = 210 \text{ lb} \qquad \textbf{Ans.}$$

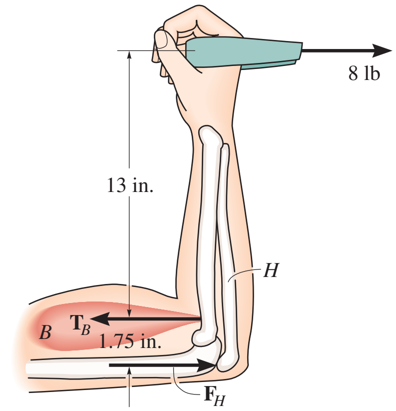
$$N_A = 262 \text{ lb}$$

$$N_C = 143 \text{ lb}$$



***5–24.**

The man is pulling a load of 8 lb with one arm held as shown. Determine the force $\mathbf{F}_H$ this exerts on the humerus bone H, and the tension developed in the biceps muscle B. Neglect the weight of the man's arm.



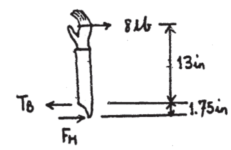
SOLUTION

$\circlearrowleft + \Sigma M_B = 0; \qquad -8(13) + F_H(1.75) = 0$

$F_H = 59.43 = 59.4 \text{ lb}$ **Ans.**

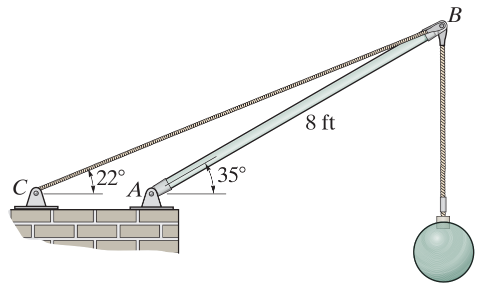
$\xrightarrow{+} \Sigma F_x = 0; \qquad 8 - T_B + 59.43 = 0$

$T_B = 67.4 \text{ lb}$ **Ans.**



5–25.

Determine the magnitude of force at the pin A and in the cable BC needed to support the 500-lb load. Neglect the weight of the boom AB.



SOLUTION

Equations of Equilibrium: The force in cable BC can be obtained directly by summing moments about point A.

$\circlearrowleft + \Sigma M_A = 0;$ $\quad F_{BC} \sin 13°(8) - 500 \cos 35°(8) = 0$

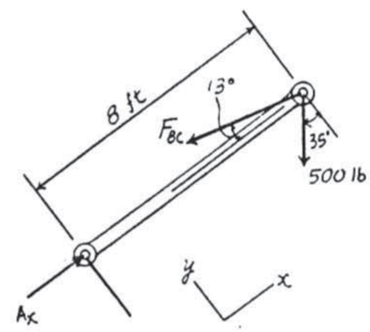
$F_{BC} = 1820.7 \text{ lb} = 1.82 \text{ kip}$ **Ans.**

$\overset{+}{\nearrow} \Sigma F_x = 0;$ $\quad A_x - 1820.7 \cos 13° - 500 \sin 35° = 0$

$A_x = 2060.9 \text{ lb}$

$\nwarrow + \Sigma F_y = 0;$ $\quad A_y + 1820.7 \sin 13° - 500 \cos 35° = 0$

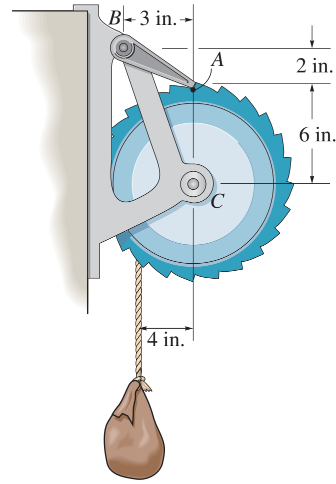
$A_y = 0$



Thus, $\quad F_A = A_x = 2060.9 \text{ lb} = 2.06 \text{ kip}$ **Ans.**

5–26.

The winch consists of a drum of radius 4 in., which is pin connected at its center C. At its outer rim is a ratchet gear having a mean radius of 6 in. The pawl AB serves as a two-force member (short link) and keeps the drum from rotating. If the suspended load is 500 lb, determine the horizontal and vertical components of reaction at the pin C.



SOLUTION

Equations of Equilibrium: The force in short link AB can be obtained directly by summing moments about point C.

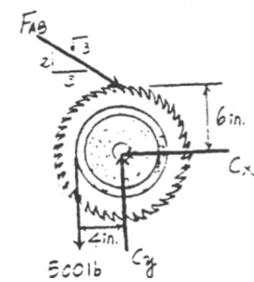
$$\circlearrowleft + \Sigma M_C = 0; \qquad 500(4) - F_{AB}\left(\frac{3}{\sqrt{13}}\right)(6) = 0 \qquad F_{AB} = 400.62 \text{ lb}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 400.62\left(\frac{3}{\sqrt{13}}\right) - C_x = 0$$

$$C_x = 333 \text{ lb} \qquad \textbf{Ans.}$$

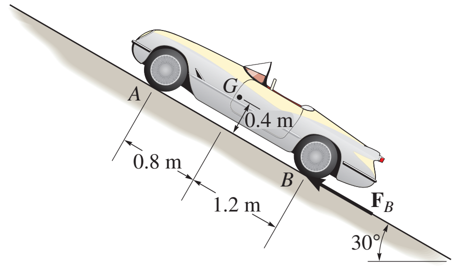
$$+\uparrow \Sigma F_y = 0; \qquad C_y - 500 - 400.62\left(\frac{2}{\sqrt{13}}\right) = 0$$

$$C_y = 722 \text{ lb} \qquad \textbf{Ans.}$$



5–27.

The sports car has a mass of 1.5 Mg and mass center at G. If the front two springs each have a stiffness of $k_A = 58$ kN/m and the rear two springs each have a stiffness of $k_B = 65$ kN/m, determine their compression when the car is parked on the 30° incline. Also, what friction force $\mathbf{F}_B$ must be applied to each of the rear wheels to hold the car in equilibrium? *Hint:* First determine the normal force at A and B, then determine the compression in the springs.

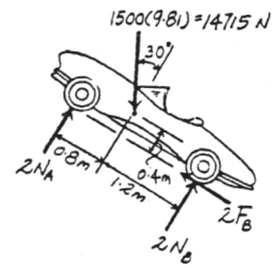


SOLUTION

Equations of Equilibrium: The normal reaction N_A can be obtained directly by summing moments about point B.

$$\circlearrowleft + \Sigma M_B = 0; \qquad 14\,715 \cos 30°(1.2) - 14\,715 \sin 30°(0.4) - 2N_A(2) = 0$$

$$N_A = 3087.32 \text{ N}$$



$$\nwarrow + \Sigma F_{x'} = 0; \qquad 2F_B - 14\,715 \sin 30° = 0$$

$$F_B = 3678.75 \text{ N} = 3.68 \text{ kN} \qquad \textbf{Ans.}$$

$$\nearrow + \Sigma F_{y'} = 0; \qquad 2N_B + 2(3087.32) - 14\,715 \cos 30° = 0$$

$$N_B = 3284.46 \text{ N}$$

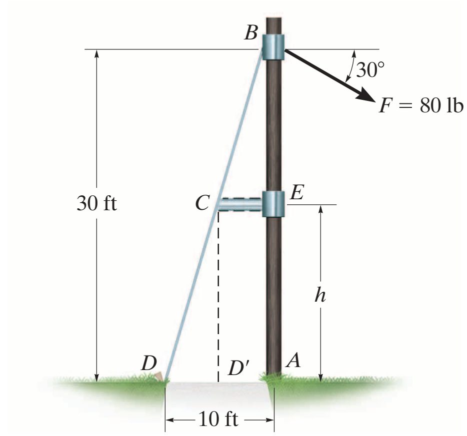
Spring Force Formula: The compression of the sping can be determined using the spring formula $x = \dfrac{F_{sp}}{k}$.

$$x_A = \frac{3087.32}{58(10^3)} = 0.05323 \text{ m} = 53.2 \text{ mm} \qquad \textbf{Ans.}$$

$$x_B = \frac{3284.46}{65(10^3)} = 0.05053 \text{ m} = 50.5 \text{ mm} \qquad \textbf{Ans.}$$

***5–28.**

The telephone pole of negligible thickness is subjected to the force of 80 lb directed as shown. It is supported by the cable BCD and can be assumed pinned at its base A. In order to provide clearance for a sidewalk right of way, where D is located, the strut CE is attached at C, as shown by the dashed lines (cable segment CD is removed). If the tension in CD' is to be twice the tension in BCD, determine the height h for placement of the strut CE.



SOLUTION

$\circlearrowleft + \Sigma M_A = 0; \qquad -80(30)\cos 30° + \frac{1}{\sqrt{10}} T_{BCD}(30) = 0$

$T_{BCD} = 219.089 \text{ lb}$

Require $T_{CD'} = 2(219.089) = 438.178 \text{ lb}$

$\circlearrowleft + \Sigma M_A = 0; \qquad 438.178(d) - 80\cos 30°(30) = 0$

$d = 4.7434 \text{ ft}$

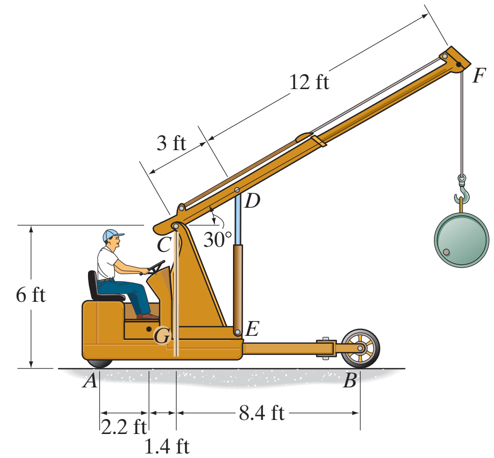
$$\frac{30 - h}{4.7434} = \frac{30}{10}$$

$300 - 10h = 142.3025$

$h = 15.8 \text{ ft}$ **Ans.**

5–29.

The floor crane and the driver have a total weight of 2500 lb with a center of gravity at G. If the crane is required to lift the 500-lb drum, determine the normal reaction on *both* the wheels at A and *both* the wheels at B when the boom is in the position shown.



SOLUTION

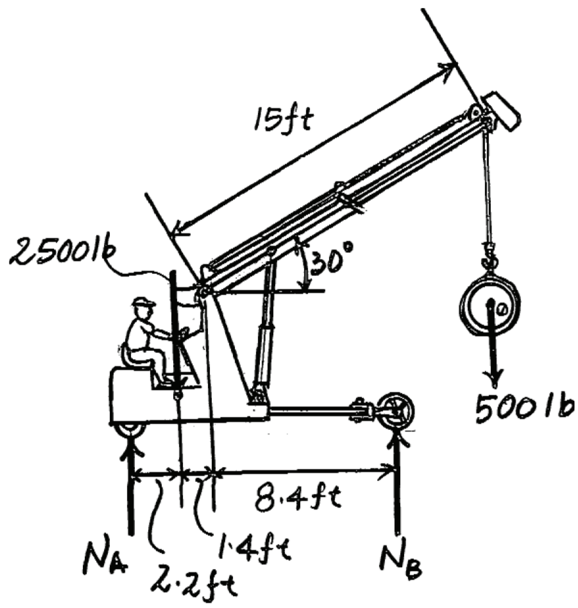
Equations of Equilibrium: From the free-body diagram of the floor crane, Fig. *a*,

$\circlearrowleft + \Sigma M_B = 0;\ 2500(1.4 + 8.4) - 500(15\cos 30° - 8.4) - N_A(2.2 + 1.4 + 8.4) = 0$

$$N_A = 1850.40 \text{ lb} = 1.85 \text{ kip} \qquad \textbf{Ans.}$$

$+\uparrow \Sigma F_y = 0; \qquad 1850.40 - 2500 - 500 + N_B = 0$

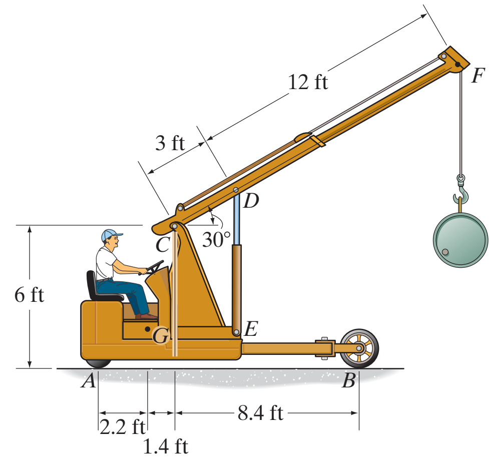
$$N_B = 1149.60 \text{ lb} = 1.15 \text{ kip} \qquad \textbf{Ans.}$$



(a)

5–30.

The floor crane and the driver have a total weight of 2500 lb with a center of gravity at G. Determine the largest weight of the drum that can be lifted without causing the crane to overturn when its boom is in the position shown.

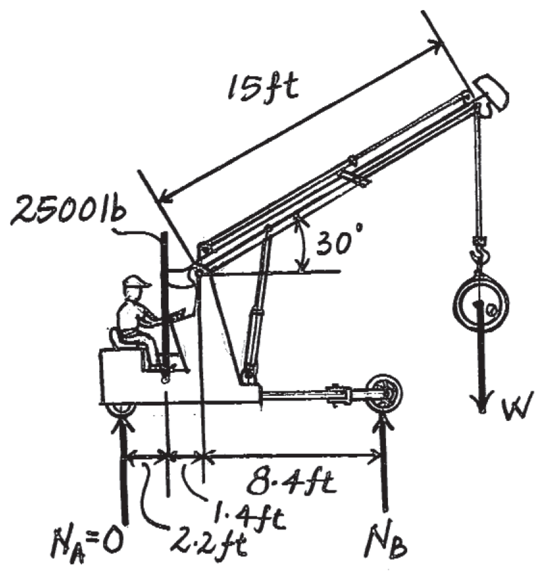


SOLUTION

Equations of Equilibrium: Since the floor crane tends to overturn about point B, the wheel at A will leave the ground and $N_A = 0$. From the free - body diagram of the floor crane, Fig. a, W can be obtained by writing the moment equation of equilibrium about point B.

$\circlearrowleft + \Sigma M_B = 0; \qquad 2500(1.4 + 8.4) - W(15 \cos 30° - 8.4) = 0$

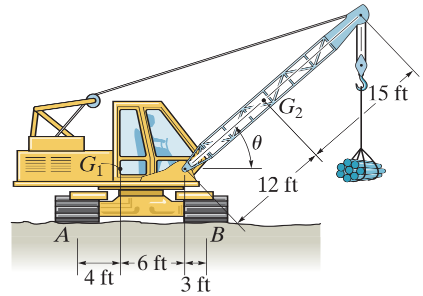
$W = 5337.25 \text{ lb} = 5.34 \text{ kip}$ **Ans.**



(a)

5–31.

The mobile crane has a weight of 120,000 lb and center of gravity at G_1; the boom has a weight of 30,000 lb and center of gravity at G_2. Determine the smallest angle of tilt θ of the boom, without causing the crane to overturn if the suspended load is $W = 40{,}000$ lb. Neglect the thickness of the tracks at A and B.



SOLUTION

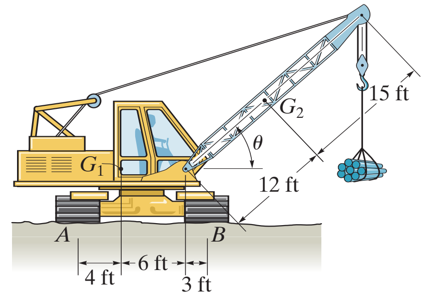
When tipping occurs, $R_A = 0$

$$\circlearrowleft + \Sigma M_B = 0; \qquad -(30\,000)(12\cos\theta - 3) - (40\,000)(27\cos\theta - 3) + (120\,000)(9) = 0$$

$$\theta = \cos^{-1}(0.896) = 26.4^\circ \qquad \textbf{Ans.}$$

***5–32.**

The mobile crane has a weight of 120,000 lb and center of gravity at G_1; the boom has a weight of 30,000 lb and center of gravity at G_2. If the suspended load has a weight of $W = 16{,}000$ lb, determine the normal reactions at the tracks A and B. For the calculation, neglect the thickness of the tracks and take $\theta = 30°$.



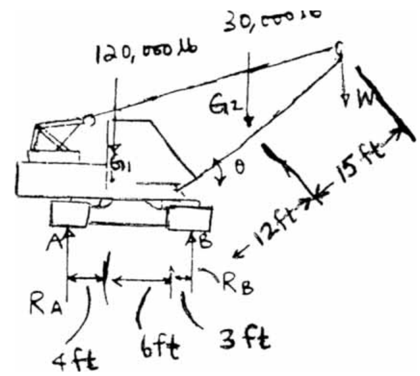
SOLUTION

$\zeta + \Sigma M_B = 0; \quad -(30\,000)(12\cos 30° - 3) - (16\,000)(27\cos 30° - 3) - R_A(13) + (120\,000)(9) = 0$

$R_A = 40\,931 \text{ lb} = 40.9 \text{ kip}$ **Ans.**

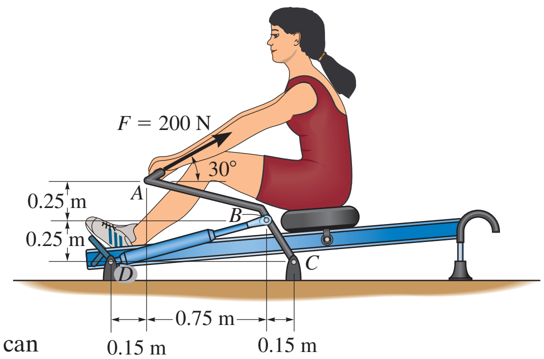
$+\uparrow \Sigma F_y = 0; \quad 40\,931 + R_B - 120\,000 - 30\,000 - 16\,000 = 0$

$R_B = 125 \text{ kip}$ **Ans.**



5–33.

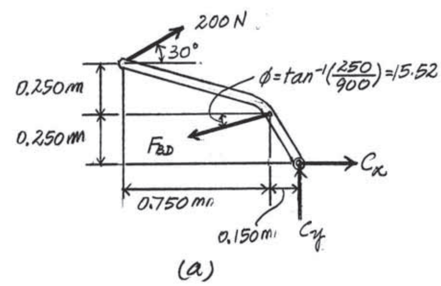
The woman exercises on the rowing machine. If she exerts a holding force of $F = 200$ N on handle ABC, determine the horizontal and vertical components of reaction at pin C and the force developed along the hydraulic cylinder BD on the handle.



SOLUTION

Equations of Equilibrium: Since the hydraulic cylinder is pinned at both ends, it can be considered as a two-force member and therefore exerts a force $\mathbf{F}_{BD}$ directed along its axis on the handle, as shown on the free-body diagram in Fig. *a*. From the free-body diagram, F_{BD} can be obtained by writing the moment equation of equilibrium about point C.

$$\circlearrowleft + \Sigma M_C = 0; \qquad F_{BD} \cos 15.52°(250) + F_{BD} \sin 15.52°(150) - 200 \cos 30°(250 + 250)$$
$$-200 \sin 30°(750 + 150) = 0$$
$$F_{BD} = 628.42 \text{ N} = 628 \text{ N} \qquad \textbf{Ans.}$$



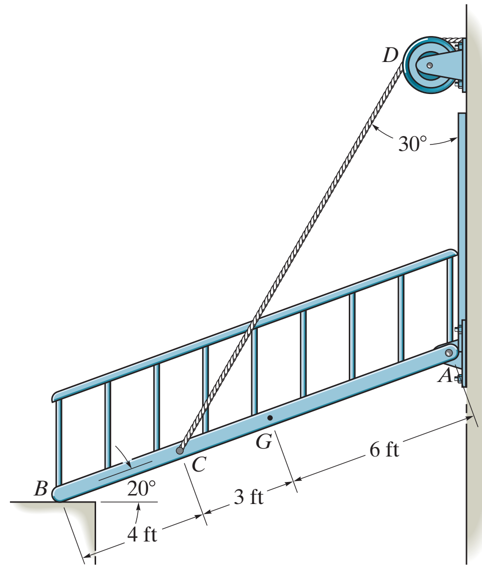
Using the above result and writing the force equations of equilibrium along the x and y axes, we have

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad C_x + 200 \cos 30° - 628.42 \cos 15.52° = 0$$
$$C_x = 432.29 \text{ N} = 432 \text{ N} \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad 200 \sin 30° - 628.42 \sin 15.52° + C_y = 0$$
$$C_y = 68.19 \text{ N} = 68.2 \text{ N} \qquad \textbf{Ans.}$$

5–34.

The ramp of a ship has a weight of 200 lb and a center of gravity at G. Determine the cable force in CD needed to just start lifting the ramp, (i.e., so the reaction at B becomes zero). Also, determine the horizontal and vertical components of force at the hinge (pin) at A.



SOLUTION

$\circlearrowleft + \Sigma M_A = 0;$ $\quad -F_{CD}\cos 30°(9\cos 20°) + F_{CD}\sin 30°(9\sin 20°) + 200(6\cos 20°) = 0$

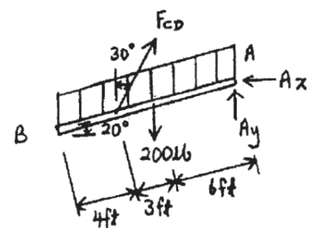
$F_{CD} = 194.9 = 195 \text{ lb}$ **Ans.**

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad 194.9\sin 30° - A_x = 0$

$A_x = 97.5 \text{ lb}$ **Ans.**

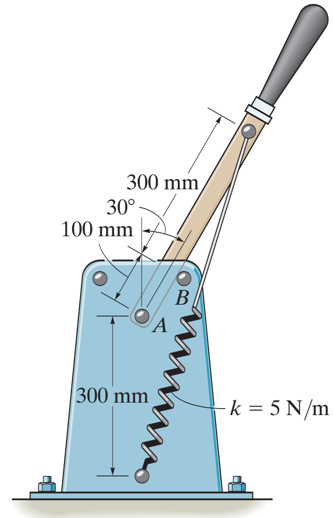
$+\uparrow \Sigma F_y = 0;$ $\quad A_y - 200 + 194.9\cos 30° = 0$

$A_y = 31.2 \text{ lb}$ **Ans.**



5–35.

The toggle switch consists of a cocking lever that is pinned to a fixed frame at A and held in place by the spring which has an unstretched length of 200 mm. Determine the magnitude of the resultant force at A and the normal force on the peg at B when the lever is in the position shown.



SOLUTION

$l = \sqrt{(0.3)^2 + (0.4)^2 - 2(0.3)(0.4)\cos 150^\circ} = 0.67664 \text{ m}$

$\frac{\sin\theta}{0.3} = \frac{\sin 150^\circ}{0.67664};\qquad \theta = 12.808^\circ$

$F_s = ks = 5(0.67664 - 0.2) = 2.3832 \text{ N}$

$\circlearrowleft + \Sigma M_A = 0; \qquad -(2.3832 \sin 12.808^\circ)(0.4) + N_B(0.1) = 0$

$N_B = 2.11327 \text{ N} = 2.11 \text{ N}$ **Ans.**

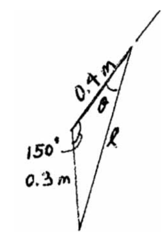
$\nearrow + \Sigma F_x = 0; \qquad A_x - 2.3832 \cos 12.808^\circ = 0$

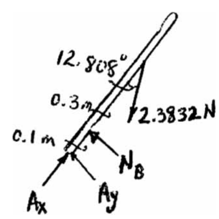
$A_x = 2.3239 \text{ N}$

$+\nwarrow \Sigma F_y = 0; \qquad A_y + 2.11327 - 2.3832 \sin 12.808^\circ = 0$

$A_y = -1.5850 \text{ N}$

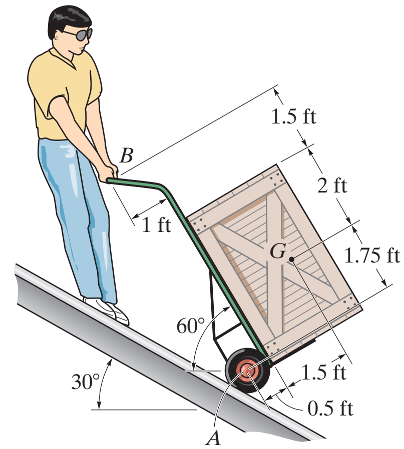
$F_A = \sqrt{(2.3239)^2 + (-1.5850)^2} = 2.81 \text{ N}$ **Ans.**





***5–36.**

The worker uses the hand truck to move material down the ramp. If the truck and its contents are held in the position shown and have a weight of 100 lb with center of gravity at G, determine the resultant normal force of both wheels on the ground A and the magnitude of the force required at the grip B.



SOLUTION

$\circlearrowleft + \Sigma M_B = 0;\quad (N_A \cos 30°)(5.25) + N_A \sin 30°(0.5)$

$\quad - 100 \sin 30°(3.5) - 100 \cos 30°(2.5) = 0$

$\quad N_A = 81.621 \text{ lb} = 81.6 \text{ lb}$ **Ans.**

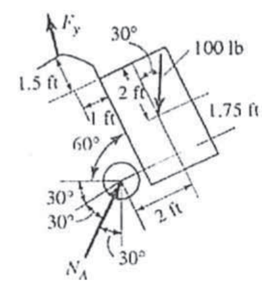
$+\searrow \Sigma F_x = 0;\quad -B_x + 100 \cos 30° - 81.621 \sin 30° = 0$

$\quad B_x = 45.792 \text{ lb}$

$\nearrow + \Sigma F_y = 0;\quad B_y - 100 \sin 30° + 81.621 \cos 30° = 0$

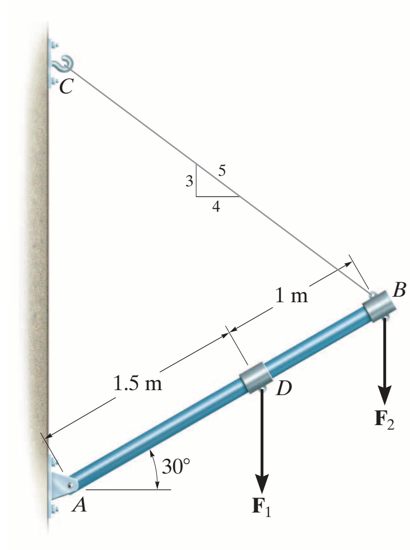
$\quad B_y = -20.686 \text{ lb}$

$F_B = \sqrt{(45.792)^2 + (-20.686)^2} = 50.2 \text{ lb}$ **Ans.**



5–37.

The boom supports the two vertical loads. Neglect the size of the collars at D and B and the thickness of the boom, and compute the horizontal and vertical components of force at the pin A and the force in cable CB. Set $F_1 = 800$ N and $F_2 = 350$ N.



SOLUTION

$\circlearrowleft + \Sigma M_A = 0;$ $$-800(1.5\cos 30^\circ) - 350(2.5\cos 30^\circ) + \frac{4}{5}F_{CB}(2.5\sin 30^\circ) + \frac{3}{5}F_{CB}(2.5\cos 30^\circ) = 0$$

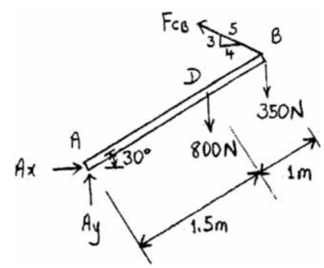
$$F_{CB} = 781.6 = 782\text{ N}$$ **Ans.**

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $$A_x - \frac{4}{5}(781.6) = 0$$

$$A_x = 625\text{ N}$$ **Ans.**

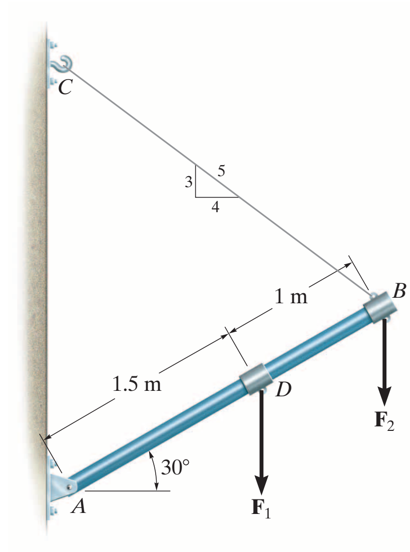
$+\uparrow \Sigma F_y = 0;$ $$A_y - 800 - 350 + \frac{3}{5}(781.6) = 0$$

$$A_y = 681\text{ N}$$ **Ans.**



5–38.

The boom is intended to support two vertical loads, $\mathbf{F}_1$ and $\mathbf{F}_2$. If the cable CB can sustain a maximum load of 1500 N before it fails, determine the critical loads if $F_1 = 2F_2$. Also, what is the magnitude of the maximum reaction at pin A?



SOLUTION

$\circlearrowleft + \Sigma M_A = 0;$ $\quad -2F_2(1.5 \cos 30°) - F_2(2.5 \cos 30°)$

$$+\frac{4}{5}(1500)(2.5 \sin 30°) + \frac{3}{5}(1500)(2.5 \cos 30°) = 0$$

$F_2 = 724 \text{ N}$ **Ans.**

$F_1 = 2F_2 = 1448 \text{ N}$

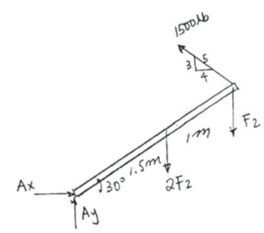
$F_1 = 1.45 \text{ kN}$ **Ans.**

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad A_x - \frac{4}{5}(1500) = 0$

$A_x = 1200 \text{ N}$

$+\uparrow \Sigma F_y = 0;$ $\quad A_y - 724 - 1448 + \frac{3}{5}(1500) = 0$

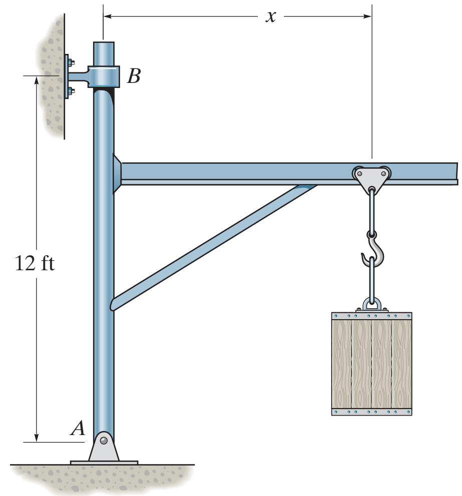
$A_y = 1272 \text{ N}$



$F_A = \sqrt{(1200)^2 + (1272)^2} = 1749 \text{ N} = 1.75 \text{ kN}$ **Ans.**

5–39.

The jib crane is pin connected at A and supported by a smooth collar at B. If $x = 8$ ft, determine the reactions on the jib crane at the pin A and smooth collar B. The load has a weight of 5000 lb.



SOLUTION

Equations of Equilibrium: Referring to the FBD of the jib crane shown in Fig. a, we notice that $\mathbf{N}_B$ and $\mathbf{A}_Y$ can be obtained directly by writing the moment equation of equilibrium about point A and force equation of equilibrium along y axis, respectively.

$\circlearrowleft + \Sigma M_A = 0;\qquad N_B(12) - 5000(8) = 0$

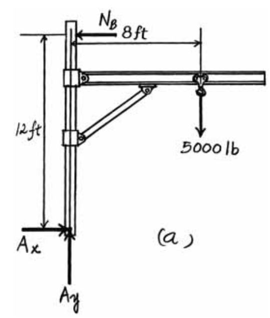
$N_B = 3333.33 \text{ lb} = 3333 \text{ lb} = 3.33 \text{ kip}$ **Ans.**

$+\uparrow \Sigma F_y = 0;\qquad A_y - 5000 = 0 \qquad A_y = 5000 \text{ lb} = 5.00 \text{ kip}$ **Ans.**

Using the result of $\mathbf{N}_B$ to write the force equation of equilibrium along x axis,

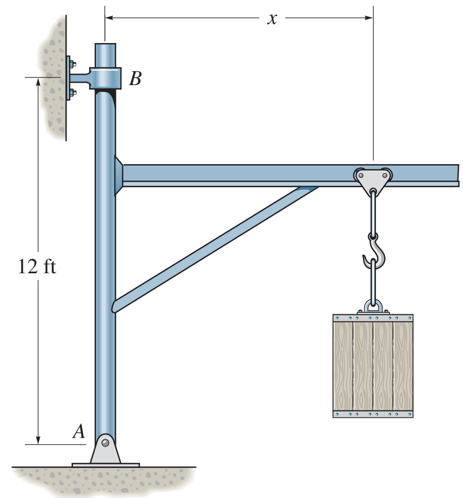
$\overset{+}{\rightarrow} \Sigma F_x = 0;\qquad A_x - 3333.33 = 0$

$A_x = 3333.33 \text{ lb} = 3333 \text{ lb} = 3.33 \text{ kip}$ **Ans.**



***5–40.**

The jib crane is pin connected at A and supported by a smooth collar at B. Determine the roller placement x of the 5000-lb load so that it gives the maximum and minimum reactions at the supports. Calculate these reactions in each case. Neglect the weight of the crane. Require 4 ft $\leq x \leq$ 10 ft.



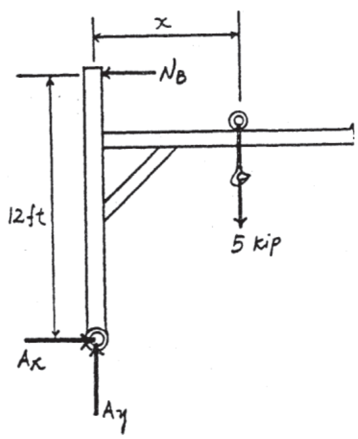
SOLUTION

Equations of Equilibrium:

$\circlearrowleft + \Sigma M_A = 0; \quad N_B(12) - 5x = 0 \quad N_B = 0.4167x$ **(1)**

$+\uparrow \Sigma F_y = 0; \quad A_y - 5 = 0 \quad A_y = 5.00 \text{ kip}$ **(2)**

$\xrightarrow{+} \Sigma F_x = 0; \quad A_x - 0.4167x = 0 \quad A_x = 0.4167x$ **(3)**



By observation, the **maximum support reactions** occur when

$$x = 10 \text{ ft}$$ **Ans.**

With $x = 10$ ft, from Eqs. (1), (2) and (3), the **maximum support reactions** are

$$A_x = N_B = 4.17 \text{ kip} \qquad A_y = 5.00 \text{ kip}$$ **Ans.**

By observation, the **minimum support reactions** occur when

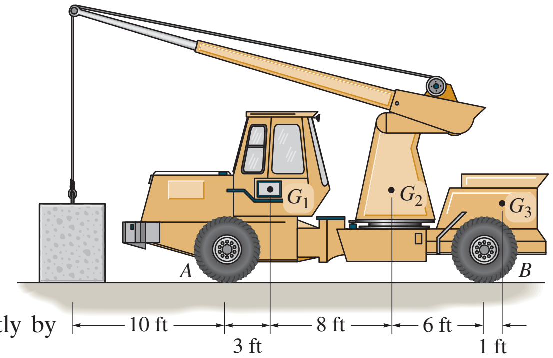
$$x = 4 \text{ ft}$$ **Ans.**

With $x = 4$ ft, from Eqs. (1), (2) and (3), the **minimum support reactions** are

$$A_x = N_B = 1.67 \text{ kip} \qquad A_y = 5.00 \text{ kip}$$ **Ans.**

5–41.

The crane consists of three parts, which have weights of $W_1 = 3500$ lb, $W_2 = 900$ lb, $W_3 = 1500$ lb and centers of gravity at G_1, G_2, and G_3, respectively. Neglecting the weight of the boom, determine (a) the reactions on each of the four tires if the load is hoisted at constant velocity and has a weight of 800 lb, and (b), with the boom held in the position shown, the maximum load the crane can lift without tipping over.



SOLUTION

Equations of Equilibrium: The normal reaction N_B can be obtained directly by summing moments about point A.

$$\zeta + \Sigma M_A = 0; \qquad 2N_B(17) + W(10) - 3500(3) - 900(11) - 1500(18) = 0$$

$$N_B = 1394.12 - 0.2941W \qquad \textbf{(1)}$$

Using the result $N_B = 22788.24 - 0.5882W$,

$$+\uparrow \Sigma F_y = 0; \qquad 2N_A + (22788.24 - 0.5882W) - W - 3500 - 900 - 1500 = 0$$

$$N_A = 0.7941W + 1555.88 \qquad \textbf{(2)}$$

a) Set $W = 800$ lb and substitute into Eqs. (1) and (2) yields

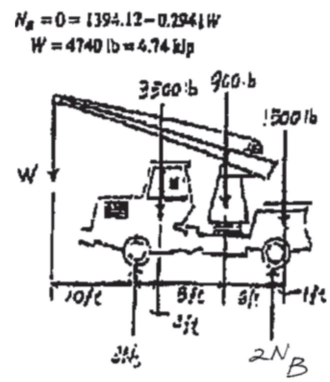
$$N_A = 0.7941(800) + 1555.88 = 2191.18 \text{ lb} = 2.19 \text{ kip} \qquad \textbf{Ans.}$$

$$N_B = 1394.12 - 0.2941(800) = 1158.82 \text{ lb} = 1.16 \text{ kip} \qquad \textbf{Ans.}$$

b) When the crane is about to tip over, the normal reaction on $N_B = 0$. From Eq. (1),

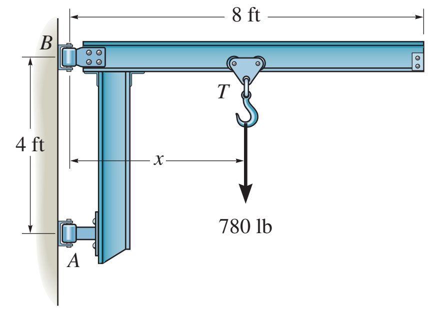
$$N_B = 0 = 1394.12 - 0.2941W$$

$$W = 4740 \text{ lb} = 4.74 \text{ kip} \qquad \textbf{Ans.}$$



5–42.

The cantilevered jib crane is used to support the load of 780 lb. If $x = 5$ ft, determine the reactions at the supports. Note that the supports are collars that allow the crane to rotate freely about the vertical axis. The collar at B supports a force in the vertical direction, whereas the one at A does not.



SOLUTION

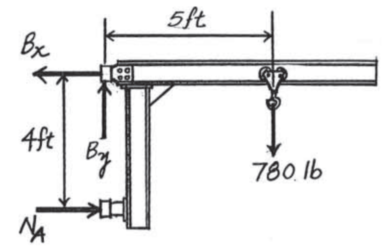
Equations of Equilibrium: Referring to the FBD of the jib crane shown in Fig. *a*, we notice that $\mathbf{N}_A$ and $\mathbf{B}_y$ can be obtained directly by writing the moment equation of equilibrium about point B and force equation of equilibrium along the y axis, respectively.

$\circlearrowleft + \Sigma M_B = 0; \quad N_A(4) - 780(5) = 0 \quad N_A = 975 \text{ lb}$ **Ans.**

$+\uparrow \Sigma F_y = 0; \quad B_y - 780 = 0 \quad B_y = 780$ **Ans.**

Using the result of $\mathbf{N}_A$ to write the force equation of equilibrium along x axis,

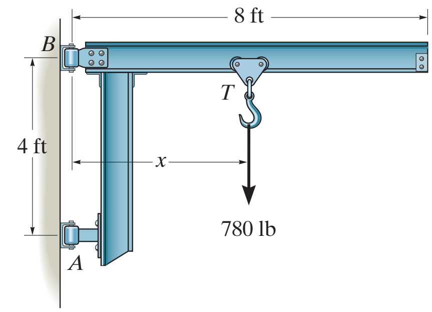
$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad 975 - B_x = 0 \quad B_x = 975 \text{ lb}$ **Ans.**



(a)

5–43.

The cantilevered jib crane is used to support the load of 780 lb. If the trolley T can be placed anywhere between $1.5 \text{ ft} \le x \le 7.5 \text{ ft}$, determine the maximum magnitude of reaction at the supports A and B. Note that the supports are collars that allow the crane to rotate freely about the vertical axis. The collar at B supports a force in the vertical direction, whereas the one at A does not.



SOLUTION

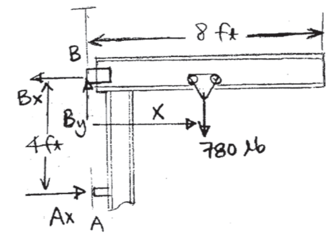
Require $x = 7.5$ ft

$\circlearrowleft + \Sigma M_A = 0; \qquad -780(7.5) + B_x(4) = 0$

$B_x = 1462.5 \text{ lb}$

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad A_x - 1462.5 = 0$

$A_x = 1462.5 = 1462 \text{ lb}$ **Ans.**



$+\uparrow \Sigma F_y = 0; \qquad B_y - 780 = 0$

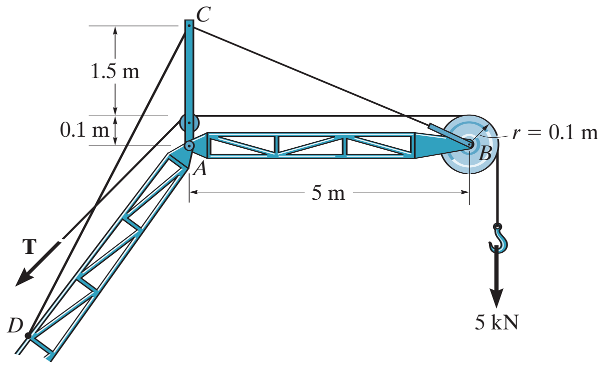
$B_y = 780 \text{ lb}$

$F_B = \sqrt{(1462.5)^2 + (780)^2}$

$= 1657.5 \text{ lb} = 1.66 \text{ kip}$ **Ans.**

***5–44.**

The upper portion of the crane boom consists of the jib AB, which is supported by the pin at A, the guy line BC, and the backstay CD, each cable being separately attached to the mast at C. If the 5-kN load is supported by the hoist line, which passes over the pulley at B, determine the magnitude of the resultant force the pin exerts on the jib at A for equilibrium, the tension in the guy line BC, and the tension T in the hoist line. Neglect the weight of the jib. The pulley at B has a radius of 0.1 m.

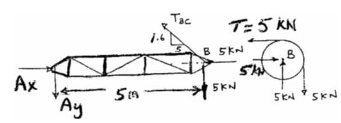


SOLUTION

From pulley, tension in the hoist line is

$$\circlearrowleft + \Sigma M_B = 0; \qquad T(0.1) - 5(0.1) = 0;$$

$$T = 5 \text{ kN} \qquad \textbf{Ans.}$$



From the jib,

$$\circlearrowleft + \Sigma M_A = 0; \qquad -5(5) + T_{BC}\left(\frac{1.6}{\sqrt{27.56}}\right)(5) = 0$$

$$T_{BC} = 16.4055 = 16.4 \text{ kN} \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad -A_y + (16.4055)\left(\frac{1.6}{\sqrt{27.56}}\right) - 5 = 0$$

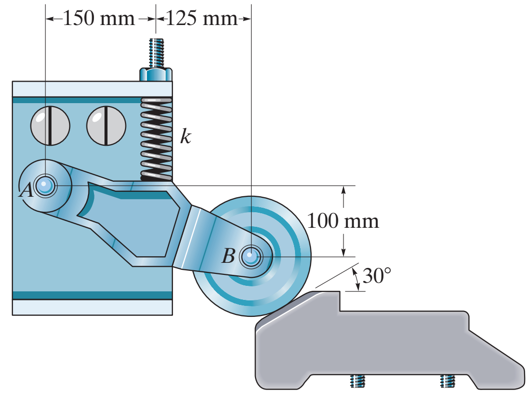
$$A_y = 0$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad A_x - 16.4055\left(\frac{5}{\sqrt{27.56}}\right) - 5 = 0$$

$$F_A = F_x = 20.6 \text{ kN} \qquad \textbf{Ans.}$$

5–45.

The device is used to hold an elevator door open. If the spring has a stiffness of $k = 40$ N/m and it is compressed 0.2 m, determine the horizontal and vertical components of reaction at the pin A and the resultant force at the wheel bearing B.



SOLUTION

$$F_s = ks = (40)(0.2) = 8 \text{ N}$$

$$\circlearrowleft + \Sigma M_A = 0; \quad -(8)(150) + F_B(\cos 30°)(275) - F_B(\sin 30°)(100) = 0$$

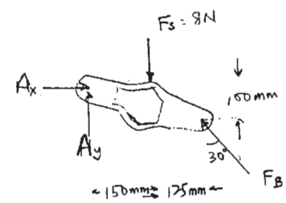
$$F_B = 6.37765 \text{ N} = 6.38 \text{ N}$$ **Ans.**

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad A_x - 6.37765 \sin 30° = 0$$

$$A_x = 3.19 \text{ N}$$ **Ans.**

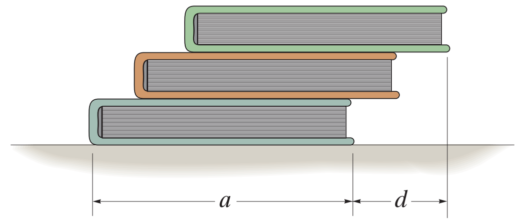
$$+\uparrow \Sigma F_y = 0; \quad A_y - 8 + 6.37765 \cos 30° = 0$$

$$A_y = 2.48 \text{ N}$$ **Ans.**



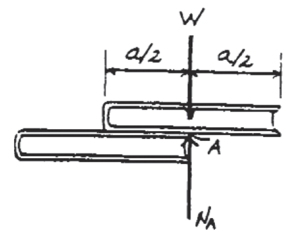
5–46.

Three uniform books, each having a weight W and length a, are stacked as shown. Determine the maximum distance d that the top book can extend out from the bottom one so the stack does not topple over.

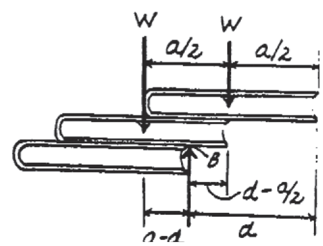


SOLUTION

Equilibrium: For top two books, the upper book will topple when the center of gravity of this book is to the right of point A. Therefore, the maximum distance from the right edge of this book to point A is $a/2$.



Equation of Equilibrium: For the entire three books, the top two books will topple about point B.

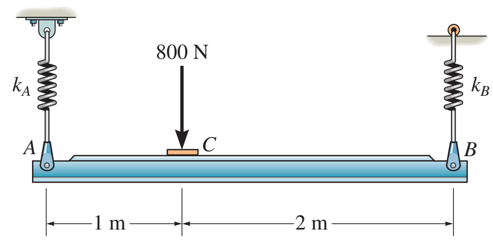


$$\circlearrowleft +\Sigma M_B = 0; \qquad W(a-d) - W\left(d - \frac{a}{2}\right) = 0$$

$$d = \frac{3a}{4}$$ **Ans.**

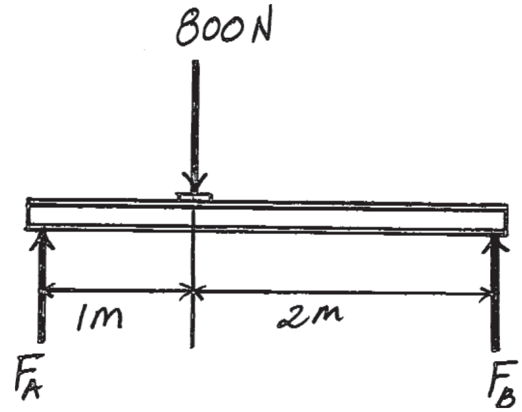
5–47.

The horizontal beam is supported by springs at its ends. Each spring has a stiffness of $k = 5$ kN/m and is originally unstretched when the beam is in the horizontal position. Determine the angle of tilt of the beam if a load of 800 N is applied at point C as shown.



SOLUTION

Equations of Equilibrium: The spring force at A and B can be obtained directly by summing moment about points B and A, respectively.



$$\circlearrowleft + \Sigma M_B = 0; \qquad 800(2) - F_A(3) = 0 \qquad F_A = 533.33 \text{ N}$$

$$\circlearrowleft + \Sigma M_A = 0; \qquad F_B(3) - 800(1) = 0 \qquad F_B = 266.67 \text{ N}$$

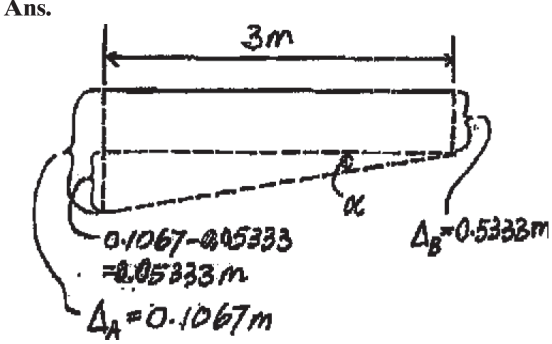
Spring Formula: Applying $\Delta = \dfrac{F}{k}$, we have

$$\Delta_A = \frac{533.33}{5(10^3)} = 0.1067 \text{ m}$$

$$\Delta_B = \frac{266.67}{5(10^3)} = 0.05333 \text{ m}$$

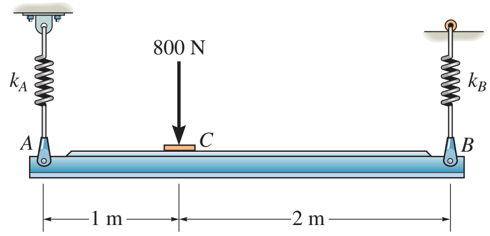
Geometry: The angle of tilt α is

$$\alpha = \tan^{-1}\left(\frac{0.05333}{3}\right) = 1.02° \qquad \textbf{Ans.}$$



***5–48.**

The horizontal beam is supported by springs at its ends. If the stiffness of the spring at A is $k_A = 5\ \text{kN/m}$, determine the required stiffness of the spring at B so that if the beam is loaded with the 800-N force it remains in the horizontal position. The springs are originally constructed so that the beam is in the horizontal position when it is unloaded.

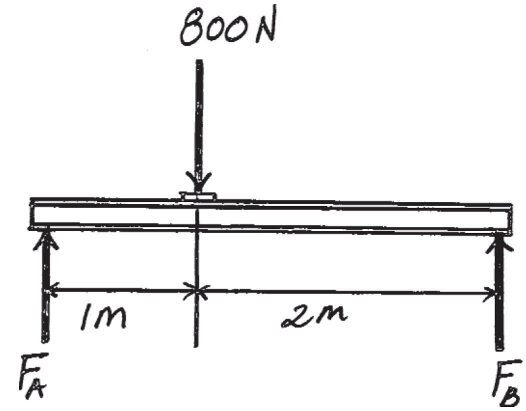


SOLUTION

Equations of Equilibrium: The spring forces at A and B can be obtained directly by summing moments about points B and A, respectively.

$$\circlearrowleft +\Sigma M_B = 0; \qquad 800(3) - F_A(3) = 0 \qquad F_A = 533.33\ \text{N}$$

$$\circlearrowleft +\Sigma M_A = 0; \qquad F_B(3) - 800(1) = 0 \qquad F_B = 266.67\ \text{N}$$



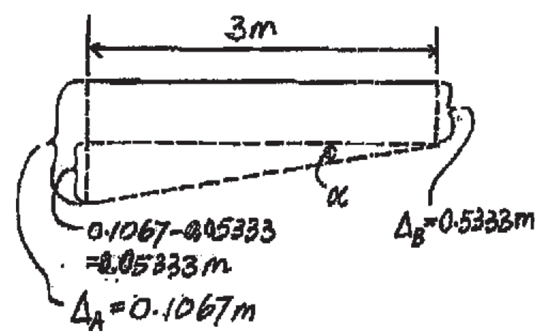
Spring Formula: Applying $\Delta = \dfrac{F}{k}$, we have

$$\Delta_A = \frac{533.33}{5(10^3)} = 0.1067\ \text{m} \qquad \Delta_B = \frac{266.67}{k_B}$$

Geometry: Requires, $\Delta_B = \Delta_A$. Then

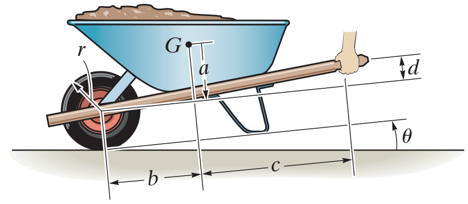
$$\frac{266.67}{k_B} = 0.1067$$

$$k_B = 2500\ \text{N/m} = 2.50\ \text{kN/m}$$ **Ans.**



5–49.

The wheelbarrow and its contents have a mass of $m = 60$ kg with a center of mass at G. Determine the normal reaction on the tire and the vertical force on each hand to hold it at $\theta = 30°$. Take $a = 0.3$ m, $b = 0.45$ m, $c = 0.75$ m and $d = 0.1$ m.



SOLUTION

Equations of Equilibrium: Referring to the FBD of the wheelbarrow shown in Fig. a, we notice that force **P** can be obtained directly by writing the moment equation of equilibrium about A.

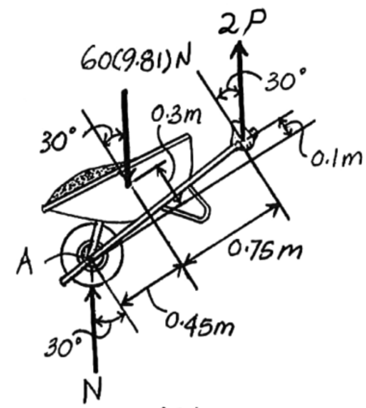
$$\circlearrowleft + \Sigma M_A = 0; \quad 60(9.81)\sin 30°(0.3) - 60(9.81)\cos 30°(0.45) + 2P\cos 30°(1.2) - 2P\sin 30°(0.1) = 0$$

$$P = 71.315 \text{ N} = 71.3 \text{ N} \qquad \textbf{Ans.}$$

Using this result to write the force equation of equilibrium along vertical,

$$+\uparrow \Sigma F_y = 0; \quad N + 2(71.315) - 60(9.81) = 0$$

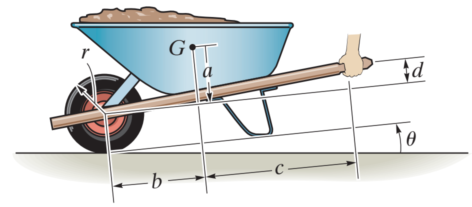
$$N = 445.97 \text{ N} = 446 \text{ N} \qquad \textbf{Ans.}$$



(a)

5–50.

The wheelbarrow and its contents have a mass m and center of mass at G. Determine the greatest angle of tilt θ without causing the wheelbarrow to tip over.

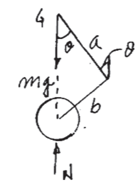


SOLUTION

Require point G to be over the wheel axle for tipping. Thus

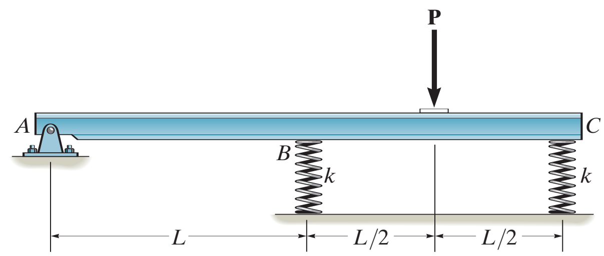
$$b \cos \theta = a \sin \theta$$

$$\theta = \tan^{-1}\frac{b}{a}$$ **Ans.**



5–51.

The rigid beam of negligible weight is supported horizontally by two springs and a pin. If the springs are uncompressed when the load is removed, determine the force in each spring when the load **P** is applied. Also, compute the vertical deflection of end C. Assume the spring stiffness k is large enough so that only small deflections occur. *Hint:* The beam rotates about A so the deflections in the springs can be related.



SOLUTION

$\circlearrowleft + \Sigma M_A = 0;\qquad F_B(L) + F_C(2L) - P\left(\frac{3}{2}L\right) = 0$

$$F_B + 2F_C = 1.5P$$

$$\frac{L}{\Delta_B} = \frac{2L}{\Delta_C}$$

$$\Delta_C = 2\Delta_B$$

$$\frac{F_C}{k} = \frac{2F_B}{k}$$

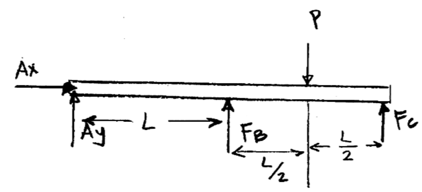
$$F_C = 2F_B$$

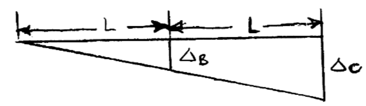
$$5F_B = 1.5P$$

$F_B = 0.3P$ **Ans.**

$F_C = 0.6P$ **Ans.**

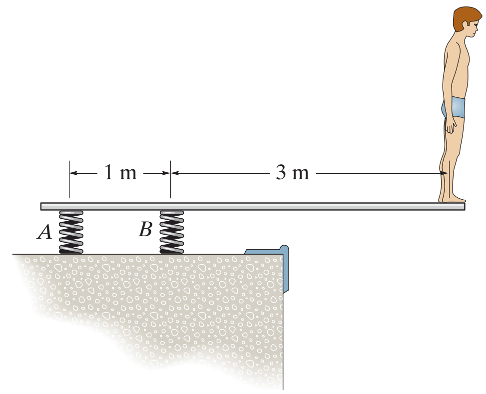
Deflection, $\quad x_C = \dfrac{0.6P}{k}$ **Ans.**





***5–52.**

A boy stands out at the end of the diving board, which is supported by two springs A and B, each having a stiffness of $k = 15\text{kN/m}$. In the position shown the board is horizontal. If the boy has a mass of 40 kg, determine the angle of tilt which the board makes with the horizontal after he jumps off. Neglect the weight of the board and assume it is rigid.

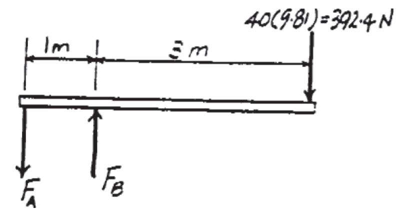


SOLUTION

Equations of Equilibrium: The spring force at A and B can be obtained directly by summing moments about points B and A, respectively.

$$\zeta + \Sigma M_B = 0; \qquad F_A(1) - 392.4(3) = 0 \qquad F_A = 1177.2 \text{ N}$$

$$\zeta + \Sigma M_A = 0; \qquad F_B(1) - 392.4(4) = 0 \qquad F_B = 1569.6 \text{ N}$$

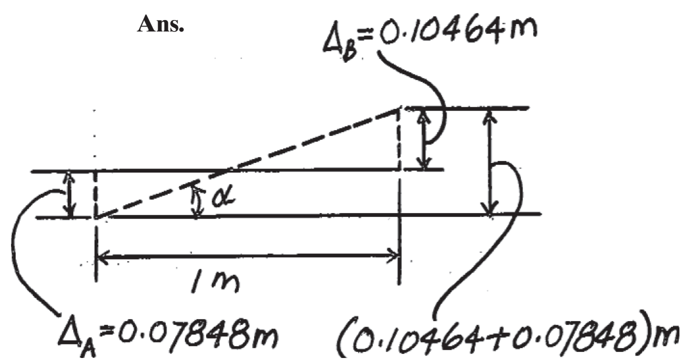


Spring Formula: Applying $\Delta = \dfrac{F}{k}$, we have

$$\Delta_A = \frac{1177.2}{15(10^3)} = 0.07848 \text{ m} \qquad \Delta_B = \frac{1569.6}{15(10^3)} = 0.10464 \text{ m}$$

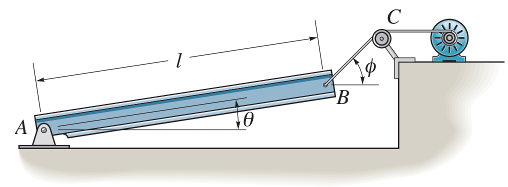
Geometry: The angle of tilt α is

$$\alpha = \tan^{-1}\left(\frac{0.10464 + 0.07848}{1}\right) = 10.4° \qquad \textbf{Ans.}$$



5–53.

The uniform beam has a weight W and length l and is supported by a pin at A and a cable BC. Determine the horizontal and vertical components of reaction at A and the tension in the cable necessary to hold the beam in the position shown.

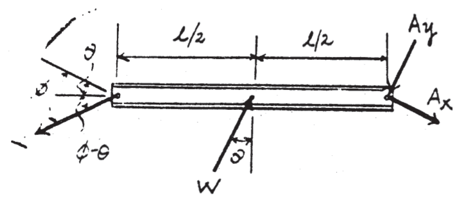


SOLUTION

Equations of Equilibrium: The tension in the cable can be obtained directly by summing moments about point A.

$$\circlearrowleft + \Sigma M_A = 0; \qquad T \sin(\phi - \theta) l - W \cos\theta\left(\frac{l}{2}\right) = 0$$

$$T = \frac{W \cos\theta}{2 \sin(\phi - \theta)} \qquad \textbf{Ans.}$$



Using the result $T = \dfrac{W \cos\theta}{2 \sin(\phi - \theta)}$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad \left(\frac{W \cos\theta}{2 \sin(\phi - \theta)}\right)\cos\phi - A_x = 0$$

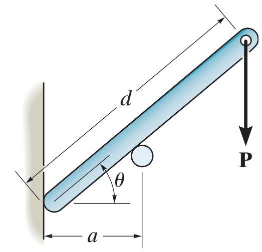
$$A_x = \frac{W \cos\phi \cos\theta}{2 \sin(\phi - \theta)} \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad A_y + \left(\frac{W \cos\theta}{2 \sin(\phi - \theta)}\right)\sin\phi - W = 0$$

$$A_y = \frac{W(\sin\phi \cos\theta - 2 \cos\phi \sin\theta)}{2 \sin(\phi - \theta)} \qquad \textbf{Ans.}$$

5–54.

Determine the distance d for placement of the load $\mathbf{P}$ for equilibrium of the smooth bar in the position θ as shown. Neglect the weight of the bar.



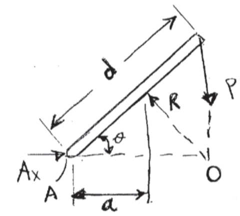
SOLUTION

$+\uparrow\Sigma F_y = 0;\qquad R\cos\theta - P = 0$

$\circlearrowleft + \Sigma M_A = 0;\qquad -P(d\cos\theta) + R\left(\dfrac{a}{\cos\theta}\right) = 0$

$$Rd\cos^2\theta = R\left(\frac{a}{\cos\theta}\right)$$

$$d = \frac{a}{\cos^3\theta}$$ **Ans.**



Also;

Require forces to be concurrent at point O.

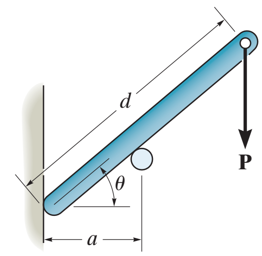
$$AO = d\cos\theta = \frac{a/\cos\theta}{\cos\theta}$$

Thus,

$$d = \frac{a}{\cos^3\theta}$$ **Ans.**

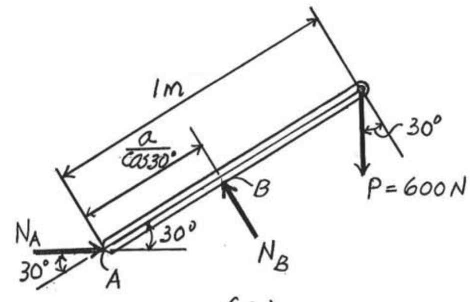
5–55.

If $d = 1$ m, and $\theta = 30°$, determine the normal reaction at the smooth supports and the required distance a for the placement of the roller if $P = 600$ N. Neglect the weight of the bar.



SOLUTION

Equations of Equilibrium: Referring to the FBD of the rod shown in Fig. a,



(a)

$\circlearrowleft + \Sigma M_A = 0;$ $$N_B = \left(\frac{a}{\cos 30°}\right) - 600 \cos 30°(1) = 0$$

$$N_B = \frac{450}{a} \qquad (1)$$

$\nwarrow^{+} \Sigma F_{y'} = 0;$ $$N_B - N_A \sin 30° - 600 \cos 30° = 0$$

$$N_B - 0.5N_A = 600 \cos 30° \qquad (2)$$

$^{+}\nearrow \Sigma F_{x'} = 0;$ $$N_A \cos 30° - 600 \sin 30° = 0$$

$$N_A = 346.41 \text{ N} = 346 \text{ N}$$ **Ans.**

Substitute this result into Eq (2),

$$N_B - 0.5(346.41) = 600 \cos 30°$$

$$N_B = 692.82 \qquad N = 693 \text{ N}$$ **Ans.**

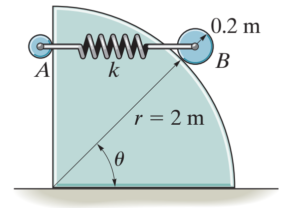
Substitute this result into Eq (1),

$$692.82 = \frac{450}{a}$$

$$a = 0.6495 \text{ m} \qquad m = 0.650 \text{ m}$$ **Ans.**

***5–56.**

The disk B has a mass of 20 kg and is supported on the smooth cylindrical surface by a spring having a stiffness of $k = 400$ N/m and unstretched length of $l_0 = 1$ m. The spring remains in the horizontal position since its end A is attached to the small roller guide which has negligible weight. Determine the angle θ for equilibrium of the roller.

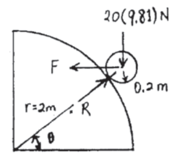


SOLUTION

$+\uparrow \Sigma F_y = 0; \qquad R \sin\theta - 20(9.81) = 0$

$\xrightarrow{+} \Sigma F_x = 0; \qquad R \cos\theta - F = 0$

$$\tan\theta = \frac{20(9.81)}{F}$$



Since $\cos\theta = \dfrac{1.0 + \frac{F}{400}}{2.2}$

$$2.2 \cos\theta = 1.0 + \frac{20(9.81)}{400 \tan\theta}$$

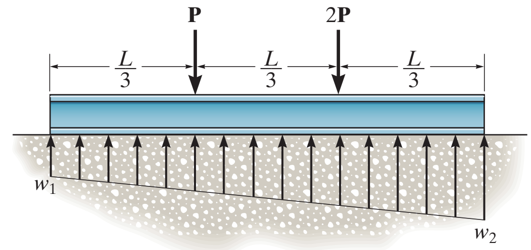
$$880 \sin\theta = 400 \tan\theta + 20(9.81)$$

Solving,

$$\theta = 27.1^\circ \quad \text{and} \quad \theta = 50.2^\circ$$ **Ans.**

5–57.

The beam is subjected to the two concentrated loads. Assuming that the foundation exerts a linearly varying load distribution on its bottom, determine the load intensities w_1 and w_2 for equilibrium if $P = 500$ lb and $L = 12$ ft.



SOLUTION

Equations of Equilibrium: Referring to the FBD of the beam shown in Fig. *a*, we notice that W_1 can be obtained directly by writing moment equations of equilibrium about point A.

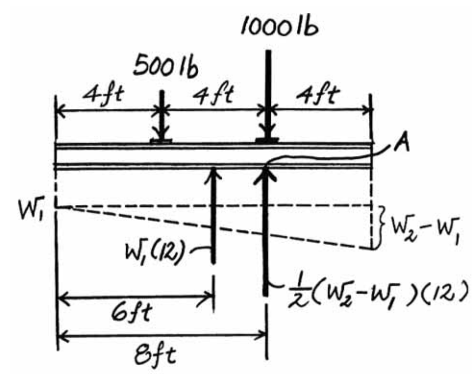
$\circlearrowleft \Sigma M_A = 0; \quad 500(4) - W_1(12)(2) = 0$

$W_1 = 83.33 \text{ lb/ft} = 83.3 \text{ lb/ft}$ **Ans.**

Using this result to write the force equation of equilibrium along y axis,

$+\uparrow \Sigma F_y = 0; \quad 83.33(12) + \frac{1}{2}(W_2 - 83.33)(12) - 500 - 1000 = 0$

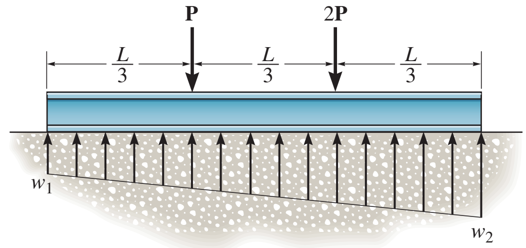
$W_2 = 166.67 \text{ lb/ft} = 167 \text{ lb/ft}$ **Ans.**



(a)

5–58.

The beam is subjected to the two concentrated loads. Assuming that the foundation exerts a linearly varying load distribution on its bottom, determine the load intensities w_1 and w_2 for equilibrium in terms of the parameters shown.

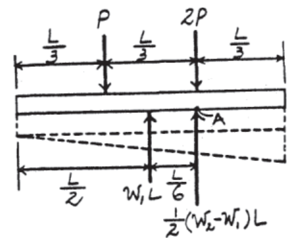


SOLUTION

Equations of Equilibrium: The load intensity w_1 can be determined directly by summing moments about point A.

$\circlearrowleft + \Sigma M_A = 0; \qquad P\left(\frac{L}{3}\right) - w_1 L\left(\frac{L}{6}\right) = 0$

$$w_1 = \frac{2P}{L}$$ **Ans.**

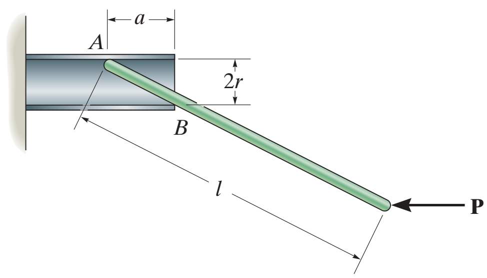


$+\uparrow \Sigma F_y = 0; \qquad \frac{1}{2}\left(w_2 - \frac{2P}{L}\right)L + \frac{2P}{L}(L) - 3P = 0$

$$w_2 = \frac{4P}{L}$$ **Ans.**

5–59.

The thin rod of length l is supported by the smooth tube. Determine the distance a needed for equilibrium if the applied load is **P**.



SOLUTION

$$\xrightarrow{+} \Sigma F_x = 0; \qquad \frac{2\,r}{\sqrt{4\,r^2 + a^2}}\,N_B - P = 0$$

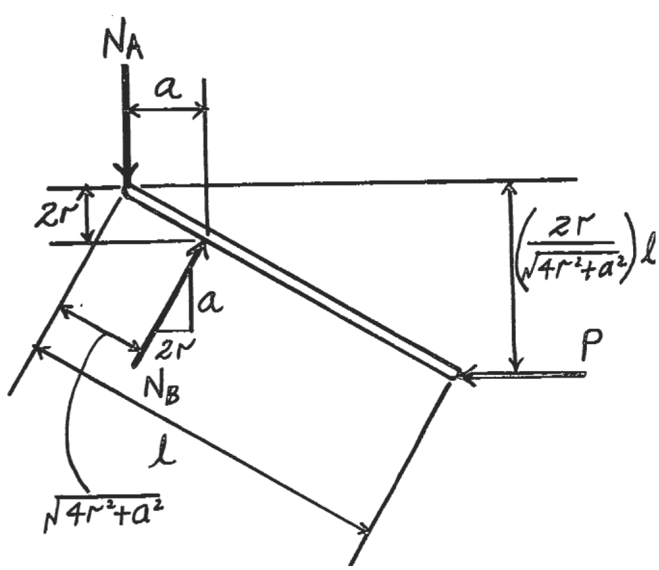
$$\circlearrowleft + \Sigma M_A = 0; \qquad -P\left(\frac{2\,r}{\sqrt{4\,r^2 + a^2}}\right)l + N_B\sqrt{4r^2 + a^2} = 0$$

$$\frac{4\,r^2\,l}{4\,r^2 + a^2} - \sqrt{4\,r^2 + a^2} = 0$$

$$4\,r^2\,l = \left(4\,r^2 + a^2\right)^{\frac{3}{2}}$$

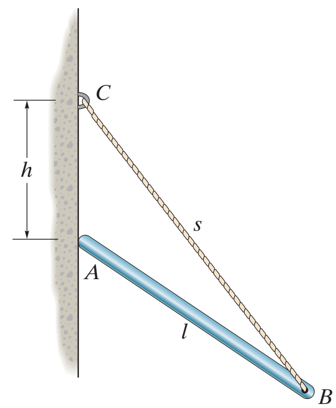
$$\left(4\,r^2\,l\right)^{\frac{2}{3}} = 4\,r^2 + a^2$$

$$a = \sqrt{(4\,r^2\,l)^{\frac{2}{3}} - 4\,r^2} \qquad \textbf{Ans.}$$



***5–60.**

The 30-N uniform rod has a length of $l = 1$ m. If $s = 1.5$ m, determine the distance h of placement at the end A along the smooth wall for equilibrium.



SOLUTION

Equations of Equilibrium: Referring to the FBD of the rod shown in Fig. *a*, write the moment equation of equilibrium about point A.

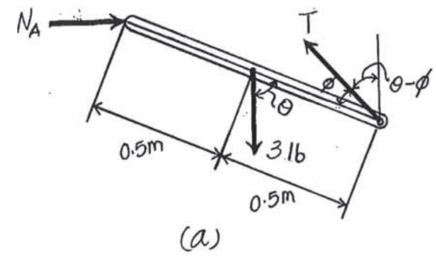
$$\zeta + \Sigma M_A = 0; \qquad T \sin\phi(1) - 3 \sin\theta(0.5) = 0$$

$$T = \frac{1.5 \sin\theta}{\sin\phi}$$

Using this result to write the force equation of equilibrium along y axis,

$$+\uparrow \Sigma F_y = 0; \qquad \left(\frac{15 \sin\theta}{\sin\phi}\right) \cos(\theta - \phi) - 3 = 0$$

$$\sin\theta \cos(\theta - \phi) - 2 \sin\phi = 0 \qquad \textbf{(1)}$$



Geometry: Applying the sine law with $\sin(180° - \theta) = \sin\theta$ by referring to Fig. *b*,

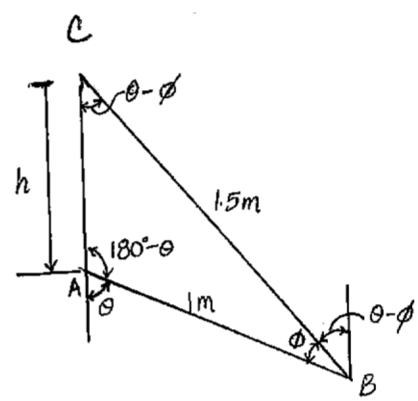
$$\frac{\sin\phi}{h} = \frac{\sin\theta}{1.5}; \qquad \sin\theta = \left(\frac{h}{1.5}\right)\sin\theta \qquad \textbf{(2)}$$

Substituting Eq. (2) into (1) yields

$$\sin\theta\left[\cos(\theta - \phi) - \frac{4}{3}h\right] = 0$$

since $\sin\theta \neq 0$, then

$$\cos(\theta - \phi) - (4/3)h \qquad \cos(\theta - \phi) = (4/3)h \qquad \textbf{(3)}$$



Again, applying law of cosine by referring to Fig. *b*,

$$l^2 = h^2 + 1.5^2 - 2(h)(1.5)\cos(\theta - \phi)$$

$$\cos(\theta - \phi) = \frac{h^2 + 1.25}{3h} \qquad \textbf{(4)}$$

Equating Eqs. (3) and (4) yields

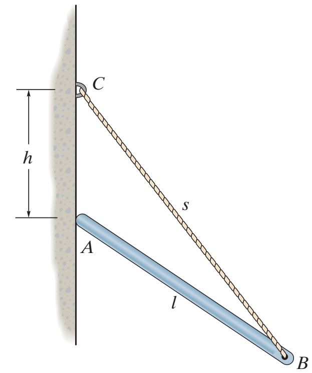
$$\frac{4}{3}h = \frac{h^2 + 1.25}{3h}$$

$$3h^2 = 1.25$$

$$h = 0.645 \text{ m} \qquad \textbf{Ans.}$$

5–61.

The uniform rod has a length l and weight W. It is supported at one end A by a smooth wall and the other end by a cord of length s which is attached to the wall as shown. Determine the placement h for equilibrium.

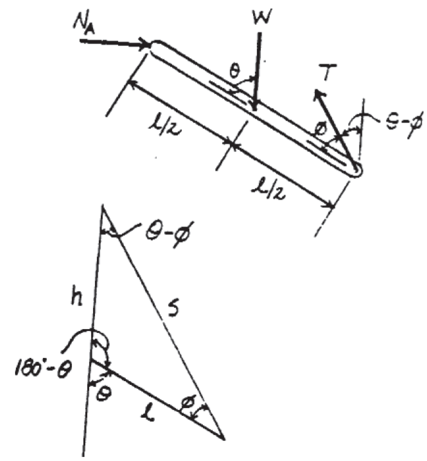


SOLUTION

Equations of Equilibrium: The tension in the cable can be obtained directly by summing moments about point A.

$$\circlearrowleft +\Sigma M_A = 0; \qquad T \sin \phi(l) - W \sin \theta\left(\frac{l}{2}\right) = 0$$

$$T = \frac{W \sin \theta}{2 \sin \phi}$$



Using the result $T = \dfrac{W \sin \theta}{2 \sin \phi}$,

$$+\uparrow \Sigma F_y = 0; \qquad \frac{W \sin \theta}{2 \sin \phi} \cos(\theta - \phi) - W = 0$$

$$\sin \theta \cos(\theta - \phi) - 2 \sin \phi = 0 \qquad \textbf{(1)}$$

Geometry: Applying the sine law with $\sin(180° - \theta) = \sin \theta$, we have

$$\frac{\sin \phi}{h} = \frac{\sin \theta}{s} \qquad \sin \phi = \frac{h}{s} \sin \theta \qquad \textbf{(2)}$$

Substituting Eq. (2) into (1) yields

$$\cos(\theta - \phi) = \frac{2h}{s} \qquad \textbf{(3)}$$

Using the cosine law,

$$l^2 = h^2 + s^2 - 2hs \cos(\theta - \phi)$$

$$\cos(\theta - \phi) = \frac{h^2 + s^2 - l^2}{2hs} \qquad \textbf{(4)}$$

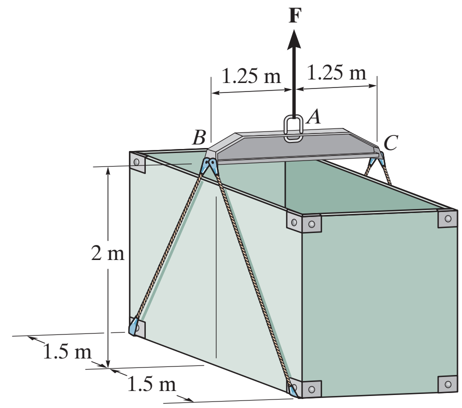
Equating Eqs. (3) and (4) yields

$$\frac{2h}{s} = \frac{h^2 + s^2 - l^2}{2hs}$$

$$h = \sqrt{\frac{s^2 - l^2}{3}} \qquad \textbf{Ans.}$$

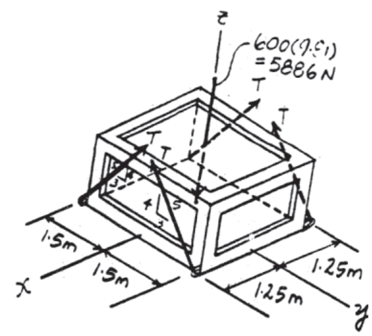
5–62.

The uniform load has a mass of 600 kg and is lifted using a uniform 30-kg strongback beam and four wire ropes as shown. Determine the tension in each segment of rope and the force that must be applied to the sling at A.



SOLUTION

Equations of Equilibrium: Due to symmetry, all wires are subjected to the same tension. This condition statisfies moment equilibrium about the x and y axes and force equilibrium along y axis.



$$\Sigma F_z = 0; \qquad 4T\left(\frac{4}{5}\right) - 5886 = 0$$

$$T = 1839.375 \text{ N} = 1.84 \text{ kN} \qquad \textbf{Ans.}$$

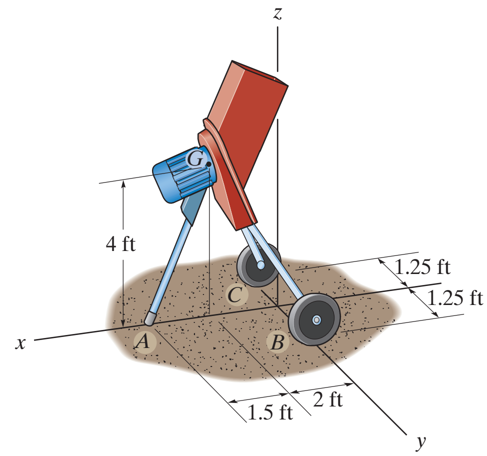
The force **F** applied to the sling A must support the weight of the load and strongback beam. Hence

$$\Sigma F_z = 0; \qquad F - 600(9.81) - 30(9.81) = 0$$

$$F = 6180.3 \text{ N} = 6.18 \text{ kN} \qquad \textbf{Ans.}$$

5–63.

The 50-lb mulching machine has a center of gravity at G. Determine the vertical reactions at the wheels C and B and the smooth contact point A.



SOLUTION

Equations of Equilibrium: From the free-body diagram of the mulching machine, Fig. *a*, N_A can be obtained by writing the moment equation of equilibrium about the y axis.

$$\Sigma M_y = 0; \quad 50(2) - N_A(1.5 + 2) = 0$$

$$N_A = 28.57 \text{ lb} = 28.6 \text{ lb} \qquad \textbf{Ans.}$$

Using the above result and writing the moment equation of equilibrium about the x axis and the force equation of equilibrium along the z axis, we have

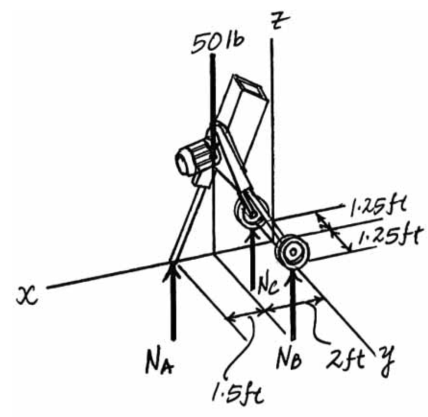
$$\Sigma M_x = 0; \quad N_B(1.25) - N_C(1.25) = 0 \qquad \textbf{(1)}$$

$$\Sigma F_z = 0; \quad N_B + N_C + 28.57 - 50 = 0 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$N_B = N_C = 10.71 \text{ lb} = 10.7 \text{ lb} \qquad \textbf{Ans.}$$

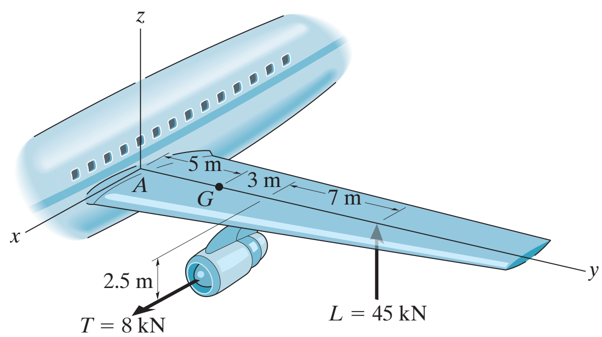
Note: If we write the force equation of equilibrium $\Sigma F_x = 0$ and $\Sigma F_y = 0$ and the moment equation of equilibrium $\Sigma M_z = 0$. This indicates that equilibrium is satisfied.



(a)

***5–64.**

The wing of the jet aircraft is subjected to a thrust of $T = 8$ kN from its engine and the resultant lift force $L = 45$ kN. If the mass of the wing is 2.1 Mg and the mass center is at G, determine the x, y, z components of reaction where the wing is fixed to the fuselage at A.



SOLUTION

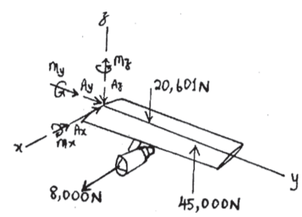
$\Sigma F_x = 0;\quad -A_x + 8000 = 0$

$A_x = 8.00$ kN **Ans.**

$\Sigma F_y = 0;\quad A_y = 0$ **Ans.**

$\Sigma F_z = 0;\quad -A_z - 20\,601 + 45\,000 = 0$

$A_z = 24.4$ kN **Ans.**



$\Sigma M_y = 0;\quad M_y - 2.5(8000) = 0$

$M_y = 20.0\ \text{kN}\cdot\text{m}$ **Ans.**

$\Sigma M_x = 0;\quad 45\,000(15) - 20\,601(5) - M_x = 0$

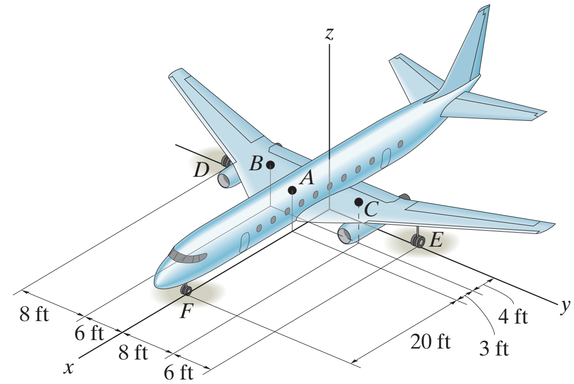
$M_x = 572\ \text{kN}\cdot\text{m}$ **Ans.**

$\Sigma M_z = 0;\quad M_z - 8000(8) = 0$

$M_z = 64.0\ \text{kN}\cdot\text{m}$ **Ans.**

5–65.

Due to an unequal distribution of fuel in the wing tanks, the centers of gravity for the airplane fuselage A and wings B and C are located as shown. If these components have weights $W_A = 45\,000$ lb, $W_B = 8000$ lb, and $W_C = 6000$ lb, determine the normal reactions of the wheels D, E, and F on the ground.

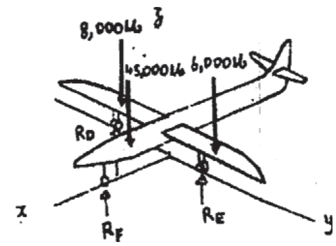


SOLUTION

$\Sigma M_x = 0; \qquad 8000(6) - R_D\,(14) - 6000(8) + R_E(14) = 0$

$\Sigma M_y = 0; \qquad 8000(4) + 45\,000(7) + 6000(4) - R_F\,(27) = 0$

$\Sigma F_z = 0; \qquad R_D + R_E + R_F - 8000 - 6000 - 45\,000 = 0$



Solving,

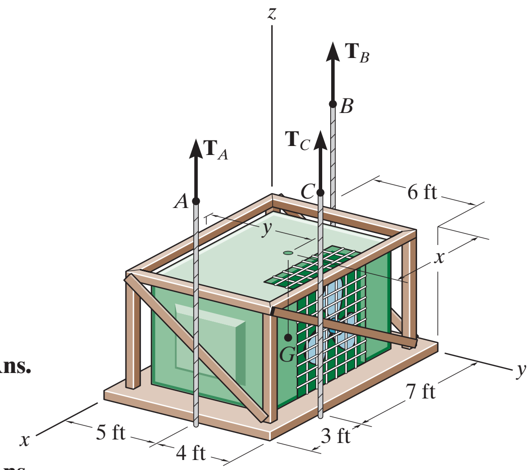
$R_D = 22.6$ kip **Ans.**

$R_E = 22.6$ kip **Ans.**

$R_F = 13.7$ kip **Ans.**

5–66.

The air-conditioning unit is hoisted to the roof of a building using the three cables. If the tensions in the cables are $T_A = 250$ lb, $T_B = 300$ lb, and $T_C = 200$ lb, determine the weight of the unit and the location (x, y) of its center of gravity G.



SOLUTION

$\Sigma F_z = 0;$ $\quad 250 + 300 + 200 - W = 0$

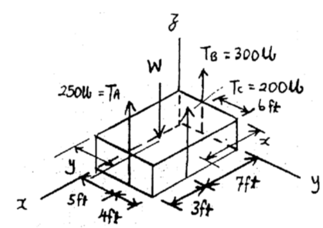
$W = 750 \text{ lb}$ **Ans.**

$\Sigma M_y = 0;$ $\quad 750(x) - 250(10) - 200(7) = 0$

$x = 5.20 \text{ ft}$ **Ans.**

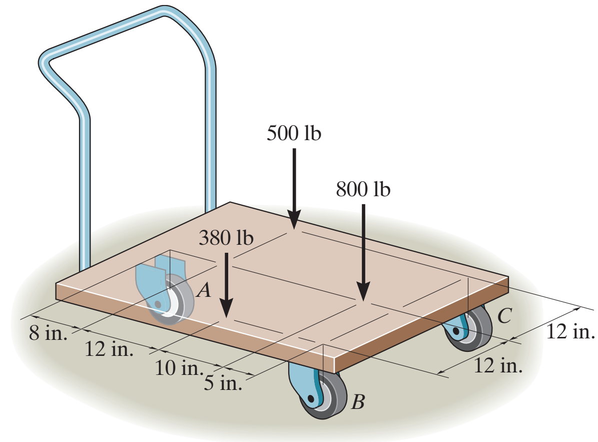
$\Sigma M_x = 0;$ $\quad 250(5) + 300(3) + 200(9) - 750(y) = 0$

$y = 5.27 \text{ ft}$ **Ans.**



5–67.

The platform truck supports the three loadings shown. Determine the normal reactions on each of its three wheels.



SOLUTION

$\Sigma M_x = 0;\qquad 380(15) + 500(27) + 800(5) - F_A(35) = 0$

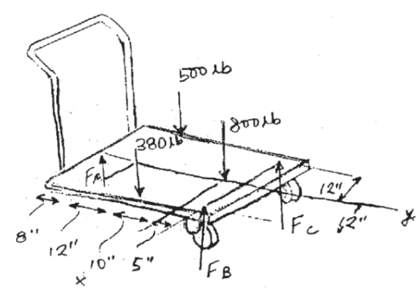
$F_A = 662.8571 = 663 \text{ lb}$ **Ans.**

$\Sigma M_y = 0;\qquad 380(12) - F_B(12) - 500(12) + F_C(12)$

$F_C - F_B = 120$

$\Sigma F_y = 0;\qquad F_B + F_C - 500 + 663 - 380 - 800 = 0$

$F_B + F_C = 1017.1429$



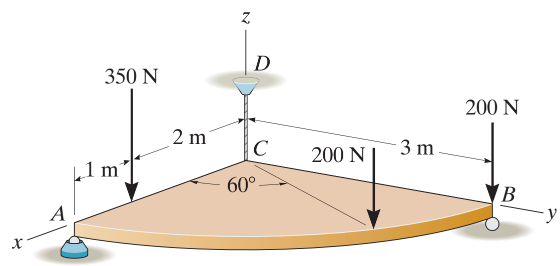
Solving,

$F_C = 569 \text{ lb}$ **Ans.**

$F_B = 449 \text{ lb}$ **Ans.**

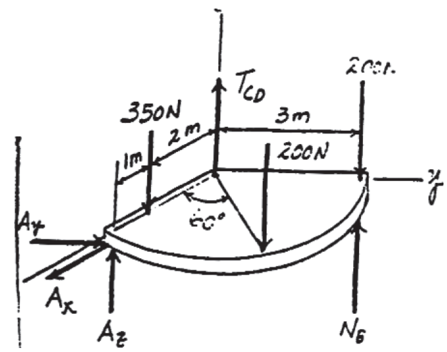
***5–68.**

Determine the force components acting on the ball-and-socket at A, the reaction at the roller B and the tension on the cord CD needed for equilibrium of the quarter circular plate.



SOLUTION

Equations of Equilibrium: The normal reactions N_B and A_z can be obtained directly by summing moments about the x and y axes, respectively.



$\Sigma M_x = 0; \qquad N_B(3) - 200(3) - 200(3 \sin 60°) = 0$

$$N_B = 373.21 \text{ N} = 373 \text{ N} \qquad \textbf{Ans.}$$

$\Sigma M_y = 0; \qquad 350(2) + 200(3 \cos 60°) - A_z(3) = 0$

$$A_z = 333.33 \text{ N} = 333 \text{ N} \qquad \textbf{Ans.}$$

$\Sigma F_z = 0; \qquad T_{CD} + 373.21 + 333.33 - 350 - 200 - 200 = 0$

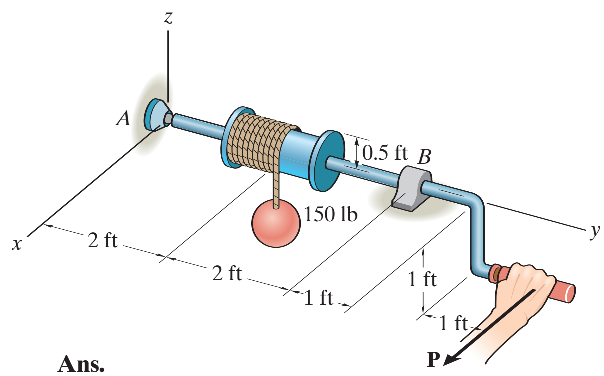
$$T_{CD} = 43.5 \text{ N} \qquad \textbf{Ans.}$$

$\Sigma F_x = 0; \qquad A_x = 0 \qquad$ **Ans.**

$\Sigma F_y = 0; \qquad A_y = 0 \qquad$ **Ans.**

5–69.

The windlass is subjected to a load of 150 lb. Determine the horizontal force **P** needed to hold the handle in the position shown, and the components of reaction at the ball-and-socket joint A and the smooth journal bearing B. The bearing at B is in proper alignment and exerts only force reactions on the windlass.



SOLUTION

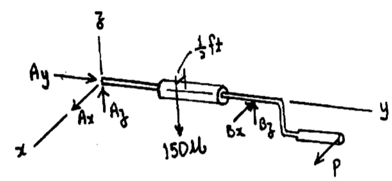
$\Sigma M_y = 0;$ $(150)(0.5) - P(1) = 0$

$P = 75 \text{ lb}$ **Ans.**

$\Sigma F_y = 0;$ $A_y = 0$ **Ans.**

$\Sigma M_x = 0;$ $-(150)(2) + B_z(4) = 0$

$B_z = 75 \text{ lb}$ **Ans.**



$\Sigma F_z = 0;$ $A_z + 75 - 150 = 0$

$A_z = 75 \text{ lb}$ **Ans.**

$\Sigma M_z = 0;$ $B_x(4) - 75(6) = 0$

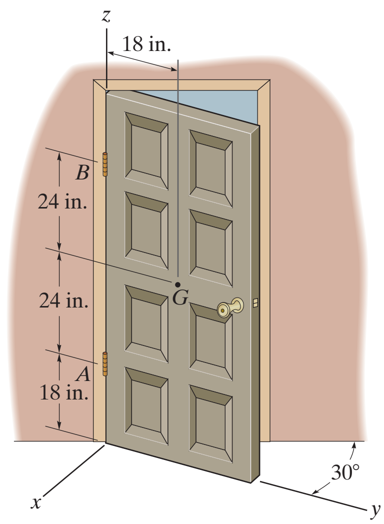
$B_x = 112.5 = 112 \text{ lb}$ **Ans.**

$\Sigma F_x = 0;$ $A_x - 112.5 + 75 = 0$

$A_x = 37.5 \text{ lb}$ **Ans.**

5–70.

The 100-lb door has its center of gravity at G. Determine the components of reaction at hinges A and B if hinge B resists only forces in the x and y directions and A resists forces in the x, y, z directions.



SOLUTION

Equations of Equilibrium: From the free-body diagram of the door, Fig. *a*, B_y, B_x, and A_z can be obtained by writing the moment equation of equilibrium about the x' and y' axes and the force equation of equilibrium along the z axis.

$\Sigma M_{x'} = 0;$ $\quad -B_y(48) - 100(18) = 0$

$B_y = -37.5 \text{ lb}$ **Ans.**

$\Sigma M_{y'} = 0;$ $\quad B_x = 0$ **Ans.**

$\Sigma F_z = 0;$ $\quad -100 + A_z = 0;$ $\quad A_z = 100 \text{ lb}$ **Ans.**

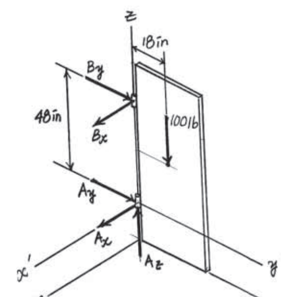
Using the above result and writing the force equations of equilibrium along the x and y axes, we have

$\Sigma F_x = 0;$ $\quad A_x = 0$ **Ans.**

$\Sigma F_y = 0;$ $\quad A_y + (-37.5) = 0$

$A_y = 37.5 \text{ lb}$ **Ans.**

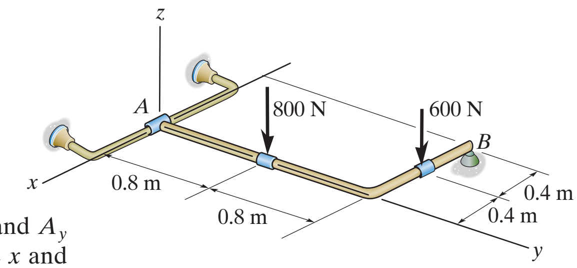
The negative sign indicates that $\mathbf{B}_y$ acts in the opposite sense to that shown on the free-body diagram. If we write the moment equation of equilibrium $\Sigma M_z = 0$, it shows that equilibrium is satisfied.



(a)

5–71.

Determine the support reactions at the smooth collar A and the normal reaction at the roller support B.



SOLUTION

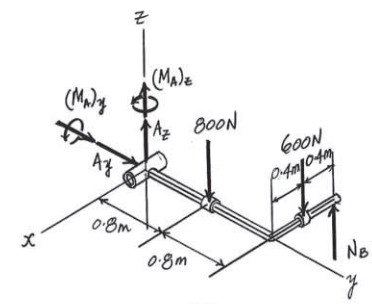
Equations of Equilibrium: From the free-body diagram, Fig. *a*, N_B, $(M_A)_z$, and A_y can be obtained by writing the moment equations of equilibrium about the x and z axes and the force equation of equilibrium along the y axis.

$\Sigma M_x = 0;\quad N_B(0.8 + 0.8) - 800(0.8) - 600(0.8 + 0.8) = 0$

$N_B = 1000 \text{ N}$ **Ans.**

$\Sigma M_z = 0;\quad (M_A)_z = 0$ **Ans.**

$\Sigma F_y = 0;\quad A_y = 0$ **Ans.**



(a)

Using the result $N_B = 1000$ N and writing the moment equation of equilibrium about the y axis and the force equation of equilibrium along the z axis, we have

$\Sigma M_y = 0;\quad (M_A)_y - 600(0.4) + 1000(0.8) = 0$

$(M_A)_y = -560 \text{ N}\cdot\text{m}$ **Ans.**

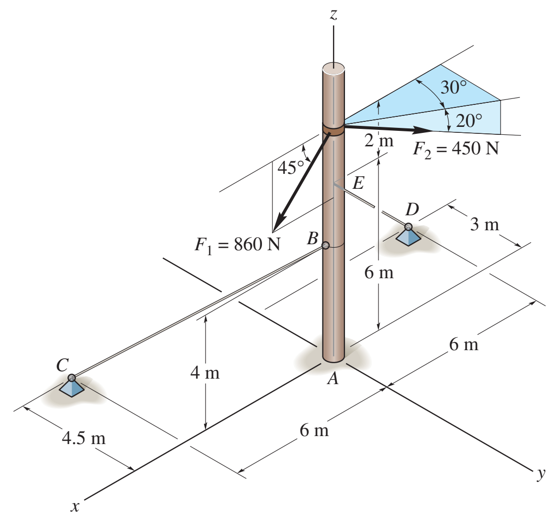
$\Sigma F_z = 0;\quad A_z + 1000 - 800 - 600 = 0$

$A_z = 400 \text{ N}$ **Ans.**

The negative sign indicates that $(\mathbf{M}_A)_y$ acts in the opposite sense to that shown on the free-body diagram. If we write the force equation of equilibrium along the x axis, $\Sigma F_x = 0$, and so equilibrium is satisfied.

***5–72.**

The pole is subjected to the two forces shown. Determine the components of reaction of A assuming it to be a ball-and-socket joint. Also, compute the tension in each of the guy wires, BC and ED.



SOLUTION

Force Vector and Position Vectors:

$$\mathbf{F}_A = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}$$

$$\mathbf{F}_1 = 860\{\cos 45°\mathbf{i} - \sin 45°\mathbf{k}\}\text{ N} = \{608.11\mathbf{i} - 608.11\mathbf{k}\}\text{ N}$$

$$\mathbf{F}_2 = 450\{-\cos 20° \cos 30°\mathbf{i} + \cos 20° \sin 30°\mathbf{k} - \sin 20°\mathbf{k}\}\text{ N}$$

$$= \{-366.21\mathbf{i} + 211.43\mathbf{j} - 153.91\mathbf{k}\}\text{ N}$$

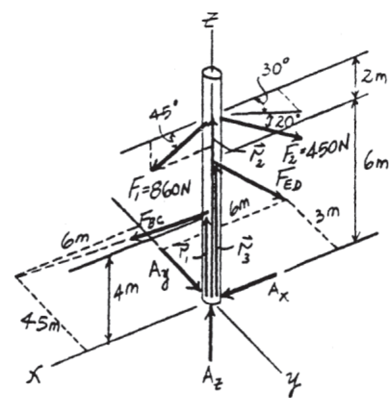
$$\mathbf{F}_{ED} = F_{ED}\left[\frac{(-6-0)\mathbf{i} + (-3-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(-6-0)^2 + (-3-0)^2 + (0-6)^2}}\right]$$

$$= -\frac{2}{3}F_{ED}\mathbf{i} - \frac{1}{3}F_{ED}\mathbf{j} - \frac{2}{3}F_{ED}\mathbf{k}$$

$$\mathbf{F}_{BC} = F_{BC}\left[\frac{(6-0)\mathbf{i} + (-4.5-0)\mathbf{j} + (0-4)\mathbf{k}}{\sqrt{(6-0)^2 + (-4.5-0)^2 + (0-4)^2}}\right]$$

$$= \frac{12}{17}F_{BC}\mathbf{i} - \frac{9}{17}F_{BC}\mathbf{j} - \frac{8}{17}F_{BC}\mathbf{k}$$

$$\mathbf{r}_1 = \{4\mathbf{k}\}\text{ m} \qquad \mathbf{r}_2 = \{8\mathbf{k}\}\text{ m} \qquad \mathbf{r}_3 = \{6\mathbf{k}\}\text{ m}$$



Equations of Equilibrium: Force equilibrium requires

$$\Sigma\mathbf{F} = \mathbf{0}; \qquad \mathbf{F}_A + \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_{ED} + \mathbf{F}_{BC} = \mathbf{0}$$

$$\left(A_x + 608.11 - 366.21 - \frac{2}{3}F_{ED} + \frac{12}{17}F_{BC}\right)\mathbf{i}$$

$$+\left(A_y + 211.43 - \frac{1}{3}F_{ED} - \frac{9}{17}F_{BC}\right)\mathbf{j}$$

$$+\left(A_z - 608.11 - 153.91 - \frac{2}{3}F_{ED} - \frac{8}{17}F_{BC}\right)\mathbf{k} = \mathbf{0}$$

Equating **i**, **j** and **k** components, we have

$$\Sigma F_x = 0; \qquad A_x + 608.11 - 366.21 - \frac{2}{3}F_{ED} + \frac{12}{17}F_{BC} = 0 \qquad \textbf{(1)}$$

$$\Sigma F_y = 0; \qquad A_y + 211.43 - \frac{1}{3}F_{ED} - \frac{9}{17}F_{BC} = 0 \qquad \textbf{(2)}$$

$$\Sigma F_z = 0; \qquad A_z - 608.11 - 153.91 - \frac{2}{3}F_{ED} - \frac{8}{17}F_{BC} = 0 \qquad \textbf{(3)}$$

***5–72. (continued)**

Moment equilibrium requires

$\Sigma \mathbf{M}_A = \mathbf{0}; \qquad \mathbf{r}_1 \times \mathbf{F}_{BC} + \mathbf{r}_2 \times (\mathbf{F}_1 + \mathbf{F}_2) + \mathbf{r}_3 \times \mathbf{F}_{ED} = \mathbf{0}$

$$4\mathbf{k} \times \left(\frac{12}{17}F_{BC}\mathbf{i} - \frac{9}{17}F_{BC}\mathbf{j} - \frac{8}{17}F_{BC}\mathbf{k}\right)$$

$$+ 8\mathbf{k} \times (241.90\mathbf{i} + 211.43\mathbf{j} - 762.02\mathbf{k})$$

$$+ 6\mathbf{k} \times \left(-\frac{2}{3}F_{ED}\mathbf{i} - \frac{1}{3}F_{ED}\mathbf{j} - \frac{2}{3}F_{ED}\mathbf{k}\right) = \mathbf{0}$$

Equating **i**, **j** and **k** components, we have

$\Sigma M_x = 0; \qquad \frac{36}{17}F_{BC} + 2F_{ED} - 1691.45 = 0$ **(4)**

$\Sigma M_y = 0; \qquad \frac{48}{17}F_{BC} - 4F_{ED} + 1935.22 = 0$ **(5)**

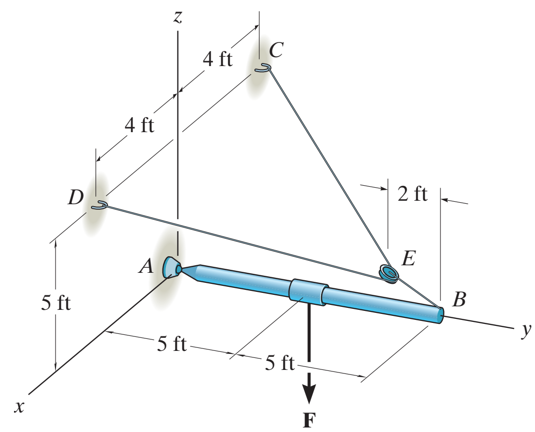
Solving Eqs. (4) and (5) yields

$F_{BC} = 205.09 \text{ N} = 205 \text{ N} \qquad F_{ED} = 628.57 \text{ N} = 629 \text{ N}$ **Ans.**

Substituting the results into Eqs. (1), (2) and (3) yields

$A_x = 32.4 \text{ N} \qquad A_y = 107 \text{ N} \qquad A_z = 1277.58 \text{ N} = 1.28 \text{ kN}$ **Ans.**

5–73.



The boom AB is held in equilibrium by a ball-and-socket joint A and a pulley and cord system as shown. Determine the x, y, z components of reaction at A and the tension in cable DEC if $\mathbf{F} = \{-1500\mathbf{k}\}$ lb.

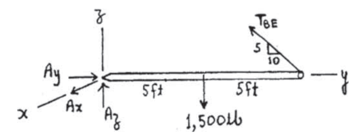
SOLUTION

From FBD of boom,

$\Sigma M_x = 0;\qquad \frac{5}{\sqrt{125}}T_{BE}(10) - 1500(5) = 0$

$T_{BE} = 1677.05 \text{ lb}$

$\Sigma F_x = 0;\qquad A_x = 0$ **Ans.**

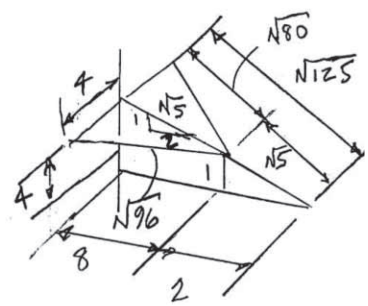


$\Sigma F_y = 0;\qquad A_y - \frac{10}{\sqrt{125}}(1677.05) = 0$

$A_y = 1500 \text{ lb} = 1.50 \text{ kip}$ **Ans.**

$\Sigma F_z = 0;\qquad A_z - 1500 + \frac{5}{\sqrt{125}}(1677.05) = 0$

$A_z = 750 \text{ lb}$ **Ans.**



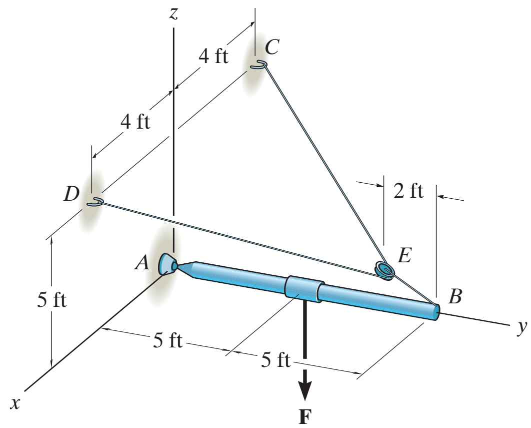
From FBD of pulley,

$\Sigma F_z = 0;\qquad 2\left(\frac{4}{\sqrt{96}}\right)T - \frac{1}{\sqrt{5}}(1677.05) = 0$

$T = 918.56 = 919 \text{ lb}$ **Ans.**

5–74.

The cable *CED* can sustain a maximum tension of 800 lb before it fails. Determine the greatest vertical force *F* that can be applied to the boom. Also, what are the *x, y, z* components of reaction at the ball-and-socket joint *A*?

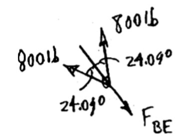


SOLUTION

From FBD of pulley;

$\Sigma F_{x'} = 0;$ $\quad 2(800)\cos 24.09° - F_{BE} = 0$

$$F_{BE} = 1460.59 \text{ lb}$$



From FBD of boom;

$\Sigma M_x = 0;$ $\quad \frac{5}{\sqrt{125}}(1460.59)(10) - F(5) = 0$

$F = 1306.39 \text{ lb} = 1.31 \text{ kip}$ **Ans.**

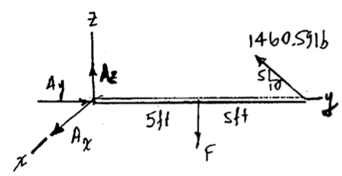
$\Sigma F_x = 0;$ $\quad A_x = 0$ **Ans.**

$\Sigma F_y = 0;$ $\quad A_y - \frac{10}{\sqrt{125}}(1460.59) = 0$

$A_y = 1306.39 \text{ lb} = 1.31 \text{ kip}$ **Ans.**

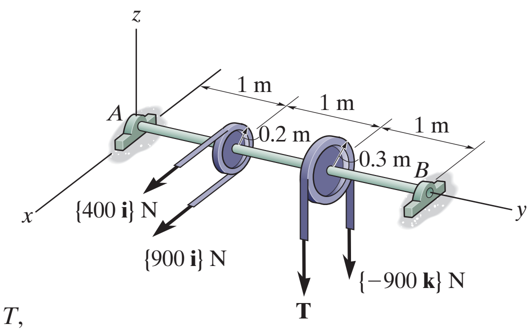
$\Sigma F_z = 0;$ $\quad A_z - 1306.39 + \frac{5}{\sqrt{125}}(1460.59) = 0$

$A_z = 653 \text{ lb}$ **Ans.**



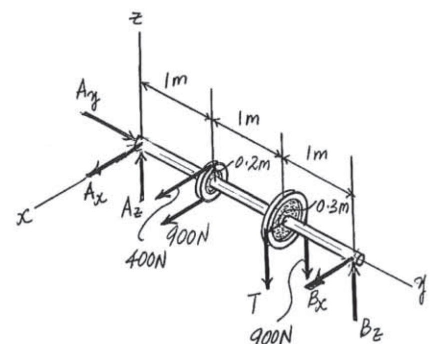
5–75.

If the pulleys are fixed to the shaft, determine the magnitude of tension **T** and the x, y, z components of reaction at the smooth thrust bearing A and smooth journal bearing B.



SOLUTION

Equations of Equilibrium: From the free-body diagram of the shaft, Fig. a, A_y, T, and B_x can be obtained by writing the force equation of equilibrium along the y axis and the moment equations of equilibrium about the y and z axes, respectively.



(a)

$\Sigma F_y = 0; \quad A_y = 0$ **Ans.**

$\Sigma M_y = 0; \quad 400(0.2) - 900(0.2) - 900(0.3) + T(0.3) = 0$

$T = 1233.33 \text{ N} = 1.23 \text{ kN}$ **Ans.**

$\Sigma M_z = 0; \quad -B_x(3) - 400(1) - 900(1) = 0$

$B_x = -433.33 \text{ N} = -433\text{N}$ **Ans.**

Using the above results and writing the moment equation of equilibrium about the x axis and the force equation of equilibrium along the x axis, we have

$\Sigma M_x = 0; \quad B_z(3) - 900(2) - 1233.33(2) = 0$

$B_z = 1422.22 \text{ N} = 1.42 \text{ kN}$ **Ans.**

$\Sigma F_x = 0; \quad 400 + 900 - 433.33 - A_x = 0$

$A_x = 866.67 \text{ N} = 867 \text{ N}$ **Ans.**

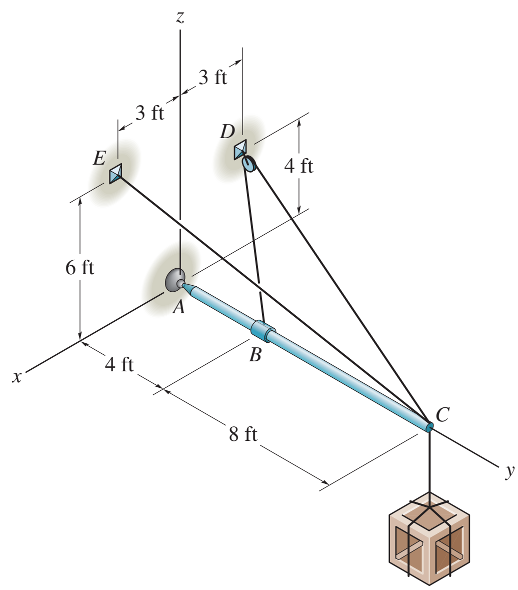
Finally, writing the force equation of equilibrium along the z axis, yields

$\Sigma F_z = 0; \quad A_z - 1233.33 - 900 + 1422.22 = 0$

$A_z = 711.11 \text{ N} = 711 \text{ N}$ **Ans.**

***5–76.**

The boom AC is supported at A by a ball-and-socket joint and by two cables BDC and CE. Cable BDC is continuous and passes over a pulley at D. Calculate the tension in the cables and the x, y, z components of reaction at A if a crate has a weight of 80 lb.



SOLUTION

$$\mathbf{F}_{CE} = F_{CE}\frac{(3\mathbf{i} - 12\mathbf{j} + 6\mathbf{k})}{\sqrt{3^2 + (-12)^2 + 6^2}}$$

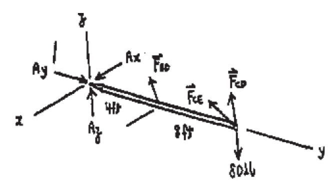
$$= \{0.2182F_{CE}\mathbf{i} - 0.8729F_{CE}\mathbf{j} + 0.4364F_{CE}\mathbf{k}\}\ \text{lb}$$

$$\mathbf{F}_{CD} = F_{BDC}\frac{(-3\mathbf{i} - 12\mathbf{j} + 4\mathbf{k})}{\sqrt{(-3)^2 + (-12)^2 + 4^2}}$$

$$= \{-0.2308F_{BDC}\mathbf{i} - 0.9231F_{BDC}\mathbf{j} + 0.3077F_{BDC}\mathbf{k}\}\ \text{lb}$$

$$\mathbf{F}_{BD} = F_{BDC}\frac{(-3\mathbf{i} - 4\mathbf{j} + 4\mathbf{k})}{\sqrt{(-3)^2 + (-4)^2 + 4^2}}$$

$$= F_{BDC}(-0.4685\mathbf{i} - 0.6247\mathbf{j} + 0.6247\mathbf{k})$$



$\Sigma M_x = 0;\quad F_{BDC}(0.6247)(4) + 0.4364F_{CE}(12) + 0.3077F_{BDC}(12) - 80(12) = 0$

$\Sigma M_z = 0;\quad 0.4685F_{BDC}(4) + 0.2308F_{BDC}(12) - 0.2182F_{CE}(12) = 0$

$F_{BDC} = 62.02 = 62.0\ \text{lb}$ **Ans.**

$F_{CE} = 109.99 = 110\ \text{lb}$ **Ans.**

$\Sigma F_x = 0;\quad A_x + 0.2182(109.99) - 0.2308(62.02) - 0.4685(62.02) = 0$

$A_x = 19.4\ \text{lb}$ **Ans.**

$\Sigma F_y = 0;\quad A_y - 0.8729(109.99) - 0.9231(62.02) - 0.6247(62.02) = 0$

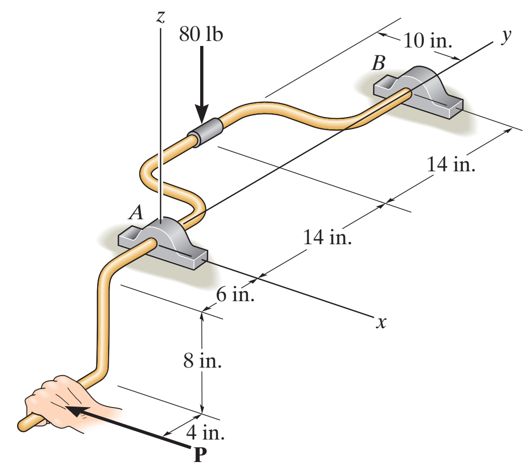
$A_y = 192\ \text{lb}$ **Ans.**

$\Sigma F_z = 0;\quad A_z + 0.4364(109.99) + 0.3077(62.02) + 0.6247(62.02) - 80 = 0$

$A_z = -25.8\ \text{lb}$ **Ans.**

5–77.

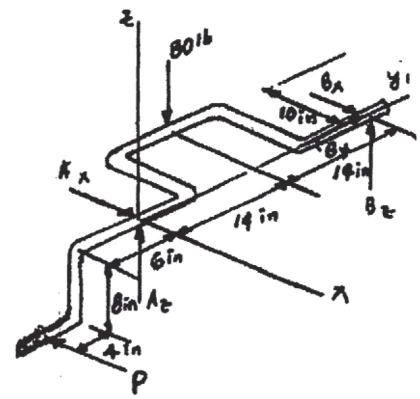
A vertical force of 80 lb acts on the crankshaft. Determine the horizontal equilibrium force P that must be applied to the handle and the x, y, z components of force at the smooth journal bearing A and the thrust bearing B. The bearings are properly aligned and exert only force reactions on the shaft.



SOLUTION

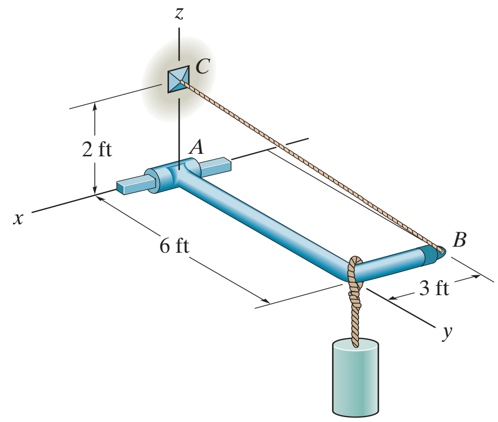
$\Sigma M_y = 0;$	$P(8) - 80(10) = 0$	$P = 100 \text{ lb}$	**Ans.**
$\Sigma M_x = 0;$	$B_z(28) - 80(14) = 0$	$B_z = 40 \text{ lb}$	**Ans.**
$\Sigma M_z = 0;$	$-B_x(28) - 100(10) = 0$	$B_x = -35.7 \text{ lb}$	**Ans.**
$\Sigma F_x = 0;$	$A_x + (-35.7) - 100 = 0$	$A_x = 136 \text{ lb}$	**Ans.**
$\Sigma F_y = 0;$	$B_y = 0$		**Ans.**
$\Sigma F_z = 0;$	$A_z + 40 - 80 = 0$	$A_z = 40 \text{ lb}$	**Ans.**

Negative sign indicates that B_x acts in the opposite sense to that shown on the FBD.



5–78.

Member AB is supported by a cable BC and at A by a *square* rod which fits loosely through the square hole at the end joint of the member as shown. Determine the components of reaction at A and the tension in the cable needed to hold the 800-lb cylinder in equilibrium.



SOLUTION

$$\mathbf{F}_{BC} = F_{BC}\left(\frac{3}{7}\mathbf{i} - \frac{6}{7}\mathbf{j} + \frac{2}{7}\mathbf{k}\right)$$

$\Sigma F_x = 0; \quad F_{BC}\left(\frac{3}{7}\right) = 0$

$F_{BC} = 0$ **Ans.**

$\Sigma F_y = 0; \quad A_y = 0$ **Ans.**

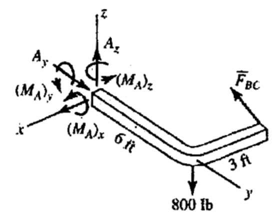
$\Sigma F_z = 0; \quad A_z = 800 \text{ lb}$ **Ans.**

$\Sigma M_x = 0; \quad (M_A)_x - 800(6) = 0$

$(M_A)_x = 4.80 \text{ kip} \cdot \text{ft}$ **Ans.**

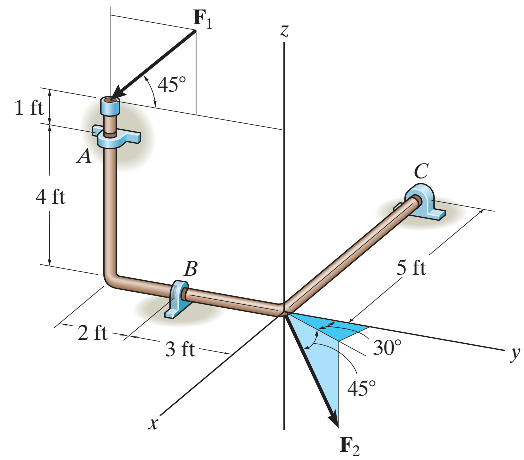
$\Sigma M_y = 0; \quad (M_A)_y = 0$ **Ans.**

$\Sigma M_z = 0; \quad (M_A)_z = 0$ **Ans.**



5–79.

The bent rod is supported at A, B, and C by smooth journal bearings. Compute the x, y, z components of reaction at the bearings if the rod is subjected to forces $F_1 = 300$ lb and $F_2 = 250$ lb. F_1 lies in the y–z plane. The bearings are in proper alignment and exert only force reactions on the rod.



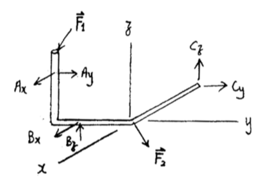
SOLUTION

$\mathbf{F}_1 = (-300 \cos 45°\mathbf{j} - 300 \sin 45°\mathbf{k})$

$= \{-212.1\mathbf{j} - 212.1\mathbf{k}\}$ lb

$\mathbf{F}_2 = (250 \cos 45° \sin 30°\mathbf{i} + 250 \cos 45° \cos 30°\mathbf{j} - 250 \sin 45°\mathbf{k})$

$= \{88.39\mathbf{i} + 153.1\mathbf{j} - 176.8\mathbf{k}\}$ lb



$\Sigma F_x = 0;\quad A_x + B_x + 88.39 = 0$

$\Sigma F_y = 0;\quad A_y + C_y - 212.1 + 153.1 = 0$

$\Sigma F_z = 0;\quad B_z + C_z - 212.1 - 176.8 = 0$

$\Sigma M_x = 0;\quad -B_z(3) - A_y(4) + 212.1(5) + 212.1(5) = 0$

$\Sigma M_y = 0;\quad C_z(5) + A_x(4) = 0$

$\Sigma M_z = 0;\quad A_x(5) + B_x(3) - C_y(5) = 0$

$A_x = 633$ lb **Ans.**

$A_y = -141$ lb **Ans.**

$B_x = -721$ lb **Ans.**

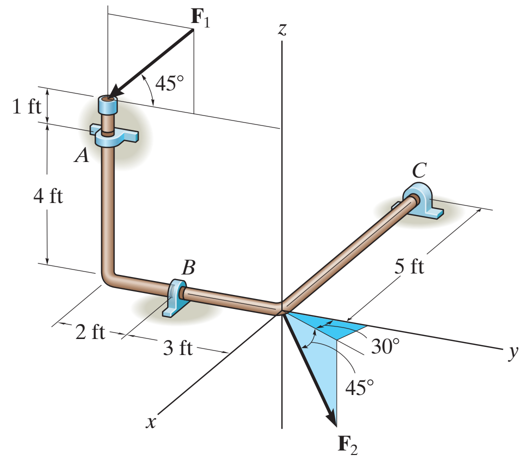
$B_z = 895$ lb **Ans.**

$C_y = 200$ lb **Ans.**

$C_z = -506$ lb **Ans.**

***5–80.**

The bent rod is supported at A, B, and C by smooth journal bearings. Determine the magnitude of $\mathbf{F}_2$ which will cause the reaction $\mathbf{C}_y$ at the bearing C to be equal to zero. The bearings are in proper alignment and exert only force reactions on the rod. Set $F_1 = 300$ lb.



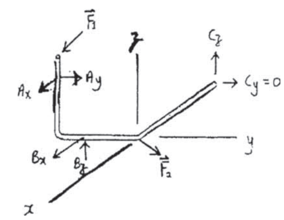
SOLUTION

$$\mathbf{F}_1 = (-300\cos 45°\mathbf{j} - 300\sin 45°\mathbf{k})$$

$$= \{-212.1\mathbf{j} - 212.1\mathbf{k}\}\text{ lb}$$

$$\mathbf{F}_2 = (F_2\cos 45°\sin 30°\mathbf{i} + F_2\cos 45°\cos 30°\mathbf{j} - F_2\sin 45°\mathbf{k})$$

$$= \{0.3536F_2\mathbf{i} + 0.6124F_2\mathbf{j} - 0.7071F_2\mathbf{k}\}\text{ lb}$$



$\Sigma F_x = 0;\qquad A_x + B_x + 0.3536F_2 = 0$

$\Sigma F_y = 0;\qquad A_y + 0.6124F_2 - 212.1 = 0$

$\Sigma F_z = 0;\qquad B_z + C_z - 0.7071F_2 - 212.1 = 0$

$\Sigma M_x = 0;\qquad -B_z(3) - A_y(4) + 212.1(5) + 212.1(5) = 0$

$\Sigma M_y = 0;\qquad C_z(5) + A_x(4) = 0$

$\Sigma M_z = 0;\qquad A_x(5) + B_x(3) = 0$

$A_x = 357$ lb

$A_y = -200$ lb

$B_x = -595$ lb

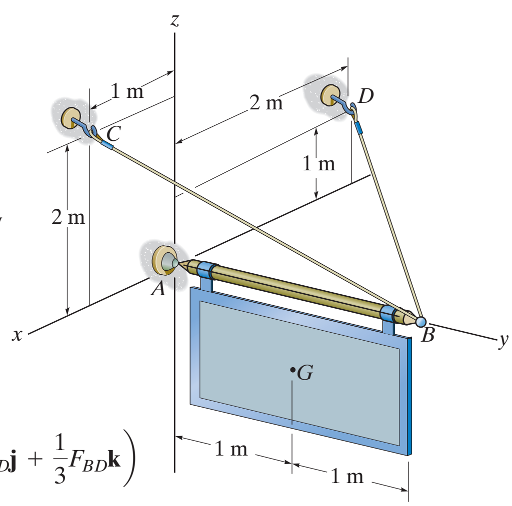
$B_z = 974$ lb

$C_z = -286$ lb

$F_2 = 674$ lb **Ans.**

5–81.

The sign has a mass of 100 kg with center of mass at G. Determine the x, y, z components of reaction at the ball-and-socket joint A and the tension in wires BC and BD.



SOLUTION

Equations of Equilibrium: Expressing the forces indicated on the free-body diagram, Fig. *a*, in Cartesian vector form, we have

$$\mathbf{F}_A = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}$$

$$\mathbf{W} = \{-100(9.81)\mathbf{k}\}\text{ N} = \{-981\text{k}\}\text{ N}$$

$$\mathbf{F}_{BD} = F_{BD}\mathbf{u}_{BD} = F_{BD}\left[\frac{(-2-0)\mathbf{i} + (0-2)\mathbf{j} + (1-0)\mathbf{k}}{\sqrt{(-2-0)^2 + (0-2)^2 + (1-0)^2}}\right] = \left(-\frac{2}{3}F_{BD}\mathbf{i} - \frac{2}{3}F_{BD}\mathbf{j} + \frac{1}{3}F_{BD}\mathbf{k}\right)$$

$$\mathbf{F}_{BC} = F_{BC}\mathbf{u}_{BC} = F_{BC}\left[\frac{(1-0)\mathbf{i} + (0-2)\mathbf{j} + (2-0)\mathbf{k}}{\sqrt{(1-0)^2 + (0-2)^2 + (2-0)^2}}\right] = \left(\frac{1}{3}F_{BC}\mathbf{i} - \frac{2}{3}F_{BC}\mathbf{j} + \frac{2}{3}F_{BC}\mathbf{k}\right)$$

Applying the forces equation of equilibrium, we have

$$\Sigma\mathbf{F} = 0; \qquad \mathbf{F}_A + \mathbf{F}_{BD} + \mathbf{F}_{BC} + \mathbf{W} = 0$$

$$(A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}) + \left(-\frac{2}{3}F_{BD}\mathbf{i} - \frac{2}{3}F_{BD}\mathbf{j} + \frac{1}{3}F_{BD}\mathbf{k}\right) + \left(\frac{1}{3}F_{BC}\mathbf{i} - \frac{2}{3}F_{BC}\mathbf{j} + \frac{2}{3}F_{BC}\mathbf{k}\right) + (-981\,k) = 0$$

$$\left(A_x - \frac{2}{3}F_{BD} + \frac{1}{3}F_{BC}\right)\mathbf{i} + \left(A_y - \frac{2}{3}F_{BD} - \frac{2}{3}F_{BC}\right)\mathbf{j} + \left(A_z + \frac{1}{3}F_{BD} + \frac{2}{3}F_{BC} - 981\right)\mathbf{k} = 0$$

Equating **i**, **j**, and **k** components, we have

$$A_x - \frac{2}{3}F_{BD} + \frac{1}{3}F_{BC} = 0 \qquad \textbf{(1)}$$

$$A_y - \frac{2}{3}F_{BD} - \frac{2}{3}F_{BC} = 0 \qquad \textbf{(2)}$$

$$A_z + \frac{1}{3}F_{BD} + \frac{2}{3}F_{BC} - 981 = 0 \qquad \textbf{(3)}$$

In order to write the moment equation of equilibrium about point A, the position vectors $\mathbf{r}_{AG}$ and $\mathbf{r}_{AB}$ must be determined first.

$$\mathbf{r}_{AG} = \{1\mathbf{j}\}\text{ m}$$

$$\mathbf{r}_{AB} = \{2\mathbf{j}\}\text{ m}$$

Thus,

$$\Sigma\mathbf{M}_A = 0; \mathbf{r}_{AB} \times (\mathbf{F}_{BC} + \mathbf{F}_{BD}) + (\mathbf{r}_{AG} \times \mathbf{W}) = 0$$

$$(2\mathbf{j}) \times \left[\left(\frac{1}{3}F_{BC} - \frac{2}{3}F_{BD}\right)\mathbf{i} - \left(\frac{2}{3}F_{BC} + \frac{2}{3}F_{BD}\right)\mathbf{j} + \left(\frac{2}{3}F_{BC} + \frac{1}{3}F_{BD}\right)\mathbf{k}\right] + (1\mathbf{j}) \times (-981\mathbf{k}) = 0$$

$$\left(\frac{4}{3}F_{BC} + \frac{2}{3}F_{BD} - 981\right)\mathbf{i} + \left(\frac{4}{3}F_{BD} - \frac{2}{3}F_{BC}\right)\mathbf{k} = 0$$

Equating **i**, **j**, and **k** components we have

$$\frac{4}{3}F_{BC} + \frac{2}{3}F_{BC} - 981 = 0 \qquad \textbf{(4)}$$

$$\frac{4}{3}F_{BC} - \frac{2}{3}F_{BC} = 0 \qquad \textbf{(5)}$$

5–81. (continued)

Soving Eqs. (1) through (5), yields

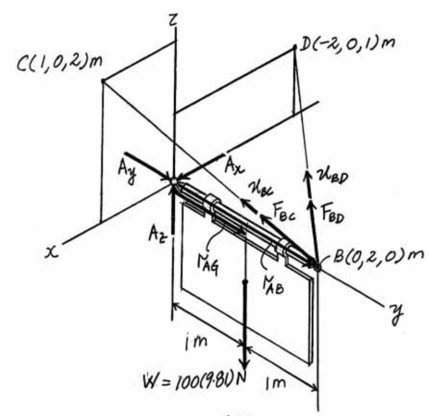
$F_{BD} = 294.3\text{ N} = 294\text{ N}$ **Ans.**

$F_{BC} = 588.6\text{ N} = 589\text{ N}$ **Ans.**

$A_x = 0$ **Ans.**

$A_y = 588.6\text{ N} = 589\text{ N}$ **Ans.**

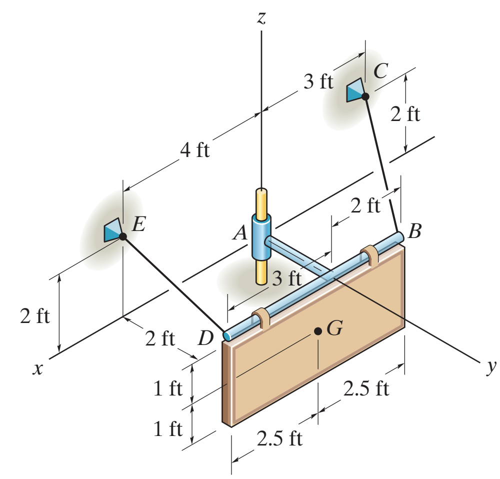
$A_z = 490.5\text{ N}$ **Ans.**



(a)

5–82.

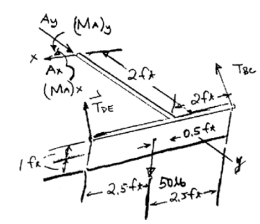
Determine the tensions in the cables and the components of reaction acting on the smooth collar at A necessary to hold the 50-lb sign in equilibrium. The center of gravity for the sign is at G.



SOLUTION

$$\mathbf{T}_{DE} = T_{DE}\left(\frac{1}{3}\mathbf{i} - \frac{2}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}\right)$$

$$\mathbf{T}_{BC} = T_{BC}\left(\frac{-1}{3}\mathbf{i} - \frac{2}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}\right)$$



$$\Sigma F_x = 0; \qquad \frac{1}{3}T_{DE} - \frac{1}{3}T_{BC} + A_x = 0$$

$$\Sigma F_z = 0; \qquad \frac{2}{3}T_{DE} + \frac{2}{3}T_{BC} - 50 = 0$$

$$\Sigma F_y = 0; \qquad -\frac{2}{3}T_{DE} - \frac{2}{3}T_{BC} + A_y = 0$$

$$\Sigma M_x = 0; \qquad (M_A)_x + \frac{2}{3}T_{DE}(2) + \frac{2}{3}T_{BC}(2) - 50(2) = 0$$

$$\Sigma M_y = 0; \qquad (M_A)_y - \frac{2}{3}T_{DE}(3) + \frac{2}{3}T_{BC}(2) + 50(0.5) = 0$$

$$\Sigma M_z = 0; \qquad -\frac{1}{3}T_{DE}(2) - \frac{2}{3}T_{DE}(3) + \frac{1}{3}T_{BC}(2) + \frac{2}{3}T_{BC}(2) = 0$$

Solving;

$T_{DE} = 32.1429 = 32.1 \text{ lb}$ **Ans.**

$T_{BC} = 42.8571 = 42.9 \text{ lb}$ **Ans.**

$A_x = 3.5714 = 3.57 \text{ lb}$ **Ans.**

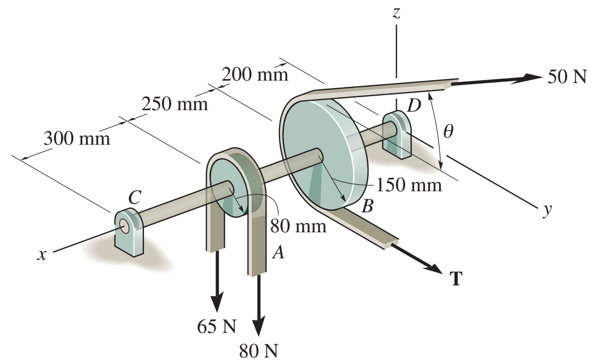
$A_y = 50 \text{ lb}$ **Ans.**

$(M_A)_x = 0$ **Ans.**

$(M_A)_y = -17.8571 = -17.9 \text{ lb} \cdot \text{ft}$ **Ans.**

5–83.

Both pulleys are fixed to the shaft and as the shaft turns with constant angular velocity, the power of pulley A is transmitted to pulley B. Determine the horizontal tension $\mathbf{T}$ in the belt on pulley B and the x, y, z components of reaction at the journal bearing C and thrust bearing D if $\theta = 0°$. The bearings are in proper alignment and exert only force reactions on the shaft.



SOLUTION

Equations of Equilibrium:

$\Sigma M_x = 0;$ $\quad 65(0.08) - 80(0.08) + T(0.15) - 50(0.15) = 0$

$T = 58.0\text{ N}$ **Ans.**

$\Sigma M_y = 0;$ $\quad (65 + 80)(0.45) - C_z(0.75) = 0$

$C_z = 87.0\text{ N}$ **Ans.**

$\Sigma M_z = 0;$ $\quad (50 + 58.0)(0.2) - C_y(0.75) = 0$

$C_y = 28.8\text{ N}$ **Ans.**

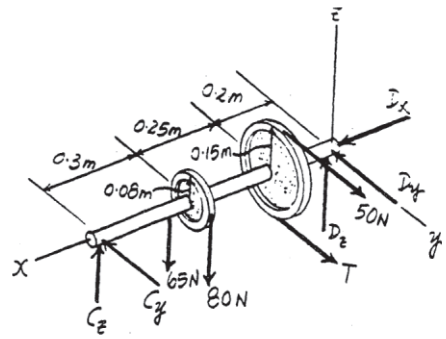
$\Sigma F_x = 0;$ $\quad D_x = 0$ **Ans.**

$\Sigma F_y = 0;$ $\quad D_y + 28.8 - 50 - 58.0 = 0$

$D_y = 79.2\text{ N}$ **Ans.**

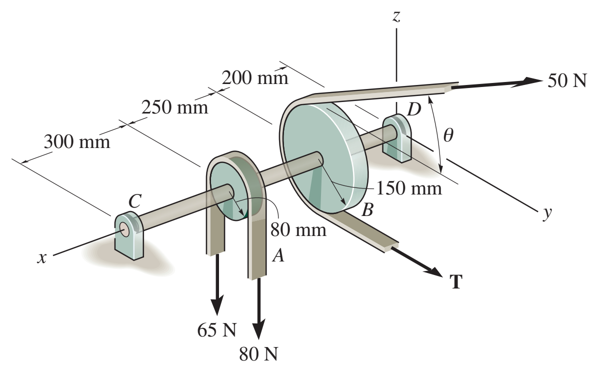
$\Sigma F_z = 0;$ $\quad D_z + 87.0 - 80 - 65 = 0$

$D_z = 58.0\text{ N}$ **Ans.**



***5–84.**

Both pulleys are fixed to the shaft and as the shaft turns with constant angular velocity, the power of pulley A is transmitted to pulley B. Determine the horizontal tension **T** in the belt on pulley B and the x, y, z components of reaction at the journal bearing C and thrust bearing D if $\theta = 45°$. The bearings are in proper alignment and exert only force reactions on the shaft.



SOLUTION

Equations of Equilibrium:

$\Sigma M_x = 0;\qquad 65(0.08) - 80(0.08) + T(0.15) - 50(0.15) = 0$

$T = 58.0\text{ N}$ **Ans.**

$\Sigma M_y = 0;\qquad (65 + 80)(0.45) - 50\sin 45°(0.2) - C_z(0.75) = 0$

$C_z = 77.57\text{ N} = 77.6\text{ N}$ **Ans.**

$\Sigma M_z = 0;\qquad 58.0(0.2) + 50\cos 45°(0.2) - C_y(0.75) = 0$

$C_y = 24.89\text{ N} = 24.9\text{ N}$ **Ans.**

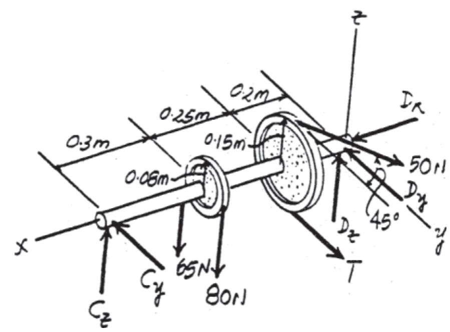
$\Sigma F_x = 0;\qquad D_x = 0$ **Ans.**

$\Sigma F_y = 0;\qquad D_y + 24.89 - 50\cos 45° - 58.0 = 0$

$D_y = 68.5\text{ N}$ **Ans.**

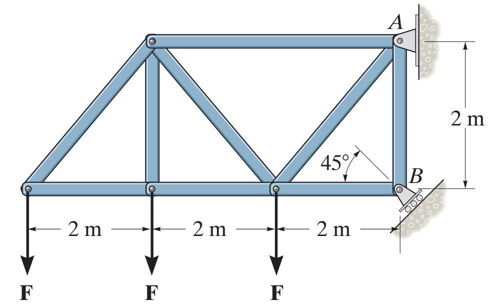
$\Sigma F_z = 0;\qquad D_z + 77.57 + 50\sin 45° - 80 - 65 = 0$

$D_z = 32.1\text{ N}$ **Ans.**



5–85.

If the roller at B can sustain a maximum load of 3 kN, determine the largest magnitude of each of the three forces **F** that can be supported by the truss.

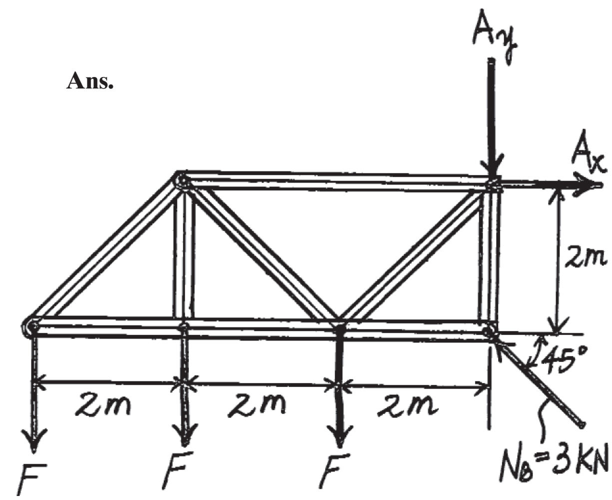


SOLUTION

Equations of Equilibrium: The unknowns A_x and A_y can be eliminated by summing moments about point A.

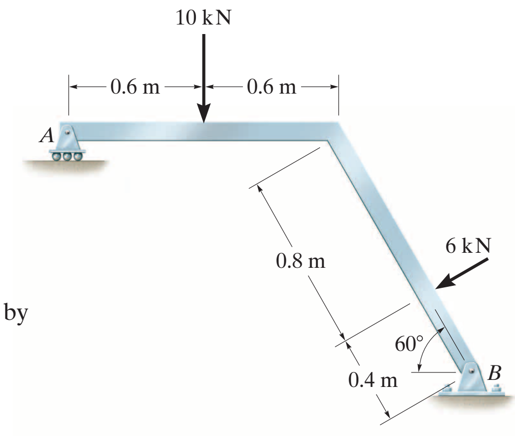
$$\zeta + \Sigma M_A = 0; \qquad F(6) + F(4) + F(2) - 3\cos 45^\circ(2) = 0$$

$$F = 0.3536 \text{ kN} = 354 \text{ N} \qquad \textbf{Ans.}$$



5–86.

Determine the normal reaction at the roller A and horizontal and vertical components at pin B for equilibrium of the member.



SOLUTION

Equations of Equilibrium: The normal reaction N_A can be obtained directly by summing moments about point B.

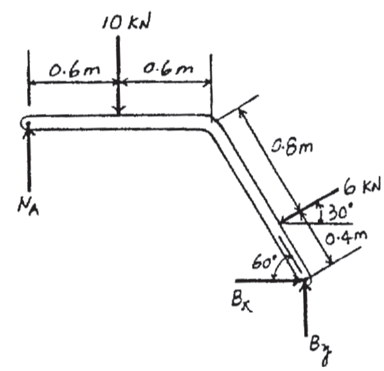
$\circlearrowleft + \Sigma M_A = 0; \qquad 10(0.6 + 1.2\cos 60°) + 6\,(0.4) - N_A(1.2 + 1.2\cos 60°) = 0$

$N_A = 8.00 \text{ kN}$ **Ans.**

$\xrightarrow{+} \Sigma F_x = 0; \qquad B_x - 6\cos 30° = 0 \qquad B_x = 5.20 \text{ kN}$ **Ans.**

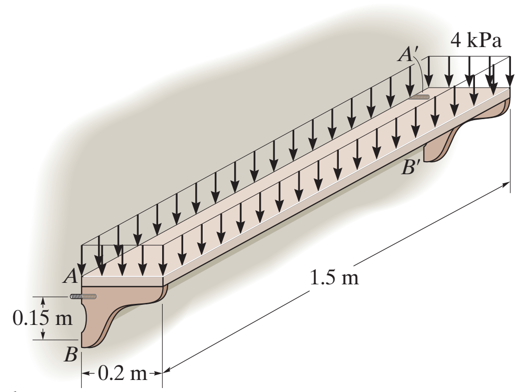
$+\uparrow \Sigma F_y = 0; \qquad B_y + 8.00 - 6\sin 30° - 10 = 0$

$B_y = 5.00 \text{ kN}$ **Ans.**



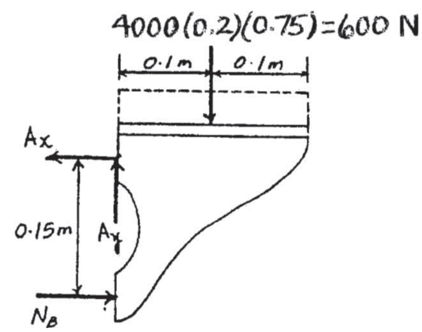
5–87.

The symmetrical shelf is subjected to a uniform load of 4 kPa. Support is provided by a bolt (or pin) located at each end A and A' and by the symmetrical brace arms, which bear against the smooth wall on both sides at B and B'. Determine the force resisted by each bolt at the wall and the normal force at B for equilibrium.



SOLUTION

Equations of Equilibrium: Each shelf's post at its end supports half of the applied load, ie, $4000\,(0.2)\,(0.75) = 600$ N. The normal reaction N_B can be obtained directly by summing moments about point A.



$\circlearrowleft + \Sigma M_A = 0; \qquad N_B\,(0.15) - 600(0.1) = 0 \qquad N_B = 400 \text{ N}$ **Ans.**

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 400 - A_x = 0 \qquad A_x = 400 \text{ N}$

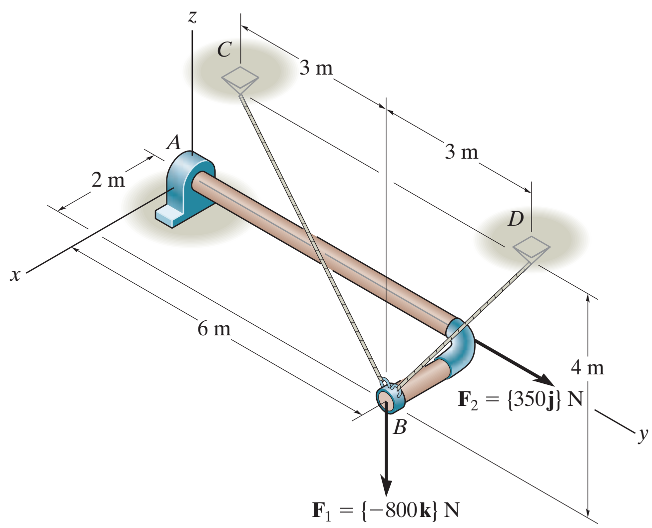
$+\uparrow \Sigma F_y = 0; \qquad A_y - 600 = 0 \qquad A_y = 600 \text{ N}$

The force resisted by the bolt at A is

$$F_A = \sqrt{A_x^2 + A_y^2} = \sqrt{400^2 + 600^2} = 721 \text{ N}$$ **Ans.**

***5–88.**

Determine the x and z components of reaction at the journal bearing A and the tension in cords BC and BD necessary for equilibrium of the rod.



SOLUTION

$\mathbf{F}_1 = \{-800\mathbf{k}\}$ N

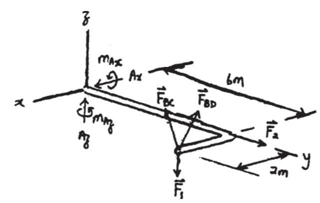
$\mathbf{F}_2 = \{350\mathbf{j}\}$ N

$$\mathbf{F}_{BC} = F_{BC}\frac{(-3\mathbf{j} + 4\mathbf{k})}{5}$$

$$= \{-0.6F_{BC}\mathbf{j} + 0.8F_{BC}\mathbf{k}\}\text{ N}$$

$$\mathbf{F}_{BD} = F_{BD}\frac{(3\mathbf{j} + 4\mathbf{k})}{5}$$

$$= \{0.6F_{BD}\mathbf{j} + 0.8F_{BD}\mathbf{k}\}\text{ N}$$



$\Sigma F_x = 0;\quad A_x = 0$ **Ans.**

$\Sigma F_y = 0;\quad 350 - 0.6F_{BC} + 0.6F_{BD} = 0$

$\Sigma F_z = 0;\quad A_z - 800 + 0.8F_{BC} + 0.8F_{BD} = 0$

$\Sigma M_x = 0;\quad (M_A)_x + 0.8F_{BD}(6) + 0.8F_{BC}(6) - 800(6) = 0$

$\Sigma M_y = 0;\quad 800(2) - 0.8F_{BC}(2) - 0.8F_{BD}(2) = 0$

$\Sigma M_z = 0;\quad (M_A)_z - 0.6F_{BC}(2) + 0.6F_{BD}(2) = 0$

$F_{BD} = 208$ N **Ans.**

$F_{BC} = 792$ N **Ans.**

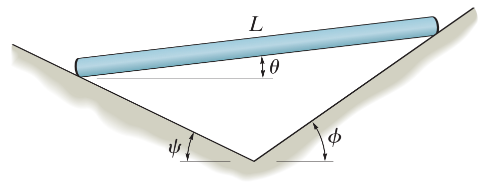
$A_z = 0$ **Ans.**

$(M_A)_x = 0$ **Ans.**

$(M_A)_z = 700\text{ N}\cdot\text{m}$ **Ans.**

5–89.

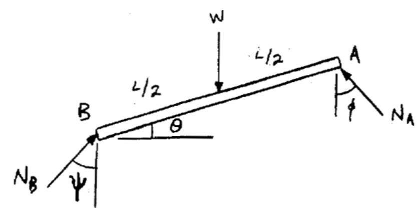
The uniform rod of length L and weight W is supported on the smooth planes. Determine its position θ for equilibrium. Neglect the thickness of the rod.



SOLUTION

$$\zeta+\Sigma M_B = 0; \quad -W\left(\frac{L}{2}\cos\theta\right) + N_A\cos\phi\,(L\cos\theta) + N_A\sin\phi\,(L\sin\theta) = 0$$

$$N_A = \frac{W\cos\theta}{2\cos(\phi-\theta)} \qquad \textbf{(1)}$$



$$\overset{+}{\rightarrow}\Sigma F_x = 0; \quad N_B\sin\psi - N_A\sin\phi = 0 \qquad \textbf{(2)}$$

$$+\uparrow\Sigma F_y = 0; \quad N_B\cos\psi + N_A\cos\phi - W = 0$$

$$N_B = \frac{W - N_A\cos\phi}{\cos\psi} \qquad \textbf{(3)}$$

Substituting Eqs. (1) and (3) into Eq. (2):

$$\left(W - \frac{W\cos\theta\cos\phi}{2\cos(\phi-\theta)}\right)\tan\psi - \frac{W\cos\theta\sin\phi}{2\cos(\phi-\theta)} = 0$$

$$2\cos(\phi-\theta)\tan\psi - \cos\theta\tan\psi\cos\phi - \cos\theta\sin\phi = 0$$

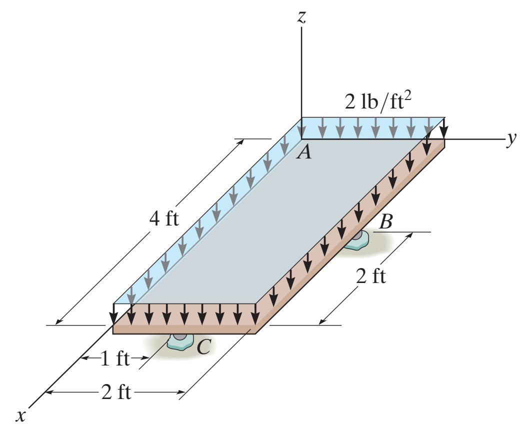
$$\sin\theta\,(2\sin\phi\tan\psi) - \cos\theta\,(\sin\phi - \cos\phi\tan\psi) = 0$$

$$\tan\theta = \frac{\sin\phi - \cos\phi\tan\psi}{2\sin\phi\tan\psi}$$

$$\theta = \tan^{-1}\left(\frac{1}{2}\cot\psi - \frac{1}{2}\cot\phi\right) \qquad \textbf{Ans.}$$

5–90.

Determine the x, y, z components of reaction at the ball supports B and C and the ball-and-socket A (not shown) for the uniformly loaded plate.



SOLUTION

$W = (4 \text{ ft})(2 \text{ ft})(2 \text{ lb/ft}^2) = 16 \text{ lb}$

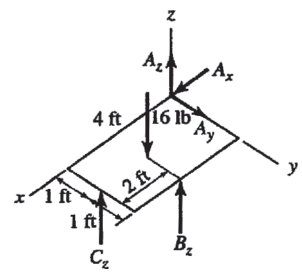
$\Sigma F_x = 0; \quad A_x = 0$ **Ans.**

$\Sigma F_y = 0; \quad A_y = 0$ **Ans.**

$\Sigma F_z = 0; \quad A_z + B_z + C_z - 16 = 0$ **(1)**

$\Sigma M_x = 0; \quad 2B_z - 16(1) + C_z(1) = 0$ **(2)**

$\Sigma M_y = 0; \quad -B_z(2) + 16(2) - C_z(4) = 0$ **(3)**

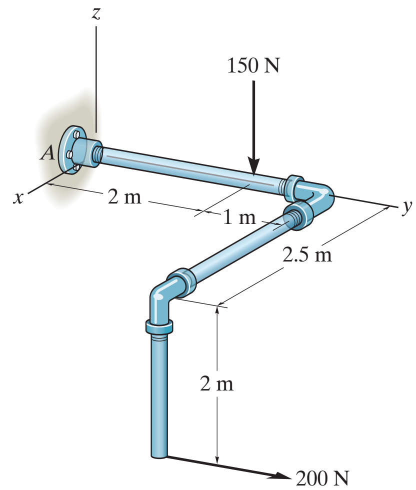


Solving Eqs. (1)—(3):

$$A_z = B_z = C_z = 5.33 \text{ lb}$$ **Ans.**

5–91.

Determine the x, y, z components of reaction at the fixed wall A. The 150-N force is parallel to the z axis and the 200-N force is parallel to the y axis.



SOLUTION

$\Sigma F_x = 0;\quad A_x = 0$ **Ans.**

$\Sigma F_y = 0;\quad A_y + 200 = 0$

$A_y = -200\text{ N}$ **Ans.**

$\Sigma F_z = 0;\quad A_z - 150 = 0$

$A_z = 150\text{ N}$ **Ans.**

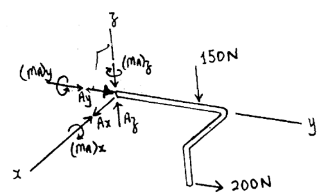
$\Sigma M_x = 0;\quad -150(2) + 200(2) - (M_A)_x = 0$

$(M_A)_x = 100\text{ N}\cdot\text{m}$ **Ans.**

$\Sigma M_y = 0;\quad (M_A)_y = 0$ **Ans.**

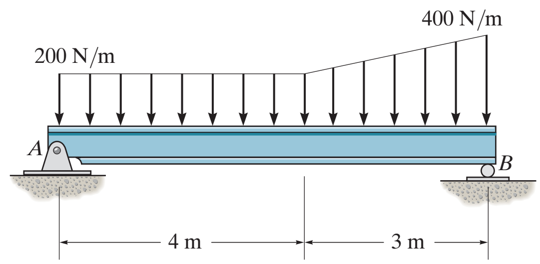
$\Sigma M_z = 0;\quad 200(2.5) - (M_A)_z = 0$

$(M_A)_z = 500\text{ N}\cdot\text{m}$ **Ans.**



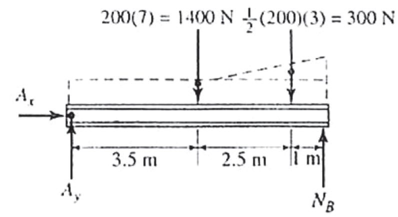
***5–92.**

Determine the reactions at the supports A and B for equilibrium of the beam.



SOLUTION

Equations of Equilibrium: The normal reaction of N_B can be obtained directly by summing moments about point A.



$+\Sigma M_A = 0; \qquad N_B(7) - 1400(3.5) - 300(6) = 0$

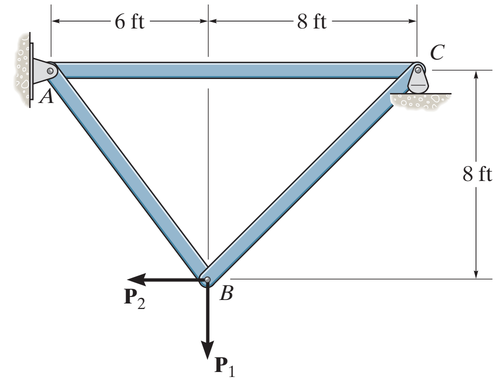
$N_B = 957.14 \text{ N} = 957 \text{ N}$ **Ans.**

$Ag - 1400 - 300 + 957 = 0 \qquad Ag = 743 \text{ N}$

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad A_z = 0$ **Ans.**

6–1.

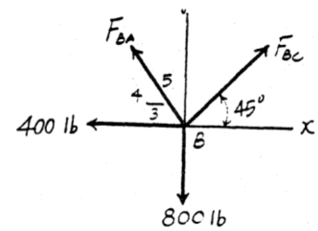
Determine the force in each member of the truss and state if the members are in tension or compression. Set $P_1 = 800$ lb and $P_2 = 400$ lb.



SOLUTION

Method of Joints: In this case, the support reactions are not required for determining the member forces.

Joint B:



$$\xrightarrow{+} \Sigma F_x = 0; \qquad F_{BC}\cos 45° - F_{BA}\left(\frac{3}{5}\right) - 400 = 0 \qquad \textbf{(1)}$$

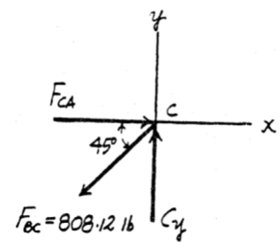
$$+\uparrow \Sigma F_y = 0; \qquad F_{BC}\sin 45° + F_{BA}\left(\frac{4}{5}\right) - 800 = 0 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$F_{BA} = 285.71 \text{ lb (T)} = 286 \text{ lb (T)}$ **Ans.**

$F_{BC} = 808.12 \text{ lb (T)} = 808 \text{ lb (T)}$ **Ans.**

Joint C:



$$\xrightarrow{+} \Sigma F_x = 0; \qquad F_{CA} - 808.12 \cos 45° = 0$$

$$F_{CA} = 571 \text{ lb (C)} \qquad \textbf{Ans.}$$

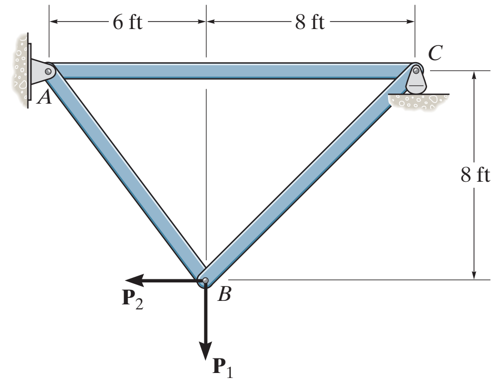
$$+\uparrow \Sigma F_y = 0; \qquad C_y - 808.12 \sin 45° = 0$$

$$C_y = 571 \text{ lb}$$

Note: The support reactions A_x and A_y can be determined by analyzing Joint A using the results obtained above.

6–2.

Determine the force on each member of the truss and state if the members are in tension or compression. Set $P_1 = 500$ lb and $P_2 = 100$ lb.



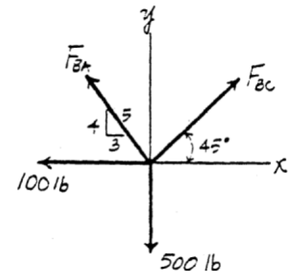
SOLUTION

Method of Joints: In this case, the support reactions are not required for determining the member forces.

Joint B:

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad F_{BC}\cos 45° - F_{BA}\left(\frac{3}{5}\right) - 100 = 0 \qquad \textbf{(1)}$$

$$+\uparrow \Sigma F_y = 0; \qquad F_{BC}\sin 45° + F_{BA}\left(\frac{4}{5}\right) - 500 = 0 \qquad \textbf{(2)}$$



Solving Eqs. (1) and (2) yields

$$F_{BA} = 285.71 \text{ lb (T)} = 286 \text{ lb (T)} \qquad \textbf{Ans.}$$

$$F_{BC} = 383.86 \text{ lb (T)} = 384 \text{ lb (T)} \qquad \textbf{Ans.}$$

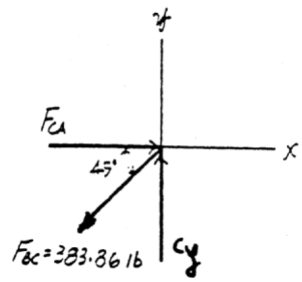
Joint C:

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad F_{CA} - 383.86\cos 45° = 0$$

$$F_{CA} = 271 \text{ lb (C)} \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad C_y - 383.86\sin 45° = 0$$

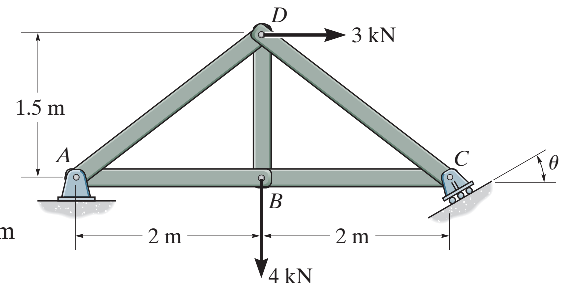
$$C_y = 271.43 \text{ lb}$$



Note: The support reactions A_x and A_y can be determined by analyzing Joint A using the results obtained above.

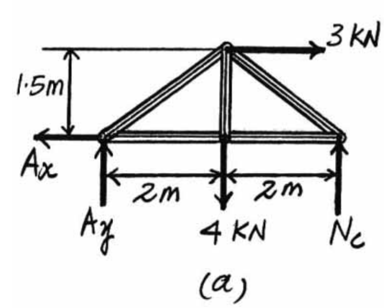
6–3.

Determine the force in each member of the truss, and state if the members are in tension or compression. Set $\theta = 0°$.



SOLUTION

Support Reactions: Applying the equations of equilibrium to the free-body diagram of the entire truss, Fig. *a*, we have



$\circlearrowleft + \Sigma M_A = 0;$ $\quad N_C(2 + 2) - 4(2) - 3(1.5) = 0$

$$N_C = 3.125 \text{ kN}$$

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad 3 - A_x = 0$

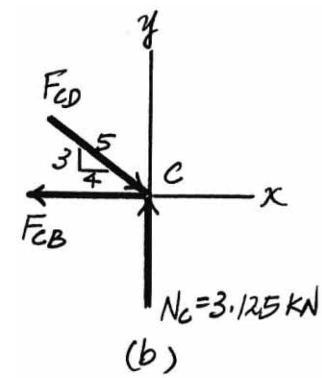
$$A_x = 3 \text{ kN}$$

$+\uparrow \Sigma F_y = 0;$ $\quad A_y + 3.125 - 4 = 0$

$$A_y = 0.875 \text{ kN}$$

Method of Joints: We will use the above result to analyze the equilibrium of joints C and A, and then proceed to analyze of joint B.

Joint C: From the free-body diagram in Fig. *b,* we can write



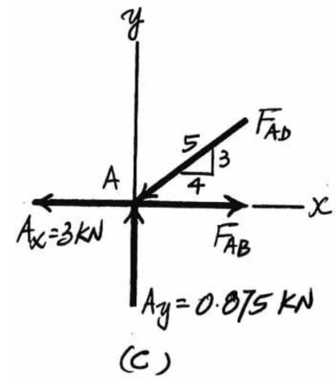
$+\uparrow \Sigma F_y = 0;$ $\quad 3.125 - F_{CD}\left(\frac{3}{5}\right) = 0$

$$F_{CD} = 5.208 \text{ kN} = 5.21 \text{ kN (C)} \qquad \textbf{Ans.}$$

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad 5.208\left(\frac{4}{5}\right) - F_{CB} = 0$

$$F_{CB} = 4.167 \text{ kN} = 4.17 \text{ kN (T)} \qquad \textbf{Ans.}$$

Joint A: From the free-body diagram in Fig. *c,* we can write



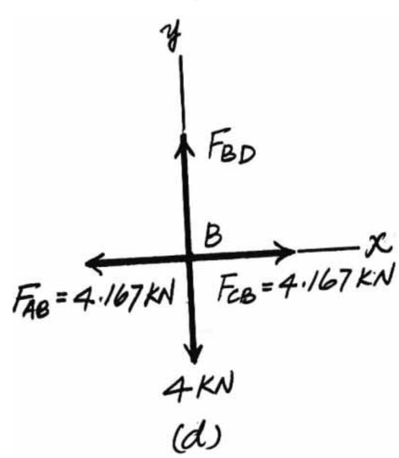
$+\uparrow \Sigma F_y = 0;$ $\quad 0.875 - F_{AD}\left(\frac{3}{5}\right) = 0$

$$F_{AD} = 1.458 \text{ kN} = 1.46 \text{ kN (C)} \qquad \textbf{Ans.}$$

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad F_{AB} - 3 - 1.458\left(\frac{4}{5}\right) = 0$

$$F_{AB} = 4.167 \text{ kN} = 4.17 \text{ kN (T)} \qquad \textbf{Ans.}$$

Joint B: From the free-body diagram in Fig. *d,* we can write



$+\uparrow \Sigma F_y = 0;$ $\quad F_{BD} - 4 = 0$

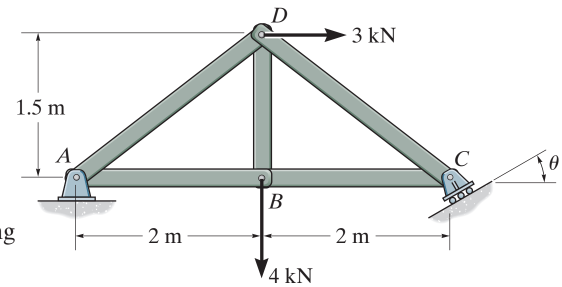
$$F_{BD} = 4 \text{ kN (T)} \qquad \textbf{Ans.}$$

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad 4.167 - 4.167 = 0 \qquad \text{(check!)}$

Note: The equilibrium analysis of joint D can be used to check the accuracy of the solution obtained above.

***6–4.**

Determine the force in each member of the truss, and state if the members are in tension or compression. Set $\theta = 30°$.



SOLUTION

Support Reactions: From the free-body diagram of the truss, Fig. *a*, and applying the equations of equilibrium, we have

$\circlearrowleft + \Sigma M_A = 0;\qquad N_C \cos 30°(2 + 2) - 3(1.5) - 4(2) = 0$

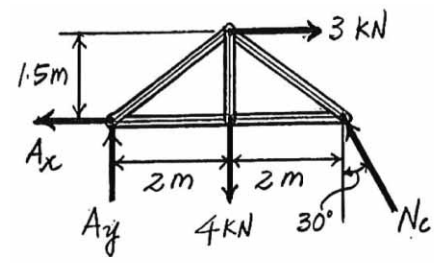
$N_C = 3.608 \text{ kN}$

$\overset{+}{\rightarrow} \Sigma F_x = 0;\qquad 3 - 3.608 \sin 30° - A_x = 0$

$A_x = 1.196 \text{ kN}$

$+\uparrow \Sigma F_y = 0;\qquad A_y + 3.608 \cos 30° - 4 = 0$

$A_y = 0.875 \text{ kN}$



(a)

Method of Joints: We will use the above result to analyze the equilibrium of joints C and A, and then proceed to analyze of joint B.

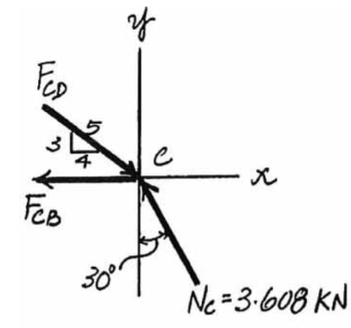
Joint C: From the free-body diagram in Fig. *b*, we can write

$+\uparrow \Sigma F_y = 0;\qquad 3.608 \cos 30° - F_{CD}\left(\frac{3}{5}\right) = 0$

$F_{CD} = 5.208 \text{ kN} = 5.21 \text{ kN (C)}$ **Ans.**

$\overset{+}{\rightarrow} \Sigma F_x = 0;\qquad 5.208\left(\frac{4}{5}\right) - 3.608 \sin 30° - F_{CB} = 0$

$F_{CB} = 2.362 \text{ kN} = 2.36 \text{ kN (T)}$ **Ans.**



(b)

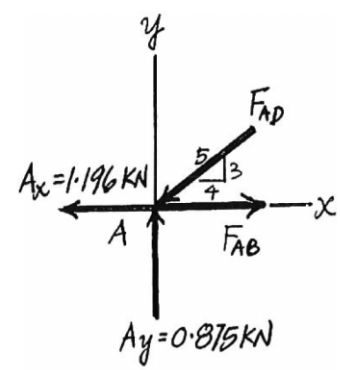
Joint A: From the free-body diagram in Fig. *c*, we can write

$+\uparrow \Sigma F_y = 0;\qquad 0.875 - F_{AD}\left(\frac{3}{5}\right) = 0$

$F_{AD} = 1.458 \text{ kN} = 1.46 \text{ kN (C)}$ **Ans.**

$\overset{+}{\rightarrow} \Sigma F_x = 0;\qquad F_{AB} - 1.458\left(\frac{4}{5}\right) - 1.196 = 0$

$F_{AB} = 2.362 \text{ kN} = 2.36 \text{ kN (T)}$ **Ans.**



(c)

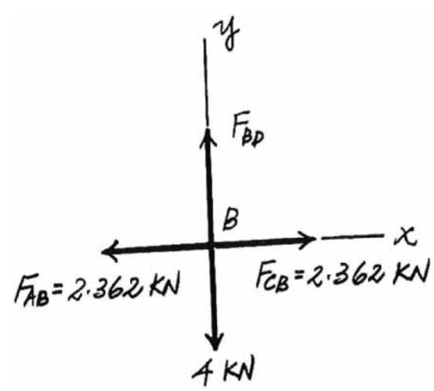
Joint B: From the free-body diagram in Fig. *d*, we can write

$+\uparrow \Sigma F_y = 0;\qquad F_{BD} - 4 = 0$

$F_{BD} = 4 \text{ kN (T)}$ **Ans.**

$\overset{+}{\rightarrow} \Sigma F_x = 0;\qquad 2.362 - 2.362 = 0 \qquad \text{(check!)}$

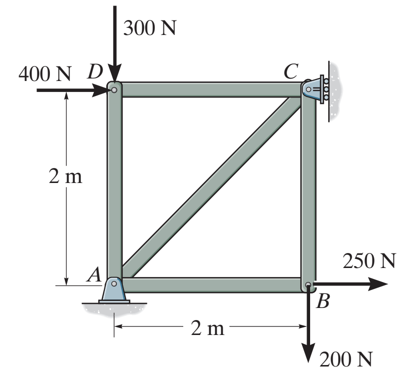
Note: The equilibrium analysis of joint D can be used to check the accuracy of the solution obtained above.



(d)

6–5.

Determine the force in each member of the truss, and state if the members are in tension or compression.



SOLUTION

Method of Joints: Here, the support reactions A and C do not need to be determined. We will first analyze the equilibrium of joints D and B, and then proceed to analyze joint C.

Joint D: From the free-body diagram in Fig. *a*, we can write

$\xrightarrow{+} \Sigma F_x = 0;$ $\quad 400 - F_{DC} = 0$

$F_{DC} = 400 \text{ N (C)}$ **Ans.**

$+\uparrow \Sigma F_y = 0;$ $\quad F_{DA} - 300 = 0$

$F_{DA} = 300 \text{ N (C)}$ **Ans.**

Joint B: From the free-body diagram in Fig. *b*, we can write

$\xrightarrow{+} \Sigma F_x = 0;$ $\quad 250 - F_{BA} = 0$

$F_{BA} = 250 \text{ N (T)}$ **Ans.**

$+\uparrow \Sigma F_y = 0;$ $\quad F_{BC} - 200 = 0$

$F_{BC} = 200 \text{ N (T)}$ **Ans.**

Joint C: From the free-body diagram in Fig. *c*, we can write

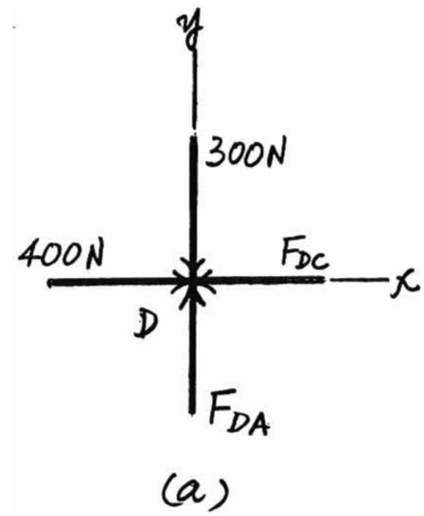
$+\uparrow \Sigma F_y = 0;$ $\quad F_{CA} \sin 45^\circ - 200 = 0$

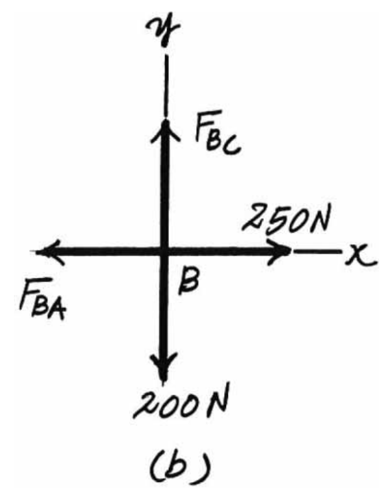
$F_{CA} = 282.84 \text{ N} = 283 \text{ N (C)}$ **Ans.**

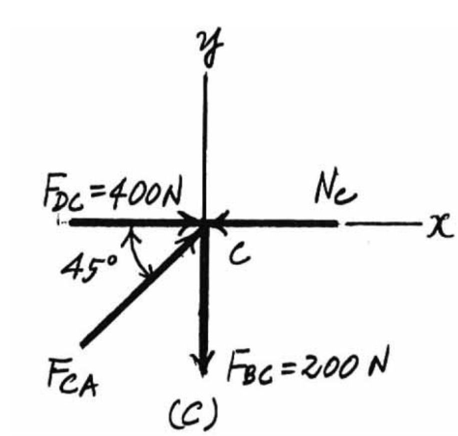
$\xrightarrow{+} \Sigma F_x = 0;$ $\quad 400 + 282.84 \cos 45^\circ - N_C = 0$

$N_C = 600 \text{ N}$

Note: The equilibrium analysis of joint A can be used to determine the components of support reaction at A.

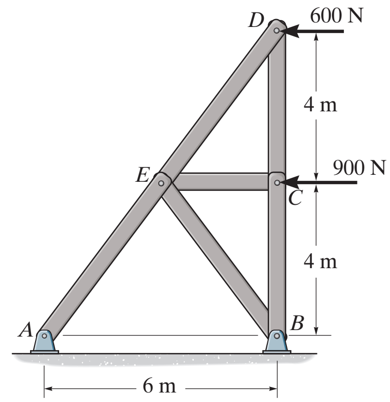






6–6.

Determine the force in each member of the truss, and state if the members are in tension or compression.



SOLUTION

Method of Joints: We will begin by analyzing the equilibrium of joint D, and then proceed to analyze joints C and E.

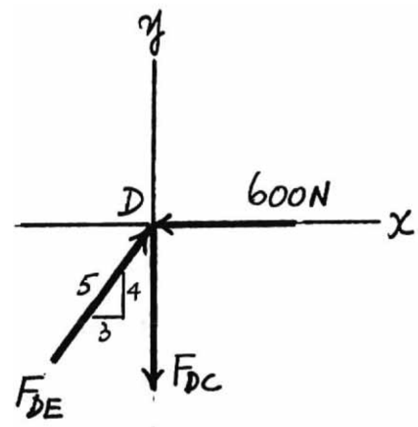
Joint D: From the free-body diagram in Fig. *a*,

$$\xrightarrow{+} \Sigma F_x = 0; \qquad F_{DE}\left(\frac{3}{5}\right) - 600 = 0$$

$$F_{DE} = 1000 \text{ N} = 1.00 \text{ kN (C)} \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad 1000\left(\frac{4}{5}\right) - F_{DC} = 0$$

$$F_{DC} = 800 \text{ N (T)} \qquad \textbf{Ans.}$$



(a)

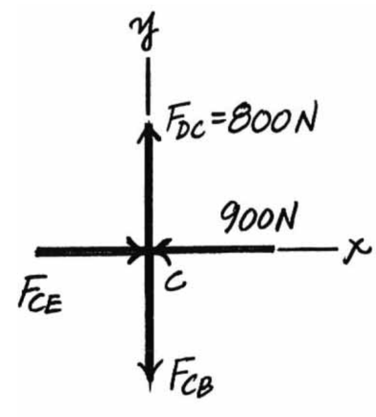
Joint C: From the free-body diagram in Fig. *b*,

$$\xrightarrow{+} \Sigma F_x = 0; \qquad F_{CE} - 900 = 0$$

$$F_{CE} = 900 \text{ N (C)} \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad 800 - F_{CB} = 0$$

$$F_{CB} = 800 \text{ N (T)} \qquad \textbf{Ans.}$$



(b)

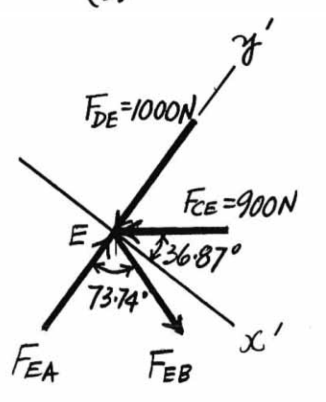
Joint E: From the free-body diagram in Fig. *c*,

$$\searrow + \Sigma F_{x'} = 0; \qquad -900 \cos 36.87° + F_{EB} \sin 73.74° = 0$$

$$F_{EB} = 750 \text{ N (T)} \qquad \textbf{Ans.}$$

$$\nearrow + \Sigma F_{y'} = 0; \qquad F_{EA} - 1000 - 900 \sin 36.87° - 750 \cos 73.74° = 0$$

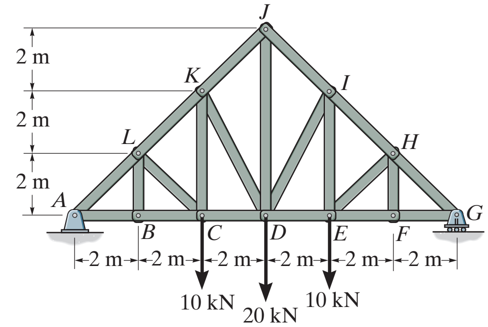
$$F_{EA} = 1750 \text{ N} = 1.75 \text{ kN (C)} \qquad \textbf{Ans.}$$



(c)

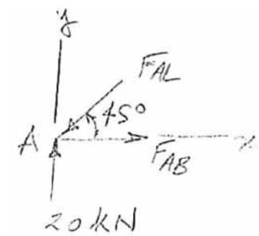
6–7.

Determine the force in each member of the *Pratt truss*, and state if the members are in tension or compression.



SOLUTION

Joint A:



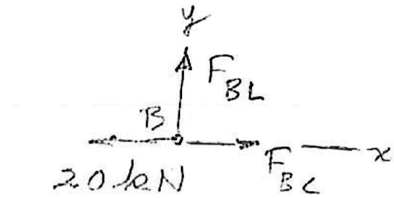
$+\uparrow \Sigma F_y = 0; \quad 20 - F_{AL} \sin 45° = 0$

$F_{AL} = 28.28 \text{ kN (C)}$

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad F_{AB} - 28.28 \cos 45° = 0$

$F_{AB} = 20 \text{ kN (T)}$

Joint B:

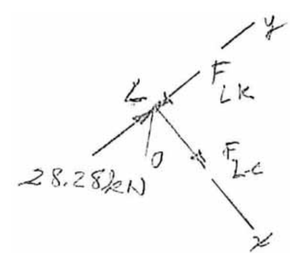


$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad F_{BC} - 20 = 0$

$F_{BC} = 20 \text{ kN (T)}$

$+\uparrow \Sigma F_y = 0; \quad F_{BL} = 0$

Joint L:

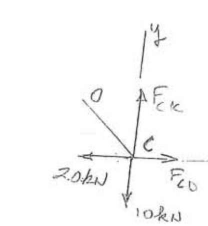


$\searrow + \Sigma F_x = 0; \quad F_{LC} = 0$

$+\nearrow \Sigma F_y = 0; \quad 28.28 - F_{LK} = 0$

$F_{LK} = 28.28 \text{ kN (C)}$

Joint C:



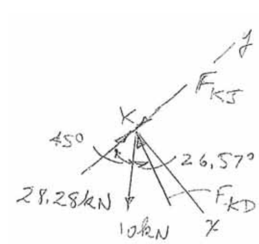
$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad F_{CD} - 20 = 0$

$F_{CD} = 20 \text{ kN (T)}$

$+\uparrow \Sigma F_y = 0; \quad F_{CK} - 10 = 0$

$F_{CK} = 10 \text{ kN (T)}$

Joint K:



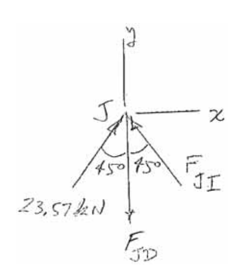
$\searrow + \Sigma F_x - 0; \quad 10 \sin 45° - F_{KD} \cos (45° - 26.57°) = 0$

$F_{KD} = 7.454 \text{ kN (L)}$

$+\nearrow \Sigma F_y = 0; \quad 28.28 - 10 \cos 45° + 7.454 \sin (45° - 26.57°) - F_{KJ} = 0$

$F_{KJ} = 23.57 \text{ kN (C)}$

Joint J:



$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad 23.57 \sin 45° - F_{JI} \sin 45° = 0$

$F_{JI} = 23.57 \text{ kN (L)}$

$+\uparrow \Sigma F_y = 0; \quad 2\,(23.57 \cos 45°) - F_{JD} = 0$

$F_{JD} = 33.3 \text{ kN (T)}$ **Ans.**

Due to Symmetry

$F_{AL} = F_{GH} = F_{LK} = F_{HI} = 28.3 \text{ kN (C)}$ **Ans.**

$F_{AB} = F_{GF} = F_{BC} = F_{FE} = F_{CD} = F_{ED} = 20 \text{ kN (T)}$ **Ans.**

$F_{BL} = F_{FH} = F_{LC} = F_{HE} = 0$ **Ans.**

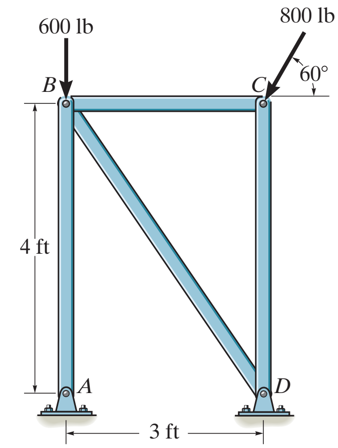
$F_{CK} = F_{EI} = 10 \text{ kN (T)}$ **Ans.**

$F_{KJ} = F_{IJ} = 23.6 \text{ kN (C)}$ **Ans.**

$F_{KD} = F_{ID} = 7.45 \text{ kN (C)}$ **Ans.**

***6–8.**

Determine the force in each member of the truss and state if the members are in tension or compression. *Hint:* The horizontal force component at A must be zero. Why?



SOLUTION

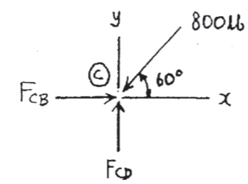
Joint C:

$\xrightarrow{+} \Sigma F_x = 0;\qquad F_{CB} - 800 \cos 60° = 0$

$F_{CB} = 400 \text{ lb (C)}$ **Ans.**

$+\uparrow \Sigma F_y = 0;\qquad F_{CD} - 800 \sin 60° = 0$

$F_{CD} = 693 \text{ lb (C)}$ **Ans.**



Joint B:

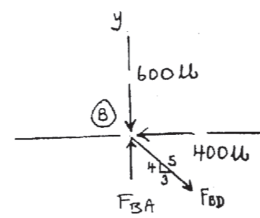
$\xrightarrow{+} \Sigma F_x = 0;\qquad \frac{3}{5}F_{BD} - 400 = 0$

$F_{BD} = 666.7 = 667 \text{ lb (T)}$ **Ans.**

$+\uparrow \Sigma F_y = 0;\qquad F_{BA} - \frac{4}{5}(666.7) - 600 = 0$

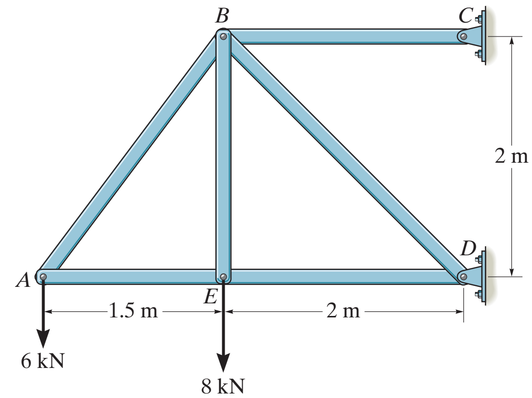
$F_{BA} = 1133 \text{ lb} = 1.13 \text{ kip (C)}$ **Ans.**

Member AB is a two-force member and exerts only a vertical force along AB at A.



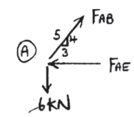
6–9.

Determine the force in each member of the truss and state if the members are in tension or compression. *Hint:* The vertical component of force at C must equal zero. Why?



SOLUTION

Joint A:



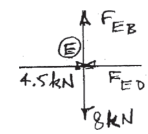
$+\uparrow \Sigma F_y = 0;\qquad \frac{4}{5}F_{AB} - 6 = 0$

$F_{AB} = 7.5\text{ kN (T)}$ **Ans.**

$\overset{+}{\rightarrow} \Sigma F_x = 0;\qquad -F_{AE} + 7.5\left(\frac{3}{5}\right) = 0$

$F_{AE} = 4.5\text{ kN (C)}$ **Ans.**

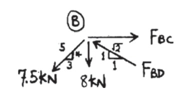
Joint E:



$\overset{+}{\rightarrow} \Sigma F_x = 0;\qquad F_{ED} = 4.5\text{ kN(C)}$ **Ans.**

$+\uparrow \Sigma F_y = 0;\qquad F_{EB} = 8\text{ kN (T)}$ **Ans.**

Joint B:



$+\uparrow \Sigma F_y = 0;\qquad \frac{1}{\sqrt{2}}(F_{BD}) - 8 - \frac{4}{5}(7.5) = 0$

$F_{BD} = 19.8\text{ kN (C)}$ **Ans.**

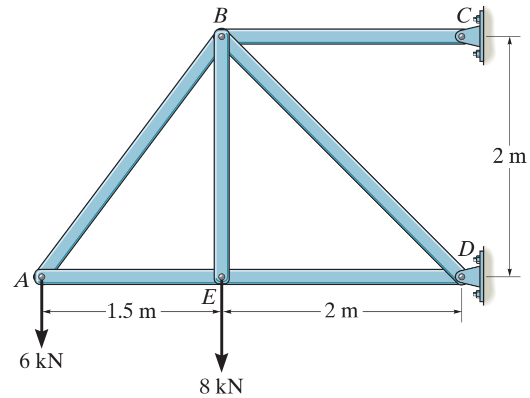
$\overset{+}{\rightarrow} \Sigma F_x = 0;\qquad F_{BC} - \frac{3}{5}(7.5) - \frac{1}{\sqrt{2}}(19.8) = 0$

$F_{BC} = 18.5\text{ kN (T)}$ **Ans.**

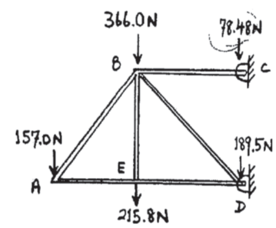
C_y is zero because BC is a two-force member .

6–10.

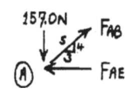
Each member of the truss is uniform and has a mass of 8 kg/m. Remove the external loads of 6 kN and 8 kN and determine the approximate force in each member due to the weight of the truss. State if the members are in tension or compression. Solve the problem by *assuming* the weight of each member can be represented as a vertical force, half of which is applied at each end of the member.



SOLUTION



Joint A:



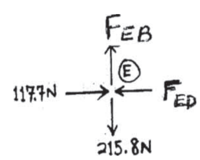
$+\uparrow \Sigma F_y = 0;$ $\qquad \frac{4}{5}F_{AB} - 157.0 = 0$

$F_{AB} = 196.2 = 196 \text{ N (T)}$ **Ans.**

$\overset{+}{\rightarrow} \Sigma F_y = 0;$ $\qquad -F_{AE} + 196.2\left(\frac{3}{5}\right) = 0$

$F_{AE} = 117.7 = 118 \text{ N (C)}$ **Ans.**

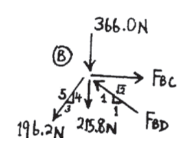
Joint E:



$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\qquad F_{ED} = 117.7 = 118 \text{ N (C)}$ **Ans.**

$+\uparrow \Sigma F_y = 0;$ $\qquad F_{EB} = 215.8 = 216 \text{ N (T)}$ **Ans.**

Joint B:



$+\uparrow \Sigma F_y = 0;$ $\qquad \frac{1}{\sqrt{2}}(F_{BD}) - 366.0 - 215.8 - \frac{4}{5}(196.2) = 0$

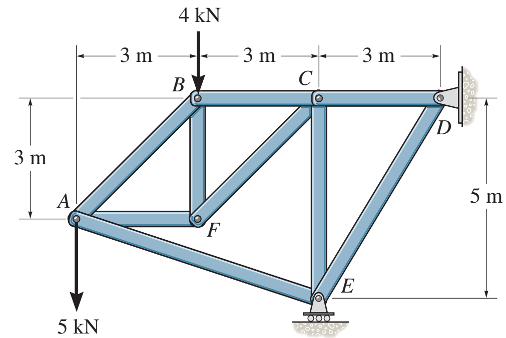
$F_{BD} = 1045 = 1.04 \text{ kN (C)}$ **Ans.**

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\qquad F_{BC} - \frac{3}{5}(196.2) - \frac{1}{\sqrt{2}}(1045) = 0$

$F_{BC} = 857 \text{ N (T)}$ **Ans.**

6–11.

Determine the force in each member of the truss and state if the members are in tension or compression.



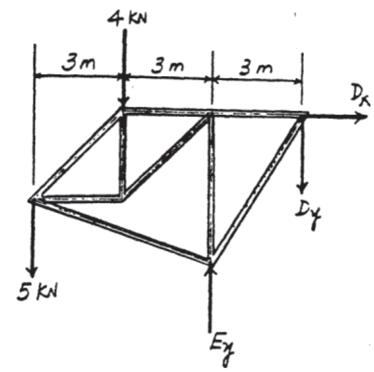
SOLUTION

Support Reactions:

$\zeta + \Sigma M_D = 0;\qquad 4(6) + 5(9) - E_y(3) = 0 \qquad E_y = 23.0\text{ kN}$

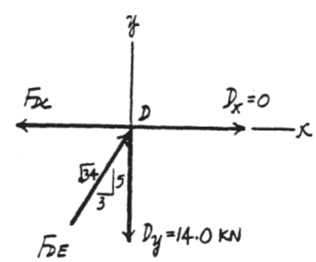
$+\uparrow \Sigma F_y = 0;\qquad 23.0 - 4 - 5 - D_y = 0 \qquad D_y = 14.0\text{ kN}$

$\xrightarrow{+} \Sigma F_x = 0 \qquad D_x = 0$



Method of Joints:

Joint D:



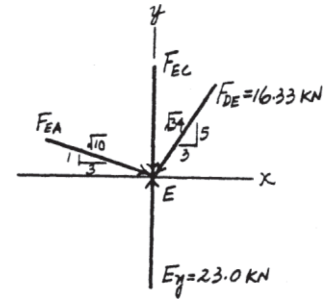
$+\uparrow \Sigma F_y = 0;\qquad F_{DE}\left(\frac{5}{\sqrt{34}}\right) - 14.0 = 0$

$F_{DE} = 16.33\text{ kN (C)} = 16.3\text{ kN (C)}$ **Ans.**

$\xrightarrow{+} \Sigma F_x = 0;\qquad 16.33\left(\frac{3}{\sqrt{34}}\right) - F_{DC} = 0$

$F_{DC} = 8.40\text{ kN (T)}$ **Ans.**

Joint E:



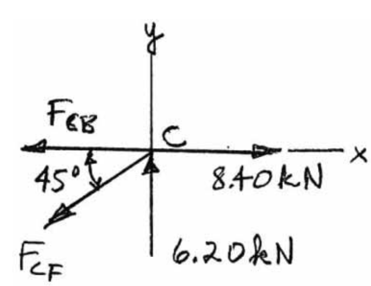
$\xrightarrow{+} \Sigma F_x = 0;\qquad F_{EA}\left(\frac{3}{\sqrt{10}}\right) - 16.33\left(\frac{3}{\sqrt{34}}\right) = 0$

$F_{EA} = 8.854\text{ kN (C)} = 8.85\text{ kN (C)}$ **Ans.**

$+\uparrow \Sigma F_y = 0;\qquad 23.0 - 16.33\left(\frac{5}{\sqrt{34}}\right) - 8.854\left(\frac{1}{\sqrt{10}}\right) - F_{EC} = 0$

$F_{EC} = 6.20\text{ kN (C)}$ **Ans.**

Joint C:



$+\uparrow \Sigma F_y = 0;\qquad 6.20 - F_{CF}\sin 45° = 0$

$F_{CF} = 8.768\text{ kN (T)} = 8.77\text{ kN (T)}$ **Ans.**

$\xrightarrow{+} \Sigma F_x = 0;\qquad 8.40 - 8.768\cos 45° - F_{CB} = 0$

$F_{CB} = 2.20\text{ kN (T)}$ **Ans.**

6–11. (continued)

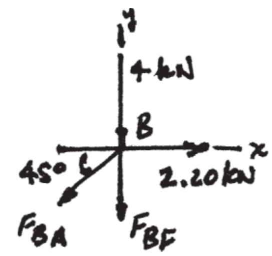
Joint B:

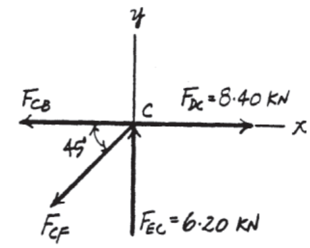
$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 2.20 - F_{BA} \cos 45° = 0$

$F_{BA} = 3.111 \text{ kN (T)} = 3.11 \text{ kN (T)}$ **Ans.**

$+\uparrow \Sigma F_y = 0; \qquad F_{BF} - 4 - 3.111 \sin 45° = 0$

$F_{BF} = 6.20 \text{ kN (C)}$ **Ans.**



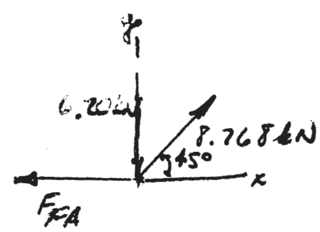


Joint F:

$+\uparrow \Sigma F_y = 0; \qquad 8.768 \sin 45° - 6.20 = 0$ **(Check!)**

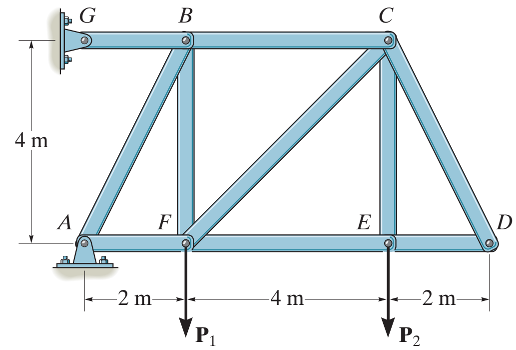
$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 8.768 \cos 45° - F_{FA} = 0$

$F_{FA} = 6.20 \text{ kN (T)}$ **Ans.**



***6–12.**

Determine the force in each member of the truss and state if the members are in tension or compression. Set $P_1 = 10$ kN, $P_2 = 15$ kN.



SOLUTION

$\circlearrowleft + \Sigma M_A = 0; \quad G_x(4) - 10(2) - 15(6) = 0$

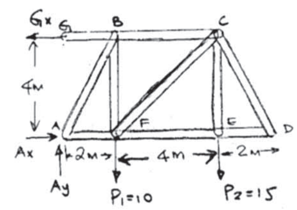
$G_x = 27.5$ kN

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad A_x - 27.5 = 0$

$A_x = 27.5$ kN

$+\uparrow \Sigma F_y = 0; \quad A_y - 10 - 15 = 0$

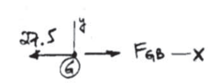
$A_y = 25$ kN



Joint G:

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad F_{GB} - 27.5 = 0$

$F_{GB} = 27.5$ kN (T) **Ans.**



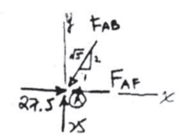
Joint A:

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad 27.5 - F_{AF} - \frac{1}{\sqrt{5}}(F_{AB}) = 0$

$+\uparrow \Sigma F_y = 0; \quad 25 - F_{AB}\left(\frac{2}{\sqrt{5}}\right) = 0$

$F_{AF} = 15.0$ kN (C) **Ans.**

$F_{AB} = 27.95 = 28.0$ kN (C) **Ans.**



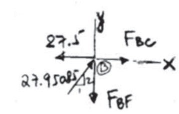
Joint B:

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad 27.95\left(\frac{1}{\sqrt{5}}\right) + F_{BC} - 27.5 = 0$

$+\uparrow \Sigma F_y = 0; \quad 27.95\left(\frac{2}{\sqrt{5}}\right) - F_{BF} = 0$

$F_{BF} = 25.0$ kN (T) **Ans.**

$F_{BC} = 15.0$ kN (T) **Ans.**



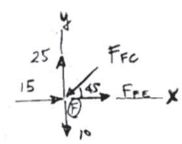
Joint F:

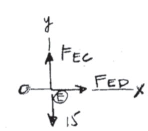
$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad 15 + F_{FE} - \frac{1}{\sqrt{2}}(F_{FC}) = 0$

$+\uparrow \Sigma F_y = 0; \quad 25 - 10 - F_{FC}\left(\frac{1}{\sqrt{2}}\right) = 0$

$F_{FC} = 21.21 = 21.2$ kN (C) **Ans.**

$F_{FE} = 0$ **Ans.**



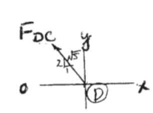


Joint E:

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad F_{ED} = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0; \quad F_{EC} - 15 = 0$

$F_{EC} = 15.0$ kN (T) **Ans.**

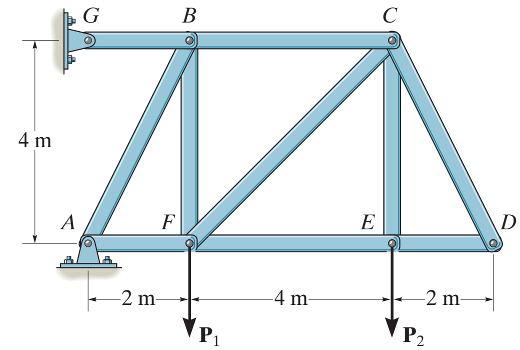


Joint D:

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad F_{DC} = 0$ **Ans.**

6–13.

Determine the force in each member of the truss and state if the members are in tension or compression. Set $P_1 = 0$, $P_2 = 20$ kN.



SOLUTION

$\zeta + \Sigma M_A = 0;$ $\quad -F_{GB}(4) + 20(6) = 0$

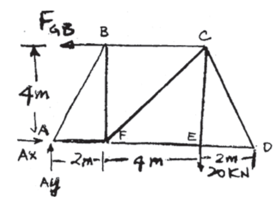
$F_{GB} = 30 \text{ kN (T)}$ **Ans.**

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad A_x - 30 = 0$

$A_x = 30 \text{ kN}$

$+\uparrow \Sigma F_y = 0;$ $\quad A_y - 20 = 0$

$A_y = 20 \text{ kN}$



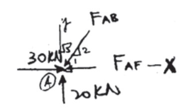
Joint A:

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad 30 - F_{AF} - \frac{1}{\sqrt{5}}(F_{AB}) = 0$

$+\uparrow \Sigma F_y = 0;$ $\quad 20 - F_{AB}\left(\frac{2}{\sqrt{5}}\right) = 0$

$F_{AF} = 20 \text{ kN (C)}$ **Ans.**

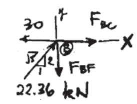
$F_{AB} = 22.36 = 22.4 \text{ kN (C)}$ **Ans.**



Joint B:

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad 22.36\left(\frac{1}{\sqrt{5}}\right) + F_{BC} - 30 = 0$

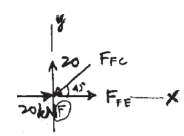
$+\uparrow \Sigma F_y = 0;$ $\quad 22.36\left(\frac{2}{\sqrt{5}}\right) - F_{BF} = 0$



$F_{BF} = 20 \text{ kN (T)}$ **Ans.**

$F_{BC} = 20 \text{ kN (T)}$ **Ans.**

Joint F:

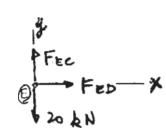


$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad 20 + F_{FE} - \frac{1}{\sqrt{2}}(F_{FC}) = 0$

$+\uparrow \Sigma F_y = 0;$ $\quad 20 - F_{FC}\left(\frac{1}{\sqrt{2}}\right) = 0$

$F_{FC} = 28.28 = 28.3 \text{ kN (C)}$ **Ans.**

$F_{FE} = 0$ **Ans.**



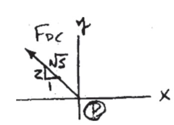
Joint E:

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad F_{ED} - 0 = 0$

$+\uparrow \Sigma F_y = 0;$ $\quad F_{EC} - 20 = 0$

$F_{ED} = 0$ **Ans.**

$F_{EC} = 20.0 \text{ kN (T)}$ **Ans.**



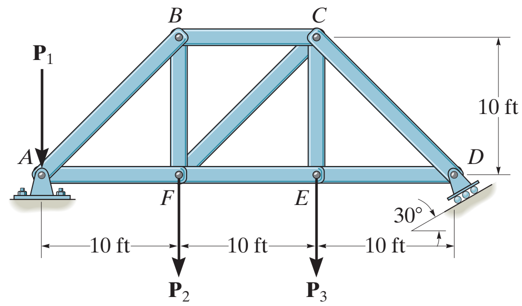
Joint D:

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad \frac{1}{\sqrt{5}}(F_{DC}) - 0 = 0$

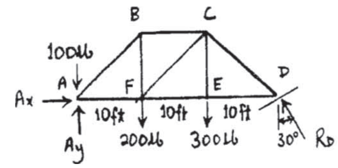
$F_{DC} = 0$ **Ans.**

6–14.

Determine the force in each member of the truss and state if the members are in tension or compression. Set $P_1 = 100$ lb, $P_2 = 200$ lb, $P_3 = 300$ lb.



SOLUTION



$\circlearrowleft + \Sigma M_A = 0;$ $\quad 200(10) + 300(20) - R_D \cos 30°(30) = 0$

$R_D = 307.9$ lb

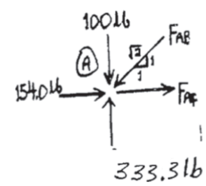
$+\uparrow \Sigma F_y = 0;$ $\quad A_y - 100 - 200 - 300 + 307.9 \cos 30° = 0$

$A_y = 333.3$ lb

$\xrightarrow{+} \Sigma F_x = 0;$ $\quad A_x - 307.9 \sin 30° = 0$

$A_x = 154.0$ lb

Joint A:



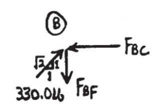
$+\uparrow \Sigma F_y = 0;$ $\quad 333.3 - 100 - \frac{1}{\sqrt{2}} F_{AB} = 0$

$F_{AB} = 330$ lb (C) **Ans.**

$\xrightarrow{+} \Sigma F_x = 0;$ $\quad 154.0 + F_{AF} - \frac{1}{\sqrt{2}}(330) = 0$

$F_{AF} = 79.37 = 79.4$ lb (T) **Ans.**

Joint B:



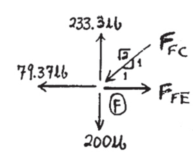
$+\uparrow \Sigma F_y = 0;$ $\quad \frac{1}{\sqrt{2}}(330) - F_{BF} = 0$

$F_{BF} = 233.3 = 233$ lb (T) **Ans.**

$\xrightarrow{+} \Sigma F_x = 0;$ $\quad \frac{1}{\sqrt{2}}(330) - F_{BC} = 0$

$F_{BC} = 233.3 = 233$ lb (C) **Ans.**

Joint F:



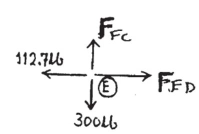
$+\uparrow \Sigma F_y = 0;$ $\quad -\frac{1}{\sqrt{2}} F_{FC} - 200 + 233.3 = 0$

$F_{FC} = 47.14 = 47.1$ lb (C) **Ans.**

$\xrightarrow{+} \Sigma F_x = 0;$ $\quad F_{FE} - 79.37 - \frac{1}{\sqrt{2}}(47.14) = 0$

$F_{FE} = 112.7 = 113$ lb (T) **Ans.**

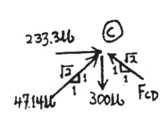
Joint E:



$\xrightarrow{+} \Sigma F_x = 0;$ $\quad F_{EC} = 300$ lb (T) **Ans.**

$+\uparrow \Sigma F_y = 0;$ $\quad F_{ED} = 112.7 = 113$ lb (T) **Ans.**

Joint C:



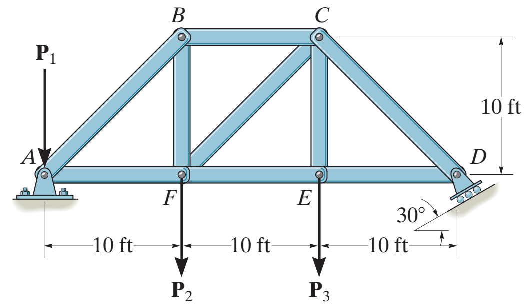
$\xrightarrow{+} \Sigma F_x = 0;$ $\quad \frac{1}{\sqrt{2}}(47.14) + 233.3 - \frac{1}{\sqrt{2}} F_{CD} = 0$

$F_{CD} = 377.1 = 377$ lb (C) **Ans.**

$+\uparrow \Sigma F_y = 0;$ $\quad \frac{1}{\sqrt{2}}(47.14) - 300 + \frac{1}{\sqrt{2}}(377.1) = 0$ **Check!**

6–15.

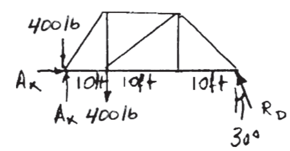
Determine the force in each member of the truss and state if the members are in tension or compression. Set $P_1 = 400$ lb, $P_2 = 400$ lb, $P_3 = 0$.



SOLUTION

$\circlearrowleft + \Sigma M_A = 0; \qquad -400(10) + R_D \cos 30°(30) = 0$

$R_D = 153.96$ lb



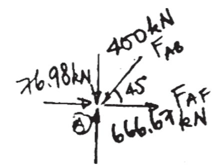
$+\uparrow \Sigma F_y = 0; \qquad A_y - 400 - 400 + 153.96 \cos 30° = 0$

$A_y = 666.67$ lb

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad A_x - 153.96 \sin 30° = 0$

$A_x = 76.98$ lb

Joint A:



$+\uparrow \Sigma F_y = 0; \qquad 666.67 - 400 - \dfrac{1}{\sqrt{2}} F_{AB} = 0$

$F_{AB} = 377.12 = 377$ lb (C) **Ans.**

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 76.98 + F_{AF} - \dfrac{1}{\sqrt{2}}(377.12) = 0$

$F_{AF} = 189.69 = 190$ lb (T) **Ans.**

Joint B:

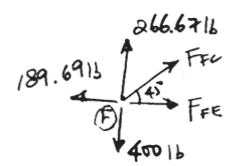
$+\uparrow \Sigma F_y = 0; \qquad \dfrac{1}{\sqrt{2}}(377.12) - F_{BF} = 0$

$F_{BF} = 266.67 = 267$ lb (T) **Ans.**

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad \dfrac{1}{\sqrt{2}}(377.12) - F_{BC} = 0$

$F_{BC} = 266.67 = 267$ lb (C) **Ans.**

Joint F:

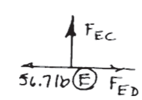


$+\uparrow \Sigma F_y = 0; \qquad \dfrac{1}{\sqrt{2}} F_{FC} - 400 + 266.67 = 0$

$F_{FC} = 188.56 = 189$ lb (T) **Ans.**

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad F_{FE} - 189.69 + \dfrac{1}{\sqrt{2}}(188.56) = 0$

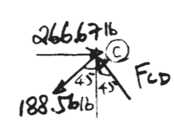
$F_{FE} = 56.35 = 56.4$ lb (T) **Ans.**



Joint E:

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad F_{ED} = 56.4$ lb (T) **Ans.**

$+\uparrow \Sigma F_y = 0; \qquad F_{EC} = 0$ **Ans.**



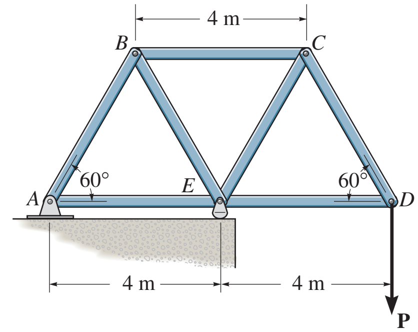
Joint C:

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad -\dfrac{1}{\sqrt{2}}(188.56) + 266.67 - \dfrac{1}{\sqrt{2}} F_{CD} = 0$

$F_{CD} = 188.56 = 189$ lb (C) **Ans.**

***6–16.**

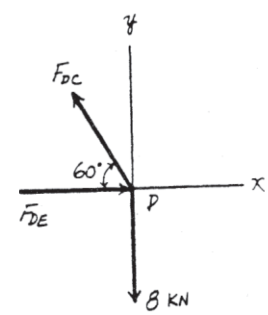
Determine the force in each member of the truss. State whether the members are in tension or compression. Set $P = 8\text{ kN}$.



SOLUTION

Method of Joints: In this case, the support reactions are not required for determining the member forces.

Joint D:



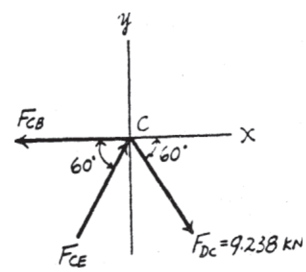
$+\uparrow \Sigma F_y = 0; \qquad F_{DC} \sin 60° - 8 = 0$

$F_{DC} = 9.238\text{ kN (T)} = 9.24\text{ kN (T)}$ **Ans.**

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad F_{DE} - 9.238 \cos 60° = 0$

$F_{DE} = 4.619\text{ kN (C)} = 4.62\text{ kN (C)}$ **Ans.**

Joint C:



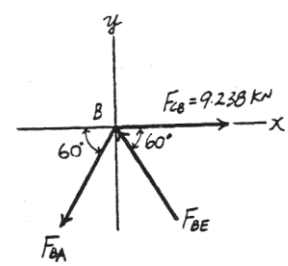
$+\uparrow \Sigma F_y = 0; \qquad F_{CE} \sin 60° - 9.238 \sin 60° = 0$

$F_{CE} = 9.238\text{ kN (C)} = 9.24\text{ kN (C)}$ **Ans.**

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 2(9.238 \cos 60°) - F_{CB} = 0$

$F_{CB} = 9.238\text{ kN (T)} = 9.24\text{ kN (T)}$ **Ans.**

Joint B:



$+\uparrow \Sigma F_y = 0; \qquad F_{BE} \sin 60° - F_{BA} \sin 60° = 0$

$F_{BE} = F_{BA} = F$

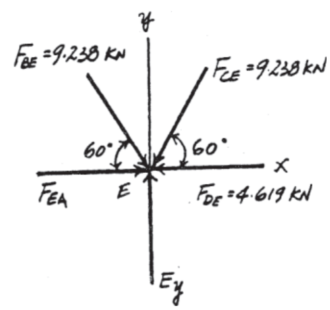
$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 9.238 - 2F \cos 60° = 0$

$F = 9.238\text{ kN}$

Thus,

$F_{BE} = 9.24\text{ kN (C)} \qquad F_{BA} = 9.24\text{ kN (T)}$ **Ans.**

Joint E:



$+\uparrow \Sigma F_y = 0; \qquad E_y - 2(9.238 \sin 60°) = 0 \qquad E_y = 16.0\text{ kN}$

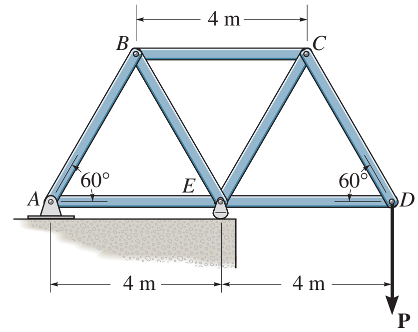
$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad F_{EA} + 9.238 \cos 60° - 9.238 \cos 60° - 4.619 = 0$

$F_{EA} = 4.62\text{ kN (C)}$ **Ans.**

Note: The support reactions A_x and A_y can be determinedd by analyzing Joint A using the results obtained above.

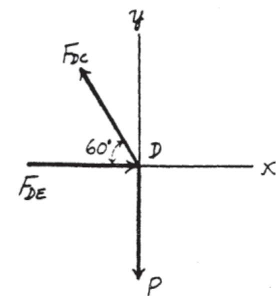
6–17.

If the maximum force that any member can support is 8 kN in tension and 6 kN in compression, determine the maximum force P that can be supported at joint D.



SOLUTION

Method of Joints: In this case, the support reactions are not required for determining the member forces.



Joint D:

$+\uparrow \Sigma F_y = 0;$ $\quad F_{DC} \sin 60° - P = 0 \qquad F_{DC} = 1.1547P \text{ (T)}$

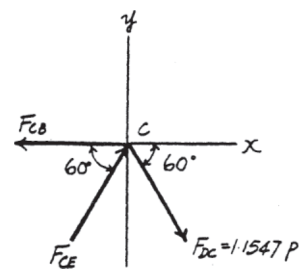
$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad F_{DE} - 1.1547P \cos 60° = 0 \qquad F_{DE} = 0.57735P \text{ (C)}$

Joint C:

$+\uparrow \Sigma F_y = 0;$ $\quad F_{CE} \sin 60° - 1.1547P \sin 60° = 0$

$$F_{CE} = 1.1547P \text{ (C)}$$

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad 2(1.1547P \cos 60°) - F_{CB} = 0 \qquad F_{CB} = 1.1547P \text{ (T)}$



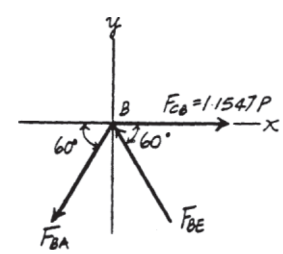
Joint B:

$+\uparrow \Sigma F_y = 0;$ $\quad F_{BE} \sin 60° - F_{BA} \sin 60° = 0 \qquad F_{BE} = F_{BA} = F$

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad 1.1547P - 2F \cos 60° = 0 \qquad F = 1.1547P$

Thus,

$$F_{BE} = 1.1547P \text{ (C)} \qquad F_{BA} = 1.1547P \text{ (T)}$$



Joint E:

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad F_{EA} + 1.1547P \cos 60° - 1.1547P \cos 60°$

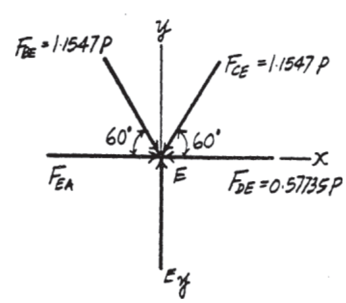
$$-0.57735P = 0$$

$$F_{EA} = 0.57735P \text{ (C)}$$

From the above analysis, the maximum compression and tension in the truss member is $1.1547P$. For this case, compression controls which requires

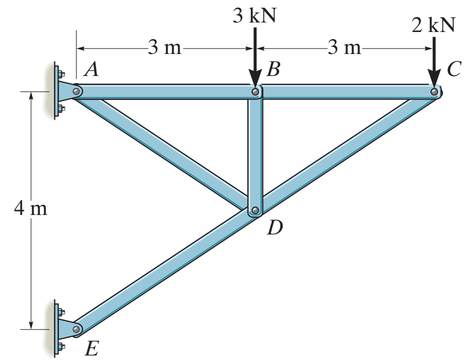
$$1.1547P = 6$$

$$P = 5.20 \text{ kN}$$ **Ans.**



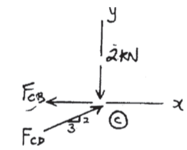
6–18.

Determine the force in each member of the truss and state if the members are in tension or compression. *Hint:* The resultant force at the pin E acts along member ED. Why?



SOLUTION

Joint C:



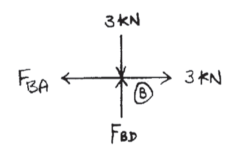
$+\uparrow \Sigma F_y = 0;$ $\qquad \dfrac{2}{\sqrt{13}} F_{CD} - 2 = 0$

$F_{CD} = 3.606 = 3.61 \text{ kN (C)}$ **Ans.**

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\qquad -F_{CD} + 3.606\left(\dfrac{3}{\sqrt{13}}\right) = 0$

$F_{CB} = 3 \text{ kN (T)}$ **Ans.**

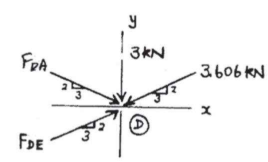
Joint B:



$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\qquad F_{BA} = 3 \text{ kN (T)}$ **Ans.**

$+\uparrow \Sigma F_y = 0;$ $\qquad F_{BD} = 3 \text{ kN (C)}$ **Ans.**

Joint D:



$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\qquad \dfrac{3}{\sqrt{13}} F_{DE} - \dfrac{3}{\sqrt{13}}(3.606) + \dfrac{3}{\sqrt{13}} F_{DA} = 0$

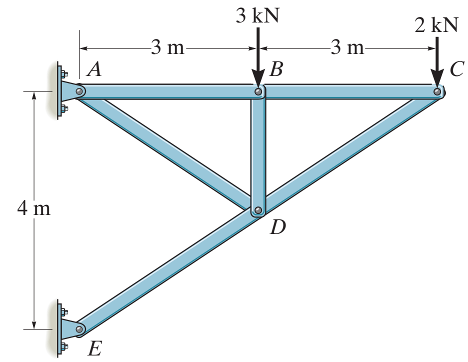
$+\uparrow \Sigma F_y = 0;$ $\qquad \dfrac{2}{\sqrt{13}}(F_{DE}) - \dfrac{2}{\sqrt{13}}(F_{DA}) - \dfrac{2}{\sqrt{13}}(3.606) - 3 = 0$

$F_{DA} = 2.70 \text{ kN (T)}$ **Ans.**

$F_{DE} = 6.31 \text{ kN (C)}$ **Ans.**

6–19.

Each member of the truss is uniform and has a mass of 8 kg/m. Remove the external loads of 3 kN and 2 kN and determine the approximate force in each member due to the weight of the truss. State if the members are in tension or compression. Solve the problem by *assuming* the weight of each member can be represented as a vertical force, half of which is applied at each end of the member.

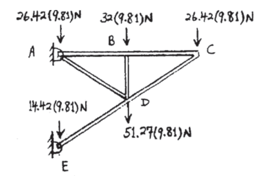


SOLUTION

Joint C:

$+\uparrow \Sigma F_y = 0;$ $\qquad \dfrac{2}{\sqrt{13}} F_{CD} - 259.2 = 0$

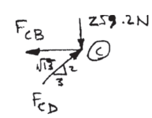
$F_{CD} = 467.3 = 467 \text{ N (C)}$ **Ans.**



$\xrightarrow{+} \Sigma F_x = 0;$ $\qquad -F_{CB} + 467.3\left(\dfrac{3}{\sqrt{13}}\right) = 0$

$F_{CB} = 388.8 = 389 \text{ N (T)}$ **Ans.**

Joint B:

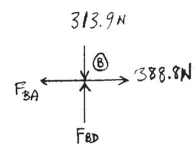


$\xrightarrow{+} \Sigma F_x = 0;$ $\qquad F_{BA} = 388.8 = 389 \text{ N (T)}$ **Ans.**

$+\uparrow \Sigma F_y = 0;$ $\qquad F_{BD} = 313.9 = 314 \text{ N (C)}$ **Ans.**

Joint D:

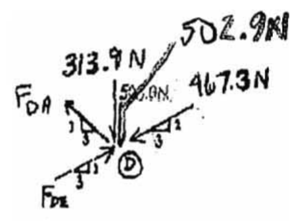
$\xrightarrow{+} \Sigma F_x = 0;$ $\qquad \dfrac{3}{\sqrt{13}} F_{DE} - \dfrac{3}{\sqrt{13}}(467.3) - \dfrac{3}{\sqrt{13}} F_{DA} = 0$

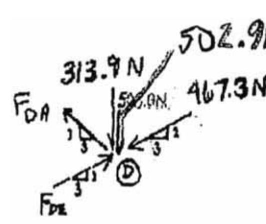


$+\uparrow \Sigma F_y = 0;$ $\qquad \dfrac{2}{\sqrt{13}}(F_{DE}) + \dfrac{2}{\sqrt{13}}(F_{DA}) - \dfrac{2}{\sqrt{13}}(467.3) - 313.9 - 502.9 = 0$

$F_{DE} = 1204 = 1.20 \text{ kN (C)}$ **Ans.**

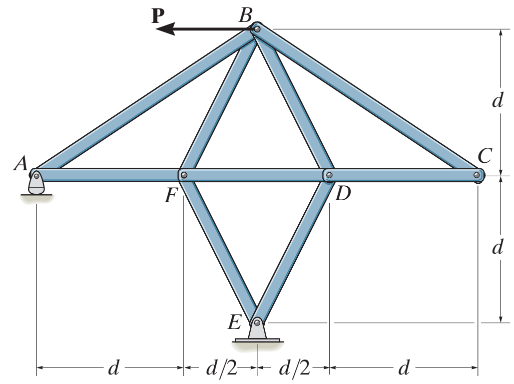
$F_{DA} = 736 \text{ N (T)}$ **Ans.**





***6–20.**

Determine the force in each member of the truss in terms of the load P, and indicate whether the members are in tension or compression.



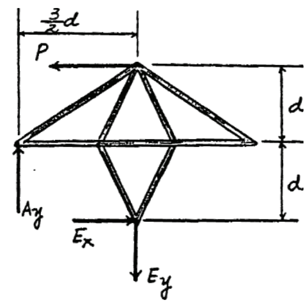
SOLUTION

Support Reactions:

$$\zeta + \Sigma M_E = 0; \quad P(2d) - A_y\left(\frac{3}{2}d\right) = 0 \qquad A_y = \frac{4}{3}P$$

$$+\uparrow \Sigma F_y = 0; \quad \frac{4}{3}P - E_y = 0 \qquad E_y = \frac{4}{3}P$$

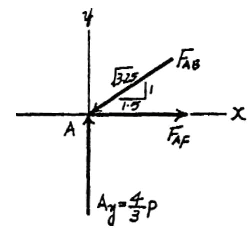
$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad E_x - P = 0 \qquad E_x = P$$



Method of Joints: By inspection of joint C, members CB and CD are zero force members. Hence

$$F_{CB} = F_{CD} = 0 \qquad \textbf{Ans.}$$

Joint A:



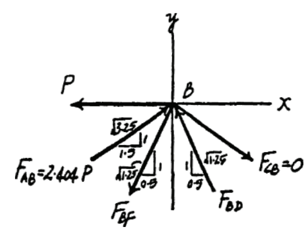
$$+\uparrow \Sigma F_y = 0; \quad F_{AB}\left(\frac{1}{\sqrt{3.25}}\right) - \frac{4}{3}P = 0$$

$$F_{AB} = 2.40P \text{ (C)} = 2.40P \text{ (C)} \qquad \textbf{Ans.}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad F_{AF} - 2.404P\left(\frac{1.5}{\sqrt{3.25}}\right) = 0$$

$$F_{AF} = 2.00P \text{ (T)} \qquad \textbf{Ans.}$$

Joint B:



$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad 2.404P\left(\frac{1.5}{\sqrt{3.25}}\right) - P - F_{BF}\left(\frac{0.5}{\sqrt{1.25}}\right) - F_{BD}\left(\frac{0.5}{\sqrt{1.25}}\right) = 0$$

$$1.00P - 0.4472F_{BF} - 0.4472F_{BD} = 0 \qquad \textbf{(1)}$$

$$+\uparrow \Sigma F_y = 0; \quad 2.404P\left(\frac{1}{\sqrt{3.25}}\right) + F_{BD}\left(\frac{1}{\sqrt{1.25}}\right) - F_{BF}\left(\frac{1}{\sqrt{1.25}}\right) = 0$$

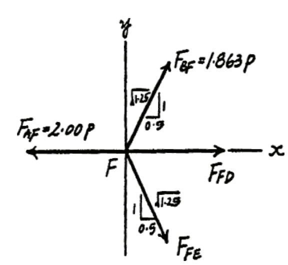
$$1.333P + 0.8944F_{BD} - 0.8944F_{BF} = 0 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yield,

$$F_{BF} = 1.863P \text{ (T)} = 1.86P \text{ (T)} \qquad \textbf{Ans.}$$

$$F_{BD} = 0.3727P \text{ (C)} = 0.373P \text{ (C)} \qquad \textbf{Ans.}$$

Joint F:



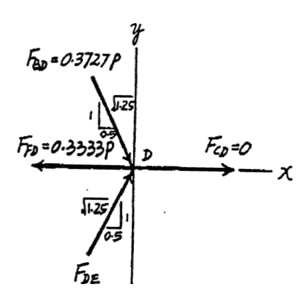
$$+\uparrow \Sigma F_y = 0; \quad 1.863P\left(\frac{1}{\sqrt{1.25}}\right) - F_{FE}\left(\frac{1}{\sqrt{1.25}}\right) = 0$$

$$F_{FE} = 1.863P \text{ (T)} = 1.86P \text{ (T)} \qquad \textbf{Ans.}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad F_{FD} + 2\left[1.863P\left(\frac{0.5}{\sqrt{1.25}}\right)\right] - 2.00P = 0$$

$$F_{FD} = 0.3333P \text{ (T)} = 0.333P \text{ (T)} \qquad \textbf{Ans.}$$

Joint D:



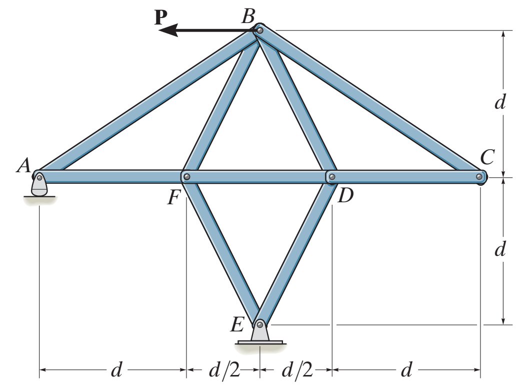
$$+\uparrow \Sigma F_y = 0; \quad F_{DE}\left(\frac{1}{\sqrt{1.25}}\right) - 0.3727P\left(\frac{1}{\sqrt{1.25}}\right) = 0$$

$$F_{DE} = 0.3727P \text{ (C)} = 0.373P \text{ (C)} \qquad \textbf{Ans.}$$

$$\overset{+}{\rightarrow} \Sigma F_y = 0; \quad 2\left[0.3727P\left(\frac{0.5}{\sqrt{1.25}}\right)\right] - 0.3333P = 0 \; (\textit{Check!})$$

6–21.

If the maximum force that any member can support is 4 kN in tension and 3 kN in compression, determine the maximum force P that can be applied at joint B. Take $d = 1$ m.



SOLUTION

Support Reactions:

$$\circlearrowleft + \Sigma M_E = 0; \quad P(2d) - A_y\left(\frac{3}{2}d\right) = 0 \qquad A_y = \frac{4}{3}P$$

$$+\uparrow \Sigma F_y = 0; \quad \frac{4}{3}P - E_y = 0 \qquad E_y = \frac{4}{3}P$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad E_x - P = 0 \qquad E_x = P$$

Method of Joints: By inspection of joint C, members CB and CD are zero force members. Hence

$$F_{CB} = F_{CD} = 0$$

Joint A:

$$+\uparrow \Sigma F_y = 0; \quad F_{AB}\left(\frac{1}{\sqrt{3.25}}\right) - \frac{4}{3}P = 0 \qquad F_{AB} = 2.404P \text{ (C)}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad F_{AF} - 2.404P\left(\frac{1.5}{\sqrt{3.25}}\right) = 0 \qquad F_{AF} = 2.00P \text{ (T)}$$

Joint B:

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad 2.404P\left(\frac{1.5}{\sqrt{3.25}}\right) - P - F_{BF}\left(\frac{0.5}{\sqrt{1.25}}\right) - F_{BD}\left(\frac{0.5}{\sqrt{1.25}}\right) = 0$$

$$1.00P - 0.4472F_{BF} - 0.4472F_{BD} = 0 \qquad \textbf{(1)}$$

$$+\uparrow \Sigma F_y = 0; \quad 2.404P\left(\frac{1}{\sqrt{3.25}}\right) + F_{BD}\left(\frac{1}{\sqrt{1.25}}\right) - F_{BF}\left(\frac{1}{\sqrt{1.25}}\right) = 0$$

$$1.333P + 0.8944F_{BD} - 0.8944F_{BF} = 0 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yield,

$$F_{BF} = 1.863P \text{ (T)} \qquad F_{BD} = 0.3727P \text{ (C)}$$

Joint F:

$$+\uparrow \Sigma F_y = 0; \quad 1.863P\left(\frac{1}{\sqrt{1.25}}\right) - F_{FE}\left(\frac{1}{\sqrt{1.25}}\right) = 0$$

$$F_{FE} = 1.863P \text{ (T)}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad F_{FD} + 2\left[1.863P\left(\frac{0.5}{\sqrt{1.25}}\right)\right] - 2.00P = 0$$

$$F_{FD} = 0.3333P \text{ (T)}$$

Joint D:

$$+\uparrow \Sigma F_y = 0; \quad F_{DE}\left(\frac{1}{\sqrt{1.25}}\right) - 0.3727P\left(\frac{1}{\sqrt{1.25}}\right) = 0$$

$$F_{DE} = 0.3727P \text{ (C)}$$

$$\overset{+}{\rightarrow} \Sigma F_y = 0; \quad 2\left[0.3727P\left(\frac{0.5}{\sqrt{1.25}}\right)\right] - 0.3333P = 0 \ (\textit{Check!})$$

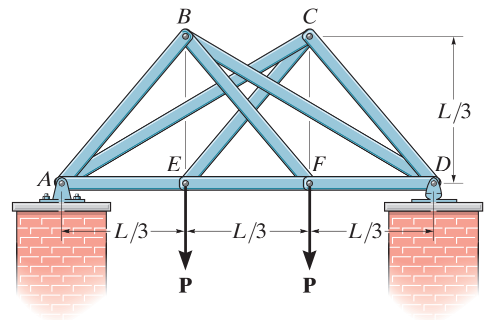
From the above analysis, the maximum compression and tension in the truss members are $2.404P$ and $2.00P$, respectively. For this case, compression controls which requires

$$2.404P = 3$$

$$P = 1.25 \text{ kN}$$

6–22.

Determine the force in each member of the double scissors truss in terms of the load P and state if the members are in tension or compression.

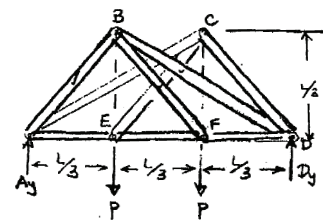


SOLUTION

$\zeta + \Sigma M_A = 0; \quad P\left(\frac{L}{3}\right) + P\left(\frac{2L}{3}\right) - (D_y)(L) = 0$

$D_y = P$

$+\uparrow \Sigma F_y = 0; \quad A_y = P$



Joint *F*:

$\xrightarrow{+} \Sigma F_x = 0; \quad F_{FD} - F_{FE} - F_{FB}\left(\frac{1}{\sqrt{2}}\right) = 0$ **(1)**

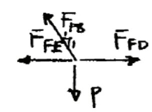
$F_{FD} - F_{FE} = P$

$\xrightarrow{+} \Sigma F_y = 0; \quad F_{FB}\left(\frac{1}{\sqrt{2}}\right) - P = 0$

$F_{FB} = \sqrt{2}P = 1.41\ P\ (\text{T})$

Similarly,

$F_{EC} = \sqrt{2}P$



Joint *C*:

$\xrightarrow{+} \Sigma F_x = 0; \quad F_{CA}\left(\frac{2}{\sqrt{5}}\right) - \sqrt{2}P\left(\frac{1}{\sqrt{2}}\right) - F_{CD}\left(\frac{1}{\sqrt{2}}\right) = 0$

$\frac{2}{\sqrt{5}}F_{CA} - \frac{1}{\sqrt{2}}F_{CD} = P$

$+\uparrow \Sigma F_y = 0; \quad F_{CA}\frac{1}{\sqrt{5}} - \sqrt{2}P\frac{1}{\sqrt{2}} + F_{CD}\frac{1}{\sqrt{2}} = 0$

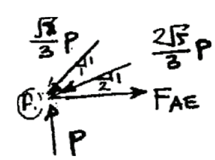
$F_{CA} = \frac{2\sqrt{5}}{3}P = 1.4907P = 1.49P\ (\text{C})$

$F_{CD} = \frac{\sqrt{2}}{3}P = 0.4714P = 0.471P\ (\text{C})$

Joint *A*:

$\xrightarrow{+} \Sigma F_x = 0; \quad F_{AE} - \frac{\sqrt{2}}{3}P\left(\frac{1}{\sqrt{2}}\right) - \frac{2\sqrt{5}}{3}P\left(\frac{2}{\sqrt{5}}\right) = 0$

$F_{AE} = \frac{5}{3}P = 1.67\ P\ (\text{T})$



Similarly,

$F_{FD} = 1.67\ P\ (\text{T})$

From Eq.(1), and Symmetry,

$F_{FE} = 0.667\ P\ (\text{T})$ **Ans.**

$F_{FD} = 1.67\ P\ (\text{T})$ **Ans.**

$F_{AB} = 0.471\ P\ (\text{C})$ **Ans.**

$F_{AE} = 1.67\ P\ (\text{T})$ **Ans.**

$F_{AC} = 1.49\ P\ (\text{C})$ **Ans.**

$F_{BF} = 1.41\ P\ (\text{T})$ **Ans.**

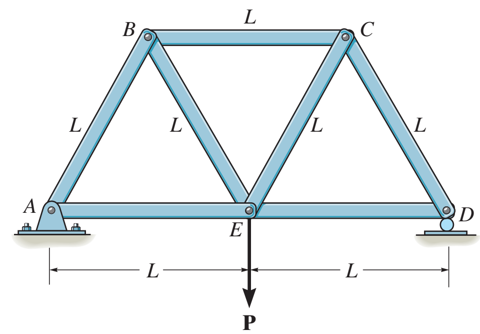
$F_{BD} = 1.49\ P\ (\text{C})$ **Ans.**

$F_{EC} = 1.41\ P\ (\text{T})$ **Ans.**

$F_{CD} = 0.471\ P\ (\text{C})$ **Ans.**

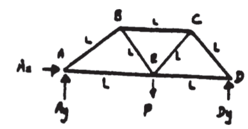
6–23.

Determine the force in each member of the truss in terms of the load P and state if the members are in tension or compression.



SOLUTION

Entire truss:



$\zeta + \Sigma M_A = 0; \qquad -P(L) + D_y\ (2\,L) = 0$

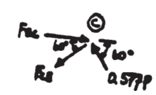
$$D_y = \frac{P}{2}$$

$+\uparrow \Sigma F_y = 0; \qquad \frac{P}{2} - P + A_y = 0$

$$A_y = \frac{P}{2}$$

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad A_x = 0$

Joint D:



$+\uparrow \Sigma F_y = 0; \qquad -F_{CD} \sin 60^\circ + \frac{P}{2} = 0$

$F_{CD} = 0.577\ P\ (\mathrm{C})$ **Ans.**

$\overset{+}{\leftarrow} \Sigma F_x = 0; \qquad F_{DB} - 0.577P \cos 60^\circ = 0$

$F_{DB} = 0.289\ P(\mathrm{T})$ **Ans.**

Joint C:

$+\uparrow \Sigma F_y = 0; \qquad 0.577\ P \sin 60^\circ - F_{CE} \sin 60^\circ = 0$

$F_{CE} = 0.577\ P\ (\mathrm{T})$ **Ans.**

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad F_{BC} - 0.577\ P \cos 60^\circ - 0.577P \cos 60^\circ = 0$

$F_{BC} = 0.577\ P\ (\mathrm{C})$ **Ans.**

Due to symmetry:

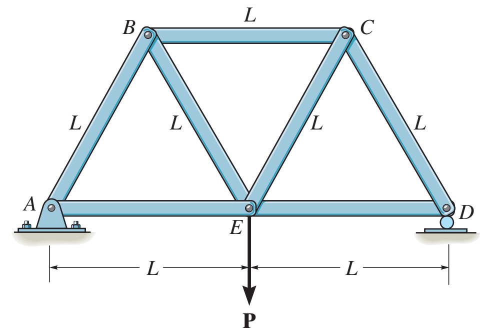
$F_{BE} = F_{CE} = 0.577\ P\ (\mathrm{T})$ **Ans.**

$F_{AB} = F_{CD} = 0.577\ P\ (\mathrm{C})$ **Ans.**

$F_{AE} = F_{DE} = 0.577\ P\ (\mathrm{T})$ **Ans.**

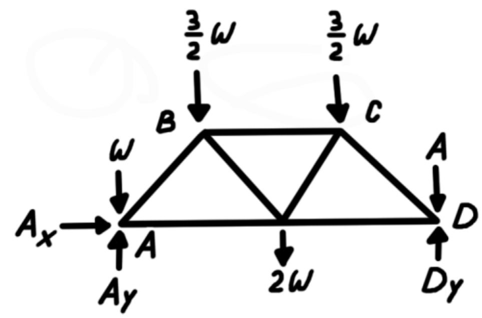
***6–24.**

Each member of the truss is uniform and has a weight W. Remove the external force **P** and determine the approximate force in each member due to the weight of the truss. State if the members are in tension or compression. Solve the problem by *assuming* the weight of each member can be represented as a vertical force, half of which is applied at each end of the member.



SOLUTION

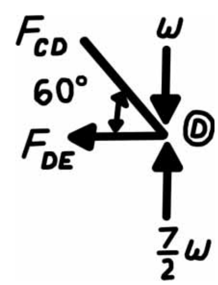
Entire truss:



$\circlearrowleft + \Sigma M_A = 0; \quad -\frac{3}{2}W\left(\frac{L}{2}\right) - 2\,W(L) - \frac{3}{2}W\left(\frac{3}{2}L\right) - W(2\,L) + D_y\,(2\,L) = 0$

$$D_y = \frac{7}{2}W$$

Joint D:



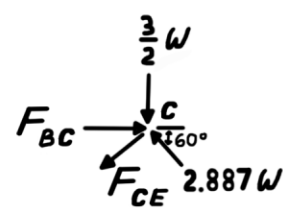
$+\uparrow \Sigma F_y = 0; \quad \frac{7}{2}W - W - F_{CD}\sin 60° = 0$

$F_{CD} = 2.887W = 2.89\,W\ \text{(C)}$ **Ans.**

$\xrightarrow{+} \Sigma F_x = 0; \quad 2.887W\cos 60° - F_{DE} = 0$

$F_{DE} = 1.44\,W\ \text{(T)}$ **Ans.**

Joint C:



$+\uparrow \Sigma F_y = 0; \quad 2.887W\sin 60° - \frac{3}{2}W - F_{CE}\sin 60° = 0$

$F_{CE} = 1.1547W = 1.15\,W\ \text{(T)}$ **Ans.**

$\xrightarrow{+} \Sigma F_x = 0; \quad F_{BC} - 1.1547W\cos 60° - 2.887W\cos 60° = 0$

$F_{BC} = 2.02\,W\ \text{(C)}$ **Ans.**

Due to symmetry:

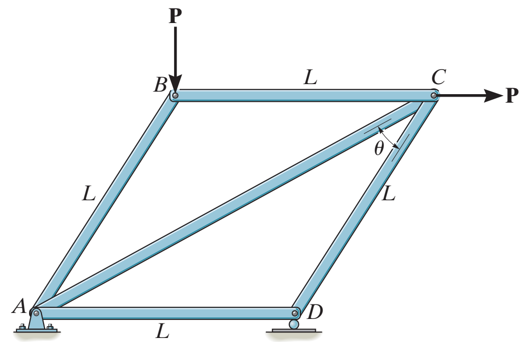
$F_{BE} = F_{CE} = 1.15\,W\ \text{(T)}$ **Ans.**

$F_{AB} = F_{CD} = 2.89\,W\ \text{(C)}$ **Ans.**

$F_{AE} = F_{DE} = 1.44\,W\ \text{(T)}$ **Ans.**

6–25.

Determine the force in each member of the truss in terms of the external loading and state if the members are in tension or compression.

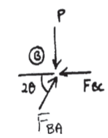


SOLUTION

Joint B:

$+\uparrow \Sigma F_y = 0;$ $\quad F_{BA} \sin 2\theta - P = 0$

$F_{BA} = P \csc 2\theta \text{ (C)}$ **Ans.**



$\xrightarrow{+} \Sigma F_x = 0;$ $\quad P \csc 2\theta(\cos 2\theta) - F_{BC} = 0$

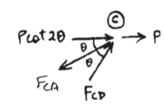
$F_{BC} = P \cot 2\theta \text{ (C)}$ **Ans.**

Joint C:

$\xrightarrow{+} \Sigma F_x = 0;$ $\quad P \cot 2\theta + P + F_{CD} \cos 2\theta - F_{CA} \cos \theta = 0$

$+\uparrow \Sigma F_y = 0;$ $\quad F_{CD} \sin 2\theta - F_{CA} \sin \theta = 0$

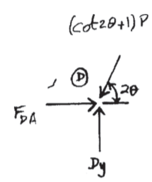
$$F_{CA} = \frac{\cot 2\theta + 1}{\cos \theta - \sin \theta \cot 2\theta} P$$



$F_{CA} = (\cot \theta \cos \theta - \sin \theta + 2 \cos \theta) P \quad \text{(T)}$ **Ans.**

$F_{CD} = (\cot 2\theta + 1) P \quad \text{(C)}$ **Ans.**

Joint D:

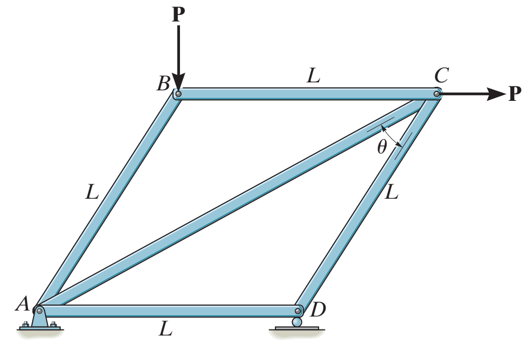


$\xrightarrow{+} \Sigma F_x = 0;$ $\quad F_{DA} - (\cot 2\theta + 1)(\cos 2\theta) P = 0$

$F_{DA} = (\cot 2\theta + 1)(\cos 2\theta)(P) \quad \text{(C)}$ **Ans.**

6–26.

The maximum allowable tensile force in the members of the truss is $(F_t)_{max} = 2$ kN, and the maximum allowable compressive force is $(F_c)_{max} = 1.2$ kN. Determine the maximum magnitude P of the two loads that can be applied to the truss. Take $L = 2$ m and $\theta = 30°$.

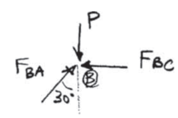


SOLUTION

$(T_t)_{max} = 2$ kN

$(F_C)_{max} = 1.2$ kN

Joint B:



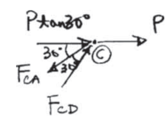
$+\uparrow \Sigma F_y = 0; \qquad F_{BA} \cos 30° - P = 0$

$$F_{BA} = \frac{P}{\cos 30°} = 1.1547\, P \text{ (C)}$$

$\xrightarrow{+} \Sigma F_x = 0; \qquad F_{AB} \sin 30° - F_{BC} = 0$

$$F_{BC} = P \tan 30° = 0.57735\, P \text{ (C)}$$

Joint C:



$+\uparrow \Sigma F_y = 0; \qquad -F_{CA} \sin 30° + F_{CD} \sin 60° = 0$

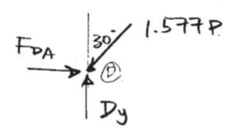
$$F_{CA} = F_{CD}\left(\frac{\sin 60°}{\sin 30°}\right) = 1.732\, F_{CD}$$

$\xrightarrow{+} \Sigma F_x = 0; \qquad P \tan 30° + P + F_{CD} \cos 60° - F_{CA} \cos 30° = 0$

$$F_{CD} = \left(\frac{\tan 30° + 1}{\sqrt{3} \cos 30° - \cos 60°}\right) P = 1.577\, P \text{ (C)}$$

$$F_{CA} = 2.732\, P \text{ (T)}$$

Joint D:



$\xrightarrow{+} \Sigma F_x = 0; \qquad F_{DA} - 1.577\, P \sin 30° = 0$

$$F_{DA} = 0.7887\, P \text{ (C)}$$

1) Assume $F_{CA} = 2 \text{ kN} = 2.732\, P$

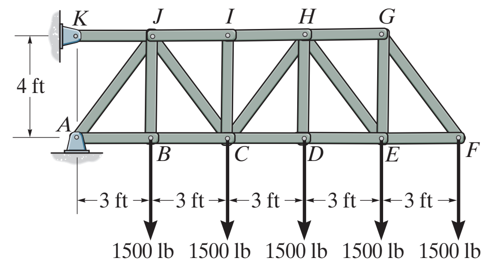
$P = 732.05$ N

$F_{CD} = 1.577(732.05) = 1154.7 \text{ N} < (F_c)_{max} = 1200$ N **(O.K.!)**

Thus, $P_{max} = 732$ N **Ans.**

6–27.

Determine the force in members HG, HE, and DE of the truss, and state if the members are in tension or compression.



SOLUTION

Method of Sections: The forces in members HG, HE, and DE are exposed by cutting the truss into two portions through section a–a and using the upper portion of the free-body diagram, Fig. a. From this free-body diagram, F_{HG} and F_{DE} can be obtained by writing the moment equations of equilibrium about points E and H, respectively. $\mathbf{F}_{HE}$ can be obtained by writing the force equation of equilibrium along the y axis.

Joint D: From the free-body diagram in Fig. a,

$\circlearrowleft + \Sigma M_E = 0; \qquad F_{HG}(4) - 1500(3) = 0$

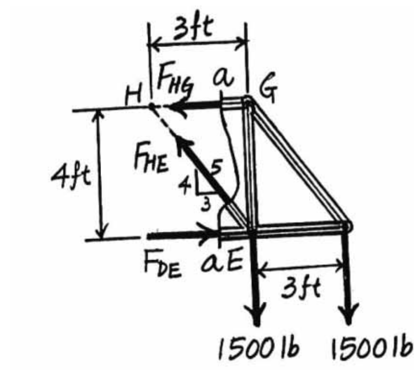
$F_{HG} = 1125 \text{ lb (T)}$ **Ans.**

$\circlearrowleft + \Sigma M_H = 0; \qquad F_{DE}(4) - 1500(6) - 1500(3) = 0$

$F_{DE} = 3375 \text{ lb (C)}$ **Ans.**

$+\uparrow \Sigma F_y = 0; \qquad F_{HE}\left(\frac{4}{5}\right) - 1500 - 1500 = 0$

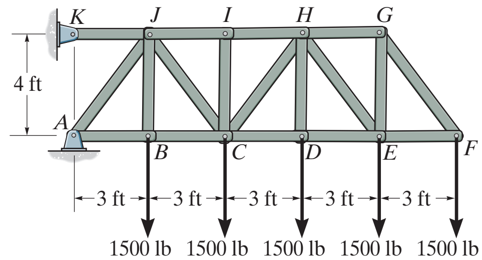
$F_{EH} = 3750 \text{ lb (T)}$ **Ans.**



(a)

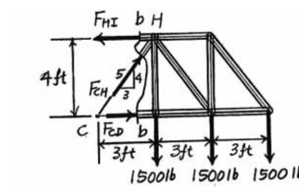
***6–28.**

Determine the force in members *CD, HI*, and *CJ* of the truss, and state if the members are in tension or compression.



SOLUTION

Method of Sections: The forces in members *HI*, *CH*, and *CD* are exposed by cutting the truss into two portions through section *b–b* on the right portion of the free-body diagram, Fig. *a*. From this free-body diagram, F_{CD} and F_{HI} can be obtained by writing the moment equations of equilibrium about points *H* and *C*, respectively. F_{CH} can be obtained by writing the force equation of equilibrium along the *y* axis.



(a)

$\circlearrowleft +\Sigma M_H = 0;$ $\quad F_{CD}(4) - 1500(6) - 1500(3) = 0$

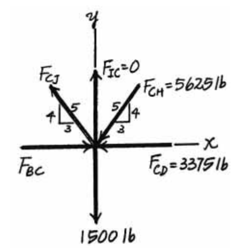
$F_{CD} = 3375 \text{ lb (C)}$ **Ans.**

$\circlearrowleft +\Sigma M_C = 0;$ $\quad F_{HI}(4) - 1500(3) - 1500(6) - 1500(9) = 0$

$F_{HI} = 6750 \text{ lb (T)}$ **Ans.**

$+\uparrow \Sigma F_y = 0;$ $\quad F_{CH}\left(\dfrac{4}{5}\right) - 1500 - 1500 = 0$

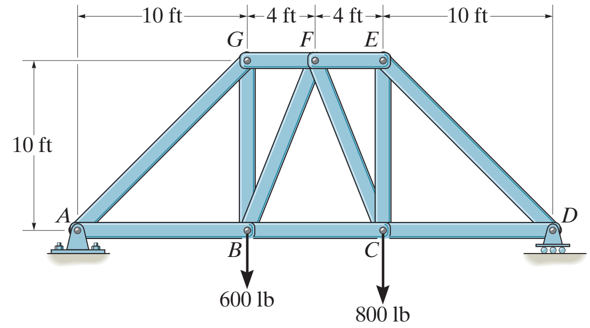
$F_{CH} = 5625 \text{ lb (C)}$ **Ans.**



(b)

6–29.

Determine the force developed in members *GB* and *GF* of the bridge truss and state if these members are in tension or compression.



SOLUTION

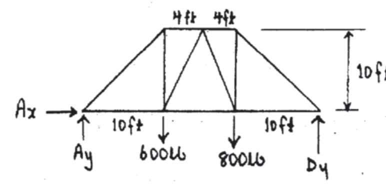
$\circlearrowleft + \Sigma M_A = 0;$ $\quad -600(10) - 800(18) + D_y(28) = 0$

$D_y = 728.571 \text{ lb}$

$\xrightarrow{+} \Sigma F_x = 0;$ $\quad A_x = 0$

$+\uparrow \Sigma F_y = 0;$ $\quad A_y - 600 - 800 + 728.571 = 0$

$A_y = 671.429 \text{ lb}$

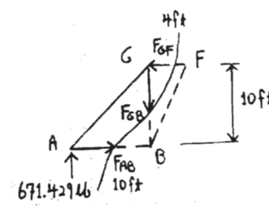


$\circlearrowleft + \Sigma M_B = 0;$ $\quad -671.429(10) + F_{GF}(10) = 0$

$F_{GF} = 671.429 \text{ lb} = 671 \text{ lb (C)}$ **Ans.**

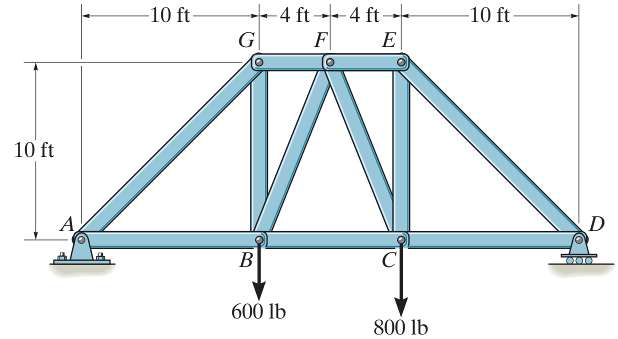
$+\uparrow \Sigma F_y = 0;$ $\quad 671.429 - F_{GB} = 0$

$F_{GB} = 671 \text{ lb (T)}$ **Ans.**



6–30.

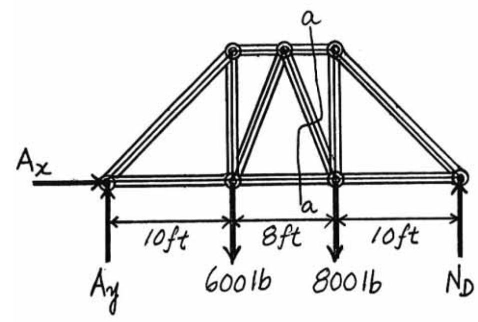
Determine the force in members *EC*, *EF*, and *FC* of the bridge truss and state if these members are in tension or compression.



SOLUTION

Support Reactions: Applying the moment equation of equilibrium about point *A* by referring to the *FBD* of the entire truss shown in Fig. *a*,

$\circlearrowleft + \Sigma M_A = 0; \qquad N_D(28) - 600(10) - 800(18) = 0 \quad N_D = 728.57 \text{ lb}$ **Ans.**



(a)

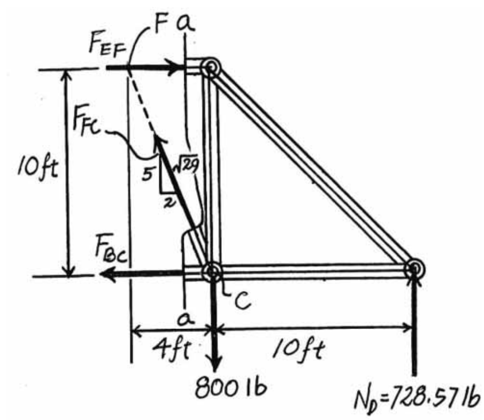
Method of Sections: Consider the *FBD* of the right portion of the truss cut through sec. *a–a*, Fig. *b*, we notice that $\mathbf{F}_{EF}$ and $\mathbf{F}_{FC}$ can be obtained directly by writing moment equation of equilibrium about joint *C* and force equation of equilibrium along *y*-axis, respectively.

$\circlearrowleft + \Sigma M_C = 0; \qquad 728.57(10) - F_{EF}(10) = 0$

$F_{EF} = 728.57 \text{ lb (C)} = 729 \text{ lb (C)}$ **Ans.**

$+\uparrow \Sigma F_y = 0; \qquad F_{FC}\left(\frac{5}{\sqrt{29}}\right) + 728.57 - 800 = 0$

$F_{FC} = 76.93 \text{ lb (T)} = 76.916 \text{ (T)}$ **Ans.**



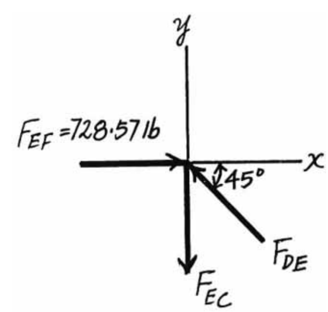
(b)

Method of joints: Using the result $\mathbf{F}_{EF}$ to consider joint *E*, Fig. *c*,

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 728.57 - F_{DE}\cos 45° = 0 \quad F_{DE} = 1030.36 \text{ lb (C)}$

$+\uparrow \Sigma F_y = 0; \qquad 1030.36 \sin 45° - F_{EC} = 0$

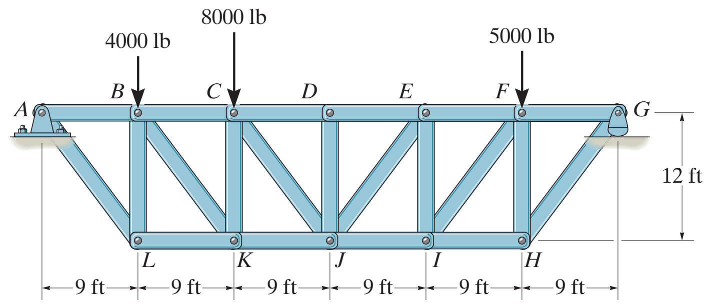
$F_{EC} = 728.57 \text{ lb (T)} = 729 \text{ lb (T)}$ **Ans.**



(c)

6–31.

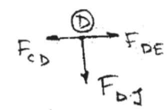
Determine the force in members *CD*, *CJ*, *KJ*, and *DJ* of the truss which serves to support the deck of a bridge. State if these members are in tension or compression.



SOLUTION

$\circlearrowleft + \Sigma M_C = 0;\qquad -9500(18) + 4000(9) + F_{KJ}(12) = 0$

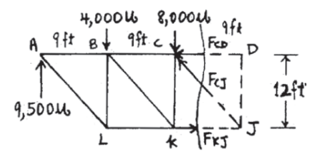
$F_{KJ} = 11\,250\ \text{lb} = 11.2\ \text{kip (T)}$ **Ans.**



$\circlearrowleft + \Sigma M_J = 0;\qquad -9500(27) + 4000(18) + 8000(9) + F_{CD}(12) = 0$

$F_{CD} = 9375\ \text{lb} = 9.38\ \text{kip (C)}$ **Ans.**

$\overset{+}{\rightarrow} \Sigma F_x = 0;\qquad -9375 + 11\,250 - \frac{3}{5}F_{CJ} = 0$

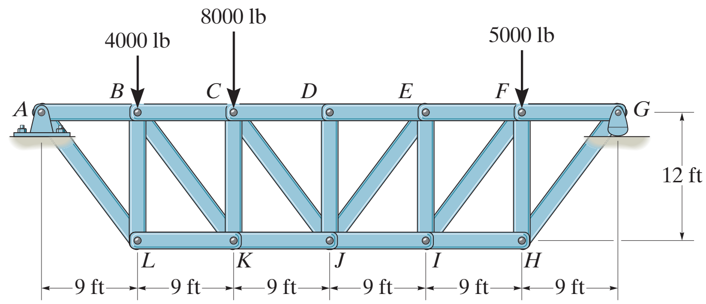


$F_{CJ} = 3125\ \text{lb} = 3.12\ \text{kip (C)}$ **Ans.**

Joint *D*: $F_{DJ} = 0$ **Ans.**

***6–32.**

Determine the force in members EI and JI of the truss which serves to support the deck of a bridge. State if these members are in tension or compression.



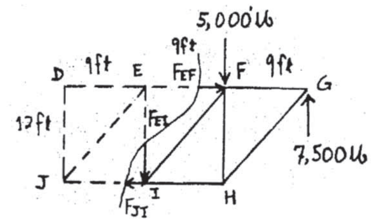
SOLUTION

$\zeta + \Sigma M_E = 0;$ $\quad -5000(9) + 7500(18) - F_{JI}(12) = 0$

$F_{JI} = 7500 \text{ lb} = 7.50 \text{ kip (T)}$ **Ans.**

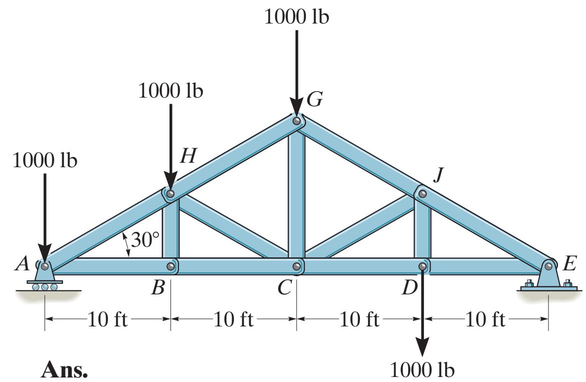
$+\uparrow \Sigma F_y = 0;$ $\quad 7500 - 5000 - F_{EI} = 0$

$F_{EI} = 2500 \text{ lb} = 2.50 \text{ kip (C)}$ **Ans.**



6–33.

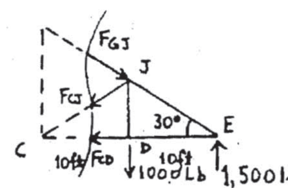
Determine the force in member *GJ* of the truss and state if this member is in tension or compression.



SOLUTION

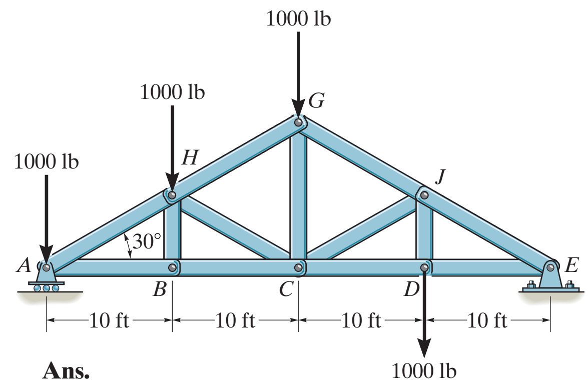
$\circlearrowleft + \Sigma M_C = 0;\qquad -1000(10) + 1500(20) - F_{GJ}\cos 30°(20\tan 30°) = 0$

$F_{GJ} = 2.00 \text{ kip (C)}$ **Ans.**



6–34.

Determine the force in member *GC* of the truss and state if this member is in tension or compression.

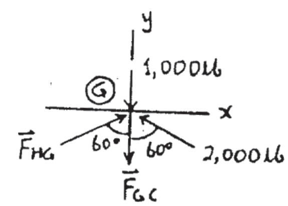


SOLUTION

$\zeta + \Sigma M_C = 0; \quad -1000(10) + 1500(20) - F_{GJ} \cos 30°(20 \tan 30°) = 0$

$F_{GJ} = 2.00 \text{ kip (C)}$ **Ans.**

Joint *G*:



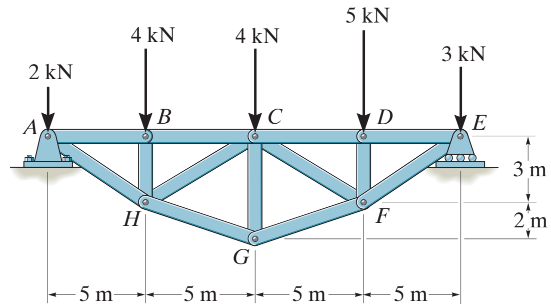
$\xrightarrow{+} \Sigma F_x = 0; \quad F_{HG} = 2000 \text{ lb}$

$+\uparrow \Sigma F_y = 0; \quad -1000 + 2(2000 \cos 60°) - F_{GC} = 0$

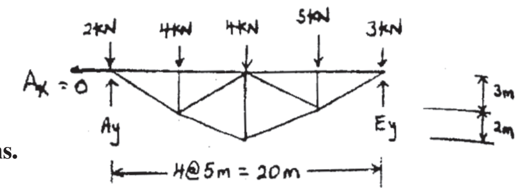
$F_{GC} = 1.00 \text{ kip (T)}$ **Ans.**

6–35.

Determine the force in members *BC*, *HC*, and *HG*. After the truss is sectioned use a single equation of equilibrium for the calculation of each force. State if these members are in tension or compression.



SOLUTION

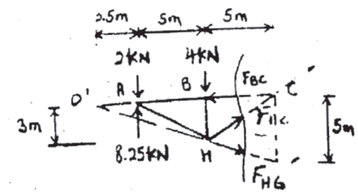


$\circlearrowleft +\Sigma M_E = 0;$ $\quad -A_y(20) + 2(20) + 4(15) + 4(10) + 5(5) = 0$

$A_y = 8.25 \text{ kN}$

$\circlearrowleft +\Sigma M_H = 0;$ $\quad -8.25(5) + 2(5) + F_{BC}(3) = 0$

$F_{BC} = 10.4 \text{ kN (C)}$ **Ans.**



$\circlearrowleft +\Sigma M_C = 0;$ $\quad -8.25(10) + 2(10) + 4(5) + \frac{5}{\sqrt{29}} F_{HG}(5) = 0$

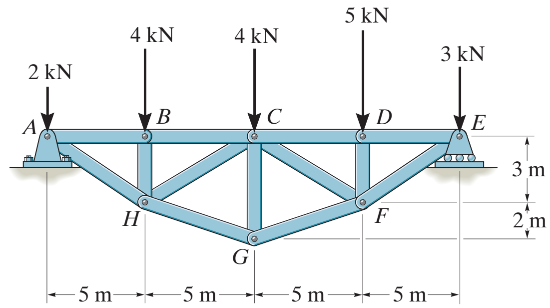
$F_{HG} = 9.1548 = 9.15 \text{ kN (T)}$ **Ans.**

$\circlearrowleft +\Sigma M_{O'} = 0;$ $\quad -2(2.5) + 8.25(2.5) - 4(7.5) + \frac{3}{\sqrt{34}} F_{HC}(12.5) = 0$

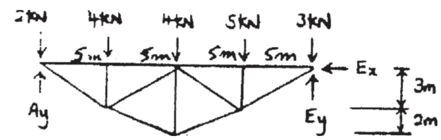
$F_{HC} = 2.24 \text{ kN (T)}$ **Ans.**

***6–36.**

Determine the force in members *CD*, *CF*, and *CG* and state if these members are in tension or compression.



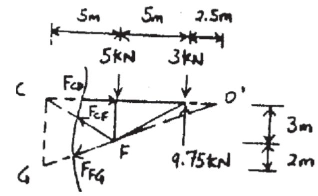
SOLUTION



$\xrightarrow{+} \Sigma F_x = 0;$ $\quad E_x = 0$

$\circlearrowleft + \Sigma M_A = 0;$ $\quad -4(5) - 4(10) - 5(15) - 3(20) + E_y(20) = 0$

$E_y = 9.75 \text{ kN}$



$\circlearrowleft + \Sigma M_C = 0;$ $\quad -5(5) - 3(10) + 9.75(10) - \frac{5}{\sqrt{29}} F_{FG}(5) = 0$

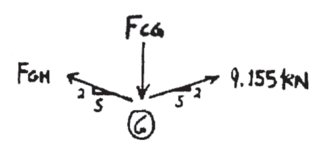
$F_{FG} = 9.155 \text{ kN (T)}$

$\circlearrowleft + \Sigma M_F = 0;$ $\quad -3(5) + 9.75(5) - F_{CD}(3) = 0$

$F_{CD} = 11.25 = 11.2 \text{ kN (C)}$ **Ans.**

$\circlearrowleft + \Sigma M_{O'} = 0;$ $\quad -9.75(2.5) + 5(7.5) + 3(2.5) - \frac{3}{\sqrt{34}} F_{CF}(12.5) = 0$

$F_{CF} = 3.21 \text{ kN (T)}$ **Ans.**



Joint *G*:

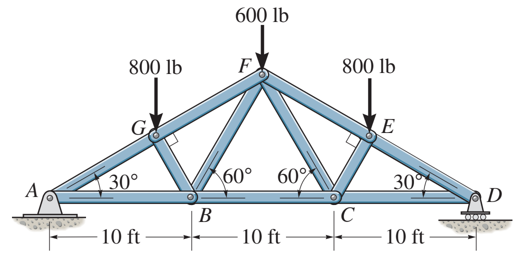
$\xrightarrow{+} \Sigma F_x = 0;$ $\quad F_{GH} = 9.155 \text{ kN (T)}$

$+\uparrow \Sigma F_y = 0;$ $\quad \frac{2}{\sqrt{29}}(9.155)(2) - F_{CG} = 0$

$F_{CG} = 6.80 \text{ kN (C)}$ **Ans.**

6–37.

Determine the force in members *GF*, *FB*, and *BC* of the *Fink truss* and state if the members are in tension or compression.

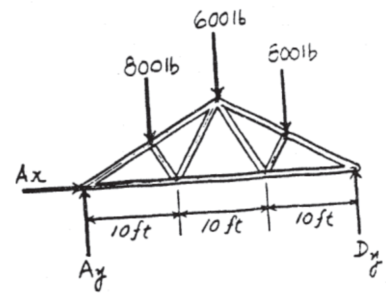


SOLUTION

Support Reactions: Due to symmetry,

$+\uparrow \Sigma F_y = 0;\qquad 2A_y - 800 - 600 - 800 = 0 \qquad A_y = 1100 \text{ lb}$

$\overset{+}{\rightarrow} \Sigma F_x = 0;\qquad A_x = 0$



Method of Sections:

$\curvearrowleft + \Sigma M_B = 0;\qquad F_{GF} \sin 30°(10) + 800(10 - 10\cos^2 30°) - 1100(10) = 0$

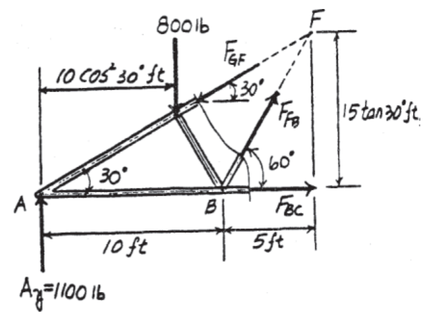
$F_{GF} = 1800 \text{ lb (C)} = 1.80 \text{ kip (C)}$ **Ans.**

$\curvearrowleft + \Sigma M_A = 0;\qquad F_{FB} \sin 60°(10) - 800(10\cos^2 30°) = 0$

$F_{FB} = 692.82 \text{ lb (T)} = 693 \text{ lb (T)}$ **Ans.**

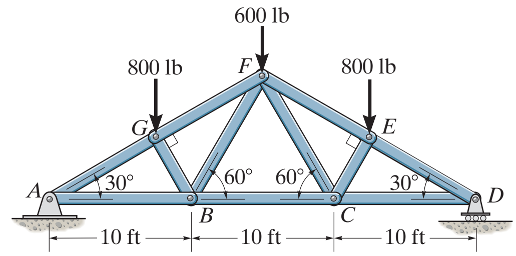
$\curvearrowleft + \Sigma M_F = 0;\qquad F_{BC}(15 \tan 30°) + 800(15 - 10\cos^2 30°) - 1100(15) = 0$

$F_{BC} = 1212.43 \text{ lb (T)} = 1.21 \text{ kip (T)}$ **Ans.**



6–38.

Determine the force in members *FE* and *EC* of the *Fink truss* and state if the members are in tension or compression.



SOLUTION

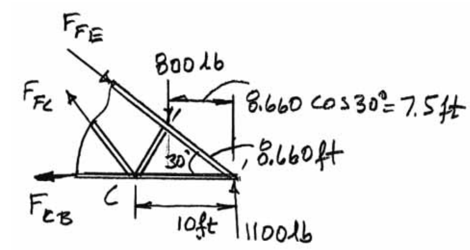
***Support Reactions*:** Due to symmetry,

$+\uparrow \Sigma F_y = 0; \qquad 2B_y - 800 - 600 - 800 = 0; \; B_y = 1100 \text{ lb}$

***Method of Sections*:**

$\circlearrowleft + \Sigma M_C = 0; \qquad 1100(10) - 800(10 - 7.5) - (F_{FE} \sin 30°)(10) = 0$

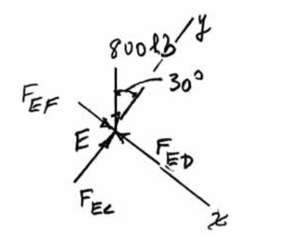
$F_{FE} = 1.80 \text{ kip (C)}$ **Ans.**



Joint *E*:

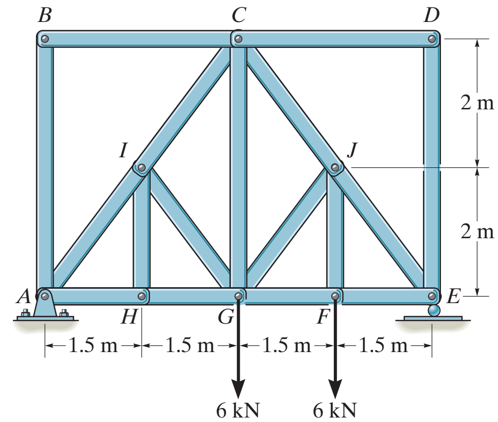
$+\uparrow \Sigma F_y = 0; \qquad F_{EC} - 800 \cos 30° = 0$

$F_{EC} = 693 \text{ lb (C)}$ **Ans.**



6–39.

Determine the force in members *IC* and *CG* of the truss and state if these members are in tension or compression. Also, indicate all zero-force members.



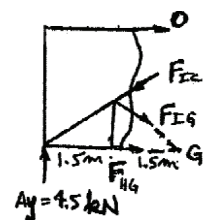
SOLUTION

By inspection of joints B, D, H and I,

$AB, BC, CD, DE, HI,$ and GI are all zero-force members. **Ans**

$\zeta + \Sigma M_G = 0; \qquad -4.5(3) + F_{IC}\left(\frac{3}{5}\right)(4) = 0$

$F_{IC} = 5.625 = 5.62 \text{ kN (C)}$ **Ans.**

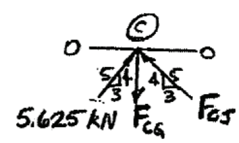


Joint *C*:

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad F_{CJ} = 5.625 \text{ kN}$

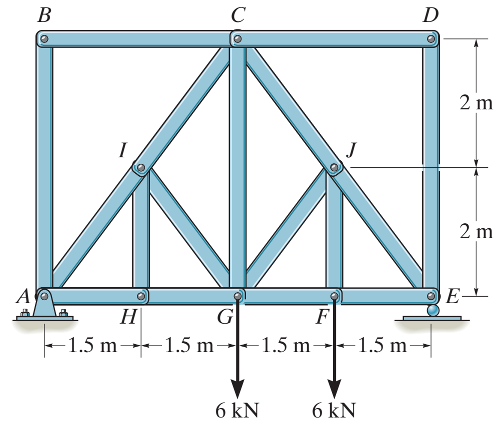
$+\uparrow \Sigma F_y = 0; \qquad \frac{4}{5}(5.625) + \frac{4}{5}(5.625) - F_{CG} = 0$

$F_{CG} = 9.00 \text{ kN (T)}$ **Ans.**



***6–40.**

Determine the force in members JE and GF of the truss and state if these members are in tension or compression. Also, indicate all zero-force members.

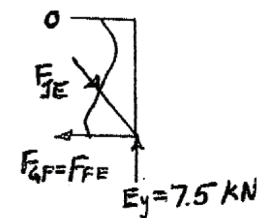


SOLUTION

By inspection of joints B, D, H and I,

AB, BC, CD, DE, HI, and GI are zero-force members. **Ans.**

Joint E:



$+\uparrow \Sigma F_y = 0;\qquad 7.5 - \frac{4}{5}F_{JE} = 0$

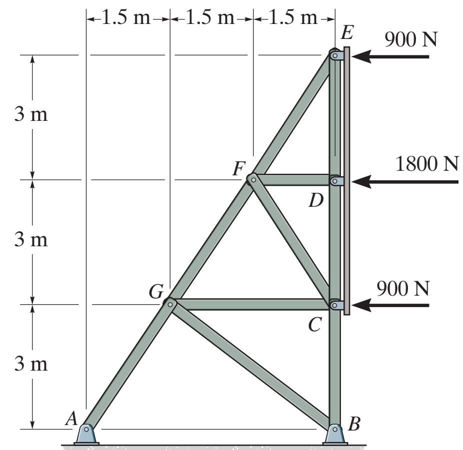
$F_{JE} = 9.375 = 9.38 \text{ kN (C)}$ **Ans.**

$\overset{+}{\rightarrow} \Sigma F_x = 0;\qquad \frac{3}{5}(9.375) - F_{GF} = 0$

$F_{GF} = 5.62 \text{ kN (T)}$ **Ans.**

6–41.

Determine the force in members FG, GC and CB of the truss used to support the sign, and state if the members are in tension or compression.



SOLUTION

Method of Sections: The forces in members FG, GC, and CB are exposed by cutting the truss into two portions through section a–a on the upper portion of the free-body diagram, Fig. a. From this free-body diagram, $\mathbf{F}_{CB}$, $\mathbf{F}_{GC}$, and $\mathbf{F}_{FG}$ can be obtained by writing the moment equations of equilibrium about points G, E, and C, respectively.

$\circlearrowleft + \Sigma M_G = 0;$ $\quad 900(6) + 1800(3) - F_{CB}(3) = 0$

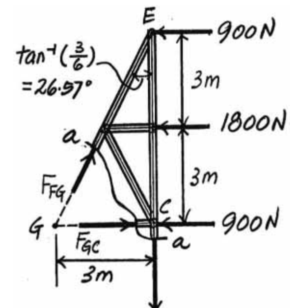
$F_{CB} = 3600 \text{ N} = 3.60 \text{ kN (T)}$ **Ans.**

$\circlearrowleft + \Sigma M_E = 0;$ $\quad F_{GC}(6) - 900(6) - 1800(3) = 0$

$F_{GC} = 1800 \text{ N} = 1.80 \text{ kN (C)}$ **Ans.**

$\circlearrowleft + \Sigma M_C = 0;$ $\quad 900(6) + 1800(3) - F_{FG} \sin 26.57°(6) = 0$

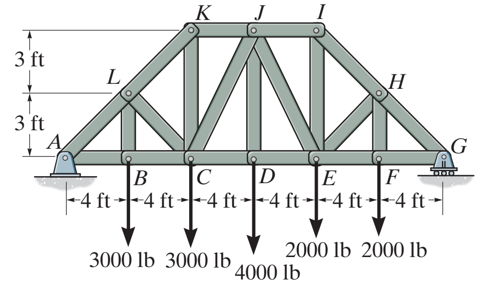
$F_{FG} = 4024.92 \text{ N} = 4.02 \text{ kN (C)}$ **Ans.**



(a)

6–42.

Determine the force in members LK, LC, and BC of the truss, and state if the members are in tension or compression.

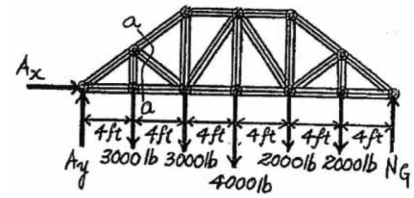


SOLUTION

Support Reactions: Applying the moment equation of equilibrium about point G by referring to the *FBD* of the entire truss shown in Fig. *a*,

$$\circlearrowleft + \Sigma M_G = 0; \qquad 2000(4) + 2000(8) + 4000(12) + 3000(16) = 3000(20) - A_y(24) = 0$$

$$A_y = 7500 \text{ lb}$$



(a)

Method of Section: Consider the *FBD* of the left portion of the truss cut through sec *a–a*, Fig. *b*, we notice that $\mathbf{F}_{LK}$, $\mathbf{F}_{LC}$ and $\mathbf{F}_{BC}$ can be obtained directly by writing moment equation of equilibrium about joint C, A, and L, respectively.

$$\circlearrowleft + \Sigma M_C = 0; \qquad F_{LK}\left(\frac{3}{5}\right)(8) + 3000(4) - 7500(8) = 0$$

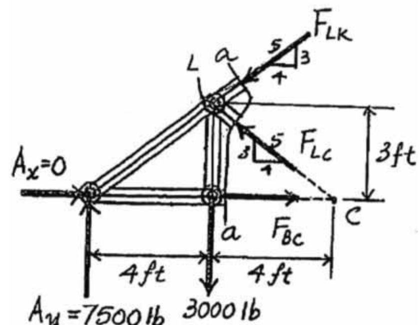
$$F_{LK} = 10\,000 \text{ lb (C)} = 10.0 \text{ kip (C)} \qquad \textbf{Ans.}$$

$$\circlearrowleft + \Sigma M_A = 0; \qquad F_{LC}\left(\frac{3}{5}\right)(8) - 3000(4) = 0$$

$$F_{LC} = 2500 \text{ lb (C)} = 2.50 \text{ kip (C)} \qquad \textbf{Ans.}$$

$$\circlearrowleft + \Sigma M_L = 0; \qquad F_{BC}(3) - 7500(4) = 0$$

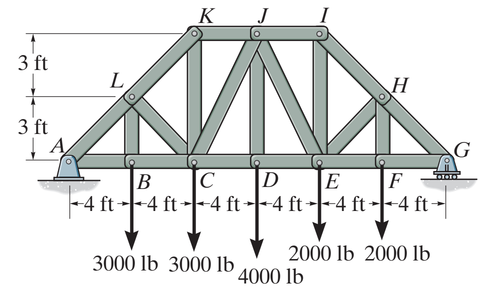
$$F_{BC} = 10\,000 \text{ lb (T)} = 10.0 \text{ kip (T)} \qquad \textbf{Ans.}$$



(b)

6–43.

Determine the force in members JI, JE, and DE of the truss, and state if the members are in tension or compression.



SOLUTION

Support Reactions: Applying the equations of equilibrium about point A to the free-body diagram of the truss, Fig. *a*, we have

$$+\Sigma M_A = 0; \qquad 3000(4) + 3000(8) + 4000(12) + 2000(16) + 2000(20) - N_G(24) = 0$$

$$N_G = 6500 \text{ lb}$$

Method of Sections: The force in members JI, JE, and DE are exposed by cutting the truss into two portions through section *b–b* on the right portion of the free-body diagram, Fig. *a*. From this free-body diagram, F_{JI} and F_{DE} can be obtained by writing the moment equations of equilibrium about points E and J, respectively. F_{JE} can be obtained by writing the force equation of equilibrium along the y axis.

$$+\Sigma M_E = 0; \qquad 6500(8) - 2000(4) - F_{JI}(6) = 0$$

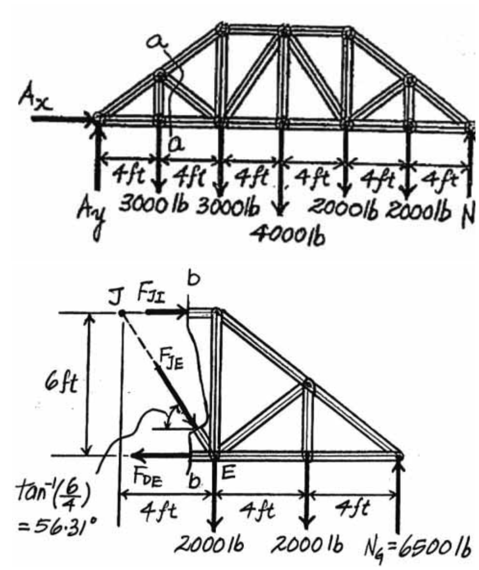
$$F_{JI} = 7333.33 \text{ lb} = 7333 \text{ lb (C)} \qquad \textbf{Ans.}$$

$$+\Sigma M_J = 0; \qquad 6500(12) - 2000(8) - 2000(4) - F_{DE}(6) = 0$$

$$F_{DE} = 9000 \text{ lb (T)} \qquad \textbf{Ans.}$$

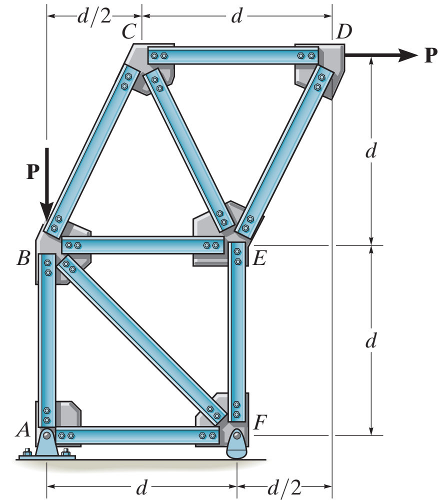
$$+\uparrow \Sigma F_y = 0; \qquad 6500 - 2000 - 2000 - F_{JE}\sin 56.31^\circ = 0$$

$$F_{JE} = 3004.63 \text{ lb} = 3005 \text{ lb (C)} \qquad \textbf{Ans.}$$



***6–44.**

The skewed truss carries the load shown. Determine the force in members *CB*, *BE*, and *EF* and state if these members are in tension or compression. Assume that all joints are pinned.



SOLUTION

$\circlearrowleft + \Sigma M_B = 0; \qquad -P(d) + F_{EF}(d) = 0$

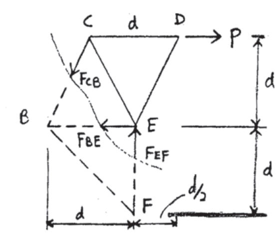
$F_{EF} = P \text{ (C)}$ **Ans.**

$\circlearrowleft + \Sigma M_E = 0; \qquad -P(d) + \left[\dfrac{d}{\sqrt{(d)^2 + \left(\frac{d}{2}\right)^2}}\right] F_{CB}(d) = 0$

$F_{CB} = 1.118\, P \text{ (T)} = 1.12\, P \text{ (T)}$ **Ans.**

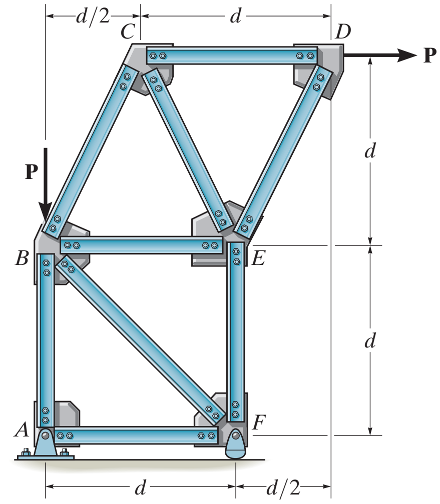
$\xrightarrow{+} \Sigma F_x = 0; \qquad P - \dfrac{0.5}{\sqrt{1.25}}(1.118\, P) - F_{BE} = 0$

$F_{BE} = 0.5P \text{ (T)}$ **Ans.**



6–45.

The skewed truss carries the load shown. Determine the force in members *AB*, *BF*, and *EF* and state if these members are in tension or compression. Assume that all joints are pinned.



SOLUTION

$\zeta + \Sigma M_F = 0; \qquad -P(2d) + P(d) + F_{AB}(d) = 0$

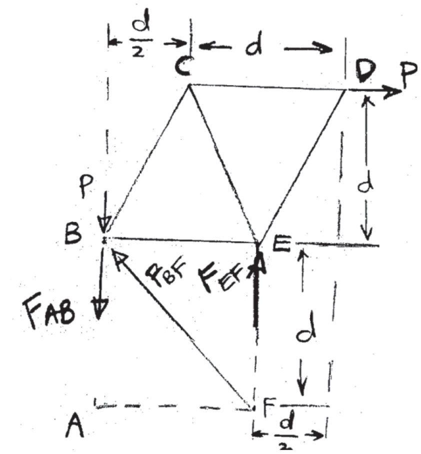
$F_{AB} = P \text{ (T)}$ **Ans.**

$\zeta + \Sigma M_B = 0; \qquad -P(d) + F_{EF}(d) = 0$

$F_{EF} = P \text{ (C)}$ **Ans.**

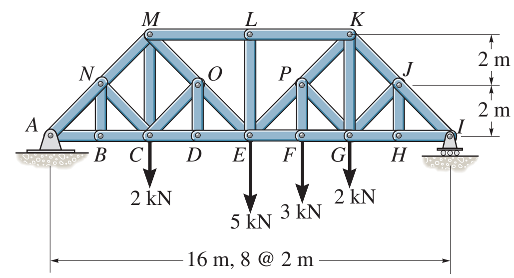
$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad P - F_{BF}\left(\frac{1}{\sqrt{2}}\right) = 0$

$F_{BF} = 1.41P \text{ (C)}$ **Ans.**



6–46.

Determine the force in members CD and CM of the *Baltimore bridge truss* and state if the members are in tension or compression. Also, indicate all zero-force members.



SOLUTION

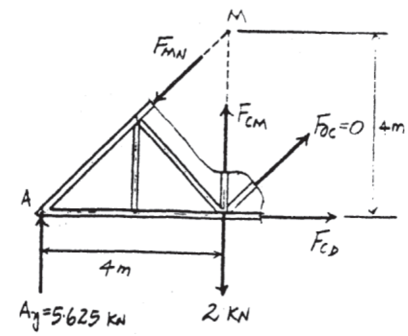
Support Reactions:

$\circlearrowleft + \Sigma M_I = 0;\qquad 2(12) + 5(8) + 3(6) + 2(4) - A_y(16) = 0$

$$A_y = 5.625 \text{ kN}$$

$\xrightarrow{+} \Sigma F_x = 0;\qquad A_x = 0$

Method of Joints: By inspection, members BN, NC, DO, OC, HJ LE and JG are zero force members. **Ans.**



Method of Sections:

$\circlearrowleft + \Sigma M_M = 0;\qquad F_{CD}(4) - 5.625(4) = 0$

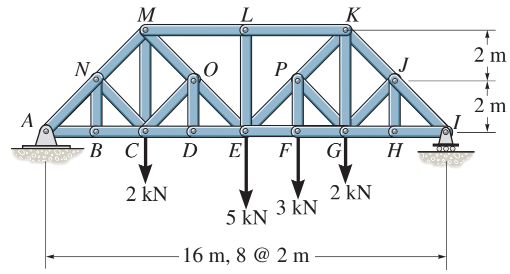
$$F_{CD} = 5.625 \text{ kN (T)}$$ **Ans.**

$\circlearrowleft + \Sigma M_A = 0;\qquad F_{CM}(4) - 2(4) = 0$

$$F_{CM} = 2.00 \text{ kN (T)}$$ **Ans.**

6–47.

Determine the force in members *EF, EP*, and *LK* of the *Baltimore bridge truss* and state if the members are in tension or compression. Also, indicate all zero-force members.



SOLUTION

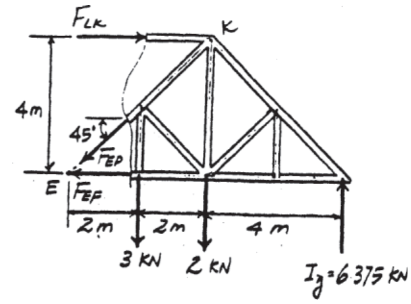
Support Reactions:

$\circlearrowleft + \Sigma M_A = 0;$ $\quad I_y(16) - 2(12) - 3(10) - 5(8) - 2(4) = 0$

$$I_y = 6.375 \text{ kN}$$

Method of Joints: By inspection, members *BN, NC, DO, OC, HJ LE* and *JG* are zero force members. **Ans.**

Method of Sections:



$\circlearrowleft + \Sigma M_K = 0;$ $\quad 3(2) + 6.375(4) - F_{EF}(4) = 0$

$$F_{EF} = 7.875 = 7.88 \text{ kN (T)}$$ **Ans.**

$\circlearrowleft + \Sigma M_E = 0;$ $\quad 6.375(8) - 2(4) - 3(2) - F_{LK}(4) = 0$

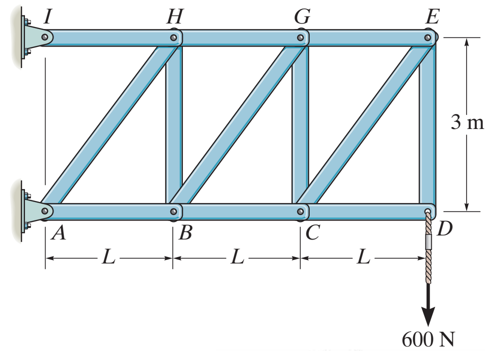
$$F_{LK} = 9.25 \text{ kN (C)}$$ **Ans.**

$+\uparrow \Sigma F_y = 0;$ $\quad 6.375 - 3 - 2 - F_{ED} \sin 45° = 0$

$$F_{ED} = 1.94 \text{ kN (T)}$$ **Ans.**

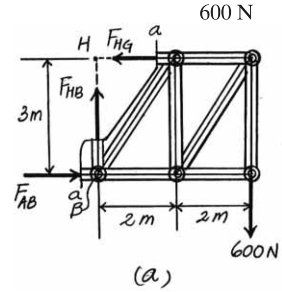
***6–48.**

The truss supports the vertical load of 600 N. If $L = 2$ m, determine the force on members HG and HB of the truss and state if the members are in tension or compression.



SOLUTION

Method of Section: Consider the FBD of the right portion of the truss cut through sec. a–a, Fig. a, we notice that $\mathbf{F}_{HB}$ and $\mathbf{F}_{HG}$ can be obtained directly by writing the force equation of equilibrium along vertical and moment equation of equilibrium about joint B, respectively.

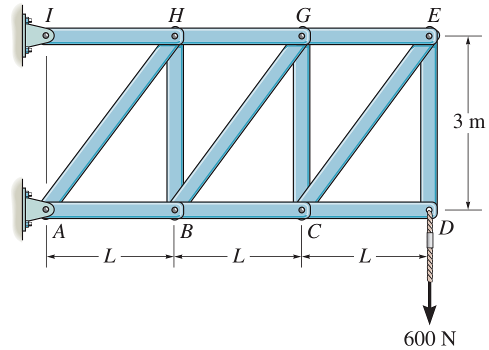


$+\uparrow \Sigma F_y = 0;\qquad F_{HB} - 600 = 0\qquad F_{HB} = 600 \text{ N (T)}$ **Ans.**

$\circlearrowleft + \Sigma M_B = 0;\qquad F_{HG}(3) - 600(4) = 0\qquad F_{HG} = 800 \text{ N (T)}$ **Ans.**

■ 6–49.

The truss supports the vertical load of 600 N. Determine the force in members *BC*, *BG*, and *HG* as the dimension *L* varies. Plot the results of *F* (ordinate with tension as positive) versus *L* (abscissa) for $0 \leq L \leq 3$ m.



SOLUTION

$+\uparrow \Sigma F_y = 0; \qquad -600 - F_{BG} \sin\theta = 0$

$$F_{BG} = -\frac{600}{\sin\theta}$$

$$\sin\theta = \frac{3}{\sqrt{L^2 + 9}}$$

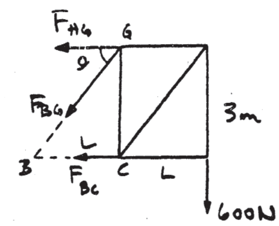
$$F_{BG} = -200\sqrt{L^2 + 9}$$

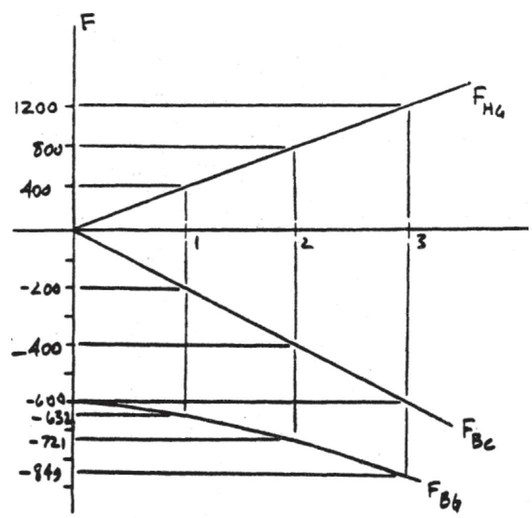
$\zeta + \Sigma M_G = 0; \qquad -F_{BC}(3) - 600(L) = 0$

$$F_{BC} = -200L$$

$\zeta + \Sigma M_B = 0; \qquad F_{HG}(3) - 600(2L) = 0$

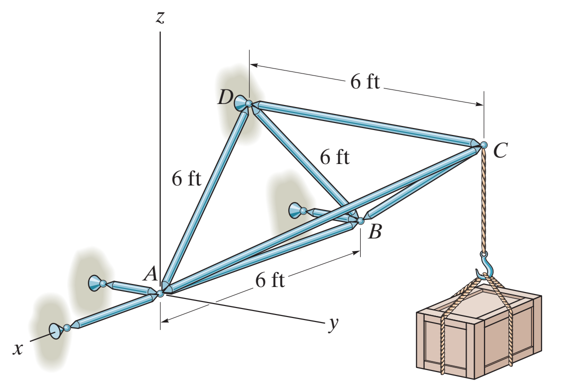
$$F_{HG} = 400L$$





6–50.

Determine the force developed in each member of the space truss and state if the members are in tension or compression. The crate has a weight of 150 lb.



SOLUTION

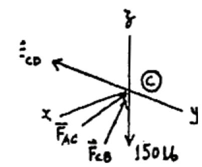
$$\mathbf{F}_{CA} = F_{CA}\left[\frac{-1\mathbf{i} + 2\mathbf{j} + 2\sin 60°\mathbf{k}}{\sqrt{8}}\right]$$

$$= -0.354\, F_{CA}\,\mathbf{i} + 0.707\, F_{CA}\mathbf{j} + 0.612\, F_{CA}\,\mathbf{k}$$

$$\mathbf{F}_{CB} = 0.354F_{CB}\,\mathbf{i} + 0.707F_{CB}\,\mathbf{j} + 0.612F_{CB}\,\mathbf{k}$$

$$\mathbf{F}_{CD} = -F_{CD}\mathbf{j}$$

$$\mathbf{W} = -150\,\mathbf{k}$$



$\Sigma F_x = 0; \qquad -0.354F_{CA} + 0.354F_{CB} = 0$

$\Sigma F_y = 0; \qquad 0.707F_{CA} + 0.707F_{CB} - F_{CD} = 0$

$\Sigma F_z = 0; \qquad 0.612F_{CA} + 0.612F_{CB} - 150 = 0$

Solving:

$F_{CA} = F_{CB} = 122.5 \text{ lb} = 122 \text{ lb (C)}$ **Ans.**

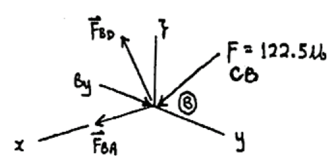
$F_{CD} = 173 \text{ lb (T)}$ **Ans.**

$\mathbf{F}_{BA} = F_{BA}\,\mathbf{i}$

$\mathbf{F}_{BD} = F_{BD}\cos 60°\mathbf{i} + F_{BD}\sin 60°\,\mathbf{k}$

$\mathbf{F}_{CB} = 122.5\,(-0.354\mathbf{i} - 0.707\,\mathbf{j} - 0.612\mathbf{k})$

$= -43.3\mathbf{i} - 86.6\mathbf{j} - 75.0\mathbf{k}$



$\Sigma F_x = 0; \qquad F_{BA} + F_{BD}\cos 60° - 43.3 = 0$

$\Sigma F_z = 0; \qquad F_{BD}\sin 60° - 75 = 0$

Solving:

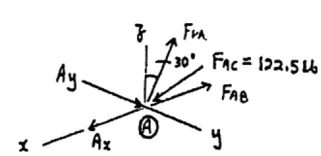
$F_{BD} = 86.6 \text{ lb (T)}$ **Ans.**

$F_{BA} = 0$ **Ans.**

$F_{AC} = 122.5(0.354\,\mathbf{i} - 0.707\mathbf{j} - 0.612\,\mathbf{k})$

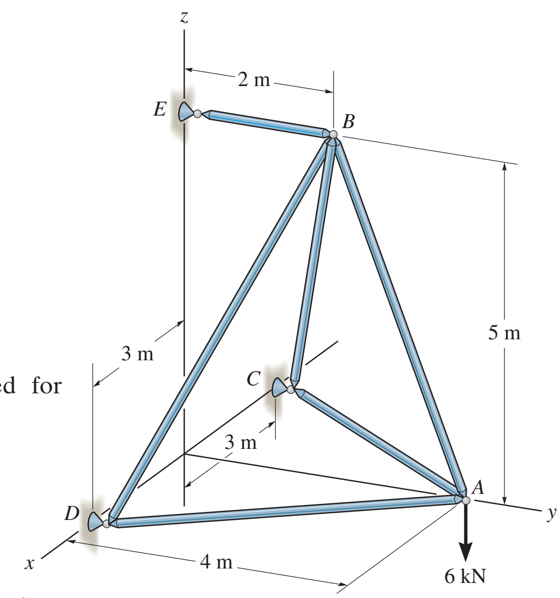
$\Sigma F_z = 0; \qquad F_{DA}\cos 30° - 0.612(122.5) = 0$

$F_{DA} = 86.6 \text{ lb (T)}$ **Ans.**



6–51.

Determine the force in each member of the space truss and state if the members are in tension or compression. *Hint:* The support reaction at E acts along member EB. Why?



SOLUTION

Method of Joints: In this case, the support reactions are not required for determining the member forces.

Joint A:

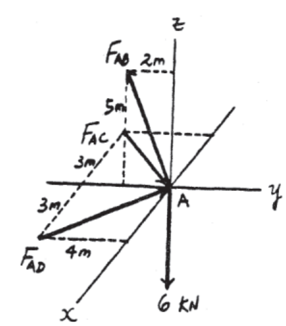
$$\Sigma F_z = 0; \qquad F_{AB}\left(\frac{5}{\sqrt{29}}\right) - 6 = 0$$

$$F_{AB} = 6.462 \text{ kN (T)} = 6.46 \text{ kN (T)} \qquad \textbf{Ans.}$$

$$\Sigma F_x = 0; \qquad F_{AC}\left(\frac{3}{5}\right) - F_{AD}\left(\frac{3}{5}\right) = 0 \qquad F_{AC} = F_{AD} \qquad \textbf{(1)}$$

$$\Sigma F_y = 0; \qquad F_{AC}\left(\frac{4}{5}\right) + F_{AD}\left(\frac{4}{5}\right) - 6.462\left(\frac{2}{\sqrt{29}}\right) = 0$$

$$F_{AC} + F_{AD} = 3.00 \qquad \textbf{(2)}$$



Solving Eqs. (1) and (2) yields

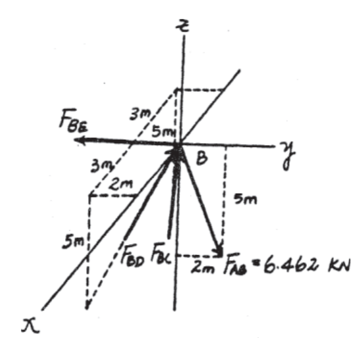
$$F_{AC} = F_{AD} = 1.50 \text{ kN (C)} \qquad \textbf{Ans.}$$

Joint B:

$$\Sigma F_x = 0; \qquad F_{BC}\left(\frac{3}{\sqrt{38}}\right) - F_{BD}\left(\frac{3}{\sqrt{38}}\right) = 0 \qquad F_{BC} = F_{BD} \qquad \textbf{(1)}$$

$$\Sigma F_z = 0; \qquad F_{BC}\left(\frac{5}{\sqrt{38}}\right) + F_{BD}\left(\frac{5}{\sqrt{38}}\right) - 6.462\left(\frac{5}{\sqrt{29}}\right) = 0$$

$$F_{BC} + F_{BD} = 7.397 \qquad \textbf{(2)}$$



Solving Eqs. (1) and (2) yields

$$F_{BC} = F_{BD} = 3.699 \text{ kN (C)} = 3.70 \text{ kN (C)} \qquad \textbf{Ans.}$$

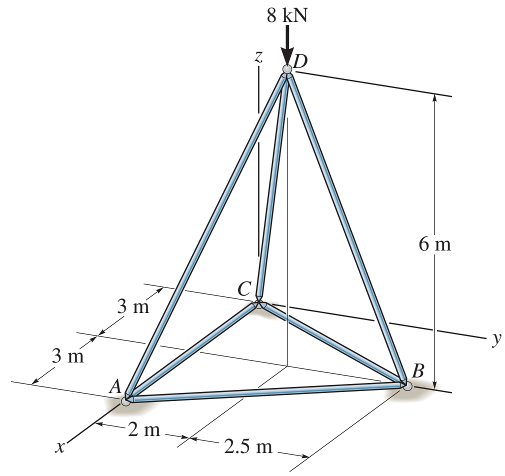
$$\Sigma F_y = 0; \qquad 2\left[3.699\left(\frac{2}{\sqrt{38}}\right)\right] + 6.462\left(\frac{2}{\sqrt{29}}\right) - F_{BE} = 0$$

$$F_{BE} = 4.80 \text{ kN (T)} \qquad \textbf{Ans.}$$

Note: The support reactions at supports C and D can be determined by analyzing joints C and D, respectively using the results obtained above.

***6–52.**

Determine the force in each member of the space truss and state if the members are in tension or compression. The truss is supported by rollers at *A*, *B*, and *C*.



SOLUTION

$\Sigma F_x = 0;\qquad \frac{3}{7}F_{DC} - \frac{3}{7}F_{DA} = 0$

$F_{DC} = F_{DA}$

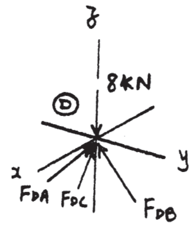
$\Sigma F_y = 0;\qquad \frac{2}{7}F_{DC} + \frac{2}{7}F_{DA} - \frac{2.5}{6.5}F_{DB} = 0$

$F_{DB} = 1.486\ \mathrm{F}_{DC}$

$\Sigma F_z = 0;\qquad -8 + 2\left(\frac{6}{7}\right)F_{DC} + \frac{6}{6.5}F_{DB} = 0$

$F_{DC} = F_{DA} = 2.59\ \text{kN (C)}$ **Ans.**

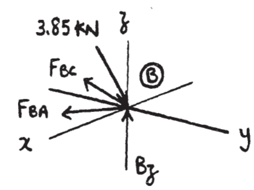
$F_{DB} = 3.85\ \text{kN (C)}$ **Ans.**



$\Sigma F_x = 0;\qquad F_{BC} = F_{BA}$

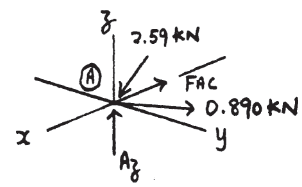
$\Sigma F_y = 0;\qquad 3.85\left(\frac{2.5}{6.5}\right) - 2\left(\frac{4.5}{\sqrt{29.25}}\right)F_{BC} = 0$

$F_{BC} = F_{BA} = 0.890\ \text{kN (T)}$ **Ans.**



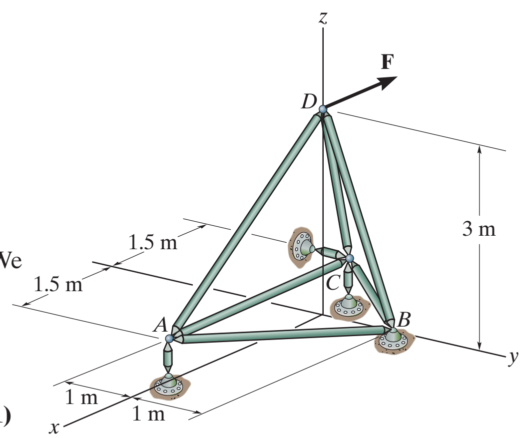
$\Sigma F_x = 0;\qquad 2.59\left(\frac{3}{7}\right) - 0.890\left(\frac{3}{\sqrt{29.25}}\right) - F_{AC} = 0$

$F_{AC} = 0.617\ \text{kN (T)}$ **Ans.**



6–53.

The space truss supports a force $\mathbf{F} = [300\mathbf{i} + 400\mathbf{j} - 500\mathbf{k}]$ N. Determine the force in each member, and state if the members are in tension or compression.



SOLUTION

Method of Joints: In this case, there is no need to compute the support reactions. We will begin by analyzing the equilibrium of joint D, and then that of joints A and C.

Joint D: From the free-body diagram, Fig. a, we can write

$$\Sigma F_x = 0; \qquad F_{DA}\left(\frac{1.5}{3.5}\right) - F_{DC}\left(\frac{1.5}{3.5}\right) + 300 = 0 \qquad \textbf{(1)}$$

$$\Sigma F_y = 0; \qquad F_{DB}\left(\frac{1}{\sqrt{10}}\right) - F_{DA}\left(\frac{1}{3.5}\right) - F_{DC}\left(\frac{1}{3.5}\right) + 400 = 0 \qquad \textbf{(2)}$$

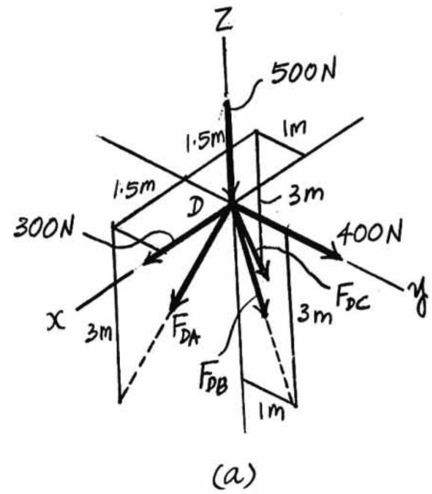
$$\Sigma F_z = 0; \qquad -F_{DA}\left(\frac{3}{3.5}\right) - F_{DC}\left(\frac{3}{3.5}\right) - F_{DB}\left(\frac{3}{\sqrt{10}}\right) - 500 = 0 \qquad \textbf{(3)}$$

Solving Eqs. (1) through (3) yields

$$F_{DB} = -895.98 \text{ N} = 896 \text{ N (C)} \qquad \textbf{Ans.}$$

$$F_{DC} = 554.17 \text{ N} = 554 \text{ N (T)} \qquad \textbf{Ans.}$$

$$F_{DA} = -145.83 \text{ N} = 146 \text{ N (C)} \qquad \textbf{Ans.}$$



(a)

Joint A: From the free-body diagram, Fig. b,

$$\Sigma F_y = 0; \qquad F_{AB}\left(\frac{2}{2.5}\right) - 145.83\left(\frac{1}{3.5}\right) = 0$$

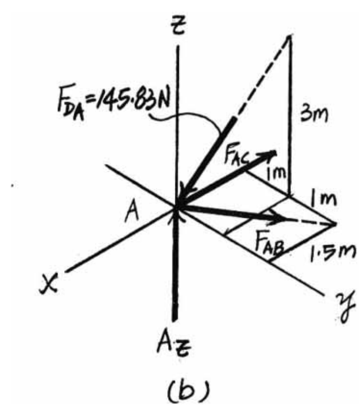
$$F_{AB} = 52.08 \text{ N} = 52.1 \text{ N (T)} \qquad \textbf{Ans.}$$

$$\Sigma F_x = 0; \qquad 145.83\left(\frac{1.5}{3.5}\right) - 52.08\left(\frac{1.5}{2.5}\right) - F_{AC} = 0$$

$$F_{AC} = 31.25 \text{ N (T)} \qquad \textbf{Ans.}$$

$$\Sigma F_z = 0; \qquad A_z - 145.83\left(\frac{3}{3.5}\right) = 0$$

$$A_z = 125 \text{ N} \qquad \textbf{Ans.}$$



(b)

Joint C: From the free-body diagram, Fig. c,

$$\Sigma F_x = 0; \qquad 31.25 + 554.17\left(\frac{1.5}{3.5}\right) - F_{CB}\left(\frac{1.5}{2.5}\right) = 0$$

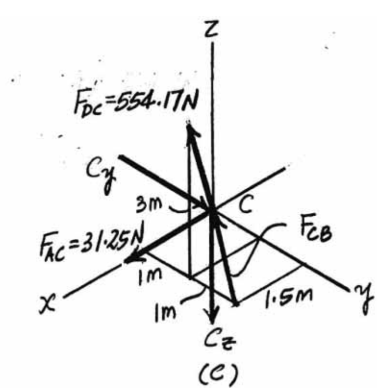
$$F_{CB} = 447.92 \text{ N} = 448 \text{ N (C)} \qquad \textbf{Ans.}$$

$$\Sigma F_y = 0; \qquad 554.17\left(\frac{1}{3.5}\right) - 447.92\left(\frac{2}{2.5}\right) + C_y = 0$$

$$C_y = 200 \text{ N}$$

$$\Sigma F_z = 0; \qquad 554.17\left(\frac{3}{3.5}\right) - C_z = 0$$

$$C_z = 475 \text{ N}$$

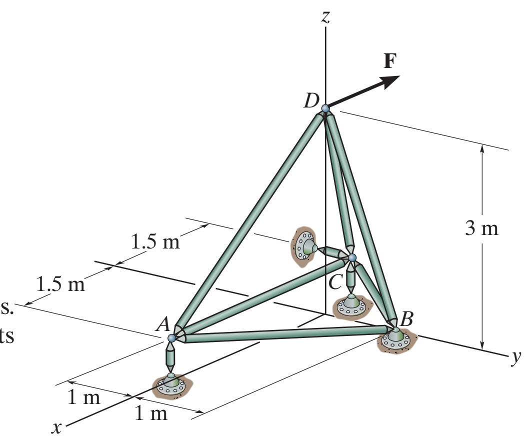


(c)

Note: The equilibrium analysis of joint B can be used to determine the components of support reaction of the ball and socket support at B.

6–54.

The space truss supports a force F = [−400i + 500j + 600k] N. Determine the force in each member, and state if the members are in tension or compression.



SOLUTION

Method of Joints: In this case, there is no need to compute the support reactions. We will begin by analyzing the equilibrium of joint D, and then that of joints A and C.

Joint D: From the free-body diagram, Fig. a, we can write

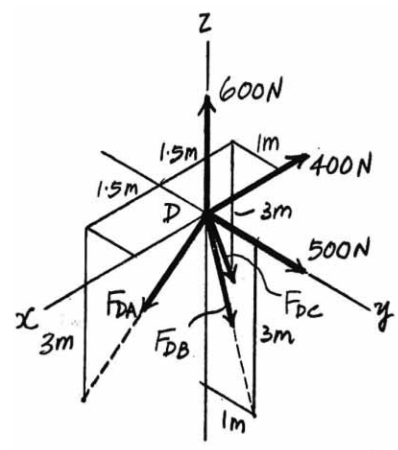
$$\Sigma F_x = 0; \qquad F_{DA}\left(\frac{1.5}{3.5}\right) - F_{DC}\left(\frac{1.5}{3.5}\right) - 400 = 0 \qquad \textbf{(1)}$$

$$\Sigma F_y = 0; \qquad F_{DB} - \left(\frac{1}{\sqrt{10}}\right) - F_{DA}\left(\frac{1}{3.5}\right) - F_{DC}\left(\frac{1}{3.5}\right) + 500 = 0 \qquad \textbf{(2)}$$

$$\Sigma F_z = 0; \qquad 600 - F_{DA}\left(\frac{3}{3.5}\right) - F_{DC}\left(\frac{3}{3.5}\right) - F_{DB}\left(\frac{3}{\sqrt{10}}\right) = 0 \qquad \textbf{(3)}$$

Solving Eqs. (1) through (3) yields

$$F_{DB} = -474.34 \text{ N} = 474 \text{ N (C)} \qquad \textbf{Ans.}$$
$$F_{DC} = 145.83 \text{ N} = 146 \text{ N (T)} \qquad \textbf{Ans.}$$
$$F_{DA} = 1079.17 \text{ N} = 1.08 \text{ kN (T)} \qquad \textbf{Ans.}$$



(a)

Joint A: From the free-body diagram, Fig. b,

$$\Sigma F_y = 0; \qquad 1079.17\left(\frac{1}{3.5}\right) - F_{AB}\left(\frac{2}{2.5}\right) = 0$$

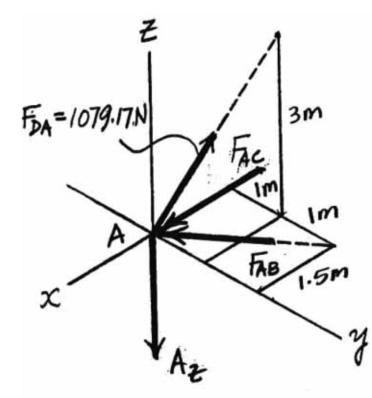
$$F_{AB} = 385.42 \text{ N} = 385 \text{ N (C)} \qquad \textbf{Ans.}$$

$$\Sigma F_x = 0; \qquad 385.42\left(\frac{1.5}{2.5}\right) - 1079.17\left(\frac{1.5}{3.5}\right) + F_{AC} = 0$$

$$F_{AC} = 231.25 \text{ N} = 231 \text{ N (C)} \qquad \textbf{Ans.}$$

$$\Sigma F_z = 0; \qquad 1079.17\left(\frac{1}{3.5}\right) - A_z = 0$$

$$A_z = 925 \text{ N} \qquad \textbf{Ans.}$$



(b)

Joint C: From the free-body diagram, Fig. c,

$$\Sigma F_x = 0; \qquad F_{CB}\left(\frac{1.5}{2.5}\right) - 231.25 + 145.83\left(\frac{1.5}{3.5}\right) = 0$$

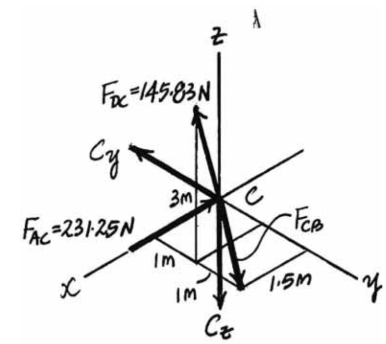
$$F_{CB} = 281.25 \text{ N} = 281 \text{ N (T)} \qquad \textbf{Ans.}$$

$$\Sigma F_y = 0; \qquad 281.25\left(\frac{1}{2.5}\right) + 145.83\left(\frac{2}{3.5}\right) - C_y = 0$$

$$C_y = 266.67 \text{ N}$$

$$\Sigma F_z = 0; \qquad 145.83\left(\frac{3}{3.5}\right) - C_z = 0$$

$$C_z = 125 \text{ N}$$

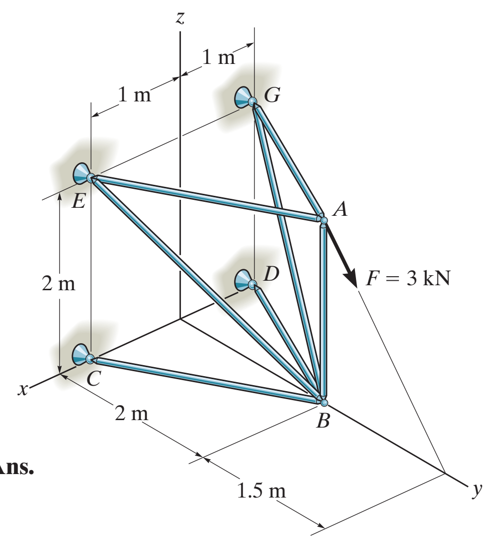


(c)

Note: The equilibrium analysis of joint B can be used to determine the components of support reaction of the ball and socket support at B.

6–55.

Determine the force in each member of the space truss and state if the members are in tension or compression. The truss is supported by ball-and-socket joints at *C*, *D*, *E*, and *G*.

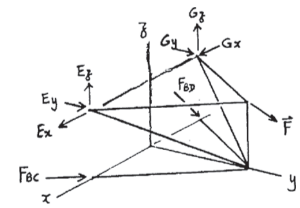


SOLUTION

$\Sigma(M_{EG})_x = 0;$ $\quad \frac{2}{\sqrt{5}}F_{BC}(2) + \frac{2}{\sqrt{5}}F_{BD}(2) - \frac{4}{5}(3)(2) = 0$

$$F_{Bc} + F_{BD} = 2.683 \text{ kN}$$

Due to symmetry: $\quad F_{BC} = F_{BD} = 1.342 = 1.34 \text{ kN (C)}$ **Ans.**



Joint *A*:

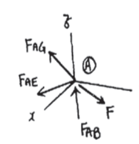
$\Sigma F_z = 0;$ $\quad F_{AB} - \frac{4}{5}(3) = 0$

$F_{AB} = 2.4 \text{ kN (C)}$ **Ans.**

$\Sigma F_x = 0;$ $\quad F_{AG} = F_{AE}$

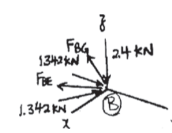
$\Sigma F_y = 0;$ $\quad \frac{3}{5}(3) - \frac{2}{\sqrt{5}}F_{AE} - \frac{2}{\sqrt{5}}F_{AG} = 0$

$F_{AG} = F_{AE} = 1.01 \text{ kN (T)}$ **Ans.**



Joint *B*:

$\Sigma F_x = 0;$ $\quad \frac{1}{\sqrt{5}}(1.342) + \frac{1}{3}F_{BE} - \frac{1}{\sqrt{5}}(1.342) - \frac{1}{3}F_{BG} = 0$



$\Sigma F_y = 0;$ $\quad \frac{2}{\sqrt{5}}(1.342) - \frac{2}{3}F_{BE} + \frac{2}{\sqrt{5}}(1.342) - \frac{2}{3}F_{BG} = 0$

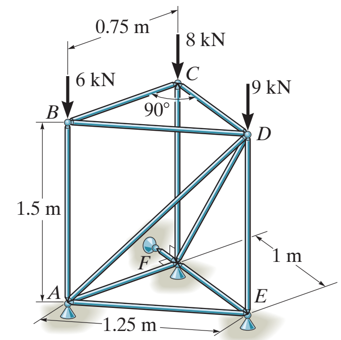
$\Sigma F_z = 0;$ $\quad \frac{2}{3}F_{BE} + \frac{2}{3}F_{BG} - 2.4 = 0$

$F_{BG} = 1.80 \text{ kN (T)}$ **Ans.**

$F_{BE} = 1.80 \text{ kN (T)}$ **Ans.**

***6–56.**

The space truss is used to support vertical forces at joints *B*, *C*, and *D*. Determine the force in each member and state if the members are in tension or compression. There is a roller at *E*, and *A* and *F* are ball-and-socket joints.



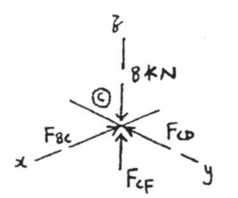
SOLUTION

Joint *C*:

$\Sigma F_x = 0;\quad F_{BC} = 0$ **Ans.**

$\Sigma F_y = 0;\quad F_{CD} = 0$ **Ans.**

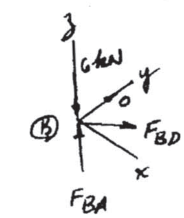
$\Sigma F_z = 0;\quad F_{CF} = 8\text{ kN (C)}$ **Ans.**



Joint *B*:

$\Sigma F_y = 0;\quad F_{BD} = 0$ **Ans.**

$\Sigma F_z = 0;\quad F_{BA} = 6\text{ kN (C)}$ **Ans.**

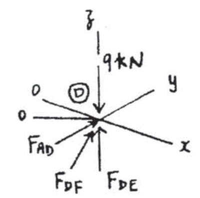


Joint *D*:

$\Sigma F_y = 0;\quad F_{AD} = 0$ **Ans.**

$\Sigma F_x = 0;\quad F_{DF} = 0$ **Ans.**

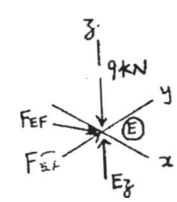
$\Sigma F_z = 0;\quad F_{DE} = 9\text{ kN (C)}$ **Ans.**



Joint *E*:

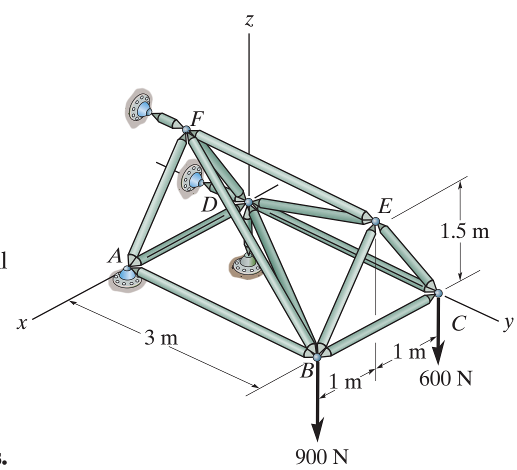
$\Sigma F_x = 0;\quad F_{EF} = 0$ **Ans.**

$\Sigma F_y = 0;\quad F_{EA} = 0$ **Ans.**



6–57.

Determine the force in members BE, BC, BF, and CE of the space truss, and state if the members are in tension or compression.



SOLUTION

Method of Joints: In this case, there is no need to compute the support reactions. We will begin by analyzing the equilibrium of joint C, and then that of joints E and B.

Joint C: From the free-body diagram, Fig. a, we can write

$$\Sigma F_z = 0; \qquad F_{CE}\left(\frac{1.5}{\sqrt{3.25}}\right) - 600 = 0$$

$$F_{CE} = 721.11 \text{ N} = 721 \text{ N (T)} \qquad \textbf{Ans.}$$

$$\Sigma F_x = 0; \qquad 721.11\left(\frac{1}{\sqrt{3.25}}\right) - F_{BC} = 0$$

$$F_{BC} = 400 \text{ N (C)} \qquad \textbf{Ans.}$$

Joint E: From the free-body diagram, Fig, b, notice that $\mathbf{F}_{EF}$, $\mathbf{F}_{ED}$, and $\mathbf{F}_{EC}$ lie in the same plane (shown shaded), and $\mathbf{F}_{BE}$ is the only force that acts outside of this plane. If the x' axis is perpendicular to this plane and the force equation of equilibrium is written along this axis, we have

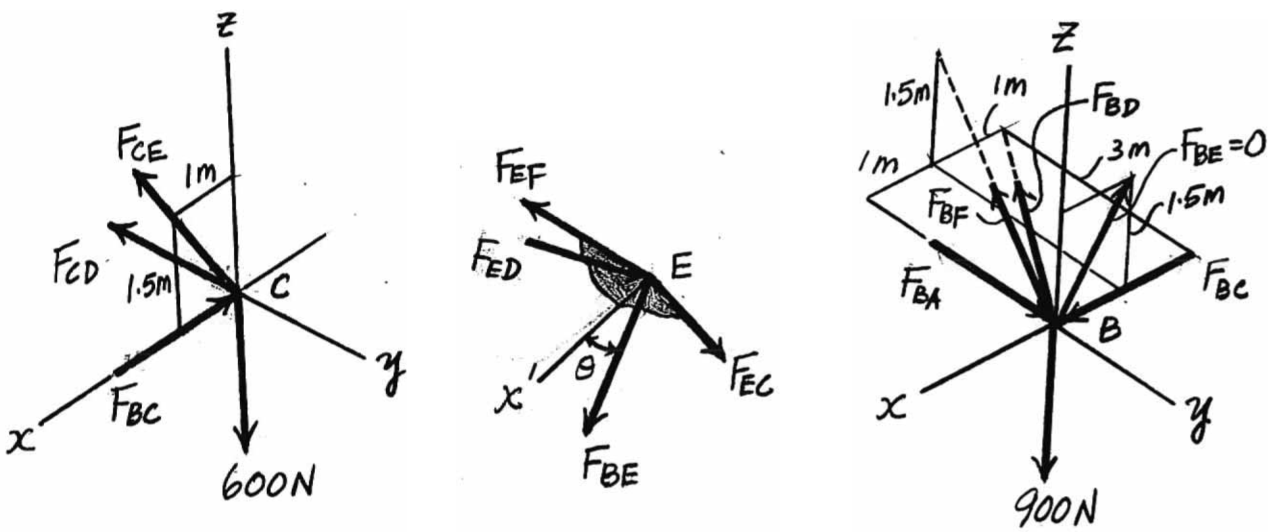
$$\Sigma F_{x'} = 0; \qquad F_{BE} = \cos\theta = 0$$

$$F_{BE} = 0 \qquad \textbf{Ans.}$$

Joint B: From the free-body diagram, Fig. c,

$$\Sigma F_z = 0; \qquad F_{BF}\left(\frac{1.5}{3.5}\right) - 900 = 0$$

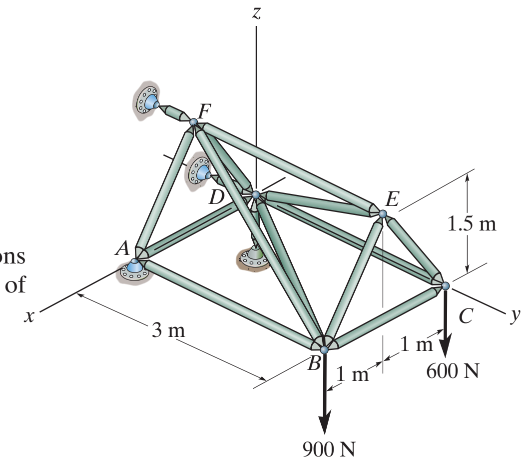
$$F_{BF} = 2100 \text{ N} = 2.10 \text{ kN (T)} \qquad \textbf{Ans.}$$



(a) (b) (c)

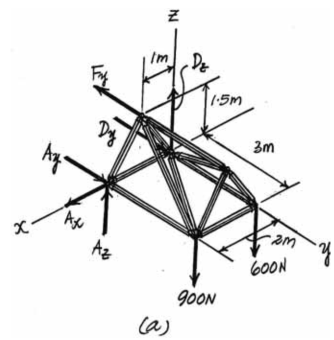
6–58.

Determine the force in members *AF, AB, AD, ED, FD*, and *BD* of the space truss, and state if the members are in tension or compression.



SOLUTION

Support Reactions: In this case, it will be easier to compute the support reactions first. From the free-body diagram of the truss, Fig. *a*, and writing the equations of equilibrium, we have



$\Sigma M_x = 0;\quad F_y(1.5) - 900(3) - 600(3) = 0 \qquad F_y = 3000\text{ N}$

$\Sigma M_y = 0;\quad 900(2) - A_z(2) = 0 \qquad A_z = 900\text{ N}$

$\Sigma M_z = 0;\quad A_y(2) - 3000(1) = 0 \qquad A_y = 1500\text{ N}$

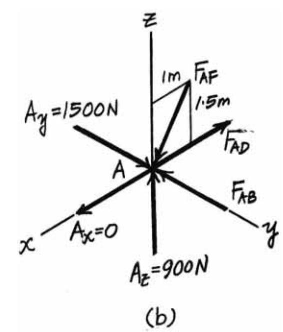
$\Sigma F_x = 0;\quad A_x = 0$

$\Sigma F_y = 0;\quad D_y + 1500 - 3000 = 0 \qquad D_y = 1500\text{ N}$

$\Sigma F_z = 0;\quad D_z + 900 - 900 - 600 = 0 \qquad D_z = 600\text{ N}$

Method of Joints: Using the above results, we will begin by analyzing the equilibrium of joint *A*, and then that of joints *C* and *D*.

Joint *A*: From the free-body diagram, Fig. *b*, we can write



$\Sigma F_y = 0;\qquad 1500 - F_{AB} = 0$

$F_{AB} = 1500\text{ N} = 1.50\text{ kN (C)}$ **Ans.**

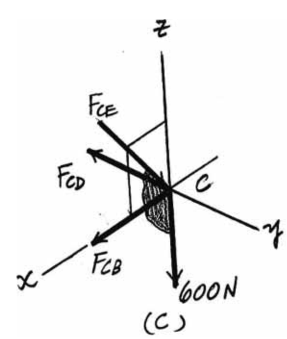
$\Sigma F_z = 0;\qquad 900 - F_{AF}\left(\dfrac{1.5}{\sqrt{3.25}}\right) = 0$

$F_{AF} = 1081.67\text{ N} = 1.08\text{ kN (C)}$ **Ans.**

$\Sigma F_x = 0;\qquad 1081.67\left(\dfrac{1}{\sqrt{3.25}}\right) - F_{AD} = 0$

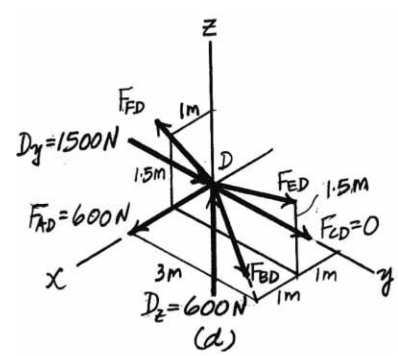
$F_{AD} = 600\text{ N (T)}$ **Ans.**

Joint *C*: From the free-body diagram of the joint in Fig. *c*, notice that $\mathbf{F}_{CE}$, $\mathbf{F}_{CB}$, and the 600-N force lie in the *x*–*z* plane (shown shaded). Thus, if we write the force equation of equilibrium along the *y* axis, we have



$\Sigma F_y = 0;\qquad F_{DC} = 0$

Joint *D*: From the free-body diagram, Fig. *d*,



$$\Sigma F_x = 0;\qquad F_{BC} - \left(\frac{2}{\sqrt{13}}\right) + F_{FD}\left(\frac{1}{3.5}\right) + F_{FD}\left(\frac{1}{\sqrt{3.25}}\right) + 600 = 0 \qquad \textbf{(1)}$$

$$\Sigma F_y = 0;\qquad F_{BD}\left(\frac{3}{\sqrt{13}}\right) + F_{ED}\left(\frac{3}{3.5}\right) + 1500 = 0 \qquad \textbf{(2)}$$

$$\Sigma F_z = 0;\qquad F_{FD}\left(\frac{1.5}{\sqrt{13}}\right) + F_{ED}\left(\frac{1.5}{3.5}\right) + 600 = 0 \qquad \textbf{(3)}$$

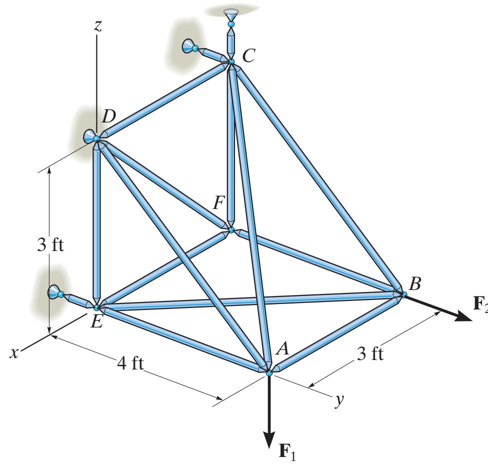
Solving Eqs. (1) through (3) yields

$F_{FD} = 0 \quad F_{ED} = -1400\text{N} = 1.40\text{ kN (C)}$ **Ans.**

$F_{BD} = -360.56\text{ N} = 361\text{ N (C)}$ **Ans.**

6–59.

The space truss is supported by a ball-and-socket joint at D and short links at C and E. Determine the force in each member and state if the members are in tension or compression. Take $\mathbf{F}_1 = \{-500\mathbf{k}\}$ lb and $\mathbf{F}_2 = \{400\mathbf{j}\}$ lb.



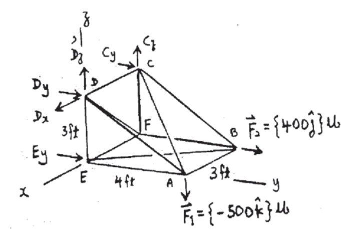
SOLUTION

$\Sigma M_z = 0; \quad -C_y(3) - 400(3) = 0$

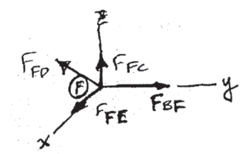
$C_y = -400$ lb

$\Sigma F_x = 0; \quad D_x = 0$

$\Sigma M_y = 0; \quad C_z = 0$



Joint F: $\Sigma F_y = 0; \quad F_{BF} = 0$ **Ans.**



Joint B:

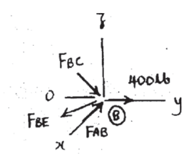
$\Sigma F_z = 0; \quad F_{BC} = 0$ **Ans.**

$\Sigma F_y = 0; \quad 400 - \frac{4}{5}F_{BE} = 0$

$F_{BE} = 500$ lb (T) **Ans.**

$\Sigma F_x = 0; \quad F_{AB} - \frac{3}{5}(500) = 0$

$F_{AB} = 300$ lb (C) **Ans.**



Joint A:

$\Sigma F_x = 0; \quad 300 - \frac{3}{\sqrt{34}}F_{AC} = 0$

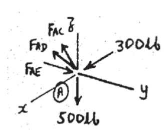
$F_{AC} = 583.1 = 583$ lb (T) **Ans.**

$\Sigma F_z = 0; \quad \frac{3}{\sqrt{34}}(583.1) - 500 + \frac{3}{5}F_{AD} = 0$

$F_{AD} = 333$ lb (T) **Ans.**

$\Sigma F_y = 0; \quad F_{AE} - \frac{4}{5}(333.3) - \frac{4}{\sqrt{34}}(583.1) = 0$

$F_{AE} = 667$ lb (C) **Ans.**

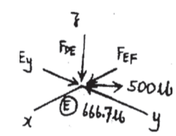


Joint E:

$\Sigma F_z = 0; \quad F_{DE} = 0$ **Ans.**

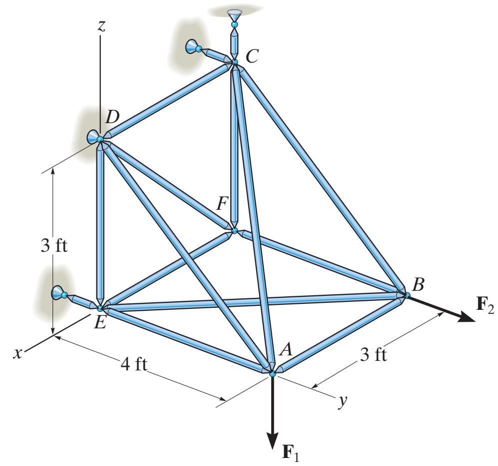
$\Sigma F_x = 0; \quad F_{EF} - \frac{3}{5}(500) = 0$

$F_{EF} = 300$ lb (C) **Ans.**

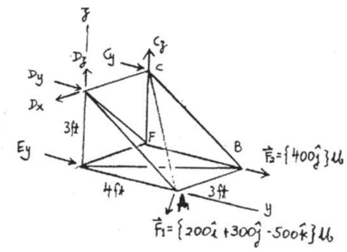


***6–60.**

The space truss is supported by a ball-and-socket joint at D and short links at C and E. Determine the force in each member and state if the members are in tension or compression. Take $\mathbf{F}_1 = \{200\mathbf{i} + 300\mathbf{j} - 500\mathbf{k}\}$ lb and $\mathbf{F}_2 = \{400\mathbf{j}\}$ lb.



SOLUTION



$\Sigma F_x = 0;\qquad D_x + 200 = 0$

$D_x = -200 \text{ lb}$

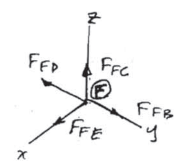
$\Sigma M_z = 0;\qquad -C_y(3) - 400(3) - 200(4) = 0$

$C_y = -666.7 \text{ lb}$

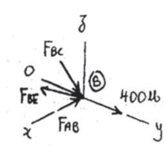
$\Sigma M_y = 0;\qquad C_z(3) - 200(3) = 0$

$C_z = 200 \text{ lb}$

Joint F: $\qquad F_{BF} = 0$ **Ans.**



Joint B:



$\Sigma F_z = 0;\qquad F_{BC} = 0$ **Ans.**

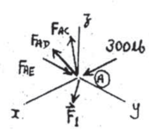
$\Sigma F_y = 0;\qquad 400 - \frac{4}{5}F_{BE} = 0$

$F_{BE} = 500 \text{ lb (T)}$ **Ans.**

$\Sigma F_x = 0;\qquad F_{AB} - \frac{3}{5}(500) = 0$

$F_{AB} = 300 \text{ lb (C)}$ **Ans.**

Joint A:



$\Sigma F_x = 0;\qquad 300 + 200 - \frac{3}{\sqrt{34}}F_{AC} = 0$

$F_{AC} = 971.8 = 972 \text{ lb (T)}$ **Ans.**

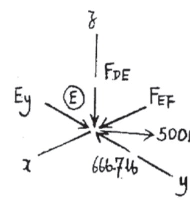
$\Sigma F_z = 0;\qquad \frac{3}{\sqrt{34}}(971.8) - 500 + \frac{3}{5}F_{AD} = 0$

$F_{AD} = 0$ **Ans.**

$\Sigma F_y = 0;\qquad F_{AE} + 300 - \frac{4}{\sqrt{34}}(971.8) = 0$

$F_{AE} = 367 \text{ lb (C)}$ **Ans.**

Joint E:



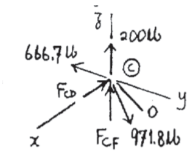
$\Sigma F_z = 0;\qquad F_{DE} = 0$ **Ans.**

$\Sigma F_x = 0;\qquad F_{EF} - \frac{3}{5}(500) = 0$

$F_{EF} = 300 \text{ lb (C)}$ **Ans.**

***6–60. (continued)**

Joint C:

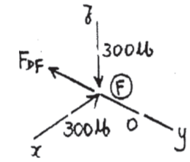


$\Sigma F_x = 0;\qquad \frac{3}{\sqrt{34}}(971.8) - F_{CD} = 0$

$F_{CD} = 500 \text{ lb (C)}$ **Ans.**

$\Sigma F_z = 0;\qquad F_{CF} - \frac{3}{\sqrt{34}}(971.8) + 200 = 0$

$F_{CF} = 300 \text{ lb (C)}$ **Ans.**



$\Sigma F_y = 0;\qquad \frac{4}{\sqrt{34}}(971.8) - 666.7 = 0$ Check!

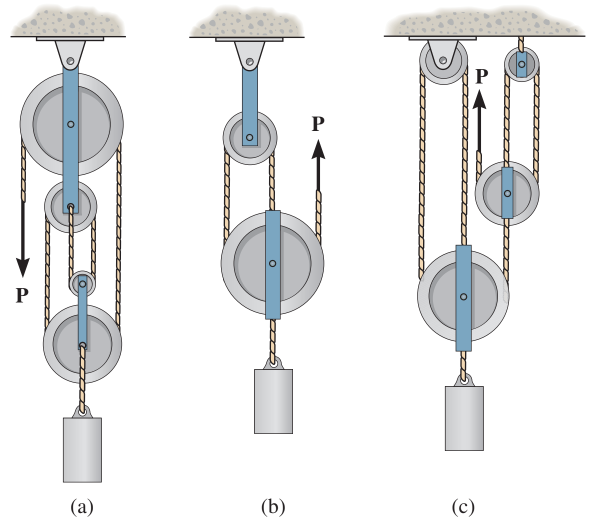
Joint F:

$\Sigma F_x = 0;\qquad \frac{3}{\sqrt{18}}F_{DF} - 300 = 0$

$F_{DF} = 424 \text{ lb (T)}$ **Ans.**

6–61.

In each case, determine the force **P** required to maintain equilibrium. The block weighs 100 lb.



SOLUTION

Equations of Equilibrium:

a) $+\uparrow \Sigma F_y = 0; \qquad 4P - 100 = 0$

$$P = 25.0 \text{ lb}$$ **Ans.**

b) $+\uparrow \Sigma F_y = 0; \qquad 3P - 100 = 0$

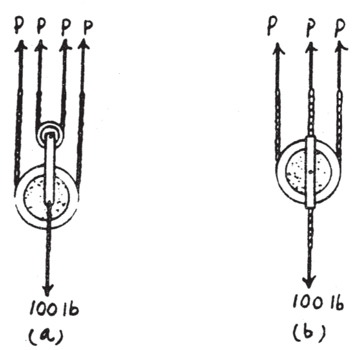
$$P = 33.3 \text{ lb}$$ **Ans.**

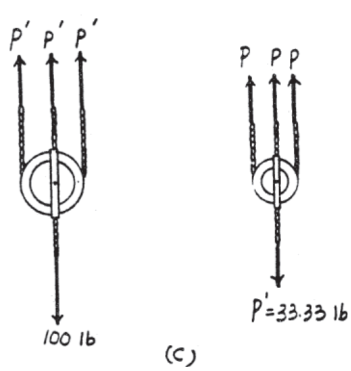
c) $+\uparrow \Sigma F_y = 0; \qquad 3P' - 100 = 0$

$$P' = 33.33 \text{ lb}$$

$+\uparrow \Sigma F_y = 0; \qquad 3P - 33.33 = 0$

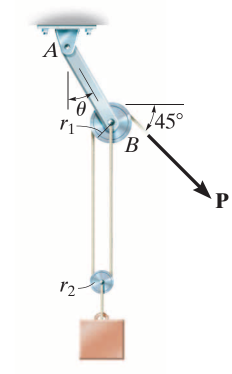
$$P = 11.1 \text{ lb}$$ **Ans.**





6–62.

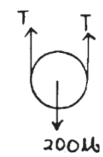
Determine the force P on the cord, and the angle θ that the pulley-supporting link AB makes with the vertical. Neglect the mass of the pulleys and the link. The block has a weight of 200 lb and the cord is attached to the pin at B. The pulleys have radii of $r_1 = 2$ in. and $r_2 = 1$ in.



SOLUTION

$+\uparrow \Sigma F_y = 0;\qquad 2T - 200 = 0$

$T = 100 \text{ lb}$ **Ans.**

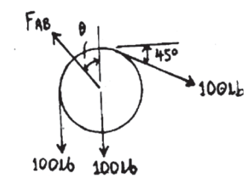


$\xrightarrow{+} \Sigma F_x = 0;\qquad 100 \cos 45° - F_{AB} \sin \theta = 0$

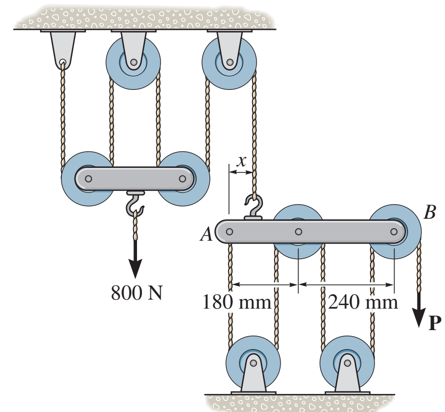
$+\uparrow \Sigma F_y = 0;\qquad F_{AB} \cos \theta - 100 - 100 - 100 \sin 45° = 0$

$\theta = 14.6°$ **Ans.**

$F_{AB} = 280 \text{ lb}$



6–63.

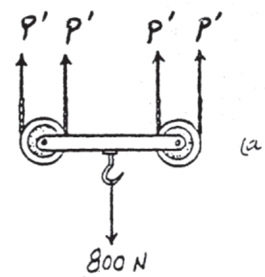


The principles of a *differential chain block* are indicated schematically in the figure. Determine the magnitude of force **P** needed to support the 800-N force. Also, find the distance x where the cable must be attached to bar AB so the bar remains horizontal. All pulleys have a radius of 60 mm.

SOLUTION

Equations of Equilibrium: From FBD(a),

$+\uparrow \Sigma F_y = 0;\qquad 4P' - 800 = 0 \qquad P' = 200\text{ N}$



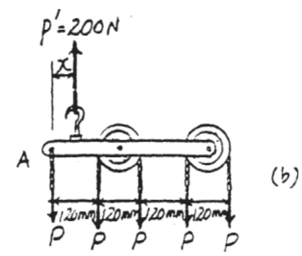
From FBD(b),

$+\uparrow \Sigma F_y = 0;\qquad 200 - 5P = 0 \qquad P = 40.0\text{ N}$ **Ans.**

$\circlearrowleft + \Sigma M_A = 0;\qquad 200(x) - 40.0(120) - 40.0(240)$

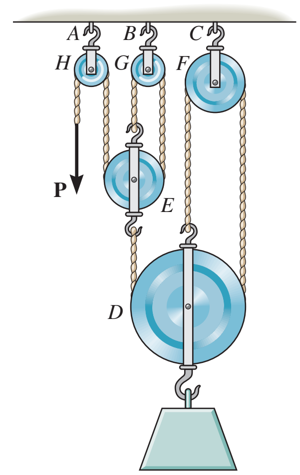
$$- 40.0(360) - 40.0(480) = 0$$

$x = 240\text{ mm}$ **Ans.**



***6–64.**

Determine the force P needed to support the 20-kg mass using the *Spanish Burton rig*. Also, what are the reactions at the supporting hooks A, B, and C?



SOLUTION

For pulley D:

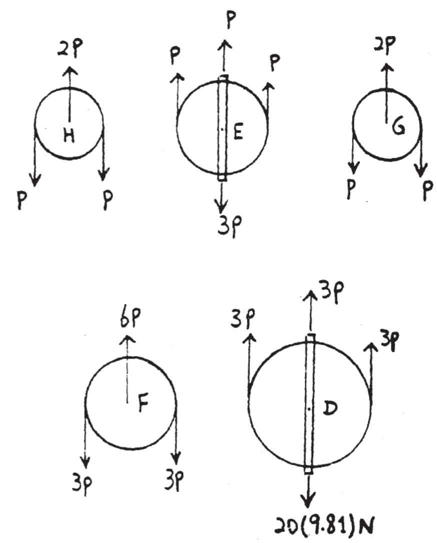
$+\uparrow \Sigma F_y = 0; \qquad 9P - 20(9.81) = 0$

$P = 21.8 \text{ N}$ **Ans.**

At A, $\qquad R_A = 2P = 43.6 \text{ N}$ **Ans.**

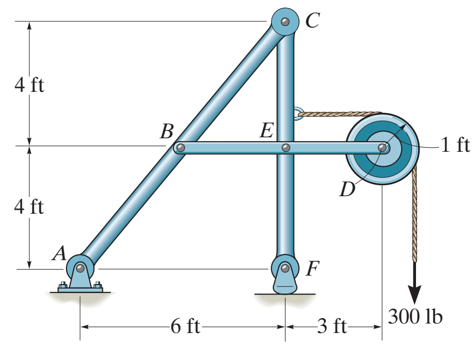
At B, $\qquad R_B = 2P = 43.6 \text{ N}$ **Ans.**

At C, $\qquad R_C = 6P = 131 \text{ N}$ **Ans.**

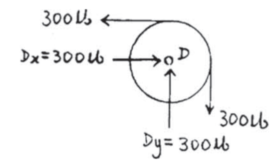


6–65.

Determine the horizontal and vertical components of force at C which member ABC exerts on member CEF.



SOLUTION



Member BED:

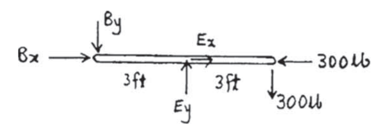
$\zeta + \Sigma M_B = 0; \qquad -300(6) + E_y(3) = 0$

$E_y = 600 \text{ lb}$

$+\uparrow \Sigma F_y = 0; \qquad -B_y + 600 - 300 = 0$

$B_y = 300 \text{ lb}$

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad B_x + E_x - 300 = 0 \qquad$ **(1)**



Member FEC:

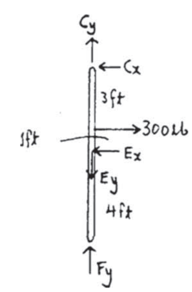
$\zeta + \Sigma M_C = 0; \qquad 300(3) - E_x(4) = 0$

$E_x = 225 \text{ lb}$

From Eq. (1) $\qquad B_x = 75 \text{ lb}$

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad -C_x + 300 - 225 = 0$

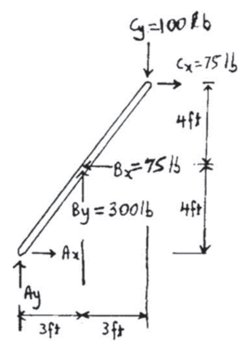
$C_x = 75 \text{ lb} \qquad$ **Ans.**



Member ABC:

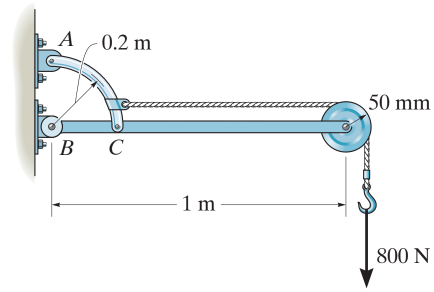
$\zeta + \Sigma M_A = 0; \qquad -75(8) - C_y(6) + 75(4) + 300(3) = 0$

$C_y = 100 \text{ lb} \qquad$ **Ans.**

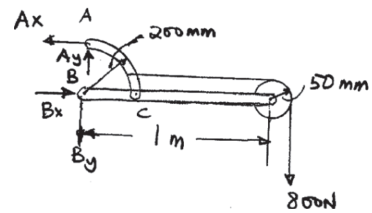


6–66.

Determine the horizontal and vertical components of force that the pins at $A, B,$ and C exert on their connecting members.



SOLUTION



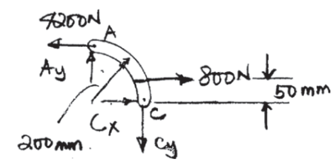
$\circlearrowleft + \Sigma M_B = 0; \quad -800(1 + 0.05) + A_x(0.2) = 0$

$A_x = 4200 \text{ N} = 4.20 \text{ kN}$ **Ans.**

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad B_x = 4200 \text{ N} = 4.20 \text{ kN}$ **Ans.**

$+\uparrow \Sigma F_y = 0; \quad A_y - B_y - 800 = 0$ **(1)**

Member AC:



$\circlearrowleft + \Sigma M_C = 0; \quad -800(50) - A_y(200) + 4200(200) = 0$

$A_y = 4000 \text{ N} = 4.00 \text{ kN}$ **Ans.**

From Eq. (1) $\quad B_y = 3.20 \text{ kN}$ **Ans.**

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad -4200 + 800 + C_x = 0$

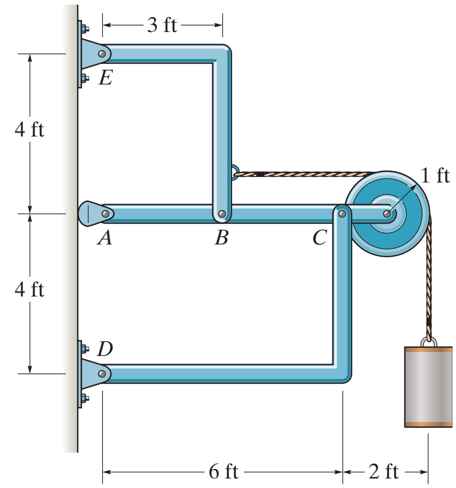
$C_x = 3.40 \text{ kN}$ **Ans.**

$+\uparrow \Sigma F_y = 0; \quad 4000 - C_y = 0$

$C_y = 4.00 \text{ kN}$ **Ans.**

6–67.

Determine the horizontal and vertical components of force at each pin. The suspended cylinder has a weight of 80 lb.



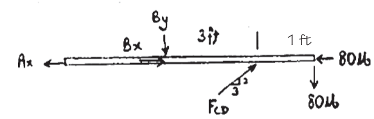
SOLUTION

$\zeta + \Sigma M_B = 0; \quad F_{CD}\left(\frac{2}{\sqrt{13}}\right)(3) - 80(4) = 0$

$F_{CD} = 192.3 \text{ lb}$

$C_x = D_x = \frac{3}{\sqrt{13}}(192.3) = 160 \text{ lb}$ **Ans.**

$C_y = D_y = \frac{2}{\sqrt{13}}(192.3) = 107 \text{ lb}$ **Ans.**

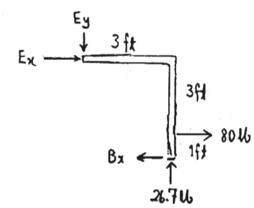


$+\uparrow \Sigma F_y = 0; \quad -B_y + \frac{2}{\sqrt{13}}(192.3) - 80 = 0$

$B_y = 26.7 \text{ lb}$ **Ans.**

$\zeta + \Sigma M_E = 0; \quad -B_x(4) + 80(3) + 26.7(3) = 0$

$B_x = 80.0 \text{ lb}$ **Ans.**



$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad E_x + 80 - 80 = 0$

$E_x = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0; \quad -E_y + 26.7 = 0$

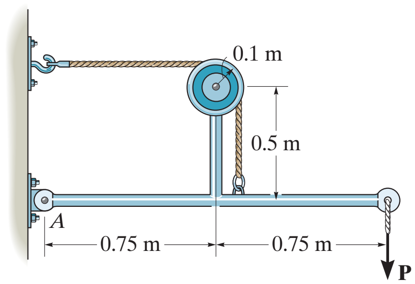
$E_y = 26.7 \text{ lb}$ **Ans.**

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad -A_x + 80 + \frac{3}{\sqrt{13}}(192.3) - 80 = 0$

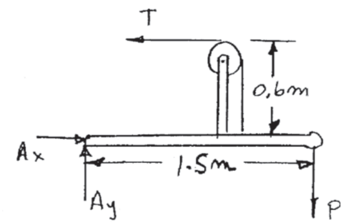
$A_x = 160 \text{ lb}$ **Ans.**

***6–68.**

Determine the greatest force P that can be applied to the frame if the largest force resultant acting at A can have a magnitude of 2 kN.



SOLUTION



$\zeta + \Sigma M_A = 0; \qquad T(0.6) - P(1.5) = 0$

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad A_x - T = 0$

$+\uparrow \Sigma F_y = 0; \qquad A_y - P = 0$

Thus, $A_x = 2.5\,P, A_y = P$

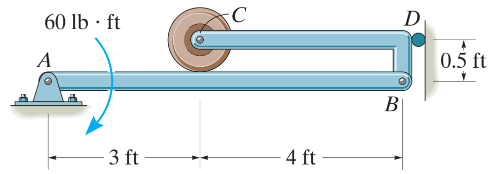
Require,

$2 = \sqrt{(2.5P)^2 + (P)^2}$

$P = 0.743 \text{ kN} = 743 \text{ N}$ **Ans.**

6–69.

Determine the force that the smooth roller C exerts on member AB. Also, what are the horizontal and vertical components of reaction at pin A? Neglect the weight of the frame and roller.



SOLUTION

$\circlearrowleft + \Sigma M_A = 0; \qquad -60 + D_x(0.5) = 0$

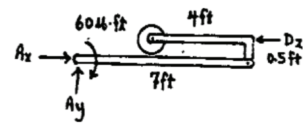
$D_x = 120 \text{ lb}$

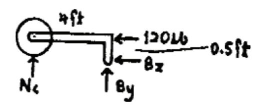
$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad A_x = 120 \text{ lb}$ **Ans.**

$+\uparrow \Sigma F_y = 0; \qquad A_y = 0$ **Ans.**

$\circlearrowleft + \Sigma M_B = 0; \qquad -N_C(4) + 120(0.5) = 0$

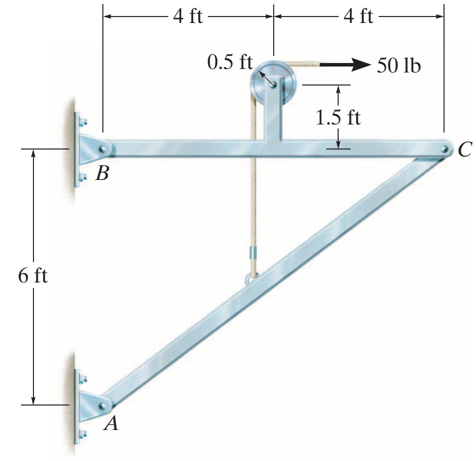
$N_C = 15.0 \text{ lb}$ **Ans.**





6–70.

Determine the horizontal and vertical components of force at pins B and C.



SOLUTION

$\circlearrowleft +\Sigma M_A = 0;\qquad -C_y(8) + C_x(6) + 50(3.5) = 0$

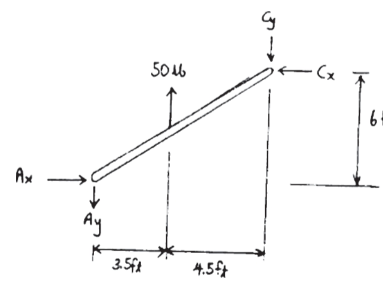
$\xrightarrow{+} \Sigma F_x = 0;\qquad A_x = C_x$

$+\uparrow \Sigma F_y = 0;\qquad 50 - A_y - C_y = 0$

$\circlearrowleft +\Sigma M_B = 0;\qquad -50(2) - 50(3.5) + C_y(8) = 0$

$C_y = 34.38 = 34.4 \text{ lb}$ **Ans.**

$C_x = 16.67 = 16.7 \text{ lb}$ **Ans.**

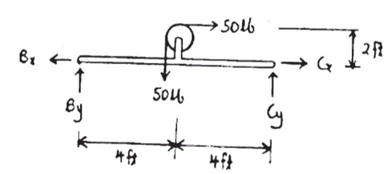


$\xrightarrow{+} \Sigma F_x = 0;\qquad 16.67 + 50 - B_x = 0$

$B_x = 66.7 \text{ lb}$ **Ans.**

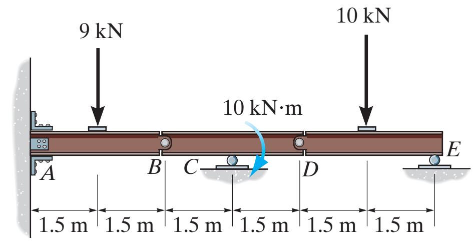
$+\uparrow \Sigma F_y = 0;\qquad B_y - 50 + 34.38 = 0$

$B_y = 15.6 \text{ lb}$ **Ans.**



6–71.

Determine the support reactions at A, C, and E on the compound beam which is pin connected at B and D.



SOLUTION

Equations of Equilibrium: First, we will consider the free-body diagram of segment DE in Fig. c.

$+\Sigma M_D = 0;$ $\quad N_E(3) - 10(1.5) = 0$

$N_E = 5 \text{ kN}$ **Ans.**

$+\Sigma M_E = 0;$ $\quad 10(1.5) - D_y(3) = 0$

$D_y = 5 \text{ kN}$

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad D_x = 0$ **Ans.**

Subsequently, the free-body diagram of segment BD in Fig. b will be considered using the results of D_x and D_y obtained above.

$+\Sigma M_B = 0;$ $\quad N_C(1.5) - 5(3) - 10 = 0$

$N_C = 16.67 \text{ kN} = 16.7 \text{ kN}$ **Ans.**

$+\Sigma M_C = 0;$ $\quad B_y(1.5) - 5(1.5) - 10 = 0$

$B_y = 11.67 \text{ kN}$

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad B_y = 0$

Finally, the free-body diagram of segment AB in Fig. a will be considered using the results of B_x and B_y obtained above.

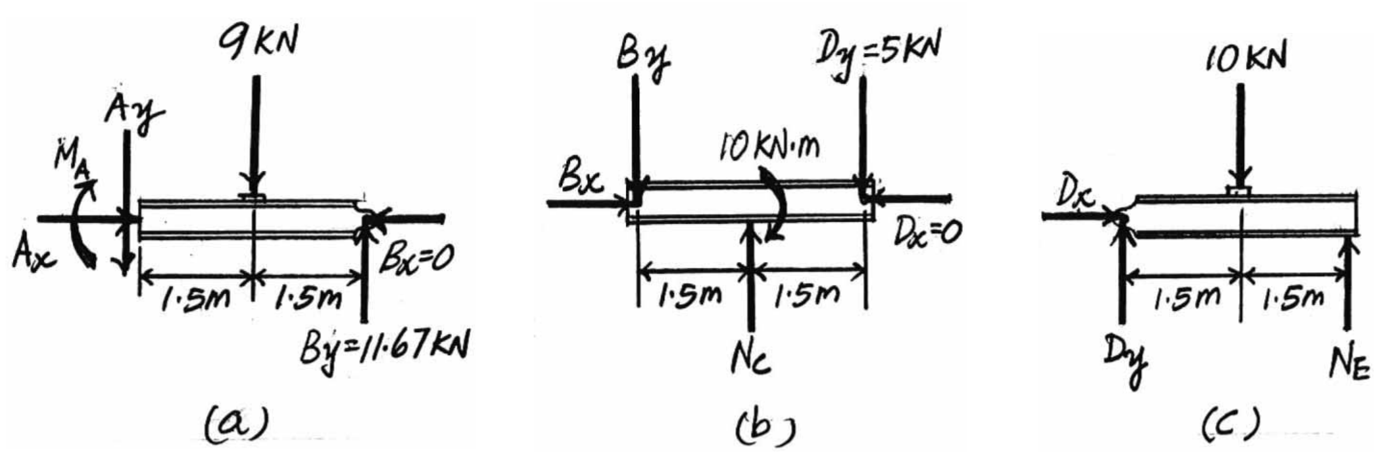
$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad A_x = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0;$ $\quad 11.67 - 9 - A_y = 0$

$A_y = 2.67 \text{ kN}$ **Ans.**

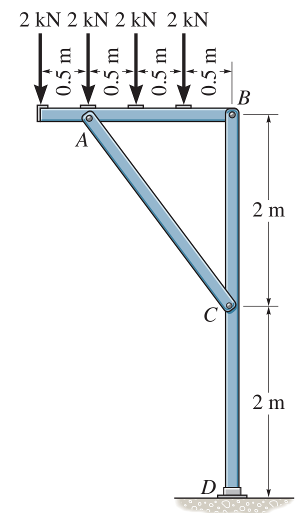
$+\Sigma M_A = 0;$ $\quad 11.67(3) - 9(1.5) - M_A = 0$

$M_A = 21.5 \text{ kN} \cdot \text{m}$ **Ans.**



***6–72.**

Determine the horizontal and vertical components of force at pins A,B, and C,and the reactions at the fixed support D of the three-member frame.



SOLUTION

Free Body Diagram: The solution for this problem will be simplified if one realizes that member AC is a two force member.

Equations of Equilibrium: For FBD(a),

$\circlearrowleft + \Sigma M_B = 0;\qquad 2(0.5) + 2(1) + 2(1.5) + 2(2) - F_{AC}\left(\frac{4}{5}\right)(1.5) = 0$

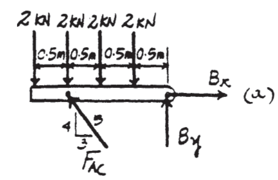
$$F_{AC} = 8.333 \text{ kN}$$

$+\uparrow \Sigma F_y = 0;\qquad B_y + 8.333\left(\frac{4}{5}\right) - 2 - 2 - 2 - 2 = 0$

$$B_y = 1.333 \text{ kN} = 1.33 \text{ kN} \qquad \textbf{Ans.}$$

$\overset{+}{\rightarrow} \Sigma F_x = 0;\qquad B_x - 8.333\left(\frac{3}{5}\right) = 0$

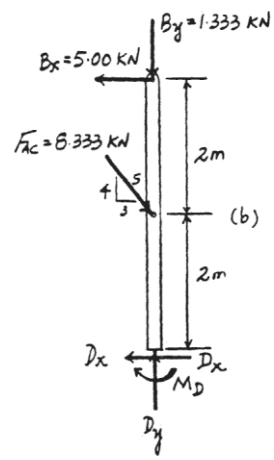
$$B_x = 5.00 \text{ kN} \qquad \textbf{Ans.}$$



For pin A and C,

$$A_x = C_x = F_{AC}\left(\frac{3}{5}\right) = 8.333\left(\frac{3}{5}\right) = 5.00 \text{ kN} \qquad \textbf{Ans.}$$

$$A_y = C_y = F_{AC}\left(\frac{4}{5}\right) = 8.333\left(\frac{4}{5}\right) = 6.67 \text{ kN} \qquad \textbf{Ans.}$$



From FBD (b),

$\circlearrowleft + \Sigma M_D = 0;\qquad 5.00(4) - 8.333\left(\frac{3}{5}\right)(2) - M_D = 0$

$$M_D = 10.0 \text{ kN} \cdot \text{m} \qquad \textbf{Ans.}$$

$+\uparrow \Sigma F_y = 0;\qquad D_y - 1.333 - 8.333\left(\frac{4}{5}\right) = 0$

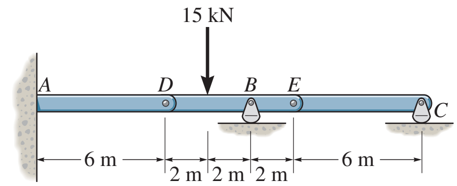
$$D_y = 8.00 \text{ kN} \qquad \textbf{Ans.}$$

$\overset{+}{\rightarrow} \Sigma F_x = 0;\qquad 8.333\left(\frac{3}{5}\right) - 5.00 - D_x = 0$

$$D_x = 0 \qquad \textbf{Ans.}$$

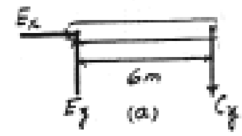
6–73.

The compound beam is fixed at A and supported by a rocker at B and C. There are hinges (pins) at D and E. Determine the reactions at the supports.



SOLUTION

Equations of Equilibrium: From FBD(a),

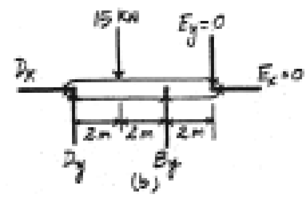


$\zeta + \Sigma M_E = 0;\quad C_y(6) = 0 \quad C_y = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0;\quad E_y - 0 = 0 \quad E_y = 0$

$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad E_x = 0$

From FBD(b),



$\zeta + \Sigma M_D = 0;\quad B_y(4) - 15(2) = 0$

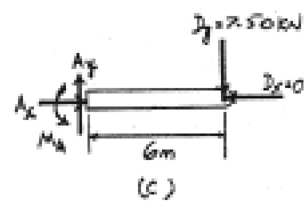
$B_y = 7.50 \text{ kN}$ **Ans.**

$+\uparrow \Sigma F_y = 0;\quad D_y + 7.50 - 15 = 0$

$D_y = 7.50 \text{ kN}$

$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad D_x = 0$

From FBD(c),



$\zeta + \Sigma M_A = 0;\quad M_A - 7.50(6) = 0$

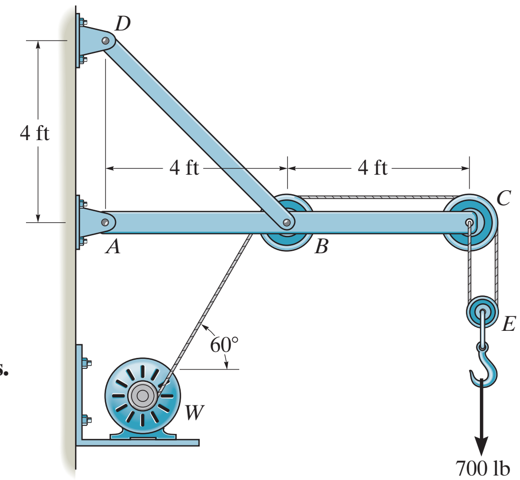
$M_A = 45.0 \text{ kN}\cdot\text{m}$ **Ans.**

$+\uparrow \Sigma F_y = 0;\quad A_y - 7.5 = 0 \quad A_y = 7.5 \text{ kN}$ **Ans.**

$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad A_x = 0$ **Ans.**

6–74.

The wall crane supports a load of 700 lb. Determine the horizontal and vertical components of reaction at the pins A and D. Also, what is the force in the cable at the winch W?

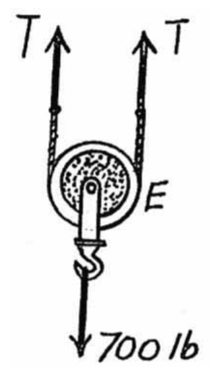


SOLUTION

Pulley E:

$+\uparrow\Sigma F_y = 0;$ $\quad 2T - 700 = 0$

$T = 350 \text{ lb}$ **Ans.**



Member ABC:

$\circlearrowleft + \Sigma M_A = 0;$ $\quad T_{BD} \sin 45°(4) - 350 \sin 60°(4) \cdot 700\,(8) = 0$

$T_{BD} = 2409 \text{ lb}$

$+\uparrow\Sigma F_y = 0;$ $\quad -A_y + 2409 \sin 45° - 350 \sin 60° - 700 = 0$

$A_y = 700 \text{ lb}$ **Ans.**

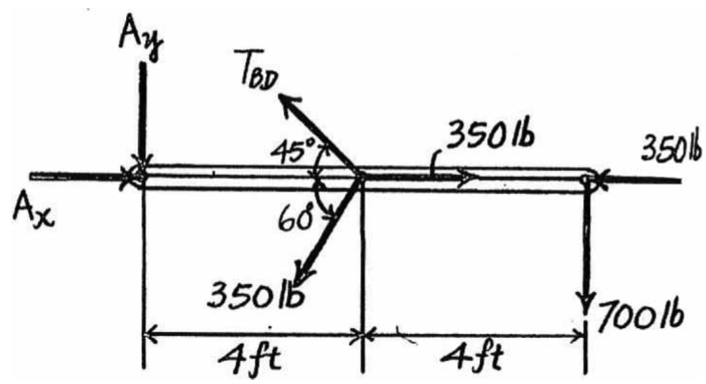
$\overset{+}{\rightarrow} \Sigma F_z = 0;$ $\quad -A_y - 2409 \cos 45° - 350 \cos 60° + 350 - 350 = 0$

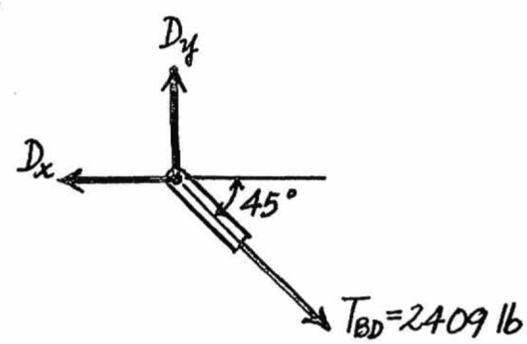
$A_z = 1.88 \text{ kip}$ **Ans.**

At D:

$D_z = 2409 \cos 45° = 1703.1 \text{ lb} = 1.70 \text{ kip}$ **Ans.**

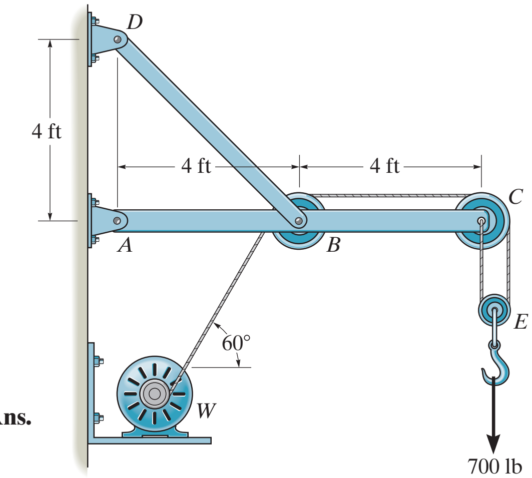
$D_y = 2409 \sin 45° = 1.70 \text{ kip}$ **Ans.**





6–75.

The wall crane supports a load of 700 lb. Determine the horizontal and vertical components of reaction at the pins A and D. Also, what is the force in the cable at the winch W? The jib ABC has a weight of 100 lb and member BD has a weight of 40 lb. Each member is uniform and has a center of gravity at its center.



SOLUTION

Pulley E:

$+\uparrow \Sigma F_y = 0;\qquad 2T - 700 = 0$

$T = 350 \text{ lb}$ **Ans.**

Member ABC:

$\circlearrowleft + \Sigma M_A = 0;\qquad B_y(4) - 700(8) - 100(4) - 350 \sin 60^\circ (4) = 0$

$B_y = 1803.1 \text{ lb}$

$+\uparrow \Sigma F_y = 0;\qquad -A_y - 350 \sin 60^\circ - 100 - 700 + 1803.1 = 0$

$A_y = 700 \text{ lb}$ **Ans.**

$\xrightarrow{+} \Sigma F_x = 0;\qquad A_x - 350 \cos 60^\circ - B_x + 350 - 350 = 0$

$A_x = B_x + 175$ **(1)**

Member DB:

$\circlearrowleft + \Sigma M_D = 0;\qquad -40(2) - 1803.1(4) + B_x(4) = 0$

$B_x = 1823.1 \text{ lb}$

$\xrightarrow{+} \Sigma F_x = 0;\qquad -D_x + 1823.1 = 0$

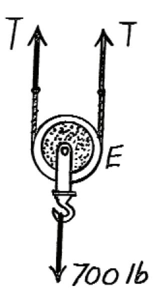
$D_x = 1.82 \text{ kip}$ **Ans.**

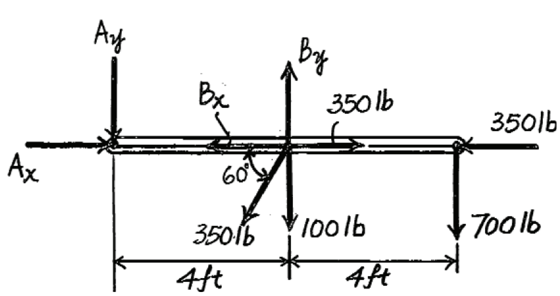
$+\uparrow \Sigma F_y = 0;\qquad D_y - 40 - 1803.1 = 0$

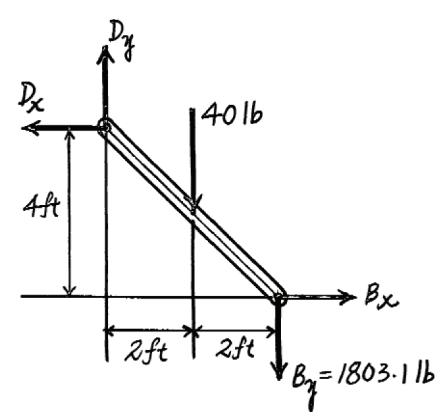
$D_y = 1843.1 = 1.84 \text{ kip}$ **Ans.**

From Eq. (1)

$A_x = 2.00 \text{ kip}$ **Ans.**

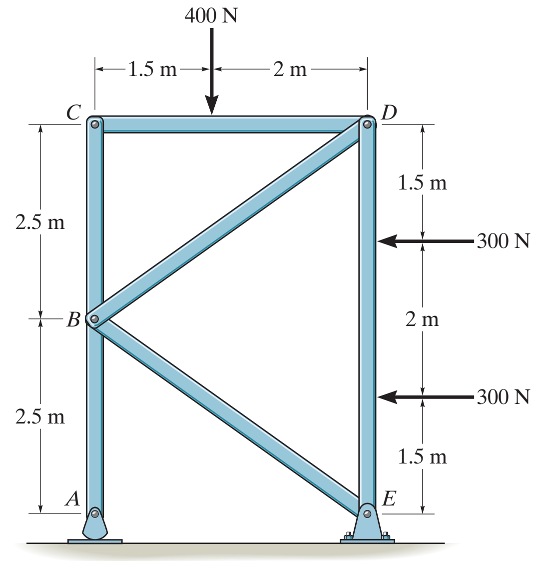






***6–76.**

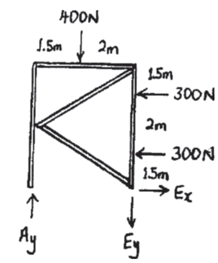
Determine the horizontal and vertical components of force which the pins at A, B, and C exert on member ABC of the frame.



SOLUTION

$\circlearrowleft + \Sigma M_E = 0; \quad -A_y(3.5) + 400(2) + 300(3.5) + 300(1.5) = 0$

$A_y = 657.1 = 657 \text{ N}$ **Ans.**



$\circlearrowleft + \Sigma M_D = 0; \quad -C_y(3.5) + 400(2) = 0$

$C_y = 228.6 = 229 \text{ N}$ **Ans.**

$\circlearrowleft + \Sigma M_B = 0; \quad C_x = 0$ **Ans.**

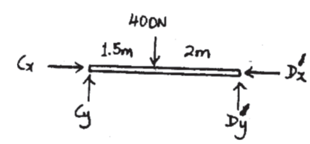
$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad F_{BD} = F_{BE}$

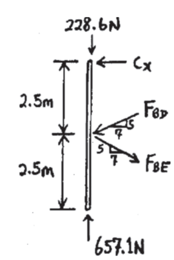
$+\uparrow \Sigma F_y = 0; \quad 657.1 - 228.6 - 2\left(\frac{5}{\sqrt{74}}\right)F_{BD} = 0$

$F_{BD} = F_{BE} = 368.7 \text{ N}$

$B_x = 0$ **Ans.**

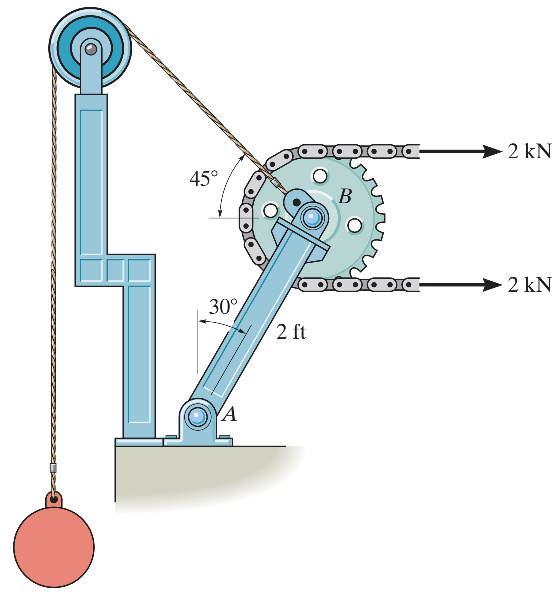
$B_y = \frac{5}{\sqrt{74}}(368.7)(2) = 429 \text{ N}$ **Ans.**





6–77.

Determine the required mass of the suspended cylinder if the tension in the chain wrapped around the freely turning gear is to be 2 kN. Also, what is the magnitude of the resultant force on pin A?



SOLUTION

$\zeta + \Sigma M_A = 0; \quad -4(2\cos 30°) + W\cos 45°(2\cos 30°) + W\sin 45°(2\sin 30°) = 0$

$W = 3.586 \text{ kN}$

$m = 3.586(1000)/9.81 = 366 \text{ kg}$ **Ans.**

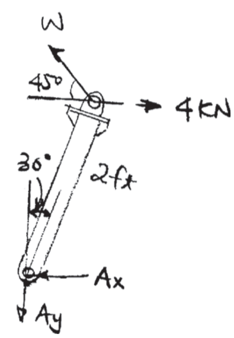
$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad 4 - 3.586\cos 45° - A_x = 0$

$A_x = 1.464 \text{ kN}$

$+\uparrow \Sigma F_y = 0; \quad 3.586\sin 45° - A_y = 0$

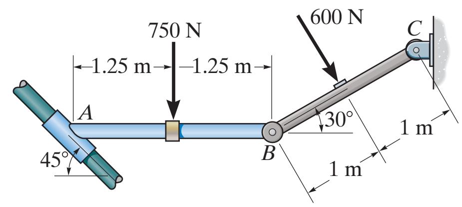
$A_y = 2.536 \text{ kN}$

$F_A = \sqrt{(1.464)^2 + (2.536)^2} = 2.93 \text{ kN}$ **Ans.**



6–78.

Determine the reactions on the collar at A and the pin at C. The collar fits over a smooth rod, and rod AB is fixed connected to the collar.



SOLUTION

Equations of Equilibrium: From the force equation of equilibrium of member AB, Fig. a, we can write

$$+\Sigma M_A = 0; \qquad M_A - 750(1.25) - B_y(2.5) = 0 \qquad \textbf{(1)}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad N_A \cos 45^\circ - B_x = 0 \qquad \textbf{(2)}$$

$$+\uparrow \Sigma F_y = 0; \qquad N_A \sin 45^\circ - 750 - B_y = 0 \qquad \textbf{(3)}$$

From the free-body diagram of member BC in Fig. b,

$$+\Sigma M_C = 0; \qquad B_x(2 \sin 30^\circ) - B_y(2 \cos 30^\circ) + 600(1) = 0 \qquad \textbf{(4)}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad B_x + 600 \sin 30^\circ - C_x = 0 \qquad \textbf{(5)}$$

$$+\uparrow \Sigma F_y = 0; \qquad B_y - C_y - 600 \cos 30^\circ = 0 \qquad \textbf{(6)}$$

Solving Eqs. (2), (3), and (4) yields

$B_y = 1844.13 \text{ N} = 1.84 \text{ kN} \qquad B_x = 2594.13 \text{ N}$

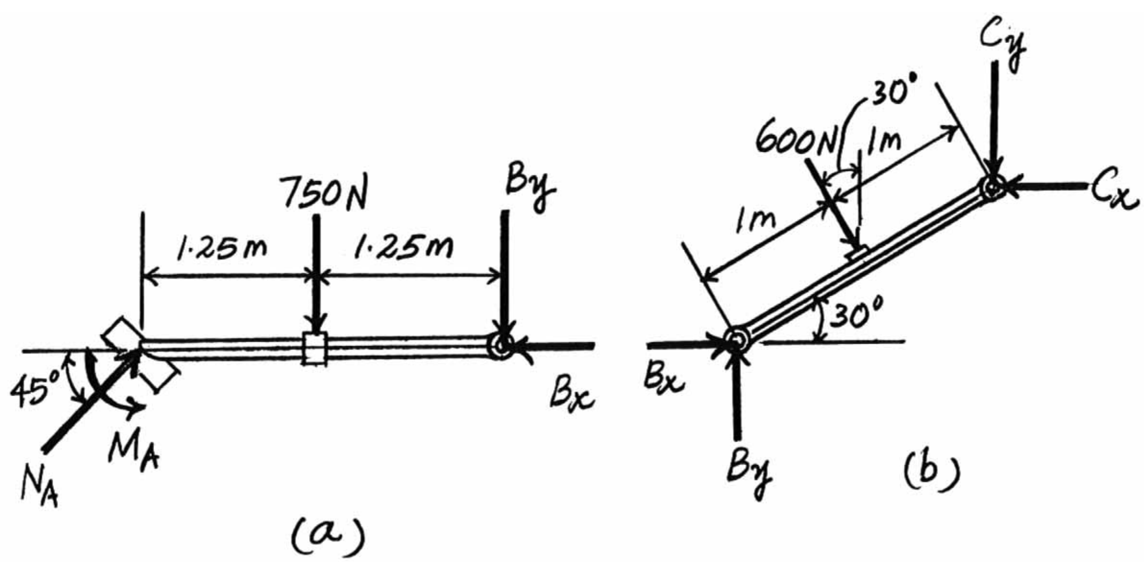
$N_A = 3668.66 \text{ N} = 3.67 \text{ kN}$ **Ans.**

Substituting the results of B_x and B_y into Eqs. (1), (5), and (6) yields

$M_A = 5547.84 \text{ N}\cdot\text{m} = 5.55 \text{ kN}\cdot\text{m}$ **Ans.**

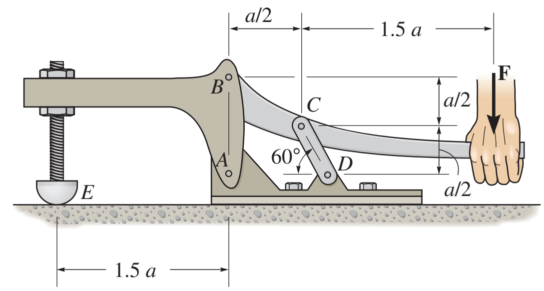
$C_x = 2894.13 \text{ N} = 2.89 \text{ kN}$ **Ans.**

$C_y = 1324.52 \text{ N} = 1.32 \text{ kN}$ **Ans.**



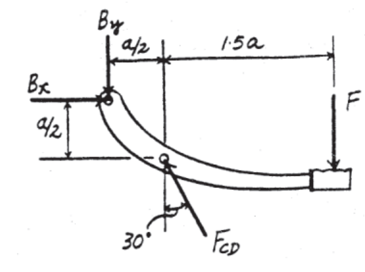
6–79.

The toggle clamp is subjected to a force **F** at the handle. Determine the vertical clamping force acting at E.



SOLUTION

Free Body Diagram: The solution for this problem will be simplified if one realizes that member CD is a two force member.



(a)

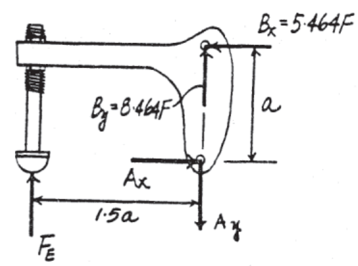
Equations of Equilibrium: From FBD (a),

$$\zeta + \Sigma M_B = 0; \qquad F_{CD}\cos 30°\left(\frac{a}{2}\right) - F_{CD}\sin 30°\left(\frac{a}{2}\right) - F(2a) = 0$$

$$F_{CD} = 10.93F$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad B_x - 10.93\sin 30° = 0$$

$$B_x = 5.464F$$



(b)

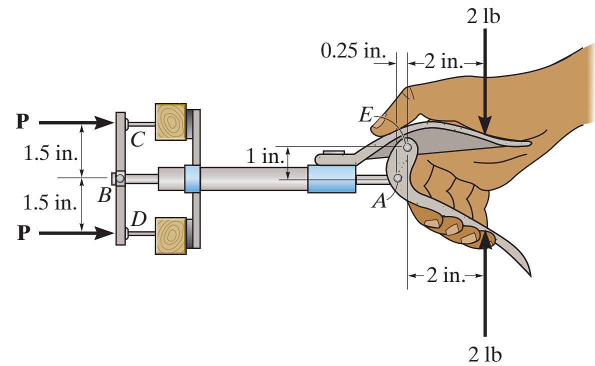
From (b),

$$\zeta + \Sigma M_A = 0; \qquad 5.464F(a) - F_E\,(1.5a) = 0$$

$$F_E = 3.64F$$ **Ans.**

*6–80.

When a force of 2 lb is applied to the handles of the brad squeezer, it pulls in the smooth rod AB. Determine the force $\mathbf{P}$ exerted on each of the smooth brads at C and D.

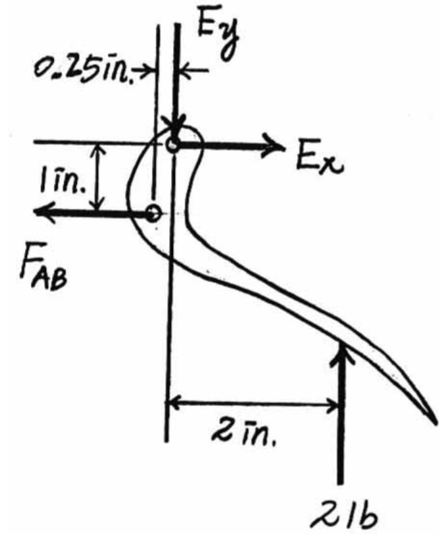


SOLUTION

Equations of Equilibrium: Applying the moment equation of equilibrium about point E to the free-body diagram of the lower handle in Fig. a, we have

$$+\Sigma M_E = 0; \qquad 2(2) - F_{AB}(1) = 0$$

$$F_{AB} = 4 \text{ lb}$$



(a)

Using the result of F_{AB} and considering the free-body diagram in Fig. b,

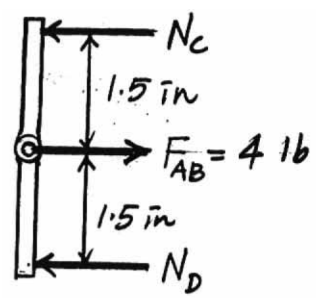
$$+\Sigma M_B = 0; \qquad N_C(1.5) - N_D(1.5) = 0$$

$$N_C = N_D \qquad \textbf{(1)}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 4 - N_C - N_D = 0 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

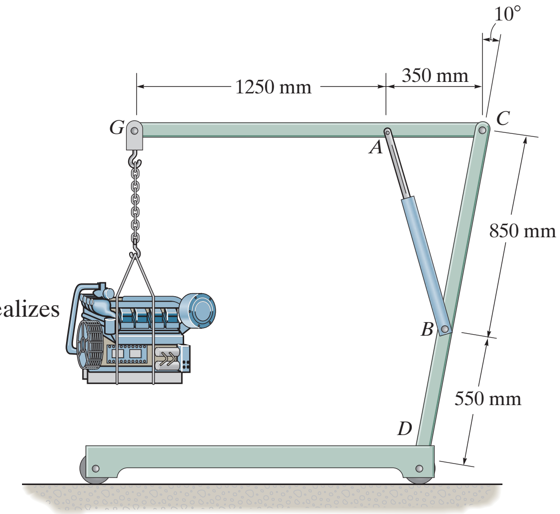
$$N_C = N_D = 2 \text{ lb} \qquad \textbf{Ans.}$$



(b)

6–81.

The engine hoist is used to support the 200-kg engine. Determine the force acting in the hydraulic cylinder AB, the horizontal and vertical components of force at the pin C, and the reactions at the fixed support D.

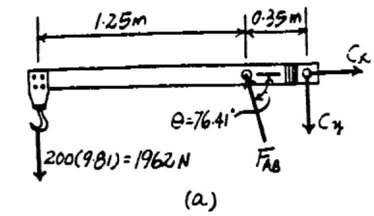


SOLUTION

Free-Body Diagram: The solution for this problem will be simplified if one realizes that member AB is a two force member. From the geometry,

$$l_{AB} = \sqrt{350^2 + 850^2 - 2(350)(850)\cos 80^\circ} = 861.21 \text{ mm}$$

$$\frac{\sin\theta}{850} = \frac{\sin 80^\circ}{861.24} \qquad \theta = 76.41^\circ$$



Equations of Equilibrium: From FBD (a),

$\circlearrowleft + \Sigma M_C = 0;\qquad 1962(1.60) - F_{AB}\sin 76.41^\circ(0.35) = 0$

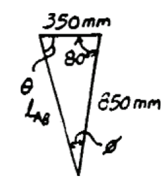
$F_{AB} = 9227.60 \text{ N} = 9.23 \text{ kN}$ **Ans.**

$\overset{+}{\rightarrow} \Sigma F_x = 0;\qquad C_x - 9227.60\cos 76.41^\circ = 0$

$C_x = 2168.65 \text{ N} = 2.17 \text{ kN}$ **Ans.**

$+\uparrow \Sigma F_y = 0;\qquad 9227.60\sin 76.41^\circ - 1962 - C_y = 0$

$C_y = 7007.14 \text{ N} = 7.01 \text{ kN}$ **Ans.**



From FBD (b),

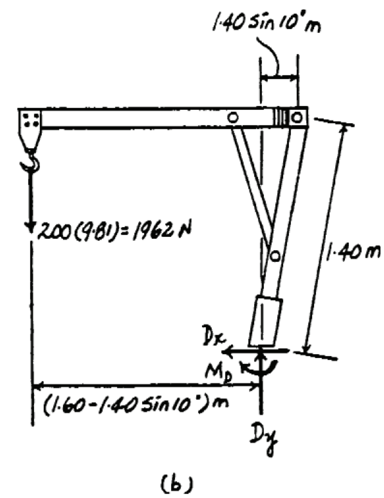
$\overset{+}{\rightarrow} \Sigma F_x = 0;\qquad D_x = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0;\qquad D_y - 1962 = 0$

$D_y = 1962 \text{ N} = 1.96 \text{ kN}$ **Ans.**

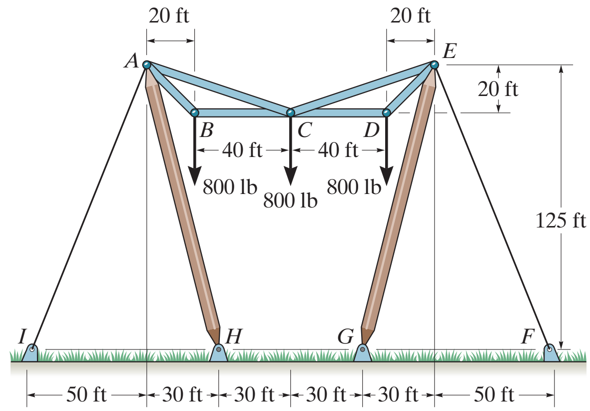
$\circlearrowleft + \Sigma M_D = 0;\qquad 1962(1.60 - 1.40\sin 10^\circ) - M_D = 0$

$M_D = 2662.22 \text{ N}\cdot\text{m} = 2.66 \text{ kN}\cdot\text{m}$ **Ans.**



6–82.

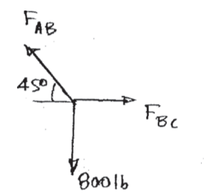
The three power lines exert the forces shown on the truss joints, which in turn are pin-connected to the poles AH and EG. Determine the force in the guy cable AI and the pin reaction at the support H.



SOLUTION

AH is a two - force member.

Joint B:



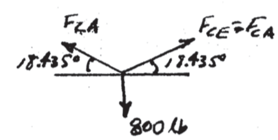
$+\uparrow \Sigma F_y = 0; \quad F_{AB} \sin 45° - 800 = 0$

$F_{AB} = 1131.37 \text{ lb}$

Joint C:

$+\uparrow \Sigma F_y = 0; \quad 2F_{CA} \sin 18.435° - 800 = 0$

$F_{CA} = 1264.91 \text{ lb}$



Joint A:

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad -T_{AI} \sin 21.801° - F_H \cos 76.504° + 1264.91 \cos 18.435° + 1131.37 \cos 45° = 0$

$+\uparrow \Sigma F_y = 0; \quad -T_{AI} \cos 21.801° + F_H \sin 76.504° - 1131.37 \sin 45° - 1264.91 \sin 18.435° = 0$

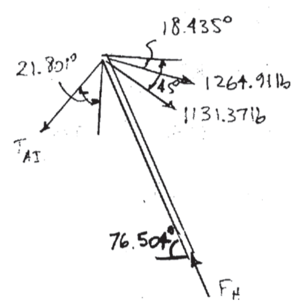
$T_{AI}(0.3714) + F_H(0.2334) = 2000$

$-T_{AI}(0.9285) + F_H(0.97239) = 1200$

Solving,

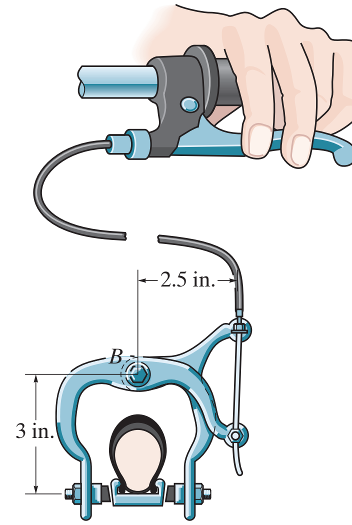
$T_{AI} = T_{EF} = 2.88 \text{ kip}$ **Ans.**

$F_H = F_G = 3.99 \text{ kip}$ **Ans.**



6–83.

By squeezing on the hand brake of the bicycle, the rider subjects the brake cable to a tension of 50 lb. If the caliper mechanism is pin-connected to the bicycle frame at B, determine the normal force each brake pad exerts on the rim of the wheel. Is this the force that stops the wheel from turning? Explain.

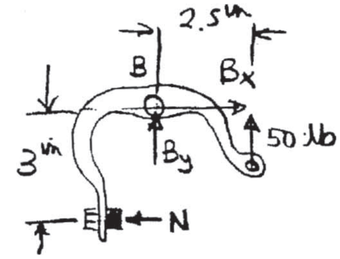


SOLUTION

$$\zeta + \Sigma M_B = 0; \quad -N(3) + 50(2.5) = 0$$

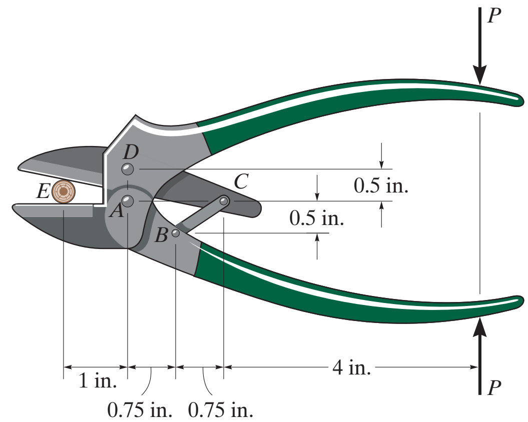
$$N = 41.7 \text{ lb} \qquad \textbf{Ans.}$$

This normal force **does not** stop the wheel from turning. A frictional force (see Chapter 8), which acts along on the wheel's rim stops the wheel. **Ans.**

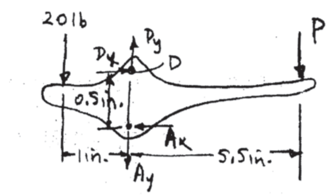


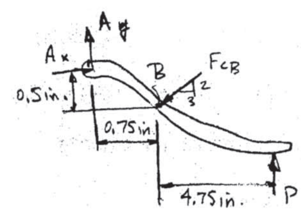
***6–84.**

Determine the required force P that must be applied at the blade of the pruning shears so that the blade exerts a normal force of 20 lb on the twig at E.



SOLUTION





$\circlearrowleft + \Sigma M_D = 0; \qquad P(5.5) + A_x(0.5) - 20(1) = 0$

$5.5P + 0.5A_x = 20$

$+\uparrow \Sigma F_y = 0; \qquad D_y - P - A_y - 20 = 0$

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad D_x = A_x$

$\circlearrowleft + \Sigma M_B = 0; \qquad A_y(0.75) + A_x(0.5) - 4.75P = 0$

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad A_x - F_{CB}\left(\frac{3}{\sqrt{13}}\right) = 0$

$+\uparrow \Sigma F_y = 0; \qquad A_y + P - F_{CB}\left(\frac{2}{\sqrt{13}}\right) = 0$

Solving:

$A_x = 13.3$ lb

$A_y = 6.46$ lb

$D_x = 13.3$ lb

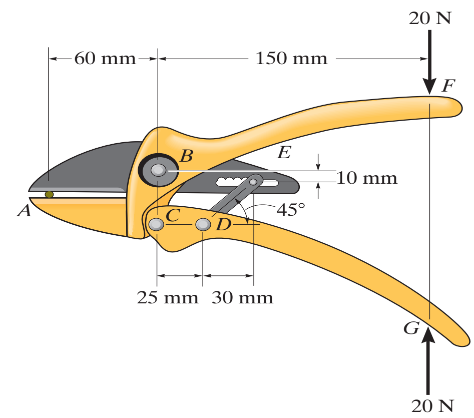
$D_y = 28.9$ lb

$P = 2.42$ lb **Ans.**

$F_{CB} = 16.0$ lb

6–85.

The pruner multiplies blade-cutting power with the compound leverage mechanism. If a 20-N force is applied to the handles, determine the cutting force generated at A. Assume that the contact surface at A is smooth.



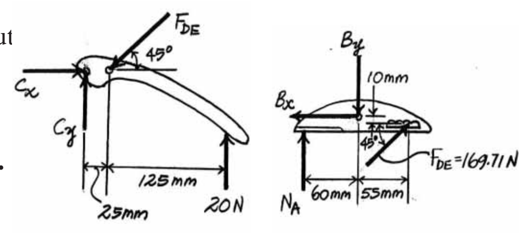
SOLUTION

Equations of Equilibrium: Applying the moment equation of equilibrium about point C to the free-body diagram of handle CDG in Fig. a, we have

$$+\Sigma M_C = 0; \qquad 20(150) - F_{DE} \sin 45°(25) = 0$$
$$F_{DE} = 169.71\,\text{N}$$

Using the result of F_{DE} and applying the moment equation of equilibrium about point B on the free-body diagram of the cutter in Fig. b, we obtain

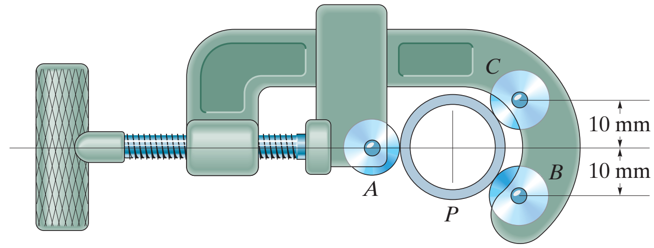
$$+\Sigma M_B = 0; \qquad 169.71 \sin 45°(55) + 169.71 \cos 45°(10) - N_A(60) = 0$$
$$F_A = 130\,\text{N} \qquad \textbf{Ans.}$$



(a) (b)

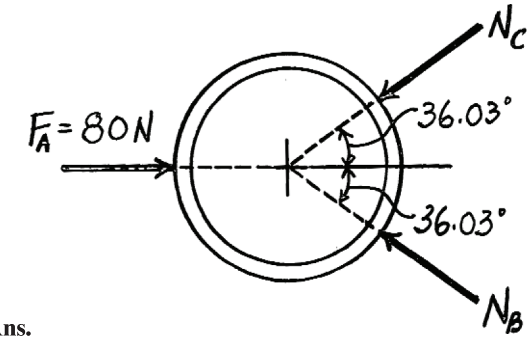
6–86.

The pipe cutter is clamped around the pipe P. If the wheel at A exerts a normal force of $F_A = 80\text{ N}$ on the pipe, determine the normal forces of wheels B and C on the pipe. The three wheels each have a radius of 7 mm and the pipe has an outer radius of 10 mm.



SOLUTION

$$\theta = \sin^{-1}\left(\frac{10}{17}\right) = 36.03°$$



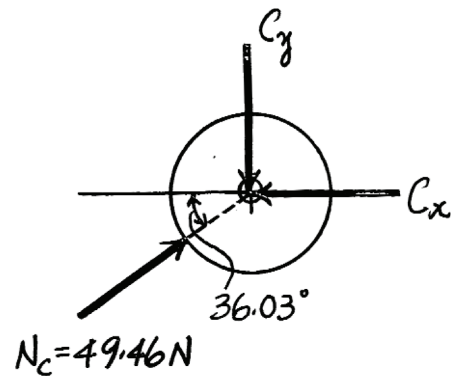
Equations of Equilibrium:

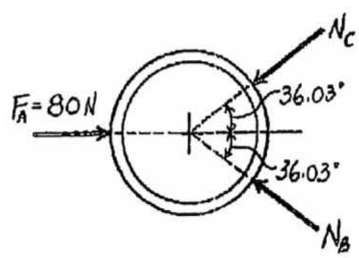
$+\uparrow \Sigma F_y = 0;$ $\quad N_B \sin 36.03° - N_C \sin 36.03° = 0$

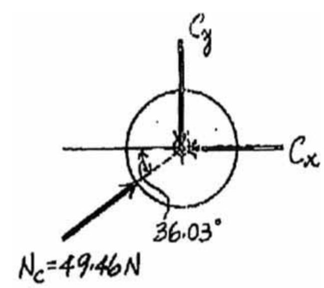
$$N_B = N_C$$

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad 80 - N_C \cos 36.03° - N_C \cos 36.03° = 0$

$$N_B = N_C = 49.5\text{ N}$$ **Ans.**

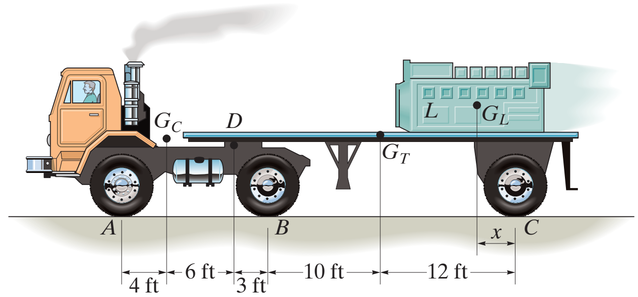






6–87.

The flat-bed trailer has a weight of 7000 lb and center of gravity at G_T. It is pin connected to the cab at D. The cab has a weight of 6000 lb and center of gravity at G_C. Determine the range of values x for the position of the 2000-lb load L so that when it is placed over the rear axle, no axle is subjected to more than 5500 lb. The load has a center of gravity at G_L.

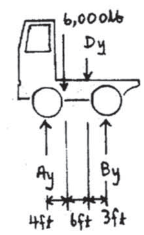


SOLUTION

Case 1: Assume $A_y = 5500$ lb

$\zeta + \Sigma M_B = 0; \qquad -5500(13) + 6000(9) + D_y(3) = 0$

$D_y = 5833.33$ lb



$+\uparrow \Sigma F_y = 0; \qquad B_y - 6000 - 5833.33 + 5500 = 0$

$B_y = 6333.33 \text{ lb} > 5500 \text{ lb}$ **(N.G!)**

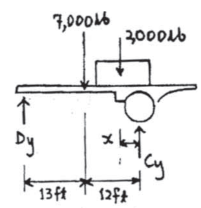
Case 2: Assume $B_y = 5500$ lb

$\zeta + \Sigma M_A = 0; \qquad 5500(13) - 6000(4) - D_y(10) = 0$

$D_y = 4750$ lb

$+\uparrow \Sigma F_y = 0; \qquad A_y - 6000 - 4750 + 5500 = 0$

$A_y = 5250$ lb



$+\uparrow \Sigma F_y = 0; \qquad 4750 - 7000 - 2000 + C_y = 0$

$C_y = 4250 \text{ lb} < 5500 \text{ lb}$ **(O.K!)**

$\zeta + \Sigma M_D = 0; \qquad -7000(13) - 2000(13 + 12 - x) + 4250(25) = 0$

$x = 17.4$ ft

Case 3: Assume $C_y = 5500$ lb

$+\uparrow \Sigma F_y = 0; \qquad D_y - 9000 + 5500 = 0$

$D_y = 3500$ lb

$\zeta + \Sigma M_C = 0; \qquad -3500(25) + 7000(12) + 2000(x) = 0$

$x = 1.75$ ft

$\zeta + \Sigma M_A = 0; \qquad -6000(4) - 3500(10) + B_y(13) = 0$

$B_y = 4538.46 \text{ lb} < 5500 \text{ lb}$ **(O. K!)**

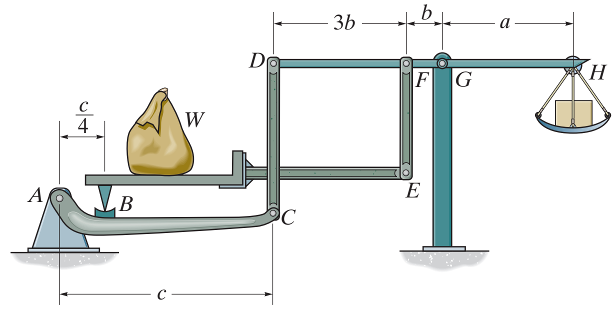
$+\uparrow \Sigma F_y = 0; \qquad A_y - 6000 - 3500 + 4538.46 = 0$

$A_y = 4961.54 \text{ lb} < 5500 \text{ lb}$ **(O. K!)**

Thus, $\qquad 1.75 \text{ ft} \le x \le 17.4 \text{ ft}$ **Ans.**

***6–88.**

Show that the weight W_1 of the counterweight at H required for equilibrium is $W_1 = (b/a)W$, and so it is independent of the placement of the load W on the platform.



SOLUTION

Equations of Equilibrium: First, we will consider the free-body diagram of member BE in Fig. a,

$$+\Sigma M_E = 0; \qquad W(x) - N_B\left(3b + \frac{3}{4}c\right) = 0$$

$$N_B = \frac{Wx}{\left(3b + \frac{3}{4}c\right)}$$

$$+\uparrow\Sigma F_y = 0; \qquad F_{EF} + \frac{Wx}{\left(3b + \frac{3}{4}c\right)} - W = 0$$

$$F_{EF} = W\left(1 - \frac{x}{3b + \frac{3}{4}c}\right)$$

Using the result of N_B and applying the moment equation of equilibrium about point A on the free-body diagram in Fig. b, we obtain

$$+\Sigma M_A = 0; \qquad F_{CD}\,(c) - \frac{Wx}{3b + \frac{3}{4}c}\left(\frac{1}{4}c\right) = 0$$

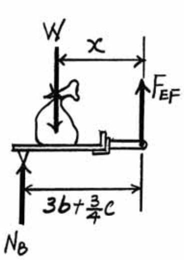
$$N_{CD} = \frac{Wx}{12b + 3c}$$

Writing the moment equation of equilibrium about point G on the free-body diagram in Fig. c, we have

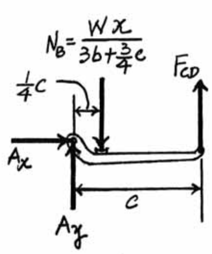
$$+\Sigma M_G = 0; \qquad \frac{Wx}{12b + 3c}(4b) + W\left(1 - \frac{x}{3b + \frac{3}{4}c}\right)(b) - W_1(a) = 0$$

$$W_1 = \frac{b}{a}W \qquad \textbf{Ans.}$$

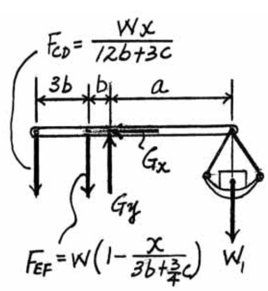
This result shows that the required weight W_1 of the counterweight is independent of the position x of the load on the platform.



(a)



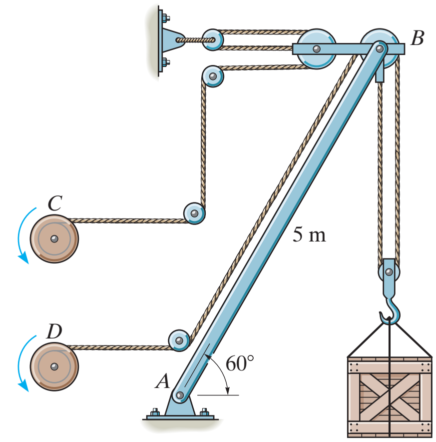
(b)



(c)

6–89.

The derrick is pin connected to the pivot at A. Determine the largest mass that can be supported by the derrick if the maximum force that can be sustained by the pin at A is 18 kN.

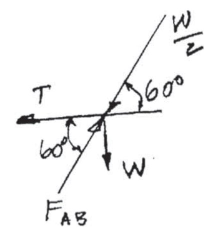


SOLUTION

AB is a two-force member.

Pin B

Require $F_{AB} = 18$ kN



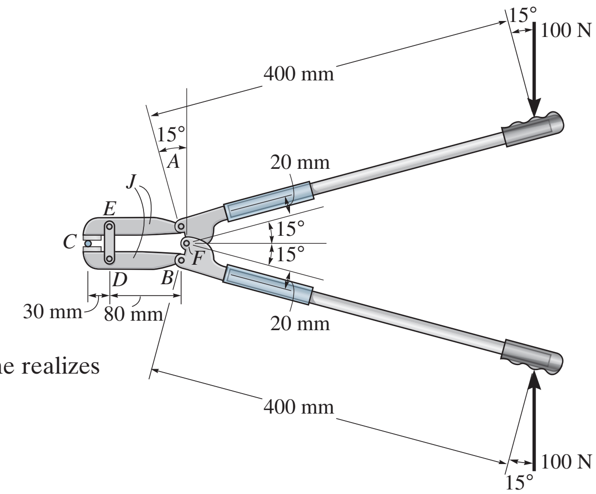
$$+\uparrow \Sigma F_y = 0; \qquad 18 \sin 60° - \frac{W}{2} \sin 60° - W = 0$$

$$W = 10.878 \text{ kN}$$

$$m = \frac{10.878}{9.81} = 1.11 \text{ Mg}$$ **Ans.**

6–90.

Determine the force that the jaws J of the metal cutters exert on the smooth cable C if 100-N forces are applied to the handles. The jaws are pinned at E and A, and D and B. There is also a pin at F.



SOLUTION

Free Body Diagram: The solution for this problem will be simplified if one realizes that member ED is a two force member.

Equations of Equilibrium: From FBD (b),

$\xrightarrow{+} \Sigma F_x = 0; \qquad A_x = 0$

From (a),

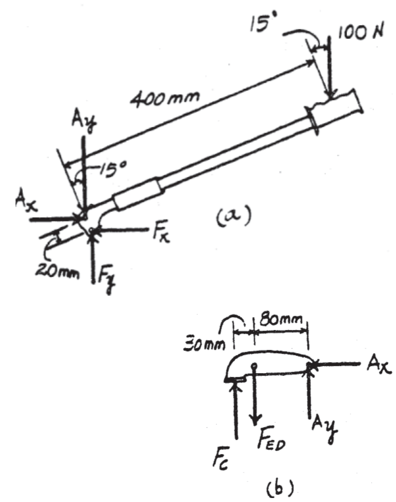
$\zeta + \Sigma M_F = 0; \qquad A_y \sin 15°(20) + 100 \sin 15°(20) - 100 \cos 15°(400) = 0$

$$A_y = 7364.10 \text{ N}$$

From FBD (b),

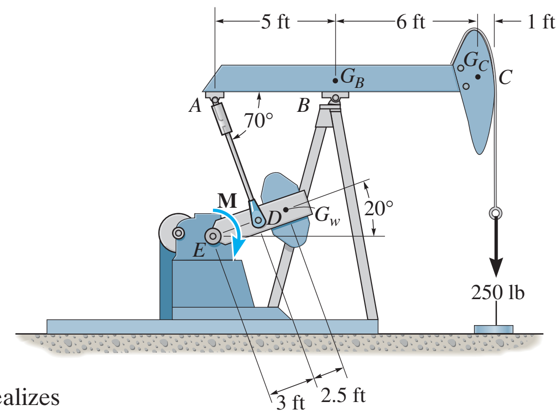
$\zeta + \Sigma M_E = 0; \qquad 7364.10(80) - F_C(30) = 0$

$$F_C = 19637.60 \text{ N} = 19.6 \text{ kN}$$ **Ans.**



6–91.

The pumping unit is used to recover oil. When the walking beam ABC is horizontal, the force acting in the wireline at the well head is 250 lb. Determine the torque **M** which must be exerted by the motor in order to overcome this load. The horse-head C weighs 60 lb and has a center of gravity at G_C. The walking beam ABC has a weight of 130 lb and a center of gravity at G_B, and the counterweight has a weight of 200 lb and a center of gravity at G_W. The pitman, AD, is pin connected at its ends and has negligible weight.



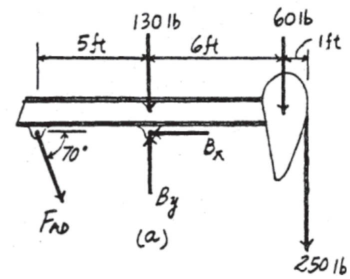
SOLUTION

Free-Body Diagram: The solution for this problem will be simplified if one realizes that the pitman AD is a two force member.

Equations of Equilibrium: From FBD (a),

$$\circlearrowleft + \Sigma M_B = 0; \qquad F_{AD} \sin 70°(5) - 60(6) - 250(7) = 0$$

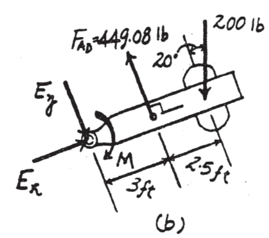
$$F_{AD} = 449.08 \text{ lb}$$



From (b),

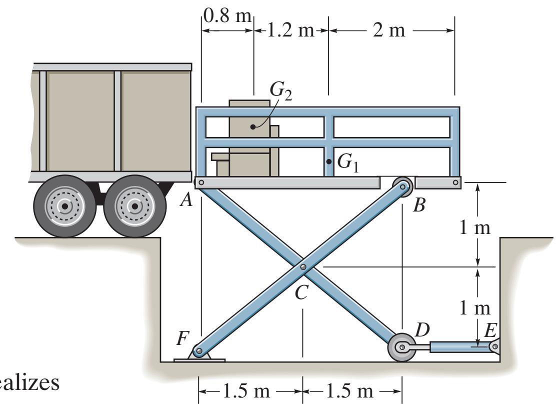
$$\circlearrowleft + \Sigma M_E = 0; \qquad 449.08(3) - 200 \cos 20°(5.5) - M = 0$$

$$M = 314 \text{ lb} \cdot \text{ft} \qquad \textbf{Ans.}$$



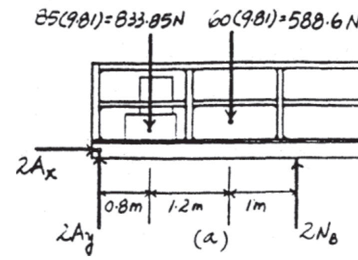
***6–92.**

The scissors lift consists of *two* sets of cross members and *two* hydraulic cylinders, DE, symmetrically located on *each side* of the platform. The platform has a uniform mass of 60 kg, with a center of gravity at G_1. The load of 85 kg, with center of gravity at G_2, is centrally located between each side of the platform. Determine the force in each of the hydraulic cylinders for equilibrium. Rollers are located at B and D.



SOLUTION

Free Body Diagram: The solution for this problem will be simplified if one realizes that the hydraulic cyclinder DE is a two force member.



Equations of Equilibrium: From FBD (a),

$$\circlearrowleft + \Sigma M_A = 0; \qquad 2N_B(3) - 833.85(0.8) - 588.6(2) = 0$$

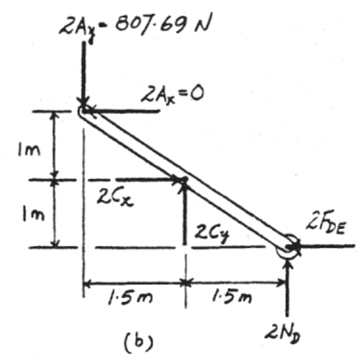
$$2N_B = 614.76 \text{ N}$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad A_x = 0$$

$$+\uparrow \Sigma F_y = 0; \qquad 2A_y + 614.76 - 833.85 - 588.6 = 0$$

$$2A_y = 807.69 \text{ N}$$

From FBD (b),



$$\circlearrowleft + \Sigma M_D = 0; \qquad 807.69(3) - 2C_y(1.5) - 2C_x(1) = 0$$

$$2C_x + 3C_y = 2423.07 \qquad \textbf{(1)}$$

From FBD (c),

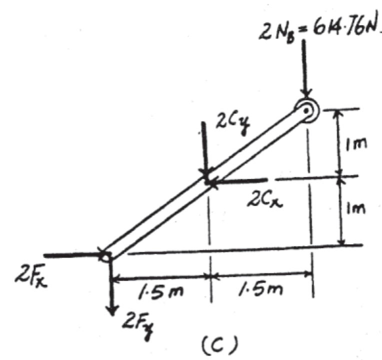
$$\circlearrowleft + \Sigma M_F = 0; \qquad 2C_x(1) - 2C_y(1.5) - 614.76(3) = 0$$

$$2C_x - 3C_y = 1844.28 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$C_x = 1066.84 \text{ N} \qquad C_y = 96.465 \text{ N}$$

From FBD (b),

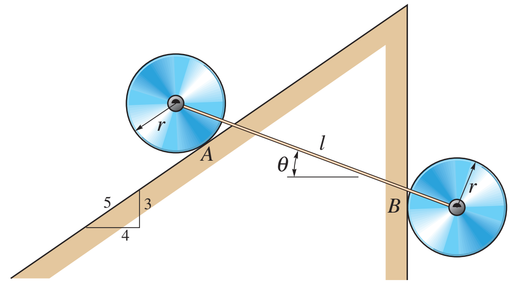


$$\xrightarrow{+} \Sigma F_x = 0; \qquad 2(1066.84) - 2F_{DE} = 0$$

$$F_{DE} = 1066.84 \text{ N} = 1.07 \text{ kN} \qquad \textbf{Ans.}$$

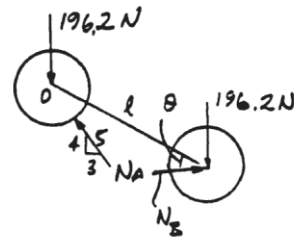
6–93.

The two disks each have a mass of 20 kg and are attached at their centers by an elastic cord that has a stiffness of $k = 2\ \text{kN/m}$. Determine the stretch of the cord when the system is in equilibrium, and the angle θ of the cord.



SOLUTION

Entire system:



$$\xrightarrow{+} \Sigma F_x = 0; \qquad N_B - N_A\left(\frac{3}{5}\right) = 0$$

$$+\uparrow \Sigma F_y = 0; \qquad N_A\left(\frac{4}{5}\right) - 2\,(196.2) = 0$$

$$\circlearrowleft + \Sigma M_O = 0; \qquad N_B\,(l \sin\theta) - 196.2\, l \cos\theta = 0$$

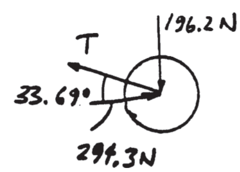
Solving,

$$N_A = 490.5\ \text{N}$$

$$N_B = 294.3\ \text{N}$$

$$\theta = 33.69^\circ = 33.7^\circ \qquad \textbf{Ans.}$$

Disk B:



$$\xrightarrow{+} \Sigma F_x = 0; \qquad -T\cos 33.69^\circ + 294.3 = 0$$

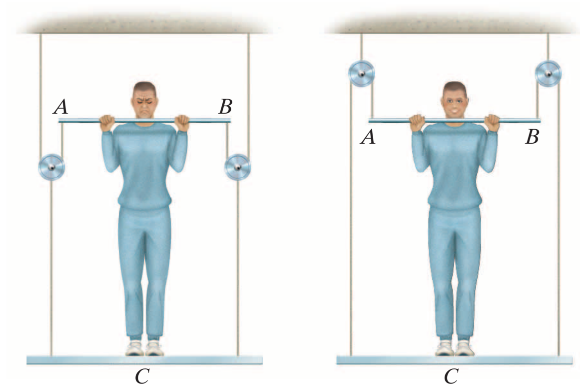
$$T = 353.70\ \text{N}$$

$$F_x = kx; \qquad 353.70 = 2000\,x$$

$$x = 0.177\ \text{m} = 177\ \text{mm} \qquad \textbf{Ans.}$$

6–94.

A man having a weight of 175 lb attempts to hold himself using one of the two methods shown. Determine the total force he must exert on bar AB in each case and the normal reaction he exerts on the platform at C. Neglect the weight of the platform.

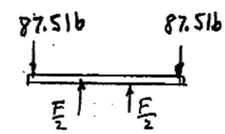


(a) (b)

SOLUTION

(a)

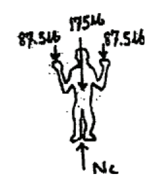
Bar:



$+\uparrow \Sigma F_y = 0; \qquad 2(F/2) - 2(87.5) = 0$

$F = 175 \text{ lb}$ **Ans.**

Man:

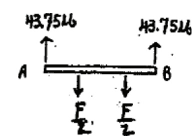


$+\uparrow \Sigma F_y = 0; \qquad N_C - 175 - 2(87.5) = 0$

$N_C = 350 \text{ lb}$ **Ans.**

(b)

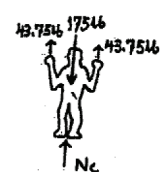
Bar:



$+\uparrow \Sigma F_y = 0; \qquad 2(43.75) - 2(F/2) = 0$

$F = 87.5 \text{ lb}$ **Ans.**

Man:

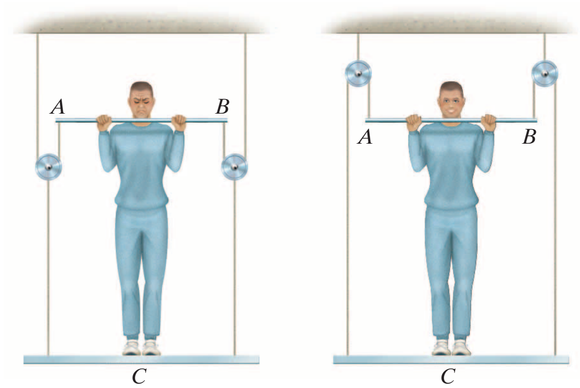


$+\uparrow \Sigma F_y = 0; \qquad N_C - 175 + 2(43.75) = 0$

$N_C = 87.5 \text{ lb}$ **Ans.**

6–95.

A man having a weight of 175 lb attempts to hold himself using one of the two methods shown. Determine the total force he must exert on bar AB in each case and the normal reaction he exerts on the platform at C. The platform has a weight of 30 lb.

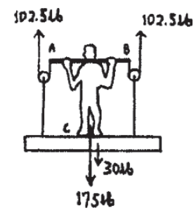


(a) (b)

SOLUTION

(a)

Bar:



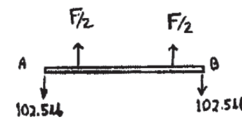
$+\uparrow\Sigma F_y = 0; \qquad 2(F/2) - 102.5 - 102.5 = 0$

$F = 205\text{ lb}$ **Ans.**

Man:

$+\uparrow\Sigma F_y = 0; \qquad N_C - 175 - 102.5 - 102.5 = 0$

$N_C = 380\text{ lb}$ **Ans.**

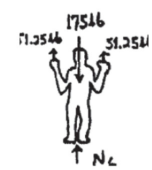


(b)

Bar:

$+\uparrow\Sigma F_y = 0; \qquad 2(F/2) - 51.25 - 51.25 = 0$

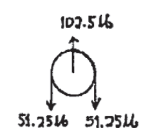
$F = 102\text{ lb}$ **Ans.**

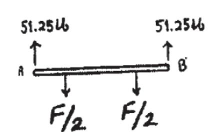


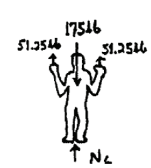
Man:

$+\uparrow\Sigma F_y = 0; \qquad N_C - 175 + 51.25 + 51.25 = 0$

$N_C = 72.5\text{ lb}$ **Ans.**

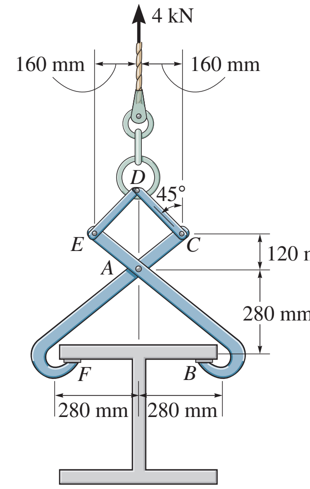






***6–96.**

The double link grip is used to lift the beam. If the beam weighs 4 kN, determine the horizontal and vertical components of force acting on the pin at A and the horizontal and vertical components of force that the flange of the beam exerts on the jaw at B.

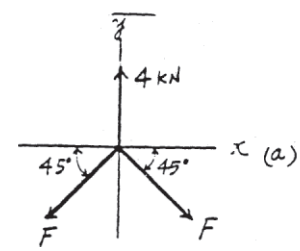


SOLUTION

Free Body Diagram: The solution for this problem will be simplified if one realizes that members ED and CD are two force members.

Equations of Equilibrium: Using method of joint, [FBD (a)],

$+\uparrow \Sigma F_y = 0; \qquad 4 - 2F \sin 45° = 0 \qquad F = 2.828 \text{ kN}$



From FBD (b),

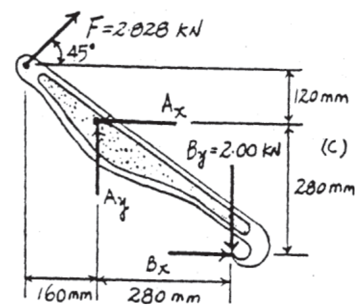
$+\uparrow \Sigma F_y = 0; \qquad 2B_y - 4 = 0 \qquad B_y = 2.00 \text{ kN}$ **Ans.**

From FBD (c),

$\circlearrowleft + \Sigma M_A = 0; \qquad B_x(280) - 2.00(280) - 2.828 \cos 45°(120)$

$- 2.828 \sin 45°(160) = 0$

$B_x = 4.00 \text{ kN}$ **Ans.**

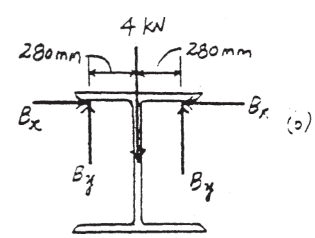


$+\uparrow \Sigma F_y = 0; \qquad A_y + 2.828 \sin 45° - 2.00 = 0$

$A_y = 0$ **Ans.**

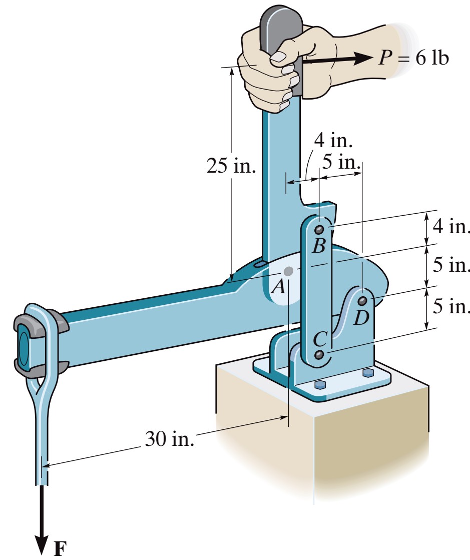
$\xrightarrow{+} \Sigma F_x = 0; \qquad 4.00 + 2.828 \cos 45° - A_x = 0$

$A_x = 6.00 \text{ kN}$ **Ans.**



6–97.

If a force of $P = 6\text{ lb}$ is applied perpendicular to the handle of the mechanism, determine the magnitude of force $\mathbf{F}$ for equilibrium. The members are pin connected at A, B, C, and D.



SOLUTION

$\zeta + \Sigma M_A = 0; \quad F_{BC}(4) - 6(25) = 0$

$F_{BC} = 37.5\text{ lb}$

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad -A_x + 6 = 0$

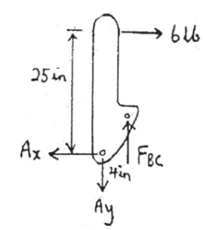
$A_x = 6\text{ lb}$

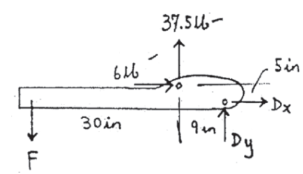
$+\uparrow \Sigma F_y = 0; \quad -A_y + 37.5 = 0$

$A_y = 37.5\text{ lb}$

$\zeta + \Sigma M_D = 0; \quad -5(6) - 37.5(9) + 39(F) = 0$

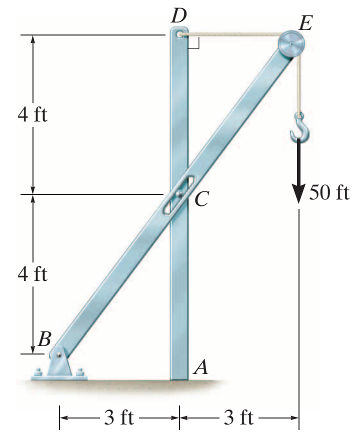
$F = 9.42\text{ lb}$ **Ans.**





6–98.

Determine the horizontal and vertical components of force at pin B and the normal force the pin at C exerts on the smooth slot. Also, determine the moment and horizontal and vertical reactions of force at A. There is a pulley at E.

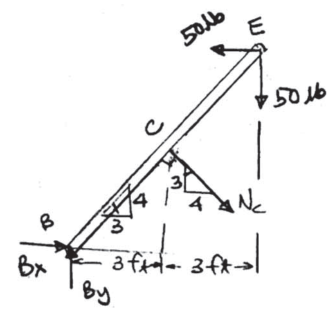


SOLUTION

BCE:

$\circlearrowleft + \Sigma M_B = 0; \quad -50(6) - N_C(5) + 50(8) = 0$

$N_C = 20 \text{ lb}$ **Ans.**



$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad B_x + 20\left(\frac{4}{5}\right) - 50 = 0$

$B_x = 34 \text{ lb}$ **Ans.**

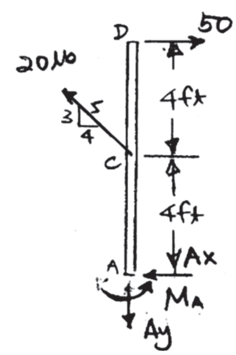
$+\uparrow \Sigma F_y = 0; \quad B_y - 20\left(\frac{3}{5}\right) - 50 = 0$

$B_y = 62 \text{ lb}$ **Ans.**

ACD:

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad -A_x - 20\left(\frac{4}{5}\right) + 50 = 0$

$A_x = 34 \text{ lb}$ **Ans.**



$+\uparrow \Sigma F_y = 0; \quad -A_y + 20\left(\frac{3}{5}\right) = 0$

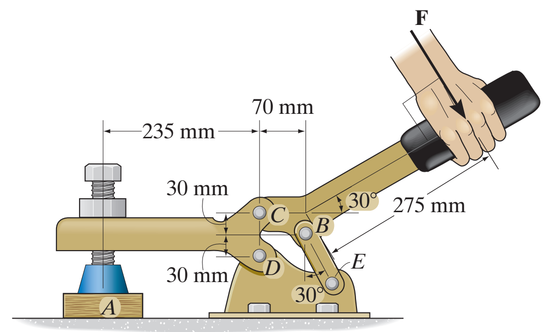
$A_y = 12 \text{ lb}$ **Ans.**

$\circlearrowleft + \Sigma M_A = 0; \quad M_A + 20\left(\frac{4}{5}\right)(4) - 50(8) = 0$

$M_A = 336 \text{ lb} \cdot \text{ft}$ **Ans.**

6–99.

If a clamping force of 300 N is required at A, determine the amount of force **F** that must be applied to the handle of the toggle clamp.

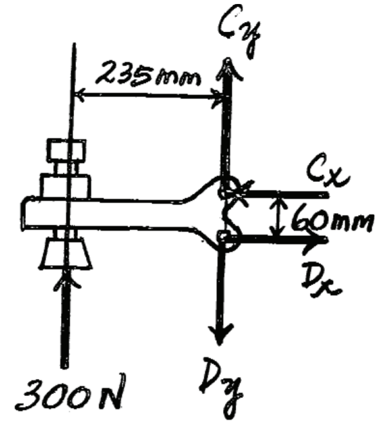


SOLUTION

Equations of Equilibrium: First, we will consider the free-body diagram of the clamp in Fig. a. Writing the moment equation of equilibrium about point D,

$$\circlearrowleft + \Sigma M_D = 0; \qquad C_x(60) - 300(235) = 0$$

$$C_x = 1175\text{ N}$$



(a)

Subsequently, the free - body diagram of the handle in Fig. b will be considered.

$$\circlearrowleft + \Sigma M_C = 0; \qquad F_{BE}\cos 30°(70) - F_{BE}\sin 30°(30) - F\cos 30°(275\cos 30° + 70)$$

$$-F\sin 30°(275\sin 30°) = 0$$

$$45.62F_{BE} - 335.62F = 0 \qquad \textbf{(1)}$$

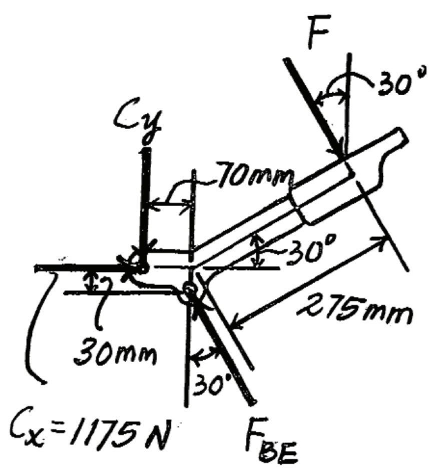
$$\xrightarrow{+} \Sigma F_x = 0; \qquad 1175 + F\sin 30° - F_{BE}\sin 30° = 0$$

$$0.5F_{BE} - 0.5F = 1175 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$F = 369.69\text{ N} = 370\text{ N} \qquad \textbf{Ans.}$$

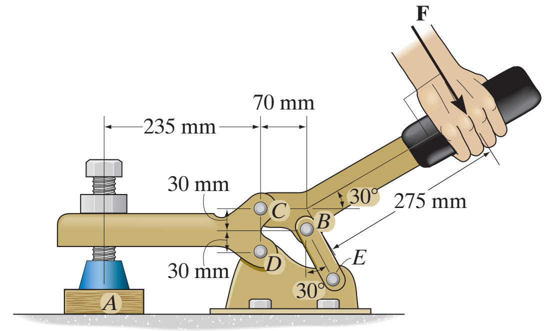
$$F_{BE} = 2719.69\text{N}$$



(b)

***6–100.**

If a force of $F = 350$ N is applied to the handle of the toggle clamp, determine the resulting clamping force at A.



SOLUTION

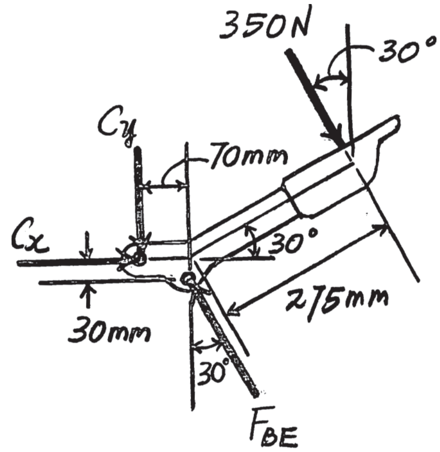
Equations of Equilibrium: First, we will consider the free-body diagram of the handle in Fig. *a*.

$$\circlearrowleft + \Sigma M_C = 0; \quad F_{BE}\cos 30°(70) - F_{BE}\sin 30°(30) - 350\cos 30°(275\cos 30° + 70) - 350\sin 30°(275\sin 30°) = 0$$

$$F_{BE} = 2574.81 \text{ N}$$

$$\xrightarrow{+} \Sigma F_x = 0; \quad C_x - 2574.81\sin 30° + 350\sin 30° = 0$$

$$C_x = 1112.41 \text{ N}$$

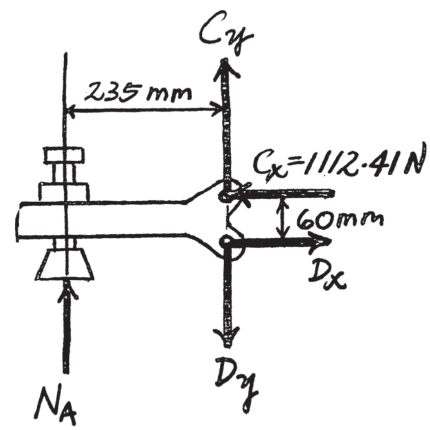


(a)

Subsequently, the free-body diagram of the clamp in Fig. *b* will be considered. Using the result of C_x and writing the moment equation of equilibrium about point D,

$$\circlearrowleft + \Sigma M_D = 0; \quad 1112.41(60) - N_A(235) = 0$$

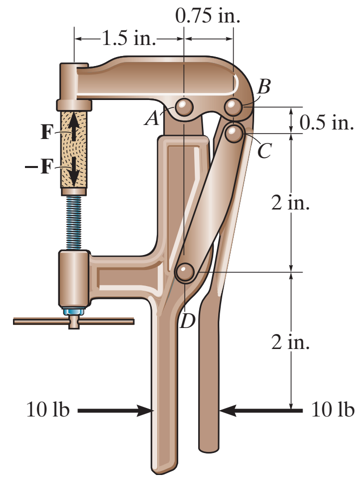
$$N_A = 284.01 \text{ N} = 284 \text{ N}$$ **Ans.**



(b)

6–101.

If a force of 10 lb is applied to the grip of the clamp, determine the compressive force F that the wood block exerts on the clamp.



SOLUTION

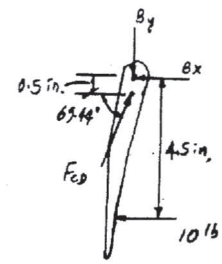
From FBD (a)

$\circlearrowleft + \Sigma M_B = 0; \qquad F_{CD} \cos 69.44°(0.5) - 10(4.5) = 0 \qquad F_{CD} = 256.32 \text{ lb}$

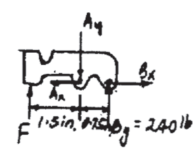
$+\uparrow \Sigma F_y = 0; \qquad 256.32 \sin 69.44° - B_y = 0 \qquad B_y = 240 \text{ lb}$

From FBD (b)

$\circlearrowleft + \Sigma M_A = 0; \qquad 240(0.75) - F(1.5) = 0 \qquad F = 120 \text{ lb}$ **Ans.**



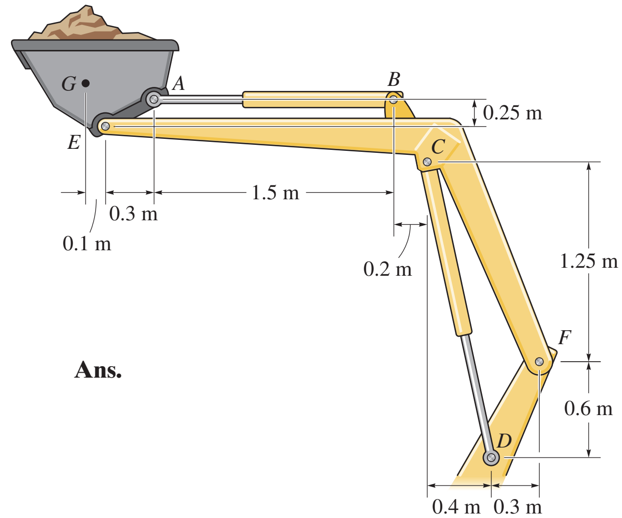
(a)



(b)

6–102.

The tractor boom supports the uniform mass of 500 kg in the bucket which has a center of mass at G. Determine the force in each hydraulic cylinder AB and CD and the resultant force at pins E and F. The load is supported equally on each side of the tractor by a similar mechanism.



SOLUTION

$\zeta + \Sigma M_E = 0; \qquad 2452.5(0.1) - F_{AB}(0.25) = 0$

$F_{AB} = 981 \text{ N}$ **Ans.**

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad -E_x + 981 = 0; \qquad E_x = 981 \text{ N}$

$+\uparrow \Sigma F_y = 0; \qquad E_y - 2452.5 = 0; \qquad E_y = 2452.5 \text{ N}$

$F_E = \sqrt{(981)^2 + (2452.5)^2} = 2.64 \text{ kN}$ **Ans.**

$\zeta + \Sigma M_F = 0; \qquad 2452.5(2.80) - F_{CD}(\cos 12.2°)(0.7) + F_{CD}(\sin 12.2°)(1.25) = 0$

$F_{CD} = 16\,349 \text{ N} = 16.3 \text{ kN}$ **Ans.**

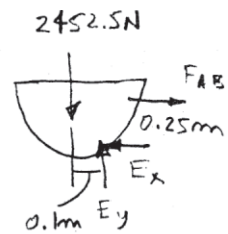
$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad F_x - 16\,349 \sin 12.2° = 0$

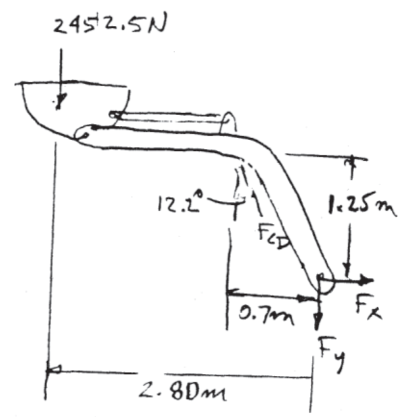
$F_x = 3455 \text{ N}$

$+\uparrow \Sigma F_y = 0; \qquad -F_y - 2452.5 + 16\,349 \cos 12.2° = 0$

$F_y = 13\,527 \text{ N}$

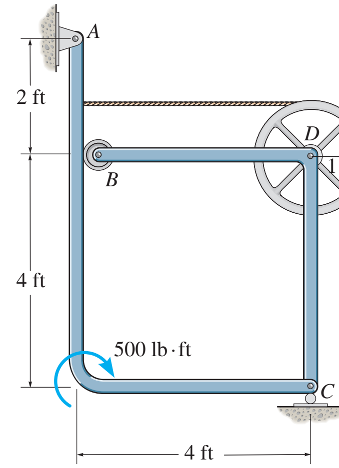
$F_F = \sqrt{(3455)^2 + (13\,527)^2} = 14.0 \text{ kN}$ **Ans.**





6–103.

The two-member frame supports the 200-lb cylinder and 500-lb · ft couple moment. Determine the force of the roller at B on member AC and the horizontal and vertical components of force which the pin at C exerts on member CB and the pin at A exerts on member AC. The roller C does not contact member CB.



SOLUTION

Equations of Equilibrium : From FBD (a),

$\zeta + \Sigma M_A = 0;\quad N_C(4) - 200(5) - 500 = 0 \quad N_C = 375 \text{ lb}$

$\overset{+}{\rightarrow} F_x = 0;\quad A_x = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0;\quad 375 - 200 - A_y = 0 \quad A_y = 175 \text{ lb}$ **Ans.**

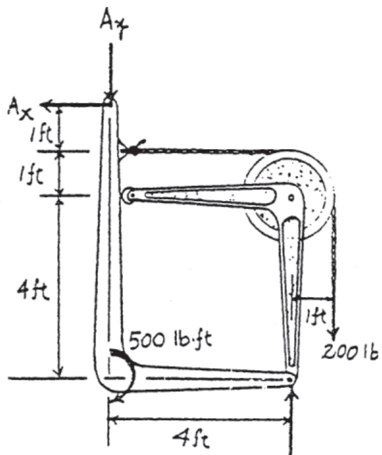
From FBD (b),

$\zeta + \Sigma M_C = 0;\quad 200(5) - 200(1) - B_x(4) = 0$

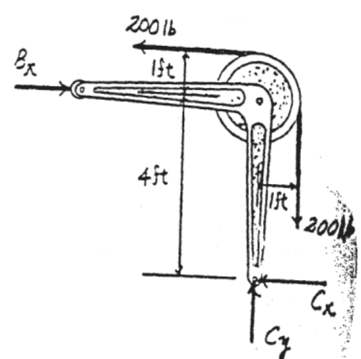
$B_x = 200 \text{ lb}$ **Ans.**

$\overset{+}{\rightarrow} F_x = 0;\quad 200 - 200 - C_x = 0 \quad C_x = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0;\quad C_y - 200 = 0 \quad C_y = 200 \text{ lb}$ **Ans.**



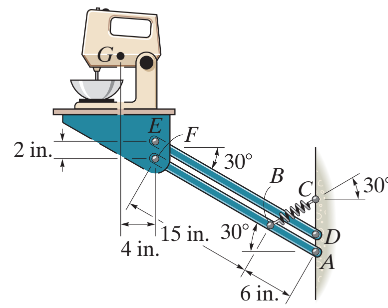
(a)



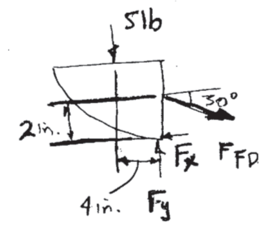
(b)

***6–104.**

The mechanism is used to hide kitchen appliances under a cabinet by allowing the shelf to rotate downward. If the mixer weighs 10 lb, is centered on the shelf, and has a mass center at G, determine the stretch in the spring necessary to hold the shelf in the equilibrium position shown. There is a similar mechanism on each side of the shelf, so that each mechanism supports 5 lb of the load. The springs each have a stiffness of $k = 4$ lb/in. spring.



SOLUTION



$\circlearrowleft + \Sigma M_F = 0;\qquad 5(4) - 2(F_{ED})(\cos 30°) = 0$

$F_{ED} = 11.547 \text{ lb}$

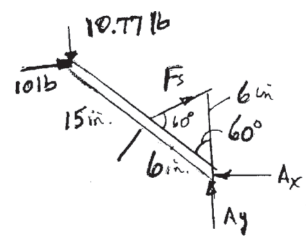
$\overset{+}{\rightarrow} \Sigma F_x = 0;\qquad -F_x + 11.547 \cos 30° = 0$

$F_x = 10.00 \text{ lb}$

$+\uparrow \Sigma F_y = 0;\qquad -5 + F_y - 11.547 \sin 30° = 0$

$F_y = 10.77 \text{ lb}$

Member FBA:



$\circlearrowleft + \Sigma M_A = 0;\qquad 10.77(21 \cos 30°) - 10(21 \sin 30°) - F_s(\sin 60°)(6) = 0$

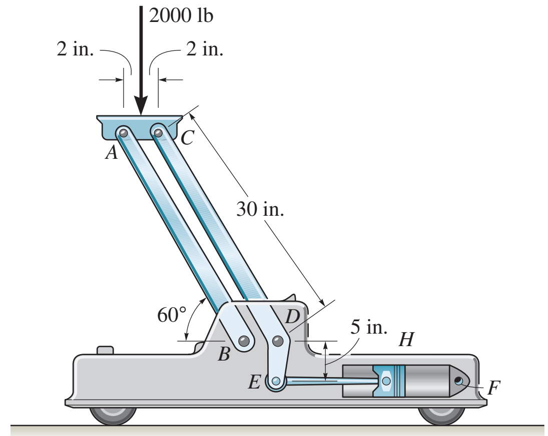
$F_s = 17.5 \text{ lb}$

$F_s = ks;\qquad 17.5 = 4x$

$x = 4.38 \text{ in.}$ **Ans.**

6–105.

The linkage for a hydraulic jack is shown. If the load on the jack is 2000 lb, determine the pressure acting on the fluid when the jack is in the position shown. All lettered points are pins. The piston at H has a cross-sectional area of $A = 2\ \text{in}^2$. *Hint:* First find the force F acting along link EH. The pressure in the fluid is $p = F/A$.



SOLUTION

$\circlearrowleft + \Sigma M_C = 0; \quad -F_{AB}(\sin 60^\circ)(4) + 2000(2) = 0$

$F_{AB} = 1154.70\ \text{lb}$

$\xrightarrow{+} \Sigma F_x = 0; \quad C_x - F_{AB} \cos 60^\circ = 0$

$C_x = 577.35\ \text{lb}$

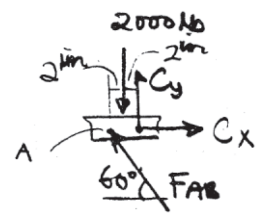
$+\uparrow \Sigma F_y = 0; \quad C_y + 1154.70 \sin 60^\circ - 2000 = 0$

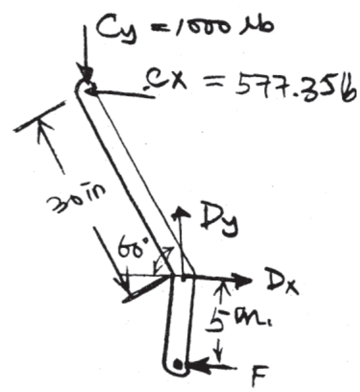
$C_y = 1000\ \text{lb}$

$\circlearrowleft + \Sigma M_D = 0; \quad -F(5) + 1000(30 \cos 60^\circ) + 577.35(30 \sin 60^\circ) = 0$

$F = 6000\ \text{lb}$

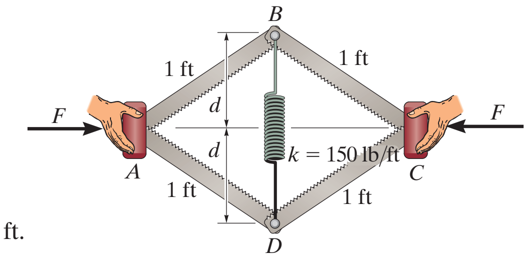
$$p = \frac{F}{A} = \frac{6000}{2} = 3000\ \text{psi}$$ **Ans.**





6–106.

If $d = 0.75$ ft and the spring has an unstretched length of 1 ft, determine the force F required for equilibrium.



SOLUTION

Spring Force Formula: The elongation of the spring is $x = 2(0.75) - 1 = 0.5$ ft. Thus, the force in the spring is given by

$$F_{sp} = kx = 150(0.5) = 75 \text{ lb}$$

Equations of Equilibrium: First, we will analyze the equilibrium of joint B. From the free-body diagram in Fig. a,

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad F_{AB} \cos 48.59° - F_{BC} \cos 48.59° = 0$$

$$F_{AB} = F_{BC} = F'$$

$$+\uparrow \Sigma F_y = 0; \qquad 2F' \sin 48.59° - 75 = 0$$

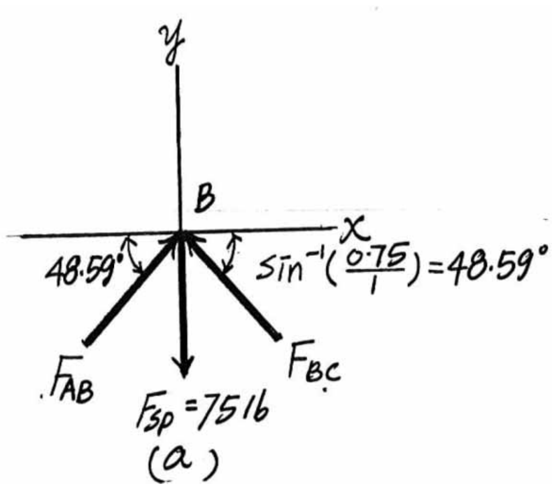
$$F' = 50 \text{ lb}$$

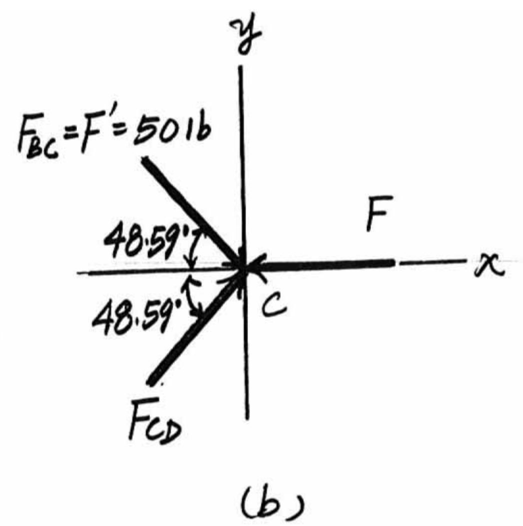
From the free-body diagram in Fig. b, using the result $F_{BC} = F' = 50$ lb, and analyzing the equilibrium of joint C, we have

$$+\uparrow \Sigma F_y = 0; \qquad F_{CD} \sin 48.59° - 50 \sin 48.59° = 0 \qquad F_{CD} = 50 \text{ lb}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 2(50 \cos 48.59°) - F = 0$$

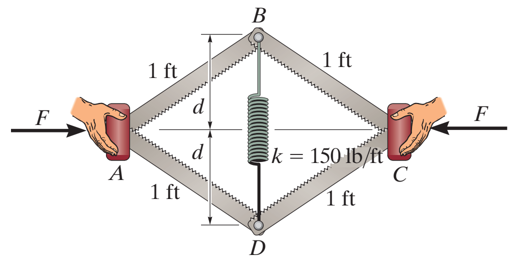
$$F = 66.14 \text{ lb} = 66.1 \text{ lbs}$$ **Ans.**





■6–107.

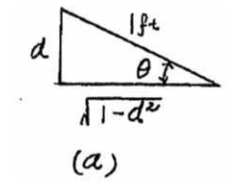
If a force of $F = 50$ lb is applied to the pads at A and C, determine the smallest dimension d required for equilibrium if the spring has an unstretched length of 1 ft.



SOLUTION

Geometry: From the geometry shown in Fig. *a*, we can write

$$\sin\theta = d \qquad \cos\theta = \sqrt{1 - d^2}$$



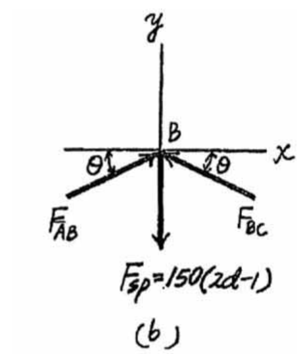
Spring Force Formula: The elongation of the spring is $x = 2d - 1$. Thus, the force in the spring is given by

$$F_{sp} = kx = 150(2d - 1)$$

***Equations of Equilibrium*:** First, we will analyze the equilibrium of joint B. From the free-body diagram in Fig. *b*,

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad F_{AB}\cos\theta - F_{BC}\cos\theta = 0 \qquad F_{AB} = F_{BC} = F'$$

$$+\uparrow\Sigma F_y = 0; \qquad 2F'(d) - 150(2d - 1) = 0 \qquad F' = \frac{150d - 75}{d}$$



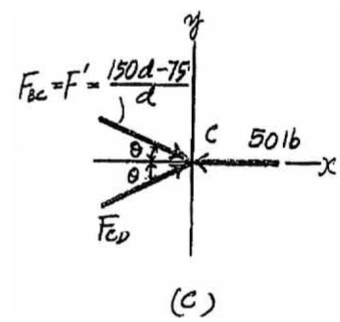
From the free-body diagram in Fig. *c*, using the result $F_{BC} = F' = \dfrac{150d - 75}{d}$, and analyzing the equilibrium of joint C, we have

$$+\uparrow\Sigma F_y = 0; \qquad F_{CD}\sin\theta - \left(\frac{150d - 75}{d}\right)\sin\theta = 0 \qquad F_{CD} = \frac{150d - 75}{d}$$

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad 2\left[\left(\frac{150d - 75}{d}\right)\left(\sqrt{1-d^2}\right)\right] - 50 = 0$$

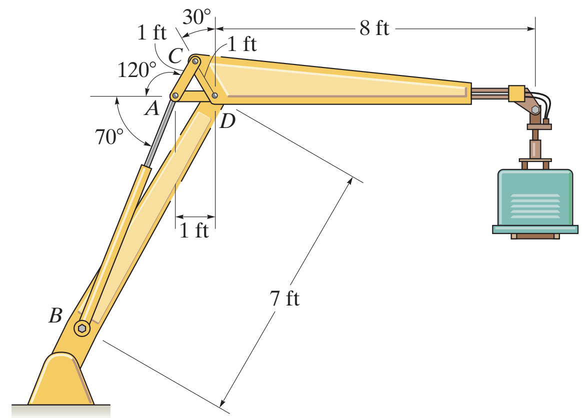
Solving the above equation using a graphing utility, we obtain **Ans.**

$d = 0.6381$ ft $= 0.638$ ft or $d = 0.9334$ ft $= 0.933$ ft



***6–108.**

The hydraulic crane is used to lift the 1400-lb load. Determine the force in the hydraulic cylinder AB and the force in links AC and AD when the load is held in the position shown.



SOLUTION

$\circlearrowleft + \Sigma M_D = 0;$ $\quad F_{CA}(\sin 60°)(1) - 1400(8) = 0$

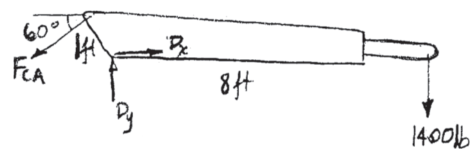
$F_{CA} = 12\,932.65 \text{ lb} = 12.9 \text{ kip}$ **Ans.**

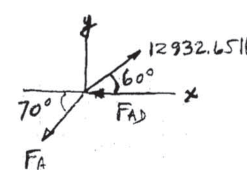
$+\uparrow \Sigma F_y = 0;$ $\quad 12\,932.65 \sin 60° - F_{AB} \sin 70° = 0$

$F_{AB} = 11\,918.79 \text{ lb} = 11.9 \text{ kip}$ **Ans.**

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad -11\,918.79 \cos 70° + 12\,932.65 \cos 60° - F_{AD} = 0$

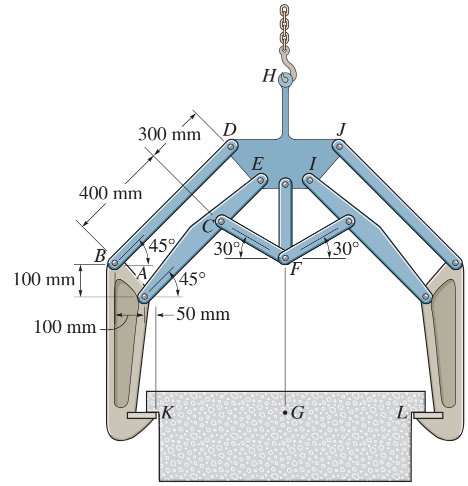
$F_{AD} = 2389.86 \text{ lb} = 2.39 \text{ kip}$ **Ans.**





6–109.

The symmetric coil tong supports the coil which has a mass of 800 kg and center of mass at G. Determine the horizontal and vertical components of force the linkage exerts on plate $DEIJH$ at points D and E. The coil exerts only vertical reactions at K and L.

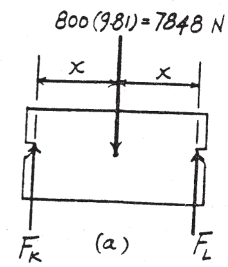


SOLUTION

Free-Body Diagram: The solution for this problem will be simplified if one realizes that links BD and CF are two-force members.

Equations of Equilibrium : From FBD (a),

$\circlearrowleft + \Sigma M_L = 0; \qquad 7848(x) - F_K(2x) = 0 \qquad F_K = 3924 \text{ N}$



From FBD (b),

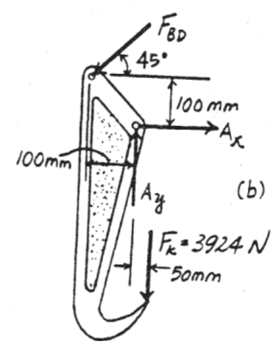
$\circlearrowleft + \Sigma M_A = 0; \qquad F_{BD}\cos 45°(100) + F_{BD}\sin 45°(100) - 3924(50) = 0$

$F_{BD} = 1387.34 \text{ N}$

$\xrightarrow{+} \Sigma F_x = 0; \qquad A_x - 1387.34\cos 45° = 0 \qquad A_x = 981 \text{ N}$

$+\uparrow \Sigma F_y = 0; \qquad A_y - 3924 - 1387.34\sin 45° = 0$

$A_y = 4905 \text{ N}$



From FBD (c),

$\circlearrowleft + \Sigma M_E = 0; \qquad 4905\sin 45°(700) - 981\sin 45°(700)$

$- F_{CF}\cos 15°(300) = 0$

$F_{CF} = 6702.66 \text{ N}$

$\xrightarrow{+} \Sigma F_x = 0; \qquad E_x - 981 - 6702.66\cos 30° = 0$

$E_x = 6785.67 \text{ N} = 6.79 \text{ kN}$ **Ans.**

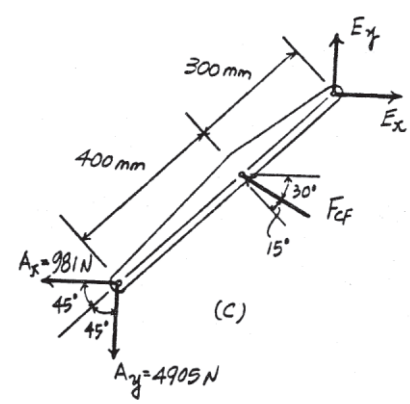
$+\uparrow \Sigma F_y = 0; \qquad E_y + 6702.66\sin 30° - 4905 = 0$

$E_y = 1553.67 \text{ N} = 1.55 \text{ kN}$ **Ans.**

At point D,

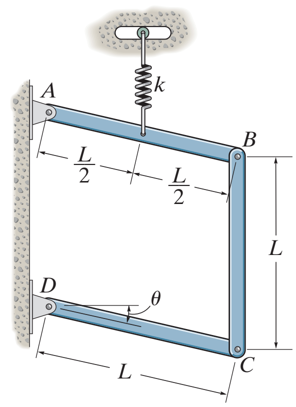
$D_x = F_{BD}\cos 45° = 1387.34\cos 45° = 981 \text{ N}$ **Ans.**

$D_y = F_{BD}\sin 45° = 1387.34\sin 45° = 981 \text{ N}$ **Ans.**



6–110.

If each of the three uniform links of the mechanism has a length $L = 3$ ft and weight of $W = 10$ lb, determine the angle θ for equilibrium. The spring has a stiffness of $k = 20$ lb/in. It always remains vertical due to the roller guide and is unstretched when $\theta = 0$.



SOLUTION

Equations of Equilibrium: Here, the spring stretches $x = 18 \sin\theta$. Thus, the force in the spring is $F_{sp} = kx = 20(18 \sin\theta) = 360 \sin\theta$. Referring to the *FBD* of member *BC* shown in Fig. *a*,

$\circlearrowleft + \uparrow \Sigma M_B = 0;$ $\qquad C_x = 0;$

then,

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\qquad B_x = 0;$

$+\uparrow \Sigma F_y = 0;$ $\qquad B_y - C_y - 10 = 0$ **(1)**

Referring to the *FBD* of member *CD* shown in Fig. *b*,

$\circlearrowleft + \Sigma M_D = 0;$ $\qquad C_y(36 \cos\theta) - 10(18 \cos\theta) = 0$

$$C_y = 5 \text{ lb}$$

Substitute this result into Eq (1),

$$B_y = 15 \text{ lb}$$

Referring to the *FBD* of member *AB* shown in Fig. *c*,

$\circlearrowleft + \Sigma M_A = 0;$ $\qquad (360 \sin\theta \cos\theta)(18) - 10 \cos\theta(18) - 15(36 \cos\theta) = 0$

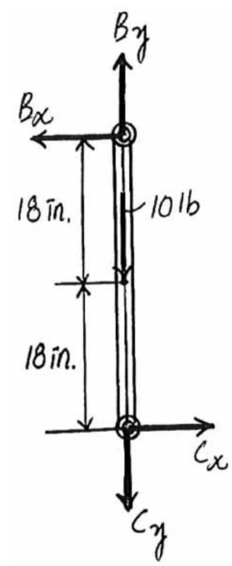
$$6480 \sin\theta \cos\theta - 180 \cos\theta - 540 \cos\theta = 0$$

Since $\cos\theta \neq 0$, then

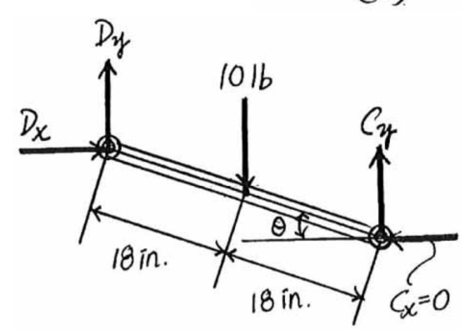
$$9 \sin\theta - 1 = 0$$

$$\sin\theta = \frac{1}{9}$$

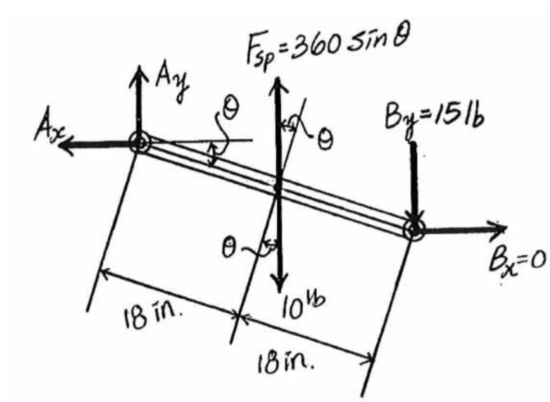
$$\theta = 6.38° \qquad \textbf{Ans.}$$



(a)



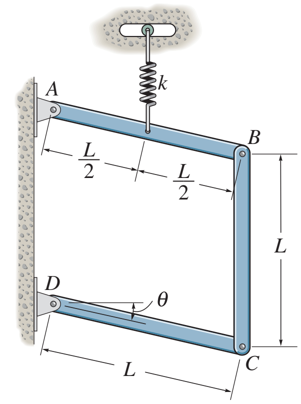
(b)



(c)

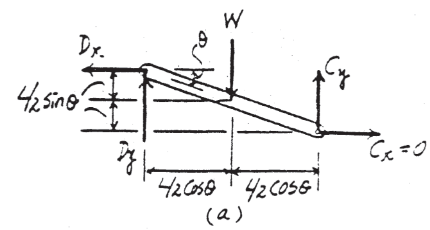
6–111.

If each of the three uniform links of the mechanism has a length L and weight W, determine the angle θ for equilibrium. The spring, which always remains vertical, is unstretched when $\theta = 0°$.



SOLUTION

Free Body Diagram: The spring stretches $x = \dfrac{L}{2}\,\theta$. Then, the spring force is $F_{sp} = kx = \dfrac{kL}{2}\sin\theta$.

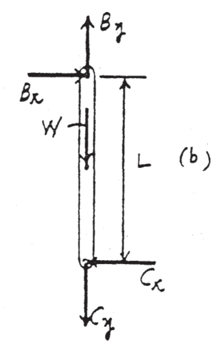


Equations of Equilibrium: From FBD (b),

$\circlearrowleft + \Sigma M_B = 0; \qquad C_x = 0$

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad B_x = 0$

$+\uparrow \Sigma F_y = 0; \qquad B_y - C_y - W = 0 \qquad$ **(1)**



From FBD (a),

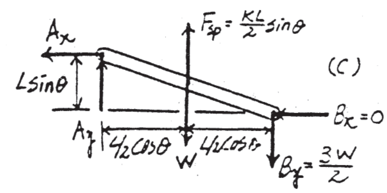
$$\circlearrowleft + \Sigma M_D = 0; \qquad C_y\,(L\cos\theta) - W\left(\frac{L}{2}\cos\theta\right) = 0$$

$$C_y = \frac{W}{2}$$

Substitute $C_y = \dfrac{W}{2}$ into Eq. (1), we have $B_y = \dfrac{3W}{2}$ from FBD (c),

$$\circlearrowleft + \Sigma M_A = 0; \qquad \frac{kL}{2}\sin\theta\left(\frac{L}{2}\cos\theta\right) - W\left(\frac{L}{2}\cos\theta\right) - \frac{3W}{2}(L\cos\theta) = 0$$

$$\theta = \sin^{-1}\left(\frac{8W}{kL}\right) \qquad \textbf{Ans.}$$



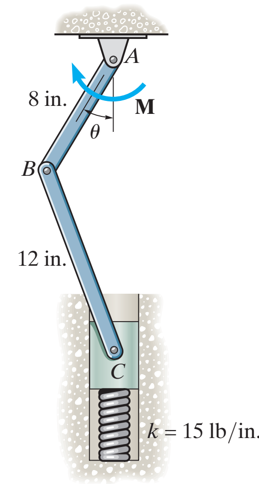
or

$$\cos\theta = 0$$

$$\theta = 90° \qquad \textbf{Ans.}$$

*6–112.

The piston C moves vertically between the two smooth walls. If the spring has a stiffness of $k = 15$ lb/in., and is unstretched when $\theta = 0°$, determine the couple **M** that must be applied to AB to hold the mechanism in equilibrium when $\theta = 30°$.



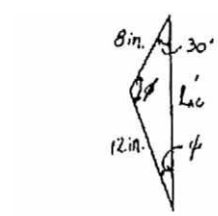
SOLUTION

Geometry:

$$\frac{\sin\psi}{8} = \frac{\sin 30°}{12} \qquad \psi = 19.47°$$

$$\phi = 180° - 30° - 19.47 = 130.53°$$

$$\frac{l'_{AC}}{\sin 130.53°} = \frac{12}{\sin 30°} \qquad l'_{AC} = 18.242 \text{ in.}$$

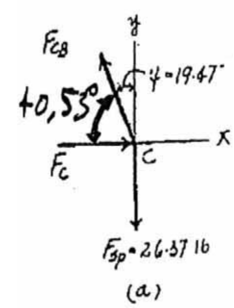


Free Body Diagram: The solution for this problem will be simplified if one realizes that member CB is a two force member. Since the spring stretches $x = l_{AC} - l'_{AC} = 20 - 18.242 = 1.758$ in. the spring force is $F_{sp} = kx = 15\,(1.758) = 26.37$ lb.

Equations of Equilibrium: Using the method of joints, [FBD (a)],

$$+\uparrow \Sigma F_y = 0; \qquad F_{CB}\cos 19.47° - 26.37 = 0$$

$$F_{CB} = 27.97 \text{ lb}$$



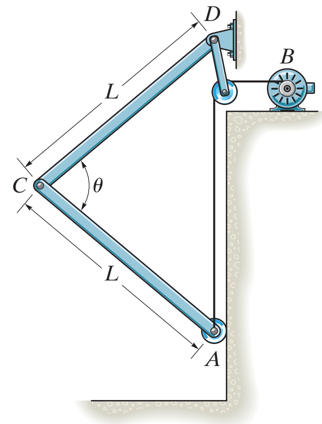
From FBD (b),

$$\circlearrowleft + \Sigma M_A = 0; \qquad 27.97\cos 40.53°\,(8) - M = 0$$

$$M = 170.08 \text{ lb}\cdot\text{in} = 14.2 \text{ lb}\cdot\text{ft}$$ **Ans.**

6–113.

The aircraft-hangar door opens and closes slowly by means of a motor, which draws in the cable AB. If the door is made in two sections (bifold) and each section has a uniform weight of 300 lb and height $L = 10$ ft, determine the force on the cable when $\theta = 90°$. The sections are pin connected at C and D and the bottom is attached to a roller that travels along the vertical track.

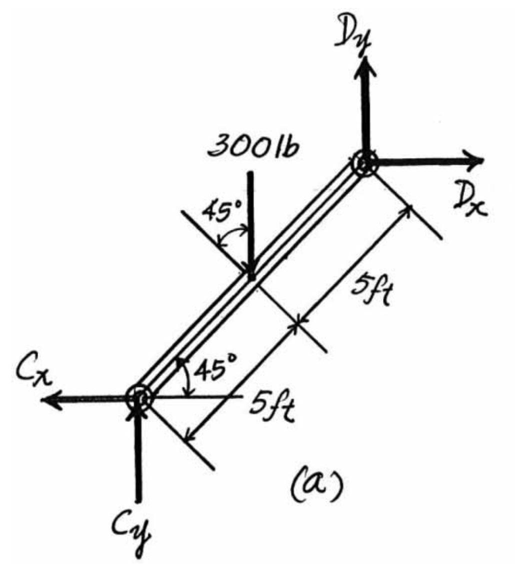


SOLUTION

Equations of Equilibrium: Referring to the *FBD* of member *CD* shown in Fig. *a*,

$$\zeta + \Sigma M_D = 0; \quad 300 \cos 45°(5) - C_y\,(10 \cos 45°) - C_x\,(10 \sin 45°) = 0$$

$$C_x + C_y = 150 \qquad \textbf{(1)}$$



Referring to the *FBD* of member *AC* shown in Fig. *b*

$$\zeta + \uparrow \Sigma M_A = 0; \quad 300 \cos 45°(5) + C_y\,(10 \cos 45°) - C_x\,(10 \sin 45°) = 0$$

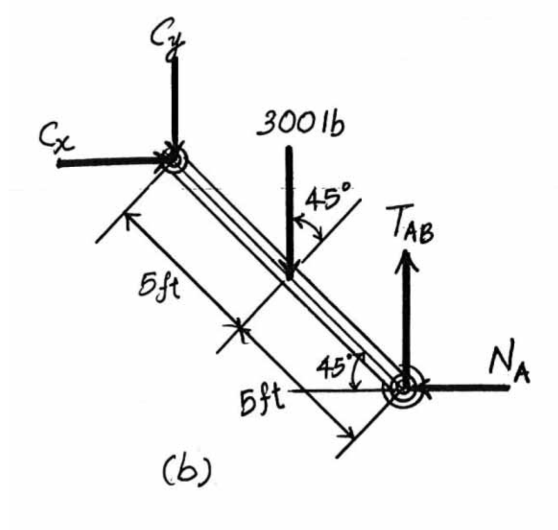
$$C_x - C_y = 150 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$C_x = 150 \text{ lb} \quad C_y = 0$$

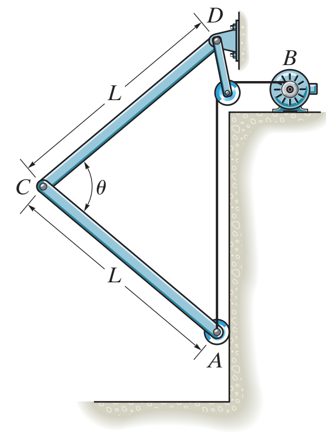
Using these results to write the force equations of equilibrium along y axis

$$+\uparrow \Sigma F_y = 0; \quad T_{AB} - 300 = 0 \qquad T_{AB} = 300 \text{ lb} \qquad \textbf{Ans.}$$



6–114.

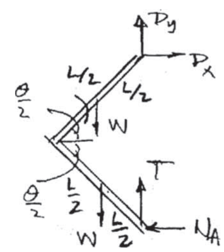
The aircraft-hangar door opens and closes slowly by means of a motor which draws in the cable AB. If the door is made in two sections (bifold) and each section has a uniform weight W and length L, determine the force in the cable as a function of the door's position θ. The sections are pin connected at C and D and the bottom is attached to a roller that travels along the vertical track.



SOLUTION

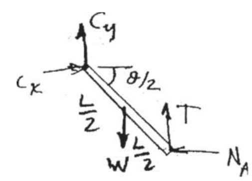
$\zeta + \Sigma M_D = 0; \qquad 2(W)\left(\frac{L}{2}\right)\cos\left(\frac{\theta}{2}\right) - 2L\left(\sin\left(\frac{\theta}{2}\right)\right)N_A = 0$

$$N_A = \frac{W}{2\tan\left(\frac{\theta}{2}\right)}$$



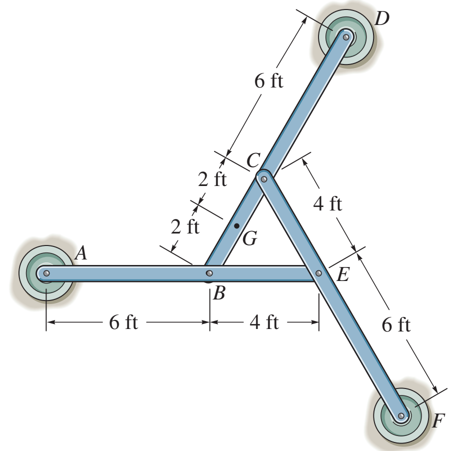
$\zeta + \Sigma M_C = 0; \qquad TL\left(\cos\left(\frac{\theta}{2}\right)\right) - \frac{W}{2\tan\left(\frac{\theta}{2}\right)}\left(L\sin\left(\frac{\theta}{2}\right)\right) - W\left(\frac{L}{2}\right)\left(\cos\left(\frac{\theta}{2}\right)\right) = 0$

$$T = W \qquad \textbf{Ans.}$$



6–115.

The three pin-connected members shown in the *top view* support a downward force of 60 lb at G. If only vertical forces are supported at the connections B, C, E and pad supports A, D, F, determine the reactions at each pad.



SOLUTION

Equations of Equilibrium : From FBD (a),

$\circlearrowleft + \Sigma M_D = 0;\qquad 60(8) + F_C(6) - F_B(10) = 0 \qquad$ **(1)**

$+\uparrow \Sigma F_y = 0;\qquad F_B + F_D - F_C - 60 = 0 \qquad$ **(2)**

From FBD (b),

$\circlearrowleft + \Sigma M_F = 0;\qquad F_E(6) - F_C(10) = 0 \qquad$ **(3)**

$+\uparrow \Sigma F_y = 0;\qquad F_C + F_F - F_E = 0 \qquad$ **(4)**

From FBD (c),

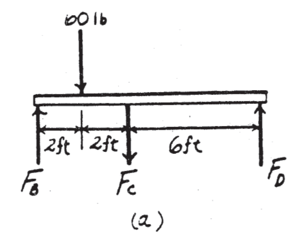
$\circlearrowleft + \Sigma M_A = 0;\qquad F_E(10) - F_B(6) = 0 \qquad$ **(5)**

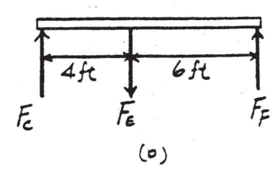
$+\uparrow \Sigma F_y = 0;\qquad F_A + F_E - F_B = 0 \qquad$ **(6)**

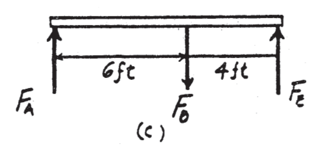
Solving Eqs. (1), (2), (3), (4), (5) and (6) yields,

$F_E = 36.73 \text{ lb} \qquad F_C = 22.04 \text{ lb} \qquad F_B = 61.22 \text{ lb}$

$F_D = 20.8 \text{ lb} \qquad F_F = 14.7 \text{ lb} \qquad F_A = 24.5 \text{ lb}$ **Ans.**

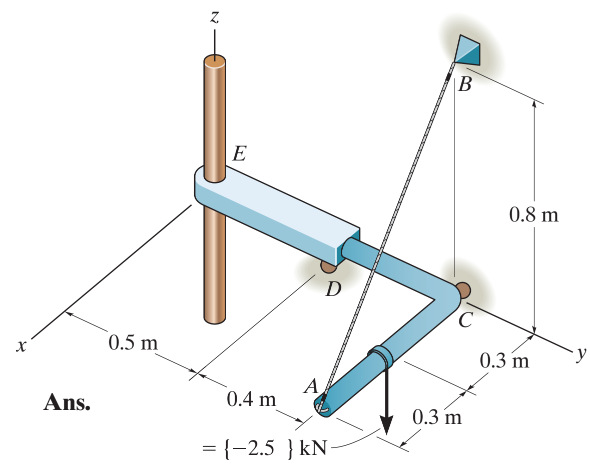






***6–116.**

The structure is subjected to the loading shown. Member AD is supported by a cable AB and roller at C and fits through a smooth circular hole at D. Member ED is supported by a roller at D and a pole that fits in a smooth snug circular hole at E. Determine the x, y, z components of reaction at E and the tension in cable AB.



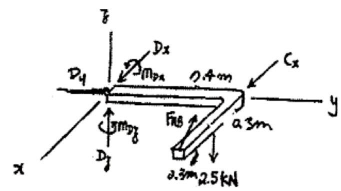
SOLUTION

$\Sigma M_y = 0;\quad -\frac{4}{5}F_{AB}(0.6) + 2.5(0.3) = 0$

$F_{AB} = 1.5625 = 1.56\text{ kN}$ **Ans.**

$\Sigma F_z = 0;\quad \frac{4}{5}(1.5625) - 2.5 + D_z = 0$

$D_z = 1.25\text{ kN}$



$\Sigma F_y = 0;\quad D_y = 0$

$\Sigma F_x = 0;\quad D_x + C_x - \frac{3}{5}(1.5625) = 0$ **(1)**

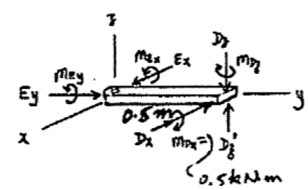
$\Sigma M_x = 0;\quad M_{Dx} + \frac{4}{5}(1.5625)(0.4) - 2.5(0.4) = 0$

$M_{Dx} = 0.5\text{ kN}\cdot\text{m}$

$\Sigma M_z = 0;\quad M_{Dz} + \frac{3}{5}(1.5625)(0.4) - C_x(0.4) = 0$ **(2)**

$\Sigma F_z = 0;\quad D_{z'} = 1.25\text{ kN}$

$\Sigma M_x = 0;\quad M_{Ex} = 0.5\text{ kN}\cdot\text{m}$ **Ans.**



$\Sigma M_y = 0;\quad M_{Ey} = 0$ **Ans.**

$\Sigma F_y = 0;\quad E_y = 0$ **Ans.**

$\Sigma M_z = 0;\quad D_x(0.5) - M_{Dz} = 0$ **(3)**

Solving Eqs. (1), (2) and (3):

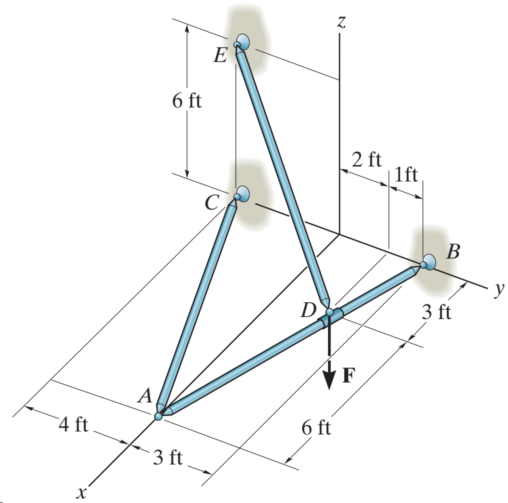
$C_x = 0.938\text{ kN}$

$M_{Dz} = 0$

$D_x = 0$

6–117.

The three-member frame is connected at its ends using ball-and-socket joints. Determine the x, y, z components of reaction at B and the tension in member ED. The force acting at D is $\mathbf{F} = \{135\mathbf{i} + 200\mathbf{j} - 180\mathbf{k}\}$ lb.



SOLUTION

AC is a two-force member.

$$\mathbf{F} = \{135\mathbf{i} + 200\mathbf{j} - 180\mathbf{k}\} \text{ lb}$$

$\Sigma M_y = 0;\qquad -\frac{6}{9}F_{DE}(3) + 180(3) = 0$

$F_{DE} = 270 \text{ lb}$ **Ans.**

$\Sigma F_z = 0;\qquad B_z + \frac{6}{9}(270) - 180 = 0$

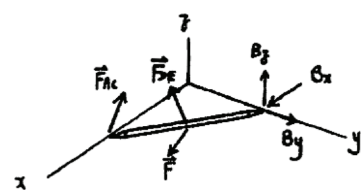
$B_z = 0$ **Ans.**

$\Sigma(M_B)_z = 0;\qquad -\frac{9}{\sqrt{97}}F_{AC}(3) - \frac{4}{\sqrt{97}}F_{AC}(9) + 135(1) + 200(3) - \frac{6}{9}(270)(3) - \frac{3}{9}(270)(1) = 0$

$F_{AC} = 16.41 \text{ lb}$

$\Sigma F_x = 0;\qquad 135 - \frac{3}{9}(270) + B_x - \frac{9}{\sqrt{97}}(16.41) = 0$

$B_x = -30 \text{ lb}$ **Ans.**

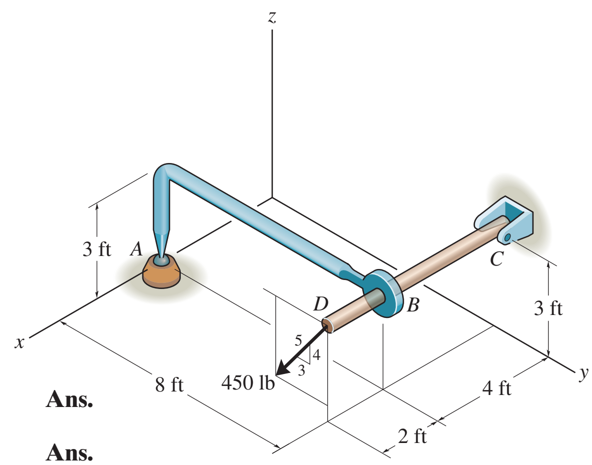


$\Sigma F_y = 0;\qquad B_y - \frac{4}{\sqrt{97}}(16.41) + 200 - \frac{6}{9}(270) = 0$

$B_y = -13.3 \text{ lb}$ **Ans.**

6–118.

The structure is subjected to the force of 450 lb which lies in a plane parallel to the y-z plane. Member AB is supported by a ball-and-socket joint at A and fits through a snug hole at B. Member CD is supported by a pin at C. Determine the x, y, z components of reaction at A and C.



SOLUTION

$\Sigma M_x = 0;\qquad M_{Cx} = 0$ **Ans.**

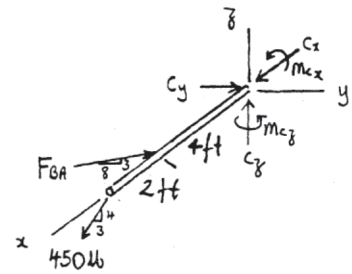
$\Sigma F_x = 0;\qquad C_x = 0$ **Ans.**

$$\Sigma F_y = 0;\qquad -450\left(\frac{3}{5}\right) + F_{BA}\left(\frac{8}{\sqrt{73}}\right) + C_y = 0$$

$$\Sigma F_z = 0;\qquad C_z + F_{BA}\left(\frac{3}{\sqrt{73}}\right) - 450\left(\frac{4}{5}\right) = 0$$

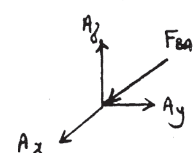
$$\Sigma M_y = 0;\qquad 450\left(\frac{4}{5}\right)(6) - F_{BA}\left(\frac{3}{\sqrt{73}}\right)(4) = 0$$

$$\Sigma M_z = 0;\qquad M_{Cz} + F_{BA}\left(\frac{8}{\sqrt{73}}\right)(4) - 450\left(\frac{3}{5}\right)(6) = 0$$



$F_{BA} = 1.538 \text{ kip} = 1.54 \text{ kip}$ **Ans.**

$C_z = -0.18 \text{ kip}$ **Ans.**



$C_y = -1.17 \text{ kip}$ **Ans.**

$M_{Cz} = -4.14 \text{ kip} \cdot \text{ft}$ **Ans.**

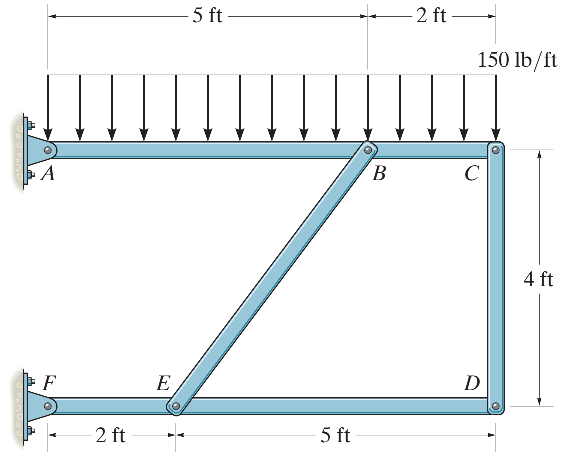
$A_x = 0$ **Ans.**

$$A_y = 1.538\left(\frac{8}{\sqrt{73}}\right) = 1.44 \text{ kip}$$ **Ans.**

$$A_z = 1.538\left(\frac{3}{\sqrt{73}}\right) = 0.540 \text{ kip}$$ **Ans.**

6–119.

Determine the resultant forces at pins B and C on member ABC of the four-member frame.



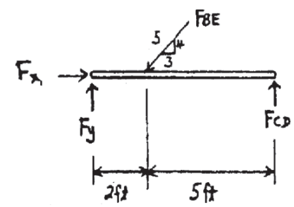
SOLUTION

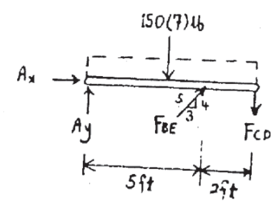
$\zeta + \Sigma M_F = 0; \qquad F_{CD}(7) - \frac{4}{5}F_{BE}(2) = 0$

$\zeta + \Sigma M_A = 0; \qquad -150(7)(3.5) + \frac{4}{5}F_{BE}(5) - F_{CD}(7) = 0$

$F_{BE} = 1531\text{ lb} = 1.53\text{ kip}$ **Ans.**

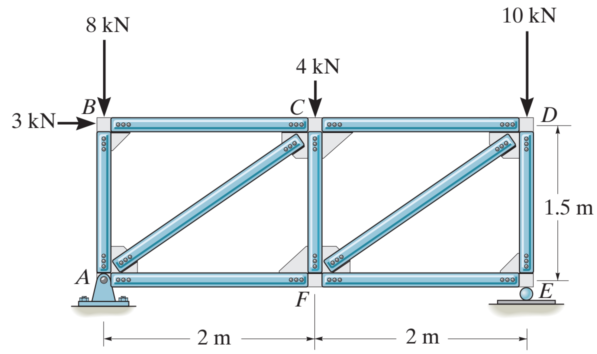
$F_{CD} = 350\text{ lb}$ **Ans.**





***6–120.**

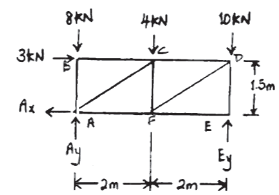
Determine the force in each member of the truss and state if the members are in tension or compression.



SOLUTION

$\zeta + \Sigma M_A = 0; \qquad -3(1.5) - 4(2) - 10(4) + E_y(4) = 0$

$E_y = 13.125 \text{ kN}$

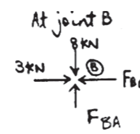


$+\uparrow \Sigma F_y = 0; \qquad A_y - 8 - 4 - 10 + 13.125 = 0$

$A_y = 8.875 \text{ kN}$

$+\uparrow \Sigma F_x = 0; \qquad A_x = 3 \text{ kN}$

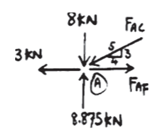
Joint *B*:



$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad F_{BC} = 3 \text{ kN (C)}$ **Ans.**

$+\uparrow \Sigma F_y = 0; \qquad F_{BA} = 8 \text{ kN (C)}$ **Ans.**

Joint *A*:



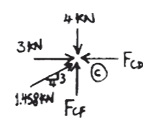
$+\uparrow \Sigma F_y = 0; \qquad 8.875 - 8 - \frac{3}{5} F_{AC} = 0$

$F_{AC} = 1.458 = 1.46 \text{ kN (C)}$ **Ans.**

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad F_{AF} - 3 - \frac{4}{5}(1.458) = 0$

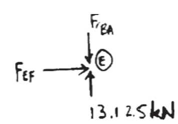
$F_{AF} = 4.17 \text{ kN (T)}$ **Ans.**

Joint *C*:



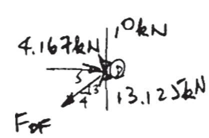
$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 3 + \frac{4}{5}(1.458) - F_{CD} = 0$

$F_{CD} = 4.167 = 4.17 \text{ kN (C)}$ **Ans.**



$+\uparrow \Sigma F_y = 0; \qquad F_{CF} - 4 + \frac{3}{5}(1.458) = 0$

$F_{CF} = 3.125 = 3.12 \text{ kN (C)}$ **Ans.**



Joint *E*:

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad F_{EF} = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0; \qquad F_{ED} = 13.125 = 13.1 \text{ kN (C)}$ **Ans.**

Joint *D*:

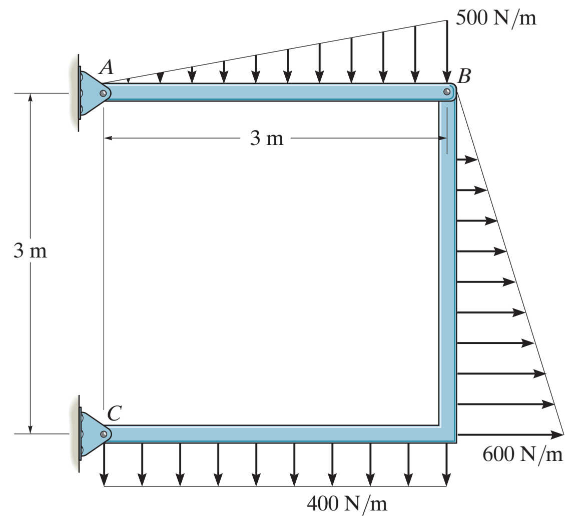
$+\uparrow \Sigma F_y = 0; \qquad 13.125 - 10 - \frac{3}{5} F_{DF} = 0$

$F_{DF} = 5.21 \text{ kN (T)}$ **Ans.**

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 4.167 - \frac{4}{5}(5.21) = 0$ **Check!**

6–121.

Determine the horizontal and vertical components of force at pins A and C of the two-member frame.



SOLUTION

Member AB:

$\zeta + \Sigma M_A = 0; \quad -750(2) + B_y(3) = 0$

$B_y = 500 \text{ N}$

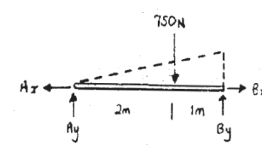
Member BC:

$\zeta + \Sigma M_C = 0; \quad -1200(1.5) - 900(1) + B_x(3) - 500(3) = 0$

$B_x = 1400 \text{ N}$

$+\uparrow \Sigma F_y = 0; \quad A_y - 750 + 500 = 0$

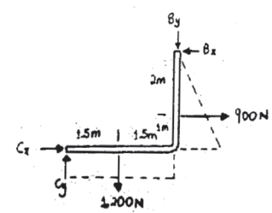
$A_y = 250 \text{ N}$ **Ans.**



Member AB:

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad -A_x + 1400 = 0$

$A_x = 1400 \text{ N} = 1.40 \text{ kN}$ **Ans.**



Member BC:

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad C_x + 900 - 1400 = 0$

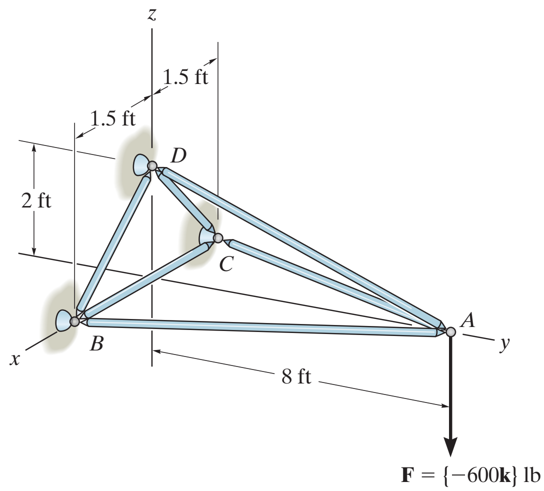
$C_x = 500 \text{ N}$ **Ans.**

$+\uparrow \Sigma F_y = 0; \quad -500 - 1200 + C_y = 0$

$C_y = 1700 \text{ N} = 1.70 \text{ kN}$ **Ans.**

6–122.

Determine the force in members *AB, AD*, and *AC* of the space truss and state if the members are in tension or compression.



SOLUTION

Method of Joints: In this case the support reactions are not required for determining the member forces.

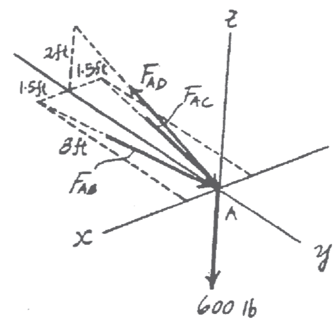
Joint *A*:

$$\Sigma F_z = 0; \qquad F_{AD}\left(\frac{2}{\sqrt{68}}\right) - 600 = 0$$

$$F_{AD} = 2473.86 \text{ lb (T)} = 2.47 \text{ kip (T)} \qquad \textbf{Ans.}$$

$$\Sigma F_x = 0; \qquad F_{AC}\left(\frac{1.5}{\sqrt{66.25}}\right) - F_{AB}\left(\frac{1.5}{\sqrt{66.25}}\right) = 0$$

$$F_{AC} = F_{AB} \qquad \textbf{(1)}$$



$$\Sigma F_y = 0; \qquad F_{AC}\left(\frac{8}{\sqrt{66.25}}\right) + F_{AB}\left(\frac{8}{\sqrt{66.25}}\right) - 2473.86\left(\frac{8}{\sqrt{68}}\right) = 0$$

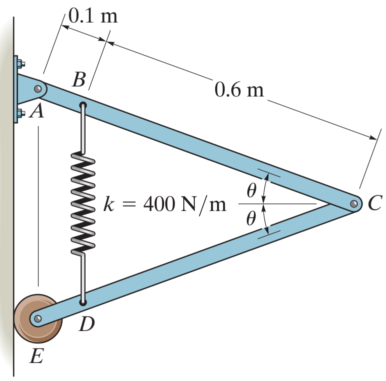
$$0.9829\,F_{AC} + 0.9829\,F_{AB} = 2400 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

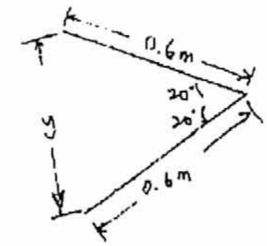
$$F_{AC} = F_{AB} = 1220.91 \text{ lb (C)} = 1.22 \text{ kip (C)} \qquad \textbf{Ans.}$$

6–123.

The spring has an unstretched length of 0.3 m. Determine the mass m of each uniform link if the angle $\theta = 20°$ for equilibrium.



SOLUTION



$$\frac{y}{2(0.6)} = \sin 20°$$

$$y = 1.2 \sin 20°$$

$$F_s = (1.2 \sin 20° - 0.3)(400) = 44.1697 \text{ N}$$

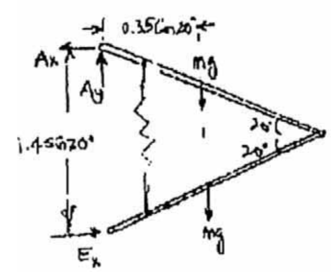
$$\zeta + \Sigma M_A = 0; \qquad E_x(1.4 \sin 20°) - 2(mg)(0.35 \cos 20°) = 0$$

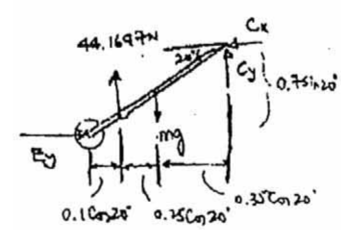
$$E_x = 1.37374(mg)$$

$$\zeta + \Sigma M_C = 0; \qquad 1.37374mg(0.7 \sin 20°) + mg(0.35 \cos 20°) - 44.1697(0.6 \cos 20°) = 0$$

$$mg = 37.860$$

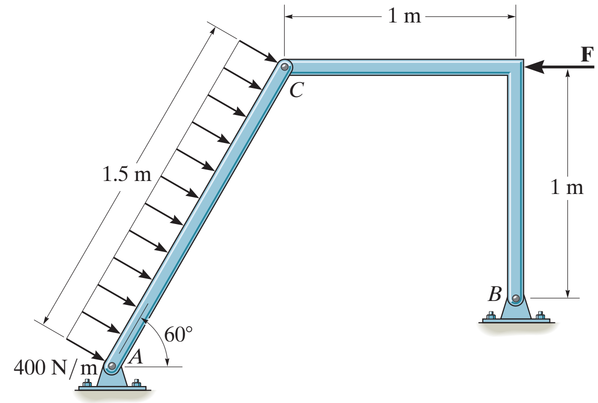
$$m = 37.860/9.81 = 3.86 \text{ kg} \qquad \textbf{Ans.}$$





***6–124.**

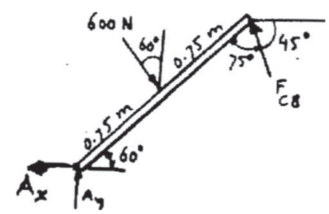
Determine the horizontal and vertical components of force that the pins A and B exert on the two-member frame. Set $F = 0$.



SOLUTION

CB is a two-force member.

Member AC:



$\circlearrowleft + \Sigma M_A = 0; \quad -600\,(0.75) + 1.5\,(F_{CB} \sin 75°) = 0$

$F_{CB} = 310.6$

Thus,

$$B_x = B_y = 310.6\left(\frac{1}{\sqrt{2}}\right) = 220 \text{ N}$$ **Ans.**

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad -A_x + 600 \sin 60° - 310.6 \cos 45° = 0$

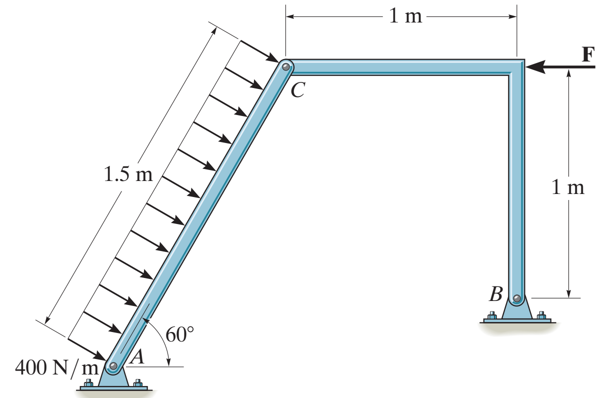
$A_x = 300 \text{ N}$ **Ans.**

$+\uparrow \Sigma F_y = 0; \quad A_y - 600 \cos 60° + 310.6 \sin 45° = 0$

$A_y = 80.4 \text{ N}$ **Ans.**

6–125.

Determine the horizontal and vertical components of force that pins A and B exert on the two-member frame. Set $F = 500$ N.



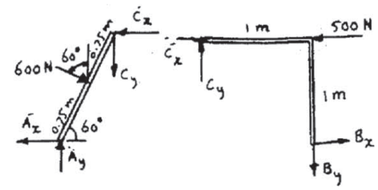
SOLUTION

Member AC:

$\zeta + \Sigma M_A = 0; \qquad -600\,(0.75) - C_y\,(1.5 \cos 60°) + C_x\,(1.5 \sin 60°) = 0$

Member CB:

$\zeta + \Sigma M_B = 0; \qquad -C_x\,(1) - C_y\,(1) + 500\,(1) = 0$



Solving,

$C_x = 402.6$ N

$C_y = 97.4$ N

Member AC:

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad -A_x + 600 \sin 60° - 402.6 = 0$

$A_x = 117$ N **Ans.**

$+\uparrow \Sigma F_y = 0; \qquad A_y - 600 \cos 60° - 97.4 = 0$

$A_y = 397$ N **Ans.**

Member CB:

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 402.6 - 500 + B_x = 0$

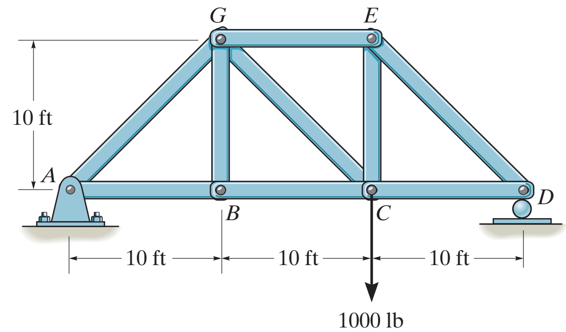
$B_x = 97.4$ N **Ans.**

$+\uparrow \Sigma F_y = 0; \qquad -B_y + 97.4 = 0$

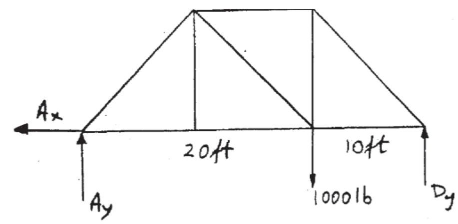
$B_y = 97.4$ N **Ans.**

6–126.

Determine the force in each member of the truss and state if the members are in tension or compression.



SOLUTION



$\zeta + \Sigma M_A = 0; \quad D_y(30) - 1000(20) = 0$

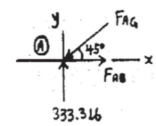
$D_y = 666.7 \text{ lb}$

$\xrightarrow{+} \Sigma F_x = 0; \quad A_x = 0$

$+\uparrow \Sigma F_y = 0; \quad A_y - 1000 + 666.7 = 0$

$A_y = 333.3 \text{ lb}$

Joint A:



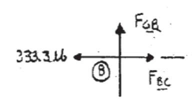
$\xrightarrow{+} \Sigma F_x = 0; \quad F_{AB} - F_{AG} \cos 45° = 0$

$+\uparrow \Sigma F_y = 0; \quad 333.3 - F_{AG} \sin 45° = 0$

$F_{AG} = 471 \text{ lb (C)}$ **Ans.**

$F_{AB} = 333.3 = 333 \text{ lb (T)}$ **Ans.**

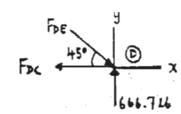
Joint B:



$\xrightarrow{+} \Sigma F_x = 0; \quad F_{BC} = 333.3 = 333 \text{ lb (T)}$ **Ans.**

$+\uparrow \Sigma F_y = 0; \quad F_{GB} = 0$ **Ans.**

Joint D:



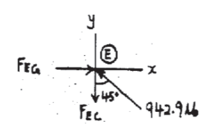
$\xrightarrow{+} \Sigma F_x = 0; \quad -F_{DC} + F_{DE} \cos 45° = 0$

$+\uparrow \Sigma F_y = 0; \quad 666.7 - F_{DE} \sin 45° = 0$

$F_{DE} = 942.9 \text{ lb} = 943 \text{ lb (C)}$ **Ans.**

$F_{DC} = 666.7 \text{ lb} = 667 \text{ lb (T)}$ **Ans.**

Joint E:



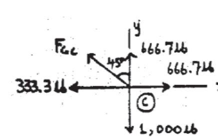
$\xrightarrow{+} \Sigma F_x = 0; \quad -942.9 \sin 45° + F_{EG} = 0$

$+\uparrow \Sigma F_y = 0; \quad -F_{EC} + 942.9 \cos 45° = 0$

$F_{EC} = 666.7 \text{ lb} = 667 \text{ lb (T)}$ **Ans.**

$F_{EG} = 666.7 \text{ lb} = 667 \text{ lb (C)}$ **Ans.**

Joint C:

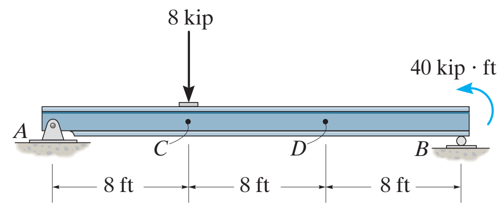


$+\uparrow \Sigma F_y = 0; \quad F_{GC} \cos 45° + 666.7 - 1000 = 0$

$F_{GC} = 471 \text{ lb (T)}$ **Ans.**

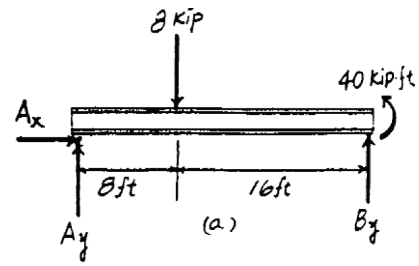
7–1.

Determine the internal normal force and shear force, and the bending moment in the beam at points *C* and *D*. Assume the support at *B* is a roller. Point *C* is located just to the right of the 8-kip load.



SOLUTION

Support Reactions: FBD (a).



$\circlearrowleft + \Sigma M_A = 0; \quad B_y(24) + 40 - 8(8) = 0 \quad B_y = 1.00 \text{ kip}$

$+\uparrow \Sigma F_y = 0; \quad A_y + 1.00 - 8 = 0 \quad A_y = 7.00 \text{ kip}$

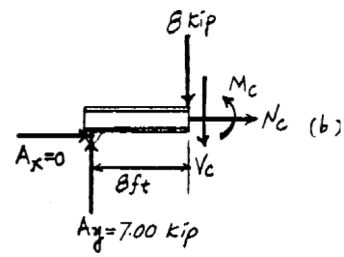
$\xrightarrow{+} \Sigma F_x = 0 \quad A_x = 0$

Internal Forces: Applying the equations of equilibrium to segment *AC* [FBD (b)], we have

$\xrightarrow{+} \Sigma F_x = 0 \quad N_C = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0; \quad 7.00 - 8 - V_C = 0 \quad V_C = -1.00 \text{ kip}$ **Ans.**

$\circlearrowleft + \Sigma M_C = 0; \quad M_C - 7.00(8) = 0 \quad M_C = 56.0 \text{ kip} \cdot \text{ft}$ **Ans.**



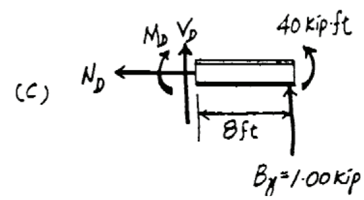
Applying the equations of equilibrium to segment *BD* [FBD (c)], we have

$\xrightarrow{+} \Sigma F_x = 0; \quad N_D = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0; \quad V_D + 1.00 = 0 \quad V_D = -1.00 \text{ kip}$ **Ans.**

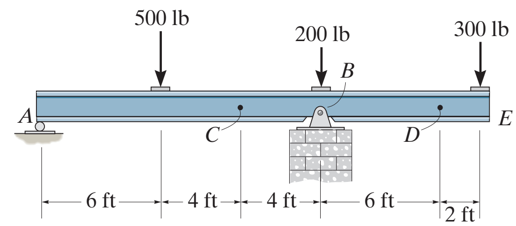
$\circlearrowleft + \Sigma M_D = 0; \quad 1.00(8) + 40 - M_D = 0$

$M_D = 48.0 \text{ kip} \cdot \text{ft}$ **Ans.**



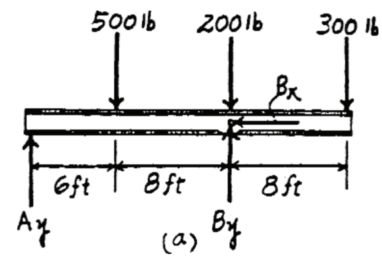
7–2.

Determine the shear force and moment at points C and D.



SOLUTION

Support Reactions: FBD (a).



$\curvearrowleft +\Sigma M_B = 0; \qquad 500(8) - 300(8) - A_y(14) = 0$

$A_y = 114.29 \text{ lb}$

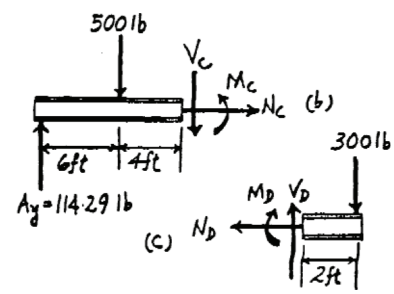
Internal Forces: Applying the equations of equilibrium to segment AC [FBD (b)], we have

$\overset{+}{\rightarrow} \Sigma F_x = 0 \qquad N_C = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0; \qquad 114.29 - 500 - V_C = 0 \qquad V_C = -386 \text{ lb}$ **Ans.**

$\curvearrowleft +\Sigma M_C = 0; \qquad M_C + 500(4) - 114.29(10) = 0$

$M_C = -857 \text{ lb} \cdot \text{ft}$ **Ans.**



Applying the equations of equilibrium to segment ED [FBD (c)], we have

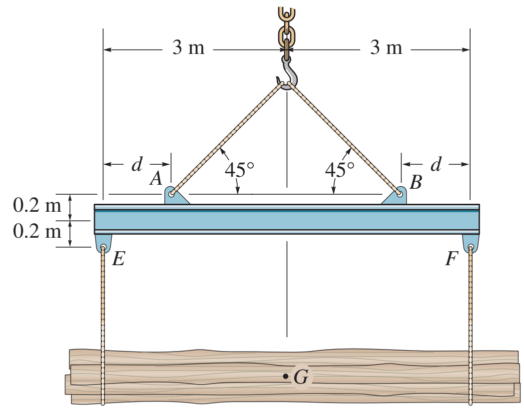
$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad N_D = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0; \qquad V_D - 300 = 0 \qquad V_D = 300 \text{ lb}$ **Ans.**

$\curvearrowleft +\Sigma M_D = 0; \qquad -M_D - 300(2) = 0 \qquad M_D = -600 \text{ lb} \cdot \text{ft}$ **Ans.**

7–3.

The strongback or lifting beam is used for materials handling. If the suspended load has a weight of 2 kN and a center of gravity of G, determine the placement d of the padeyes on the top of the beam so that there is no moment developed within the length AB of the beam. The lifting bridle has two legs that are positioned at 45°, as shown.

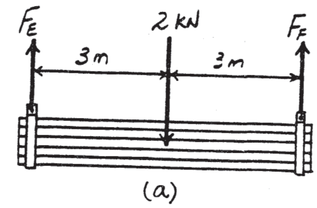


SOLUTION

Support Reactions: From FBD (a),

$$\circlearrowleft + \Sigma M_E = 0; \qquad F_F(6) - 2(3) = 0 \qquad F_E = 1.00 \text{ kN}$$

$$+\uparrow \Sigma F_y = 0; \qquad F_F + 1.00 - 2 = 0 \qquad F_F = 1.00 \text{ kN}$$



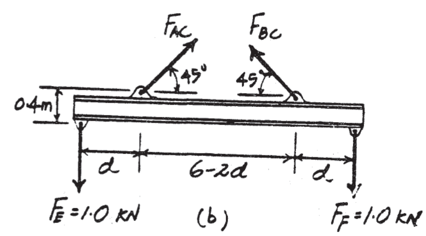
From FBD (b),

$$\xrightarrow{+} \Sigma F_x = 0; \qquad F_{AC} \cos 45° - F_{BC} \cos 45° = 0 \qquad F_{AC} = F_{BC} = F$$

$$+\uparrow \Sigma F_y = 0; \qquad 2F \sin 45° - 1.00 - 1.00 = 0$$

$$F_{AC} = F_{BC} = F = 1.414 \text{ kN}$$

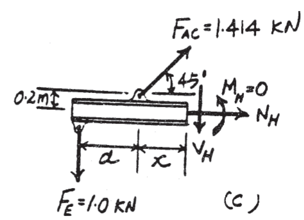
Internal Forces: This problem requires $M_H = 0$. Summing moments about point H of segment EH [FBD (c)], we have



$$\circlearrowleft + \Sigma M_H = 0; \qquad 1.00(d + x) - 1.414 \sin 45°(x)$$

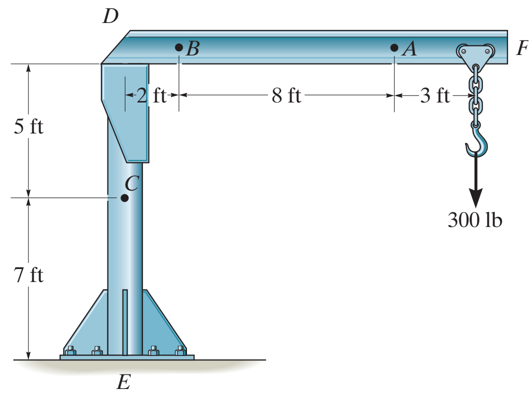
$$- 1.414 \cos 45°(0.2) = 0$$

$$d = 0.200 \text{ m} \qquad \textbf{Ans.}$$



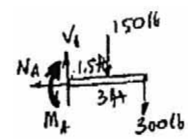
***7–4.**

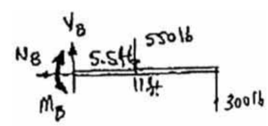
The boom DF of the jib crane and the column DE have a uniform weight of 50 lb/ft. If the hoist and load weigh 300 lb, determine the normal force, shear force, and moment in the crane at sections passing through points A, B, and C.

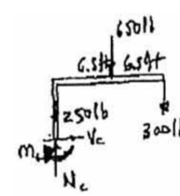


SOLUTION

$\overset{+}{\rightarrow} \Sigma F_x = 0;$	$N_A = 0$	**Ans.**
$+\uparrow \Sigma F_y = 0;$	$V_A - 450 = 0; \qquad V_A = 450 \text{ lb}$	**Ans.**
$\circlearrowleft + \Sigma M_A = 0;$	$-M_A - 150(1.5) - 300(3) = 0; \qquad M_A = -1125 \text{ lb} \cdot \text{ft}$	**Ans.**
$\overset{+}{\rightarrow} \Sigma F_x = 0;$	$N_B = 0$	**Ans.**
$+\uparrow \Sigma F_y = 0;$	$V_B - 550 - 300 = 0; \qquad V_B = 850 \text{ lb}$	**Ans.**
$\circlearrowleft + \Sigma M_B = 0;$	$-M_B - 550(5.5) - 300(11) = 0; \qquad M_B = -6325 \text{ lb} \cdot \text{ft}$	**Ans.**
$\overset{+}{\rightarrow} \Sigma F_x = 0;$	$V_C = 0$	**Ans.**
$+\uparrow \Sigma F_y = 0;$	$N_C - 650 - 300 - 250 = 0; \qquad N_C = 1200 \text{ lb}$	**Ans.**
$\circlearrowleft + \Sigma M_C = 0;$	$-M_C - 650(6.5) - 300(13) = 0; \qquad M_C = -8125 \text{ lb} \cdot \text{ft}$	**Ans.**

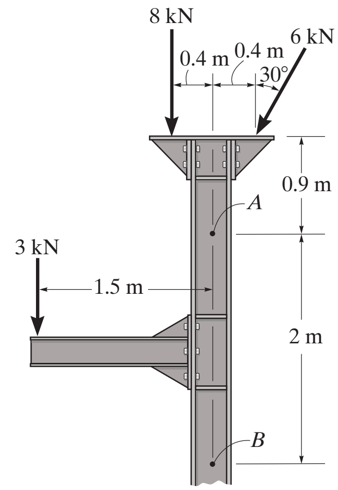






7–5.

Determine the internal normal force, shear force, and moment at points A and B in the column.



SOLUTION

Applying the equation of equilibrium to Fig. *a* gives

$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad V_A - 6 \sin 30° = 0 \qquad V_A = 3 \text{ kN}$ **Ans.**

$+\uparrow \Sigma F_y = 0;\quad N_A - 6 \cos 30° - 8 = 0 \qquad N_A = 13.2 \text{ kN}$ **Ans.**

$\circlearrowleft + \Sigma M_A = 0;\quad 8(0.4) + 6 \sin 30°(0.9) - 6 \cos 30°(0.4) - M_A = 0$

$M_A = 3.82 \text{ kN} \cdot \text{m}$ **Ans.**

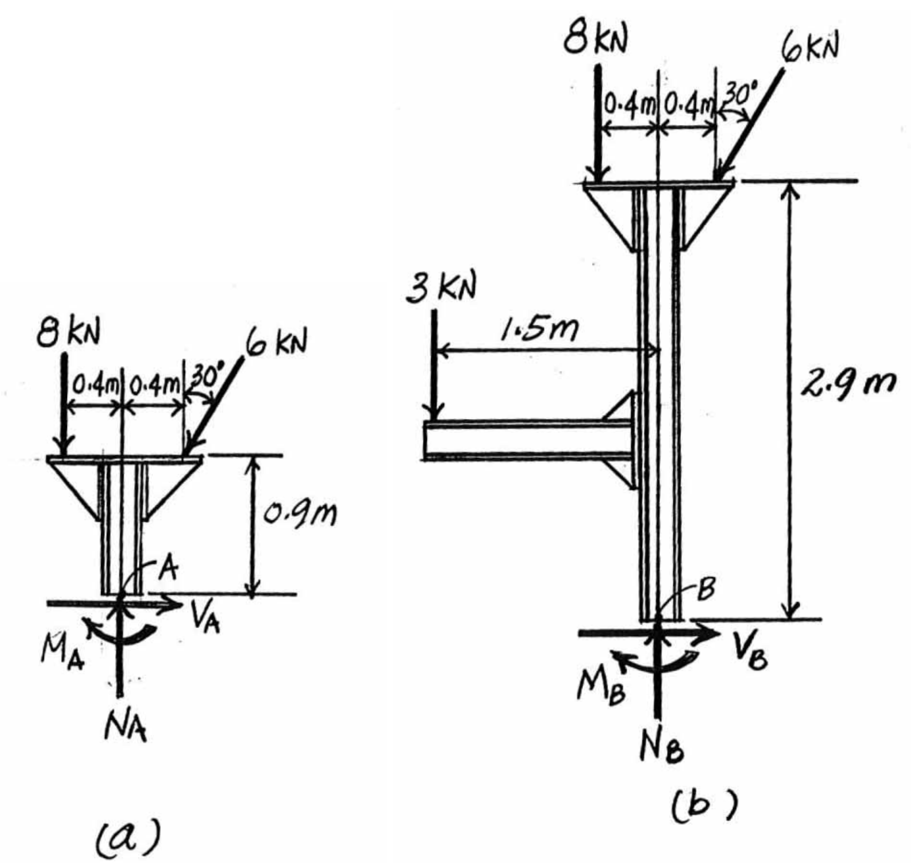
and to Fig. *b*,

$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad V_B - 6 \sin 30° = 0 \qquad V_B = 3 \text{ kN}$ **Ans.**

$+\uparrow \Sigma F_y = 0;\quad N_B - 3 - 8 - 6 \cos 30° = 0 \qquad N_B = 16.2 \text{ kN}$ **Ans.**

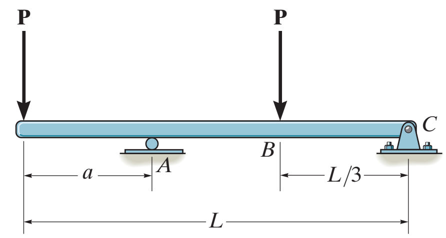
$\circlearrowleft + \Sigma M_B = 0;\quad 3(1.5) + 8(0.4) + 6 \sin 30°(2.9) - 6 \cos 30°(0.4) - M_B = 0$

$M_B = 14.3 \text{ kN} \cdot \text{m}$ **Ans.**



7–6.

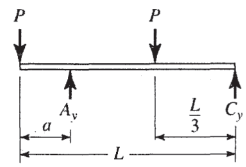
Determine the distance a as a fraction of the beam's length L for locating the roller support so that the moment in the beam at B is zero.



SOLUTION

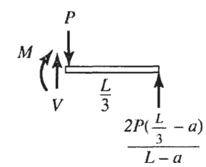
$$\circlearrowleft + \Sigma M_A = 0; \quad -P\left(\frac{2L}{3} - a\right) + C_y(L - a) + Pa = 0$$

$$C_y = \frac{2P\left(\frac{L}{3} - a\right)}{L - a}$$



$$\circlearrowleft + \Sigma M = 0; \quad M = \frac{2P\left(\frac{L}{3} - a\right)}{L - a}\left(\frac{L}{3}\right) = 0$$

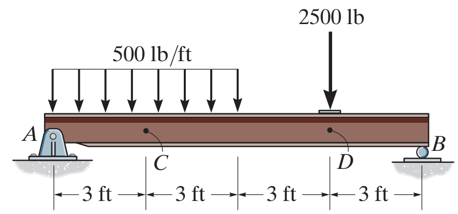
$$2PL\left(\frac{L}{3} - a\right) = 0$$



$$a = \frac{L}{3}$$ **Ans.**

7–7.

Determine the internal normal force, shear force, and moment at points C and D in the simply-supported beam. Point D is located just to the left of the 2500-lb force.



SOLUTION

With reference to Fig. *a*, we have

$\circlearrowleft + \Sigma M_A = 0; \quad B_y(12) - 500(6)(3) - 2500(9) = 0 \qquad B_y = 2625 \text{ lb}$

$\circlearrowleft + \Sigma M_B = 0; \quad 2500(3) + 500(6)(9) - A_y(12) = 0 \qquad A_y = 2875 \text{ lb}$

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad A_x = 0$

Using these results and referring to Fig. *b*, we have

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad N_C = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0; \quad 2875 - 500(3) - V_C = 0 \qquad V_C = 1375 \text{ lb}$ **Ans.**

$\circlearrowleft + \Sigma M_C = 0; \quad M_C + 500(3)(1.5) - 2875(3) = 0 \qquad M_C = 6375 \text{ lb} \cdot \text{ft}$ **Ans.**

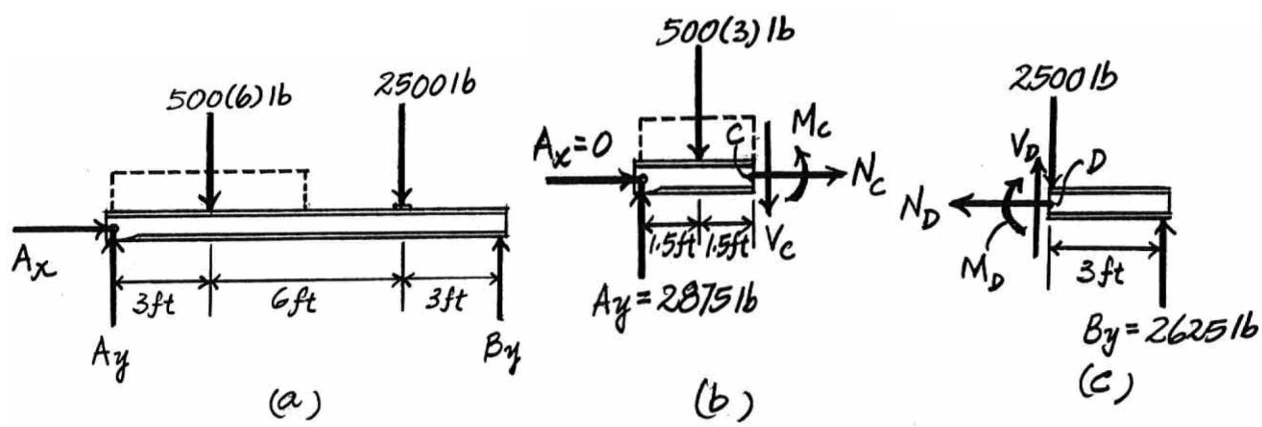
Also, by referring to Fig. *c*, we have

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad N_D = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0; \quad V_D + 2625 - 2500 = 0 \qquad V_D = -125 \text{ lb}$ **Ans.**

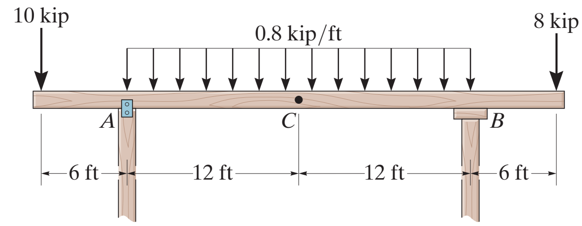
$\circlearrowleft + \Sigma M_D = 0; \quad 2625(3) - M_D = 0 \qquad M_D = 7875 \text{ lb} \cdot \text{ft}$ **Ans.**

The negative sign indicates that $\mathbf{V}_D$ acts in the opposite sense to that shown on the free-body diagram.



***7–8.**

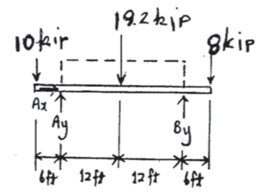
Determine the normal force, shear force, and moment at a section passing through point C. Assume the support at A can be approximated by a pin and B as a roller.



SOLUTION

$\circlearrowleft + \Sigma M_A = 0; \qquad -19.2(12) - 8(30) + B_y(24) + 10(6) = 0$

$B_y = 17.1 \text{ kip}$



$\xrightarrow{+} \Sigma F_x = 0; \qquad A_x = 0$

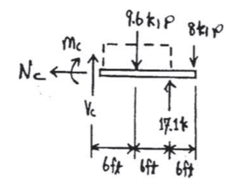
$+\uparrow \Sigma F_y = 0; \qquad A_y - 10 - 19.2 + 17.1 - 8 = 0$

$A_y = 20.1 \text{ kip}$

$\xrightarrow{+} \Sigma F_x = 0; \qquad N_C = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0; \qquad V_C - 9.6 + 17.1 - 8 = 0$

$V_C = 0.5 \text{ kip}$ **Ans.**

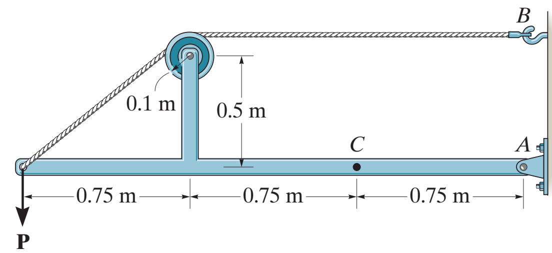


$\circlearrowleft + \Sigma M_C = 0; \qquad -M_C - 9.6(6) + 17.1(12) - 8(18) = 0$

$M_C = 3.6 \text{ kip} \cdot \text{ft}$ **Ans.**

7–9.

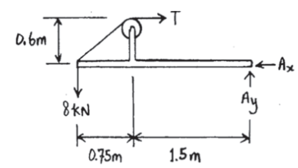
Determine the normal force, shear force, and moment at a section passing through point C. Take $P = 8$ kN.



SOLUTION

$\zeta + \Sigma M_A = 0;$ $\quad -T(0.6) + 8(2.25) = 0$

$T = 30$ kN



$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad A_x = 30$ kN

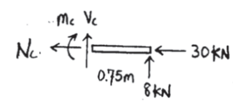
$+\uparrow \Sigma F_y = 0;$ $\quad A_y = 8$ kN

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad -N_C - 30 = 0$

$N_C = -30$ kN **Ans.**

$+\uparrow \Sigma F_y = 0;$ $\quad V_C + 8 = 0$

$V_C = -8$ kN **Ans.**

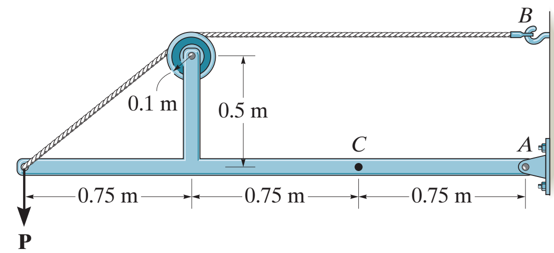


$\zeta + \Sigma M_C = 0;$ $\quad -M_C + 8(0.75) = 0$

$M_C = 6 \text{ kN} \cdot \text{m}$ **Ans.**

7–10.

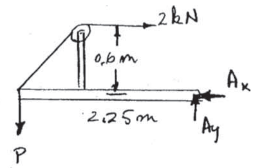
The cable will fail when subjected to a tension of 2 kN. Determine the largest vertical load P the frame will support and calculate the internal normal force, shear force, and moment at a section passing through point C for this loading.



SOLUTION

$\zeta + \Sigma M_A = 0;$ $\quad -2(0.6) + P(2.25) = 0$

$P = 0.533\text{ kN}$ **Ans.**



$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad A_x = 2\text{ kN}$

$+\uparrow \Sigma F_y = 0;$ $\quad A_y = 0.533\text{ kN}$

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad -N_C - 2 = 0$

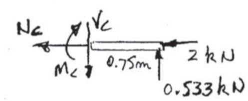
$N_C = -2\text{ kN}$ **Ans.**

$+\uparrow \Sigma F_y = 0;$ $\quad V_C - 0.533 = 0$

$V_C = -0.533\text{ kN}$ **Ans.**

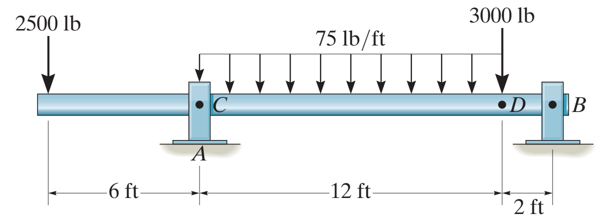
$\zeta + \Sigma M_C = 0;$ $\quad -M_C + 0.533(0.75) = 0$

$M_C = 0.400\text{ kN} \cdot \text{m}$ **Ans.**



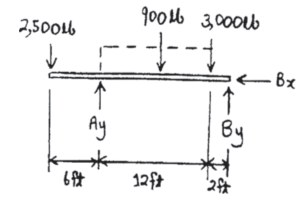
7–11.

The shaft is supported by a journal bearing at A and a thrust bearing at B. Determine the normal force, shear force, and moment at a section passing through (a) point C, which is just to the right of the bearing at A, and (b) point D, which is just to the left of the 3000-lb force.



SOLUTION

$\circlearrowleft + \Sigma M_B = 0;$ $\quad -A_y(14) + 2500(20) + 900(8) + 3000(2) = 0$



$A_y = 4514 \text{ lb}$

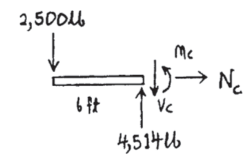
$\xrightarrow{+} \Sigma F_x = 0;$ $\quad B_x = 0$

$+\uparrow \Sigma F_y = 0;$ $\quad 4514 - 2500 - 900 - 3000 + B_y = 0$

$B_y = 1886 \text{ lb}$

$\circlearrowleft + \Sigma M_C = 0;$ $\quad 2500(6) + M_C = 0$

$M_C = -15\,000 \text{ lb} \cdot \text{ft} = -15.0 \text{ kip} \cdot \text{ft}$ **Ans.**



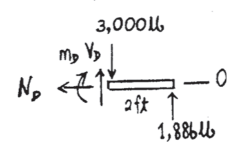
$\xrightarrow{+} \Sigma F_x = 0;$ $\quad N_C = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0;$ $\quad -2500 + 4514 - V_C = 0$

$V_C = 2014 \text{ lb} = 2.01 \text{ kip}$ **Ans.**

$\circlearrowleft + \Sigma M_D = 0;$ $\quad -M_D + 1886(2) = 0$

$M_D = 3771 \text{ lb} \cdot \text{ft} = 3.77 \text{ kip} \cdot \text{ft}$ **Ans.**



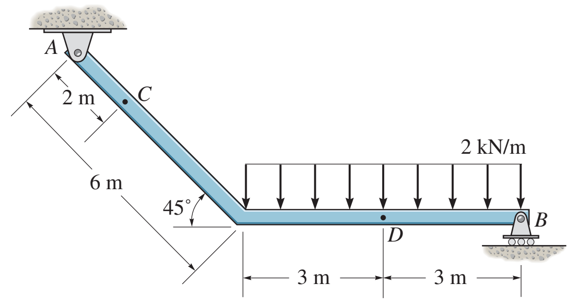
$\xrightarrow{+} \Sigma F_x = 0;$ $\quad N_D = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0;$ $\quad V_D - 3000 + 1886 = 0$

$V_D = 1114 \text{ lb} = 1.11 \text{ kip}$ **Ans.**

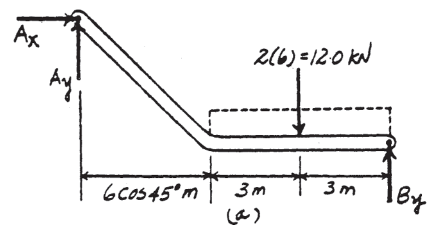
***7–12.**

Determine the internal normal force, shear force, and the moment at points C and D.



SOLUTION

Support Reactions: FBD (a).



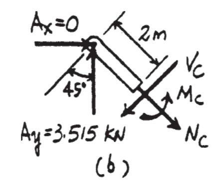
$\circlearrowleft + \Sigma M_A = 0; \qquad B_y\,(6 + 6\cos 45°) - 12.0(3 + 6\cos 45°) = 0$

$$B_y = 8.485 \text{ kN}$$

$+\uparrow \Sigma F_y = 0; \qquad A_y + 8.485 - 12.0 = 0 \qquad A_y = 3.515 \text{ kN}$

$\overset{+}{\rightarrow} \Sigma F_x = 0 \qquad A_x = 0$

Internal Forces: Applying the equations of equilibrium to segment AC [FBD (b)], we have



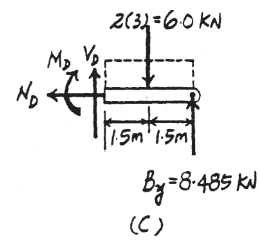
$\nearrow + \Sigma F_{x'} = 0; \qquad 3.515\cos 45° - V_C = 0 \qquad V_C = 2.49 \text{ kN}$ **Ans.**

$\nwarrow + \Sigma F_{y'} = 0; \qquad 3.515\sin 45° - N_C = 0 \qquad N_C = 2.49 \text{ kN}$ **Ans.**

$\circlearrowleft + \Sigma M_C = 0; \qquad M_C - 3.515\cos 45°(2) = 0$

$M_C = 4.97 \text{ kN}\cdot\text{m}$ **Ans.**

Applying the equations of equilibrium to segment BD [FBD (c)], we have



$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad N_D = 0$ **Ans.**

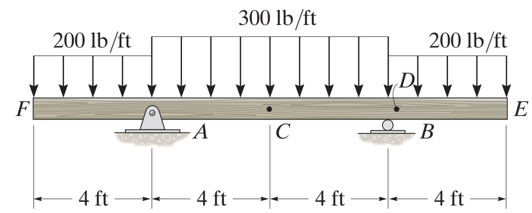
$+\uparrow \Sigma F_y = 0; \qquad V_D + 8.485 - 6.00 = 0 \qquad V_D = -2.49 \text{ kN}$ **Ans.**

$\circlearrowleft + \Sigma M_D = 0; \qquad 8.485(3) - 6(1.5) - M_D = 0$

$M_D = 16.5 \text{ kN}\cdot\text{m}$ **Ans.**

7–13.

Determine the internal normal force, shear force, and moment acting at point C and at point D, which is located just to the right of the roller support at B.

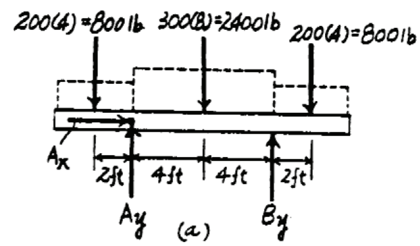


SOLUTION

Support Reactions: From FBD (a),

$\circlearrowleft + \Sigma M_A = 0;\qquad B_y(8) + 800(2) - 2400(4) - 800(10) = 0$

$B_y = 2000 \text{ lb}$



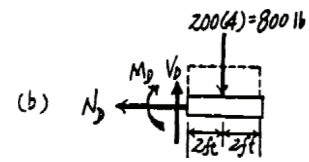
Internal Forces: Applying the equations of equilibrium to segment ED [FBD (b)], we have

$\overset{+}{\rightarrow} \Sigma F_x = 0;\qquad N_D = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0;\qquad V_D - 800 = 0 \qquad V_D = 800 \text{ lb}$ **Ans.**

$\circlearrowleft + \Sigma M_D = 0;\qquad -M_D - 800(2) = 0$

$M_D = -1600 \text{ lb} \cdot \text{ft} = -1.60 \text{ kip} \cdot \text{ft}$ **Ans.**



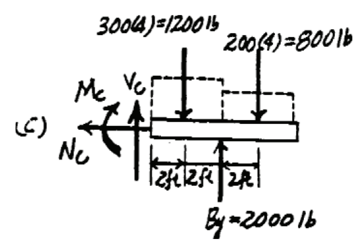
Applying the equations of equilibrium to segment EC [FBD (c)], we have

$\overset{+}{\rightarrow} \Sigma F_x = 0;\qquad N_C = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0;\qquad V_C + 2000 - 1200 - 800 = 0 \qquad V_C = 0$ **Ans.**

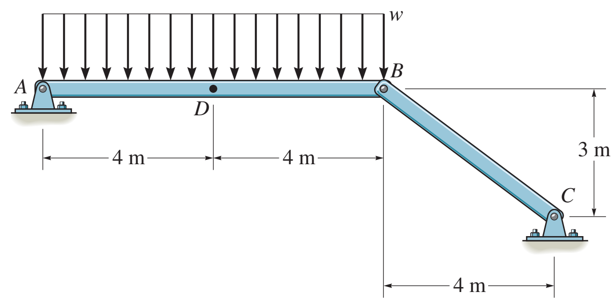
$\circlearrowleft + \Sigma M_C = 0;\qquad 2000(4) - 1200(2) - 800(6) - M_C = 0$

$M_C = 800 \text{ lb} \cdot \text{ft}$ **Ans.**

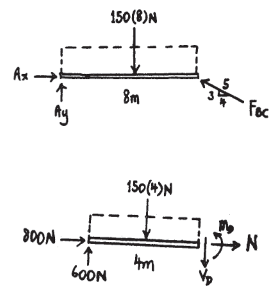


7–14.

Determine the normal force, shear force, and moment at a section passing through point D. Take $w = 150$ N/m.



SOLUTION



$\zeta + \Sigma M_A = 0; \qquad -150(8)(4) + \frac{3}{5}F_{BC}(8) = 0$

$F_{BC} = 1000 \text{ N}$

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad A_x - \frac{4}{5}(1000) = 0$

$A_x = 800 \text{ N}$

$+\uparrow \Sigma F_y = 0; \qquad A_y - 150(8) + \frac{3}{5}(1000) = 0$

$A_y = 600 \text{ N}$

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad N_D = -800 \text{ N}$ **Ans.**

$+\uparrow \Sigma F_y = 0; \qquad 600 - 150(4) - V_D = 0$

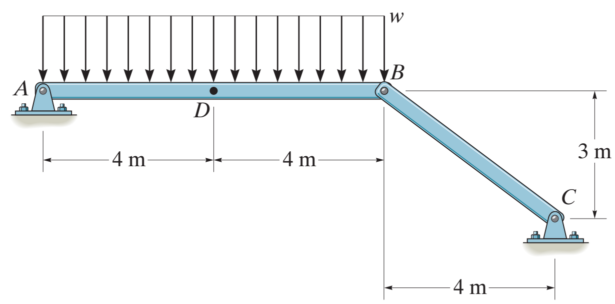
$V_D = 0$ **Ans.**

$\zeta + \Sigma M_D = 0; \qquad -600(4) + 150(4)(2) + M_D = 0$

$M_D = 1200 \text{ N} \cdot \text{m} = 1.20 \text{ kN} \cdot \text{m}$ **Ans.**

7–15.

The beam AB will fail if the maximum internal moment at D reaches $800\ \text{N}\cdot\text{m}$ or the normal force in member BC becomes 1500 N. Determine the largest load w it can support.



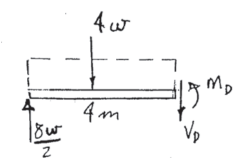
SOLUTION

Assume maximum moment occurs at D;

$\circlearrowleft + \Sigma M_D = 0; \qquad M_D - 4w(2) = 0$

$800 = 4w(2)$

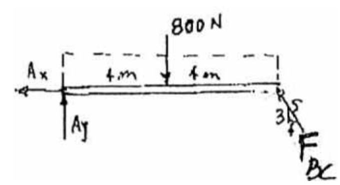
$w = 100\ \text{N/m}$



$\circlearrowleft + \Sigma M_A = 0; \qquad -800(4) + F_{BC}(0.6)(8) = 0$

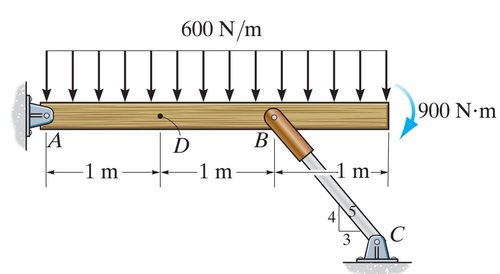
$F_{BC} = 666.7\ \text{N} < 1500\ \text{N}$ **(O.K.!)**

$w = 100\ \text{N/m}$ **Ans.**



***7–16.**

Determine the internal normal force, shear force, and moment at point D in the beam.



SOLUTION

Writing the equations of equilibrium with reference to Fig. *a,* we have

$\circlearrowleft + \Sigma M_A = 0;$ $\quad F_{BC}\left(\frac{4}{5}\right)(2) - 600(3)(1.5) - 900 = 0 \quad F_{BC} = 2250 \text{ N}$

$\circlearrowleft + \Sigma M_B = 0;$ $\quad 600(3)(0.5) - 900 - A_y(2) = 0 \quad A_y = 0$

$\xrightarrow{+} \Sigma F_x = 0;$ $\quad A_x - 2250\left(\frac{2}{5}\right) = 0 \quad A_x = 1350 \text{ N}$

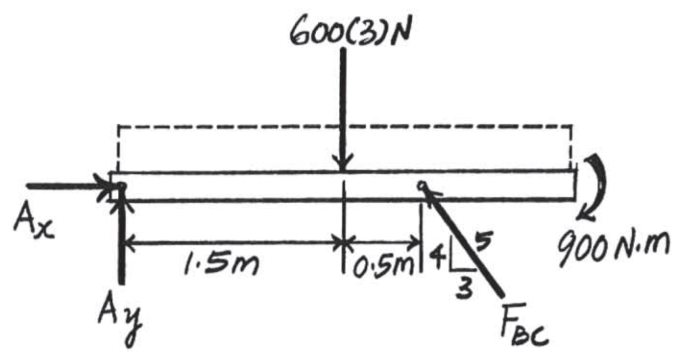
Using these results and referring to Fig. *b*, we have

$\xrightarrow{+} \Sigma F_x = 0;$ $\quad N_D + 1350 = 0 \quad N_D = -1350 \text{ N} = -1.35 \text{ kN}$ **Ans.**

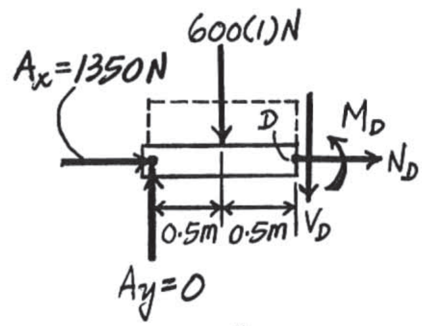
$+\uparrow \Sigma F_y = 0;$ $\quad -V_D - 600(1) = 0 \quad V_D = -600 \text{ N}$ **Ans.**

$\circlearrowleft + \Sigma M_D = 0;$ $\quad M_D + 600(1)(0.5) = 0 \quad M_D = -300 \text{ N} \cdot \text{m}$ **Ans.**

The negative sign indicates that $\mathbf{N}_D$, $\mathbf{V}_D$, and $\mathbf{M}_D$ act in the opposite sense to that shown on the free-body diagram.



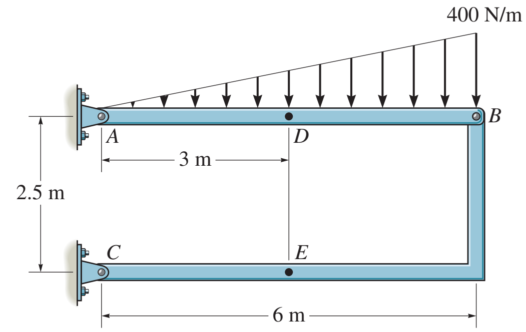
(a)



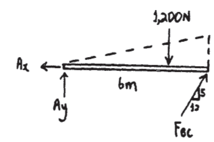
(b)

7–17.

Determine the normal force, shear force, and moment at a section passing through point E of the two-member frame.



SOLUTION

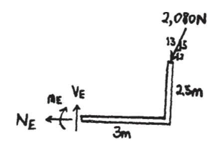


$\circlearrowleft + \Sigma M_A = 0;\qquad -1200(4) + \frac{5}{13}F_{BC}(6) = 0$

$F_{BC} = 2080\text{ N}$

$\overset{+}{\rightarrow} \Sigma F_x = 0;\qquad -N_E - \frac{12}{13}(2080) = 0$

$N_E = -1920\text{ N} = -1.92\text{ kN}$ **Ans.**



$+\uparrow \Sigma F_y = 0;\qquad V_E - \frac{5}{13}(2080) = 0$

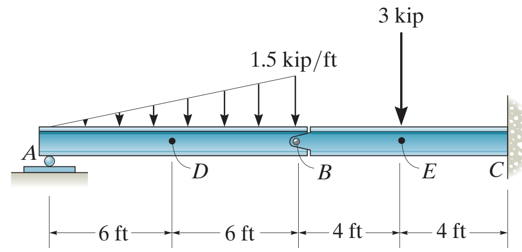
$V_E = 800\text{ N}$ **Ans.**

$\circlearrowleft + \Sigma M_E = 0;\qquad -\frac{5}{13}(2080)(3) + \frac{12}{13}(2080)(2.5) - M_E = 0$

$M_E = 2400\text{ N}\cdot\text{m} = 2.40\text{ kN}\cdot\text{m}$ **Ans.**

7–18.

Determine the normal force, shear force, and moment in the beam at sections passing through points D and E. Point E is just to the right of the 3-kip load.



SOLUTION

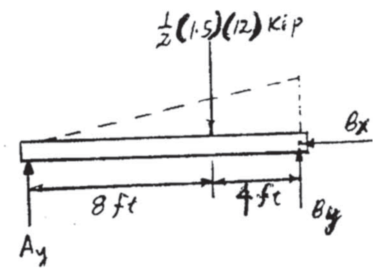
$\zeta + \Sigma M_B = 0;\quad \frac{1}{2}(1.5)(12)(4) - A_y(12) = 0$

$A_y = 3 \text{ kip}$

$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad B_x = 0$

$+\uparrow \Sigma F_y = 0;\quad B_y + 3 - \frac{1}{2}(1.5)(12) = 0$

$B_y = 6 \text{ kip}$



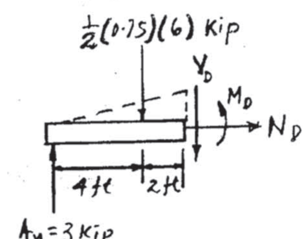
$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad N_D = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0;\quad 3 - \frac{1}{2}(0.75)(6) - V_D = 0$

$V_D = 0.75 \text{ kip}$ **Ans.**

$\zeta + \Sigma M_D = 0;\quad M_D + \frac{1}{2}(0.75)(6)(2) - 3(6) = 0$

$M_D = 13.5 \text{ kip} \cdot \text{ft}$ **Ans.**



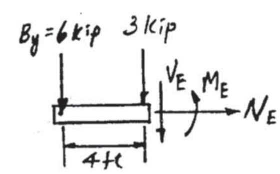
$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad N_E = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0;\quad -V_E - 3 - 6 = 0$

$V_E = -9 \text{ kip}$ **Ans.**

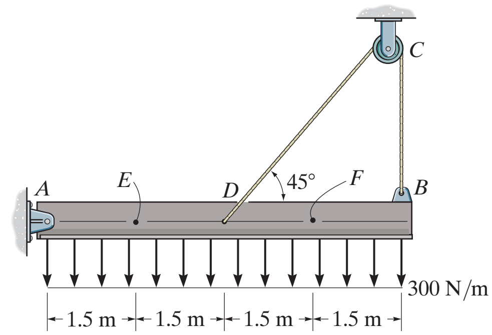
$\zeta + \Sigma M_E = 0;\quad M_E + 6(4) = 0$

$M_E = -24.0 \text{ kip} \cdot \text{ft}$ **Ans.**



7–19.

Determine the internal normal force, shear force, and moment at points E and F in the beam.



SOLUTION

With reference to Fig. *a*,

$\zeta + \Sigma M_A = 0; \quad T(6) + T \sin 45°(3) - 300(6)(3) = 0 \qquad T = 664.92 \text{ N}$

$\xrightarrow{+} \Sigma F_x = 0; \quad 664.92 \cos 45° - A_x = 0 \qquad A_x = 470.17 \text{ N}$

$+\uparrow \Sigma F_y = 0; \quad A_y + 664.92 \sin 45° + 664.92 - 300(6) = 0 \qquad A_y = 664.92 \text{ N}$

Use these result and referring to Fig. *b*,

$\xrightarrow{+} \Sigma F_x = 0; \quad N_E - 470.17 = 0$

$N_E = 470 \text{ N}$ **Ans.**

$+\uparrow \Sigma F_y = 0; \quad 664.92 - 300(1.5) - V_E = 0$

$V_E = 215 \text{ N}$ **Ans.**

$\zeta + \Sigma M_E = 0; \quad M_E + 300(1.5)(0.75) - 664.92(1.5) = 0$

$M_E = 660 \text{ N} \cdot \text{m}$ **Ans.**

Also, by referring to Fig. *c*,

$\xrightarrow{+} \Sigma F_x = 0; \quad N_F = 0$ **Ans.**

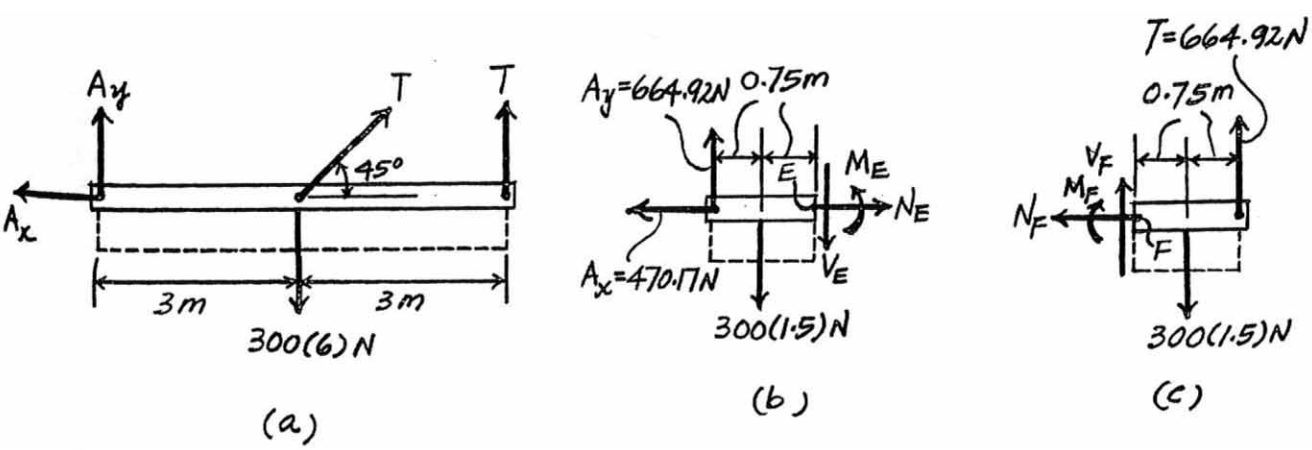
$+\uparrow \Sigma F_y = 0; \quad V_F + 664.92 - 300 = 0$

$V_F = -215 \text{ N}$ **Ans.**

$\zeta + \Sigma M_F = 0; \quad 664.92(1.5) - 300(1.5)(0.75) - M_F = 0$

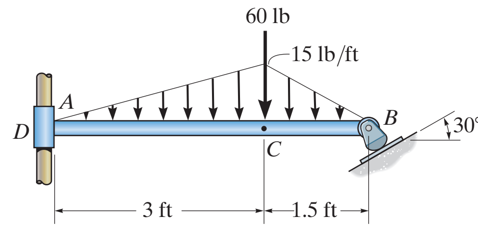
$M_F = 660 \text{ N} \cdot \text{m}$ **Ans.**

The negative sign indicates that $\mathbf{V}_F$ acts in the opposite sense to that shown on the free-body diagram.



***7–20.**

Rod AB is fixed to a smooth collar D, which slides freely along the vertical guide. Determine the internal normal force, shear force, and moment at point C. which is located just to the left of the 60-lb concentrated load.



SOLUTION

With reference to Fig. *a*, we obtain

$+\uparrow \Sigma F_y = 0; \qquad F_B \cos 30° - \frac{1}{2}(15)(3) - 60 - \frac{1}{2}(15)(1.5) = 0 \qquad F_B = 108.25 \text{ lb}$

Using this result and referring to Fig. *b*, we have

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad -N_C - 108.25 \sin 30° = 0 \qquad N_C = -54.1 \text{ lb}$ **Ans.**

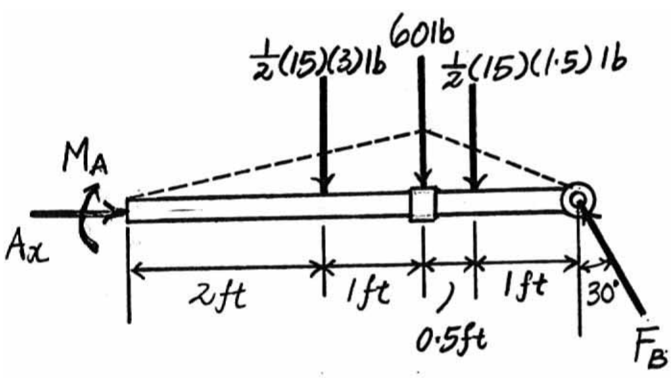
$+\uparrow \Sigma F_y = 0; \qquad V_C - 60 - \frac{1}{2}(15)(1.5) + 108.25 \cos 30° = 0$

$V_C = -22.5 \text{ lb}$ **Ans.**

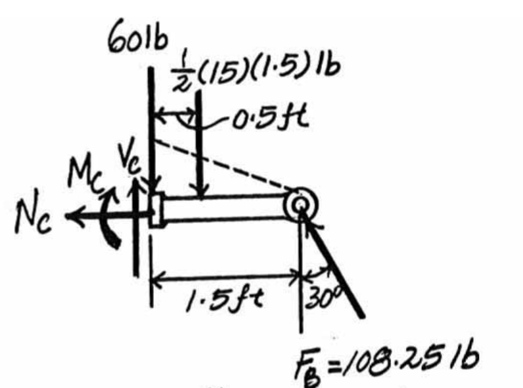
$\circlearrowleft + \Sigma M_C = 0; \qquad 108.25 \cos 30°(1.5) - \frac{1}{2}(15)(1.5)(0.5) - M_C = 0$

$M_C = 135 \text{ lb} \cdot \text{ft}$ **Ans.**

The negative signs indicates that $\mathbf{N}_C$ and $\mathbf{V}_C$ act in the opposite sense to that shown on the free-body diagram.



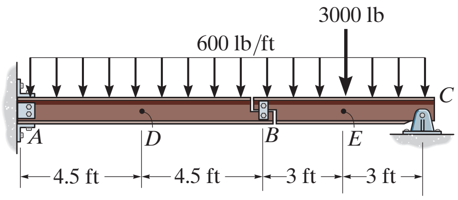
(a)



(b)

7–21.

Determine the internal normal force, shear force, and moment at points D and E in the compound beam. Point E is located just to the left of the 3000-lb force. Assume the support at A is fixed and the beam segments are connected together by a short link at B.



SOLUTION

With reference to Fig. b, we have

$\circlearrowleft + \Sigma M_C = 0; \quad 600(6)(3) + 3000(3) - F_B(6) = 0 \qquad F_B = 3300 \text{ lb}$

Using this result and referring to Fig. c, we have

$\xrightarrow{+} \Sigma F_x = 0; \quad N_D = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0; \quad V_D - 600(4.5) - 3300 = 0 \qquad V_D = 6 \text{ kip}$ **Ans.**

$\circlearrowleft + \Sigma M_D = 0; \quad -M_D - 600(4.5)(2.25) - 3300(4.5) = 0$

$M_D = -20925 \text{ lb} \cdot \text{ft} = -20.9 \text{ kip} \cdot \text{ft}$ **Ans.**

Also, by referring to Fig. d, we can write

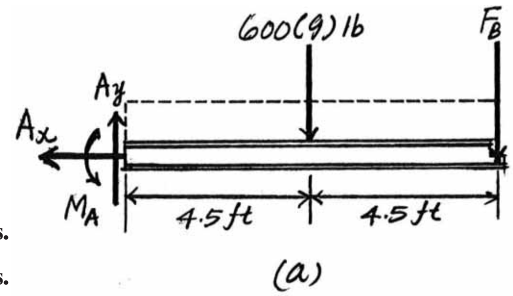
$\xrightarrow{+} \Sigma F_x = 0; \quad N_E = 0$ **Ans.**

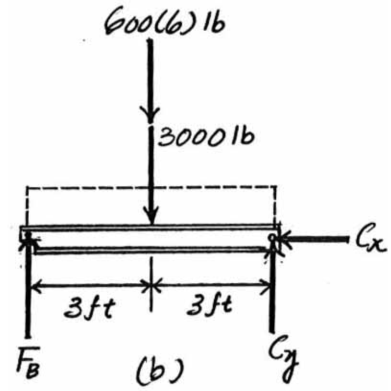
$+\uparrow \Sigma F_y = 0; \quad 3300 - 600(3) - V_E = 0 \qquad V_E = 1500 \text{ lb} = 1.5 \text{ kip}$ **Ans.**

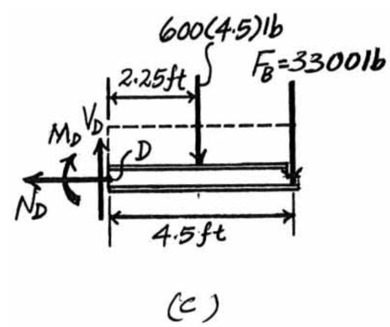
$\circlearrowleft + \Sigma M_E = 0; \quad M_E + 600(3)(1.5) - 3300(3) = 0$

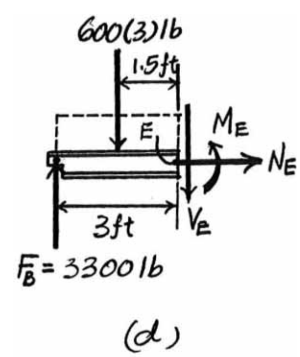
$M_E = 7200 \text{ lb} \cdot \text{ft} = 7.2 \text{ kip} \cdot \text{ft}$ **Ans.**

The negative sign indicates that $\mathbf{M}_D$ acts in the opposite sense to that shown in the free-body diagram.



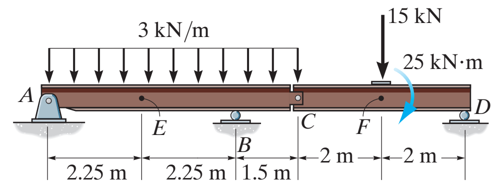






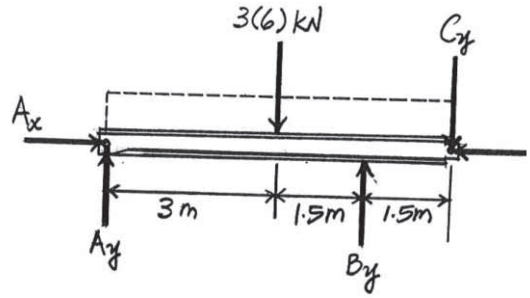
7–22.

Determine the internal normal force, shear force, and moment at points E and F in the compound beam. Point F is located just to the left of the 15-kN force and 25-kN · m couple moment.



SOLUTION

With reference to Fig. b, we have



(a)

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad C_x = 0$

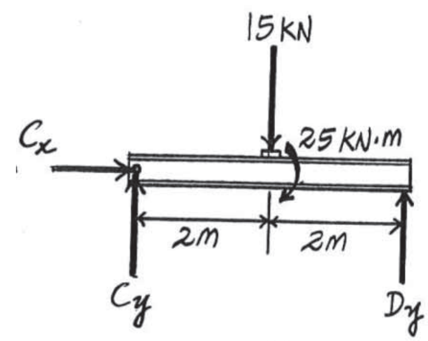
$\circlearrowleft + \Sigma M_C = 0; \qquad D_y(4) - 15(2) - 25 = 0 \qquad D_y = 13.75 \text{ kN}$

$\circlearrowleft + \Sigma M_D = 0; \qquad 15(2) - 25 - C_y(4) = 0 \qquad C_y = 1.25 \text{ kN}$

Using these results and referring to Fig. a, we have

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad A_x = 0$

$\circlearrowleft + \Sigma M_B = 0; \qquad 3(6)(1.5) - 1.25(1.5) - A_y(4.5) = 0 \qquad A_y = 5.583 \text{ kN}$



(b)

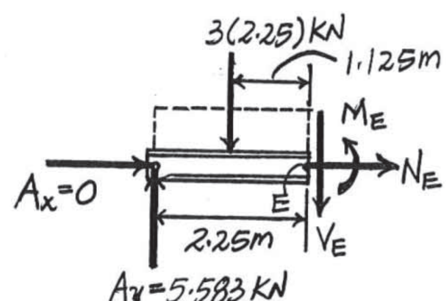
With these results and referring to Fig. c,

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad N_E = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0; \qquad 5.583 - 3(2.25) - V_E = 0 \qquad V_E = -1.17 \text{ kN}$ **Ans.**

$\circlearrowleft + \Sigma M_E = 0; \qquad M_E + 3(2.25) - 3(2.25)(8.125) = 0$

$M_E = 4.97 \text{ kN} \cdot \text{m}$ **Ans.**



(C)

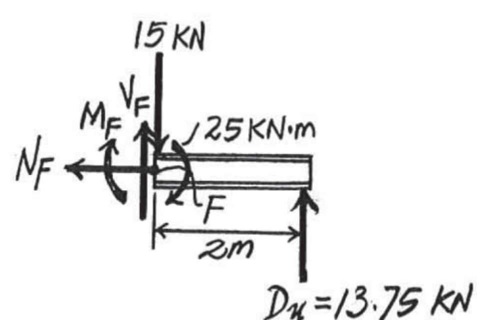
Also, using the result of D_y referring to Fig. d, we have

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad N_F = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0; \qquad V_F - 15 + 13.75 = 0 \qquad V_F = 1.25 \text{ kN}$ **Ans.**

$\circlearrowleft + \Sigma M_F = 0; \qquad 13.75(2) - 25 - M_F = 0 \qquad M_F = 2.5 \text{ kN} \cdot \text{m}$ **Ans.**

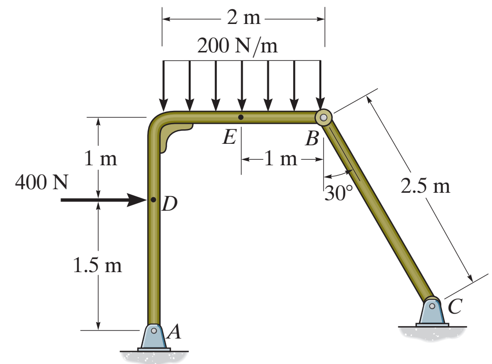
The negative sign indicates that $\mathbf{V}_E$ acts in the opposite sense to that shown in the free-body diagram.



(d)

7–23.

Determine the internal normal force, shear force, and moment at points D and E in the frame. Point D is located just above the 400-N force.



SOLUTION

With reference to Fig. *a,* we have

$$\zeta + \Sigma M_A = 0; \qquad F_B \cos 30°(2) + F_B \sin 30°(2.5) - 200(2)(1) - 400(1.5) = 0$$

$$F_B = 335.34 \text{ N}$$

Using this result and referring to Fig. *b*, we have

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad V_D - 335.34 \sin 30° = 0 \qquad V_D = 168 \text{ N} \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad 335.34 \cos 30° - 200(2) - N_D = 0 \qquad N_D = -110 \text{ N} \qquad \textbf{Ans.}$$

$$\zeta + \Sigma M_D = 0; \qquad 335.34 \cos 30°(2) + 335.34 \sin 30°(1) - 200(2)(1) - M_D = 0$$

$$M_D = 348 \text{ N} \cdot \text{m} \qquad \textbf{Ans.}$$

Also, by referring to Fig. *c,* we can write

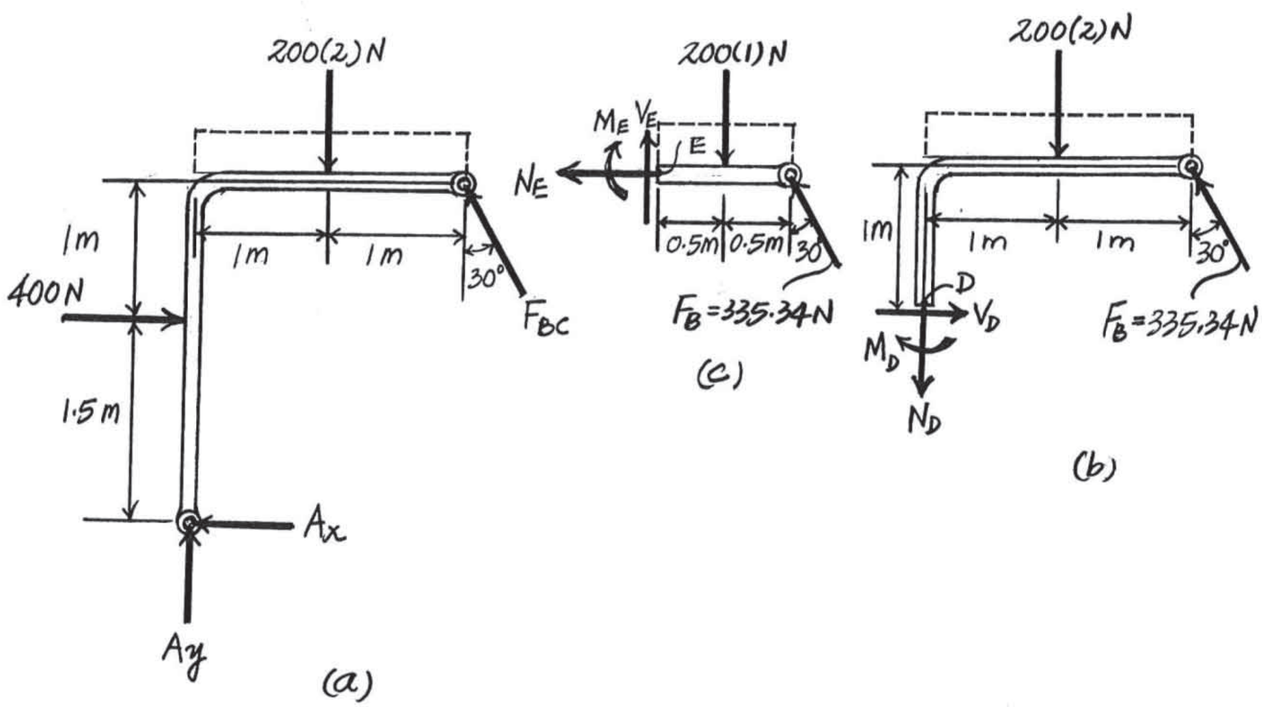
$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad -N_E - 335.34 \sin 30° = 0 \qquad N_E = -168 \text{ N} \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad V_E + 335.34 \cos 30° - 200(1) = 0 \qquad V_E = -90.4 \text{ N} \qquad \textbf{Ans.}$$

$$\zeta + \Sigma M_E = 0; \qquad 335.34 \cos 30°(1) - 200(1)(0.5) - M_E = 0$$

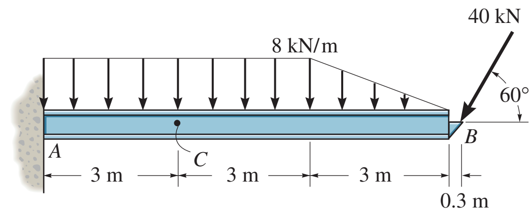
$$M_E = 190 \text{ N} \cdot \text{m} \qquad \textbf{Ans.}$$

The negative sign indicates that $\mathbf{N}_D$, $\mathbf{N}_E$, and $\mathbf{V}_E$ acts in the opposite sense to that shown in the free-body diagram.



***7–24.**

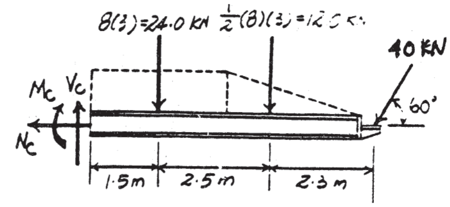
Determine the internal normal force, shear force, and bending moment at point C.



SOLUTION

Free body Diagram: The support reactions at A need not be computed.

Internal Forces: Applying equations of equilibrium to segment BC, we have



$\xrightarrow{+} \Sigma F_x = 0;$ $\quad -40 \cos 60° - N_C = 0 \qquad N_C = -20.0 \text{ kN}$ **Ans.**

$+\uparrow \Sigma F_y = 0;$ $\quad V_C - 24.0 - 12.0 - 40 \sin 60° = 0$

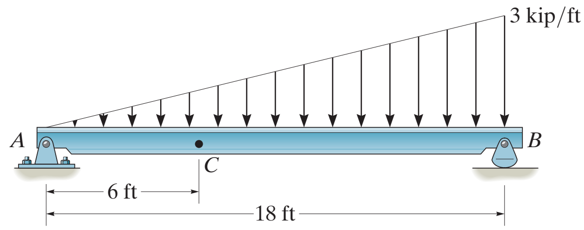
$$V_C = 70.6 \text{ kN}$$ **Ans.**

$\circlearrowleft + \Sigma M_C = 0;$ $\quad -24.0(1.5) - 12.0(4) - 40 \sin 60°(6.3) - M_C = 0$

$$M_C = -302 \text{ kN} \cdot \text{m}$$ **Ans.**

7–25.

Determine the shear force and moment acting at a section passing through point C in the beam.



SOLUTION

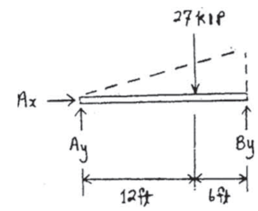
$\zeta + \Sigma M_B = 0;$ $\quad -A_y(18) + 27(6) = 0$

$A_y = 9 \text{ kip}$

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad A_x = 0$

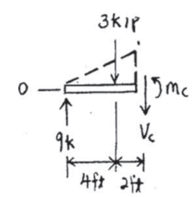
$\zeta + \Sigma M_C = 0;$ $\quad -9(6) + 3(2) + M_C = 0$

$M_C = 48 \text{ kip} \cdot \text{ft}$ **Ans.**



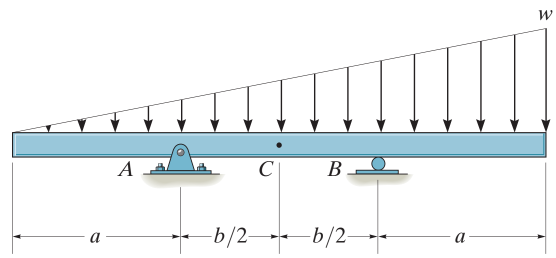
$+\uparrow \Sigma F_y = 0;$ $\quad 9 - 3 - V_C = 0$

$V_C = 6 \text{ kip}$ **Ans.**



7–26.

Determine the ratio of a/b for which the shear force will be zero at the midpoint C of the beam.



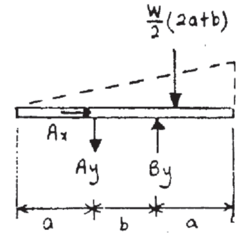
SOLUTION

$\zeta + \Sigma M_B = 0;\qquad -\frac{w}{2}(2a+b)\left[\frac{2}{3}(2a+b) - (a+b)\right] + A_y(b) = 0$

$$A_y = \frac{w}{6b}(2a+b)(a-b)$$

$\overset{+}{\rightarrow} \Sigma F_x = 0;\qquad A_x = 0$

$+\uparrow \Sigma F_y = 0;\qquad -\frac{w}{6b}(2a+b)(a-b) - \frac{w}{4}\left(a+\frac{b}{2}\right) - V_C = 0$



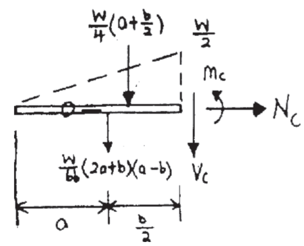
Since $V_C = 0$,

$$-\frac{1}{6b}(2a+b)(a-b) = \frac{1}{4}(2a+b)\left(\frac{1}{2}\right)$$

$$-\frac{1}{6b}(a-b) = \frac{1}{8}$$

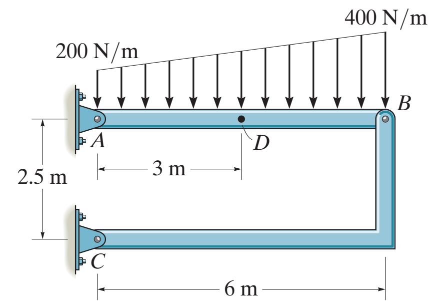
$$-a+b = \frac{3}{4}b$$

$$\frac{a}{b} = \frac{1}{4}\qquad \textbf{Ans.}$$



7–27.

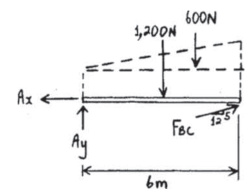
Determine the normal force, shear force, and moment at a section passing through point D of the two-member frame.



SOLUTION

$\circlearrowleft + \Sigma M_A = 0; \qquad -1200(3) - 600(4) + \frac{5}{13} F_{BC}(6) = 0$

$$F_{BC} = 2600 \text{ N}$$



$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad A_x = \frac{12}{13}(2600) = 2400 \text{ N}$

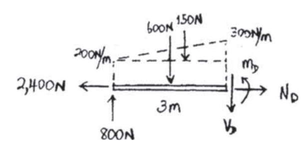
$+\uparrow \Sigma F_y = 0; \qquad A_y - 1200 - 600 + \frac{5}{13}(2600) = 0$

$$A_y = 800 \text{ N}$$

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad N_D = 2400 \text{ N} = 2.40 \text{ kN}$ **Ans.**

$+\uparrow \Sigma F_y = 0; \qquad 800 - 600 - 150 - V_D = 0$

$V_D = 50 \text{ N}$ **Ans.**

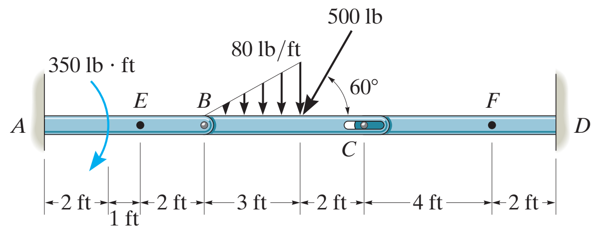


$\circlearrowleft + \Sigma M_D = 0; \qquad -800(3) + 600(1.5) + 150(1) + M_D = 0$

$M_D = 1350 \text{ N}\cdot\text{m} = 1.35 \text{ kN}\cdot\text{m}$ **Ans.**

***7–28.**

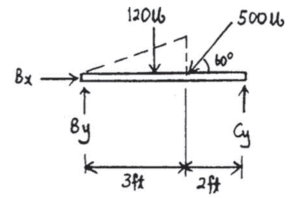
Determine the normal force, shear force, and moment at sections passing through points E and F. Member BC is pinned at B and there is a smooth slot in it at C. The pin at C is fixed to member CD.



SOLUTION

$\circlearrowleft +\Sigma M_B = 0;$ $\quad -120(2) - 500 \sin 60°(3) + C_y(5) = 0$

$C_y = 307.8 \text{ lb}$

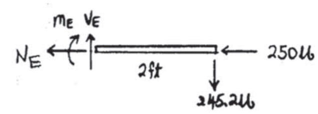


$\xrightarrow{+} \Sigma F_x = 0;$ $\quad B_x - 500 \cos 60° = 0$

$B_x = 250 \text{ lb}$

$+\uparrow \Sigma F_y = 0;$ $\quad B_y - 120 - 500 \sin 60° + 307.8 = 0$

$B_y = 245.2 \text{ lb}$



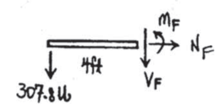
$\xrightarrow{+} \Sigma F_x = 0;$ $\quad -N_E - 250 = 0$

$N_E = -250 \text{ lb}$ **Ans.**

$+\uparrow \Sigma F_y = 0;$ $\quad V_E = 245 \text{ lb}$ **Ans.**

$\circlearrowleft +\Sigma M_E = 0;$ $\quad -M_E - 245.2(2) = 0$

$M_E = -490 \text{ lb} \cdot \text{ft}$ **Ans.**



$\xrightarrow{+} \Sigma F_x = 0;$ $\quad N_F = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0;$ $\quad -307.8 - V_F = 0$

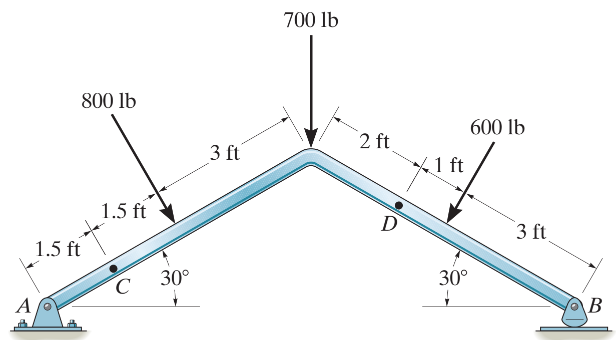
$V_F = -308 \text{ lb}$ **Ans.**

$\circlearrowleft +\Sigma M_F = 0;$ $\quad 307.8(4) + M_F = 0$

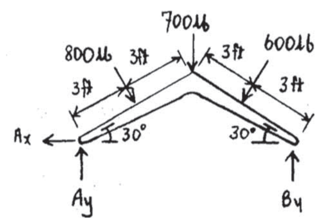
$M_F = -1231 \text{ lb} \cdot \text{ft} = -1.23 \text{ kip} \cdot \text{ft}$ **Ans.**

7–29.

Determine the normal force, shear force, and moment acting at a section passing through point C.



SOLUTION



$\circlearrowleft + \Sigma M_A = 0;\quad -800\,(3) - 700(6\cos 30°) - 600\cos 30°(6\cos 30° + 3\cos 30°)$

$+ 600\sin 30°(3\sin 30°) + B_y\,(6\cos 30° + 6\cos 30°) = 0$

$B_y = 927.4\text{ lb}$

$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad 800\sin 30° - 600\sin 30° - A_x = 0$

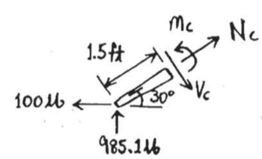
$A_x = 100\text{ lb}$

$+\uparrow \Sigma F_y = 0;\quad A_y - 800\cos 30° - 700 - 600\cos 30° + 927.4 = 0$

$A_y = 985.1\text{ lb}$

$\nearrow + \Sigma F_x = 0;\quad N_C - 100\cos 30° + 985.1\sin 30° = 0$

$N_C = -406\text{ lb}$ **Ans.**



$+\nwarrow \Sigma F_y = 0;\quad 100\sin 30° + 985.1\cos 30° - V_C = 0$

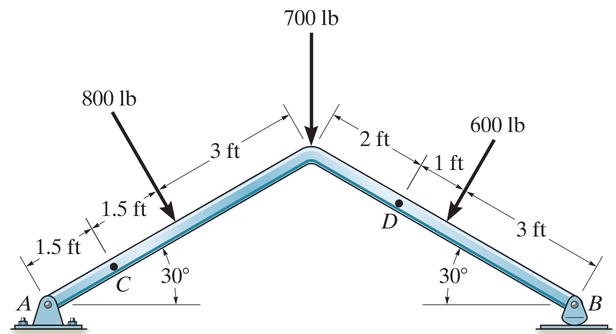
$V_C = 903\text{ lb}$ **Ans.**

$\circlearrowleft + \Sigma M_C = 0;\quad -985.1(1.5\cos 30°) - 100(1.5\sin 30°) + M_C = 0$

$M_C = 1355\text{ lb}\cdot\text{ft} = 1.35\text{ kip}\cdot\text{ft}$ **Ans.**

7–30.

Determine the normal force, shear force, and moment acting at a section passing through point D.

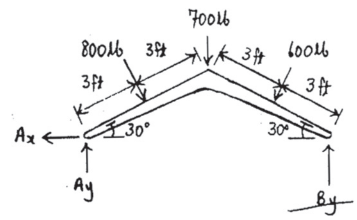


SOLUTION

$\zeta + \Sigma M_A = 0;$ $\quad -800(3) - 700(6\cos 30°) - 600\cos 30°(6\cos 30° + 3\cos 30°)$

$+ 600\sin 30°(3\sin 30°) + B_y(6\cos 30° + 6\cos 30°) = 0$

$B_y = 927.4 \text{ lb}$



$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad 800\sin 30° - 600\sin 30° - A_x = 0$

$A_x = 100 \text{ lb}$

$+\uparrow \Sigma F_y = 0;$ $\quad A_y - 800\cos 30° - 700 - 600\cos 30° + 927.4 = 0$

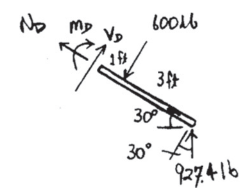
$A_y = 985.1 \text{ lb}$

$+\nwarrow \Sigma F_x = 0;$ $\quad N_D - 927.4\sin 30° = 0$

$N_D = -464 \text{ lb}$ **Ans.**

$\nearrow + \Sigma F_y = 0;$ $\quad V_D - 600 + 927.4\cos 30° = 0$

$V_D = -203 \text{ lb}$ **Ans.**

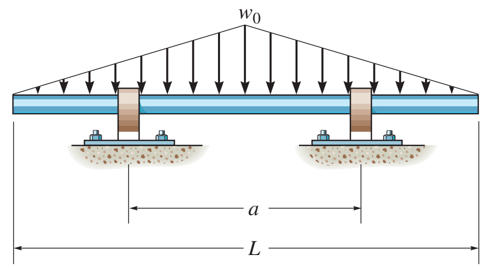


$\zeta + \Sigma M_D = 0;$ $\quad -M_D - 600(1) + 927.4(4\cos 30°) = 0$

$M_D = 2612 \text{ lb} \cdot \text{ft} = 2.61 \text{ kip} \cdot \text{ft}$ **Ans.**

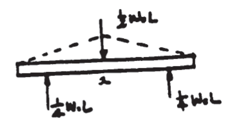
7–31.

Determine the distance a between the supports in terms of the shaft's length L so that the bending moment in the *symmetric* shaft is zero at the shaft's center. The intensity of the distributed load at the center of the shaft is w_0. The supports are journal bearings.



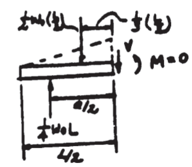
SOLUTION

Support reactions: FBD(a)



Moments Function:

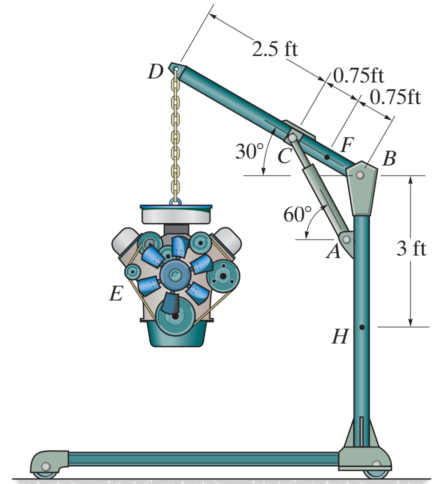
$$\zeta + \Sigma M = 0; \qquad 0 + \frac{1}{2}(w_0)\left(\frac{L}{2}\right)\left(\frac{1}{3}\right)\left(\frac{L}{2}\right) - \frac{1}{4}w_0 L\left(\frac{a}{2}\right) = 0$$



$$a = \frac{L}{3}$$ **Ans.**

7–32.

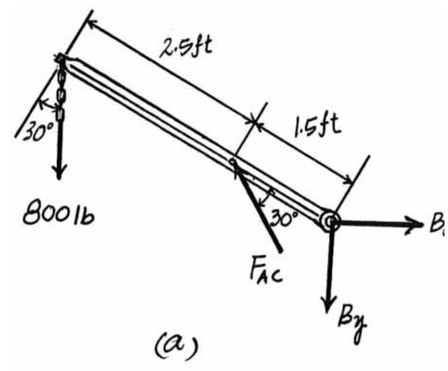
If the engine weighs 800 lb, determine the internal normal force, shear force, and moment at points F and H in the floor crane.



SOLUTION

With reference to Fig. a,

$\zeta + \Sigma M_B = 0;$ $\quad 800 \cos 30°(4) - F_{AC} \sin 30°(1.5) = 0 \quad$ $F_{AC} = 3695.04$ lb



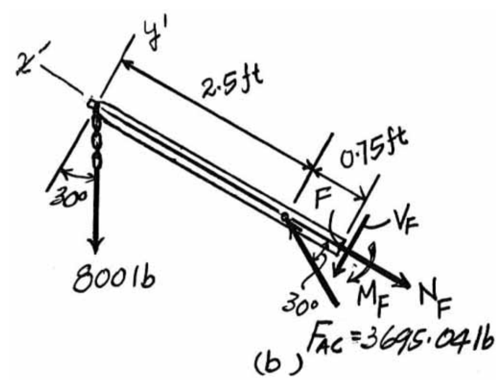
Using this result and referring to Fig. b, we have

$+\nwarrow \Sigma F_{x'} = 0;$ $\quad 3695.04 \cos 30° - 800 \sin 30° - N_F = 0 \quad$ $N_F = 2800$ lb **Ans.**

$+\nearrow \Sigma F_{y'} = 0;$ $\quad 3695.04 \sin 30° - 800 \cos 30° - V_F = 0 \quad$ $V_F = 1155$ lb **Ans.**

$\zeta + \Sigma M_F = 0;$ $\quad 800 \cos 30°(3.25) - 3695.04 \sin 30°(0.75) - M_F = 0$

$M_F = 866 \text{ lb} \cdot \text{ft}$ **Ans.**

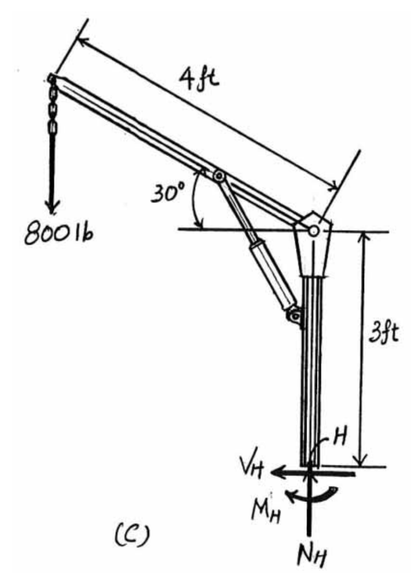


Also, referring to Fig. c, we can write

$\xrightarrow{+} \Sigma F_x = 0;$ $\quad V_H = 0$ **Ans.**

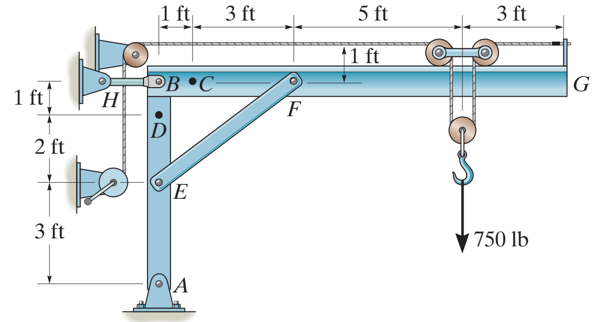
$+\uparrow \Sigma F_y = 0;$ $\quad N_H - 800 = 0 \quad$ $N_H = 800$ lb **Ans.**

$\zeta + \Sigma M_H = 0;$ $\quad 800(4 \cos 30°) - M_H = 0 \quad$ $M_H = 2771 \text{ lb} \cdot \text{ft}$ **Ans.**



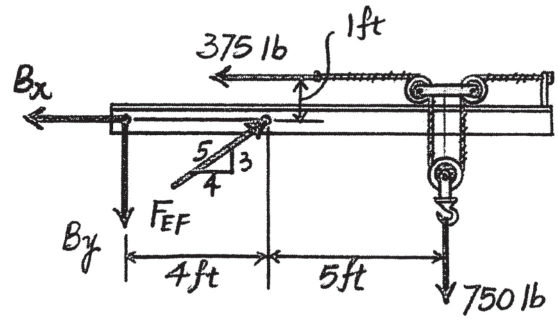
7–33.

The jib crane supports a load of 750 lb from the trolley which rides on the top of the jib. Determine the internal normal force, shear force, and moment in the jib at point C when the trolley is at the position shown. The crane members are pinned together at B, E and F and supported by a short link BH.



SOLUTION

Member BFG:



$\zeta + \Sigma M_B = 0; \qquad F_{EF}\left(\frac{3}{5}\right)(4) - 750\,(9) + 375\,(1) = 0$

$F_{EF} = 2656.25 \text{ lb}$

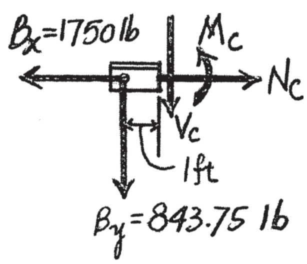
$\xrightarrow{+} \Sigma F_x = 0; \qquad -B_x + 2656.25\left(\frac{4}{5}\right) - 375 = 0$

$B_x = 1750 \text{ lb}$

$+\uparrow \Sigma F_y = 0; \qquad -B_y + 2656.25\left(\frac{3}{5}\right) - 750 = 0$

$B_y = 843.75 \text{ lb}$

Segment BC:



$\xrightarrow{+} \Sigma F_x = 0; \qquad N_C - 1750 = 0$

$N_C = 1.75 \text{ kip}$ **Ans.**

$+\uparrow \Sigma F_y = 0; \qquad -843.75 - V_C = 0$

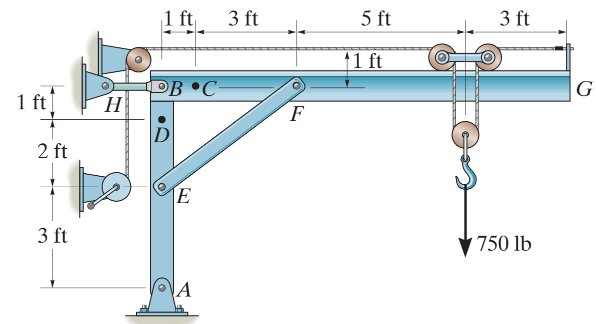
$V_C = -844 \text{ lb}$ **Ans.**

$\zeta + \Sigma M_C = 0; \qquad M_C + 843.75\,(1) = 0$

$M_C = -844 \text{ lb} \cdot \text{ft}$ **Ans.**

7–34.

The jib crane supports a load of 750 lb from the trolley which rides on the top of the jib. Determine the internal normal force, shear force, and moment in the column at point D when the trolley is at the position shown. The crane members are pinned together at B, E and F and supported by a short link BH.

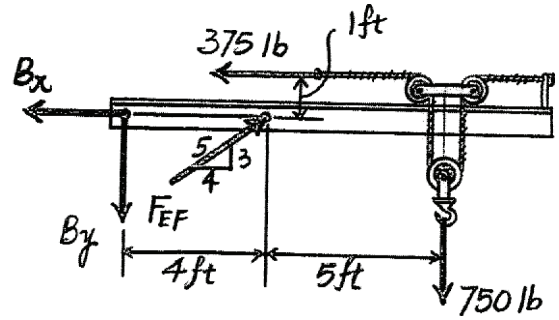


SOLUTION

Member *BFG*:

$\circlearrowleft +\Sigma M_B = 0; \qquad F_{EF}\left(\frac{3}{5}\right)(4) - 750\,(9) + 375(1) = 0$

$F_{EF} = 2656.25 \text{ lb}$



Entire Crane:

$\circlearrowleft +\Sigma M_A = 0; \qquad T_B\,(6) - 750\,(9) + 375(7) = 0$

$T_B = 687.5 \text{ lb}$

$\xrightarrow{+} \Sigma F_x = 0; \qquad A_x - 687.5 - 375 = 0$

$A_x = 1062.5 \text{ lb}$

$+\uparrow \Sigma F_y = 0; \qquad A_y - 750 = 0$

$A_y = 750 \text{ lb}$

Segment AED:

$+\uparrow \Sigma F_y = 0; \qquad N_D + 750 - 2656.25\left(\frac{3}{5}\right) = 0$

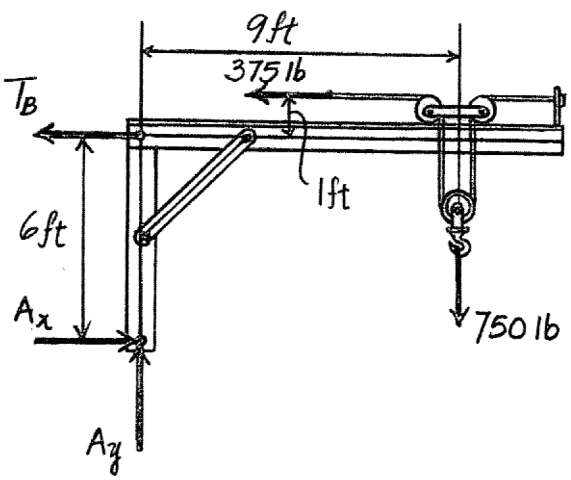
$N_D = 844 \text{ lb}$ **Ans.**

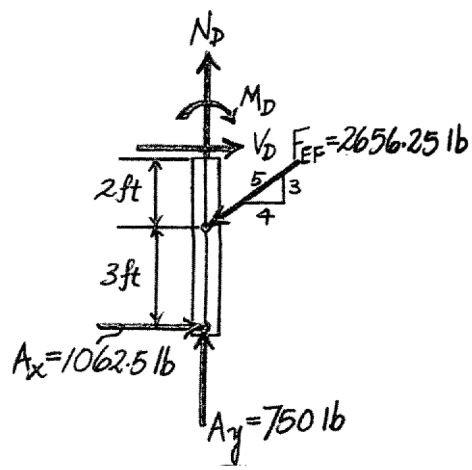
$\xrightarrow{+} \Sigma F_x = 0; \qquad 1062.5 - 2656.25\left(\frac{4}{5}\right) + V_D = 0$

$V_D = 1.06 \text{ kip}$ **Ans.**

$\circlearrowleft +\Sigma M_D = 0; \qquad -M_D - 2656.25\left(\frac{4}{5}\right)(2) + 1062.5\,(5) = 0$

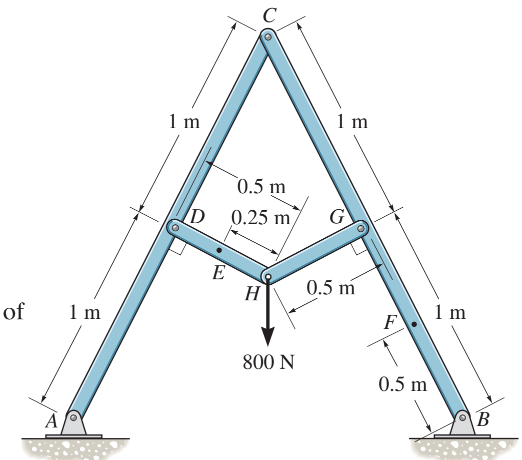
$M_D = 1.06 \text{ kip} \cdot \text{ft}$ **Ans.**





7–35.

Determine the internal normal force, shear force, and bending moment at points E and F of the frame.



SOLUTION

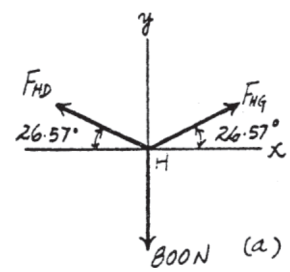
Support Reactions: Members HD and HG are two force members. Using method of joint [FBD (a)], we have

$\overset{+}{\rightarrow} \Sigma F_x = 0 \qquad F_{HG} \cos 26.57^\circ - F_{HD} \cos 26.57^\circ = 0$

$$F_{HD} = F_{HG} = F$$

$+\uparrow \Sigma F_y = 0; \qquad 2F \sin 26.57^\circ - 800 = 0$

$$F_{HD} = F_{HG} = F = 894.43 \text{ N}$$

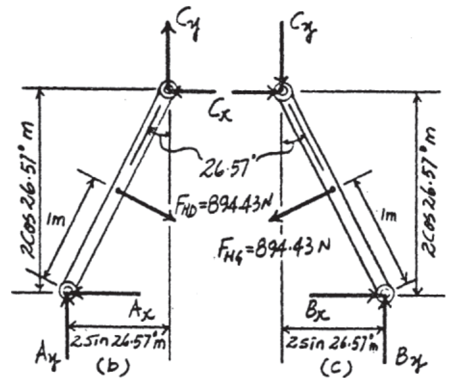


From FBD (b),

$\circlearrowleft + \Sigma M_A = 0; \qquad C_x\,(2 \cos 26.57^\circ) + C_y\,(2 \sin 26.57^\circ) - 894.43(1) = 0 \qquad$ **(1)**

From FBD (c),

$\circlearrowleft + \Sigma M_A = 0; \qquad 894.43(1) - C_x\,(2 \cos 26.57^\circ) + C_y\,(2 \sin 26.57^\circ) = 0 \qquad$ **(2)**



Solving Eqs. (1) and (2) yields,

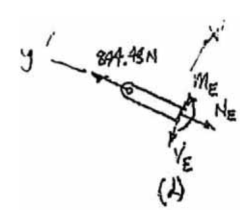
$$C_y = 0 \qquad C_x = 500 \text{ N}$$

Internal Forces: Applying the equations of equilibrium to segment DE [FBD (d)], we have

$\nearrow\!+ \; \Sigma F_{x'} = 0; \qquad V_E = 0 \qquad$ **Ans.**

$\nwarrow\!+ \Sigma F_{y'} = 0; \qquad 894.43 - N_E = 0 \qquad N_E = 894 \text{ N} \qquad$ **Ans.**

$\circlearrowleft + \Sigma M_E = 0; \qquad M_E = 0 \qquad$ **Ans.**



Applying the equations of equilibrium to segment CF [FBD (e)], we have

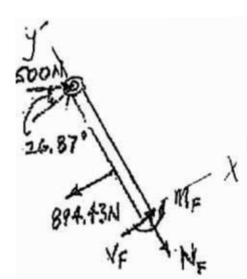
$+\nearrow \Sigma F_{x'} = 0; \qquad V_F + 500 \cos 26.57^\circ - 894.43 = 0$

$$V_F = 447 \text{ N} \qquad \textbf{Ans.}$$

$\nwarrow\!+ \Sigma F_{y'} = 0; \qquad -N_F - 500 \sin 26.57^\circ = 0 \qquad N_F = -224 \text{ N} \qquad$ **Ans.**

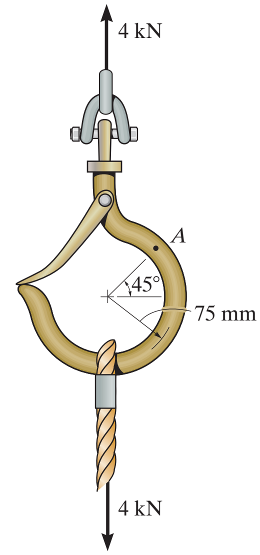
$\circlearrowleft + \Sigma M_F = 0; \qquad M_F + 894.43(0.5) - 500 \cos 26.57^\circ(1.5) = 0$

$$M_F = 224 \text{ N} \cdot \text{m} \qquad \textbf{Ans.}$$



***7–36.**

The hook supports the 4-kN load. Determine the internal normal force, shear force, and moment at point A.



SOLUTION

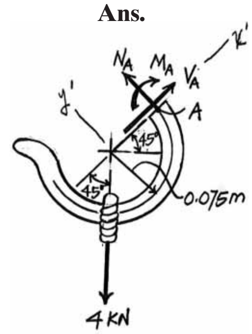
With reference to Fig. a,

$+\nearrow\Sigma F_{x'} = 0; \quad V_A - 4\cos 45° = 0 \quad V_A = 2.83\text{ kN}$ **Ans.**

$+\nwarrow\Sigma F_{y'} = 0; \quad N_A - 4\sin 45° = 0 \quad N_A = 2.83\text{ kN}$ **Ans.**

$\circlearrowleft + \Sigma M_A = 0; \quad 4\sin 45°(0.075) - M_A = 0$

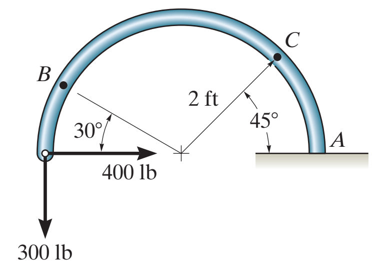
$M_A = 0.212\text{ kN}\cdot\text{m} = 212\text{ N}\cdot\text{m}$ **Ans.**



(a)

7–37.

Determine the normal force, shear force, and moment acting at sections passing through point B on the curved rod.



SOLUTION

$\nearrow+\Sigma F_x = 0;$ $\quad 400 \sin 30° - 300 \cos 30° + N_B = 0$

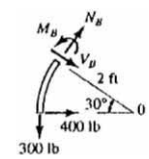
$N_B = 59.8 \text{ lb}$ **Ans.**

$+\searrow\Sigma F_y = 0;$ $\quad V_B + 400 \cos 30° + 300 \sin 30° = 0$

$V_B = -496 \text{ lb}$ **Ans.**

$\circlearrowleft+\Sigma M_B = 0;$ $\quad M_B + 400(2 \sin 30°) + 300(2 - 2 \cos 30°) = 0$

$M_B = -480 \text{ lb} \cdot \text{ft}$ **Ans.**



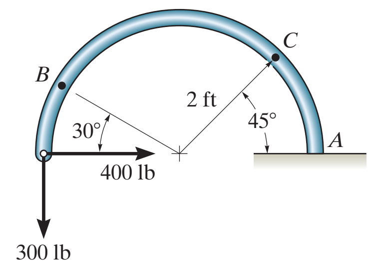
Also,

$\circlearrowleft+\Sigma M_O = 0;$ $\quad -59.81(2) + 300(2) + M_B = 0$

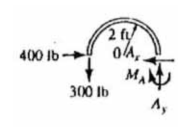
$M_B = -480 \text{ lb} \cdot \text{ft}$ **Ans.**

7–38.

Determine the normal force, shear force, and moment acting at sections passing through point C on the curved rod.



SOLUTION



$\xrightarrow{+} \Sigma F_x = 0;$ $\quad A_x = 400 \text{ lb}$

$+\uparrow \Sigma F_y = 0;$ $\quad A_y = 300 \text{ lb}$

$\circlearrowleft + \Sigma M_A = 0;$ $\quad M_A - 300(4) = 0$

$M_A = 1200 \text{ lb} \cdot \text{ft}$

$+\nwarrow \Sigma F_x = 0;$ $\quad N_C + 400 \sin 45° + 300 \cos 45° = 0$

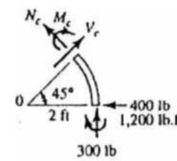
$N_C = -495 \text{ lb}$ **Ans.**

$\nearrow + \Sigma F_x = 0;$ $\quad V_C - 400 \cos 45° + 300 \sin 45° = 0$

$V_C = 70.7 \text{ lb}$ **Ans.**

$\circlearrowleft + \Sigma M_C = 0;$ $\quad -M_C - 1200 - 400(2 \sin 45°) + 300(2 - 2 \cos 45°) = 0$

$M_C = -1590 \text{ lb} \cdot \text{ft} = -1.59 \text{ kip} \cdot \text{ft}$ **Ans.**



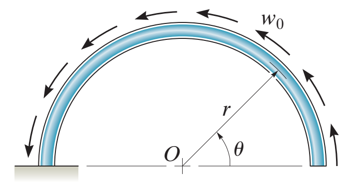
Also,

$\circlearrowleft + \Sigma M_O = 0;$ $\quad 495.0(2) + 300(2) + M_C = 0$

$M_C = -1590 \text{ lb} \cdot \text{ft} = -1.59 \text{ kip} \cdot \text{ft}$ **Ans.**

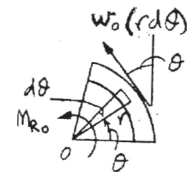
7–39.

The semicircular arch is subjected to a uniform distributed load along its axis of w_0 per unit length. Determine the internal normal force, shear force, and moment in the arch at $\theta = 45°$.



SOLUTION

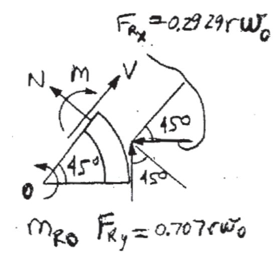
Resultants of distributed load:



$$F_{Rx} = \int_0^\theta w_0\,(r\,d\theta)\sin\theta = r\,w_0\,(-\cos\theta)\Big|_0^\theta = r\,w_0\,(1 - \cos\theta)$$

$$F_{Ry} = \int_0^\theta w_0\,(r\,d\theta)\cos\theta = r\,w_0\,(\sin\theta)\Big|_0^\theta = r\,w_0\,(\sin\theta)$$

$$M_{Ro} = \int_0^\theta w_0\,(r\,d\theta)\,r = r^2\,w_0\theta$$



At $\theta = 45°$

$+\swarrow\Sigma F_x = 0;$ $\quad -V + F_{Rx}\cos\theta - F_{Ry}\sin\theta = 0$

$$V = 0.2929\,r\,w_0\cos 45° - 0.707\,r\,w_0\sin 45°$$

$$V = -0.293\,r\,w_0$$ **Ans.**

$+\nwarrow\Sigma F_y = 0;$ $\quad N + F_{Ry}\cos\theta + F_{Rx}\sin\theta = 0$

$$N = -0.707\,r\,w_0\cos 45° - 0.2929\,r\,w_0\sin 45°$$

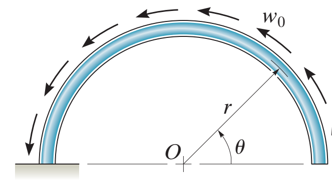
$$N = -0.707\,r\,w_0$$ **Ans.**

$\circlearrowleft + \Sigma M_o = 0;$ $\quad -M + r^2\,w_0\left(\frac{\pi}{4}\right) + (-0.707\,r\,w_0)(r) = 0$

$$M = -0.0783\,r^2\,w_0$$ **Ans.**

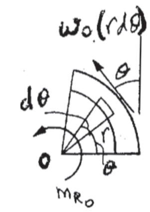
***7–40.**

The semicircular arch is subjected to a uniform distributed load along its axis of w_0 per unit length. Determine the internal normal force, shear force, and moment in the arch at $\theta = 120°$.



SOLUTION

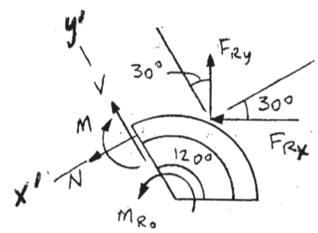
Resultants of distributed load:



$$F_{Rx} = \int_0^\theta w_0\,(r\,d\theta)\sin\theta = r\,w_0(-\cos\theta)\Big|_0^\theta = r\,w_0(1-\cos\theta)$$

$$F_{Ry} = \int_0^\theta w_0(r\,d\theta)\cos\theta = r\,w_0\,(\sin\theta)\Big|_0^\theta = r\,w_0\,(\sin\theta)$$

$$M_{Ro} = \int_0^\theta w_0\,(r\,d\theta)\,r = r^2\,w_0\theta$$



At $\theta = 120°$,

$$F_{Rx} = r\,w_0\,(1-\cos 120°) = 1.5\,r\,w_0$$

$$F_{Ry} = r\,w_0 \sin 120° = 0.86603\,r\,w_0$$

$+\swarrow\Sigma F_{x'} = 0;$ $\quad N + 1.5\,r\,w_0\cos 30° - 0.86603\;r\,w_0\sin 30° = 0$

$N = -0.866\,r\,w_0$ **Ans.**

$+\nwarrow\Sigma F_{y'} = 0;$ $\quad V + 1.5\,r\,w_0\sin 30° + 0.86603\;r\,w_0\cos 30° = 0$

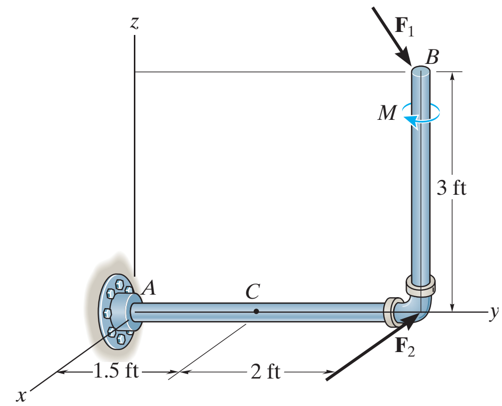
$V = -1.5\,rw_0$ **Ans.**

$\circlearrowleft+\Sigma M_o = 0;$ $\quad -M + r^2 w_0\,(\pi)\left(\dfrac{120°}{180°}\right) + (-0.866\,r\,w_0)r = 0$

$M = 1.23\,r^2 w_0$ **Ans.**

7–41.

Determine the x, y, z components of force and moment at point C in the pipe assembly. Neglect the weight of the pipe. The load acting at (0, 3.5 ft, 3 ft) is $\mathbf{F}_1 = \{-24\mathbf{i} - 10\mathbf{k}\}$ lb and $\mathbf{M} = \{-30\mathbf{k}\}$ lb · ft and at point (0, 3.5 ft, 0) $\mathbf{F}_2 = \{-80\mathbf{i}\}$ lb.



SOLUTION

Free body Diagram: The support reactions need not be computed.

Internal Forces: Applying the equations of equilibrium to segment BC, we have

$\Sigma F_x = 0;$ $(V_C)_x - 24 - 80 = 0$ $(V_C)_x = 104 \text{ lb}$ **Ans.**

$\Sigma F_y = 0;$ $N_C = 0$ **Ans.**

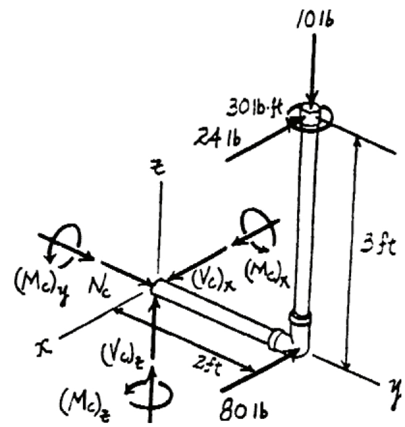
$\Sigma F_z = 0;$ $(V_C)_z - 10 = 0$ $(V_C)_z = 10.0 \text{ lb}$ **Ans.**

$\Sigma M_x = 0;$ $(M_C)_x - 10(2) = 0$ $(M_C)_x = 20.0 \text{ lb} \cdot \text{ft}$ **Ans.**

$\Sigma M_y = 0;$ $(M_C)_y - 24(3) = 0$ $(M_C)_y = 72.0 \text{ lb} \cdot \text{ft}$ **Ans.**

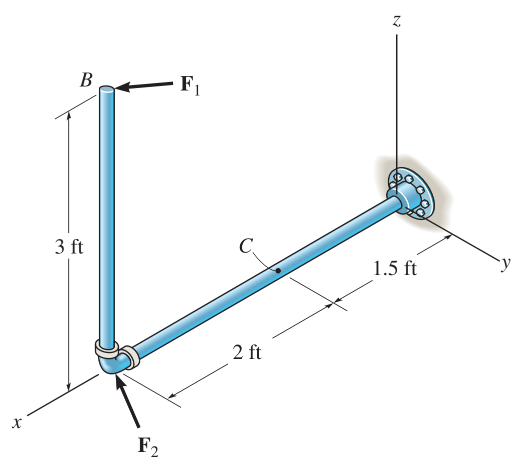
$\Sigma M_z = 0;$ $(M_C)_z + 24(2) + 80(2) - 30 = 0$

$(M_C)_z = -178 \text{ lb} \cdot \text{ft}$ **Ans.**



7–42.

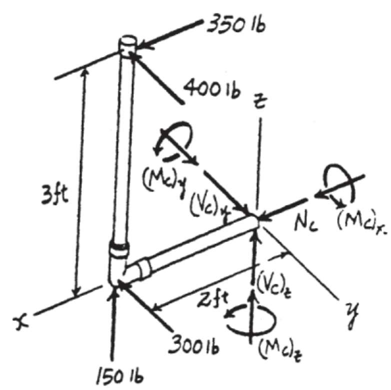
Determine the x, y, z components of force and moment at point C in the pipe assembly. Neglect the weight of the pipe. Take $\mathbf{F}_1 = \{350\mathbf{i} - 400\mathbf{j}\}$ lb and $\mathbf{F}_2 = \{-300\mathbf{j} + 150\mathbf{k}\}$ lb.



SOLUTION

Free body Diagram: The support reactions need not be computed.

Internal Forces: Applying the equations of equilibrium to segment BC, we have



$\Sigma F_x = 0;$ $\quad N_C + 350 = 0 \qquad N_C = -350 \text{ lb}$ **Ans.**

$\Sigma F_y = 0;$ $\quad (V_C)_y - 400 - 300 = 0 \qquad (V_C)_y = 700 \text{ lb}$ **Ans.**

$\Sigma F_z = 0;$ $\quad (V_C)_z + 150 = 0 \qquad (V_C)_z = -150 \text{ lb}$ **Ans.**

$\Sigma M_x = 0;$ $\quad (M_C)_x + 400(3) = 0$

$(M_C)_x = -1200 \text{ lb}\cdot\text{ft} = -1.20 \text{ kip}\cdot\text{ft}$ **Ans.**

$\Sigma M_y = 0;$ $\quad (M_C)_y + 350(3) - 150(2) = 0$

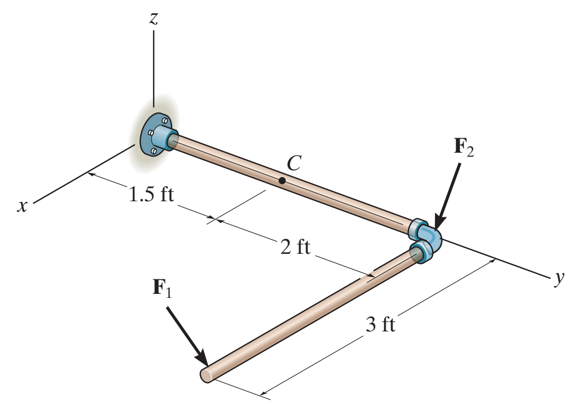
$(M_C)_y = -750 \text{ lb}\cdot\text{ft}$ **Ans.**

$\Sigma M_z = 0;$ $\quad (M_C)_z - 300(2) - 400(2) = 0$

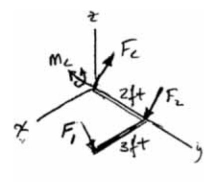
$(M_C)_z = 1400 \text{ lb}\cdot\text{ft} = 1.40 \text{ kip}\cdot\text{ft}$ **Ans.**

7–43.

Determine the x, y, z components of internal loading at a section passing through point C in the pipe assembly. Neglect the weight of the pipe. Take $\mathbf{F_1} = \{350\mathbf{j} - 400\mathbf{k}\}$ lb and $\mathbf{F_2} = \{150\mathbf{i} - 200\mathbf{k}\}$ lb.



SOLUTION



$\Sigma\mathbf{F}_R = 0;\qquad \mathbf{F}_C + \mathbf{F}_1 + \mathbf{F}_2 = 0$

$\mathbf{F}_C = \{-170\mathbf{i} - 50\mathbf{j} + 600\mathbf{k}\}$ lb

$C_x = -150$ lb **Ans.**

$C_y = -350$ lb **Ans.**

$C_z = 600$ lb **Ans.**

$\Sigma\mathbf{M}_R = 0;\qquad \mathbf{M}_C + \mathbf{r}_{C1} \times \mathbf{F}_1 + \mathbf{r}_{C2} \times \mathbf{F}_2 = 0$

$$\mathbf{M}_C + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 2 & 0 \\ 0 & 350 & -400 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 2 & 0 \\ 150 & -150 & -200 \end{vmatrix} = 0$$

$\mathbf{M}_C = \{1200\mathbf{i} - 1200\mathbf{j} - 750\mathbf{k}\}\ \text{lb}\cdot\text{ft}$

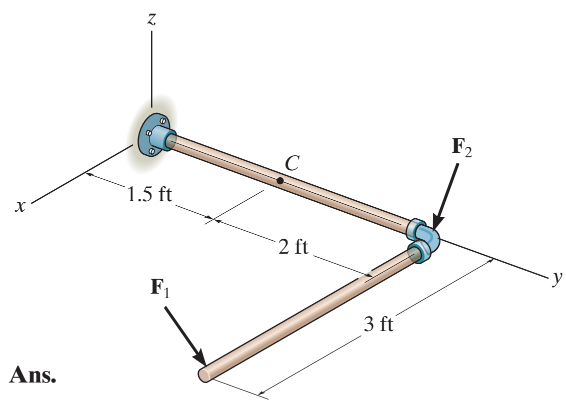
$M_{Cx} = 1.20\ \text{kip}\cdot\text{ft}$ **Ans.**

$M_{Cy} = -1.20\ \text{kip}\cdot\text{ft}$ **Ans.**

$M_{Cz} = -750\ \text{lb}\cdot\text{ft}$ **Ans.**

***7–44.**

Determine the x, y, z components of internal loading at a section passing through point C in the pipe assembly. Neglect the weight of the pipe. Take $\mathbf{F_1} = \{-80\mathbf{i} + 200\mathbf{j} - 300\mathbf{k}\}$ lb and $\mathbf{F_2} = \{250\mathbf{i} - 150\mathbf{j} - 200\mathbf{k}\}$ lb.



SOLUTION

$\Sigma \mathbf{F}_R = 0;$ $\quad \mathbf{F}_C + \mathbf{F}_1 + \mathbf{F}_2 = 0$

$$\mathbf{F}_C = \{-170\mathbf{i} - 50\mathbf{j} + 500\mathbf{k}\}\ \text{lb}$$

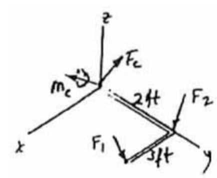
$C_x = -170$ lb **Ans.**

$C_y = -50$ lb **Ans.**

$C_z = 500$ lb **Ans.**

$\Sigma \mathbf{M}_R = 0;$ $\quad \mathbf{M}_C + \mathbf{r}_{C1} \times \mathbf{F}_1 + \mathbf{r}_{C2} \times \mathbf{F}_2 = 0$

$$\mathbf{M}_C + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 2 & 0 \\ -80 & 200 & -300 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 2 & 0 \\ 250 & -150 & -200 \end{vmatrix} = 0$$



$$\mathbf{M}_C = \{1000\mathbf{i} - 900\mathbf{j} - 260\mathbf{k}\}\ \text{lb} \cdot \text{ft}$$

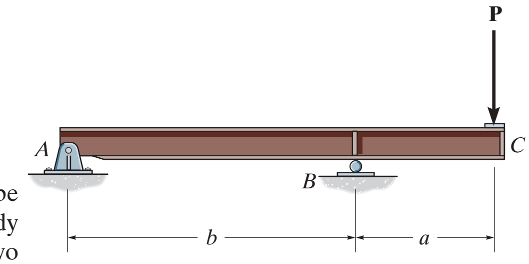
$M_{Cx} = 1$ kip · ft **Ans.**

$M_{Cy} = -900$ lb · ft **Ans.**

$M_{Cz} = -260$ lb · ft **Ans.**

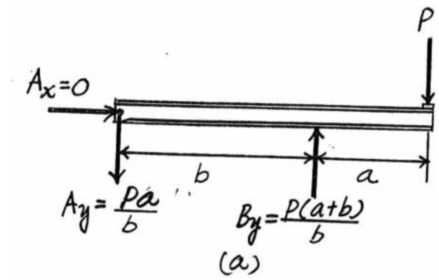
7–45.

Draw the shear and moment diagrams for the overhang beam.



SOLUTION

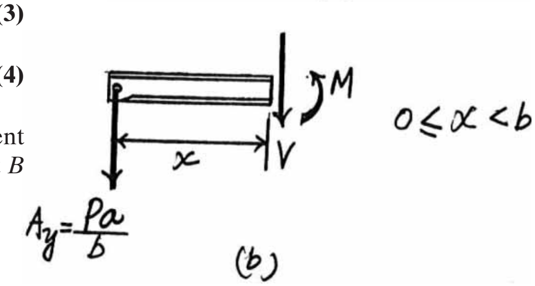
Since the loading discontinues at B, the shear stress and moment equation must be written for regions $0 \le x < b$ and $b < x \le a + b$ of the beam. The free-body diagram of the beam's segment sectioned through an arbitrary point in these two regions are shown in Figs. *b* and *c*.



Region $0 \le x < b$, Fig. *b*

$+\uparrow \Sigma F_y = 0; \qquad -\dfrac{Pa}{b} - V = 0 \qquad V = -\dfrac{Pa}{b} \qquad \textbf{(1)}$

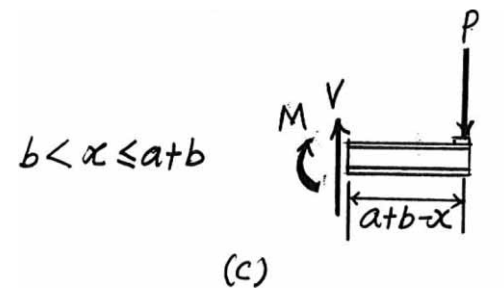
$\circlearrowleft + \Sigma M = 0; \qquad M + \dfrac{Pa}{b}x = 0 \qquad M = -\dfrac{Pa}{b}x \qquad \textbf{(2)}$



Region $b < x \le a + b$, Fig. *c*

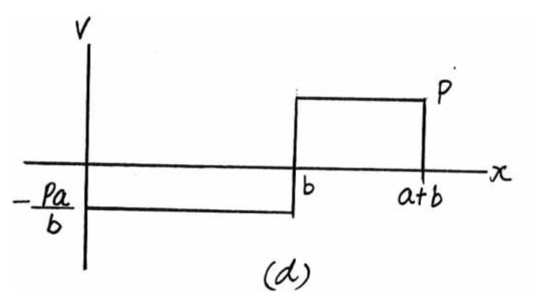
$\Sigma F_y = 0; \qquad V - P = 0 \qquad V = P \qquad \textbf{(3)}$

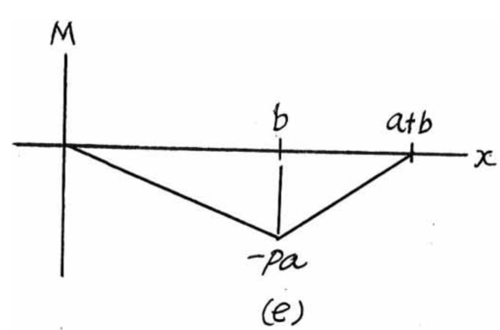
$\circlearrowleft + \Sigma M = 0; \qquad -M - P(a + b - x) = 0 \qquad M = -P(a + b - x) \qquad \textbf{(4)}$



The shear diagram in Fig. *d* is plotted using Eqs. (1) and (3), while the moment diagram shown in Fig. *e* is plotted using Eqs. (2) and (4). The value of moment at B is evaluated using either Eqs. (2) or (4) by substituting $x = b$; i.e.,

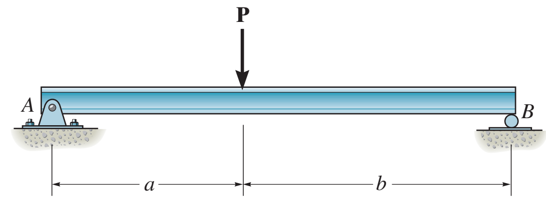
$$M|_{x=b} = -\frac{Pa}{b}(b) = -Pa \text{ or } M|_{x=b} = -P(a + b - b) = -Pa$$





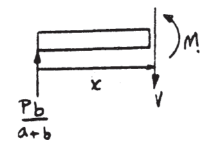
7–46.

Draw the shear and moment diagrams for the beam (a) in terms of the parameters shown; (b) set $P = 600$ lb, $a = 5$ ft, $b = 7$ ft.



SOLUTION

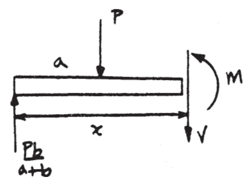
(a) For $0 \le x < a$



$+\uparrow \Sigma F_y = 0;$ $\qquad \dfrac{P\,b}{a+b} - V = 0$

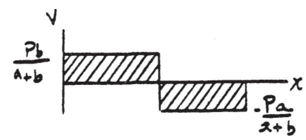
$$V = \frac{P\,b}{a+b}$$ **Ans.**

$\circlearrowleft + \Sigma M = 0;$ $\qquad M - \dfrac{P\,b}{a+b}x = 0$



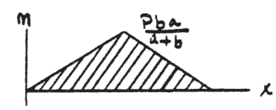
$$M = \frac{P\,b}{a+b}x$$ **Ans.**

For $a < x \le (a+b)$



$+\uparrow \Sigma F_y = 0;$ $\qquad \dfrac{P\,b}{a+b} - P - V = 0$

$$V = -\frac{P\,a}{a+b}$$ **Ans.**



$\circlearrowleft + \Sigma M = 0;$ $\qquad -\dfrac{P\,b}{a+b}x + P(x-a) + M = 0$

$$M = P\,a - \frac{P\,a}{a+b}x$$ **Ans.**

(b) For $P = 600$ lb, $a = 5$ ft, $b = 7$ ft

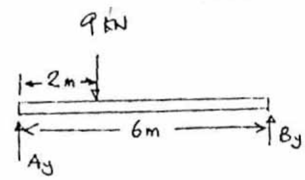
7–46. (continued)

(b)

$\circlearrowleft + \Sigma M_B = 0;$ $\quad A_y(6) - 9(4) = 0$

$A_y = 6\text{ kN}$

$+\uparrow \Sigma F_y = 0;$ $\quad B_y = 3\text{ kN}$



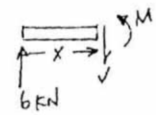
For $0 \leq x \leq 2\text{ m}$

$+\uparrow \Sigma F_y = 0;$ $\quad 6 - V = 0$

$V = 6\ \text{kN}$ **Ans.**

$\circlearrowleft + \Sigma M = 0;$ $\quad 6x - M = 0$

$M = 6x\ \text{kN} \cdot \text{m}$ **Ans.**



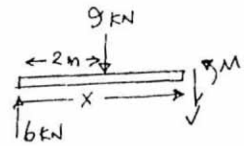
Fo $2 < x \leq 6\text{ m}$

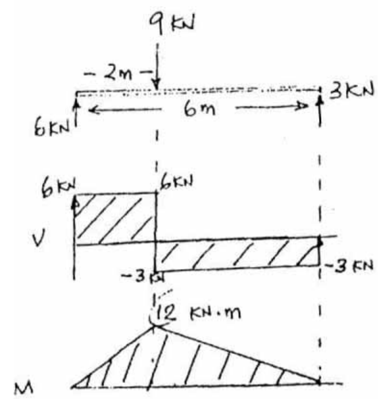
$+\uparrow \Sigma F_y = 0;$ $\quad 6 - 9 - V = 0$

$V = -3\ \text{kN}$ **Ans.**

$\circlearrowleft + \Sigma M = 0;$ $\quad 6x - 9(x - 2) - M = 0$

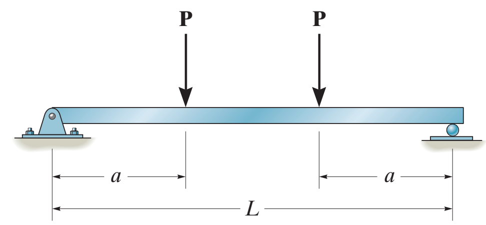
$M = 18 - 3x\ \text{kN} \cdot \text{m}$ **Ans.**





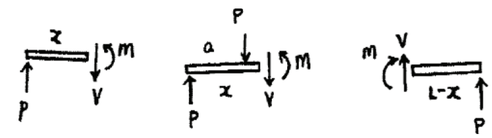
7–47.

Draw the shear and moment diagrams for the beam (a) in terms of the parameters shown; (b) set $P = 800$ lb, $a = 5$ ft, $L = 12$ ft.



SOLUTION

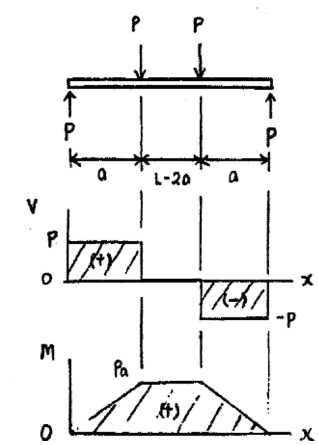
(a) For $0 \leq x < a$



$+\uparrow \Sigma F_y = 0; \quad V = P$ **Ans.**

$\zeta + \Sigma M = 0; \quad M = Px$ **Ans.**

For $a < x < L-a$



$+\uparrow \Sigma F_y = 0; \quad V = 0$ **Ans.**

$\zeta + \Sigma M = 0; \quad -Px + P(x - a) + M = 0$

$M = Pa$ **Ans.**

For $L-a < x \leq L$

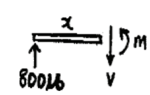
$+\uparrow \Sigma F_y = 0; \quad V = -P$ **Ans.**

$\zeta + \Sigma M = 0; \quad -M + P(L - x) = 0$

$M = P(L - x)$ **Ans.**

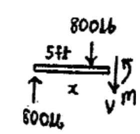
(b) Set $P = 800$ lb, $a = 5$ ft, $L = 12$ ft

For $0 \leq x < 5$ ft



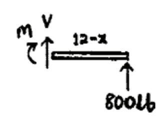
$+\uparrow \Sigma F_y = 0; \quad V = 800$ lb **Ans.**

$\zeta + \Sigma M = 0; \quad M = 800x$ lb · ft **Ans.**



For 5 ft $< x <$ 7 ft

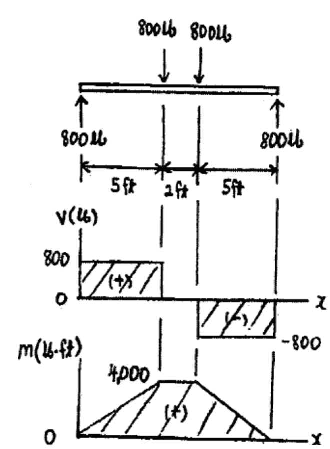
$+\uparrow \Sigma F_y = 0; \quad V = 0$ **Ans.**



$\zeta + \Sigma M = 0; \quad -800x + 800(x - 5) + M = 0$

$M = 4000$ lb · ft **Ans.**

For 7 ft $< x \leq$ 12 ft



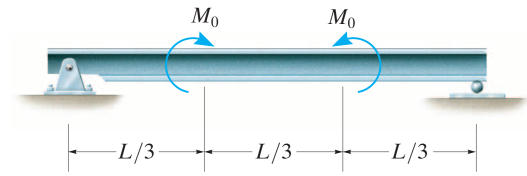
$+\uparrow \Sigma F_y = 0; \quad V = -800$ lb **Ans.**

$\zeta + \Sigma M = 0; \quad -M + 800(12 - x) = 0$

$M = (9600 - 800x)$ lb · ft **Ans.**

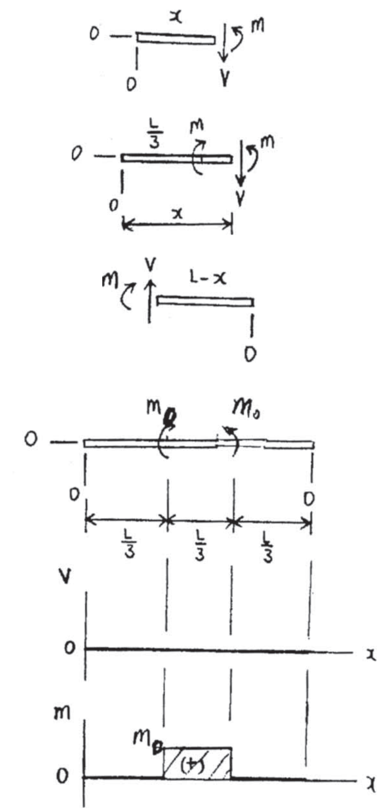
***7–48.**

Draw the shear and moment diagrams for the beam (a) in terms of the parameters shown; (b) set $M_0 = 500\ \text{N}\cdot\text{m}$, $L = 8\ \text{m}$.



SOLUTION

(a)



For $0 \leq x \leq \dfrac{L}{3}$

$+\uparrow \Sigma F_y = 0; \qquad V = 0$ **Ans.**

$\circlearrowleft + \Sigma M = 0; \qquad M = 0$ **Ans.**

For $\dfrac{L}{3} < x < \dfrac{2L}{3}$

$+\uparrow \Sigma F_y = 0; \qquad V = 0$ **Ans.**

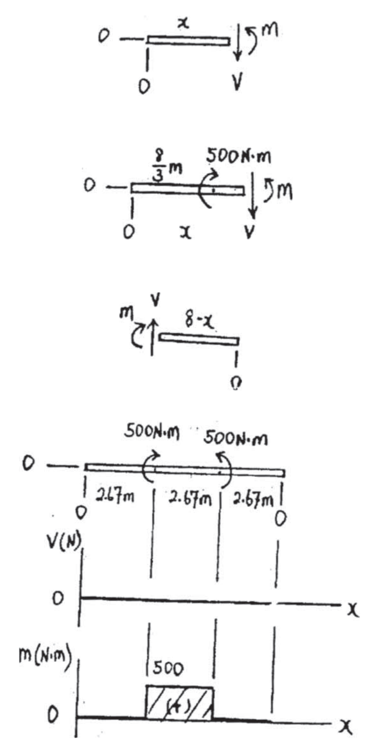
$\circlearrowleft + \Sigma M = 0; \qquad M = M_0$ **Ans.**

For $\dfrac{2L}{3} < x \leq L$

$+\uparrow \Sigma F_y = 0; \qquad V = 0$ **Ans.**

$\circlearrowleft + \Sigma M = 0; \qquad M = 0$ **Ans.**

(b)



Set $M_0 = 500\ \text{N}\cdot\text{m}, L = 8\ \text{m}$

For $0 \leq x < \dfrac{8}{3}\ \text{m}$

$+\uparrow \Sigma F_y = 0; \qquad V = 0$ **Ans.**

$\circlearrowright + \Sigma M = 0; \qquad M = 0$ **Ans.**

For $\dfrac{8}{3}\ \text{m} < x < \dfrac{16}{3}\ \text{m}$

$+\uparrow \Sigma F_y = 0; \qquad V = 0$ **Ans.**

$\circlearrowright + \Sigma M = 0; \qquad M = 500\ \text{N}\cdot\text{m}$ **Ans.**

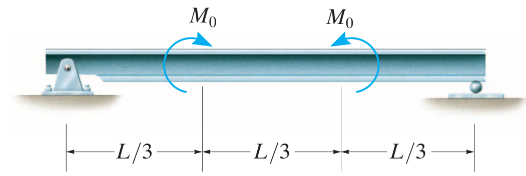
For $\dfrac{16}{3}\ \text{m} < x \leq 8\ \text{m}$

$+\uparrow \Sigma F_y = 0; \qquad V = 0$ **Ans.**

$\circlearrowright + \Sigma M = 0; \qquad M = 0$ **Ans.**

7–49.

If $L = 9$ m, the beam will fail when the maximum shear force is $V_{max} = 5$ kN or the maximum bending moment is $M_{max} = 2$ kN·m. Determine the magnitude M_0 of the largest couple moments it will support.



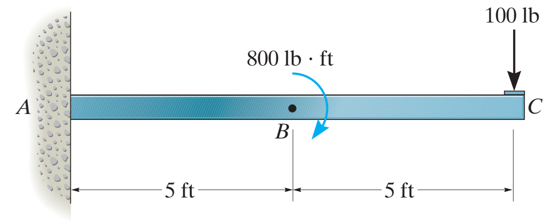
SOLUTION

See solution to Prob. 7–48 a.

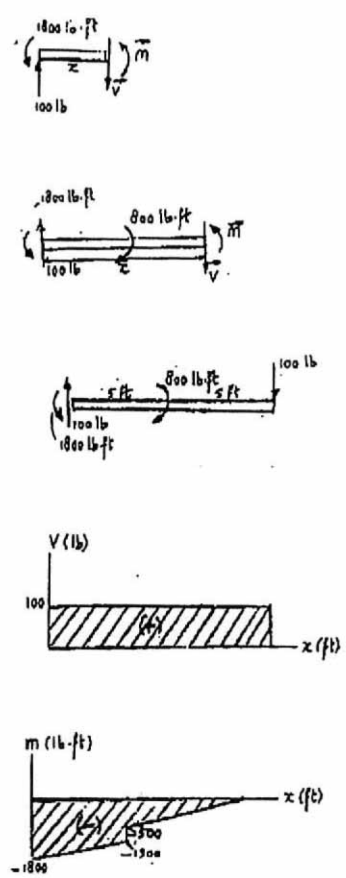
$M_{max} = M_0 = 2\ \text{kN} \cdot \text{m}$ **Ans.**

7–50.

Draw the shear and moment diagrams for the beam.

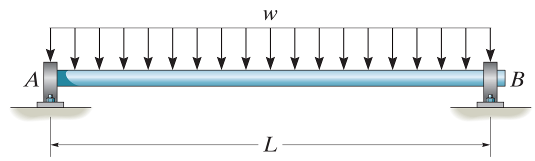


SOLUTION



7–51.

The shaft is supported by a thrust bearing at A and a journal bearing at B. If $L = 10$ ft, the shaft will fail when the maximum moment is $M_{\text{max}} = 5$ kip · ft. Determine the largest uniform distributed load w the shaft will support.



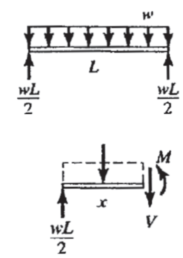
SOLUTION

For $0 \leq x \leq L$

$$\zeta + \Sigma M = 0; \qquad -\frac{wL}{2}x + wx\left(\frac{x}{2}\right) + M = 0$$

$$M = \frac{wL}{2}x - \frac{wx^2}{2}$$

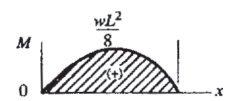
$$M = \frac{w}{2}(Lx - x^2)$$



From the moment diagram

$$M_{max} = \frac{wL^2}{8}$$

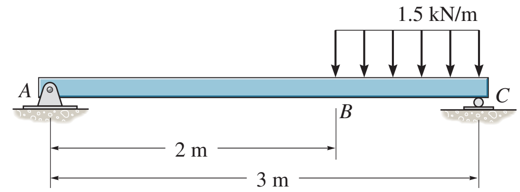
$$5000 = \frac{w(10)^2}{8}$$



$$w = 400 \text{ lb/ft}$$ **Ans.**

***7–52.**

Draw the shear and moment diagrams for the beam.

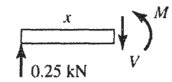


SOLUTION

Support Reactions:

$\zeta + \Sigma M_A = 0; \qquad C_y(3) - 1.5(2.5) = 0 \qquad C_y = 1.25 \text{ kN}$

$+\uparrow \Sigma F_y = 0; \qquad A_y - 1.5 + 1.25 = 0 \qquad A_y = 0.250 \text{ kN}$

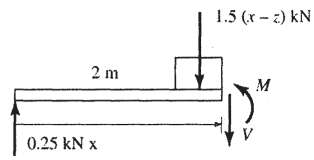


(a)

Shear and Moment Functions: For $\mathbf{0 \le x < 2\ m}$ [FBD (a)],

$+\uparrow \Sigma F_y = 0; \qquad 0.250 - V = 0 \qquad V = 0.250 \text{ kN}$ **Ans.**

$\zeta + \Sigma M = 0; \qquad M - 0.250x = 0 \qquad M = (0.250x) \text{ kN} \cdot \text{m}$ **Ans.**



(b)

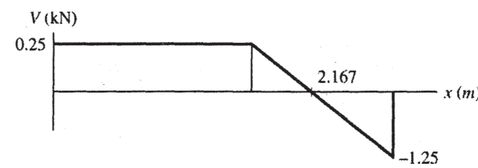
For $\mathbf{2\ m < x \le 3\ m}$ [FBD (b)],

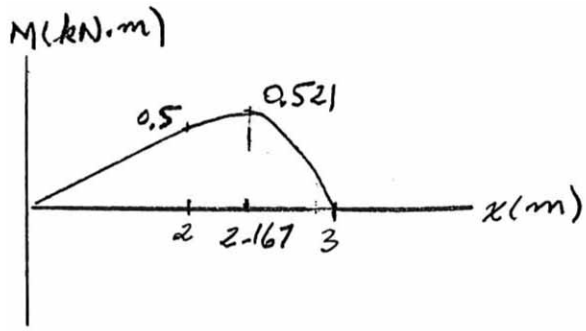
$+\uparrow \Sigma F_y = 0; \qquad 0.25 - 1.5(x - 2) - V = 0$

$V = \{3.25 - 1.50x\} \text{ kN}$ **Ans.**

$\zeta + \Sigma M = 0; \qquad 0.25x - 1.5(x - 2)\left(\dfrac{x - 2}{2}\right) - M = 0$

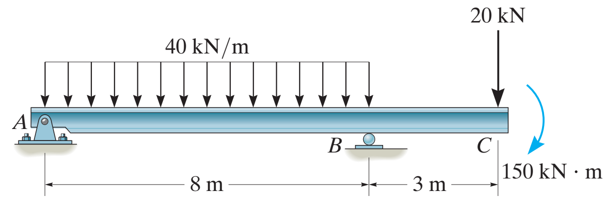
$M = \{-0.750x^2 + 3.25x - 3.00\} \text{ kN} \cdot \text{m}$ **Ans.**





7–53.

Draw the shear and moment diagrams for the beam.



SOLUTION

$0 \le x < 8$

$+\uparrow \Sigma F_y = 0; \qquad 133.75 - 40x - V = 0$

$V = 133.75 - 40x$ **Ans.**

$\circlearrowleft + \Sigma M = 0; \qquad M + 40x\left(\frac{x}{2}\right) - 133.75x = 0$

$M = 133.75x - 20x^2$ **Ans.**

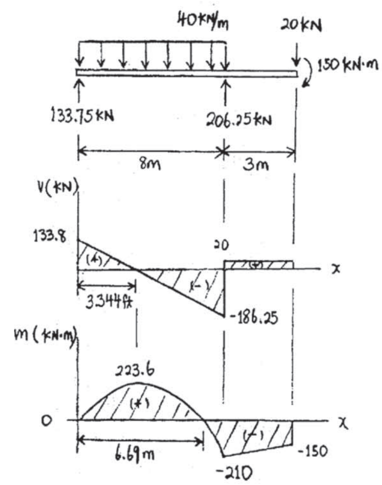
$8 < x \le 11$

$+\uparrow \Sigma F_y = 0; \qquad V - 20 = 0$

$V = 20$ **Ans.**

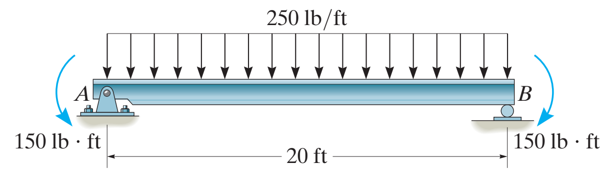
$\circlearrowright + \Sigma M = 0; \qquad M + 20(11 - x) + 150 = 0$

$M = 20x - 370$ **Ans.**



7–54.

Draw the shear and moment diagrams for the beam.



SOLUTION

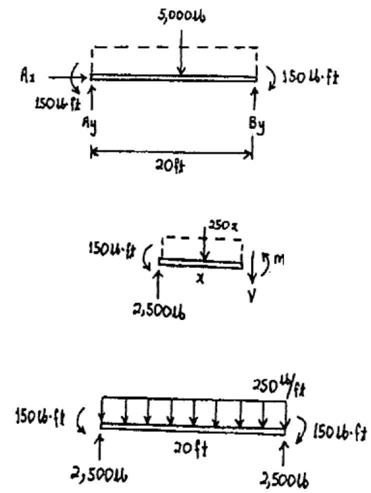
$\circlearrowleft + \Sigma M_A = 0;$ $\quad -5000(10) + B_y(20) = 0$

$B_y = 2500 \text{ lb}$

$\xrightarrow{+} \Sigma F_x = 0;$ $\quad A_x = 0$

$+\uparrow \Sigma F_y = 0;$ $\quad A_y - 5000 + 2500 = 0$

$A_y = 2500 \text{ lb}$



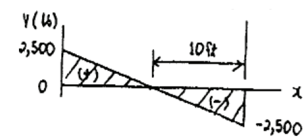
For $0 \leq x \leq 20$ ft

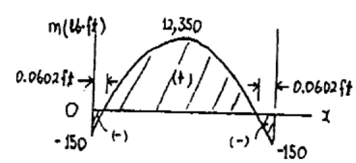
$+\uparrow \Sigma F_y = 0;$ $\quad 2500 - 250x - V = 0$

$V = 250(10 - x)$ **Ans.**

$\circlearrowleft + \Sigma M = 0;$ $\quad -2500(x) + 150 + 250x\left(\frac{x}{2}\right) + M = 0$

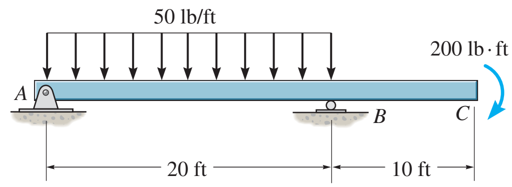
$M = 25(100x - 5x^2 - 6)$ **Ans.**





7–55.

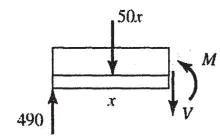
Draw the shear and bending-moment diagrams for the beam.



SOLUTION

Support Reactions:

$\circlearrowleft + \Sigma M_B = 0;$ $\quad 1000(10) - 200 - A_y(20) = 0 \qquad A_y = 490 \text{ lb}$



(a)

Shear and Moment Functions: For $0 \leq x < 20$ ft [FBD (a)],

$+\uparrow \Sigma F_y = 0;$ $\quad 490 - 50x - V = 0$

$V = \{490 - 50.0x\}$ lb **Ans.**

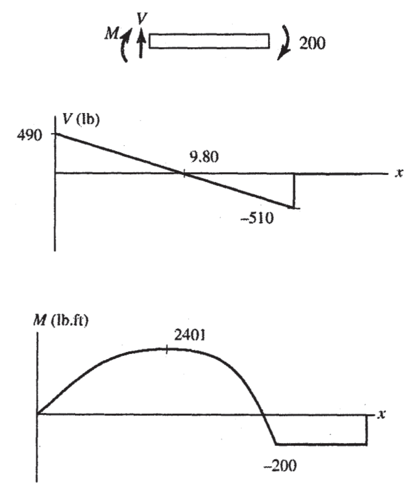
$\circlearrowleft + \Sigma M = 0;$ $\quad M + 50x\left(\frac{x}{2}\right) - 490x = 0$

$M = \{490x - 25.0x^2\}$ lb · ft **Ans.**

For 20 ft $< x \leq 30$ ft [FBD (b)],

$+\uparrow \Sigma F_y = 0;$ $\quad V = 0$ **Ans.**

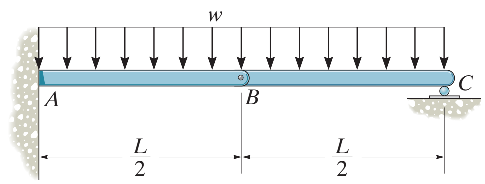
$\circlearrowleft + \Sigma M = 0;$ $\quad -200 - M = 0 \quad M = -200$ lb · ft **Ans.**



(b)

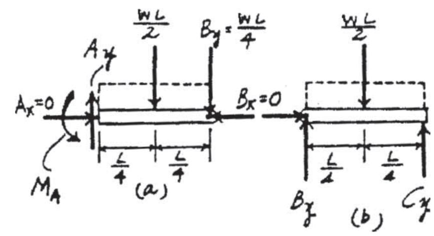
7–56.

Draw the shear and bending-moment diagrams for beam ABC. Note that there is a pin at B.



SOLUTION

Support Reactions: From FBD (a),

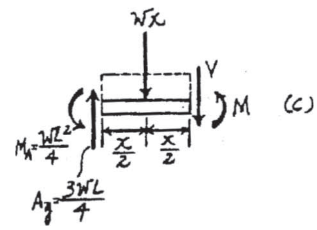


$$\zeta + \Sigma M_C = 0; \qquad \frac{wL}{2}\left(\frac{L}{4}\right) - B_y\left(\frac{L}{2}\right) = 0 \qquad B_y = \frac{wL}{4}$$

From FBD (b),

$$+\uparrow \Sigma F_y = 0; \qquad A_y - \frac{wL}{2} - \frac{wL}{4} = 0 \qquad A_y = \frac{3wL}{4}$$

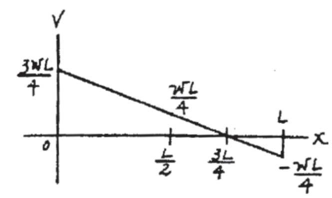
$$\zeta + \Sigma M_A = 0; \qquad M_A - \frac{wL}{2}\left(\frac{L}{4}\right) - \frac{wL}{4}\left(\frac{L}{2}\right) = 0 \qquad M_A = \frac{wL^2}{4}$$



Shear and Moment Functions: For $0 \leq x \leq L$ [FBD (c)],

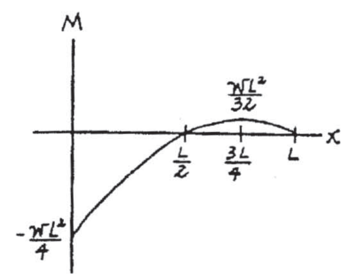
$$+\uparrow \Sigma F_y = 0; \qquad \frac{3wL}{4} - wx - V = 0$$

$$V = \frac{w}{4}(3L - 4x) \qquad \textbf{Ans.}$$



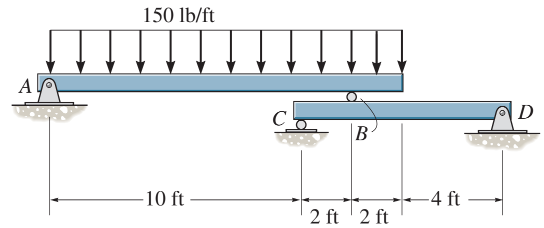
$$\zeta + \Sigma M = 0; \qquad \frac{3wL}{4}(x) - wx\left(\frac{x}{2}\right) - \frac{wL^2}{4} - M = 0$$

$$M = \frac{w}{4}\left(3Lx - 2x^2 - L^2\right) \qquad \textbf{Ans.}$$



7–57.

Draw the shear and bending-moment diagrams for each of the two segments of the compound beam.



SOLUTION

Support Reactions: From FBD (a),

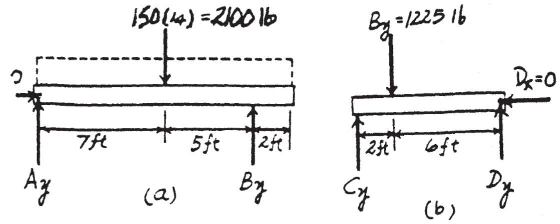
$\zeta + \Sigma M_A = 0;\quad B_y\,(12) - 2100(7) = 0 \qquad B_y = 1225 \text{ lb}$

$+\uparrow \Sigma F_y = 0;\quad A_y + 1225 - 2100 = 0 \qquad A_y = 875 \text{ lb}$

From FBD (b),

$\zeta + \Sigma M_D = 0;\quad 1225(6) - C_y(8) = 0 \qquad C_y = 918.75 \text{ lb}$

$+\uparrow \Sigma F_y = 0;\quad D_y + 918.75 - 1225 = 0 \qquad D_y = 306.25 \text{ lb}$



Shear and Moment Functions: Member AB.

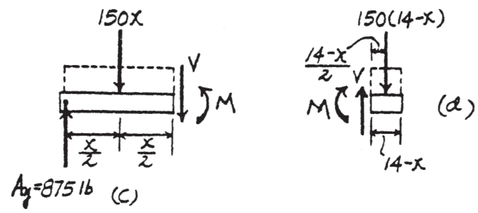
For $\mathbf{0 \le x < 12}$ **ft** [FBD (c)],

$+\uparrow \Sigma F_y = 0;\qquad 875 - 150x - V = 0$

$V = \{875 - 150x\} \text{ lb}$ **Ans.**

$\zeta + \Sigma M = 0;\qquad M + 150x\left(\dfrac{x}{2}\right) - 875x = 0$

$M = \{875x - 75.0x^2\} \text{ lb}\cdot\text{ft}$ **Ans.**



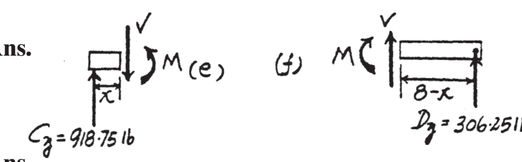
For $\mathbf{12}$ **ft** $< x \le \mathbf{14}$ **ft** [FBD (d)],

$+\uparrow \Sigma F_y = 0;\qquad V - 150(14 - x) = 0$

$V = \{2100 - 150x\} \text{ lb}$ **Ans.**

$\zeta + \Sigma M = 0;\qquad -150(14 - x)\left(\dfrac{14 - x}{2}\right) - M = 0$

$M = \{-75.0x^2 + 2100x - 14700\} \text{ lb}\cdot\text{ft}$ **Ans.**



For member CBD, $\mathbf{0 \le x < 2}$ **ft** [FBD (e)],

$+\uparrow \Sigma F_y = 0;\qquad 918.75 - V = 0 \qquad V = 919 \text{ lb}$ **Ans.**

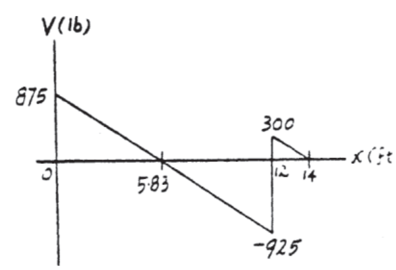
$\zeta + \Sigma M = 0;\quad 918.75x - M = 0 \qquad M = \{919x\} \text{ lb}\cdot\text{ft}$ **Ans.**

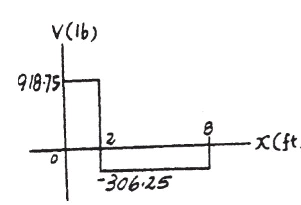
For $\mathbf{2}$ **ft** $< x \le \mathbf{8}$ **ft** [FBD (f)],

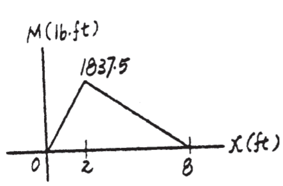
$+\uparrow \Sigma F_y = 0;\qquad V + 306.25 = 0 \qquad V = 306 \text{ lb}$ **Ans.**

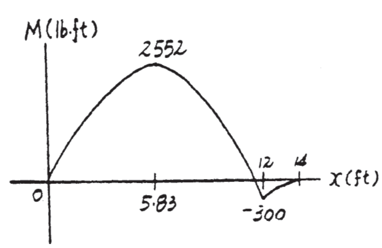
$+\Sigma M = 0;\qquad 306.25(8 - x) - M = 0$

$M = \{2450 - 306x\} \text{ lb}\cdot\text{ft}$ **Ans.**



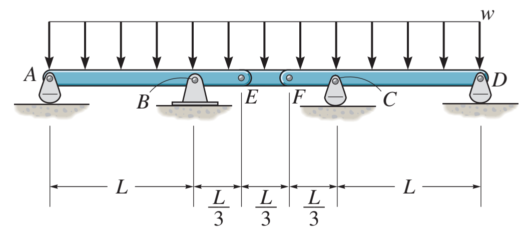






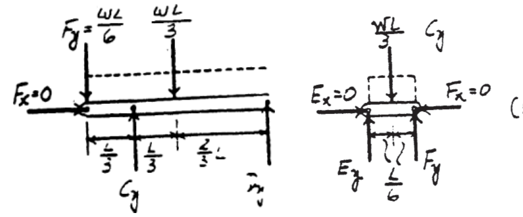
***7–58.**

Draw the shear and moment diagrams for the compound beam. The beam is pin-connected at E and F.



SOLUTION

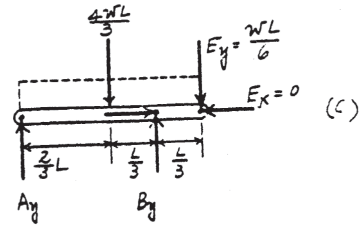
Support Reactions: From FBD (b),



$$\zeta + \Sigma M_E = 0; \qquad F_y\left(\frac{L}{3}\right) - \frac{wL}{3}\left(\frac{L}{6}\right) = 0 \qquad F_y = \frac{wL}{6}$$

$$+\uparrow \Sigma F_y = 0; \qquad E_y + \frac{wL}{6} - \frac{wL}{3} = 0 \qquad E_y = \frac{wL}{6}$$

From FBD (a),

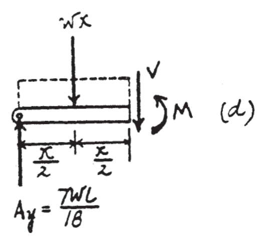


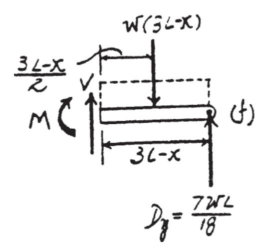
$$\zeta + \Sigma M_C = 0; \qquad D_y\,(L) + \frac{wL}{6}\left(\frac{L}{3}\right) - \frac{4wL}{3}\left(\frac{L}{3}\right) = 0 \qquad D_y = \frac{7wL}{18}$$

From FBD (c),

$$\zeta + \Sigma M_B = 0; \qquad \frac{4wL}{3}\left(\frac{L}{3}\right) - \frac{wL}{6}\left(\frac{L}{3}\right) - A_y\,(L) = 0 \qquad A_y = \frac{7wL}{18}$$

$$+\uparrow \Sigma F_y = 0; \qquad B_y + \frac{7wL}{18} - \frac{4wL}{3} - \frac{wL}{6} = 0 \qquad B_y = \frac{10wL}{9}$$

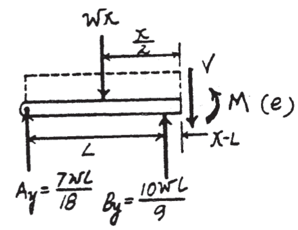




Shear and Moment Functions: For $\mathbf{0 \leq x < L}$ [FBD (d)],

$$+\uparrow \Sigma F_y = 0; \qquad \frac{7wL}{18} - wx - V = 0$$

$$V = \frac{w}{18}(7L - 18x) \qquad \textbf{Ans.}$$



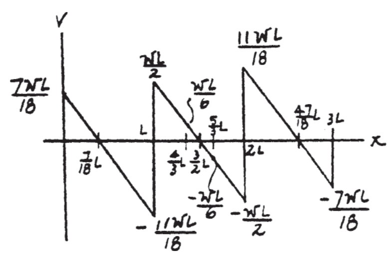
$$\zeta + \Sigma M = 0; \qquad M + wx\left(\frac{x}{2}\right) - \frac{7wL}{18}x = 0$$

$$M = \frac{w}{18}(7Lx - 9x^2) \qquad \textbf{Ans.}$$

For $\mathbf{L \leq x < 2L}$ [FBD (e)],

$$+\uparrow \Sigma F_y = 0; \qquad \frac{7wL}{18} + \frac{10wL}{9} - wx - V = 0$$

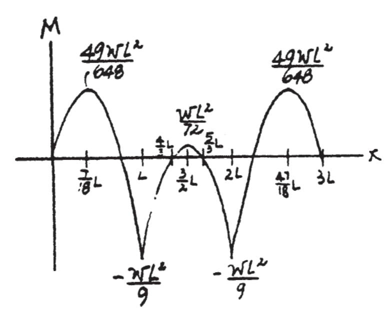
$$V = \frac{w}{2}(3L - 2x) \qquad \textbf{Ans.}$$



$$\zeta + \Sigma M = 0; \qquad M + wx\left(\frac{x}{2}\right) - \frac{7wL}{18}x - \frac{10wL}{9}(x - L) = 0$$

$$M = \frac{w}{18}(27Lx - 20L^2 - 9x^2) \qquad \textbf{Ans.}$$

For $\mathbf{2L < x \leq 3L}$ [FBD (f)],



$$+\uparrow \Sigma F_y = 0; \qquad V + \frac{7wL}{18} - w(3L - x) = 0$$

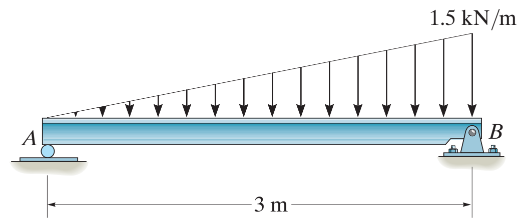
$$V = \frac{w}{18}(47L - 18x) \qquad \textbf{Ans.}$$

$$\zeta + \Sigma M = 0; \quad \frac{7wL}{18}(3L - x) - w(3L - x)\left(\frac{3L - x}{2}\right) - M = 0$$

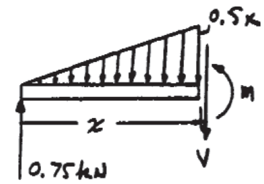
$$M = \frac{w}{18}(47Lx - 9x^2 - 60L^2) \qquad \textbf{Ans.}$$

7–59.

Draw the shear and moment diagrams for the beam.



SOLUTION



$+\uparrow \Sigma F_y = 0; \qquad 0.75 - \frac{1}{2}x\,(0.5x) - V = 0$

$V = 0.75 - 0.25x^2$

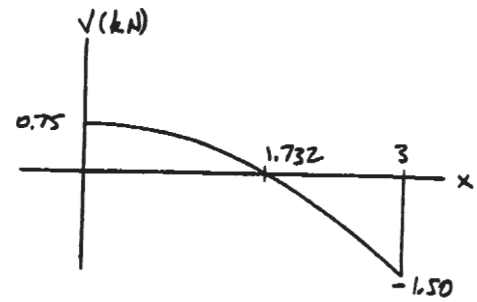
$V = 0 = 0.75 - 0.25x^2$

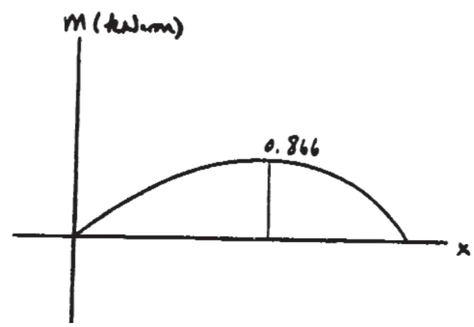
$x = 1.732 \text{ m}$

$\circlearrowleft + \Sigma M = 0; \qquad M + \left(\frac{1}{2}\right)(0.5\,x)\,(x)\left(\frac{1}{3}x\right) - 0.75\,x = 0$

$M = 0.75\,x - 0.08333\,x^3$

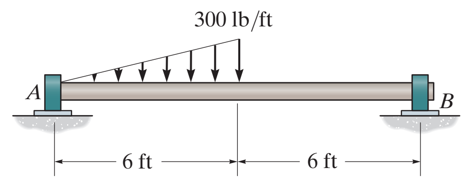
$M_{max} = 0.75(1.732) - 0.08333(1.732)^3 = 0.866$





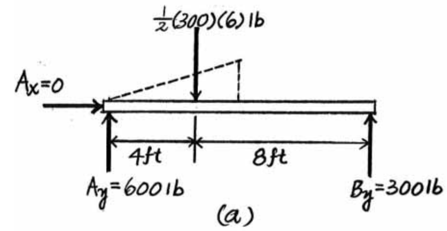
***7–60.**

The shaft is supported by a smooth thrust bearing at A and a smooth journal bearing at B. Draw the shear and moment diagrams for the shaft.



SOLUTION

Since the loading is discontinuous at the midspan, the shear and moment equations must be written for regions $0 \le x < 6$ ft and $6 \text{ ft} < x \le 12$ ft of the beam. The free-body diagram of the beam's segment sectioned through the arbitrary points within these two regions are shown in Figs. *b* and *c*.

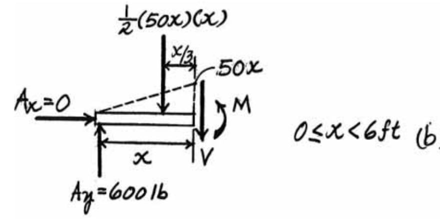


Region $0 \le x < 6$ ft, Fig. *b*

$$+\uparrow \Sigma F_y = 0; \qquad 600 - \frac{1}{2}(50x)(x) - V = 0 \qquad V = \{600 - 25x^2\}\text{ lb} \qquad \textbf{(1)}$$

$$\circlearrowleft + \Sigma M = 0; \qquad M + \frac{1}{2}(50x)(x)\left(\frac{x}{3}\right) - 600(x) = 0$$

$$M = \{600x - 8.333x^3\}\text{ lb}\cdot\text{ft} \qquad \textbf{(2)}$$



Region $6 \text{ ft} < x \le 12$ ft, Fig. *c*

$$+\uparrow \Sigma F_y = 0; \qquad V + 300 = 0 \qquad V = -300\text{ lb} \qquad \textbf{(3)}$$

$$\circlearrowleft + \Sigma M = 0; \qquad 300(12 - x) - M = 0 \qquad M = \{300(12 - x)\}\text{ lb}\cdot\text{ft} \qquad \textbf{(4)}$$

The shear diagram shown in Fig. *d* is plotted using Eqs. (1) and (3). The location at which the shear is equal to zero is obtained by setting $V = 0$ in Eq. (1).

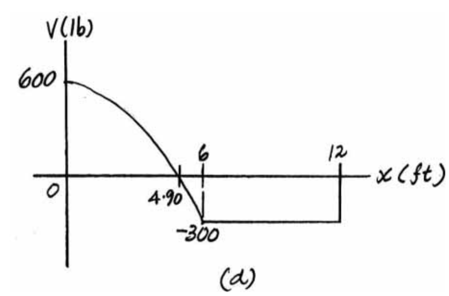
$$0 = 600 - 25x^2 \qquad x = 4.90\text{ ft}$$

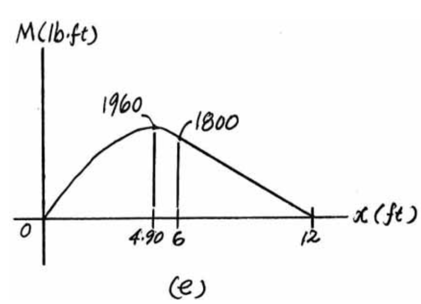
The moment diagram shown in Fig. *e* is plotted using Eqs. (2) and (4). The value of the moment at $x = 4.90$ ft ($V = 0$) is evaluated using Eq. (2).

$$M|_{x=4.90\text{ ft}} = 600(4.90) - 8.333(4.90^3) = 1960\text{ lb}\cdot\text{ft}$$

The value of the moment at $x = 6$ ft is evaluated using either Eq. (2) or Eq. (4).

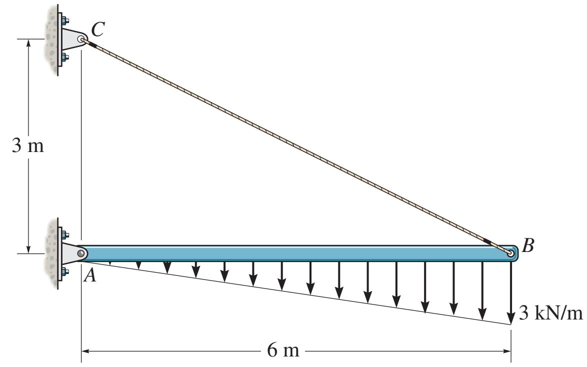
$$M|_{x=6\text{ ft}} = 300(12 - 6) = 1800\text{ lb}\cdot\text{ft}$$





7–61.

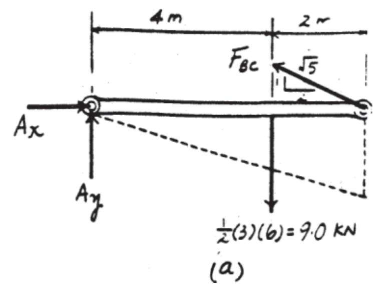
Draw the shear and moment diagrams for the beam.



SOLUTION

Support Reactions: From FBD (a),

$\circlearrowleft + \Sigma M_B = 0; \quad 9.00(2) - A_y(6) = 0 \qquad A_y = 3.00\text{ kN}$



Shear and Moment Functions: For $0 \le x \le 6$ m [FBD (b)],

$$+\uparrow \Sigma F_y = 0; \qquad 3.00 - \frac{x^2}{4} - V = 0$$

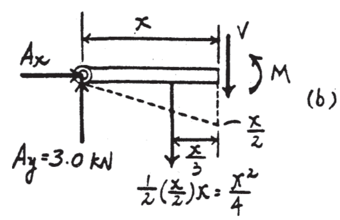
$$V = \left\{3.00 - \frac{x^2}{4}\right\}\text{ kN} \qquad \textbf{Ans.}$$

The maximum moment occurs when $V = 0$, then

$$0 = 3.00 - \frac{x^2}{4} \qquad x = 3.464\text{ m}$$

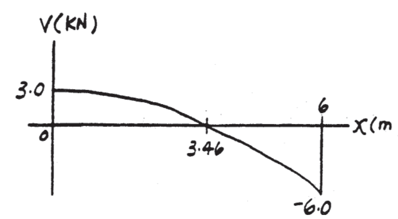
$$\circlearrowleft + \Sigma M = 0; \qquad M + \left(\frac{x^2}{4}\right)\left(\frac{x}{3}\right) - 3.00x = 0$$

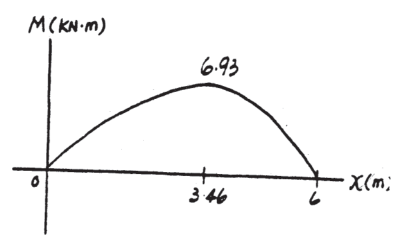
$$M = \left\{3.00x - \frac{x^3}{12}\right\}\text{ kN}\cdot\text{m} \qquad \textbf{Ans.}$$



Thus,

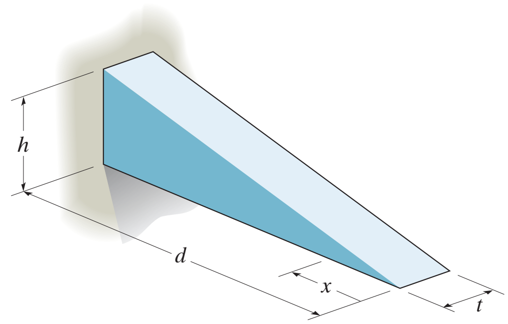
$$M_{\max} = 3.00(3.464) - \frac{3.464^3}{12} = 6.93\text{ kN}\cdot\text{m}$$





7–62.

The cantilevered beam is made of material having a specific weight γ. Determine the shear and moment in the beam as a function of x.

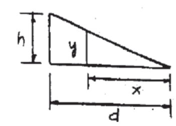


SOLUTION

By similar triangles

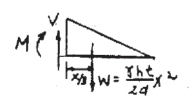
$$\frac{y}{x} = \frac{h}{d} \qquad y = \frac{h}{d}x$$

$$W = \gamma V = \gamma\left(\frac{1}{2}yxt\right) = \gamma\left[\frac{1}{2}\left(\frac{h}{d}x\right)xt\right] = \frac{\gamma ht}{2d}x^2$$



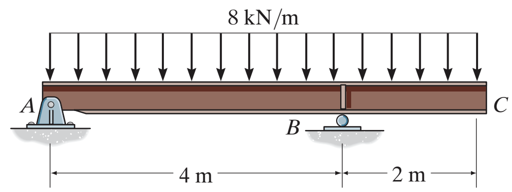
$$+\uparrow \Sigma F_y = 0; \qquad V - \frac{\gamma ht}{2d}x^2 = 0 \qquad V = \frac{\gamma ht}{2d}x^2$$ **Ans.**

$$\circlearrowleft + \Sigma M = 0; \qquad -M - \frac{\gamma ht}{2d}x^2\left(\frac{x}{3}\right) = 0 \qquad M = -\frac{\gamma ht}{6d}x^3$$ **Ans.**



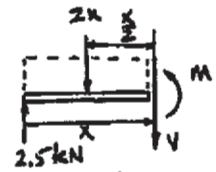
7–63.

Draw the shear and moment diagrams for the overhang beam.



SOLUTION

$0 \leq x < 5$ m:



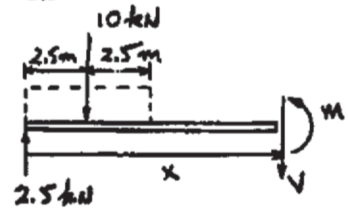
$+\uparrow \Sigma F_y = 0; \qquad 2.5 - 2x - V = 0$

$V = 2.5 - 2x$

$\circlearrowleft + \Sigma M = 0; \qquad M + 2x\left(\frac{1}{2}x\right) - 2.5x = 0$

$M = 2.5x - x^2$

$5 \leq x < 10$ m:

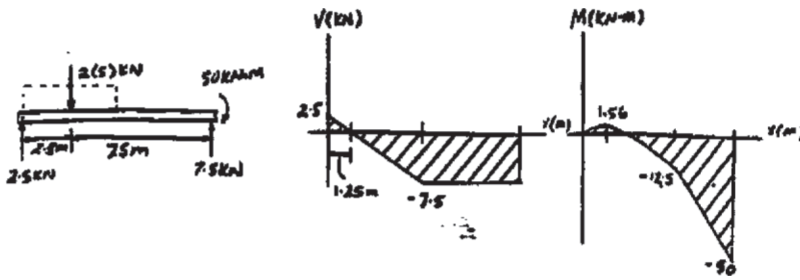


$+\uparrow \Sigma F_y = 0; \qquad 2.5 - 10 - V = 0$

$V = -7.5$

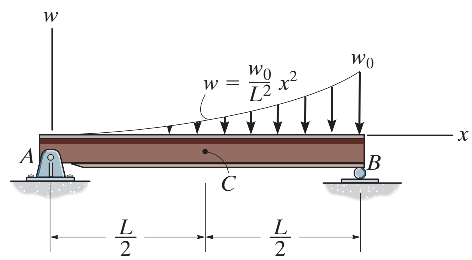
$\circlearrowleft + \Sigma M = 0; \qquad M + 10(x - 2.5) - 2.5x = 0$

$M = -7.5x - 25$



***7–64.**

Draw the shear and moment diagrams for the beam.



SOLUTION

The free-body diagram of the beam's segment sectioned through an arbitrary point shown in Fig. b will be used to write the shear and moment equations. The magnitude of the resultant force of the parabolic distributed loading and the location of its point of application are given in the inside back cover of the book.

Referring to Fig. b, we have

$$+\uparrow \Sigma F_y = 0; \quad \frac{w_0 L}{12} - \frac{1}{3}\left(\frac{w_0}{L^2}x^2\right)x - V = 0 \qquad V = \frac{w_0}{12L^2}\left(L^3 - 4x^3\right) \quad \textbf{(1)}$$

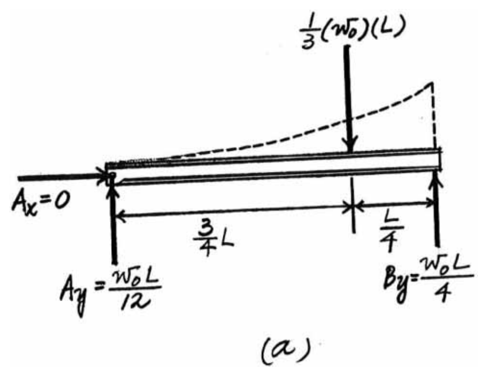
$$\circlearrowleft + \Sigma M = 0; \quad M + \frac{1}{3}\left(\frac{w_0}{L^2}x^2\right)(x)\left(\frac{x}{4}\right) - \frac{w_0 L}{12}x = 0 \quad M = \frac{w_0}{12L^2}\left(L^3 x - x^4\right) \quad \textbf{(2)}$$

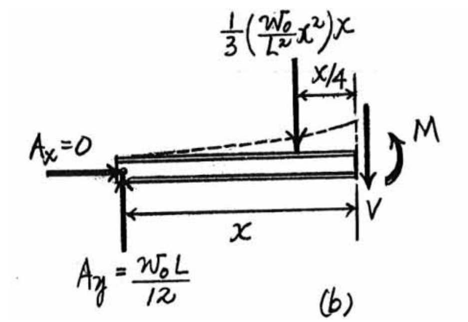
The shear and moment diagrams shown in Figs. c and d are plotted using Eqs. (1) and (2), respectively. The location at which the shear is equal to zero can be obtained by setting $V = 0$ in Eq. (1).

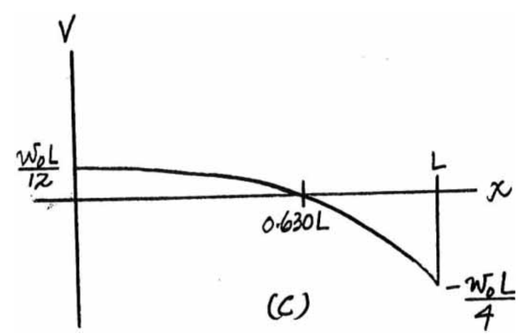
$$0 = \frac{w_0}{12L^2}\left(L^3 - 4x^3\right) \qquad x = 0.630L$$

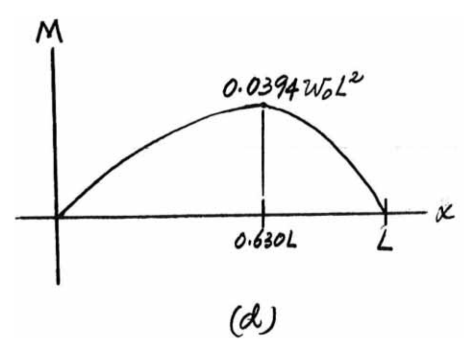
The value of the moment at $x = 0.630L$ is evaluated using Eq. (2).

$$M\big|_{x=0.630L} = \frac{w_0}{12L^2}\left[L^3(0.630L) - (0.630L)^4\right] = 0.0394 w_0 L^2$$



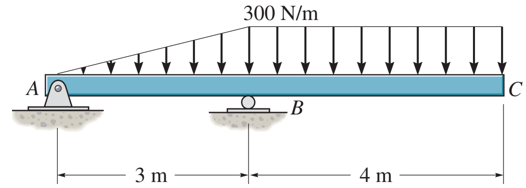






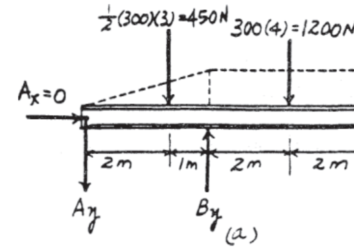
7–65.

Draw the shear and bending-moment diagrams for the beam.



SOLUTION

Support Reactions: From FBD (a),

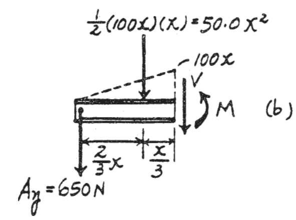


$\circlearrowleft + \Sigma M_B = 0; \qquad A_y(3) + 450(1) - 1200(2) = 0 \qquad A_y = 650\text{ N}$

Shear and Moment Functions: For $0 \leq x < 3$ m [FBD (b)],

$+\uparrow \Sigma F_y = 0; \qquad -650 - 50.0x^2 - V = 0$

$$V = \{-650 - 50.0x^2\}\text{ N} \qquad \textbf{Ans.}$$



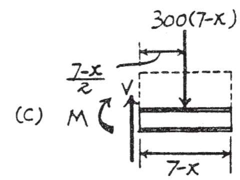
$\circlearrowleft + \Sigma M = 0; \qquad M + (50.0x^2)\left(\frac{x}{3}\right) + 650x = 0$

$$M = \{-650x - 16.7x^3\}\text{ N}\cdot\text{m} \qquad \textbf{Ans.}$$

For $3\text{ m} < x \leq 7$ m [FBD (c)],

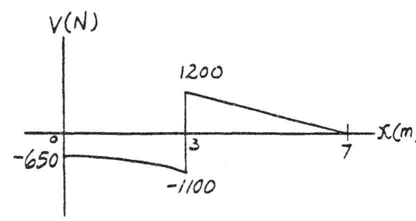
$+\uparrow \Sigma F_y = 0; \qquad V - 300(7 - x) = 0$

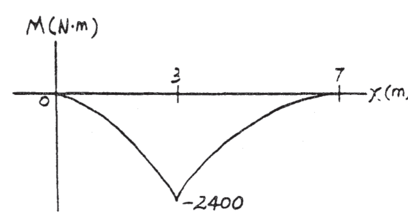
$$V = \{2100 - 300x\}\text{ N} \qquad \textbf{Ans.}$$



$\circlearrowleft + \Sigma M = 0; \qquad -300(7 - x)\left(\frac{7 - x}{2}\right) - M = 0$

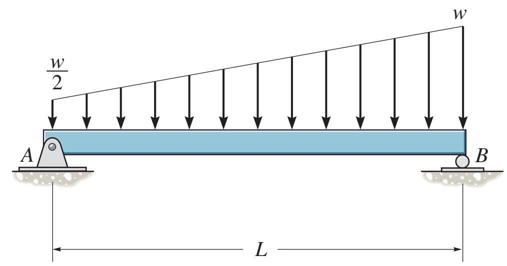
$$M = \{-150(7 - x)^2\}\text{ N}\cdot\text{m} \qquad \textbf{Ans.}$$





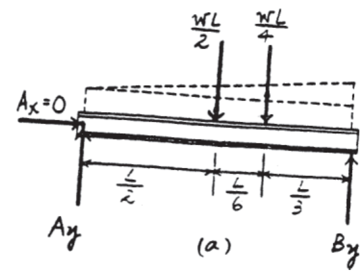
7–66.

Draw the shear and moment diagrams for the beam.



SOLUTION

Support Reactions: From FBD (a),



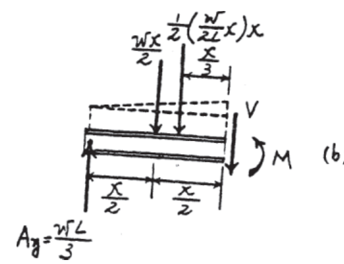
$$\zeta + \Sigma M_B = 0; \qquad \frac{wL}{4}\left(\frac{L}{3}\right) + \frac{wL}{2}\left(\frac{L}{2}\right) - A_y\,(L) = 0 \qquad A_y = \frac{wL}{3}$$

Shear and Moment Functions: For $0 \leq x \leq L$ [FBD (b)],

$$+\uparrow \Sigma F_y = 0; \qquad \frac{wL}{3} - \frac{w}{2}x - \frac{1}{2}\left(\frac{w}{2L}x\right)x - V = 0$$

$$V = \frac{w}{12L}(4L^2 - 6Lx - 3x^2) \qquad \textbf{Ans.}$$

The maximum moment occurs when $V = 0$, then

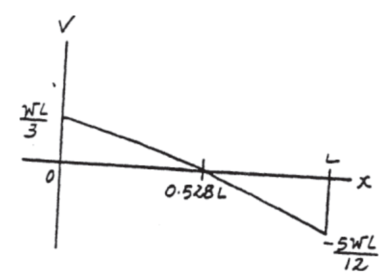


$$0 = 4L^2 - 6Lx - 3x^2 \qquad x = 0.5275L$$

$$\zeta + \Sigma M = 0; \qquad M + \frac{1}{2}\left(\frac{w}{2L}x\right)x\left(\frac{x}{3}\right) + \frac{wx}{2}\left(\frac{x}{2}\right) - \frac{wL}{3}(x) = 0$$

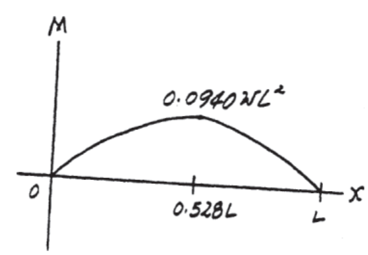
$$M = \frac{w}{12L}(4L^2x - 3Lx^2 - x^3) \qquad \textbf{Ans.}$$

Thus,



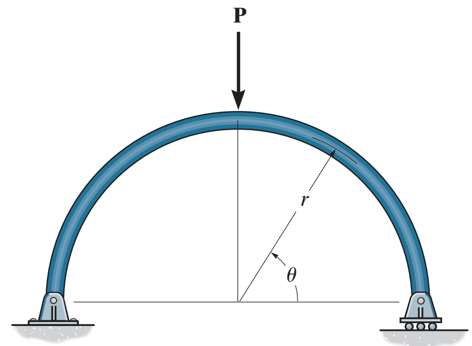
$$M_{\max} = \frac{w}{12L}[4L^2(0.5275L) - 3L(0.5275L)^2 - (0.5275L)^3]$$

$$= 0.0940wL^2 \qquad \textbf{Ans.}$$



7–67.

Determine the internal normal force, shear force, and moment in the curved rod as a function of θ, where $0^\circ \leq \theta \leq 90^\circ$.



SOLUTION

With reference to Fig. *a*,

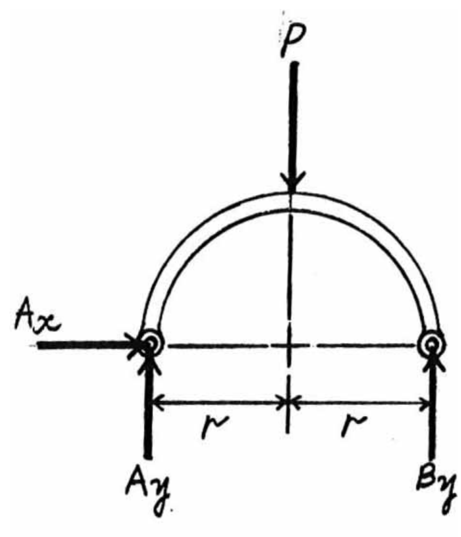
$\zeta + \Sigma M_A = 0; \qquad B_y(2r) - p(r) = 0 \qquad B_y = p/2$

Using this result and referring to Fig. *b*, we have

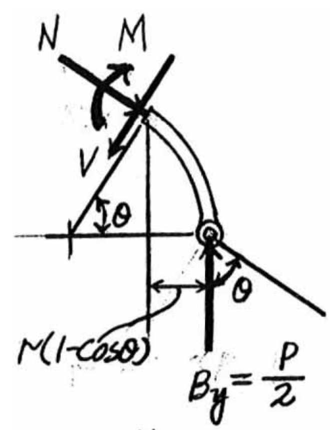
$\Sigma F_{x'} = 0; \qquad \dfrac{p}{2}\sin\theta - V = 0 \qquad V = \dfrac{p}{2}\sin\theta$ **Ans.**

$\Sigma F_{y'} = 0; \qquad \dfrac{p}{2}\cos\theta - N = 0 \qquad N = \dfrac{p}{2}\cos\theta$ **Ans.**

$\zeta + \Sigma M = 0; \qquad \dfrac{p}{2}[r(1 - \cos\theta)] - M = 0 \qquad M = \dfrac{pr}{2}(1 - \cos\theta)$ **Ans.**



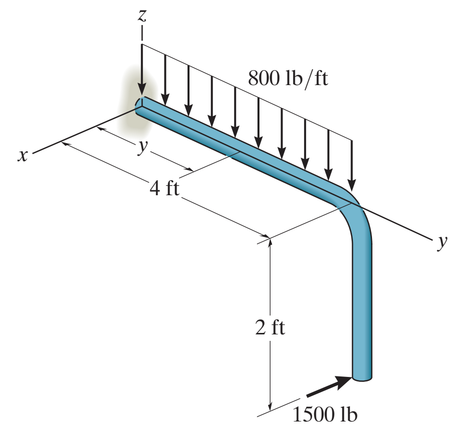
(a)



(b)

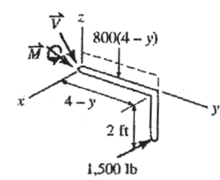
***7–68.**

Express the x, y, z components of internal loading in the rod as a function of y, where $0 \le y \le 4$ ft.



SOLUTION

For $0 \le y \le 4$ ft



$\Sigma F_x = 0;$ $\quad V_x = 1500 \text{ lb} = 1.5 \text{ kip}$ **Ans.**

$\Sigma F_y = 0;$ $\quad N_y = 0$ **Ans.**

$\Sigma F_z = 0;$ $\quad V_z = 800(4 - y) \text{ lb}$ **Ans.**

$\Sigma M_x = 0;$ $\quad M_x - 800(4 - y)\left(\dfrac{4 - y}{2}\right) = 0$

$M_x = 400(4 - y)^2 \text{ lb} \cdot \text{ft}$ **Ans.**

$\Sigma M_y = 0;$ $\quad M_y + 1500(2) = 0$

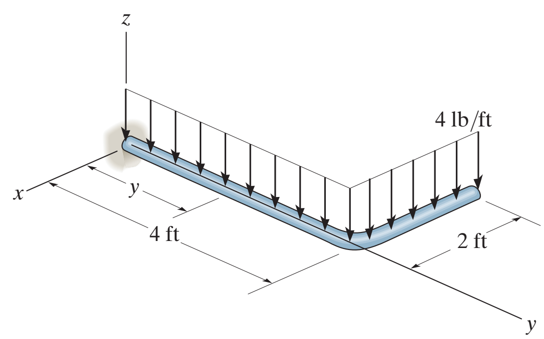
$M_y = -3000 \text{ lb} \cdot \text{ft} = -3 \text{ kip} \cdot \text{ft}$ **Ans.**

$\Sigma M_z = 0;$ $\quad M_z + 1500(4 - y) = 0$

$M_z = -1500(4 - y) \text{ lb} \cdot \text{ft}$ **Ans.**

7–69.

Express the internal shear and moment components acting in the rod as a function of *y*, where $0 \leq y \leq 4$ ft.



SOLUTION

Shear and Moment Functions:

$\Sigma F_x = 0;\qquad V_x = 0$ **Ans.**

$\Sigma F_z = 0;\qquad V_z - 4(4 - y) - 8.00 = 0$

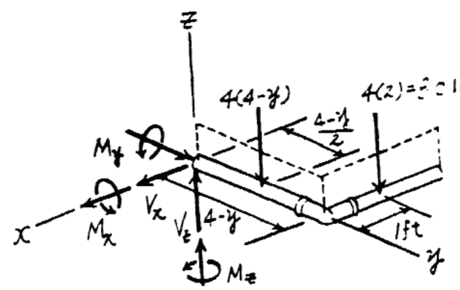
$$V_z = \{24.0 - 4y\}\text{ lb}$$ **Ans.**

$\Sigma M_x = 0;\qquad M_x - 4(4 - y)\left(\frac{4 - y}{2}\right) - 8.00(4 - y) = 0$

$$M_x = \{2y^2 - 24y + 64.0\}\text{ lb}\cdot\text{ft}$$ **Ans.**

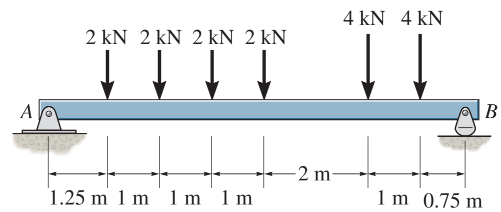
$\Sigma M_y = 0;\qquad M_y - 8.00(1) = 0\qquad M_y = 8.00\text{ lb}\cdot\text{ft}$ **Ans.**

$\Sigma M_z = 0;\qquad M_z = 0$ **Ans.**



7–70.

Draw the shear and moment diagrams for the beam.



SOLUTION

Support Reactions:

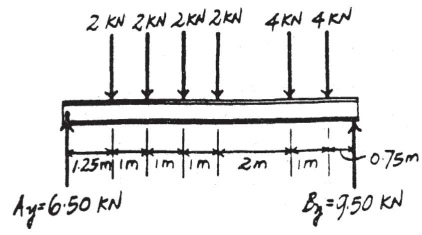
$$\zeta + \Sigma M_A = 0; \quad B_y(8) - 4(7.25) - 4(6.25) - 2(4.25)$$

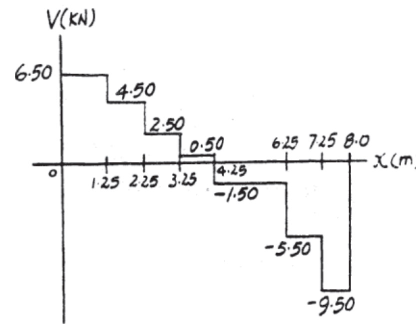
$$-2(3.25) - 2(2.25) - 2(1.25) = 0$$

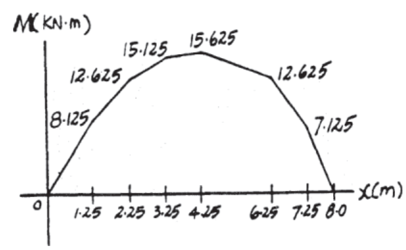
$$B_y = 9.50 \text{ kN}$$

$$+\uparrow \Sigma F_y = 0; \quad A_y + 9.50 - 2 - 2 - 2 - 2 - 4 - 4 = 0$$

$$A_y = 6.50 \text{ kN}$$

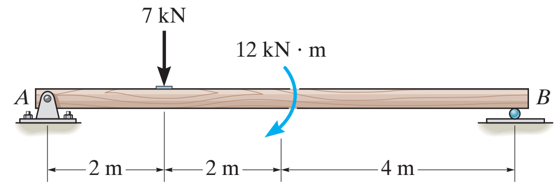




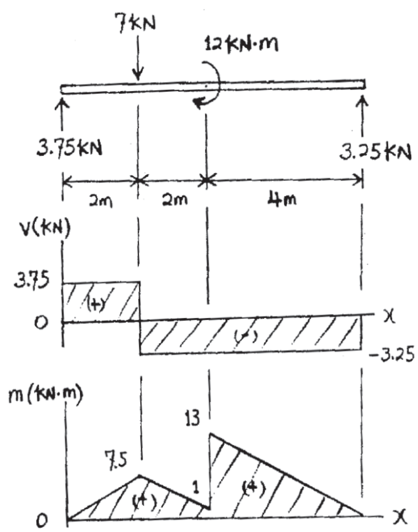


7–71.

Draw the shear and moment diagrams for the beam.

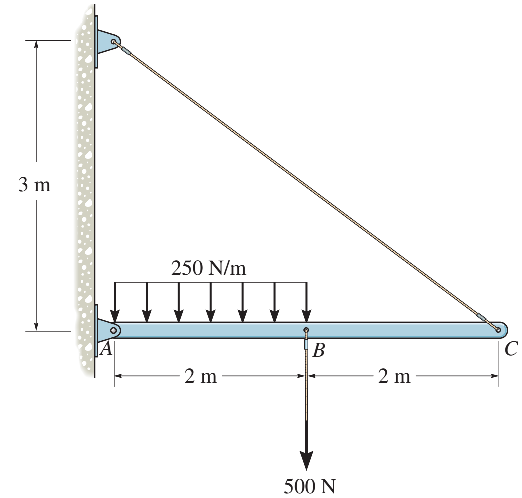


SOLUTION



***7–72.**

Draw the shear and moment diagrams for the beam.

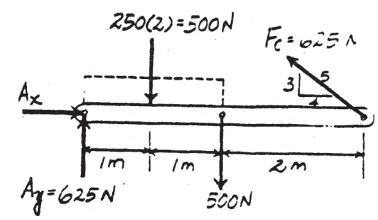


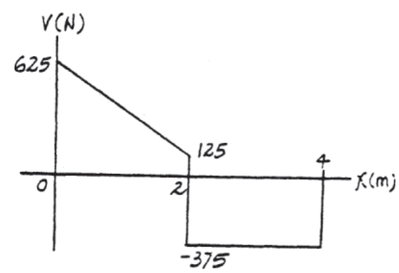
SOLUTION

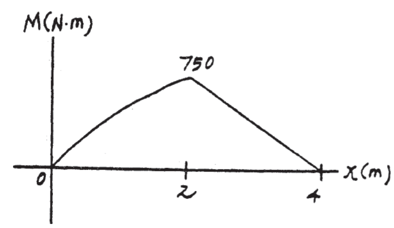
Support Reactions:

$$\zeta + \Sigma M_A = 0; \qquad F_C\left(\frac{3}{5}\right)(4) - 500(2) - 500(1) = 0 \qquad F_C = 625 \text{ N}$$

$$+\uparrow \Sigma F_y = 0; \qquad A_y + 625\left(\frac{3}{5}\right) - 500 - 500 = 0 \qquad A_y = 625 \text{ N}$$

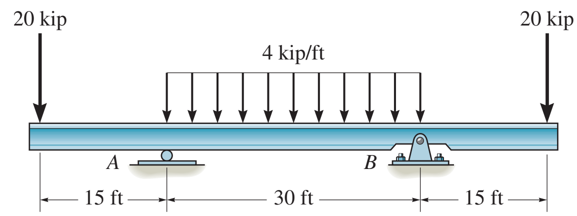




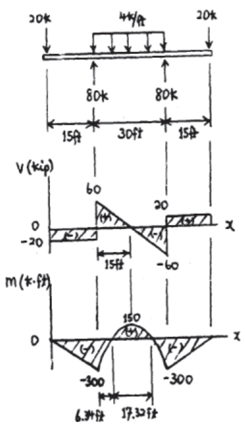


7–73.

Draw the shear and moment diagrams for the beam.

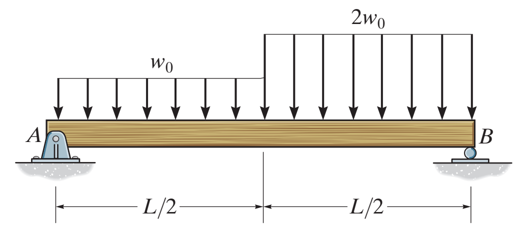


SOLUTION

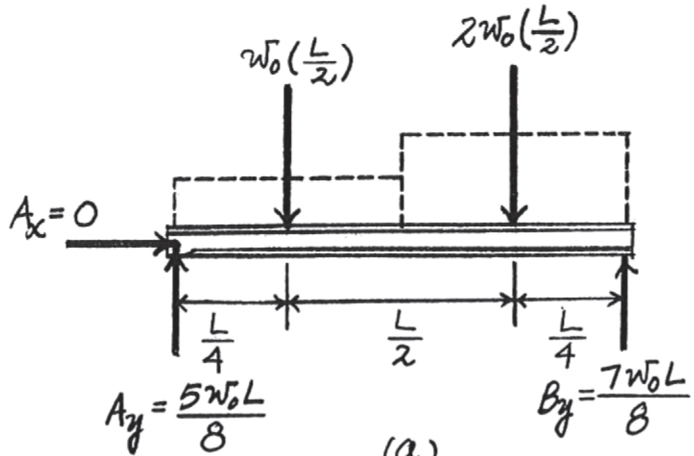


7–74.

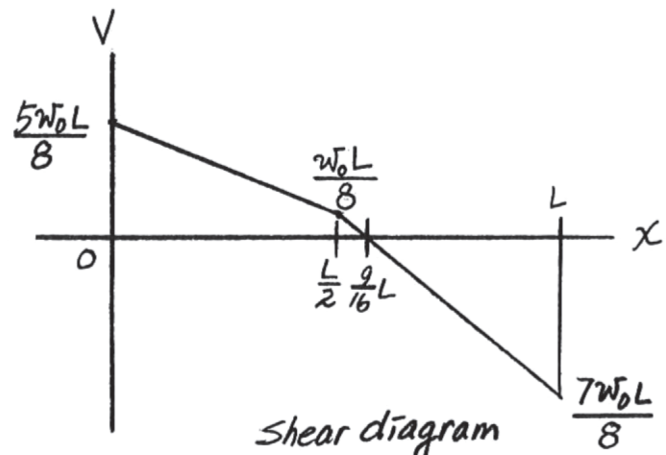
Draw the shear and moment diagrams for the simply-supported beam.



SOLUTION

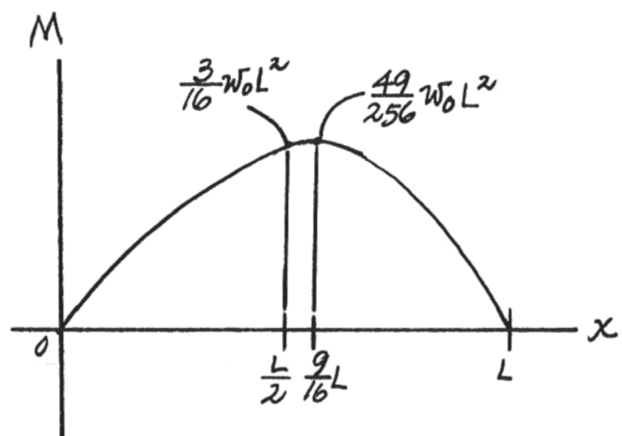


(a)



Shear diagram

(b)

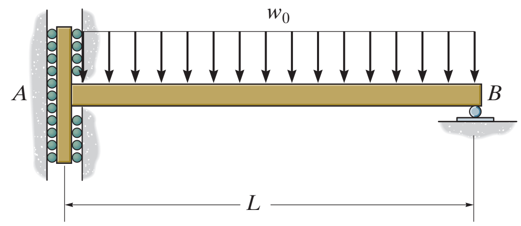


Moment diagram

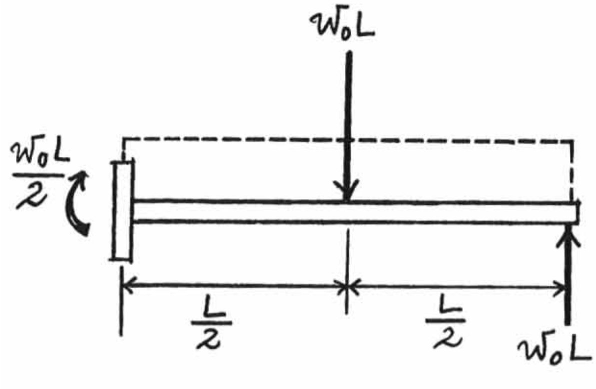
(c)

7–75.

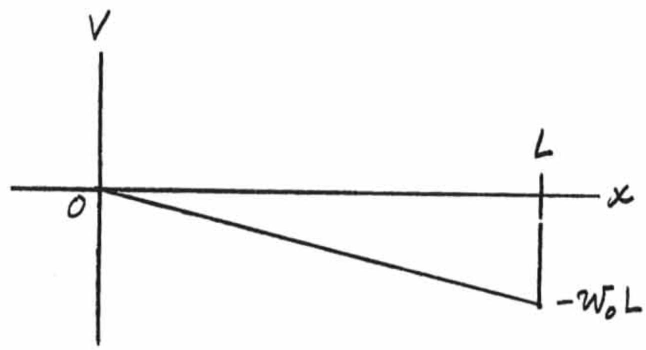
Draw the shear and moment diagrams for the beam. The support at A offers no resistance to vertical load.



SOLUTION

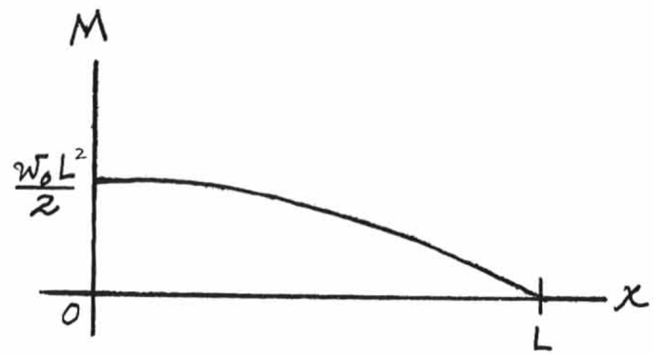


(a)



Shear diagram

(b)

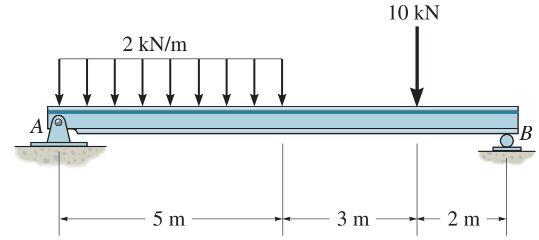


Moment diagram

(c)

***7–76.**

Draw the shear and moment diagrams for the beam.

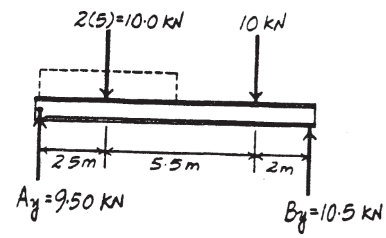


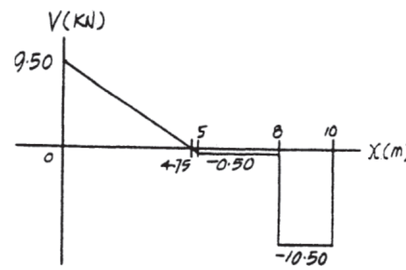
SOLUTION

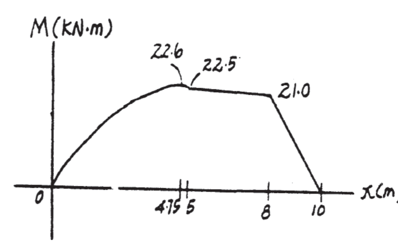
Support Reactions:

$\circlearrowleft + \Sigma M_A = 0; \quad B_y(10) - 10.0(2.5) - 10(8) = 0 \qquad B_y = 10.5 \text{ kN}$

$+\uparrow \Sigma F_y = 0; \quad A_y + 10.5 - 10.0 - 10 = 0 \qquad A_y = 9.50 \text{ kN}$

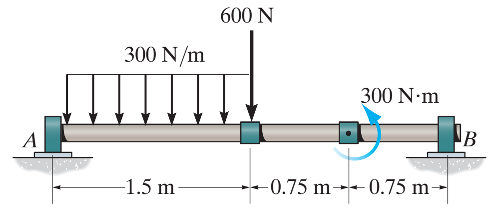




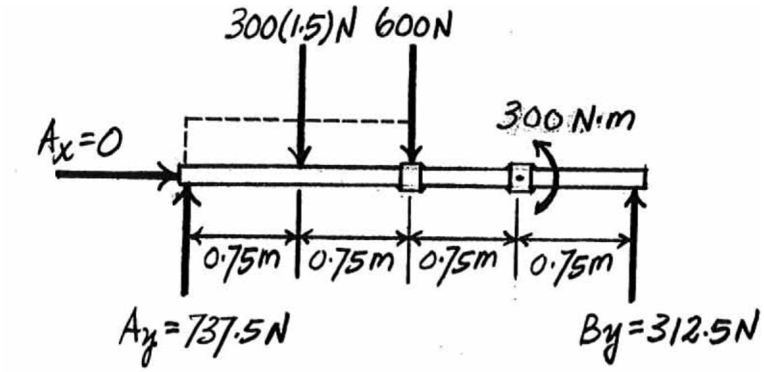


7–77.

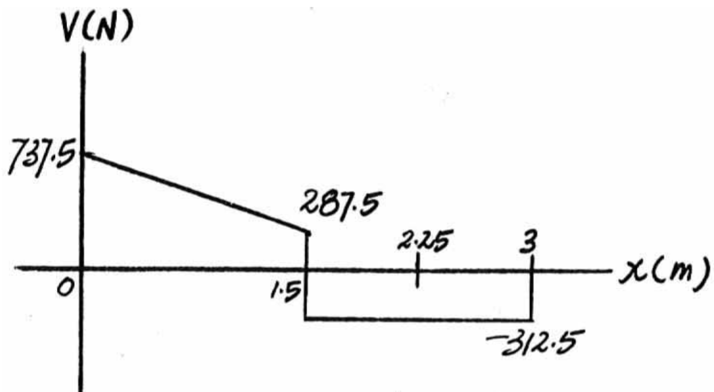
The shaft is supported by a thrust bearing at A and a journal bearing at B. Draw the shear and moment diagrams for the shaft.



SOLUTION

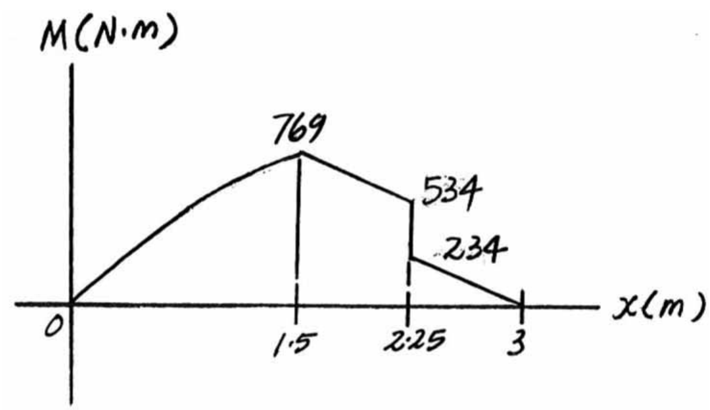


(a)



shear diagram

(b)

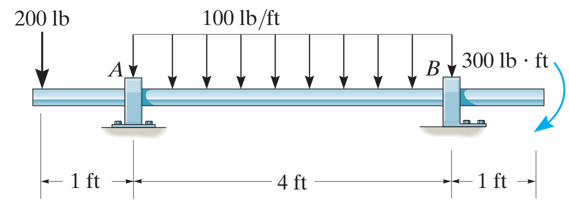


Moment diagram

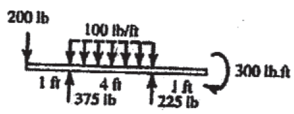
(c)

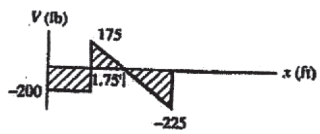
7–78.

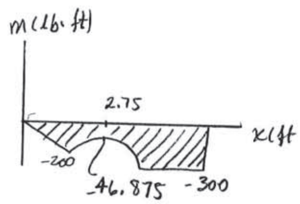
Draw the shear and moment diagrams for the shaft. The support at A is a journal bearing and at B it is a thrust bearing.



SOLUTION

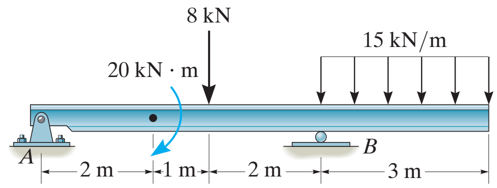




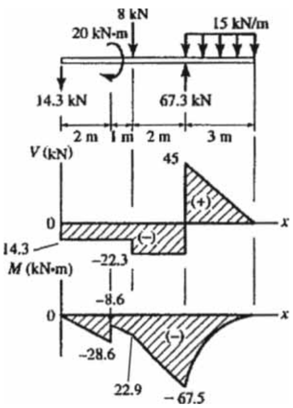


7–79.

Draw the shear and moment diagrams for the beam.

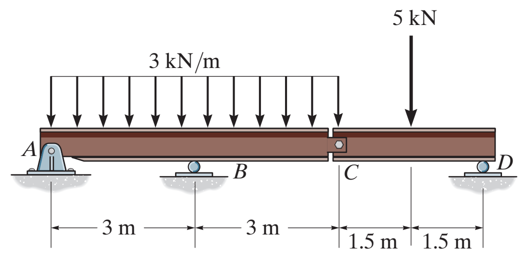


SOLUTION

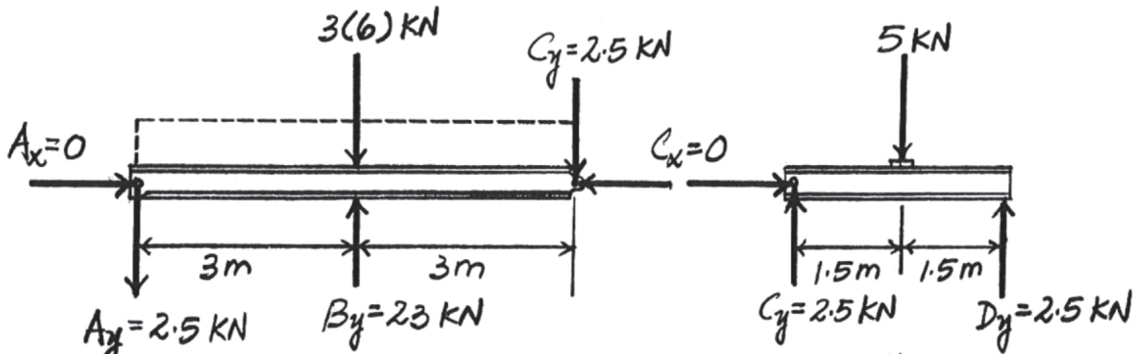


***7–80.**

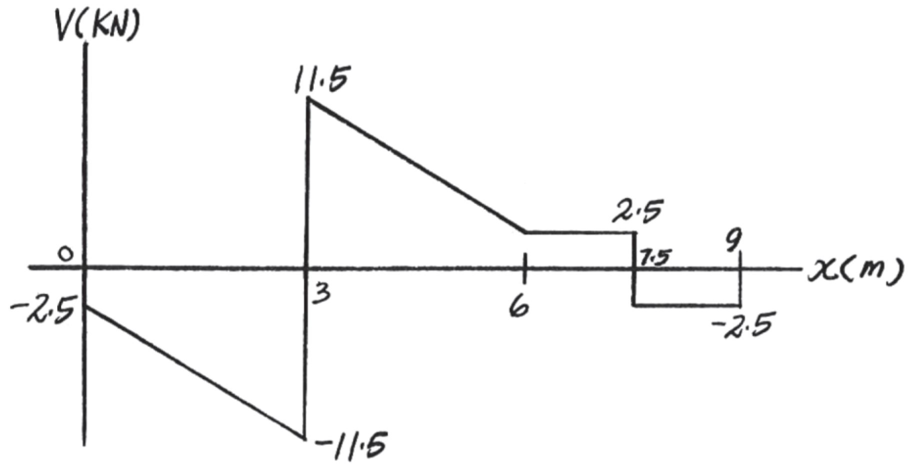
Draw the shear and moment diagrams for the compound supported beam.



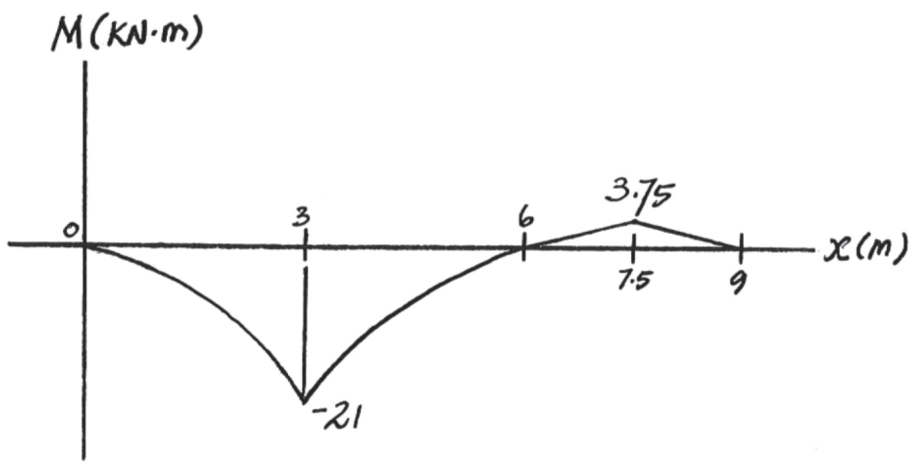
SOLUTION



(a) (b)



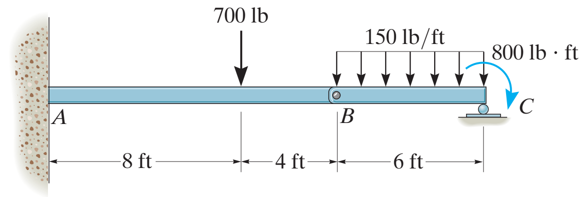
Shear diagram
(c)



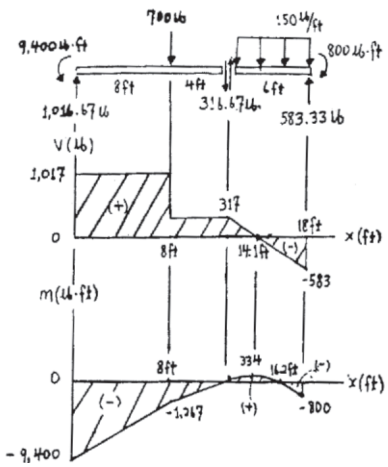
Moment diagram
(d)

7–81.

The beam consists of two segments pin connected at B. Draw the shear and moment diagrams for the beam.

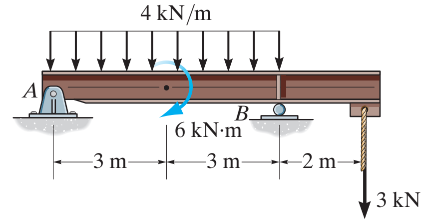


SOLUTION

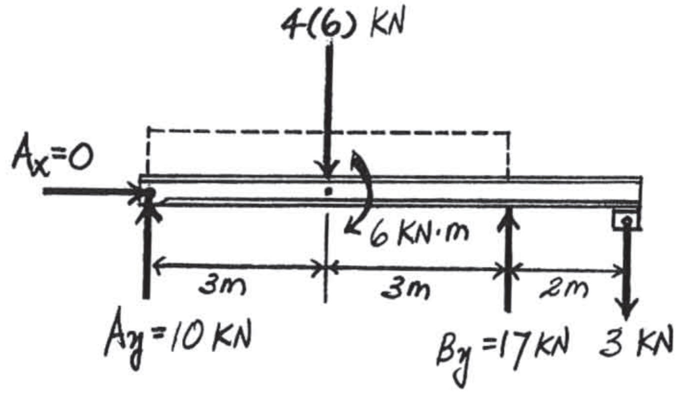


7–82.

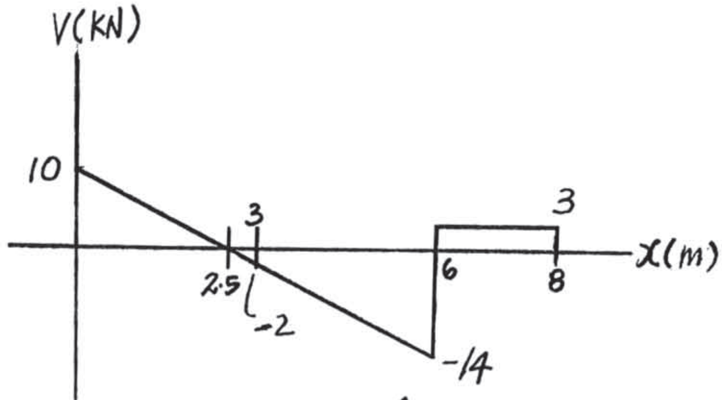
Draw the shear and moment diagrams for the overhang beam.



SOLUTION

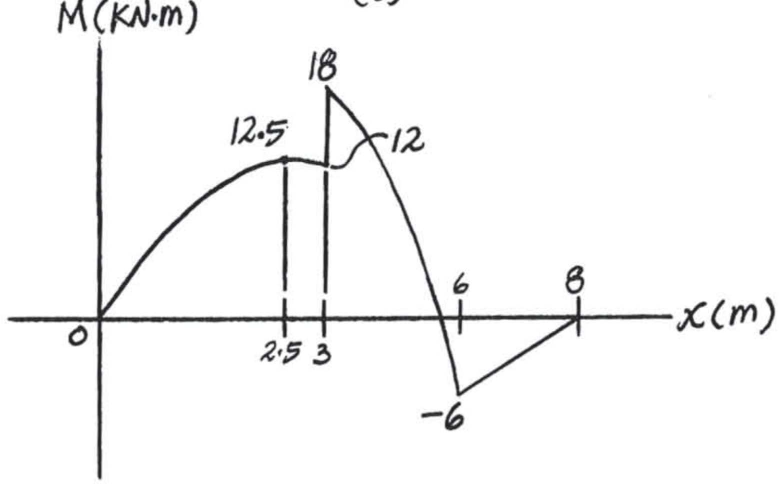


(a)



Shear diagram

(b)

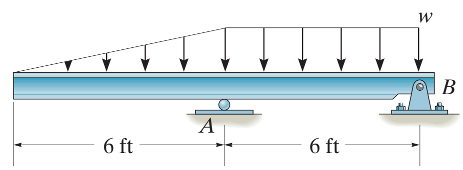


Moment diagram

(c)

7–83.

The beam will fail when the maximum moment is $M_{max} = 30$ kip · ft or the maximum shear is $V_{max} = 8$ kip. Determine the largest distributed load w the beam will support.



SOLUTION

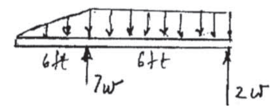
$V_{max} = 4w; \qquad 8 = 4w$

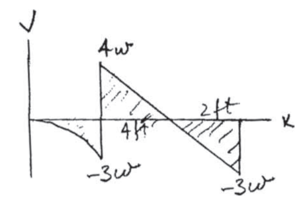
$w = 2$ kip/ft

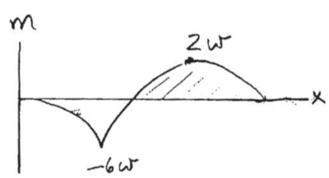
$M_{max} = -6w; \qquad -30 = -6w$

$w = 5$ kip/ft

Thus, $w = 2$ kip/ft **Ans.**

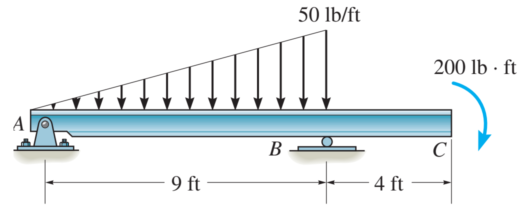




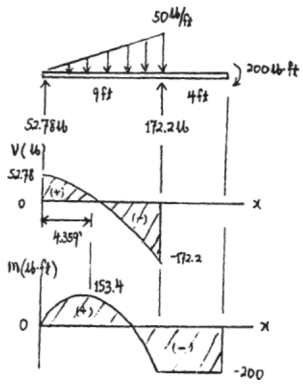


***7–84.**

Draw the shear and moment diagrams for the beam.

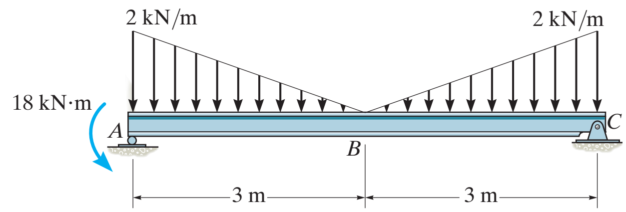


SOLUTION

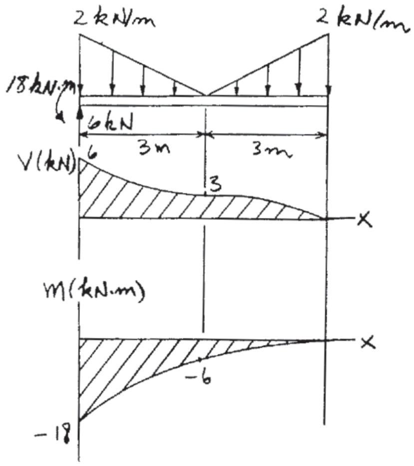


7–85.

Draw the shear and moment diagrams for the beam.

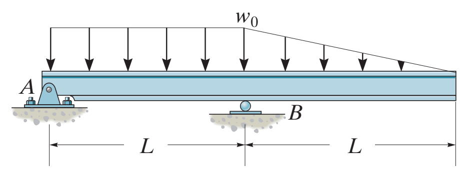


SOLUTION



7–86.

Draw the shear and moment diagrams for the beam.



SOLUTION

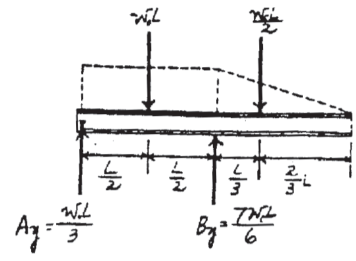
Support Reactions:

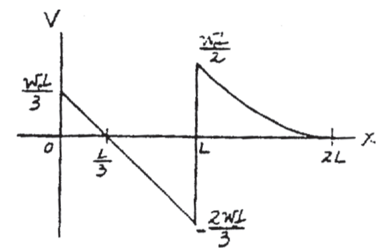
$$\circlearrowleft + \Sigma M_A = 0; \qquad B_y(L) - w_0 L\left(\frac{L}{2}\right) - \frac{w_0 L}{2}\left(\frac{4L}{3}\right) = 0$$

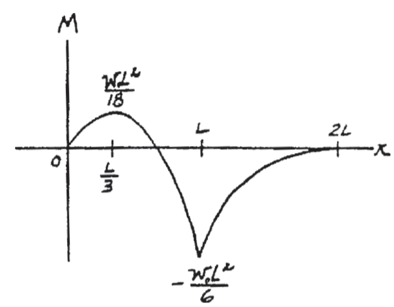
$$B_y = \frac{7w_0 L}{6}$$

$$+\uparrow \Sigma F_y = 0; \qquad A_y + \frac{7w_0 L}{6} - w_0 L - \frac{w_0 L}{2} = 0$$

$$A_y = \frac{w_0 L}{3}$$

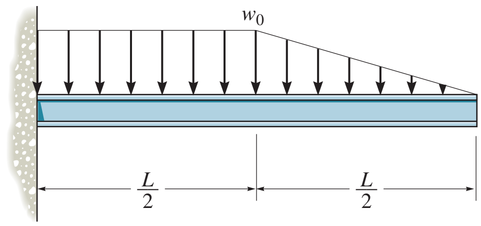






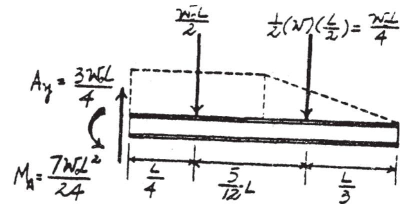
7–87.

Draw the shear and moment diagrams for the beam.



SOLUTION

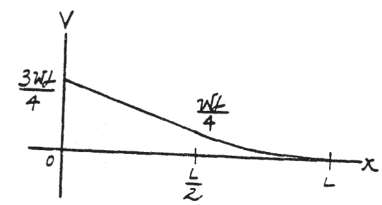
Support Reactions:

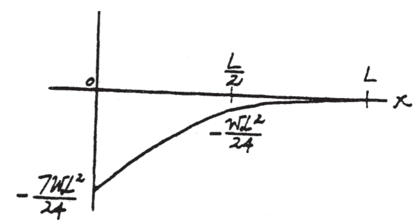


$$\zeta + \Sigma M_A = 0; \qquad M_A - \frac{w_0 L}{2}\left(\frac{L}{4}\right) - \frac{w_0 L}{4}\left(\frac{2L}{3}\right) = 0$$

$$M_A = \frac{7w_0 L^2}{24}$$

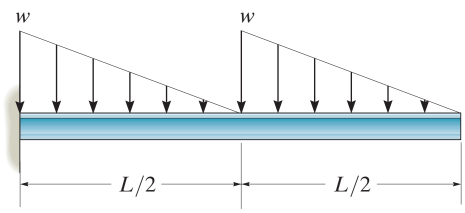
$$+\uparrow \Sigma F_y = 0; \qquad A_y - \frac{w_0 L}{2} - \frac{w_0 L}{4} = 0 \qquad A_y = \frac{3w_0 L}{4}$$



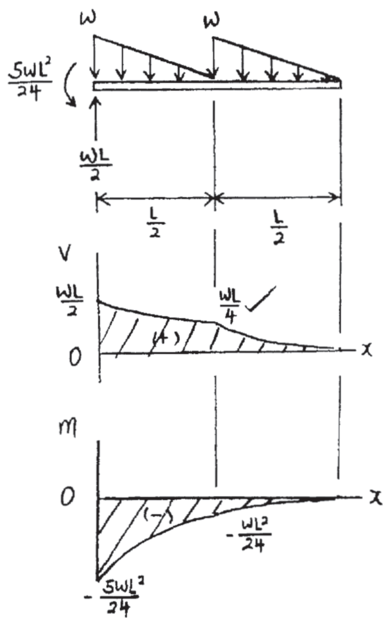


***7–88.**

Draw the shear and moment diagrams for the beam.

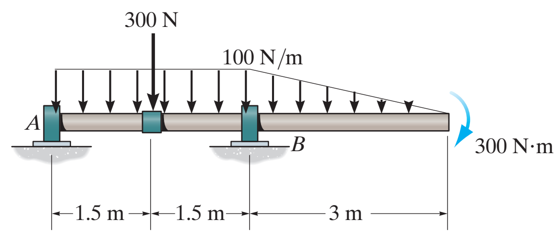


SOLUTION

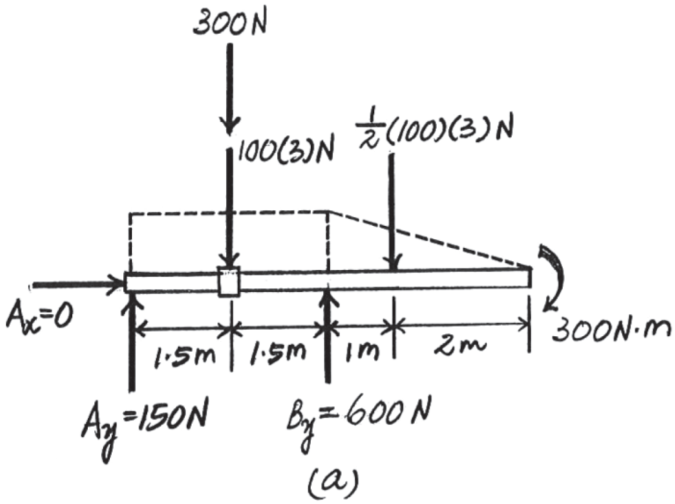


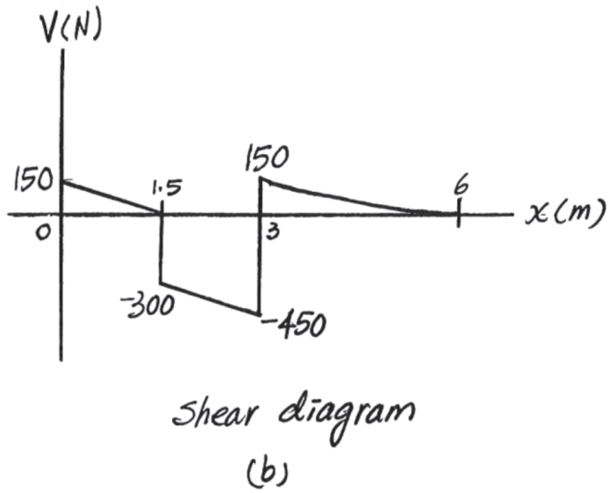
7–89.

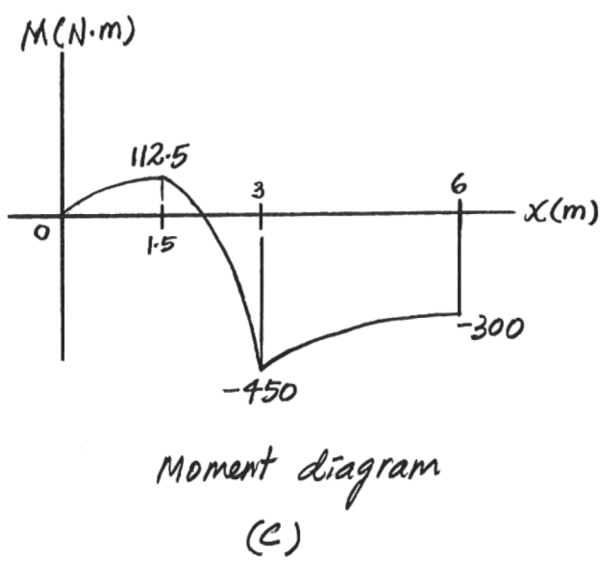
The shaft is supported by a smooth thrust bearing at A and a smooth journal bearing at B. Draw the shear and moment diagrams for the shaft.



SOLUTION

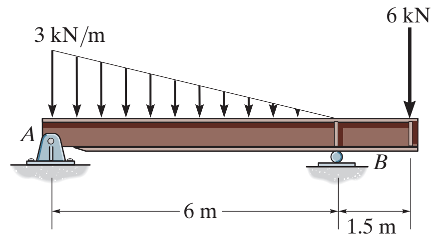






7–90.

Draw the shear and moment diagrams for the overhang beam.



SOLUTION

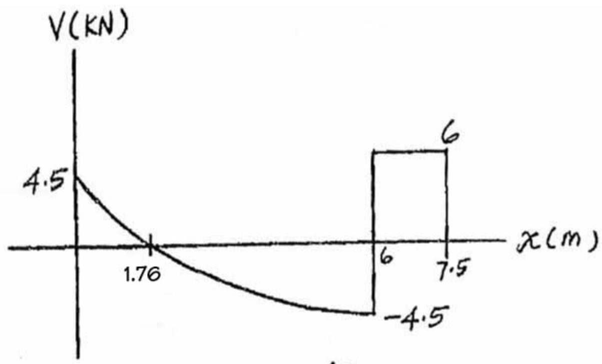
The maximum span moment occurs at the position where shear is equal to zero within the region $0 \le x < 6\,\text{m}$ of the beam. This location can be obtained using the method of sections. By setting $V = 0$, Fig. b, we have

$$+\uparrow \Sigma F_y = 0; \qquad 4.5 - \frac{1}{2}\left(\frac{1}{2}x\right)x - \frac{1}{2}(6 - x)(x) = 0 \qquad x = 1.76\text{ m} \qquad \textbf{Ans.}$$

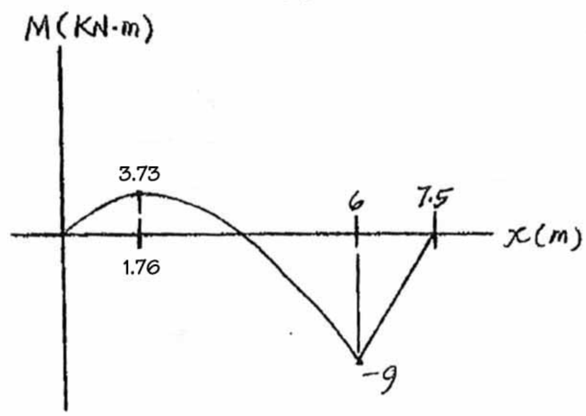
Using this result,

$$+\Sigma M = 0; \qquad M|_{x=1.76\text{m}} + \frac{1}{2}(6 - 1.76)(1.76)\left(\frac{1.76}{2}\right) + \frac{1}{4}(1.76)(1.76)\left[\frac{2}{3}(1.76)\right] - 4.5(1.76) = 0$$

$$M|_{x=1.76\text{m}} = 3.73\text{ kN}\cdot\text{m}$$



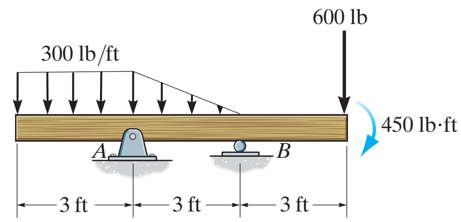
Shear diagram
(c)



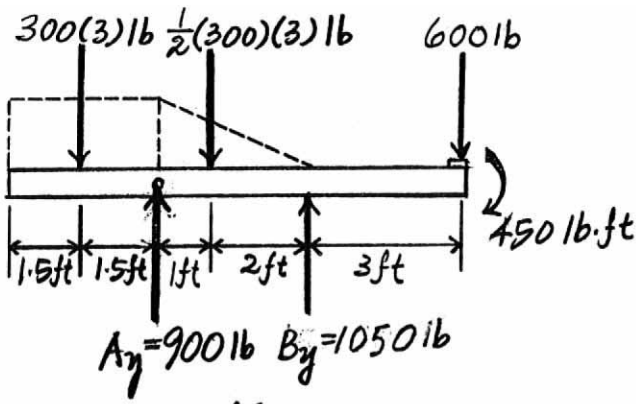
Moment diagram
(d)

7–91.

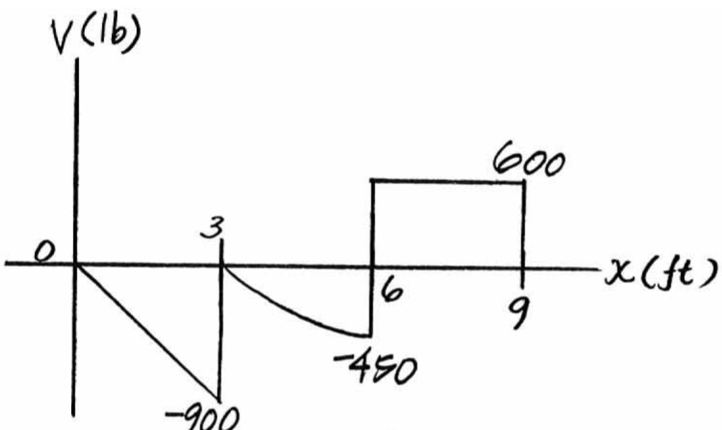
Draw the shear and moment diagrams for the overhang beam.



SOLUTION

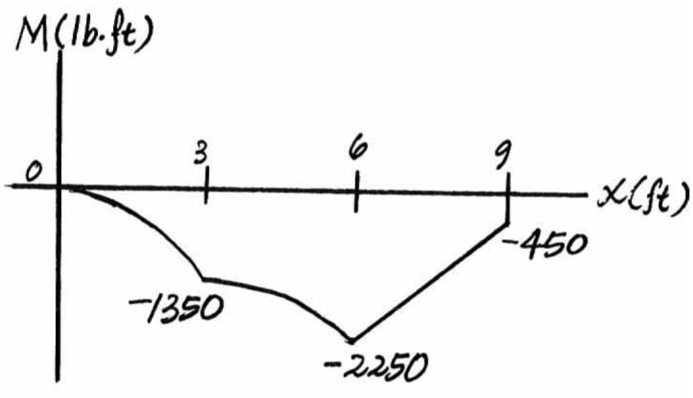


(a)



Shear diagram

(b)

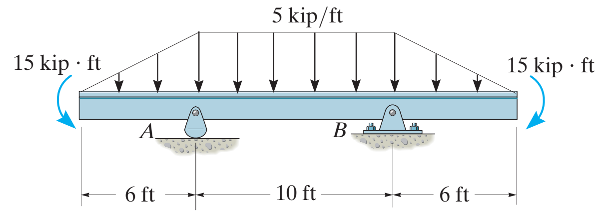


moment diagram

(c)

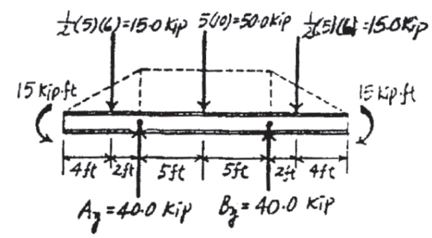
***7–92.**

Draw the shear and moment diagrams for the beam.



SOLUTION

Support Reactions: From FBD (a),



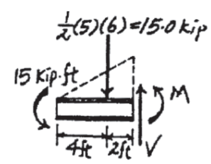
$\zeta + \Sigma M_A = 0;\qquad B_y(10) + 15.0(2) + 15 - 50.0(5) - 15.0(12) - 15 = 0$

$B_y = 40.0 \text{ kip}$

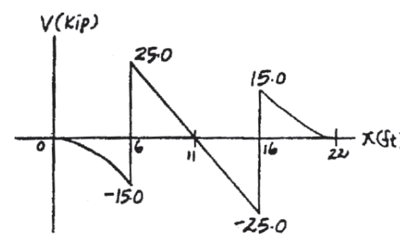
$+\uparrow \Sigma F_y = 0;\qquad A_y + 40.0 - 15.0 - 50.0 - 15.0 = 0$

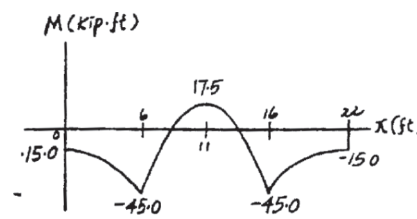
$A_y = 40.0 \text{ kip}$

Shear and Moment Diagrams: The value of the moment at supports A and B can be evaluated using the method of sections [FBD (c)].



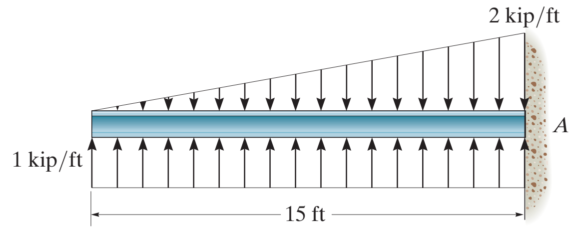
$\zeta + \Sigma M = 0;\qquad M + 15.0(2) + 15 = 0 \qquad M = -45.0 \text{ kip} \cdot \text{ft}$





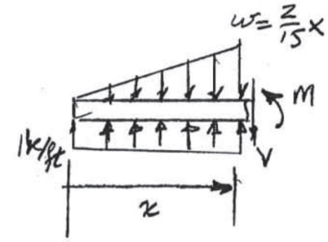
7–93.

Draw the shear and moment diagrams for the beam.



SOLUTION

Shear and Moment Functions: For $0 \leq x < 15$ ft

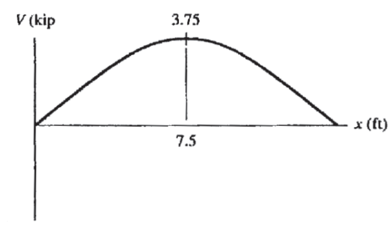


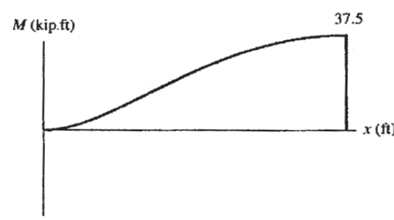
$+\uparrow \Sigma F_y = 0; \qquad 1x - x^2/15 - V = 0$

$V = \{x - x^2/15\}$ N **Ans.**

$\zeta + \Sigma M = 0; \qquad M + (x^2/15)\left(\frac{x}{3}\right) - 1x(x/2) = 0$

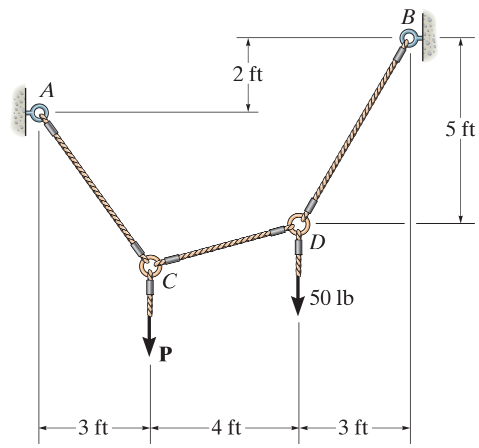
$M = \{x^2/2 - x^3/45\}$ N $\cdot$ m **Ans.**





7–94.

Determine the tension in each segment of the cable and the cable's total length. Set $P = 80$ lb.



SOLUTION

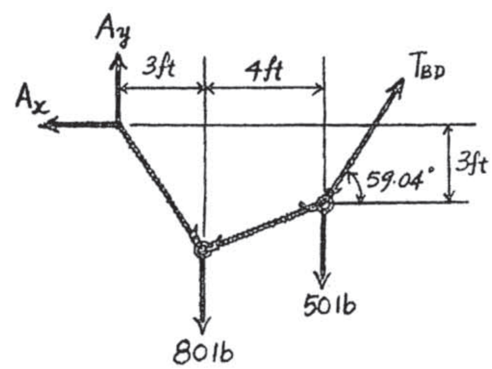
From FBD (a)

$\zeta + \Sigma M_A = 0;$ $\quad T_{BD} \cos 59.04°(3) + T_{BD} \sin 59.04°(7) - 50(7) - 80(3) = 0$

$T_{BD} = 78.188 \text{ lb} = 78.2 \text{ lb}$ **Ans.**

$\xrightarrow{+} \Sigma F_x = 0;$ $\quad 78.188 \cos 59.04° - A_x = 0$ $\quad A_x = 40.227$ lb

$+\uparrow \Sigma F_x = 0;$ $\quad A_y + 78.188 \sin 59.04° - 80 - 50 = 0$ $\quad A_y = 62.955$ lb



(a)

Joint A:

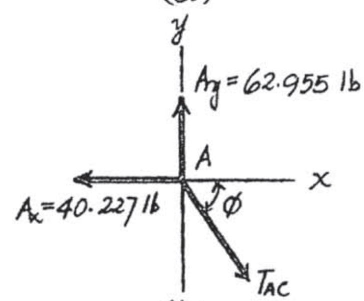
$\xrightarrow{+} \Sigma F_x = 0;$ $\quad T_{AC} \cos \phi - 40.227 = 0$ **(1)**

$+\uparrow \Sigma F_y = 0;$ $\quad -T_{AC} \sin \phi + 62.955 = 0$ **(2)**

Solving Eqs. (1) and (2) yields:

$\phi = 57.42°$

$T_{AC} = 74.7$ lb **Ans.**



(b)

Joint D:

$\xrightarrow{+} \Sigma F_x = 0;$ $\quad 78.188 \cos 59.04° - T_{CD} \cos \theta = 0$ **(3)**

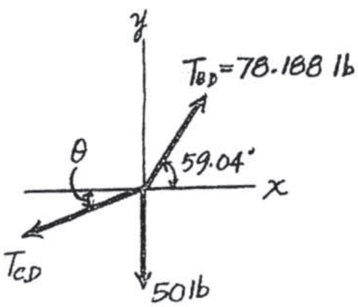
$+\uparrow \Sigma F_y = 0;$ $\quad 78.188 \sin 59.04° - T_{CD} \sin \theta - 50 = 0$ **(4)**

Solving Eqs. (3) and (4) yields:

$\theta = 22.96°$

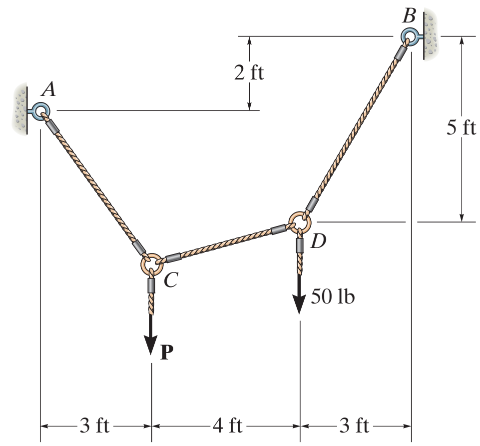
$T_{CD} = 43.7$ lb **Ans.**

Total length of the cable: $\quad l = \dfrac{5}{\sin 59.04°} + \dfrac{4}{\cos 22.96°} + \dfrac{3}{\cos 57.42°} = 15.7 \text{ ft}$ **Ans.**

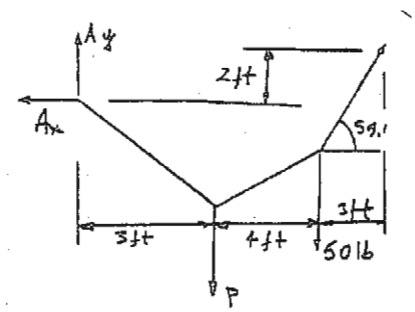


7–95.

If each cable segment can support a maximum tension of 75 lb, determine the largest load P that can be applied.



SOLUTION



$\circlearrowleft + \Sigma M_A = 0;\qquad -T_{BD}(\cos 59.04°)\,2 + T_{BD}(\sin 59.04°)(10) - 50(7) - P(3) = 0$

$T_{BD} = 0.39756\,P + 46.383$

$\overset{+}{\rightarrow} \Sigma F_x = 0;\qquad -A_x + T_{BD}\cos 59.04° = 0$

$+\uparrow \Sigma F_y = 0;\qquad A_y - P - 50 + T_{BD}\sin 59.04° = 0$

Assume maximum tension is in cable BD.

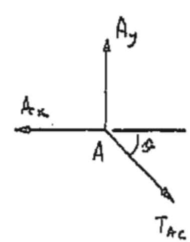
$T_{BD} = 75 \text{ lb}$

$P = 71.98 \text{ lb}$

$A_x = 38.59 \text{ lb}$

$A_y = 57.670 \text{ lb}$

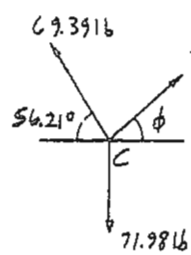
Pin A:



$T_{AC} = \sqrt{(38.59)^2 + (57.670)^2} = 69.39 \text{ lb} < 75 \text{ lb}\qquad$ OK

$\theta = \tan^{-1}\left(\dfrac{57.670}{38.59}\right) = 56.21°$

Joint C:



$\overset{+}{\rightarrow} \Sigma F_x = 0;\qquad T_{CD}\cos\phi - 69.39\cos 56.21° = 0$

$+\uparrow \Sigma F_y = 0;\qquad T_{CD}\sin\phi + 69.39\sin 56.21° - 71.98 = 0$

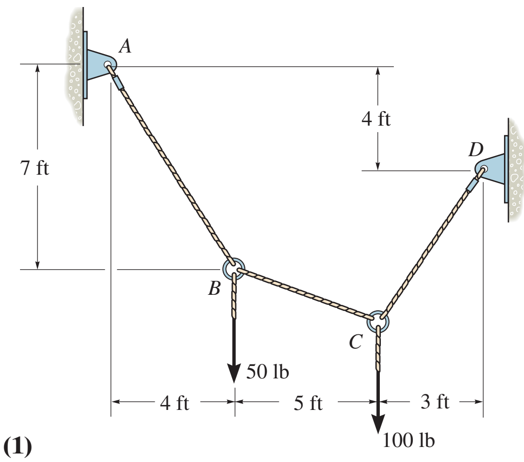
$T_{CD} = 41.2 \text{ lb} < 75 \text{ lb}\qquad$ OK

$\phi = 20.3°$

Thus, $\qquad P = 72.0 \text{ lb}\qquad$ **Ans.**

***7–96.**

Determine the tension in each segment of the cable and the cable's total length.



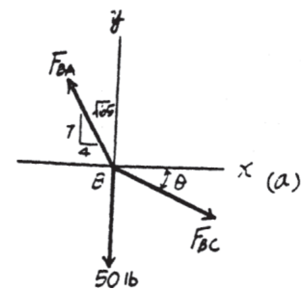
SOLUTION

Equations of Equilibrium: Applying method of joints, we have

Joint B

$\xrightarrow{+} \Sigma F_x = 0; \qquad F_{BC} \cos\theta - F_{BA}\left(\frac{4}{\sqrt{65}}\right) = 0 \qquad$ **(1)**

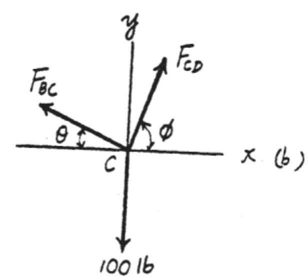
$+\uparrow \Sigma F_y = 0; \qquad F_{BA}\left(\frac{7}{\sqrt{65}}\right) - F_{BC} \sin\theta - 50 = 0 \qquad$ **(2)**



Joint C

$\xrightarrow{+} \Sigma F_x = 0; \qquad F_{CD} \cos\phi - F_{BC} \cos\theta = 0 \qquad$ **(3)**

$+\uparrow \Sigma F_y = 0; \qquad F_{BC} \sin\theta + F_{CD} \sin\phi - 100 = 0 \qquad$ **(4)**



Geometry:

$$\sin\theta = \frac{y}{\sqrt{y^2 + 25}} \qquad \cos\theta = \frac{5}{\sqrt{y^2 + 25}}$$

$$\sin\phi = \frac{3 + y}{\sqrt{y^2 + 6y + 18}} \qquad \cos\phi = \frac{3}{\sqrt{y^2 + 6y + 18}}$$

Substitute the above results into Eqs. (1), (2), (3) and (4) and solve. We have

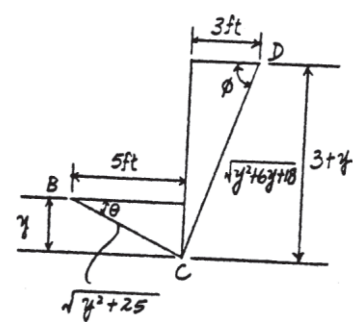
$F_{BC} = 46.7 \text{ lb} \qquad F_{BA} = 83.0 \text{ lb} \qquad F_{CD} = 88.1 \text{ lb}$ **Ans.**

$$y = 2.679 \text{ ft}$$

The total length of the cable is

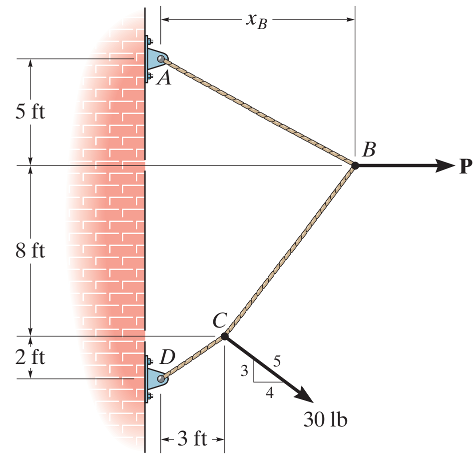
$$l = \sqrt{7^2 + 4^2} + \sqrt{5^2 + 2.679^2} + \sqrt{3^2 + (2.679 + 3)^2}$$

$= 20.2 \text{ ft}$ **Ans.**



7–97.

The cable supports the loading shown. Determine the horizontal distance x_B the force at point B acts from A. Set $P = 40$ lb.



SOLUTION

At B

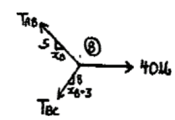
$\overset{+}{\rightarrow}\Sigma F_x = 0;$

$$40 - \frac{x_B}{\sqrt{x_B^2 + 25}}T_{AB} - \frac{x_B - 3}{\sqrt{(x_B - 3)^2 + 64}}T_{BC} = 0$$

$+\uparrow \Sigma F_y = 0;$

$$\frac{5}{\sqrt{x_B^2 + 25}}T_{AB} - \frac{8}{\sqrt{(x_B - 3)^2 + 64}}T_{BC} = 0$$

$$\frac{13x_B - 15}{\sqrt{(x_B - 3)^2 + 64}}T_{BC} = 200 \qquad \textbf{(1)}$$



At C

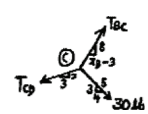
$\overset{+}{\rightarrow}\Sigma F_x = 0;$

$$\frac{4}{5}(30) + \frac{x_B - 3}{\sqrt{(x_B - 3)^2 + 64}}T_{BC} - \frac{3}{\sqrt{13}}T_{CD} = 0$$

$+\uparrow \Sigma F_y = 0;$

$$\frac{8}{\sqrt{(x_B - 3)^2 + 64}}T_{BC} - \frac{2}{\sqrt{13}}T_{CD} - \frac{3}{5}(30) = 0$$

$$\frac{30 - 2x_B}{\sqrt{(x_B - 3)^2 + 64}}T_{BC} = 102 \qquad \textbf{(2)}$$



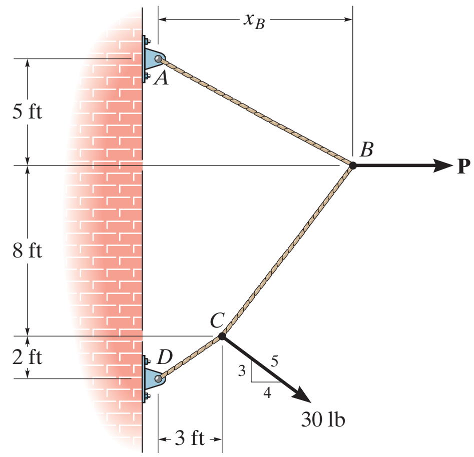
Solving Eqs. (1) & (2)

$$\frac{13x_B - 15}{30 - 2x_B} = \frac{200}{102}$$

$$x_B = 4.36 \text{ ft}$$ **Ans.**

7–98.

The cable supports the loading shown. Determine the magnitude of the horizontal force **P** so that $x_B = 6$ ft.



SOLUTION

At B

$\stackrel{+}{\rightarrow} \Sigma F_x = 0;$ $$P - \frac{6}{\sqrt{61}} T_{AB} - \frac{3}{\sqrt{73}} T_{BC} = 0$$

$+\uparrow \Sigma F_y = 0;$ $$\frac{5}{\sqrt{61}} T_{AB} - \frac{8}{\sqrt{73}} T_{BC} = 0$$

$$5P - \frac{63}{\sqrt{73}} T_{BC} = 0 \qquad \textbf{(1)}$$

At C

$\stackrel{+}{\rightarrow} \Sigma F_x = 0;$ $$\frac{4}{5}(30) + \frac{3}{\sqrt{73}} T_{BC} - \frac{3}{\sqrt{13}} T_{CD} = 0$$

$+\uparrow \Sigma F_y = 0;$ $$\frac{8}{\sqrt{73}} T_{BC} - \frac{2}{\sqrt{13}} T_{CD} - \frac{3}{5}(30) = 0$$

$$\frac{18}{\sqrt{73}} T_{BC} = 102 \qquad \textbf{(2)}$$

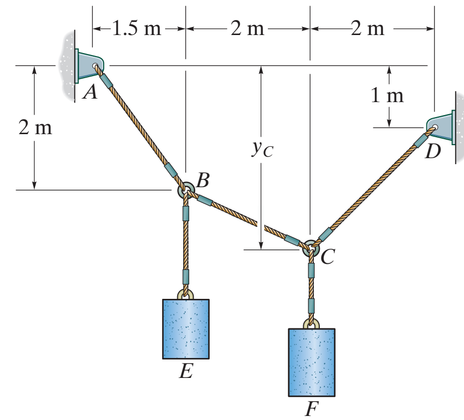
Solving Eqs. (1) & (2)

$$\frac{63}{18} = \frac{5P}{102}$$

$$P = 71.4 \text{ lb} \qquad \textbf{Ans.}$$

7–99.

If cylinders E and F have a mass of 20 kg and 40 kg, respectively, determine the tension developed in each cable and the sag y_C.



SOLUTION

First, $\mathbf{T}_{AB}$ will be obtained by considering the equilibrium of the free-body diagram shown in Fig. *a*. Subsequently, the result of T_{AB} will be used to analyze the equilibrium of joint B followed by joint C. Referring to Fig. *a*, we have

$$\circlearrowleft+\Sigma M_D = 0; \qquad 40(9.81)(2) + 20(9.81)(4) - T_{AB}\left(\frac{3}{5}\right)(1) - T_{AB}\left(\frac{4}{5}\right)(4) = 0$$

$$T_{AB} = 413.05 \text{ N} = 413 \text{ N} \qquad \textbf{Ans.}$$

Using the free-body diagram shown in Fig. *b*, we have

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad T_{BC}\cos\theta - 413.05\left(\frac{3}{5}\right) = 0$$

$$+\uparrow\Sigma F_y = 0; \qquad 413.05\left(\frac{4}{5}\right) - 20(9.81) - T_{BC}\sin\theta = 0$$

Solving,

$$\theta = 28.44^\circ$$

$$T_{BC} = 281.85\,\text{N} = 282\,\text{N} \qquad \textbf{Ans.}$$

Using the result of θ and the geometry of the cable, y_C is given by

$$\frac{y_C - 2}{2} = \tan\theta = 28.44^\circ$$

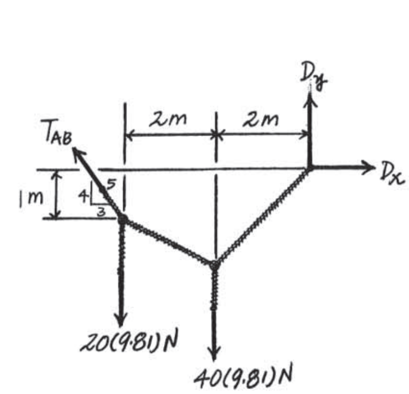
$$y_C = 3.083 \text{ m} = 3.08 \text{ m} \qquad \textbf{Ans.}$$

Using the results of y_C, θ, and T_{BC} and analyzing the equilibrium of joint C, Fig. *c*, we have

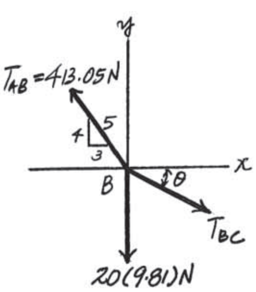
$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad T_{CD}\cos 46.17^\circ - 281.85\cos 28.44^\circ = 0$$

$$T_{CD} = 357.86 \text{ N} = 358 \text{ N} \qquad \textbf{Ans.}$$

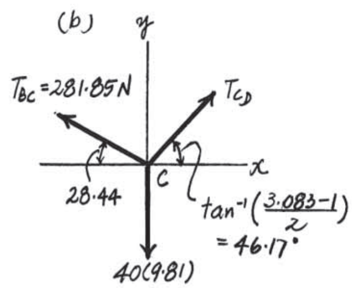
$$+\uparrow\Sigma F_y = 0; \qquad 281.85\sin 28.44^\circ + 357.86\sin 46.17^\circ - 40(9.81) = 0 \qquad \textbf{\textit{(Check!)}}$$



(a)



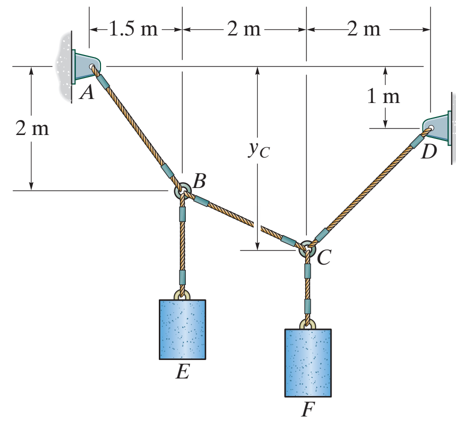
(b)



(c)

***7–100.**

If cylinder E has a mass of 20 kg and each cable segment can sustain a maximum tension of 400 N, determine the largest mass of cylinder F that can be supported. Also, what is the sag y_C?



SOLUTION

We will assume that cable AB is subjected to the greatest tension, i.e., $T_{AB} = 400$ N. Based on this assumption, M_F can be obtained by considering the equilibrium of the free-body diagram shown in Fig. *a*. We have

$$\circlearrowleft + \Sigma M_D = 0; \qquad M_F(9.81)(2) + 20(9.81)(4) - 400\left(\frac{3}{5}\right)(1) - 400\left(\frac{4}{5}\right)(4) = 0$$

$$M_F = 37.47 \text{ kg} \qquad \textbf{Ans.}$$

Analyzing the equilibrium of joint B and referring to the free-body diagram shown in Fig. *b*, we have

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad T_{BC} \cos\theta - 400\left(\frac{3}{5}\right) = 0$$

$$+\uparrow \Sigma F_y = 0; \qquad 400\left(\frac{4}{5}\right) - 20(9.81) - T_{BC} \sin\theta = 0$$

Solving,

$$\theta = 27.29^\circ$$

$$T_{BC} = 270.05 \text{ N}$$

Using these results and analyzing the equilibrium of joint C,

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad T_{CD} \cos\phi - 270.05 \cos 27.29^\circ = 0$$

$$+\uparrow \Sigma F_y = 0; \qquad T_{CD} \sin\phi + 270.05 \sin 27.29^\circ - 37.47(9.81) = 0$$

Solving,

$$\phi = 45.45^\circ \qquad T_{CD} = 342.11 \text{ N}$$

By comparing the above results, we realize that cable AB is indeed subjected to the greatest tension. Thus,

$$M_F = 37.5 \text{ kg} \qquad \textbf{Ans.}$$

Using the result of either θ or ϕ, the geometry of the cable gives

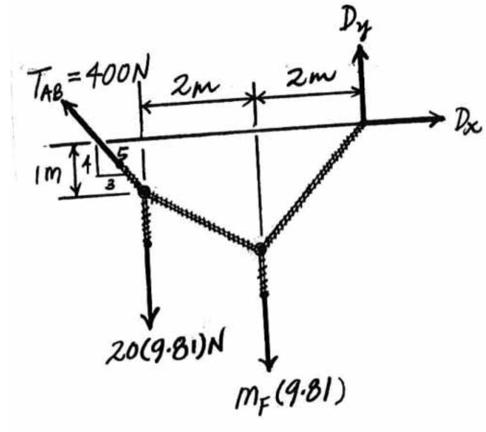
$$\frac{y_C - 2}{2} = \tan\theta = \tan 27.29^\circ$$

$$y_C = 3.03 \text{ m} \qquad \textbf{Ans.}$$

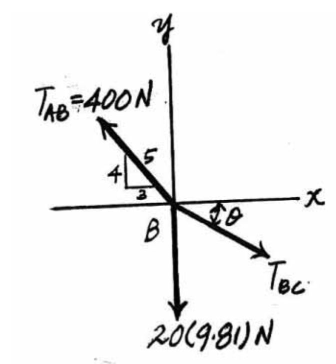
or

$$\frac{y_C - 1}{2} = \tan\phi = \tan 45.45^\circ$$

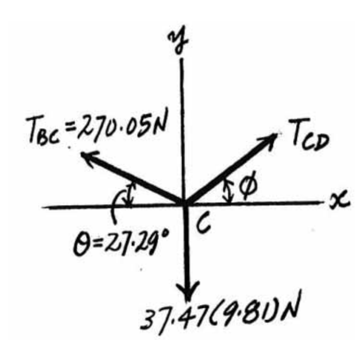
$$y_C = 3.03 \text{ m} \qquad \textbf{Ans.}$$



(a)



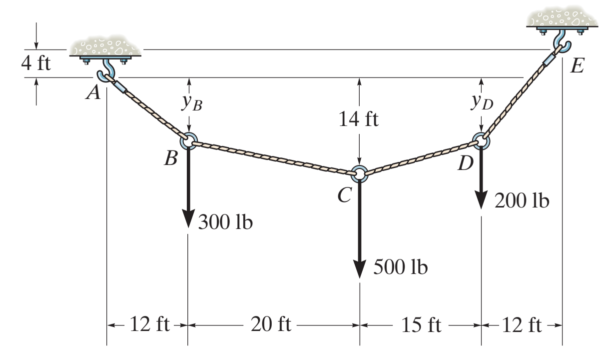
(b)



(c)

7–101.

The cable supports the three loads shown. Determine the sags y_B and y_D of points B and D and the tension in each segment of the cable.



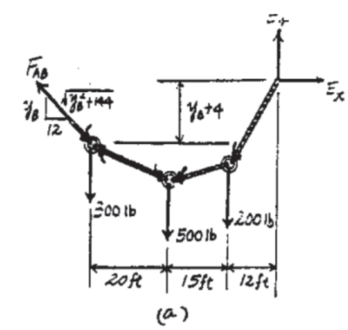
SOLUTION

Equations of Equilibrium: From FBD (a),

$$\zeta+\Sigma M_E = 0; \qquad -F_{AB}\left(\frac{y_B}{\sqrt{y_B^2+144}}\right) - F_{AB}\left(\frac{12}{\sqrt{y_B^2+144}}\right)(y_B+4)$$

$$+\,200(12) + 500(27) + 300(47) = 0$$

$$F_{AB}\left(\frac{47y_B}{\sqrt{y_B^2+144}}\right) - F_{AB}\left(\frac{12(y_B+4)}{\sqrt{y_B^2+144}}\right) = 30000 \qquad \textbf{(1)}$$

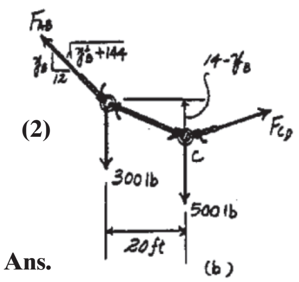


From FBD (b),

$$\zeta+\Sigma M_C = 0; \qquad -F_{AB}\left(\frac{y_B}{\sqrt{y_B^2+144}}\right)(20) + F_{AB}\left(\frac{12}{\sqrt{y_B^2+144}}\right)(14-y_B)$$

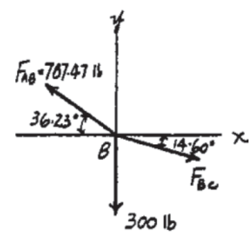
$$+\,300(20) = 0$$

$$F_{AB}\left(\frac{20y_B}{\sqrt{y_B^2+144}}\right) - F_{AB}\left(\frac{12(14-y_B)}{\sqrt{y_B^2+144}}\right) = 6000 \qquad \textbf{(2)}$$



Solving Eqs. (1) and (2) yields

$y_B = 8.792\text{ ft} = 8.79\text{ ft} \qquad F_{AB} = 787.47\text{ lb} = 787\text{ lb}$ **Ans.**



Method of Joints:

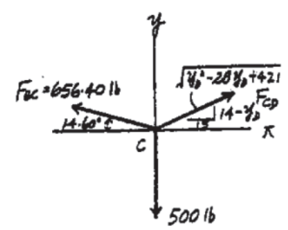
Joint B

$\overset{+}{\rightarrow}\Sigma F_x = 0; \quad F_{BC}\cos 14.60° - 787.47\cos 36.23° = 0$

$F_{BC} = 656.40\text{ lb} = 656\text{ lb}$ **Ans.**

$+\uparrow\Sigma F_y = 0; \quad 787.47\sin 36.23°$

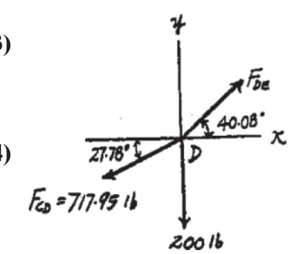
$- 656.40\sin 14.60° - 300 = 0$ **(*Checks!*)**



Joint C

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \quad F_{CD}\left(\frac{15}{\sqrt{y_D^2+28y_D+421}}\right) - 656.40\cos 14.60° = 0 \qquad \textbf{(3)}$$

$$+\uparrow\Sigma F_y = 0; \quad F_{CD}\left(\frac{14-y_D}{\sqrt{y_D^2-28y_D+421}}\right) + 656.40\sin 14.60° - 500 = 0 \qquad \textbf{(4)}$$



Solving Eqs. (1) and (2) yields

$y_D = 6.099\text{ ft} = 8.79\text{ ft} \qquad F_{CD} = 717.95\text{ lb} = 718\text{ lb}$ **Ans.**

Joint B

$\overset{+}{\rightarrow}\Sigma F_x = 0; \quad F_{DE}\cos 40.08° - 717.95\cos 27.78° = 0$

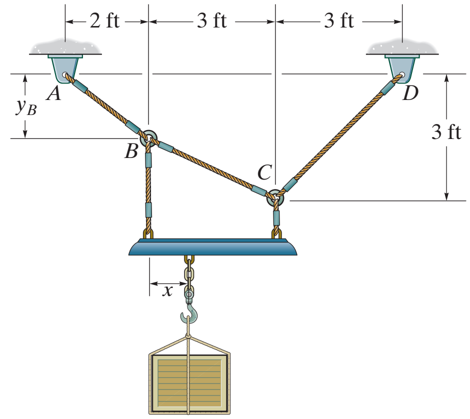
$F_{DE} = 830.24\text{ lb} = 830\text{ lb}$ **Ans.**

$+\uparrow\Sigma F_y = 0; \quad 830.24\sin 40.08°$

$- 717.95\sin 27.78° - 200 = 0$ **(*Checks!*)**

7–102.

If $x = 2$ ft and the crate weighs 300 lb, which cable segment AB, BC, or CD has the greatest tension? What is this force and what is the sag y_B?

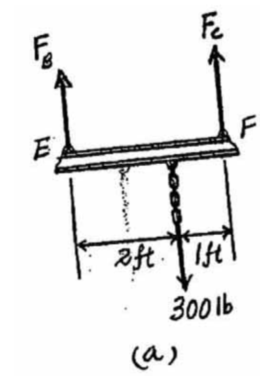


SOLUTION

The forces $\mathbf{F}_B$ and $\mathbf{F}_C$ exerted on joints B and C will be obtained by considering the equilibrium on the free-body diagram, Fig. *a*.

$+\Sigma M_E = 0; \qquad F_C(3) - 300(2) = 0 \qquad F_C = 200 \text{ lb}$

$+\Sigma M_F = 0; \qquad 300(1) - F_B(3) = 0 \qquad F_B = 200 \text{ lb}$



Referring to Fig. *b*, we have

$+\Sigma M_A = 0; \qquad T_{CD} \sin 45°(8) - 200(5) - 100(2) = 0$

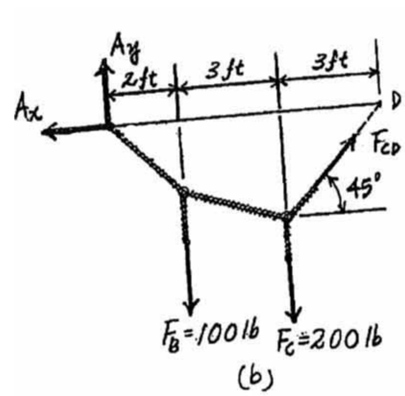
$$T_{CD} = 212.13 \text{ lb} = 212 \text{ lb (max)}$$

Using these results and analyzing the equilibrium of joint C, Fig. *c*, we obtain

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 212.13 \cos 45° - T_{BC} \cos\theta = 0$

$+\uparrow \Sigma F_y = 0; \qquad T_{BC} \sin\theta + 212.13 \sin 45° - 200 = 0$

$$T_{AB} = T_{CD} = 212 \text{ lb (max)}$$ **Ans.**



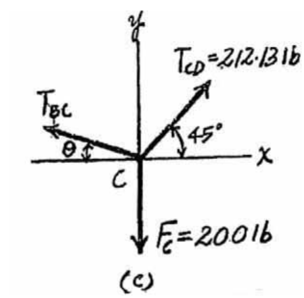
Solving,

$T_{BC} = 158.11 \text{ lb} \qquad \theta = 18.43°$

Using these results to analyze the equilibrium of joint B, Fig. *d*, we have

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 158.11 \cos 18.43° - T_{AB} \cos\phi = 0$

$+\uparrow \Sigma F_y = 0; \qquad T_{AB} \sin\phi - 100 - 158.11 \sin 18.43° = 0$



Solving,

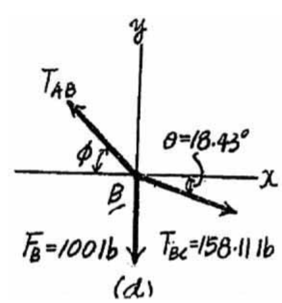
$$\phi = 45°$$

$$T_{AB} = 212.13 \text{ lb} = 212 \text{ lb (max)}$$

Thus, both cables AB and CD are subjected to maximum tension. The sag y_B is given by

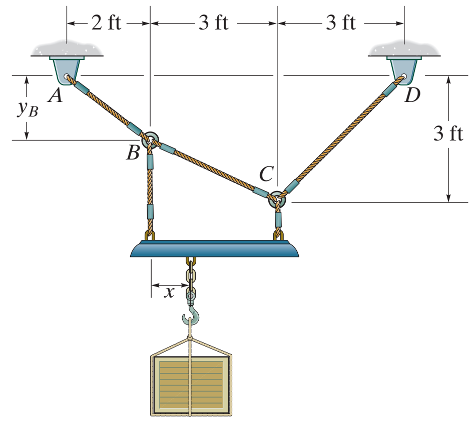
$$\frac{y_B}{2} = \tan\phi = \tan 45°$$

$$y_B = 2 \text{ ft}$$ **Ans.**



7–103.

If $y_B = 1.5$ ft, determine the largest weight of the crate and its placement x so that neither cable segment AB, BC, or CD is subjected to a tension that exceeds 200 lb.



SOLUTION

The forces $\mathbf{F}_B$ and $\mathbf{F}_C$ exerted on joints B and C will be obtained by considering the equilibrium on the free-body diagram, Fig. *a*.

$$\circlearrowleft + \Sigma M_E = 0; \quad F_C(3) - w(x) = 0 \qquad F_C = \frac{wx}{3}$$

$$\circlearrowleft + \Sigma M_F = 0; \quad w(3 - x) - F_B(3) = 0 \qquad F_B = \frac{w}{3}(3 - x)$$

Since the horizontal component of tensile force developed in each cable is constant, cable CD, which has the greatest angle with the horizontal, will be subjected to the greatest tension. Thus, we will set $T_{CD} = 200$ lb.

First, we will analyze the equilibrium of joint C, Fig. *b*.

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad 200 \cos 45° - T_{BC} \cos 26.57° = 0 \qquad T_{BC} = 158.11 \text{ lb}$$

$$+\uparrow \Sigma F_y = 0; \quad 200 \sin 45° + 158.11 \sin 26.57° - \frac{wx}{3} = 0$$

$$\frac{wx}{3} = 212.13 \qquad \textbf{(1)}$$

Using the result of T_{BC} to analyze the equilibrium of joint B, Fig. *c*, we have

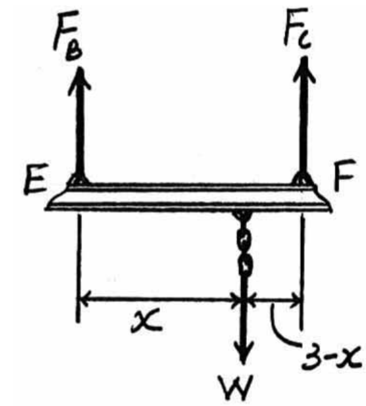
$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad 158.11 \cos 26.57° - T_{AB}\left(\frac{4}{5}\right) = 0 \qquad T_{AB} = 176.78 \text{ lb}$$

$$+\uparrow \Sigma F_y = 0; \quad 176.78\left(\frac{3}{5}\right) - 158.11 \sin 26.57° - \frac{w}{3}(3 - x) = 0$$

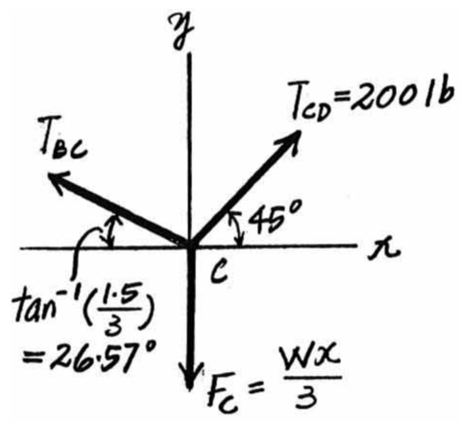
$$\frac{w}{3}(3 - x) = 35.36 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2)

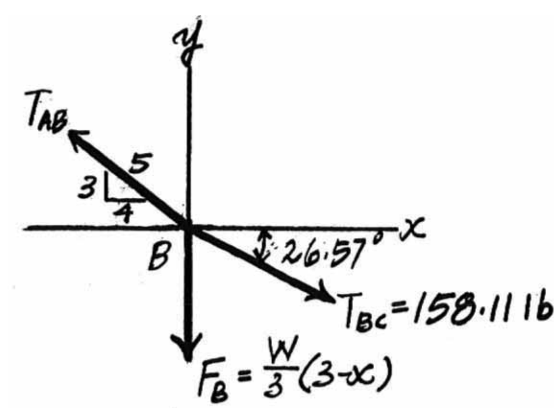
$$x = 2.57 \text{ ft} \qquad w = 247 \text{ lb} \qquad \textbf{Ans.}$$



(a)



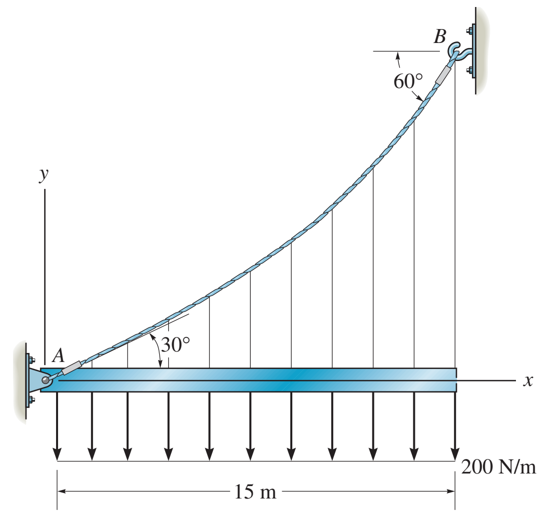
(b)



(c)

***7–104.**

The cable AB is subjected to a uniform loading of 200 N/m. If the weight of the cable is neglected and the slope angles at points A and B are 30° and 60°, respectively, determine the curve that defines the cable shape and the maximum tension developed in the cable.

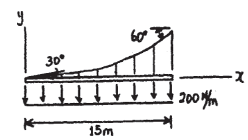


SOLUTION

$$y = \frac{1}{F_H}\int\left(\int 200\,dx\right)dx$$

$$y = \frac{1}{F_H}(100x^2 + C_1 x + C_2)$$

$$\frac{dy}{dx} = \frac{1}{F_H}(200x + C_1)$$



At $x = 0$, $\quad y = 0; \quad C_2 = 0$

At $x = 0$, $\quad \frac{dy}{dx} = \tan 30°; \quad C_1 = F_H \tan 30°$

$$y = \frac{1}{F_H}(100x^2 + F_H \tan 30° x)$$

At $x = 15$ m, $\quad \frac{dy}{dx} = \tan 60°; \quad F_H = 2598\text{ N}$

$$y = (38.5x^2 + 577x)(10^{-3})\text{ m} \qquad \textbf{Ans.}$$

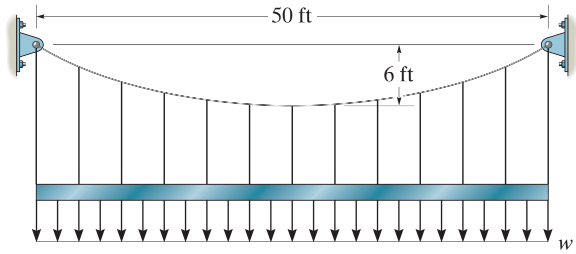
$$\theta_{max} = 60°$$

$$T_{max} = \frac{F_H}{\cos\theta_{max}} = \frac{2598}{\cos 60°} = 5196\text{ N}$$

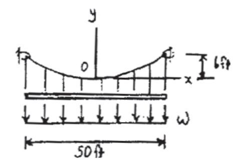
$$T_{max} = 5.20\text{ kN} \qquad \textbf{Ans.}$$

7–105.

Determine the maximum uniform loading w, measured in lb/ft, that the cable can support if it is capable of sustaining a maximum tension of 3000 lb before it will break.



SOLUTION



$$y = \frac{1}{F_H}\int\left(\int w\,dx\right)dx$$

At $x = 0,\ \dfrac{dy}{dx} = 0$

At $x = 0,\ y = 0$

$$C_1 = C_2 = 0$$

$$y = \frac{w}{2F_H}x^2$$

At $x = 25$ ft, $y = 6$ ft $\qquad F_H = 52.08\,w$

$$\left.\frac{dy}{dx}\right|_{max} = \tan\theta_{max} = \left.\frac{w}{F_H}x\right|_{x=25\text{ ft}}$$

$$\theta_{max} = \tan^{-1}(0.48) = 25.64^\circ$$

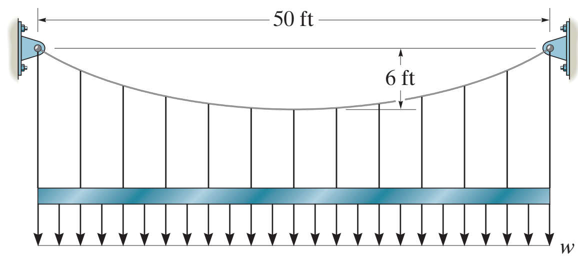
$$T_{max} = \frac{F_H}{\cos\theta_{max}} = 3000$$

$$F_H = 2705\text{ lb}$$

$w = 51.9$ lb/ft **Ans.**

7–106.

The cable is subjected to a uniform loading of $w = 250$ lb/ft. Determine the maximum and minimum tension in the cable.



SOLUTION

From Example 7–12:

$$F_H = \frac{w_0 L^2}{8h} = \frac{250\,(50)^2}{8\,(6)} = 13\,021 \text{ lb}$$

$$\theta_{max} = \tan^{-1}\left(\frac{w_0 L}{2F_H}\right) = \tan^{-1}\left(\frac{250\,(50)}{2(13\,021)}\right) = 25.64^\circ$$

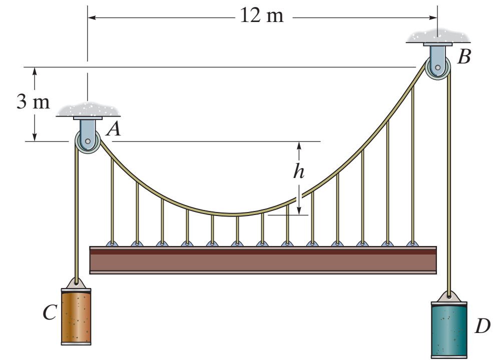
$$T_{max} = \frac{F_H}{\cos\theta_{max}} = \frac{13\,021}{\cos 25.64^\circ} = 14.4 \text{ kip}$$ **Ans.**

The minimum tension occurs at $\theta = 0^\circ$.

$$T_{min} = F_H = 13.0 \text{ kip}$$ **Ans.**

7–107.

Cylinders C and D are attached to the end of the cable. If D has a mass of 600 kg, determine the required mass of C, the maximum sag h of the cable, and the length of the cable between the pulleys A and B. The beam has a mass per unit length of 50 kg/m.



SOLUTION

From the free-body diagram shown in Fig. a, we can write

$\circlearrowleft + \Sigma M_A = 0;\quad 600(9.81)\ \sin\theta_B(12) - 600(9.81)\ \cos\theta_B(3) - 50(12)(9.81)(6) = 0$

$\theta_B = 43.05^\circ$

$\xrightarrow{+} \Sigma F_x = 0;\quad 600(9.81)\ \cos 43.05^\circ - m_C(9.81)\ \cos\theta_A = 0$

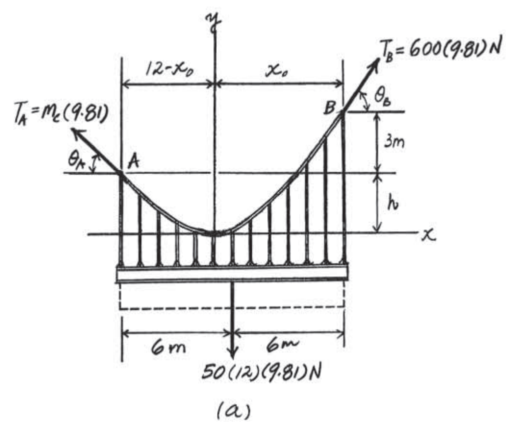
$+\uparrow \Sigma F_y = 0;\quad m_C(9.81)\ \sin\theta_A + 600(9.81)\sin 43.05^\circ - 50(12)(9.81) = 0$

Solving,

$m_C = 477.99\text{ kg} = 478\text{ kg}$ **Ans.**

$\theta_A = 23.47^\circ$ **Ans.**

Thus, $F_H = T_B\cos\theta_B = 4301.00\text{ N}$. As shown in Fig. a, the origin of the $x - y$ coordinate system is set at the lowest point of the cable. Using Eq. (1) of Example 7–12,



$$y = \frac{w_0}{2F_H}x^2 = \frac{50(9.81)}{2(4301.00)}x^2$$

$$y = 0.05702x^2$$

Using Eq. (4) and applying two other boundary conditions $y = (h + 3)$ m at $x = x_0$ and $y = h$ at $x = -(12 - x_0)$, we have

$$h + 3 = 0.05702x_0^{\,2}$$

$$h = 0.05702[-(12 - x_0)]^2$$

Solving these equations yields

$h = 0.8268\text{ m} = 0.827\text{ m}$ **Ans.**

$x_0 = 8.192\text{ m}$

The differential length of the cable is

$$ds = \sqrt{dx^2 + dy^2} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx = \sqrt{1 + 0.01301x^2\,dx}$$

Thus, the total length of the cable is

$$L = \int ds = \int_{-3.808\text{ m}}^{8.192\text{ m}} \sqrt{1 + 0.01301x^2}$$

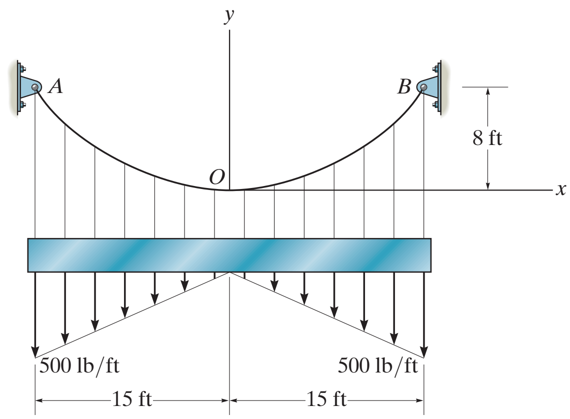
$$= 0.1140\int_{-3.808\text{ m}}^{8.192\text{ m}} \sqrt{76.89 + x^2\,dx}$$

$$= 0.1140\left\{\frac{1}{2}\left[x\sqrt{76.89 + x^2} + 76.89\ln\left(x + \sqrt{76.89 + x^2}\right)\right]\right\}\Bigg|_{-3.808\text{ m}}^{8.192\text{ m}}$$

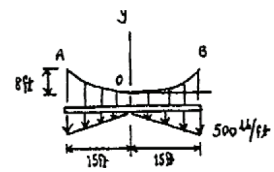
$= 13.2\text{ m}$ **Ans.**

***7–108.**

The cable is subjected to the triangular loading. If the slope of the cable at point O is zero, determine the equation of the curve $y = f(x)$ which defines the cable shape OB, and the maximum tension developed in the cable.



SOLUTION



$$y = \frac{1}{F_H}\int\left(\int w(x)dx\right)dx$$

$$= \frac{1}{F_H}\int\left(\int \frac{500}{15}xdx\right)dx$$

$$= \frac{1}{F_H}\int\left(\frac{50}{3}x^2 + C_1\right)dx$$

$$= \frac{1}{F_H}\left(\frac{50}{9}x^3 + C_1x + C_2\right)$$

$$\frac{dy}{dx} = \frac{50}{3F_H}x^2 + \frac{C_1}{F_H}$$

at $x = 0$, $\quad \frac{dy}{dx} = 0 \quad C_1 = 0$

at $x = 0$, $\quad y = 0 \quad C_2 = 0$

$$y = \frac{50}{9F_H}x^3$$

at $x = 15$ ft, $\quad y = 8$ ft $\quad F_H = 2344$ lb

$y = 2.37(10^{-3})x^3$ **Ans.**

$$\left.\frac{dy}{dx}\right|_{max} = \tan\theta_{max} = \left.\frac{50}{3(2344)}x^2\right|_{x = 15\text{ ft}}$$

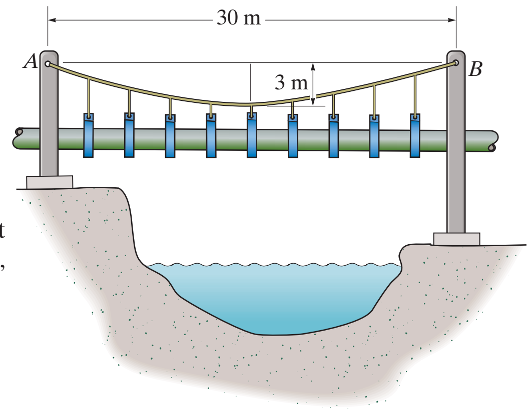
$$\theta_{max} = \tan^{-1}(1.6) = 57.99^\circ$$

$$T_{max} = \frac{F_H}{\cos\theta_{max}} = \frac{2344}{\cos 57.99^\circ} = 4422 \text{ lb}$$

$T_{max} = 4.42$ kip **Ans.**

7–109.

If the pipe has a mass per unit length of 1500 kg/m, determine the maximum tension developed in the cable.



SOLUTION

As shown in Fig. *a*, the origin of the *x*, *y* coordinate system is set at the lowest point of the cable. Here, $w(x) = w_0 = 1500(9.81) = 14.715(10^3)$ N/m. Using Eq. 7–12, we can write

$$y = \frac{1}{F_H}\int\left(\int w_0 dx\right)dx$$

$$= \frac{1}{F_H}\left(\frac{14.715(10^3)}{2}x^2 + c_1 x + c_2\right)$$

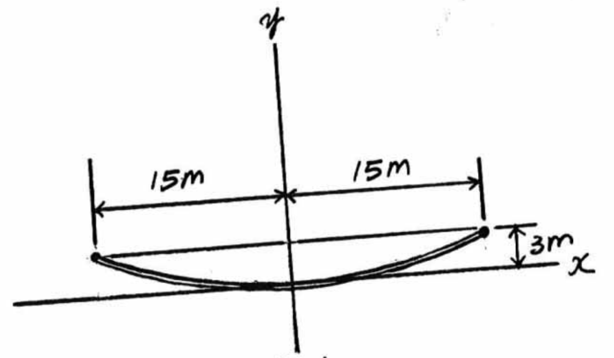
Applying the boundary condition $\frac{dy}{dx} = 0$ at $x = 0$, results in $c_1 = 0$.

Applying the boundary condition $y = 0$ at $x = 0$ results in $c_2 = 0$. Thus,

$$y = \frac{7.3575(10^3)}{F_H}x^2$$

Applying the boundary condition $y = 3$ m at $x = 15$ m, we have

$$3 = \frac{7.3575(10^3)}{F_H}(15)^2 \qquad F_H = 551.81(10^3)\text{ N}$$



(a)

Substituting this result into Eq. (1), we have

$$\frac{dy}{dx} = 0.02667x$$

The maximum tension occurs at either points at *A* or *B* where the cable has the greatest angle with the horizontal. Here,

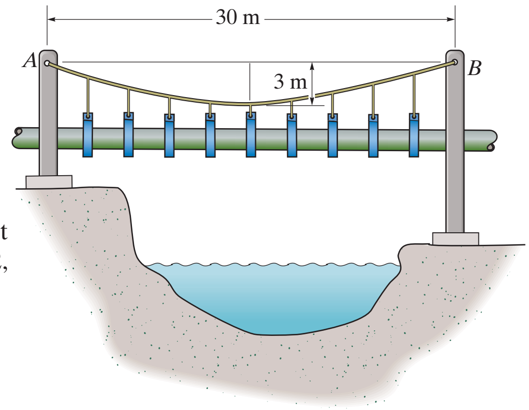
$$\theta_{max} = \tan^{-1}\left(\frac{dy}{dx}\bigg|_{15\text{ m}}\right) = \tan^{-1}[0.02667(15)] = 21.80°$$

Thus,

$$T_{max} = \frac{F_H}{\cos\theta_{max}} = \frac{551.8(10^3)}{\cos 21.80°} = 594.32(10^3)\text{ N} = 594\text{ kN}$$

7–110.

If the pipe has a mass per unit length of 1500 kg/m, determine the minimum tension developed in the cable.



SOLUTION

As shown in Fig. *a*, the origin of the *x*, *y* coordinate system is set at the lowest point of the cable. Here, $w(x) = w_0 = 1500(9.81) = 14.715(10^3)$ N/m. Using Eq. 7–12, we can write

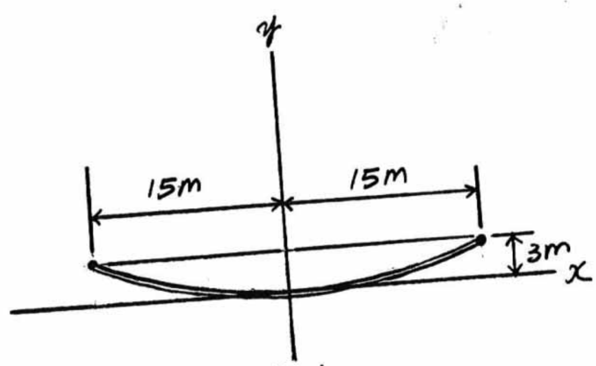
$$y = \frac{1}{F_H}\int\left(\int w_0 dx\right)dx$$

$$= \frac{1}{F_H}\left(\frac{14.715(10^3)}{2}x^2 + c_1x + c_2\right)$$

Applying the boundary condition $\frac{dy}{dx} = 0$ at $x = 0$, results in $c_1 = 0$.

Applying the boundary condition $y = 0$ at $x = 0$ results in $c_2 = 0$. Thus,

$$y = \frac{7.3575(10^3)}{F_H}x^2$$



(a)

Applying the boundary condition $y = 3$ m at $x = 15$ m, we have

$$3 = \frac{7.3575(10^3)}{F_H}(15)^2 \qquad F_H = 551.81(10^3)\text{ N}$$

Substituting this result into Eq. (1), we have

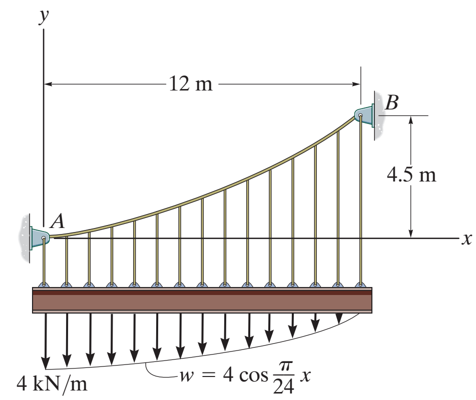
$$\frac{dy}{dx} = 0.02667x$$

The minimum tension occurs at the lowest point of the cable, where $\theta = 0°$. Thus,

$$T_{\min} = F_H = 551.81(10^3)\text{ N} = 552\text{ kN}$$

7–111.

If the slope of the cable at support A is zero, determine the deflection curve $y = f(x)$ of the cable and the maximum tension developed in the cable.



SOLUTION

Using Eq. 7–12,

$$y = \frac{1}{F_H}\int\left(\int w(x)dx\right)dx$$

$$y = \frac{1}{F_H}\int\left(\int 4\cos\frac{\pi}{24} \times dx\right)dx$$

$$y = \frac{1}{F_H}\int\frac{24}{\pi}\left[4(10^3)\right]\sin\frac{\pi}{24}x + C_1$$

$$y = -\frac{24}{\pi}\left[\frac{96(10^3)}{\pi F_H}\cos\frac{\pi}{24}x\right] + C_1x + C_2$$

Applying the boundary condition $\frac{dy}{dx} = 0$ at $x = 0$ results in $C_1 = 0$.
Applying the boundary condition $y = 0$ at $x = 0$, we have

$$0 = -\frac{24}{\pi}\left[\frac{96(10^3)}{\pi F_H}\cos 0°\right] + C_2$$

$$C_2 = \frac{2304(10^3)}{\pi^2 F_H}$$

Thus,

$$y = \frac{2304(10^3)}{\pi^2 F_H}\left[1 - \cos\frac{\pi}{24}x\right]$$

Applying the boundary condition $y = 4.5$ m at $x = 12$ m, we have

$$4.5 = \frac{2304(10^3)}{\pi^2 F_H}\left[1 - \cos\frac{\pi}{24}(12)\right]$$

$$F_H = 51.876(10^3)\text{ N}$$

Substituting this result into Eqs. (1) and (2), we obtain

$$\frac{dy}{dx} = \frac{96(10^3)}{\pi(51.876)(10^3)}\sin\frac{\pi}{24}x$$

$$= 0.5890\sin\frac{\pi}{24}x$$

and

$$y = \frac{2304(10^3)}{\pi^2(51.876)(10^3)}\left[1 - \cos\frac{\pi}{24}x\right]$$

$$= 4.5\left(1 - \cos\frac{\pi}{24}x\right)\text{ m}$$ **Ans.**

The maximum tension occurs at point B where the cable makes the greatest angle with the horizontal. Here,

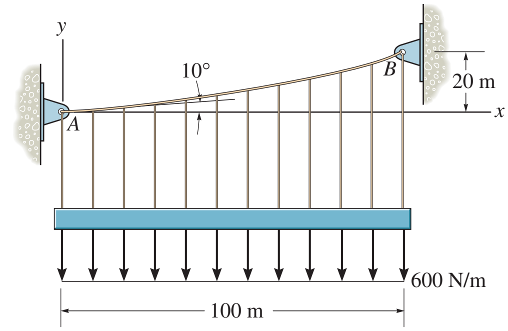
$$\theta_{max} = \tan^{-1}\left(\left.\frac{dy}{dx}\right|_{x = 12\text{ m}}\right) = \tan^{-1}\left[0.5890\sin\left(\frac{\pi}{24}(12)\right)\right] = 30.50°$$

Thus,

$$T_{max} = \frac{F_H}{\cos\theta_{max}} = \frac{51.876(10^3)}{\cos 30.50°} = 60.207(10^3)\text{ N} = 60.2\text{ kN}$$ **Ans.**

***7–112.**

Determine the maximum tension developed in the cable if it is subjected to a uniform load of 600 N/m.



SOLUTION

The Equation of The Cable:

$$y = \frac{1}{F_H}\int\left(\int w(x)dx\right)dx$$

$$= \frac{1}{F_H}\left(\frac{w_0}{2}x^2 + C_1x + C_2\right) \qquad \textbf{(1)}$$

$$\frac{dy}{dx} = \frac{1}{F_H}(w_0x + C_1) \qquad \textbf{(2)}$$

Boundary Conditions:

$y = 0$ at $x = 0$, then from Eq. (1) $0 = \frac{1}{F_H}(C_2) \qquad C_2 = 0$

$\frac{dy}{dx} = \tan 10°$ at $x = 0$, then from Eq. (2) $\tan 10° = \frac{1}{F_H}(C_1) \qquad C_1 = F_H \tan 10°$

Thus,
$$y = \frac{w_0}{2F_H}x^2 + \tan 10° x \qquad \textbf{(3)}$$

$y = 20$ m at $x = 100$ m, then from Eq. (3)

$$20 = \frac{600}{2F_H}(100^2) + \tan 10°(100) \qquad F_H = 1\,267\,265.47 \text{ N}$$

and

$$\frac{dy}{dx} = \frac{w_0}{F_H}x + \tan 10°$$

$$= \frac{600}{1\,267\,265.47}x + \tan 10°$$

$$= 0.4735(10^{-3})x + \tan 10°$$

$\theta = \theta_{max}$ at $x = 100$ m and the maximum tension occurs when $\theta = \theta_{max}$.

$$\tan\theta_{max} = \left.\frac{dy}{dx}\right|_{x=100\text{ m}} = 0.4735(10^{-3})(100) + \tan 10°$$

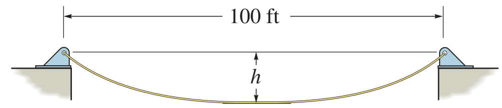
$$\theta_{max} = 12.61°$$

The maximum tension in the cable is

$$T_{max} = \frac{F_H}{\cos\theta_{max}} = \frac{1\,267\,265.47}{\cos 12.61°} = 1\,298\,579.01 \text{ N} = 1.30 \text{ MN}$$ **Ans.**

7–113.

The cable weighs 6 lb/ft and is 150 ft in length. Determine the sag h so that the cable spans 100 ft. Find the minimum tension in the cable.



SOLUTION

Deflection Curve of The Cable:

$$x = \int \frac{ds}{\left[1 + (1/F_H^2)\left(\int w_0\, ds\right)^2\right]^{\frac{1}{2}}} \qquad \text{where } w_0 = 6 \text{ lb/ft}$$

Performing the integration yields

$$x = \frac{F_H}{6}\left\{\sinh^{-1}\left[\frac{1}{F_H}(6s + C_1)\right] + C_2\right\} \qquad \textbf{(1)}$$

From Eq. 7–14

$$\frac{dy}{dx} = \frac{1}{F_H}\int w_0\, ds = \frac{1}{F_H}(6s + C_1) \qquad \textbf{(2)}$$

Boundary Conditions:

$\frac{dy}{dx} = 0$ at $s = 0$. From Eq. (2) $0 = \frac{1}{F_H}(0 + C_1) \qquad C_1 = 0$

Then, Eq. (2) becomes

$$\frac{dy}{dx} = \tan\theta = \frac{6s}{F_H} \qquad \textbf{(3)}$$

$s = 0$ at $x = 0$ and use the result $C_1 = 0$. From Eq. (1)

$$x = \frac{F_H}{6}\left\{\sinh^{-1}\left[\frac{1}{F_H}(0 + 0)\right] + C_2\right\} \qquad C_2 = 0$$

Rearranging Eq. (1), we have

$$s = \frac{F_H}{6}\sinh\left(\frac{6}{F_H}x\right) \qquad \textbf{(4)}$$

Substituting Eq. (4) into (3) yields

$$\frac{dy}{dx} = \sinh\left(\frac{6}{F_H}x\right)$$

Performing the integration

$$y = \frac{F_H}{6}\cosh\left(\frac{6}{F_H}x\right) + C_3 \qquad \textbf{(5)}$$

$y = 0$ at $x = 0$. From Eq. (5) $0 = \frac{F_H}{6}\cosh 0 + C_3$, thus, $C_3 = -\frac{F_H}{6}$

Then, Eq. (5) becomes

$$y = \frac{F_H}{6}\left[\cosh\left(\frac{6}{F_H}x\right) - 1\right] \qquad \textbf{(6)}$$

7–113. (continued)

$s = 75$ ft at $x = 50$ ft. From Eq. (4)

$$75 = \frac{F_H}{6}\sinh\left[\frac{6}{F_H}(50)\right]$$

By trial and error

$$F_H = 184.9419 \text{ lb}$$

$y = h$ at $x = 50$ ft. From Eq. (6)

$$h = \frac{184.9419}{6}\left\{\cosh\left[\frac{6}{184.9419}(50)\right] - 1\right\} = 50.3 \text{ ft} \qquad \textbf{Ans.}$$

The minimum tension occurs at $\theta = \theta_{\min} = 0°$. Thus,

$$T_{\min} = \frac{F_H}{\cos\theta_{\min}} = \frac{184.9419}{\cos 0°} = 185 \text{ lb} \qquad \textbf{Ans.}$$

7–114.

A telephone line (cable) stretches between two points which are 150 ft apart and at the same elevation. The line sags 5 ft and the cable has a weight of 0.3 lb/ft. Determine the length of the cable and the maximum tension in the cable.

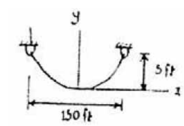
SOLUTION

$$w = 0.3\text{ lb/ft}$$

From Example 7–13,

$$s = \frac{F_H}{w}\sinh\left(\frac{w}{F_H}x\right)$$

$$y = \frac{F_H}{w}\left[\cosh\left(\frac{w}{F_H}x\right) - 1\right]$$



At $x = 75$ ft, $y = 5$ ft, $w = 0.3$ lb/ft

$$5 = \frac{F_H}{w}\left[\cosh\left(\frac{75w}{F_H}\right) - 1\right]$$

$$F_H = 169.0\text{ lb}$$

$$\left.\frac{dy}{dx}\right|_{\max} = \tan\theta_{\max} = \left.\sinh\left(\frac{w}{F_H}x\right)\right|_{x=75\text{ ft}}$$

$$\theta_{\max} = \tan^{-1}\left[\sinh\left(\frac{75(0.3)}{169}\right)\right] = 7.606°$$

$$T_{\max} = \frac{F_H}{\cos\theta_{\max}} = \frac{169}{\cos 7.606°} = 170\text{ lb}$$ **Ans.**

$$s = \frac{169.0}{0.3}\sinh\left[\frac{0.3}{169.0}(75)\right] = 75.22$$

$$L = 2s = 150\text{ ft}$$ **Ans.**

7–115.

A cable has a weight of 2 lb/ft. If it can span 100 ft and has a sag of 12 ft, determine the length of the cable. The ends of the cable are supported from the same elevation.

SOLUTION

From Eq. (5) of Example 7–13:

$$h = \frac{F_H}{w_0}\left[\cosh\left(\frac{w_0 L}{2F_H}\right) - 1\right]$$

$$12 = \frac{F_H}{2}\left[\cosh\left(\frac{2(100)}{2F_H}\right) - 1\right]$$

$$24 = F_H\left[\cosh\left(\frac{100}{F_H}\right) - 1\right]$$

$$F_H = 212.2 \text{ lb}$$

From Eq. (3) of Example 7–13:

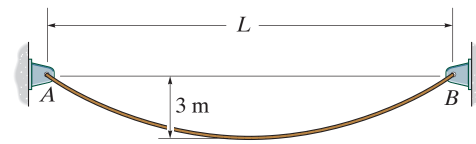
$$s = \frac{F_H}{w_0}\sinh\left(\frac{w_0}{F_H}x\right)$$

$$\frac{l}{2} = \frac{212.2}{2}\sinh\left(\frac{2(50)}{212.2}\right)$$

$$l = 104 \text{ ft}$$ **Ans.**

7–116.

The 10 kg/m cable is suspended between the supports A and B. If the cable can sustain a maximum tension of 1.5 kN and the maximum sag is 3 m, determine the maximum distance L between the supports



SOLUTION

The origin of the x, y coordinate system is set at the lowest point of the cable. Here $w_0 = 10(9.81)$ N/m $= 98.1$ N/m. Using Eq. (4) of Example 7–13,

$$y = \frac{F_H}{w_0}\left[\cosh\left(\frac{w_0}{F_H}x\right) - 1\right]$$

$$y = \frac{F_H}{98.1}\left[\cosh\left(\frac{98.1x}{F_H}\right) - 1\right]$$

Applying the boundary equation $y = 3$ m at $x = \frac{L}{2}$, we have

$$3 = \frac{F_H}{98.1}\left[\cosh\left(\frac{49.05L}{F_H}\right) - 1\right]$$

The maximum tension occurs at either points A or B where the cable makes the greatest angle with the horizontal. From Eq. (1),

$$\tan\theta_{max} = \sinh\left(\frac{49.05L}{F_H}\right)$$

By referring to the geometry shown in Fig. b, we have

$$\cos\theta_{max} = \frac{1}{\sqrt{1 + \sinh^2\left(\frac{49.05L}{F_H}\right)}} = \frac{1}{\cosh\left(\frac{49.05L}{F_H}\right)}$$

Thus,

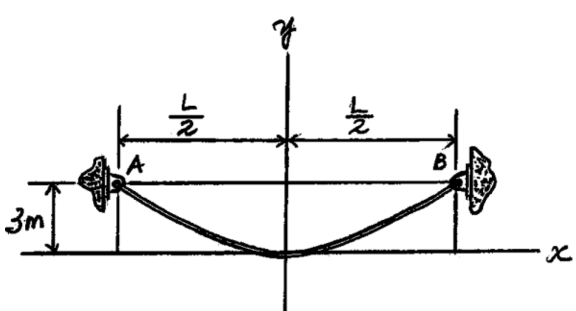
$$T_{max} = \frac{F_H}{\cos\theta_{max}}$$

$$1500 = F_H\cosh\left(\frac{49.05L}{F_H}\right) \qquad \textbf{(3)}$$

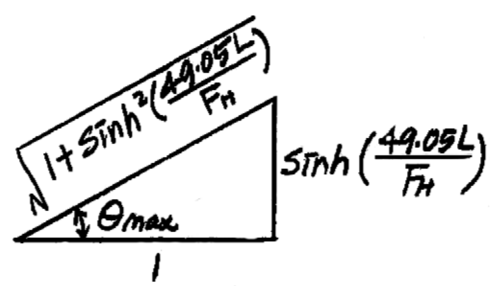
Solving Eqs. (2) and (3) yields

$L = 16.8$ m **Ans.**

$F_H = 1205.7$ N



(a)



(b)

7–117.

Show that the deflection curve of the cable discussed in Example 7–13 reduces to Eq. 4 in Example 7–12 when the *hyperbolic cosine function* is expanded in terms of a series and only the first two terms are retained. (The answer indicates that the *catenary* may be replaced by a parabola in the analysis of problems in which the sag is small. In this case, the cable weight is assumed to be uniformly distributed along the horizontal.)

SOLUTION

$$\cosh x = 1 + \frac{x^2}{21} + \cdots$$

Substituting into

$$y = \frac{F_H}{w_0}\left[\cosh\left(\frac{w_0}{F_H}x\right) - 1\right]$$

$$= \frac{F_H}{w_0}\left[1 + \frac{w_0^2 x^2}{2F_H^2} + \cdots - 1\right]$$

$$= \frac{w_0 x^2}{2F_H}$$

Using Eq. (3) in Example 7–12,

$$F_H = \frac{w_0 L^2}{8h}$$

We get $y = \dfrac{4h}{L^2}x^2$ **QED**

■**7–118.**

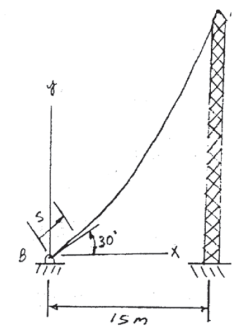
A cable has a weight of 5 lb/ft. If it can span 300 ft and has a sag of 15 ft, determine the length of the cable. The ends of the cable are supported at the same elevation.

SOLUTION

$$x = \int \frac{ds}{\left\{1 + \frac{1}{F_H^2}(w_0\, ds)^2\right\}^{\frac{1}{2}}}$$

Performing the integration yields:

$$x = \frac{F_H}{4.905}\left\{\sinh^{-1}\left[\frac{1}{F_H}(4.905s + C_1)\right] + C_2\right\} \qquad \textbf{(1)}$$



rom Eq. 7-13

$$\frac{dy}{dx} = \frac{1}{F_H}\int w_0\, ds$$

$$\frac{dy}{dx} = \frac{1}{F_H}(4.905s + C_1)$$

At $s = 0$; $\frac{dy}{dx} = \tan 30°$. Hence $C_1 = F_H \tan 30°$

$$\frac{dy}{dx} = \frac{4.905s}{F_H} + \tan 30° \qquad \textbf{(2)}$$

Applying boundary conditions at $x = 0$; $s = 0$ to Eq.(1) and using the result $C_1 = F_H \tan 30°$ yields $C_2 = \; -\sinh^{-1}(\tan 30°)$. Hence

$$x = \frac{F_H}{4.905}\left\{\sinh^{-1}\left[\frac{1}{F_H}(4.905s + F_H \tan 30°)\right] - \sinh^{-1}(\tan 30°)\right\} \qquad \textbf{(3)}$$

At $x = 15$ m; $s = 25$ m. From Eq.(3)

$$15 = \frac{F_H}{4.905}\left\{\sinh^{-1}\left[\frac{1}{F_H}(4.905(25) + \mathrm{F_H} \tan 30°)\right] - \sinh^{-1}(\tan 30°)\right\}$$

By trial and error $F_H = 73.94$ N

At point A, $s = 25$ m From Eq.(2)

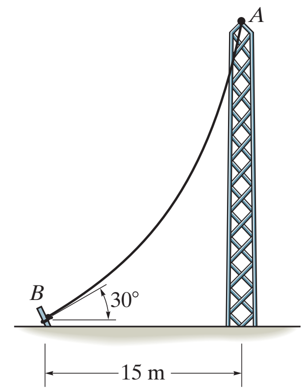
$$\tan\theta_A = \left.\frac{dy}{dx}\right|_{s=25\text{ m}} = \frac{4.905(25)}{73.94} + \tan 30° \qquad \theta_A = 65.90°$$

$$(F_v)_A = F_H \tan\theta_{\mathrm{A}} = 73.94 \tan 65.90° = 165 \text{ N} \qquad \textbf{Ans.}$$

$$(F_H)_A = F_H = 73.9 \text{ N} \qquad \textbf{Ans.}$$

■7–119.

The cable has a mass of 0.5 kg/m, and is 25 m long. Determine the vertical and horizontal components of force it exerts on the top of the tower.

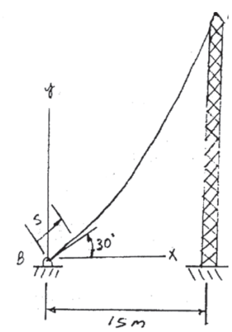


SOLUTION

$$x = \int \frac{ds}{\left\{1 + \frac{1}{F_H^2}(w_0\,ds)^2\right\}^{\frac{1}{2}}}$$

Performing the integration yields:

$$x = \frac{F_H}{4.905}\left\{\sinh^{-1}\left[\frac{1}{F_H}(4.905s + C_1)\right] + C_2\right\} \qquad \textbf{(1)}$$



rom Eq. 7-13

$$\frac{dy}{dx} = \frac{1}{F_H}\int w_0\,ds$$

$$\frac{dy}{dx} = \frac{1}{F_H}(4.905s + C_1)$$

At $s = 0$; $\frac{dy}{dx} = \tan 30°$. Hence $C_1 = F_H \tan 30°$

$$\frac{dy}{dx} = \frac{4.905s}{F_H} + \tan 30° \qquad \textbf{(2)}$$

Applying boundary conditions at $x = 0$; $s = 0$ to Eq.(1) and using the result $C_1 = F_H \tan 30°$ yields $C_2 = -\sinh^{-1}(\tan 30°)$. Hence

$$x = \frac{F_H}{4.905}\left\{\sinh^{-1}\left[\frac{1}{F_H}(4.905s + F_H \tan 30°)\right] - \sinh^{-1}(\tan 30°)\right\} \qquad \textbf{(3)}$$

At $x = 15$ m; $s = 25$ m. From Eq.(3)

$$15 = \frac{F_H}{4.905}\left\{\sinh^{-1}\left[\frac{1}{F_H}(4.905(25) + \mathrm{F_H} \tan 30°)\right] - \sinh^{-1}(\tan 30°)\right\}$$

By trial and error $F_H = 73.94$ N

At point A, $s = 25$ m From Eq.(2)

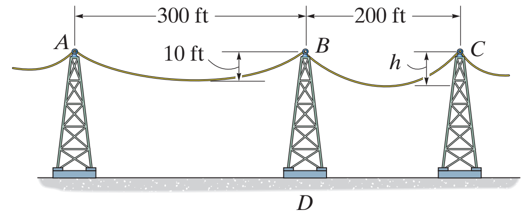
$$\tan\theta_A = \left.\frac{dy}{dx}\right|_{s=25\text{ m}} = \frac{4.905(25)}{73.94} + \tan 30° \qquad \theta_A = 65.90°$$

$$(F_v)_A = F_H \tan\theta_A = 73.94 \tan 65.90° = 165 \text{ N} \qquad \textbf{Ans.}$$

$$(F_H)_A = F_H = 73.9 \text{ N} \qquad \textbf{Ans.}$$

***7–120.**

The power transmission cable weighs 10 lb/ft. If the resultant horizontal force on tower BD is required to be zero, determine the sag h of cable BC.



SOLUTION

The origin of the x, y coordinate system is set at the lowest point of the cables. Here, $w_0 = 10$ lb/ft. Using Eq. 4 of Example 7–13,

$$y = \frac{F_H}{w_0}\left[\cosh\left(\frac{w_0}{F_H}x\right) - 1\right]$$

$$y = \frac{F_H}{10}\left[\cosh\left(\frac{10}{F_H}x\right) - 1\right] \text{ ft}$$

Applying the boundary condition of cable AB, $y = 10$ ft at $x = 150$ ft,

$$10 = \frac{(F_H)_{AB}}{10}\left[\cosh\left(\frac{10(150)}{(F_H)_{AB}}\right) - 1\right]$$

Solving by trial and error yields

$$(F_H)_{AB} = 11266.63 \text{ lb}$$

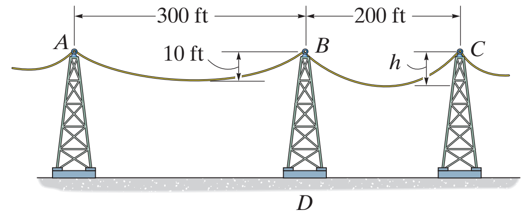
Since the resultant horizontal force at B is required to be zero, $(F_H)_{BC} = (F_H)_{AB} = 11266.62$ lb. Applying the boundary condition of cable BC $y = h$ at $x = -100$ ft to Eq. (1), we obtain

$$h = \frac{11266.62}{10}\left\{\cosh\left[\frac{10(-100)}{11266.62}\right] - 1\right\}$$

$$= 4.44 \text{ ft}$$ **Ans.**

7–121.

The power transmission cable weighs 10 lb/ft. If $h = 10$ ft, determine the resultant horizontal and vertical forces the cables exert on tower BD.



SOLUTION

The origin of the x, y coordinate system is set at the lowest point of the cables. Here, $w_0 = 10$ lb/ft. Using Eq. 4 of Example 7–13,

$$y = \frac{F_H}{w0}\left[\cosh\left(\frac{w_0}{F_H}x\right) - 1\right]$$

$$y = \frac{F_H}{10}\left[\cosh\left(\frac{10}{F_H}x\right) - 1\right] \text{ft}$$

Applying the boundary condition of cable AB, $y = 10$ ft at $x = 150$ ft,

$$10 = \frac{(F_H)_{AB}}{10}\left[\cosh\left(\frac{10(150)}{(F_H)_{AB}}\right) - 1\right]$$

Solving by trial and error yields

$$(F_H)_{AB} = 11266.63 \text{ lb}$$

Applying the boundary condition of cable BC, $y = 10$ ft at $x = -100$ ft to Eq. (2), we have

$$10 = \frac{(F_H)_{BC}}{10}\left[\cosh\left(\frac{10(100)}{(F_H)_{BC}}\right) - 1\right]$$

Solving by trial and error yields

$$(F_H)_{BC} = 5016.58 \text{ lb}$$

Thus, the resultant horizontal force at B is

$$(F_H)_R = (F_H)_{AB} - (F_H)_{BC} = 11266.63 - 5016.58 = 6250 \text{ lb} = 6.25 \text{ kip} \qquad \textbf{Ans.}$$

Using Eq. (1), $\tan(\theta_B)_{AB} = \sin h\left[\frac{10(150)}{11266.63}\right] = 0.13353$ and $\tan(\theta_B)_{BC} = \sin h\left[\frac{10(-100)}{5016.58}\right] = 0.20066$. Thus, the vertical force of cables AB and BC acting on point B are

$$(F_v)_{AB} = (F_H)_{AB}\tan(\theta_B)_{AB} = 11266.63(0.13353) = 1504.44 \text{ lb}$$

$$(F_v)_{BC} = (F_H)_{BC}\tan(\theta_B)_{BC} = 5016.58(0.20066) = 1006.64 \text{ lb}$$

The resultant vertical force at B is therefore

$$(F_v)_R = (F_v)_{AB} + (F_v)_{BC} = 1504.44 + 1006.64$$

$$= 2511.07 \text{ lb} = 2.51 \text{ kip} \qquad \textbf{Ans.}$$

7–122.

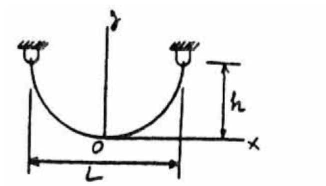
A uniform cord is suspended between two points having the same elevation. Determine the sag-to-span ratio so that the maximum tension in the cord equals the cord's total weight.

SOLUTION

From Example 7–15.

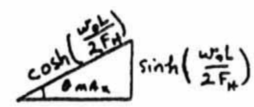
$$s = \frac{F_H}{w_0}\sinh\left(\frac{w_0}{F_H}x\right)$$

$$y = \frac{F_H}{w_0}\left[\cosh\left(\frac{w_0}{F_H}x\right) - 1\right]$$



At $x = \frac{L}{2}$,

$$\left.\frac{dy}{dx}\right|_{max} = \tan\theta_{max} = \sinh\left(\frac{w_0 L}{2F_H}\right)$$



$$\cos\theta_{max} = \frac{1}{\cosh\left(\frac{w_0 L}{2F_H}\right)}$$

$$T_{max} = \frac{F_H}{\cos\theta_{max}}$$

$$w_0(2s) = F_H\cosh\left(\frac{w_0 L}{2F_H}\right)$$

$$2F_H\sinh\left(\frac{w_0 L}{2F_H}\right) = F_H\cosh\left(\frac{w_0 L}{2F_H}\right)$$

$$\tanh\left(\frac{w_0 L}{2F_H}\right) = \frac{1}{2}$$

$$\frac{w_0 L}{2F_H} = \tanh^{-1}(0.5) = 0.5493$$

when $x = \frac{L}{2}$, $y = h$

$$h = \frac{F_H}{w_0}\left[\cosh\left(\frac{w_0}{F_H}x\right) - 1\right]$$

$$h = \frac{F_H}{w_0}\left\{\frac{1}{\sqrt{1 - \tanh^2\left(\frac{w_{0L}}{2F_H}\right)}} - 1\right\} = 0.1547\left(\frac{F_H}{w_0}\right)$$

$$\frac{0.1547\,L}{2h} = 0.5493$$

$$\frac{h}{L} = 0.141$$ **Ans.**

■ **7–123.**

A 50-ft cable is suspended between two points a distance of 15 ft apart and at the same elevation. If the minimum tension in the cable is 200 lb, determine the total weight of the cable and the maximum tension developed in the cable.

SOLUTION

$T_{min} = F_H = 200 \text{ lb}$

From Example 7–13:

$$s = \frac{F_H}{w_0}\sinh\left(\frac{w_0\,x}{F_H}\right)$$

$$\frac{50}{2} = \frac{200}{w_0}\sinh\left(\frac{w_0}{200}\left(\frac{15}{2}\right)\right)$$

Solving,

$$w_0 = 79.9 \text{ lb/ft}$$

$$\text{Total weight} = w_0\,l = 79.9\,(50) = 4.00 \text{ kip} \qquad \textbf{Ans.}$$

$$\left.\frac{dy}{dx}\right|_{max} = \tan\theta_{max} = \frac{w_0\,s}{F_H}$$

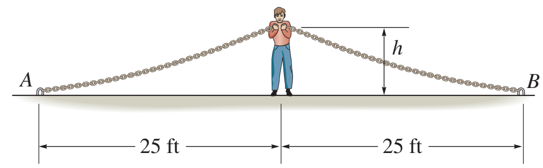
$$\theta_{max} = \tan^{-1}\left[\frac{79.9\,(25)}{200}\right] = 84.3^\circ$$

Then,

$$T_{max} = \frac{F_H}{\cos\theta_{max}} = \frac{200}{\cos 84.3^\circ} = 2.01 \text{ kip} \qquad \textbf{Ans.}$$

***7–124.**

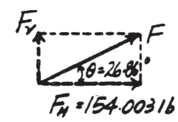
The man picks up the 52-ft chain and holds it just high enough so it is completely off the ground. The chain has points of attachment A and B that are 50 ft apart. If the chain has a weight of 3 lb/ft, and the man weighs 150 lb, determine the force he exerts on the ground. Also, how high h must he lift the chain? *Hint*: The slopes at A and B are zero.



SOLUTION

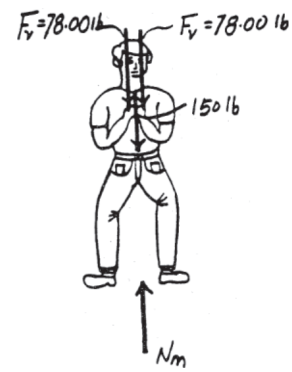
Deflection Curve of The Cable:

$$x = \int \frac{ds}{[1 + (1/F_H^2)(\int w_0\, ds)^2]^{\frac{1}{2}}} \qquad \text{where } w_0 = 3 \text{ lb/ft}$$



Performing the integration yields

$$x = \frac{F_H}{3}\left\{\sinh^{-1}\left[\frac{1}{F_H}(3s + C_1)\right] + C_2\right\} \tag{1}$$



From Eq. 7–14

$$\frac{dy}{dx} = \frac{1}{F_H}\int w_0\, ds = \frac{1}{F_H}(3s + C_1) \tag{2}$$

Boundary Conditions:

$$\frac{dy}{dx} = 0 \text{ at } s = 0. \text{ From Eq. (2)} \qquad 0 = \frac{1}{F_H}(0 + C_1) \qquad C_1 = 0$$

Then, Eq. (2) becomes

$$\frac{dy}{dx} = \tan\theta = \frac{3s}{F_H} \tag{3}$$

$s = 0$ at $x = 0$ and use the result $C_1 = 0$. From Eq. (1)

$$x = \frac{F_H}{3}\left\{\sinh^{-1}\left[\frac{1}{F_H}(0 + 0)\right] + C_2\right\} \qquad C_2 = 0$$

Rearranging Eq. (1), we have

$$s = \frac{F_H}{3}\sinh\left(\frac{3}{F_H}x\right) \tag{4}$$

Substituting Eq. (4) into (3) yields

$$\frac{dy}{dx} = \sinh\left(\frac{3}{F_H}x\right)$$

Performing the integration

$$y = \frac{F_H}{3}\cosh\left(\frac{3}{F_H}x\right) + C_3 \tag{5}$$

$$y = 0 \text{ at } x = 0. \text{ From Eq. (5)}\ 0 = \frac{F_H}{3}\cosh 0 + C_3, \text{thus, } C_3 = -\frac{F_H}{3}$$

***7–124. (continued)**

Then, Eq. (5) becomes

$$y = \frac{F_H}{3}\left[\cosh\left(\frac{3}{F_H}x\right) - 1\right] \qquad \textbf{(6)}$$

$s = 26$ ft at $x = 25$ ft. From Eq. (4)

$$26 = \frac{F_H}{3}\sinh\left[\frac{3}{F_H}(25)\right]$$

$$F_H = 154.003 \text{ lb}$$

By trial and error

$y = h$ at $x = 25$ ft. From Eq. (6)

$$h = \frac{154.003}{3}\left\{\cosh\left[\frac{3}{154.003}(25)\right] - 1\right\} = 6.21 \text{ ft} \qquad \textbf{Ans.}$$

From Eq. (3)

$$\left.\frac{dy}{dx}\right|_{s=26\text{ ft}} = \tan\theta = \frac{3(26)}{154.003} = 0.5065 \qquad \theta = 26.86°$$

The vertical force F_V that each chain exerts on the man is

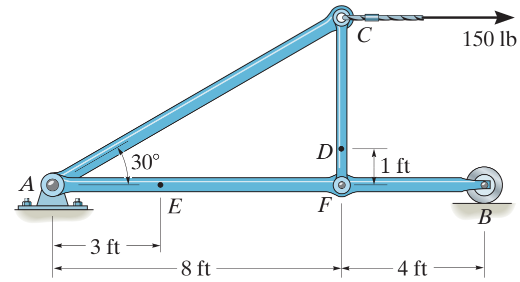
$$F_V = F_H \tan\theta = 154.003 \tan 26.86° = 78.00 \text{ lb}$$

Equation of Equilibrium: By considering the equilibrium of the man,

$$+\uparrow \Sigma F_y = 0;\ \ N_m - 150 - 2(78.00) = 0 \qquad N_m = 306 \text{ lb} \qquad \textbf{Ans.}$$

7–125.

Determine the internal normal force, shear force, and moment at points D and E of the frame.



SOLUTION

$\zeta + \Sigma M_A = 0; \quad F_{CD}(8) - 150(8 \tan 30°) = 0$

$$F_{CD} = 86.60 \text{ lb}$$

Since member CF is a two- force member

$$V_D = M_D = 0$$ **Ans.**

$$N_D = F_{CD} = 86.6 \text{ lb}$$ **Ans.**

$\zeta + \Sigma M_A = 0; \quad B_y(12) - 150(8 \tan 30°) = 0$

$$B_y = 57.735 \text{ lb}$$

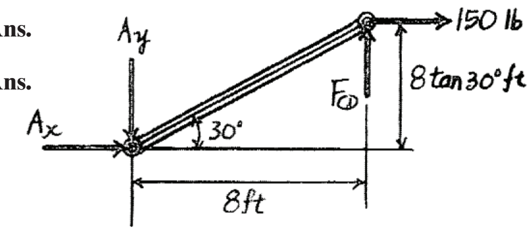
$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad N_E = 0$ **Ans.**

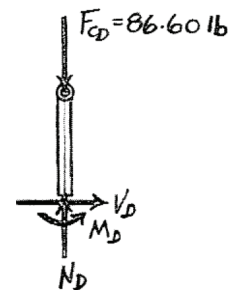
$+\uparrow \Sigma F_y = 0; \quad V_E + 57.735 - 86.60 = 0$

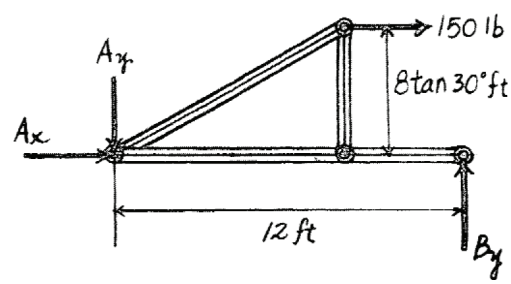
$$V_E = 28.9 \text{ lb}$$ **Ans.**

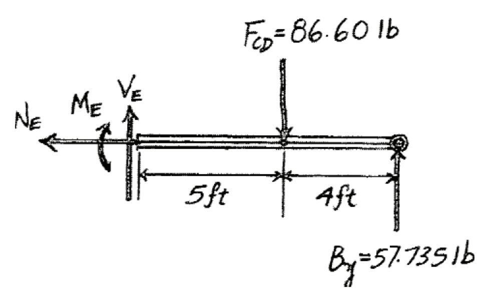
$\zeta + \Sigma M_E = 0; \quad 57.735(9) - 86.60(5) - M_E = 0$

$$M_E = 86.6 \text{ lb} \cdot \text{ft}$$ **Ans.**



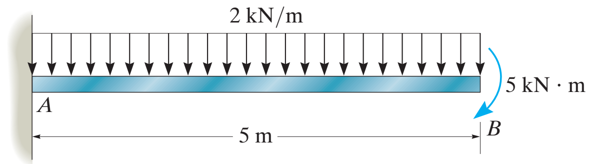






7–126.

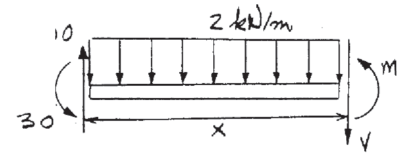
Draw the shear and moment diagrams for the beam.



SOLUTION

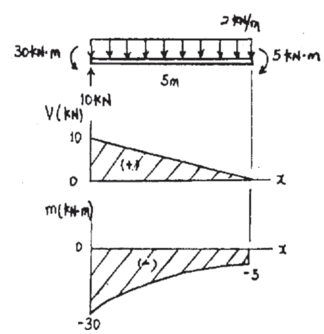
$+\uparrow \Sigma F_y = 0; \qquad -V + 10 - 2x = 0$

$V = 10 - 2x$ **Ans.**



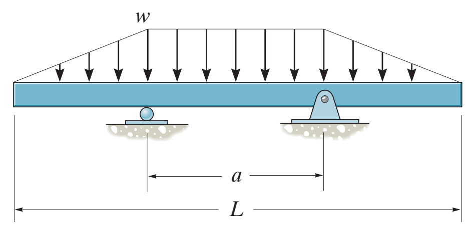
$\circlearrowleft + \Sigma M = 0; \qquad M + 30 - 10x + 2x\left(\frac{x}{2}\right) = 0$

$M = 10x - x^2 - 30$ **Ans.**



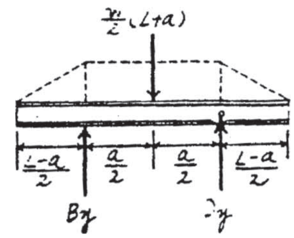
7–127.

Determine the distance a between the supports in terms of the beam's length L so that the moment in the *symmetric* beam is zero at the beam's center.



SOLUTION

Support Reactions: From FBD (a),



$$\zeta + \Sigma M_C = 0; \qquad \frac{w}{2}(L+a)\left(\frac{a}{2}\right) - B_y\,(a) = 0 \qquad B_y = \frac{w}{4}\,(L+a)$$

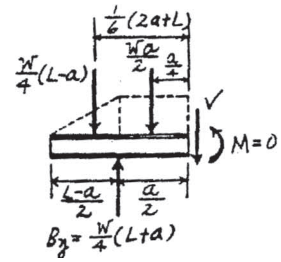
Free body Diagram: The FBD for segment AC sectioned through point C is drawn.

Internal Forces: This problem requires $M_C = 0$. Summing moments about point C [FBD (b)], we have

$$\zeta + \Sigma M_C = 0; \qquad \frac{wa}{2}\left(\frac{a}{4}\right) + \frac{w}{4}\,(L-a)\left[\frac{1}{6}\,(2a+L)\right] - \frac{w}{4}(L+a)\left(\frac{a}{2}\right) = 0$$

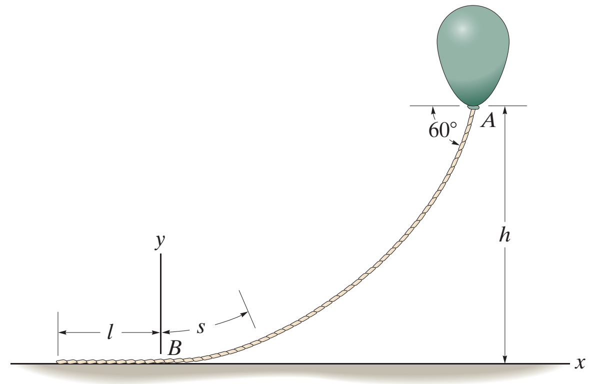
$$2a^2 + 2aL - L^2 = 0$$

$$a = 0.366L \qquad \textbf{Ans.}$$



***7–128.**

The balloon is held in place using a 400-ft cord that weighs 0.8 lb/ft and makes a 60° angle with the horizontal. If the tension in the cord at point A is 150 lb, determine the length of the cord, l, that is lying on the ground and the height h. *Hint*: Establish the coordinate system at B as shown.



SOLUTION

Deflection Curve of The Cable:

$$x = \int \frac{ds}{\left[1 + \left(1/F_H^2\right)\left(\int w_0\, ds\right)^2\right]^{\frac{1}{2}}} \qquad \text{where } w_0 = 0.8 \text{ lb/ft}$$

Performing the integration yields

$$x = \frac{F_H}{0.8}\left\{\sinh^{-1}\left[\frac{1}{F_H}(0.8s + C_1)\right] + C_2\right\} \qquad \textbf{(1)}$$

From Eq. 7–14

$$\frac{dy}{dx} = \frac{1}{F_H}\int w_0\, ds = \frac{1}{F_H}(0.8s + C_1) \qquad \textbf{(2)}$$

Boundary Conditions:

$\frac{dy}{dx} = 0$ at $s = 0$. From Eq. (2)

$$0 = \frac{1}{F_H}(0 + C_1) \qquad C_1 = 0$$

Then, Eq. (2) becomes

$$\frac{dy}{dx} = \tan\theta = \frac{0.8s}{F_H} \qquad \textbf{(3)}$$

$s = 0$ at $x = 0$ and use the result $C_1 = 0$. From Eq. (1)

$$x = \frac{F_H}{0.8}\left\{\sinh^{-1}\left[\frac{1}{F_H}(0 + 0)\right] + C_2\right\} \qquad C_2 = 0$$

Rearranging Eq. (1), we have

$$s = \frac{F_H}{0.8}\sinh\left(\frac{0.8}{F_H}x\right) \qquad \textbf{(4)}$$

Substituting Eq. (4) into (3) yields

$$\frac{dy}{dx} = \sinh\left(\frac{0.8}{F_H}x\right)$$

Performing the integration

$$y = \frac{F_H}{0.8}\cosh\left(\frac{0.8}{F_H}x\right) + C_3 \qquad \textbf{(5)}$$

$y = 0$ at $x = 0$. From Eq. (5) $0 = \frac{F_H}{0.8}\cosh 0 + C_3$, thus, $C_3 = -\frac{F_H}{0.8}$

Then, Eq. (5) becomes

$$y = \frac{F_H}{0.8}\left[\cosh\left(\frac{0.8}{F_H}x\right) - 1\right] \qquad \textbf{(6)}$$

The tension developed at the end of the cord is $T = 150$ lb and $\theta = 60°$. Thus

***7–128. (continued)**

$$T = \frac{F_H}{\cos\theta} \qquad 150 = \frac{F_H}{\cos 60^\circ} \qquad F_H = 75.0 \text{ lb}$$

From Eq. (3)

$$\frac{dy}{dx} = \tan 60^\circ = \frac{0.8s}{75} \qquad s = 162.38 \text{ ft}$$

Thus,

$$l = 400 - 162.38 = 238 \text{ ft} \qquad \textbf{Ans.}$$

Substituting $s = 162.38$ ft into Eq. (4).

$$162.38 = \frac{75}{0.8}\sinh\left(\frac{0.8}{75}x\right)$$

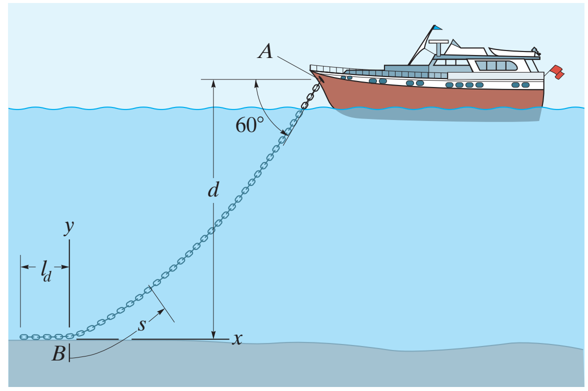
$$x = 123.46 \text{ ft}$$

$y = h$ at $x = 123.46$ ft. From Eq. (6)

$$h = \frac{75.0}{0.8}\left[\cosh\left[\frac{0.8}{75.0}(123.46)\right] - 1\right] = 93.75 \text{ ft} \qquad \textbf{Ans.}$$

7–129.

The yacht is anchored with a chain that has a total length of 40 m and a mass per unit length of 18 kg/m, and the tension in the chain at A is 7 kN. Determine the length of chain l_d which is lying at the bottom of the sea. What is the distance d? Assume that buoyancy effects of the water on the chain are negligible. *Hint:* Establish the origin of the coordinate system at B as shown in order to find the chain length BA.



SOLUTION

Component of force at A is

$$F_H = T\cos\theta = 7000\cos 60^\circ = 3500 \text{ N}$$

From Eq. (1) of Example 7 - 13

$$x = \frac{3500}{18\,(9.81)}\left(\sinh^{-1}\left[\frac{1}{3500}(18)(9.81)s + C_1\right] + C_2\right)$$

Since $\dfrac{dy}{dx} = 0, s = 0$, then

$$\frac{dy}{dx} = \frac{1}{F_H}(w_0\, s + C_1); \qquad C_1 = 0$$

Also $x = 0, s = 0$, so that $C_2 = 0$ and the above equation becomes

$$x = 19.82\left(\sinh^{-1}\left(\frac{s}{19.82}\right)\right) \qquad \textbf{(1)}$$

or,

$$s = 19.82\left(\sinh\left(\frac{x}{19.82}\right)\right) \qquad \textbf{(2)}$$

From Example 7 - 13

$$\frac{dy}{dx} = \frac{w_0\, s}{F_H} = \frac{18\,(9.81)}{3500}s = \frac{s}{19.82} \qquad \textbf{(3)}$$

Substituting Eq. (2) into Eq. (3). Integrating.

$$\frac{dy}{dx} = \sinh\left(\frac{x}{19.82}\right) \qquad \text{y} = 19.82\cosh\left(\frac{x}{19.82}\right) + C_3$$

Since $x = 0, y = 0$, then $C_3 = -19.82$

Thus,

$$y = 19.82\left(\cosh\left(\frac{x}{19.82}\right) - 1\right) \qquad \textbf{(4)}$$

Slope of the cable at point A is

$$\frac{dy}{dx} = \tan 60^\circ = 1.732$$

Using Eq. (3),

$$s_{AB} = 19.82\,(1.732) = 34.33 \text{ m}$$

Length of chain on the ground is thus

$$l_d = 40 - 34.33 = 5.67 \text{ m} \qquad \textbf{Ans.}$$

From Eq. (1), with $s = 34.33$ m

$$x = 19.82\left(\sinh^{-1}\left(\frac{34.33}{19.82}\right)\right) = 26.10 \text{ m}$$

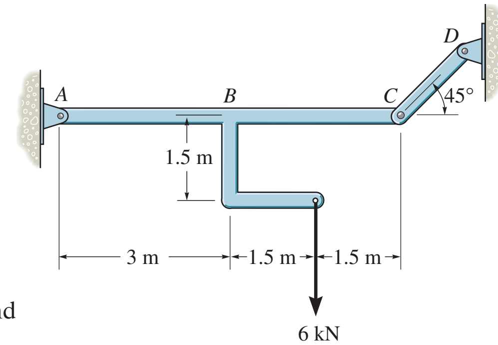
Using Eq. (4),

$$y = 19.82\left(\cosh\left(\frac{26.10}{19.82}\right) - 1\right)$$

$$d = y = 19.8 \text{ m} \qquad \textbf{Ans.}$$

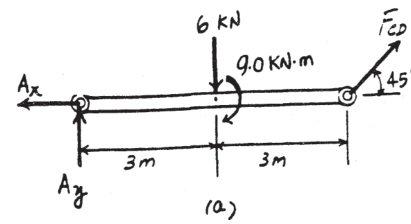
7–130.

Draw the shear and moment diagrams for the beam ABC.



SOLUTION

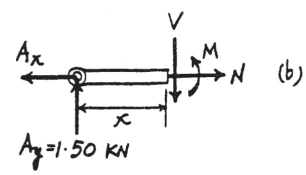
Support Reactions: The 6 kN load can be replacde by an equivalent force and couple moment at B as shown on FBD (a).



$\zeta + \Sigma M_A = 0;$ $\quad F_{CD} \sin 45°(6) - 6(3) - 9.00 = 0 \quad F_{CD} = 6.364 \text{ kN}$

$+\uparrow \Sigma F_y = 0;$ $\quad A_y + 6.364 \sin 45° - 6 = 0 \quad A_y = 1.50 \text{ kN}$

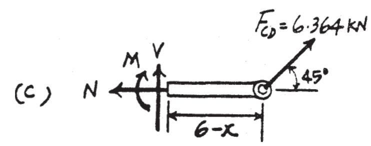
Shear and Moment Functions: For $\mathbf{0 \leq x < 3\ m}$ [FBD (b)],



$+\uparrow \Sigma F_y = 0;$ $\quad 1.50 - V = 0 \quad V = 1.50 \text{ kN}$ **Ans.**

$\zeta + \Sigma M = 0;$ $\quad M - 1.50x = 0 \quad M = \{1.50x\} \text{ kN} \cdot \text{m}$ **Ans.**

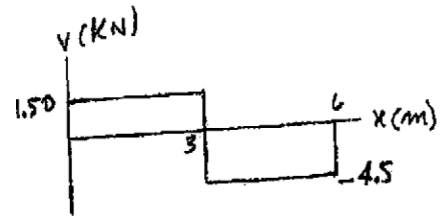
For $\mathbf{3\ m < x \leq 6\ m}$ [FBD (c)],

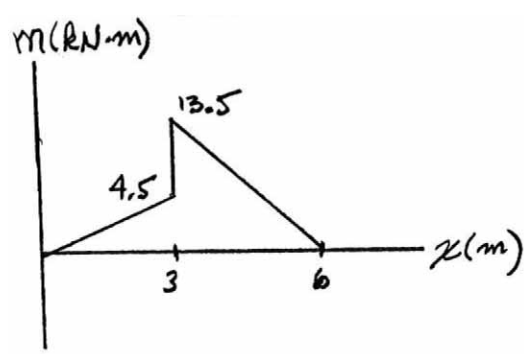


$+\uparrow \Sigma F_y = 0;$ $\quad V + 6.364 \sin 45° = 0 \quad V = -4.50 \text{ kN}$ **Ans.**

$\zeta + \Sigma M = 0;$ $\quad 6.364 \sin 45°(6 - x) - M = 0$

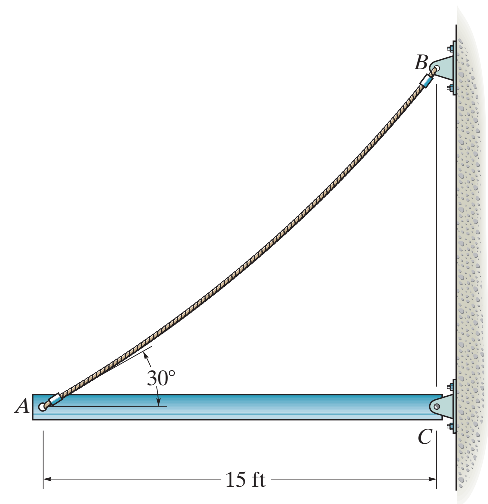
$$M = \{27.0 - 4.50x\} \text{ kN} \cdot \text{m}$$ **Ans.**





7–131.

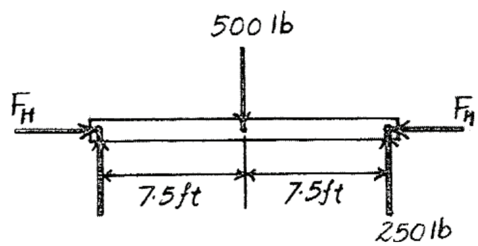
The uniform beam weighs 500 lb and is held in the horizontal position by means of cable AB, which has a weight of 5 lb/ft. If the slope of the cable at A is 30°, determine the length of the cable.



SOLUTION

$$T = \frac{250}{\sin 30^\circ} = 500 \text{ lb}$$

$$F_H = 500 \cos 30^\circ = 433.0 \text{ lb}$$



From Example 7 - 13

$$\frac{dy}{dx} = \frac{1}{F_H}(w_0\, s + C_1)$$

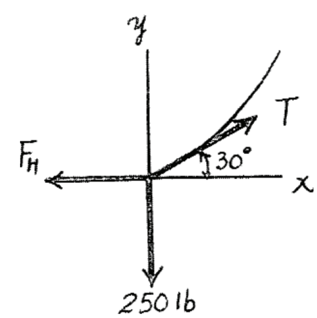
At $s = 0, \dfrac{dy}{dx} = \tan 30^\circ = 0.577$

$$\therefore C_1 = 433.0\,(0.577) = 250$$

$$x = \frac{F_H}{w_0}\left\{\sinh^{-1}\left[\frac{1}{F_H}(w_0 s + C_1)\right] + C_2\right\}$$

$$= \frac{433.0}{5}\left\{\sinh^{-1}\left[\frac{1}{433.0}(5s + 250)\right] + C_2\right\}$$

$$s = 0 \text{ at } x = 0, \qquad C_2 = -0.5493$$



Thus,

$$x = 86.6\left\{\sinh^{-1}\left[\frac{1}{433.0}(5s + 250)\right] - 0.5493\right\}$$

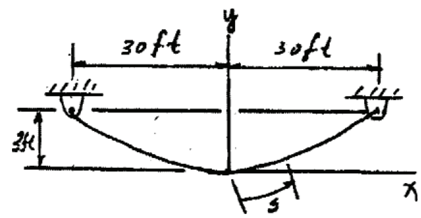
When $x = 15$ ft.

$$s = 18.2 \text{ ft}$$ **Ans.**

***7–132.**

A chain is suspended between points at the same elevation and spaced a distance of 60 ft apart. If it has a weight per unit length of 0.5 lb/ft and the sag is 3 ft, determine the maximum tension in the chain.

SOLUTION



$$x = \int \frac{ds}{\left\{1 + \frac{1}{F_H^2}\int (w_0\,ds)^2\right\}^{\frac{1}{2}}}$$

Performing the integration yields:

$$x = \frac{F_H}{0.5}\left\{\sin h^{-1}\left[\frac{1}{F_H}(0.5s + C_1)\right] + C_2\right\} \qquad \textbf{(1)}$$

From Eq. 7-14

$$\frac{dy}{dx} = \frac{1}{F_H}\int w_0\,ds$$

$$\frac{dy}{dx} = \frac{1}{F_H}(0.5s + C_1)$$

At $s = 0;$ $\quad \frac{dy}{dx} = 0 \qquad$ hence $C_1 = 0$

$$\frac{dy}{dx} = \tan\theta = \frac{0.5s}{F_H} \qquad \textbf{(2)}$$

Applying boundary conditions at $x = 0; s = 0$ to Eq. (1) and using the result $C_1 = 0$ yields $C_2 = 0$. Hence

$$s = \frac{F_H}{0.5}\sinh\left(\frac{0.5}{F_H}x\right) \qquad \textbf{(3)}$$

Substituting Eq. (3) into (2) yields:

$$\frac{dy}{dx} = \sinh\left(\frac{0.5x}{F_H}\right) \qquad \textbf{(4)}$$

Performing the integration

$$y = \frac{F_H}{0.5}\cosh\left(\frac{0.5}{F_H}x\right) + C_3$$

Applying boundary conditions at $x = 0;\ y = 0$ yields $C_3 = -\frac{F_H}{0.5}$. Therefore $y = \frac{F_H}{0.5}\left[\cosh\left(\frac{0.5}{F_H}x\right) - 1\right]$

At $x = 30$ ft; $\quad y = 3$ ft; $\qquad 3 = \frac{F_H}{0.5}\left[\cosh\left(\frac{0.5}{F_H}(30)\right) - 1\right]$

By trial and error $F_H = 75.25$ lb

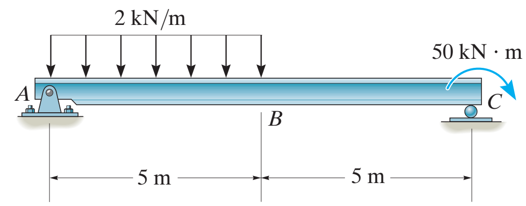
At $x = 30$ ft; $\quad \theta = \theta_{max}$. From Eq. (4)

$$\tan\theta_{max} = \left.\frac{dy}{dx}\right|_{x=30\text{ ft}} = \sinh\left(\frac{0.5(30)}{75.25}\right) \qquad \theta_{max} = 11.346°$$

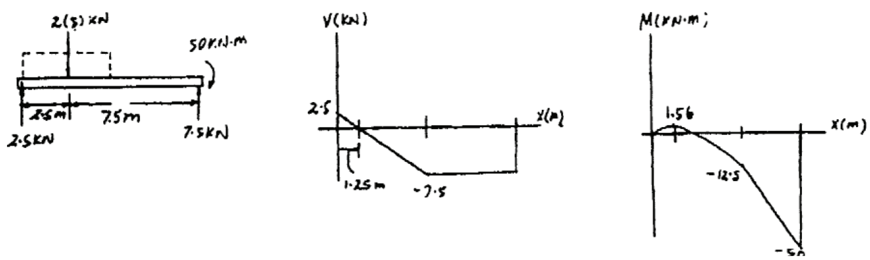
$$T_{max} = \frac{F_H}{\cos\theta_{max}} = \frac{75.25}{\cos 11.346°} = 76.7\text{ lb} \qquad \textbf{Ans.}$$

7–133.

Draw the shear and moment diagrams for the beam.

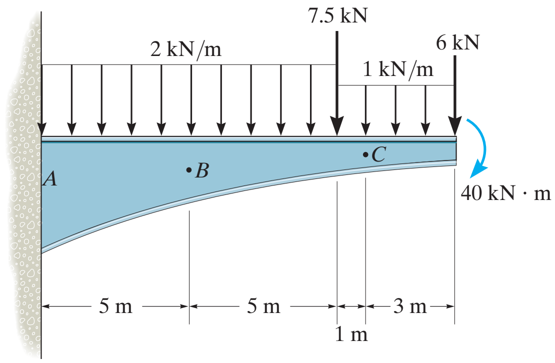


SOLUTION



7–134.

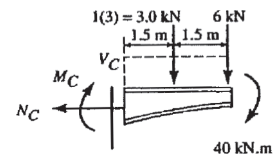
Determine the normal force, shear force, and moment at points *B* and *C* of the beam.



SOLUTION

Free body Diagram: The support reactions need not be computed for this case.

Internal Forces: Applying the equations of equilibrium to segment *DC* [FBD (a)], we have



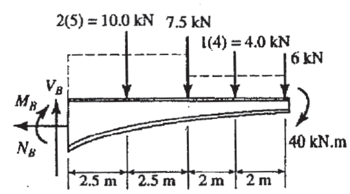
$\xrightarrow{+} \Sigma F_x = 0; \qquad N_C = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0; \qquad V_C - 3.00 - 6 = 0 \quad V_C = 9.00 \text{ kN}$ **Ans.**

$\circlearrowleft + \Sigma M_C = 0; \qquad -M_C - 3.00(1.5) - 6(3) - 40 = 0$

$M_C = -62.5 \text{ kN} \cdot \text{m}$ **Ans.**

Applying the equations of equilibrium to segment *DB* [FBD (b)], we have



$\xrightarrow{+} \Sigma F_x = 0; \qquad N_B = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0; \qquad V_B - 10.0 - 7.5 - 4.00 - 6 = 0$

$V_B = 27.5 \text{ kN}$ **Ans.**

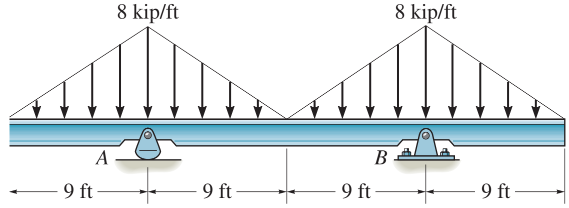
$\circlearrowleft + \Sigma M_B = 0; \qquad -M_B - 10.0(2.5) - 7.5(5)$

$- 4.00(7) - 6(9) - 40 = 0$

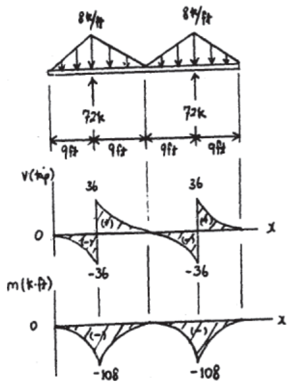
$M_B = -184.5 \text{ kN} \cdot \text{m}$ **Ans.**

7–135.

Draw the shear and moment diagrams for the beam.

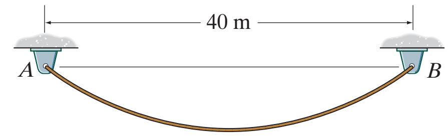


SOLUTION



***7–136.**

If the 45-m-long cable has a mass per unit length of 5 kg/m, determine the equation of the catenary curve of the cable and the maximum tension developed in the cable.



SOLUTION

As shown in Fig. *a*, the orgin of the *x, y* coordinate system is set at the lowest point of the cable. Here, $w(s) = 5(9.81)\ \text{N/m} = 49.05\ \text{N/m}$.

$$\frac{d^2y}{dx^2} = \frac{49.05}{F_H}\sqrt{1 + \left(\frac{dy}{dx}\right)^2}$$

Set $u = \dfrac{dy}{dx}$, then $\dfrac{du}{dx} = \dfrac{d^2y}{dx^2}$, then

$$\frac{du}{\sqrt{1 + u^2}} = \frac{49.05}{F_H}dx$$

Integrating,

$$\ln\left(u + \sqrt{1 + u^2}\right) = \frac{49.05}{F_H}x + C_1$$

Applying the boundary condition $u = \dfrac{dy}{dx} = 0$ at $x = 0$ results in $C_1 = 0$. Thus,

$$\ln\left(u + \sqrt{1 + u^2}\right) = \frac{49.05}{F_H}x$$

$$u + \sqrt{1 + u^2} = e^{\frac{49.05}{F_H}x}$$

$$\frac{dy}{dx} = u = \frac{e^{\frac{49.05}{F_H}x} - e^{-\frac{49.05}{F_H}x}}{2}$$

Since $\sinh x = \dfrac{e^x - e^{-x}}{2}$, then

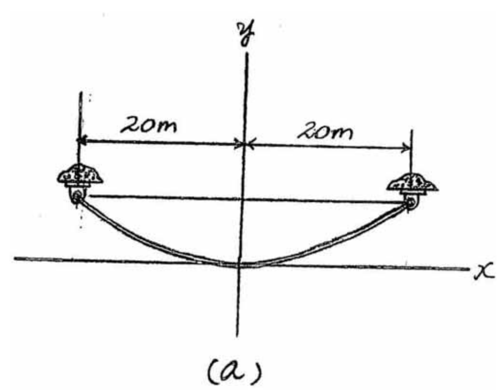
$$\frac{dy}{dx} = \sinh\frac{49.05}{F_H}x$$

Integrating,

$$y = \frac{F_H}{49.05}\cosh\left(\frac{49.05}{F_H}x\right) + C_2$$

Applying the boundary equation $y = 0$ at $x = 0$ results in $C_2 = -\dfrac{F_H}{49.05}$. Thus,

$$y = \frac{F_H}{49.05}\left[\cosh\left(\frac{49.05}{F_H}x\right) - 1\right]\ \text{m}$$



If we write the force equation of equilibrium along the *x* and *y* axes by referring to the free-body diagram shown in Fig. *b*,

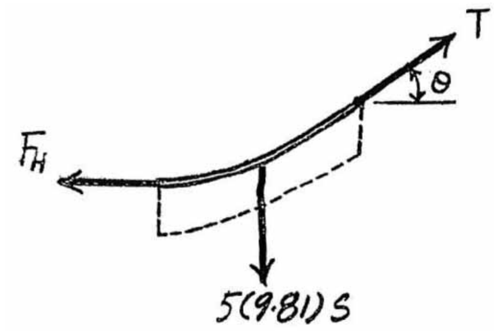
$\xrightarrow{+} \Sigma F_x = 0; \qquad T\cos\theta - F_H = 0$

$+\uparrow \Sigma F_y = 0; \qquad T\sin\theta - 5(9.81)s = 0$

***7–136. (continued)**

Eliminating T,

$$\frac{dy}{dx} = \tan\theta = \frac{49.05s}{F_H} \qquad \textbf{(3)}$$



(b)

Equating Eqs. (1) and (3) yields

$$\frac{49.05s}{F_H} = \sinh\left(\frac{49.05}{F_H}x\right)$$

$$s = \frac{F_H}{49.05} = \sinh\left(\frac{49.05}{F_H}\right)$$

Thus, the length of the cable is

$$L = 45 = 2\left\{\frac{F_H}{49.05}\sinh\left(\frac{49.05}{F_H}(20)\right)\right\}$$

Solving by trial and error,

$$F_H = 1153.41 \text{ N}$$

Substituting this result into Eq. (2),

$$y = 23.5\,[\cosh 0.0425x - 1] \text{ m} \qquad \textbf{Ans.}$$

The maximum tension occurs at either points A or B where the cable makes the greatest angle with the horizontal. Here

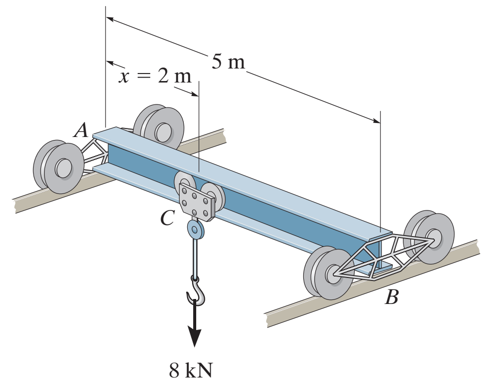
$$\theta_{\max} = \tan^{-1}\left(\frac{dy}{dx}\bigg|_{x=20\text{m}}\right) = \tan^{-1}\left\{\sinh\left(\frac{49.05}{F_H}(20)\right)\right\} = 43.74^\circ$$

Thus,

$$T_{\max} = \frac{F_H}{\cos\theta_{\max}} = \frac{1153.41}{\cos 43.74^\circ} = 1596.36 \text{ N} = 1.60 \text{ kN} \qquad \textbf{Ans.}$$

7–137.

The traveling crane consists of a 5-m-long beam having a uniform mass per unit length of 20 kg/m. The chain hoist and its supported load exert a force of 8 kN on the beam when $x = 2$ m. Draw the shear and moment diagrams for the beam. The guide wheels at the ends A and B exert only vertical reactions on the beam. Neglect the size of the trolley at C.

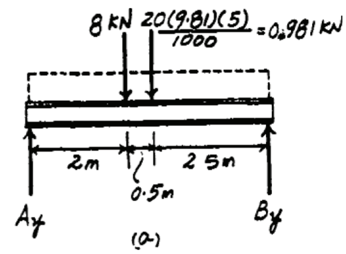


SOLUTION

Support Reactions: From FBD (a),

$$\circlearrowleft + \Sigma M_A = 0; \qquad B_y(5) - 8(2) - 0.981(2.5) = 0 \qquad B_y = 3.6905 \text{ kN}$$

$$+\uparrow \Sigma F_y = 0; \qquad A_y + 3.6905 - 8 - 0.981 = 0 \qquad A_y = 5.2905 \text{ kN}$$



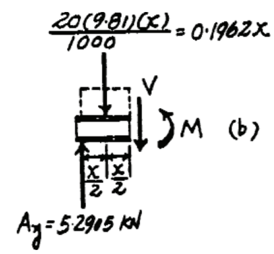
Shear and Moment Functions: For $0 \leq x < 2$ m [FBD (b)],

$$+\uparrow \Sigma F_y = 0; \qquad 5.2905 - 0.1962x - V = 0$$

$$V = \{5.29 - 0.196x\} \text{ kN} \qquad \textbf{Ans.}$$

$$\circlearrowleft + \Sigma M = 0; \qquad M + 0.1962x\left(\frac{x}{2}\right) - 5.2905x = 0$$

$$M = \{5.29x - 0.0981x^2\} \text{ kN} \cdot \text{m} \qquad \textbf{Ans.}$$



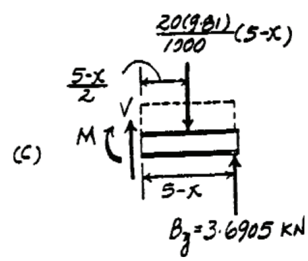
For $2 \text{ m} < x \leq 5$ m [FBD (c)],

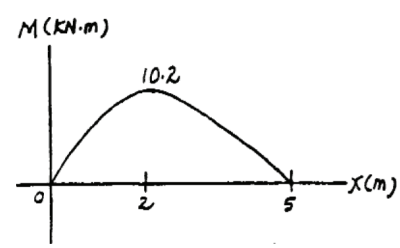
$$+\uparrow \Sigma F_y = 0; \qquad V + 3.6905 - \frac{20(9.81)}{1000}(5 - x) = 0$$

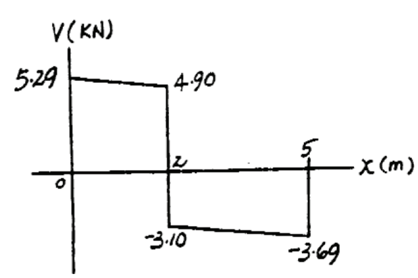
$$V = \{-0.196x - 2.71\} \text{ kN} \qquad \textbf{Ans.}$$

$$\circlearrowleft + \Sigma M = 0; \qquad 3.6905(5 - x) - \frac{20(9.81)}{1000}(5 - x)\left(\frac{5 - x}{2}\right) - M = 0$$

$$M = \{16.0 - 2.71x - 0.0981x^2\} \text{ kN} \cdot \text{m} \qquad \textbf{Ans.}$$

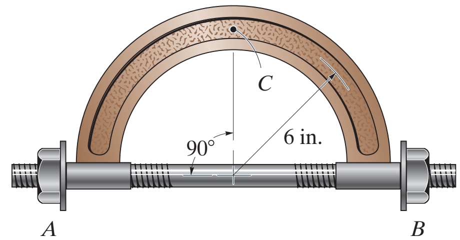






7–138.

The bolt shank is subjected to a tension of 80 lb. Determine the internal normal force, shear force, and moment at point C.

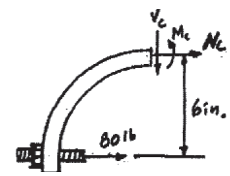


SOLUTION

$\xrightarrow{+} \Sigma F_x = 0; \qquad N_C + 80 = 0 \qquad N_C = -80 \text{ lb}$ **Ans.**

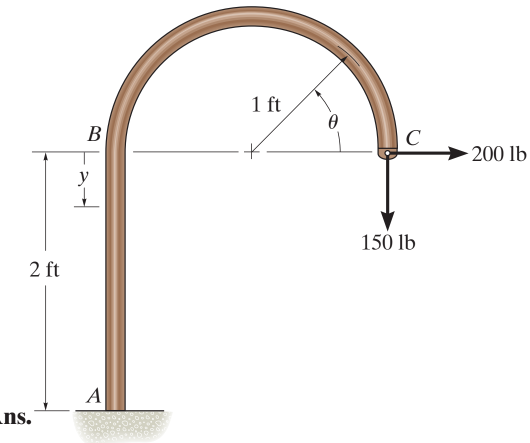
$+\uparrow \Sigma F_y = 0; \qquad V_C = 0$ **Ans.**

$\circlearrowleft + \Sigma M_C = 0; \qquad M_C + 80(6) = 0 \qquad M_C = -480 \text{lb} \cdot \text{in.}$ **Ans.**



7–139.

Determine the internal normal force, shear force, and the moment as a function of $0° \leq \theta \leq 180°$ and $0 \leq y \leq 2$ ft for the member loaded as shown.



SOLUTION

For $0° \leq \theta \leq 180°$:

$+\nearrow \Sigma F_x = 0; \quad V + 200 \cos\theta - 150 \sin\theta = 0$

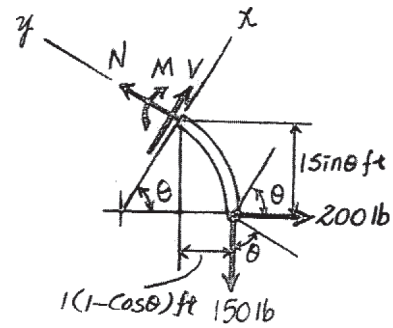
$V = 150 \sin\theta - 200 \cos\theta$ **Ans.**

$+\nwarrow \Sigma F_y = 0; \quad N - 200 \sin\theta - 150 \cos\theta = 0$

$N = 150 \cos\theta + 200 \sin\theta$ **Ans.**

$\circlearrowleft + \Sigma M = 0; \quad -M - 150(1)(1 - \cos\theta) + 200(1)\sin\theta = 0$

$M = 150 \cos\theta + 200 \sin\theta - 150$ **Ans.**



At section B, $\theta = 180°$, thus

$V_B = 200$ lb

$N_B = -150$ lb

$M_B = -300$ lb · ft

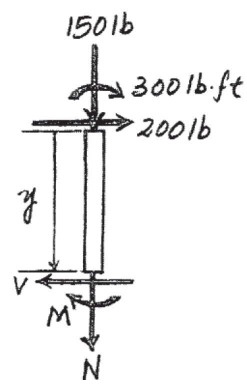
For $0 \leq y \leq 2$ ft:

$\xrightarrow{+} \Sigma F_x = 0; \quad V = 200$ lb **Ans.**

$+\uparrow \Sigma F_y = 0; \quad N = -150$ lb **Ans.**

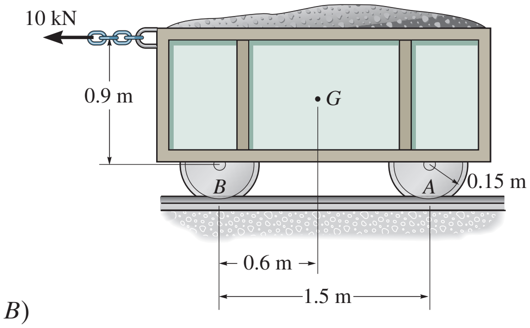
$\circlearrowleft + \Sigma M = 0; \quad -M - 300 - 200\,y = 0$

$M = -300 - 200\,y$ **Ans.**



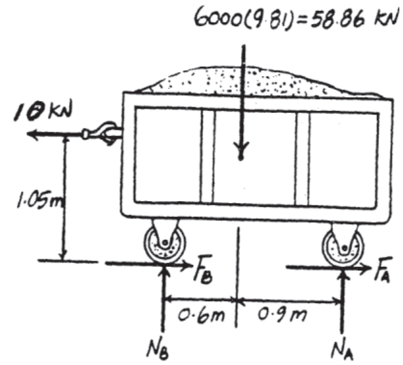
8–1.

The mine car and its contents have a total mass of 6 Mg and a center of gravity at G. If the coefficient of static friction between the wheels and the tracks is $\mu_s = 0.4$ when the wheels are locked, find the normal force acting on the front wheels at B and the rear wheels at A when the brakes at both A and B are locked. Does the car move?



SOLUTION

Equations of Equilibrium: The normal reactions acting on the wheels at (A and B) are independent as to whether the wheels are locked or not. Hence, the normal reactions acting on the wheels are the same for both cases.



$\zeta + \Sigma M_B = 0; \qquad N_A\,(1.5) + 10(1.05) - 58.86(0.6) = 0$

$$N_A = 16.544 \text{ kN} = 16.5 \text{ kN} \qquad \textbf{Ans.}$$

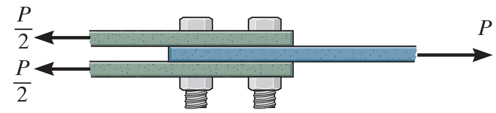
$+\uparrow \Sigma F_y = 0; \qquad N_B + 16.544 - 58.86 = 0$

$$N_B = 42.316 \text{ kN} = 42.3 \text{ kN} \qquad \textbf{Ans.}$$

When both wheels at A and B are locked, then $(F_A)_{max} = \mu_s N_A = 0.4(16.544) = 6.6176 \text{ kN}$ and $(F_B)_{max} = \mu_s N_B = 0.4(42.316) = 16.9264 \text{ kN}$. Since $(F_A)_{max} + (F_B)_{max} = 23.544 \text{ kN} > 10 \text{ kN}$, the wheels do not slip. Thus, **the mine car does not move**. **Ans.**

8–2.

Determine the maximum force P the connection can support so that no slipping occurs between the plates. There are four bolts used for the connection and each is tightened so that it is subjected to a tension of 4 kN. The coefficient of static friction between the plates is $\mu_s = 0.4$.

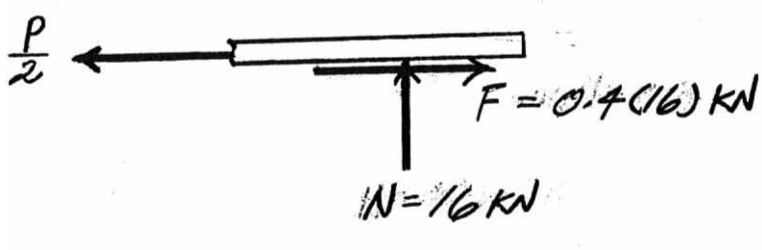


SOLUTION

Free-Body Diagram: The normal reaction acting on the contacting surface is equal to the sum total tension of the bolts. Thus, $N = 4(4)\text{ kN} = 16\text{ kN}$. When the plate is on the verge of slipping, the magnitude of the friction force acting on each contact surface can be computed using the friction formula $F = \mu_s N = 0.4(16)\text{ kN}$. As indicated on the free-body diagram of the upper plate, **F** acts to the right since the plate has a tendency to move to the left.

Equations of Equilibrium:

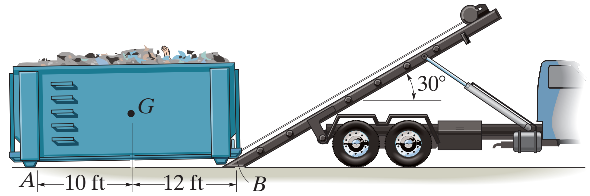
$\xrightarrow{+} \Sigma F_x = 0; \qquad 0.4(16) - \frac{P}{2} = 0 \qquad p = 12.8\text{ kN}$ **Ans.**



(a)

8–3.

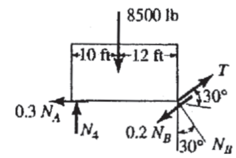
The winch on the truck is used to hoist the garbage bin onto the bed of the truck. If the loaded bin has a weight of 8500 lb and center of gravity at G, determine the force in the cable needed to begin the lift. The coefficients of static friction at A and B are $\mu_A = 0.3$ and $\mu_B = 0.2$, respectively. Neglect the height of the support at A.



SOLUTION

$\zeta + \Sigma M_B = 0;\qquad 8500(12) - N_A(22) = 0$

$N_A = 4636.364 \text{ lb}$



$\overset{+}{\rightarrow} \Sigma F_x = 0;\qquad T \cos 30°$

$- 0.2N_B \cos 30° - N_B \sin 30° - 0.3(4636.364) = 0$

$T(0.86603) - 0.67321\, N_B = 1390.91$

$+\uparrow \Sigma F_y = 0;\qquad 4636.364 - 8500 + T \sin 30° + N_B \cos 30°$

$- 0.2N_B \sin 30° = 0$

$T(0.5) + 0.766025\, N_B = 3863.636$

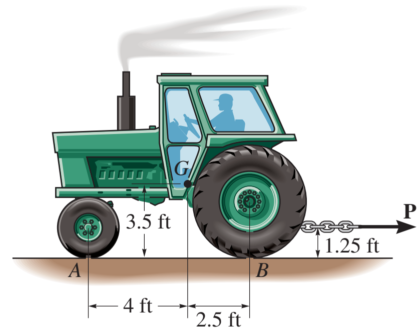
Solving:

$T = 3666.5 \text{ lb} = 3.67 \text{ kip}$ **Ans.**

$N_B = 2650.6 \text{ lb}$

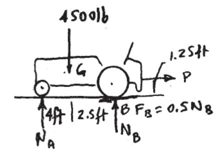
***8–4.**

The tractor has a weight of 4500 lb with center of gravity at G. The driving traction is developed at the rear wheels B, while the front wheels at A are free to roll. If the coefficient of static friction between the wheels at B and the ground is $\mu_s = 0.5$, determine if it is possible to pull at $P = 1200$ lb without causing the wheels at B to slip or the front wheels at A to lift off the ground.



SOLUTION

Slipping:



$\zeta + \Sigma M_A = 0; \qquad -4500(4) - P(1.25) + N_B(6.5) = 0$

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad P = 0.5\, N_B$

$P = 1531.9$ lb

$N_B = 3063.8$ lb

Tipping $(N_A = 0)$

$\zeta + \Sigma M_B = 0; \qquad -P(1.25) + 4500(2.5) = 0$

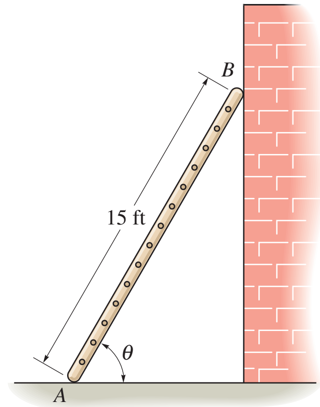
$P = 9000$ lb

Since $P_{Req'd} = 1200 \text{ lb} < 1531.9 \text{ lb}$

It is possible to pull the load without slipping or tipping. **Ans.**

8–5.

The 15-ft ladder has a uniform weight of 80 lb and rests against the smooth wall at B. If the coefficient of static friction at A is $\mu_A = 0.4$, determine if the ladder will slip. Take $\theta = 60°$.



SOLUTION

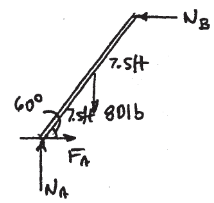
$\circlearrowleft + \Sigma M_A = 0;\qquad N_B(15 \sin 60°) - 80(7.5)\cos 60° = 0$

$N_B = 23.094 \text{ lb}$

$\overset{+}{\rightarrow} \Sigma F_x = 0;\qquad F_A = 23.094 \text{ lb}$

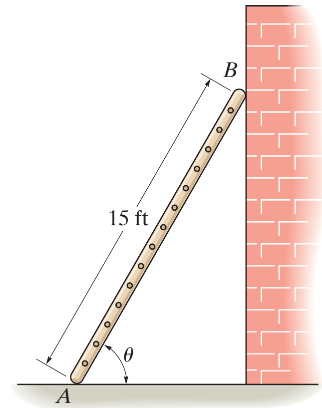
$+\uparrow \Sigma F_y = 0;\qquad N_A = 80 \text{ lb}$

$(F_A)_{max} = 0.4(80) = 32 \text{ lb} > 23.094 \text{ lb}$ **(O.K!)**



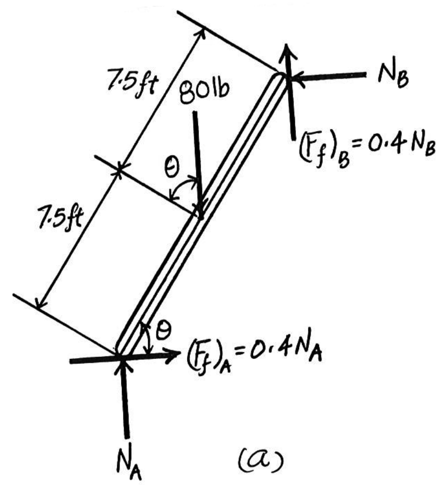
The ladder will not slip. **Ans.**

8–6.



The ladder has a uniform weight of 80 lb and rests against the wall at B. If the coefficient of static friction at A and B is $\mu = 0.4$, determine the smallest angle θ at which the ladder will not slip.

SOLUTION



Free-Body Diagram: Since the ladder is required to be on the verge to slide down, the frictional force at A and B must act to the right and upward respectively and their magnitude can be computed using friction formula as indicated on the FBD, Fig. *a*.

$(F_f)_A = \mu N_A = 0.4\, N_A \; (F_f)_B = \mu N_B = 0.4\, N_B$

Equations of Equlibrium: Referring to Fig. *a*.

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 0.4N_A - N_B = 0 \qquad N_B = 0.4\, N_A$ **(1)**

$+\uparrow \Sigma F_y = 0; \qquad N_A + 0.4N_B - 80 = 0$ **(2)**

Solving Eqs. (1) and (2) yields

$$N_A = 68.97 \text{ lb} \qquad N_B = 27.59 \text{ lb}$$

Using these results,

$\circlearrowleft + \Sigma M_A = 0; \qquad 0.4(27.59)(15\cos\theta) + 27.59(15\sin\theta) - 80\cos\theta(7.5) = 0$

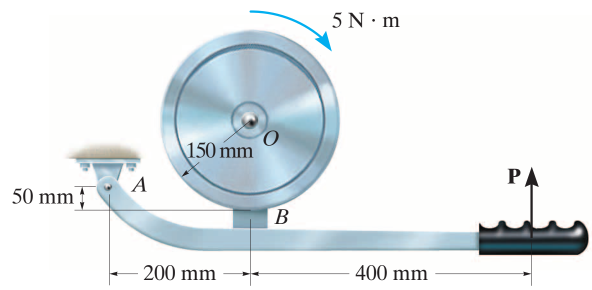
$$413.79\sin\theta - 434.48\cos\theta = 0$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{434.48}{413.79} = 1.05$$

$$\theta = 46.4^\circ$$ **Ans.**

8–7.

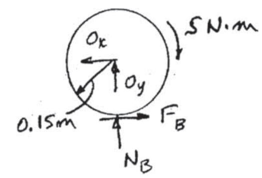
The block brake consists of a pin-connected lever and friction block at B. The coefficient of static friction between the wheel and the lever is $\mu_s = 0.3$, and a torque of $5\ \text{N}\cdot\text{m}$ is applied to the wheel. Determine if the brake can hold the wheel stationary when the force applied to the lever is (a) $P = 30$ N, (b) $P = 70$ N.



SOLUTION

To hold lever:

$\circlearrowleft + \Sigma M_O = 0; \qquad F_B(0.15) - 5 = 0; \qquad F_B = 33.333\ \text{N}$



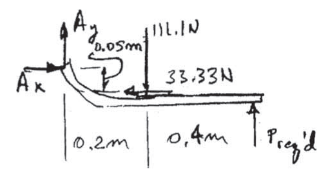
Require

$$N_B = \frac{33.333\ \text{N}}{0.3} = 111.1\ \text{N}$$

Lever,

$\circlearrowleft + \Sigma M_A = 0; \qquad P_{\text{Reqd.}}(0.6) - 111.1(0.2) - 33.333(0.05) = 0$

$P_{\text{Reqd.}} = 39.8\ \text{N}$

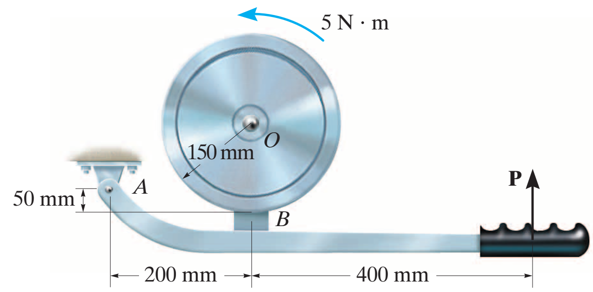


a) $P = 30\ \text{N} < 39.8\ \text{N}$ No **Ans.**

b) $P = 70\ \text{N} > 39.8\ \text{N}$ Yes **Ans.**

***8–8.**

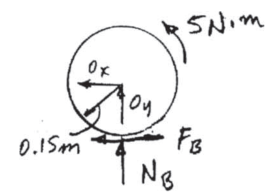
The block brake consists of a pin-connected lever and friction block at B. The coefficient of static friction between the wheel and the lever is $\mu_s = 0.3$, and a torque of $5\ \text{N}\cdot\text{m}$ is applied to the wheel. Determine if the brake can hold the wheel stationary when the force applied to the lever is (a) $P = 30$ N, (b) $P = 70$ N.



SOLUTION

To hold lever:

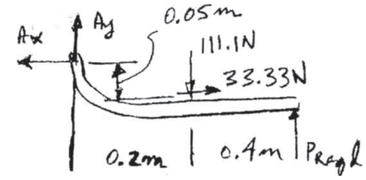
$$\zeta + \Sigma M_O = 0; \qquad -F_B(0.15) + 5 = 0; \qquad F_B = 33.333\text{ N}$$



Require

$$N_B = \frac{33.333\text{ N}}{0.3} = 111.1\text{ N}$$

Lever,



$$\zeta + \Sigma M_A = 0; \qquad P_{\text{Reqd.}}(0.6) - 111.1(0.2) + 33.333(0.05) = 0$$

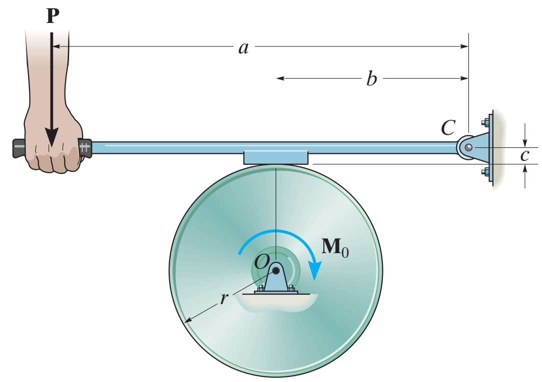
$$P_{\text{Reqd.}} = 34.26\text{ N}$$

a) $P = 30\text{ N} < 34.26\text{ N}$ No **Ans.**

b) $P = 70\text{ N} > 34.26\text{ N}$ Yes **Ans.**

8–9.

The block brake is used to stop the wheel from rotating when the wheel is subjected to a couple moment $\mathbf{M}_0$. If the coefficient of static friction between the wheel and the block is μ_s, determine the smallest force P that should be applied.



SOLUTION

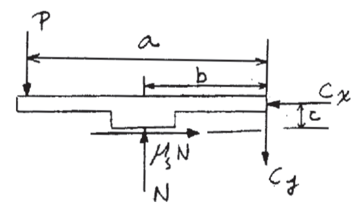
$\circlearrowleft + \Sigma M_C = 0; \qquad Pa - Nb + \mu_s Nc = 0$

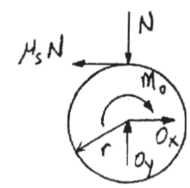
$$N = \frac{Pa}{(b - \mu_s c)}$$

$\circlearrowleft + \Sigma M_O = 0; \qquad \mu_s Nr - M_0 = 0$

$$\mu_s P\left(\frac{a}{b - \mu_s c}\right)r = M_0$$

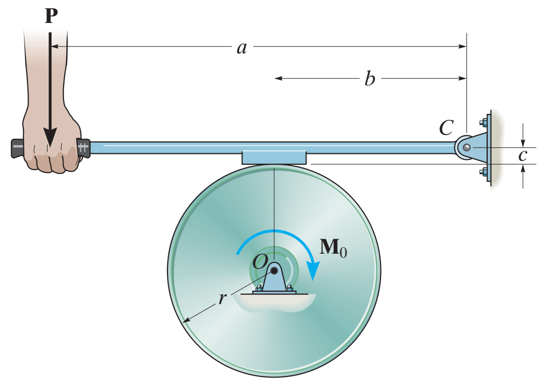
$$P = \frac{M_0}{\mu_s ra}(b - \mu_s c) \qquad \textbf{Ans.}$$





8–10.

The block brake is used to stop the wheel from rotating when the wheel is subjected to a couple moment $\mathbf{M}_0$. If the coefficient of static friction between the wheel and the block is μ_s, show that the brake is self locking, i.e., the required force $P \leq 0$, provided $b/c \leq \mu_s$.



SOLUTION

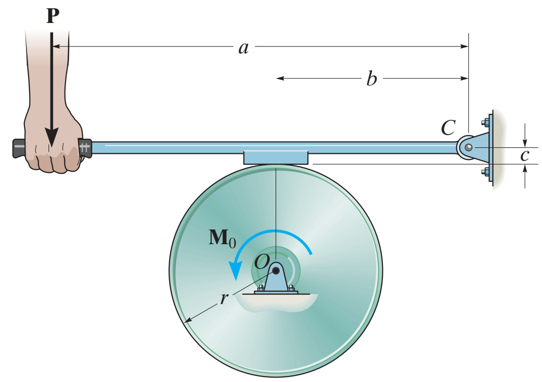
Require $P \leq 0$. Then, from Soln. 8–9

$$b \leq \mu_s c$$

$$\mu_s \geq \frac{b}{c} \qquad \textbf{Ans.}$$

8–11.

The block brake is used to stop the wheel from rotating when the wheel is subjected to a couple moment $\mathbf{M}_0$. If the coefficient of static friction between the wheel and the block is μ_s, determine the smallest force P that should be applied.



SOLUTION

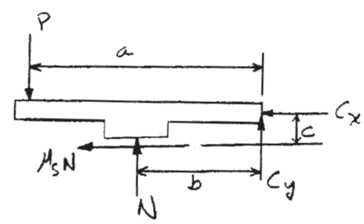
$\zeta + \Sigma M_C = 0; \qquad Pa - Nb - \mu_s Nc = 0$

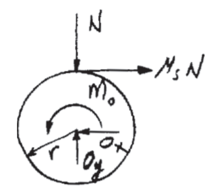
$$N = \frac{Pa}{(b + \mu_s c)}$$

$\zeta + \Sigma M_O = 0; \qquad \mu_s Nr - M_0 = 0$

$$\mu_s P\left(\frac{a}{b + \mu_s c}\right)r = M_0$$

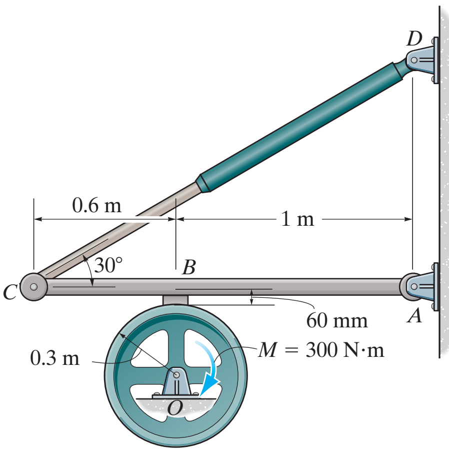
$$P = \frac{M_0}{\mu_s ra}(b + \mu_s c)$$ **Ans.**





***8–12.**

If a torque of $M = 300\ \text{N}\cdot\text{m}$ is applied to the flywheel, determine the force that must be developed in the hydraulic cylinder CD to prevent the flywheel from rotating. The coefficient of static friction between the friction pad at B and the flywheel is $\mu_s = 0.4$.



SOLUTION

Free-BodyDiagram: First we will consider the equilibrium of the flywheel using the free-body diagram shown in Fig. *a*. Here, the frictional force $\mathbf{F}_B$ must act to the left to produce the counterclockwise moment opposing the impending clockwise rotational motion caused by the $300\ \text{N}\cdot\text{m}$ couple moment. Since the wheel is required to be on the verge of slipping, then $F_B = \mu_s N_B = 0.4\ N_B$. Subsequently, the free-body diagram of member ABC shown in Fig. *b* will be used to determine $\mathbf{F}_{CD}$.

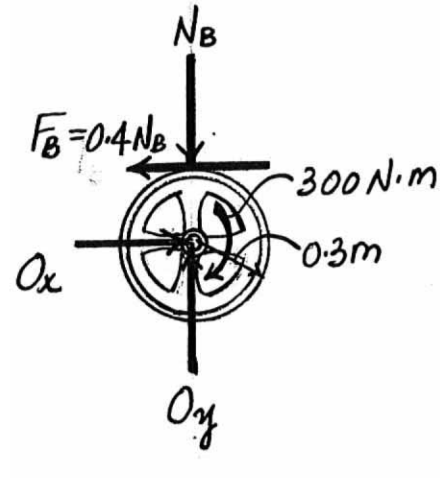
Equations of Equilibrium: We have

$$\circlearrowleft + \Sigma M_O = 0; \qquad 0.4\ N_B(0.3) - 300 = 0 \qquad N_B = 2500\ \text{N}$$

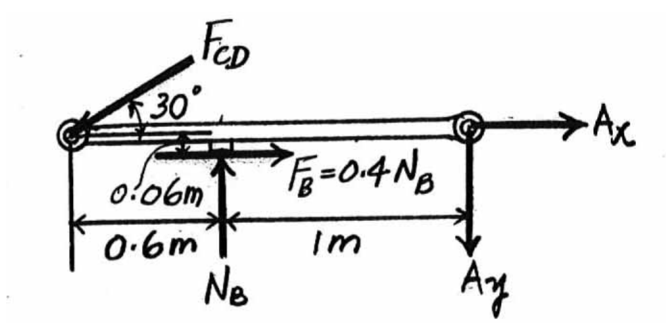
Using this result,

$$\circlearrowleft + \Sigma M_A = 0; \qquad F_{CD} \sin 30°(1.6) + 0.4(2500)(0.06) - 2500(1) = 0$$

$$F_{CD} = 3050\ \text{N} = 3.05\ \text{kN}$$ **Ans.**



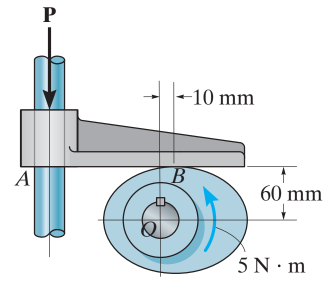
(a)



(b)

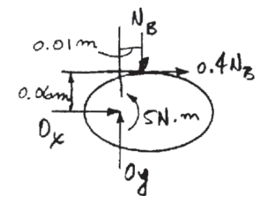
8–13.

The cam is subjected to a couple moment of $5\ \text{N}\cdot\text{m}$. Determine the minimum force P that should be applied to the follower in order to hold the cam in the position shown. The coefficient of static friction between the cam and the follower is $\mu_s = 0.4$. The guide at A is smooth.



SOLUTION

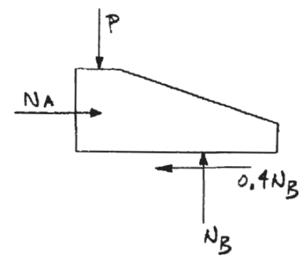
Cam:



$\circlearrowleft + \Sigma M_O = 0; \qquad 5 - 0.4\, N_B\,(0.06) - 0.01\,(N_B) = 0$

$N_B = 147.06\ \text{N}$

Follower:

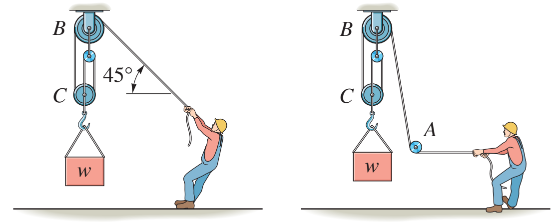


$+\uparrow \Sigma F_y = 0; \qquad 147.06 - P = 0$

$P = 147\ \text{N}$ **Ans.**

8–14.

Determine the maximum weight W the man can lift with constant velocity using the pulley system, without and then with the "leading block" or pulley at A. The man has a weight of 200 lb and the coefficient of static friction between his feet and the ground is $\mu_s = 0.6$.



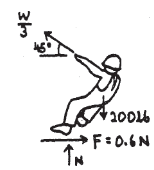
(a) (b)

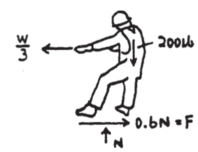
SOLUTION

a) $+\uparrow \Sigma F_y = 0; \qquad \dfrac{W}{3}\sin 45° + N - 200 = 0$

$\xrightarrow{+} \Sigma F_x = 0; \qquad -\dfrac{W}{3}\cos 45° + 0.6\,N = 0$

$W = 318 \text{ lb}$ **Ans.**





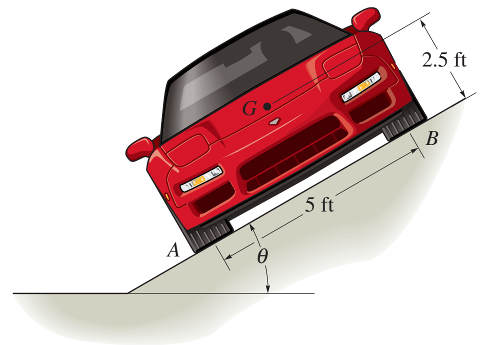
b) $+\uparrow \Sigma F_y = 0; \qquad N = 200 \text{ lb}$

$\xrightarrow{+} \Sigma F_x = 0; \qquad 0.6(200) = \dfrac{W}{3}$

$W = 360 \text{ lb}$ **Ans.**

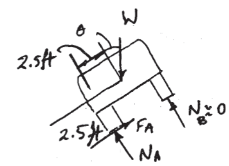
8–15.

The car has a mass of 1.6 Mg and center of mass at G. If the coefficient of static friction between the shoulder of the road and the tires is $\mu_s = 0.4$, determine the greatest slope θ the shoulder can have without causing the car to slip or tip over if the car travels along the shoulder at constant velocity.



SOLUTION

Tipping:

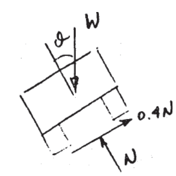


$\circlearrowleft + \Sigma M_A = 0; \qquad -W\cos\theta(2.5) + W\sin\theta(2.5) = 0$

$\tan\theta = 1$

$\theta = 45°$

Slipping:



$\nearrow + \Sigma F_x = 0; \qquad 0.4\,N - W\sin\theta = 0$

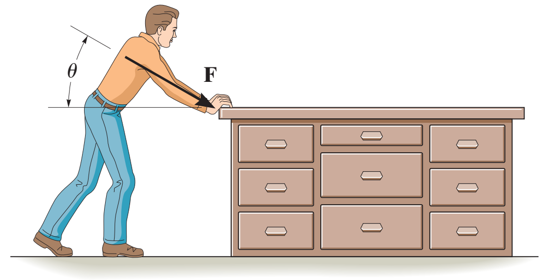
$\nwarrow + \Sigma F_y = 0; \qquad N - W\cos\theta = 0$

$\tan\theta = 0.4$

$\theta = 21.8°$ **Ans.** (car slips before it tips)

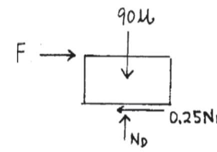
***8–16.**

The uniform dresser has a weight of 90 lb and rests on a tile floor for which $\mu_s = 0.25$. If the man pushes on it in the horizontal direction $\theta = 0°$, determine the smallest magnitude of force **F** needed to move the dresser. Also, if the man has a weight of 150 lb, determine the smallest coefficient of static friction between his shoes and the floor so that he does not slip.



SOLUTION

Dresser:



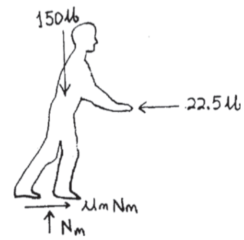
$+\uparrow \Sigma F_y = 0; \qquad N_D - 90 = 0$

$N_D = 90 \text{ lb}$

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad F - 0.25(90) = 0$

$F = 22.5 \text{ lb}$ **Ans.**

Man:



$+\uparrow \Sigma F_y = 0; \qquad N_m - 150 = 0$

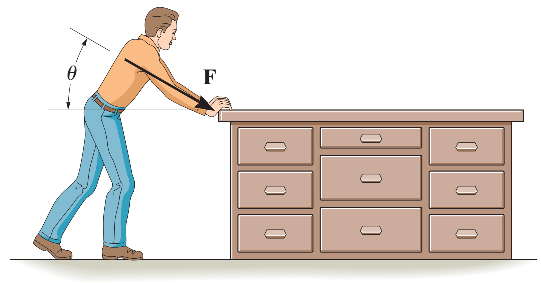
$N_m = 150 \text{ lb}$

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad -22.5 + \mu_m(150) = 0$

$\mu_m = 0.15$ **Ans.**

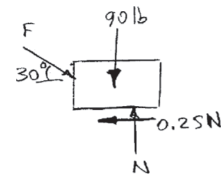
8–17.

The uniform dresser has a weight of 90 lb and rests on a tile floor for which $\mu_s = 0.25$. If the man pushes on it in the direction $\theta = 30°$, determine the smallest magnitude of force **F** needed to move the dresser. Also, if the man has a weight of 150 lb, determine the smallest coefficient of static friction between his shoes and the floor so that he does not slip.



SOLUTION

Dresser:



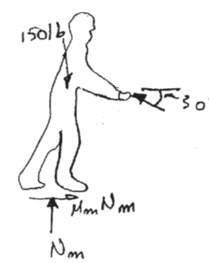
$+\uparrow \Sigma F_y = 0; \qquad N - 90 - F \sin 30° = 0$

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad F \cos 30° - 0.25\, N = 0$

$N = 105.1 \text{ lb}$

$F = 30.363 \text{ lb} = 30.4 \text{ lb}$ **Ans.**

Man:



$+\uparrow \Sigma F_y = 0; \qquad N_m - 150 + 30.363 \sin 30° = 0$

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad F_m - 30.363 \cos 30° = 0$

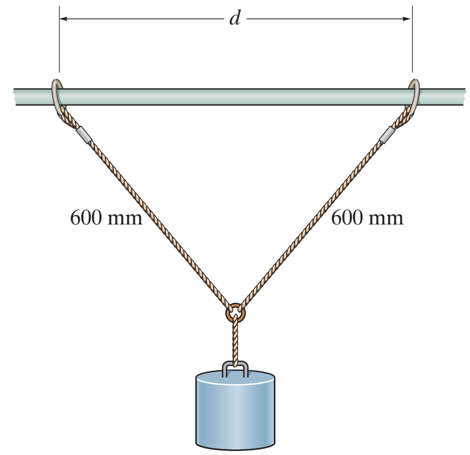
$N_m = 134.82 \text{ lb}$

$F_m = 26.295 \text{ lb}$

$$\mu_m = \frac{F_m}{N_m} = \frac{26.295}{134.82} = 0.195$$ **Ans.**

8–18.

The 5-kg cylinder is suspended from two equal-length cords. The end of each cord is attached to a ring of negligible mass that passes along a horizontal shaft. If the rings can be separated by the greatest distance $d = 400$ mm and still support the cylinder, determine the coefficient of static friction between each ring and the shaft.



SOLUTION

Equilibrium of the Cylinder: Referring to the FBD shown in Fig. *a*,

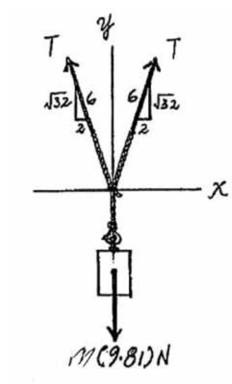
$$+\uparrow \Sigma F_y = 0; \qquad 2\left[T\left(\frac{\sqrt{32}}{6}\right)\right] - m(9.81) = 0 \qquad T = 5.2025\,m$$

Equilibrium of the Ring: Since the ring is required to be on the verge to slide, the frictional force can be computed using friction formula $F_f = \mu N$ as indicated in the *FBD* of the ring shown in Fig. *b*. Using the result of *I*,

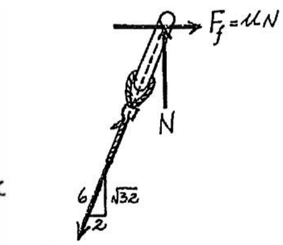
$$+\uparrow \Sigma F_y = 0; \qquad N - 5.2025\,m\left(\frac{\sqrt{32}}{6}\right) = 0 \qquad N = 4.905\,m$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad \mu(4.905\,m) - 5.2025\,m\left(\frac{2}{6}\right) = 0$$

$$\mu = 0.354 \qquad \textbf{Ans.}$$



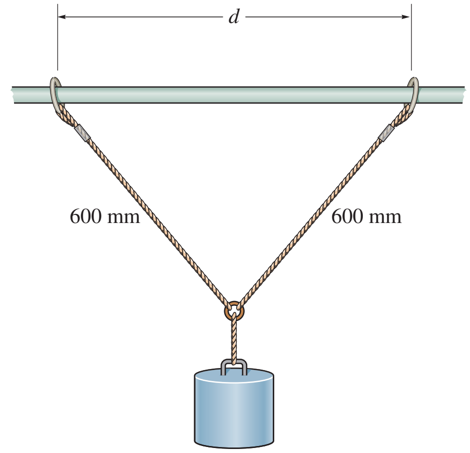
(a)



(b)

8–19.

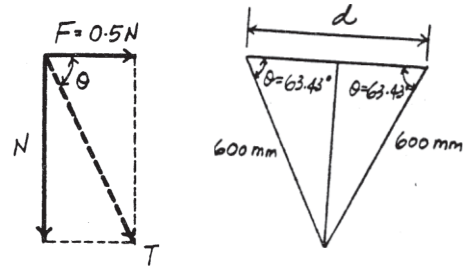
The 5-kg cylinder is suspended from two equal-length cords. The end of each cord is attached to a ring of negligible mass, which passes along a horizontal shaft. If the coefficient of static friction between each ring and the shaft is $\mu_s = 0.5$, determine the greatest distance d by which the rings can be separated and still support the cylinder.



SOLUTION

Friction: When the ring is on the verge to sliding along the rod, slipping will have to occur. Hence, $F = \mu N = 0.5N$. From the force diagram (T is the tension developed by the cord)

$$\tan\theta = \frac{N}{0.5N} = 2 \qquad \theta = 63.43^\circ$$

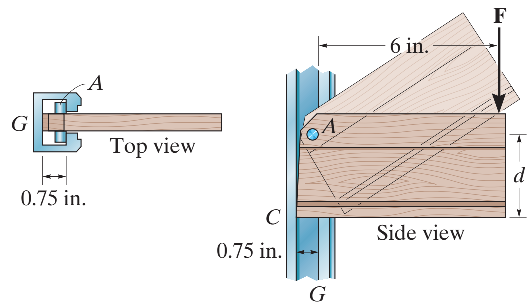


Geometry:

$$d = 2(600\cos 63.43^\circ) = 537\text{ mm}$$ **Ans.**

***8–20.**

The board can be adjusted vertically by tilting it up and sliding the smooth pin A along the vertical guide G. When placed horizontally, the bottom C then bears along the edge of the guide, where $\mu_s = 0.4$. Determine the largest dimension d which will support any applied force **F** without causing the board to slip downward.



SOLUTION

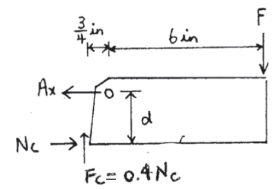
$$+\uparrow \Sigma F_y = 0; \qquad 0.4N_C - F = 0$$

$$\circlearrowleft + \Sigma M_A = 0; \qquad -F(6) + d(N_C) - 0.4N_C(0.75) = 0$$

Thus,

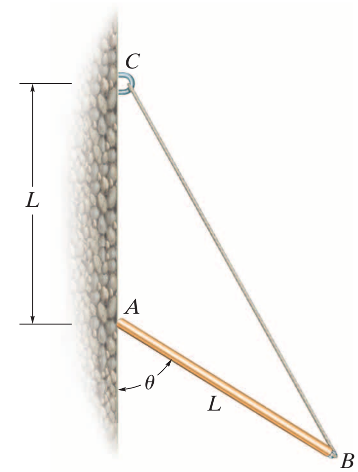
$$-0.4N_C(6) + d(N_C) - 0.4N_C(0.75) = 0$$

$$d = 2.70 \text{ in.}$$ **Ans.**



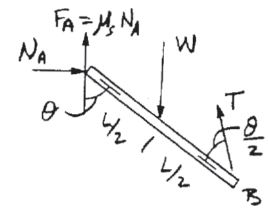
8–21.

The uniform pole has a weight W and length L. Its end B is tied to a supporting cord, and end A is placed against the wall, for which the coefficient of static friction is μ_s. Determine the largest angle θ at which the pole can be placed without slipping.



SOLUTION

$$\circlearrowleft + \Sigma M_B = 0; \qquad -N_A(L\cos\theta) - \mu_s N_A(L\sin\theta) + W\left(\frac{L}{2}\sin\theta\right) = 0 \qquad \textbf{(1)}$$



$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad N_A - T\sin\frac{\theta}{2} = 0 \qquad \textbf{(2)}$$

$$+\uparrow \Sigma F_y = 0; \qquad \mu_s N_A - W + T\cos\frac{\theta}{2} = 0 \qquad \textbf{(3)}$$

Substitute Eq. (2) into Eq. (3): $\mu_s T\sin\frac{\theta}{2} - W + T\cos\frac{\theta}{2} = 0$

$$W = T\left(\cos\frac{\theta}{2} + \mu_s\sin\frac{\theta}{2}\right) \qquad \textbf{(4)}$$

Substitute Eqs. (2) and (3) into Eq. (1):

$$T\sin\frac{\theta}{2}\cos\theta - T\cos\frac{\theta}{2}\sin\theta + \frac{W}{2}\sin\theta = 0 \qquad \textbf{(5)}$$

Substitute Eq. (4) into Eq. (5):

$$\sin\frac{\theta}{2}\cos\theta - \cos\frac{\theta}{2}\sin\theta + \frac{1}{2}\cos\frac{\theta}{2}\sin\theta + \frac{1}{2}\mu_s\sin\frac{\theta}{2}\sin\theta = 0$$

$$-\sin\frac{\theta}{2} + \frac{1}{2}\left(\cos\frac{\theta}{2} + \mu_s\sin\frac{\theta}{2}\right)\sin\theta = 0$$

$$\cos\frac{\theta}{2} + \mu_s\sin\frac{\theta}{2} = \frac{1}{\cos\frac{\theta}{2}}$$

$$\cos^2\frac{\theta}{2} + \mu_s\sin\frac{\theta}{2}\cos\frac{\theta}{2} = 1$$

$$\mu_s\sin\frac{\theta}{2}\cos\frac{\theta}{2} = \sin^2\frac{\theta}{2}$$

$$\tan\frac{\theta}{2} = \mu_s$$

$$\theta = 2\tan^{-1}\mu_s \qquad \textbf{Ans.}$$

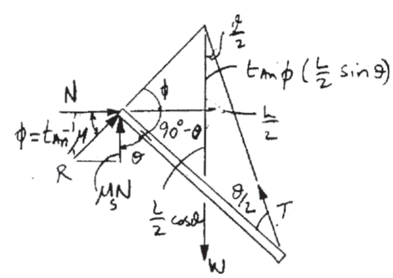
Also, because we have a three – force member,

$$\frac{L}{2} = \frac{L}{2}\cos\theta + \tan\phi\left(\frac{L}{2}\sin\theta\right)$$

$$1 = \cos\theta + \mu_s\sin\theta$$

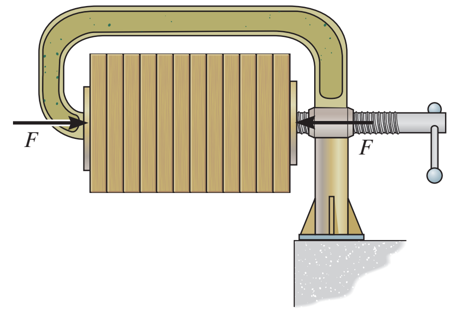
$$\mu_s = \frac{1-\cos\theta}{\sin\theta} = \tan\frac{\theta}{2}$$

$$\theta = 2\tan^{-1}\mu_s \qquad \textbf{Ans.}$$



8–22.

If the clamping force is $F = 200$ N and each board has a mass of 2 kg, determine the maximum number of boards the clamp can support. The coefficient of static friction between the boards is $\mu_s = 0.3$, and the coefficient of static friction between the boards and the clamp is $\mu_s' = 0.45$.

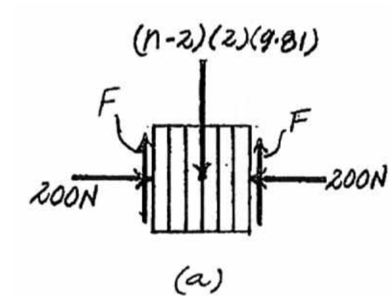


SOLUTION

Free-Body Diagram: The boards could be on the verge of slipping between the two boards at the ends or between the clamp. Let n be the number of boards between the clamp. Thus, the number of boards between the two boards at the ends is $n - 2$. If the boards slip between the two end boards, then $F = \mu_s N = 0.3(200) = 60$ N.

Equations of Equilibrium: Referring to the free-body diagram shown in Fig. a, we have

$$+\uparrow \Sigma F_y = 0; \qquad 2(60) - (n - 2)(2)(9.81) = 0 \qquad n = 8.12$$

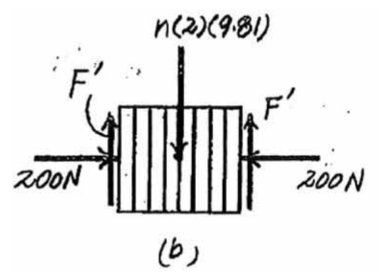


If the end boards slip at the clamp, then $F' = \mu_s' N = 0.45(200) = 90$ N. By referring to the free-body diagram shown in Fig. b, we have

$$\circlearrowleft + \uparrow \Sigma F_y = 0; \qquad 2(90) - n(2)(9.81) = 0 \qquad n = 9.17$$

Thus, the maximum number of boards that can be supported by the clamp will be the smallest value of n obtained above, which gives

$$n = 8 \qquad \textbf{Ans.}$$



$$\circlearrowleft + \Sigma M_{\text{clamp}} = 0; \qquad 60 - (2)(9.81)(n - 1)2 = 0$$

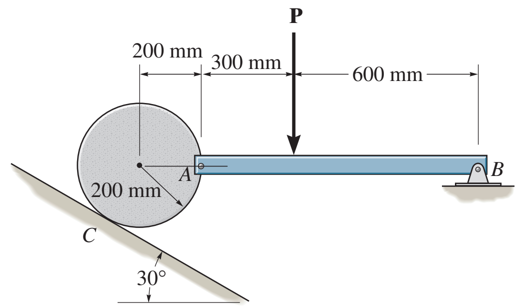
$$60 - 9.81(n - 1) = 0$$

$$n = 7.12$$

$$n = 7 \qquad \textbf{Ans.}$$

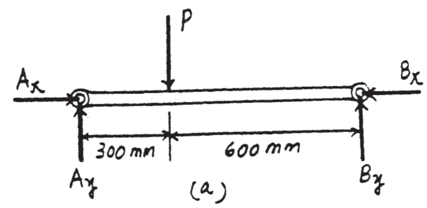
8–23.

A 35-kg disk rests on an inclined surface for which $\mu_s = 0.2$. Determine the maximum vertical force $\mathbf{P}$ that may be applied to link AB without causing the disk to slip at C.



SOLUTION

Equations of Equilibrium: From FBD (a),

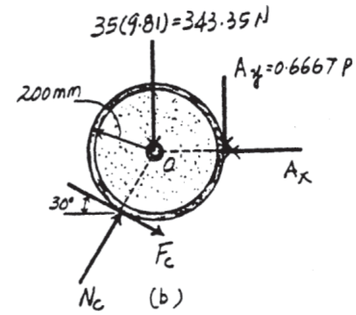


$\zeta + \Sigma M_B = 0; \qquad P(600) - A_y(900) = 0 \qquad A_y = 0.6667P$

From FBD (b),

$+\uparrow \Sigma F_y = 0 \qquad N_C \sin 60° - F_C \sin 30° - 0.6667P - 343.35 = 0$ **(1)**

$\zeta + \Sigma M_O = 0; \qquad F_C(200) - 0.6667P(200) = 0$ **(2)**



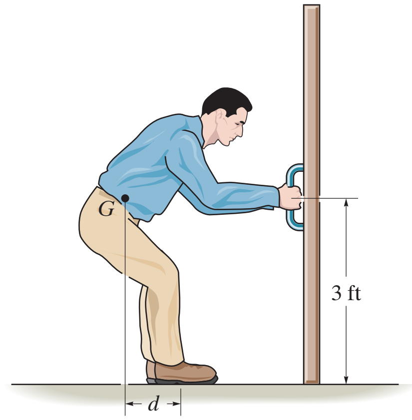
Friction: If the disk is on the verge of moving, slipping would have to occur at point C. Hence, $F_C = \mu_s N_C = 0.2N_C$. Substituting this value into Eqs. (1) and (2) and solving, we have

$P = 182 \text{ N}$ **Ans.**

$N_C = 606.60 \text{ N}$

***8–24.**

The man has a weight of 200 lb, and the coefficient of static friction between his shoes and the floor is $\mu_s = 0.5$. Determine where he should position his center of gravity G at d in order to exert the maximum horizontal force on the door. What is this force?



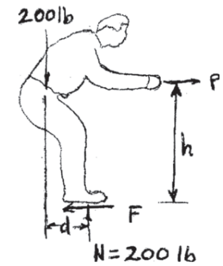
SOLUTION

$F_{max} = 0.5\,N = 0.5(200) = 100 \text{ lb}$

$\xrightarrow{+} \Sigma F_x = 0; \qquad P - 100 = 0; \qquad P = 100 \text{ lb}$ **Ans.**

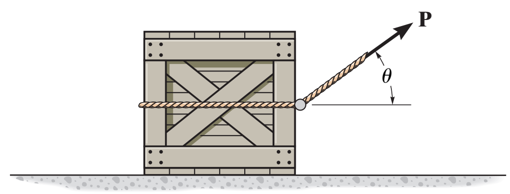
$\circlearrowleft + \Sigma M_O = 0; \qquad 200(d) - 100(3) = 0$

$d = 1.50 \text{ ft}$ **Ans.**



8–25.

The crate has a weight of $W = 150$ lb, and the coefficients of static and kinetic friction are $\mu_s = 0.3$ and $\mu_k = 0.2$, respectively. Determine the friction force on the floor if $\theta = 30°$ and $P = 200$ lb.



SOLUTION

Equations of Equilibrium: Referring to the FBD of the crate shown in Fig. *a*,

$+\uparrow \Sigma F_y = 0;\quad N + 200 \sin 30° - 150 = 0 \quad N = 50 \text{ lb}$

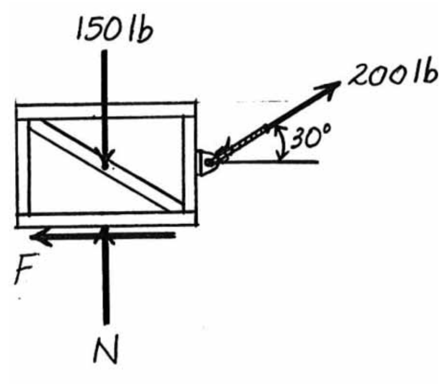
$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad 200 \cos 30° - F = 0 \quad F = 173.20 \text{ lb}$

Friction Formula: Here, the maximum frictional force that can be developed is

$$(F_f)_{\text{max}} = \mu_s N = 0.3(50) = 15 \text{ lb}$$

Since $F = 173.20 \text{ lb} > (F_f)_{\text{max}}$, the crate will slide. Thus the frictional force developed is

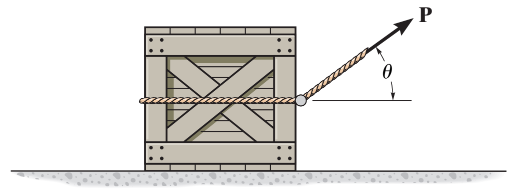
$$F_f = \mu_k N = 0.2(50) = 10 \text{ lb} \qquad \textbf{Ans.}$$



(a)

8–26.

The crate has a weight of $W = 350$ lb, and the coefficients of static and kinetic friction are $\mu_s = 0.3$ and $\mu_k = 0.2$, respectively. Determine the friction force on the floor if $\theta = 45°$ and $P = 100$ lb.



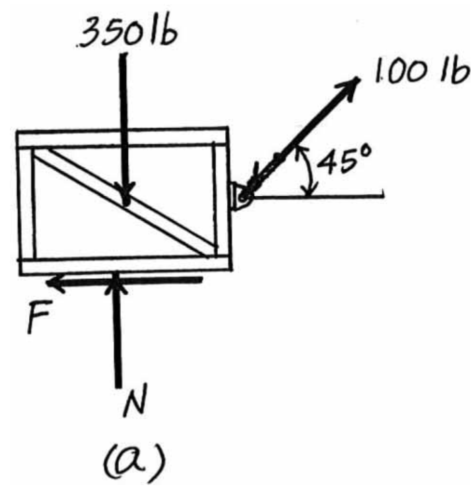
SOLUTION

Equations of Equilibrium: Referring to the FBD of the crate shown in Fig. *a*,

$$+\uparrow \Sigma F_y = 0; \qquad N + 100 \sin 45° - 350 = 0$$

$$N = 279.29 \text{ lb}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 100 \cos 45° - F = 0 \qquad F = 70.71 \text{ lb}$$



Friction Formula: Here, the maximum frictional force that can be developed is

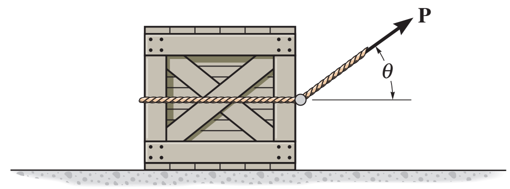
$$(F_f)_{max} = \mu_s N = 0.3(279.29) = 83.79 \text{ lb}$$

Since $F = 70.71 \text{ lb} < (F_f)_{max}$, the crate will not slide. Thus, the frictional force developed is

$$F_f = F = 70.7 \text{ lb} \qquad \textbf{Ans.}$$

8–27.

The crate has a weight W and the coefficient of static friction at the surface is $\mu_s = 0.3$. Determine the orientation of the cord and the smallest possible force **P** that has to be applied to the cord so that the crate is on the verge of moving.

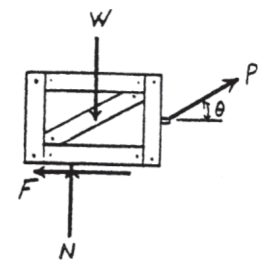


SOLUTION

Equations of Equilibrium:

$+\uparrow \Sigma F_y = 0; \qquad N + P\sin\theta - W = 0 \qquad$ **(1)**

$\xrightarrow{+} \Sigma F_x = 0; \qquad P\cos\theta - F = 0 \qquad$ **(2)**



Friction: If the crate is on the verge of moving, slipping will have to occur. Hence, $F = \mu_s N = 0.3N$. Substituting this value into Eqs. (1)and (2) and solving, we have

$$P = \frac{0.3W}{\cos\theta + 0.3\sin\theta} \qquad N = \frac{W\cos\theta}{\cos\theta + 0.3\sin\theta}$$

In order to obtain the minimum $P, \dfrac{dP}{d\theta} = 0$.

$$\frac{dP}{d\theta} = 0.3W\left[\frac{\sin\theta - 0.3\cos\theta}{(\cos\theta + 0.3\sin\theta)^2}\right] = 0$$

$$\sin\theta - 0.3\cos\theta = 0$$

$$\theta = 16.70° = 16.7° \qquad \textbf{Ans.}$$

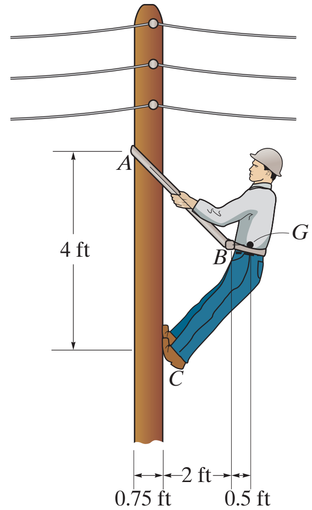
$$\frac{d^2P}{d\theta^2} = 0.3W\left[\frac{(\cos\theta + 0.3\sin\theta)^2 + 2(\sin\theta - 0.3\cos\theta)^2}{(\cos\theta + 0.3\sin\theta)^3}\right]$$

At $\theta = 16.70°, \dfrac{d^2P}{d\theta^2} = 0.2873W > 0$. Thus, $\theta = 16.70°$ will result in a minimum P.

$$P = \frac{0.3W}{\cos 16.70° + 0.3\sin 16.70°} = 0.287W \qquad \textbf{Ans.}$$

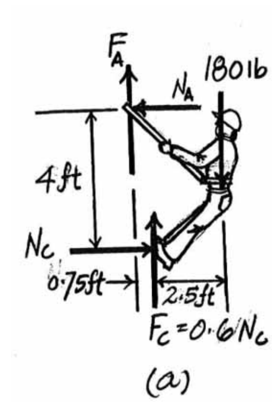
***8–28.**

If the coefficient of static friction between the man's shoes and the pole is $\mu_s = 0.6$, determine the minimum coefficient of static friction required between the belt and the pole at A in order to support the man. The man has a weight of 180 lb and a center of gravity at G.



SOLUTION

Free-Body Diagram: The man's shoe and the belt have a tendency to slip downward. Thus, the frictional forces $\mathbf{F}_A$ and $\mathbf{F}_C$ must act upward as indicated on the free-body diagram of the man shown in Fig. *a*. Here, $\mathbf{F}_C$ is required to develop to its maximum, thus $F_C = (\mu_s)_C N_C = 0.6N_C$.



Equations of Equilibrium: Referring to Fig. *a*, we have

$$\circlearrowleft +\Sigma M_A = 0; \quad N_C(4) + 0.6N_C(0.75) - 180(3.25) = 0$$

$$N_C = 131.46 \text{ lb}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad 131.46 - N_A = 0 \qquad N_A = 131.46 \text{ lb}$$

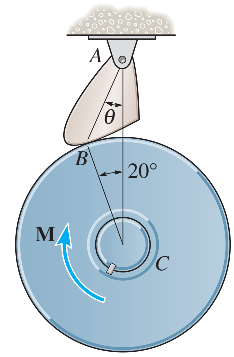
$$+\uparrow \Sigma F_y = 0; \quad F_A + 0.6(131.46) - 180 = 0 \qquad F_A = 101.12 \text{ lb}$$

To prevent the belt from slipping the coefficient of static friction at contact point A must be at least

$$(\mu_s)_A = \frac{F_A}{N_A} = \frac{101.12}{131.46} = 0.769 \qquad \textbf{Ans.}$$

8–29.

The friction pawl is pinned at A and rests against the wheel at B. It allows freedom of movement when the wheel is rotating counterclockwise about C. Clockwise rotation is prevented due to friction of the pawl which tends to bind the wheel. If $(\mu_s)_B = 0.6$, determine the design angle θ which will prevent clockwise motion for any value of applied moment M. *Hint:* Neglect the weight of the pawl so that it becomes a two-force member.

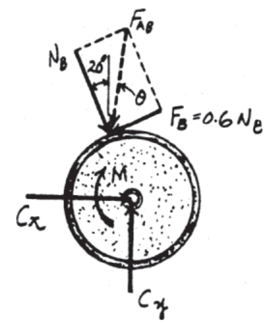


SOLUTION

Friction: When the wheel is on the verge of rotating, slipping would have to occur. Hence, $F_B = \mu N_B = 0.6N_B$. From the force diagram (F_{AB} is the force developed in the two force member AB)

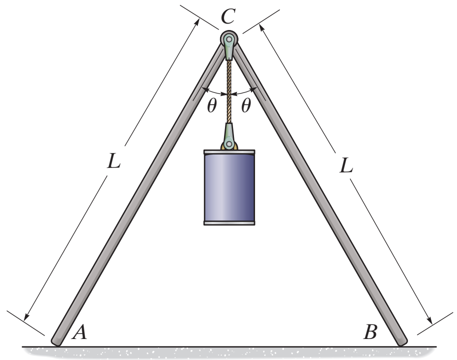
$$\tan(20° + \theta) = \frac{0.6N_B}{N_B} = 0.6$$

$$\theta = 11.0° \qquad \textbf{Ans.}$$



8–30.

If $\theta = 30°$ determine the minimum coefficient of static friction at A and B so that equilibrium of the supporting frame is maintained regardless of the mass of the cylinder C. Neglect the mass of the rods.



SOLUTION

Free-Body Diagram: Due to the symmetrical loading and system, ends A and B of the rod will slip simultaneously. Since end B tends to move to the right, the friction force $\mathbf{F}_B$ must act to the left as indicated on the free-body diagram shown in Fig. *a*.

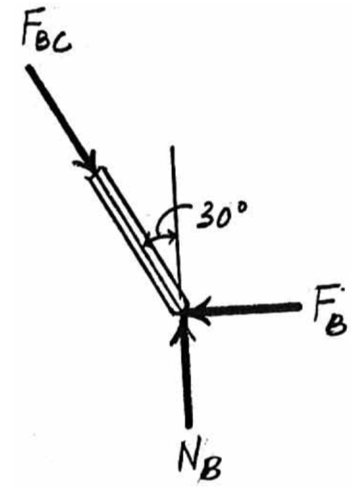
Equations of Equilibrium: We have

$$\xrightarrow{+} \Sigma F_x = 0; \qquad F_{BC} \sin 30° - F_B = 0 \qquad F_B = 0.5F_{BC}$$

$$+\uparrow \Sigma F_y = 0; \qquad N_B - F_{BC} \cos 30° = 0 \qquad N_B = 0.8660\, F_{BC}$$

Therefore, to prevent slipping the coefficient of static friction ends A and B must be at least

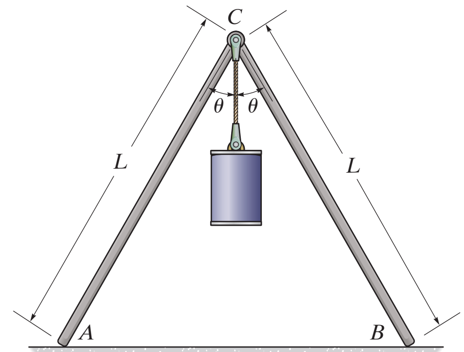
$$\mu_s = \frac{F_B}{N_B} = \frac{0.5F_{BC}}{0.8660F_{BC}} = 0.577$$ **Ans.**



(a)

8–31.

If the coefficient of static friction at A and B is $\mu_s = 0.6$, determine the maximum angle θ so that the frame remains in equilibrium, regardless of the mass of the cylinder. Neglect the mass of the rods.



SOLUTION

Free-Body Diagram: Due to the symmetrical loading and system, ends A and B of the rod will slip simultaneously. Since end B is on the verge of sliding to the right, the friction force F_B must act to the left such that $F_B = \mu_s N_B = 0.6 N_B$ as indicated on the free-body diagram shown in Fig. *a*.

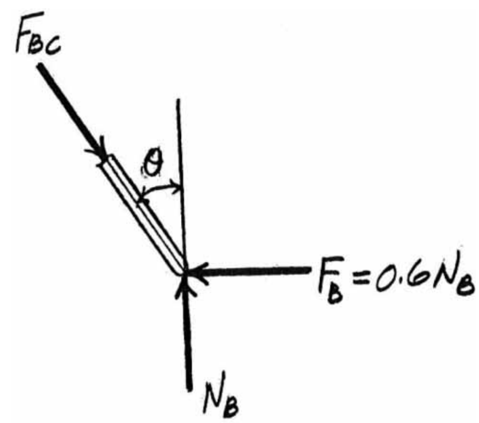
Equations of Equilibrium: We have

$+\uparrow \Sigma F_y = 0; \qquad N_B - F_{BC} \cos\theta = 0 \qquad N_B = F_{BC} \cos\theta$

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad F_{BC} \sin\theta - 0.6(F_{BC} \cos\theta) = 0$

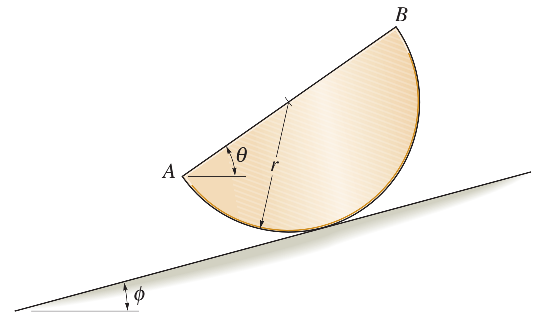
$\tan\theta = 0.6$

$\theta = 31.0°$ **Ans.**



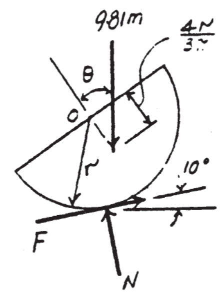
***8–32.**

The semicylinder of mass m and radius r lies on the rough inclined plane for which $\phi = 10°$ and the coefficient of static friction is $\mu_s = 0.3$. Determine if the semicylinder slides down the plane, and if not, find the angle of tip θ of its base AB.



SOLUTION

Equations of Equilibrium:



$$\circlearrowleft + \Sigma M_O = 0; \qquad F(r) - 9.81m\sin\theta\left(\frac{4r}{3\pi}\right) = 0 \qquad \textbf{(1)}$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad F\cos 10° - N\sin 10° = 0 \qquad \textbf{(2)}$$

$$+\uparrow \Sigma F_y = 0 \qquad F\sin 10° + N\cos 10° - 9.81m = 0 \qquad \textbf{(3)}$$

Solving Eqs. (1), (2) and (3) yields

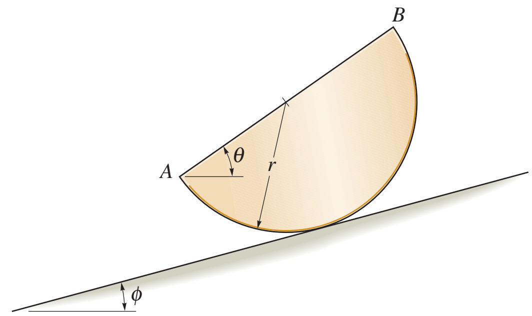
$$N = 9.661m \qquad F = 1.703m$$

$$\theta = 24.2° \qquad \textbf{Ans.}$$

Friction: The maximum friction force that can be developed between the semicylinder and the inclined plane is $(F)_{max} = \mu N = 0.3(9.661m) = 2.898m$. Since $F_{max} > F = 1.703m$, **the semicylinder will not slide down the plane**. **Ans.**

8–33.

The semicylinder of mass m and radius r lies on the rough inclined plane. If the inclination $\phi = 15°$, determine the smallest coefficient of static friction which will prevent the semicylinder from slipping.

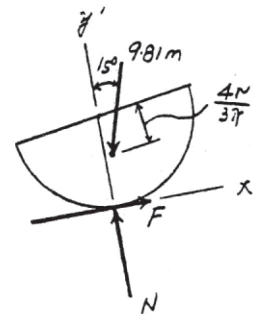


SOLUTION

Equations of Equilibrium:

$+\nearrow \Sigma F_{x'} = 0; \qquad F - 9.81m \sin 15° = 0 \qquad F = 2.539m$

$\nwarrow + \Sigma F_{y'} = 0; \qquad N - 9.81m \cos 15° = 0 \qquad N = 9.476m$



Friction: If the semicylinder is on the verge of moving, slipping would have to occur. Hence,

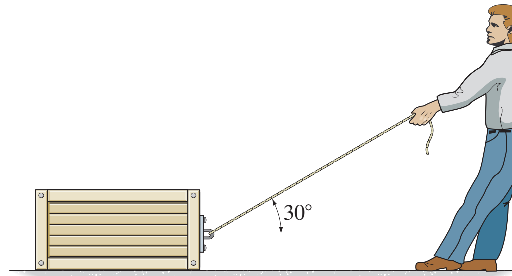
$$F = \mu_s N$$

$$2.539m = \mu_s (9.476m)$$

$$\mu_s = 0.268 \qquad \textbf{Ans.}$$

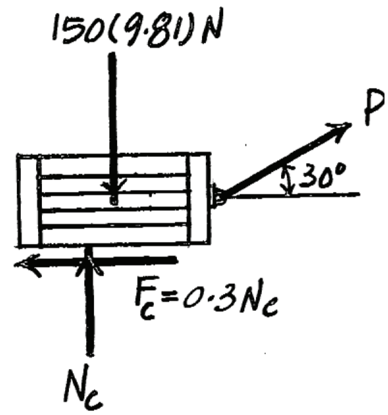
8–34.

The coefficient of static friction between the 150-kg crate and the ground is $\mu_s = 0.3$, while the coefficient of static friction between the 80-kg man's shoes and the ground is $\mu_s' = 0.4$. Determine if the man can move the crate.



SOLUTION

Free-Body Diagram: Since **P** tends to move the crate to the right, the frictional force $\mathbf{F}_C$ will act to the left as indicated on the free-body diagram shown in Fig. *a*. Since the crate is required to be on the verge of sliding the magnitude of $\mathbf{F}_C$ can be computed using the friction formula, i.e. $F_C = \mu_s N_C = 0.3\ N_C$. As indicated on the free-body diagram of the man shown in Fig. *b*, the frictional force $\mathbf{F}_m$ acts to the right since force **P** has the tendency to cause the man to slip to the left.



(a)

Equations of Equilibrium: Referring to Fig. *a*,

$$+\uparrow \Sigma F_y = 0; \qquad N_C + P \sin 30° - 150(9.81) = 0$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad P \cos 30° - 0.3N_C = 0$$

Solving,

$$P = 434.49 \text{ N}$$

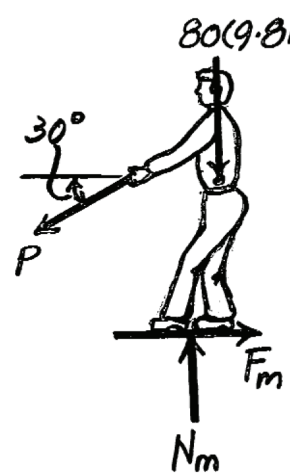
$$N_C = 1254.26 \text{ N}$$

Using the result of P and referring to Fig. *b*, we have

$$+\uparrow \Sigma F_y = 0; \qquad N_m - 434.49 \sin 30° - 80(9.81) = 0 \qquad N_m = 1002.04 \text{ N}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad F_m - 434.49 \cos 30° = 0 \qquad F_m = 376.28 \text{ N}$$

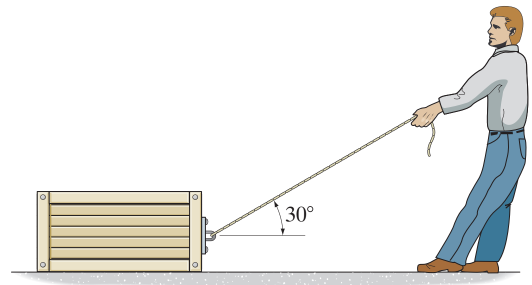
Since $F_m < F_{max} = \mu_s' N_m = 0.4(1002.04) = 400.82$ N, the man does not slip. Thus, **he can move the crate.** **Ans.**



(b)

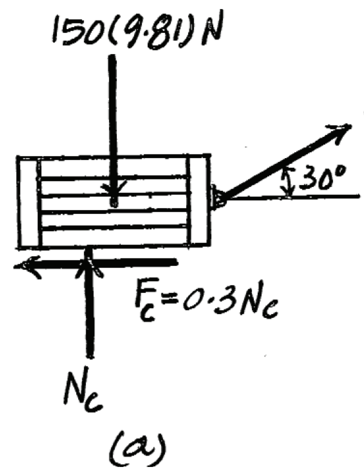
8–35.

If the coefficient of static friction between the crate and the ground is $\mu_s = 0.3$, determine the minimum coefficient of static friction between the man's shoes and the ground so that the man can move the crate.



SOLUTION

Free-Body Diagram: Since force $\mathbf{P}$ tends to move the crate to the right, the frictional force $\mathbf{F}_C$ will act to the left as indicated on the free-body diagram shown in Fig. *a*. Since the crate is required to be on the verge of sliding, $F_C = \mu_s N_C = 0.3\,N_C$. As indicated on the free-body diagram of the man shown in Fig. *b*, the frictional force $\mathbf{F}_m$ acts to the right since force $\mathbf{P}$ has the tendency to cause the man to slip to the left.



Equations of Equilibrium: Referring to Fig. *a*,

$$+\uparrow \Sigma F_y = 0; \qquad N_C + P\sin 30^\circ - 150(9.81) = 0$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad P\cos 30^\circ - 0.3N_C = 0$$

Solving yields

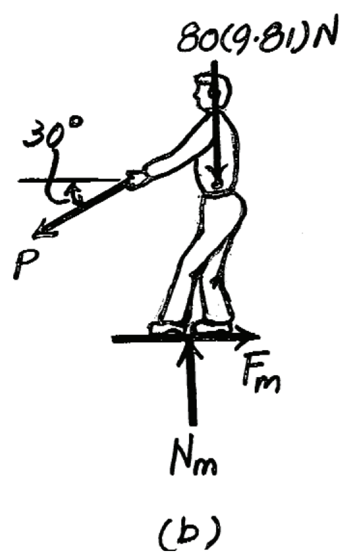
$$P = 434.49 \text{ N}$$

$$N_C = 1245.26 \text{ N}$$

Using the result of $\mathbf{P}$ and referring to Fig. *b*,

$$+\uparrow \Sigma F_y = 0; \qquad N_m - 434.49\sin 30^\circ - 80(9.81) = 0 \qquad N_m = 1002.04 \text{ N}$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad F_m - 434.49\cos 30^\circ = 0 \qquad F_m = 376.28 \text{ N}$$

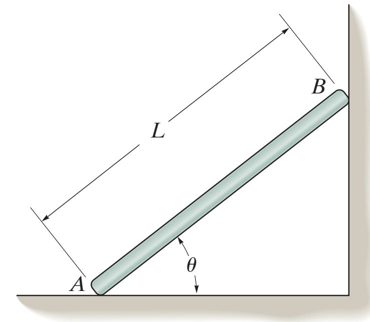


Thus, the required minimum coefficient of static friction between the man's shoes and the ground is given by

$$\mu_s{}' = \frac{F_m}{N_m} = \frac{376.28}{1002.04} = 0.376 \qquad \textbf{Ans.}$$

***8–36.**

The thin rod has a weight W and rests against the floor and wall for which the coefficients of static friction are μ_A and μ_B, respectively. Determine the smallest value of θ for which the rod will not move.



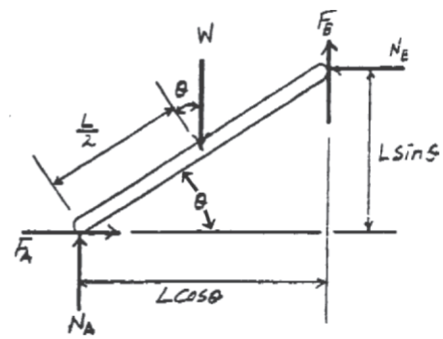
SOLUTION

Equations of Equilibrium:

$\xrightarrow{+} \Sigma F_x = 0; \qquad F_A - N_B = 0$ **(1)**

$+\uparrow \Sigma F_y = 0 \qquad N_A + F_B - W = 0$ **(2)**

$\circlearrowleft + \Sigma M_A = 0; \qquad N_B(L \sin\theta) + F_B(\cos\theta)L - W\cos\theta\left(\frac{L}{2}\right) = 0$ **(3)**



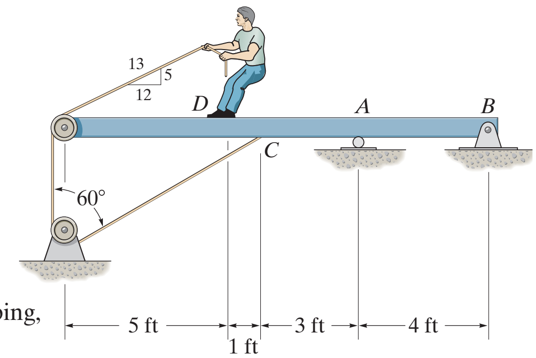
Friction: If the rod is on the verge of moving, slipping will have to occur at points A and B. Hence, $F_A = \mu_A N_A$ and $F_B = \mu_B N_B$. Substituting these values into Eqs. (1), (2), and (3) and solving we have

$$N_A = \frac{W}{1 + \mu_A \mu_B} \qquad N_B = \frac{\mu_A W}{1 + \mu_A \mu_B}$$

$$\theta = \tan^{-1}\left(\frac{1 - \mu_A \mu_B}{2\mu_A}\right)$$ **Ans.**

8–37.

The 80-lb boy stands on the beam and pulls on the cord with a force large enough to just cause him to slip. If the coefficient of static friction between his shoes and the beam is $(\mu_s)_D = 0.4$, determine the reactions at A and B. The beam is uniform and has a weight of 100 lb. Neglect the size of the pulleys and the thickness of the beam.

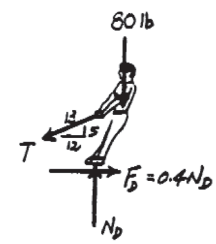


SOLUTION

Equations of Equilibrium and Friction: When the boy is on the verge of slipping, then $F_D = (\mu_s)_D N_D = 0.4N_D$. From FBD (a),

$$+\uparrow \Sigma F_y = 0; \qquad N_D - T\left(\frac{5}{13}\right) - 80 = 0 \qquad \textbf{(1)}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 0.4N_D - T\left(\frac{12}{13}\right) = 0 \qquad \textbf{(2)}$$



(a)

Solving Eqs. (1) and (2) yields

$$T = 41.6 \text{ lb} \qquad N_D = 96.0 \text{ lb}$$

Hence, $F_D = 0.4(96.0) = 38.4$ lb. From FBD (b),

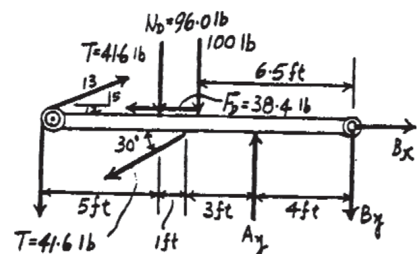
$$\circlearrowleft + \Sigma M_B = 0; \qquad 100(6.5) + 96.0(8) - 41.6\left(\frac{5}{13}\right)(13)$$

$$+ 41.6(13) + 41.6 \sin 30°(7) - A_y(4) = 0$$

$$A_y = 474.1 \text{ lb} = 474 \text{ lb} \qquad \textbf{Ans.}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad B_x + 41.6\left(\frac{12}{13}\right) - 38.4 - 41.6 \cos 30° = 0$$

$$B_x = 36.0 \text{ lb} \qquad \textbf{Ans.}$$



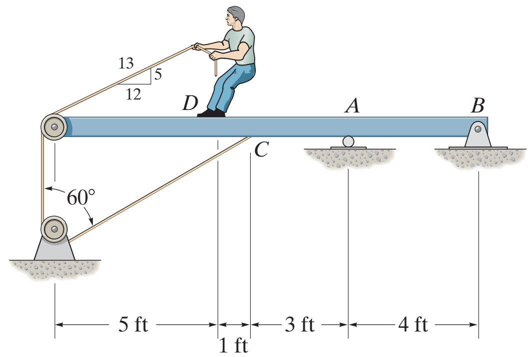
(b)

$$+\uparrow \Sigma F_y = 0; \qquad 474.1 + 41.6\left(\frac{5}{13}\right) - 41.6 - 41.6 \sin 30° - 96.0 - 100 - B_y = 0$$

$$B_y = 231.7 \text{ lb} = 232 \text{ lb} \qquad \textbf{Ans.}$$

8–38.

The 80-lb boy stands on the beam and pulls with a force of 40 lb. If $(\mu_s)_D = 0.4$, determine the frictional force between his shoes and the beam and the reactions at A and B. The beam is uniform and has a weight of 100 lb. Neglect the size of the pulleys and the thickness of the beam.

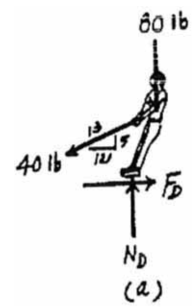


SOLUTION

Equations of Equilibrium and Friction: From FBD (a),

$$+\uparrow \Sigma F_y = 0; \qquad N_D - 40\left(\frac{5}{13}\right) - 80 = 0 \qquad N_D = 95.38 \text{ lb}$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad F_D - 40\left(\frac{12}{13}\right) = 0 \qquad F_D = 36.92 \text{ lb}$$



Since $(F_D)_{\max} = (\mu_s)N_D = 0.4(95.38) = 38.15 \text{ lb} > F_D$, then the boy does not slip. Therefore, the friction force developed is

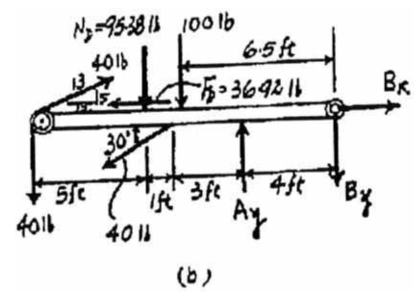
$$F_D = 36.92 \text{ lb} = 36.9 \text{ lb} \qquad \textbf{Ans.}$$

From FBD (b),

$$\circlearrowleft + \Sigma M_B = 0; \qquad 100(6.5) + 95.38(8) - 40\left(\frac{5}{13}\right)(13)$$

$$+ 40(13) + 40 \sin 30°(7) - A_y(4) = 0$$

$$A_y = 468.27 \text{ lb} = 468 \text{ lb} \qquad \textbf{Ans.}$$



$$\xrightarrow{+} \Sigma F_x = 0; \qquad B_x + 40\left(\frac{12}{13}\right) - 36.92 - 40 \cos 30° = 0$$

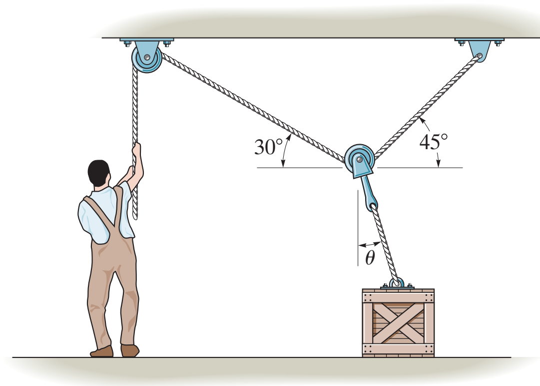
$$B_x = 34.64 \text{ lb} = 34.6 \text{ lb} \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad 468.27 + 40\left(\frac{5}{13}\right) - 40 - 40 \sin 30° - 95.38 - 100 - B_y = 0$$

$$B_y = 228.27 \text{ lb} = 228 \text{ lb} \qquad \textbf{Ans.}$$

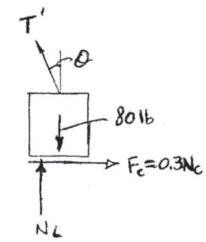
8–39.

Determine the smallest force the man must exert on the rope in order to move the 80-kg crate. Also, what is the angle θ at this moment? The coefficient of static friction between the crate and the floor is $\mu_s = 0.3$.



SOLUTION

Crate:



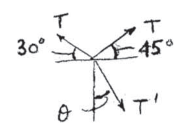
$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad 0.3N_C - T' \sin\theta = 0 \qquad \textbf{(1)}$$

$$+\uparrow\Sigma F_y = 0; \qquad N_C + T' \cos\theta - 80(9.81) = 0 \qquad \textbf{(2)}$$

Pulley:

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad -T\cos 30^\circ + T\cos 45^\circ + T' \sin\theta = 0$$

$$+\uparrow\Sigma F_y = 0; \qquad T\sin 30^\circ + T\sin 45^\circ - T' \cos\theta = 0$$



Thus,

$$T = 6.29253\,T' \sin\theta$$

$$T = 0.828427\,T' \cos\theta$$

$$\theta = \tan^{-1}\left(\frac{0.828427}{6.29253}\right) = 7.50^\circ \qquad \textbf{Ans.}$$

$$T = 0.82134\,T' \qquad \textbf{(3)}$$

From Eqs. (1) and (2),

$$N_C = 239 \text{ N}$$

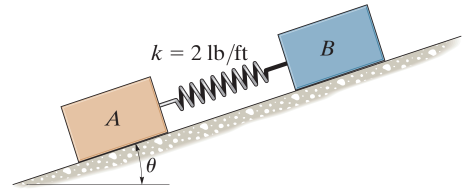
$$T' = 550 \text{ N}$$

So that

$$T = 452 \text{ N} \qquad \textbf{Ans.}$$

***8–40.**

Two blocks A and B have a weight of 10 lb and 6 lb, respectively. They are resting on the incline for which the coefficients of static friction are $\mu_A = 0.15$ and $\mu_B = 0.25$. Determine the incline angle θ for which both blocks begin to slide. Also find the required stretch or compression in the connecting spring for this to occur. The spring has a stiffness of $k = 2$ lb/ft.

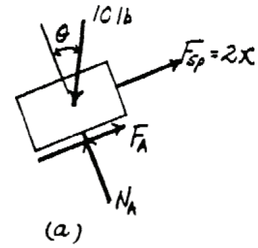


SOLUTION

Equations of Equilibrium: Using the spring force formula, $F_{sp} = kx = 2x$, from FBD (a),

$+\nearrow\Sigma F_{x'} = 0; \qquad 2x + F_A - 10 \sin\theta = 0$ **(1)**

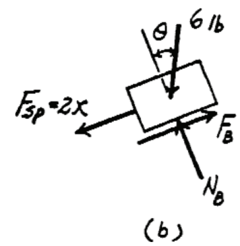
$\nwarrow + \Sigma F_{y'} = 0; \qquad N_A - 10 \cos\theta = 0$ **(2)**



From FBD (b),

$+\nearrow\Sigma F_{x'} = 0; \qquad F_B - 2x - 6 \sin\theta = 0$ **(3)**

$\nwarrow + \Sigma F_{y'} = 0; \qquad N_B - 6 \cos\theta = 0$ **(4)**



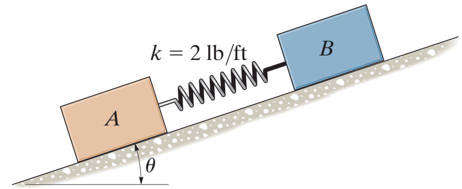
Friction: If block A and B are on the verge to move, slipping would have to occur at point A and B. Hence. $F_A = \mu_{sA} N_A = 0.15 N_A$ and $F_B = \mu_{sB} N_B = 0.25 N_B$. Substituting these values into Eqs. (1), (2), (3) and (4) and solving we have

$$\theta = 10.6^\circ \qquad x = 0.184 \text{ ft}$$ **Ans.**

$$N_A = 9.829 \text{ lb} \qquad N_B = 5.897 \text{ lb}$$

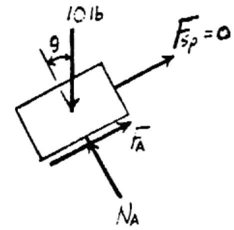
8–41.

Two blocks A and B have a weight of 10 lb and 6 lb, respectively. They are resting on the incline for which the coefficients of static friction are $\mu_A = 0.15$ and $\mu_B = 0.25$. Determine the angle θ which will cause motion of one of the blocks. What is the friction force under each of the blocks when this occurs? The spring has a stiffness of $k = 2$ lb/ft and is originally unstretched.



SOLUTION

Equations of Equilibrium: Since neither block A nor block B is moving yet, the spring force $F_{sp} = 0$. From FBD (a),

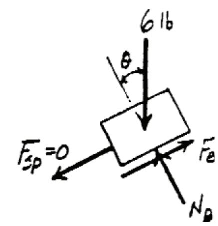


(a)

$$+\nearrow\Sigma F_{x'} = 0; \qquad F_A - 10\sin\theta = 0 \tag{1}$$

$$\nwarrow+\Sigma F_{y'} = 0; \qquad N_A - 10\cos\theta = 0 \tag{2}$$

From FBD (b),



$$+\nearrow\Sigma F_{x'} = 0; \qquad F_B - 6\sin\theta = 0 \tag{3}$$

$$\nwarrow+\Sigma F_{y'} = 0; \qquad N_B - 6\cos\theta = 0 \tag{4}$$

Friction: Assuming block A is on the verge of slipping, then

$$F_A = \mu_A N_A = 0.15N_A \tag{5}$$

Solving Eqs. (1), (2), (3), (4), and (5) yields

$$\theta = 8.531^\circ \qquad N_A = 9.889 \text{ lb} \qquad F_A = 1.483 \text{ lb}$$

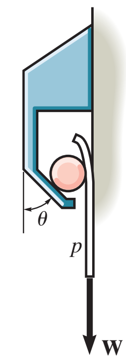
$$F_B = 0.8900 \text{ lb} \qquad N_B = 5.934 \text{ lb}$$

Since $(F_B)_{max} = \mu_B N_B = 0.25(5.934) = 1.483 \text{ lb} > F_B$, block B does not slip. Therefore, the above assumption is correct. Thus

$$\theta = 8.53^\circ \qquad F_A = 1.48 \text{ lb} \qquad F_B = 0.890 \text{ lb} \qquad \textbf{Ans.}$$

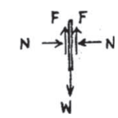
8–42.

The friction hook is made from a fixed frame which is shown colored and a cylinder of negligible weight. A piece of paper is placed between the smooth wall and the cylinder. If $\theta = 20°$, determine the smallest coefficient of static friction μ at all points of contact so that any weight W of paper p can be held.



SOLUTION

Paper:

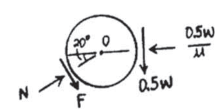


$+\uparrow \Sigma F_y = 0; \qquad F = 0.5W$

$F = \mu N; \qquad F = \mu N$

$$N = \frac{0.5W}{\mu}$$

Cylinder:



$\circlearrowleft + \Sigma M_O = 0; \qquad F = 0.5W$

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad N \cos 20° + F \sin 20° - \dfrac{0.5W}{\mu} = 0$

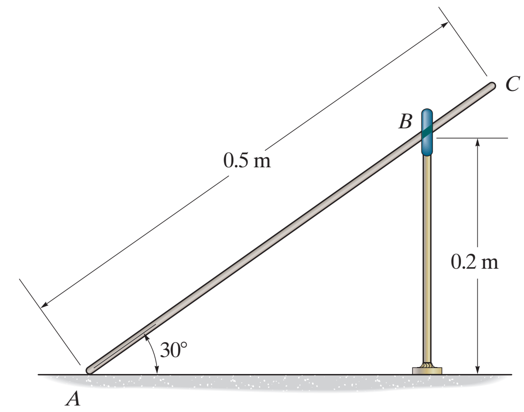
$+\uparrow \Sigma F_y = 0; \qquad N \sin 20° - F \cos 20° - 0.5W = 0$

$F = \mu N; \qquad \mu^2 \sin 20° + 2\mu \cos 20° - \sin 20° = 0$

$\mu = 0.176$ **Ans.**

8–43.

The uniform rod has a mass of 10 kg and rests on the inside of the smooth ring at B and on the ground at A. If the rod is on the verge of slipping, determine the coefficient of static friction between the rod and the ground.



SOLUTION

$\circlearrowleft + \Sigma m_A = 0; \qquad N_B(0.4) - 98.1(0.25 \cos 30°) = 0$

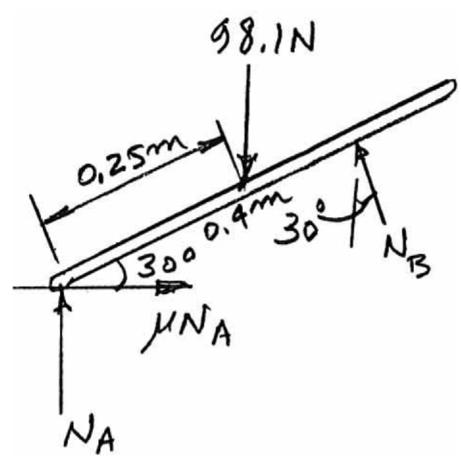
$N_B = 53.10 \text{ N}$

$+\uparrow \Sigma F_y = 0; \qquad N_A - 98.1 + 53.10 \cos 30° = 0$

$N_A = 52.12 \text{ N}$

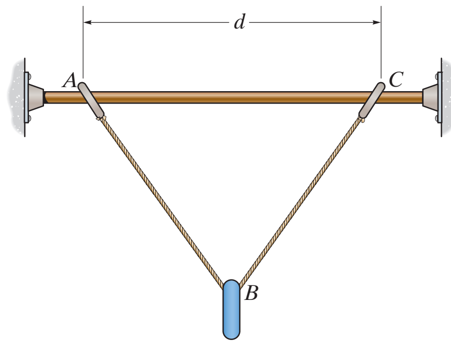
$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad \mu(52.12) - 53.10 \sin 30° = 0$

$\mu = 0.509$ **Ans.**



***8–44.**

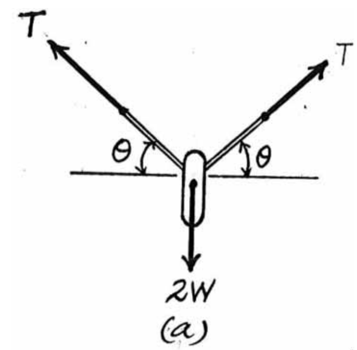
The rings A and C each weigh W and rest on the rod, which has a coefficient of static friction of μ_s. If the suspended ring at B has a weight of $2W$, determine the largest distance d between A and C so that no motion occurs. Neglect the weight of the wire. The wire is smooth and has a total length of l.



SOLUTION

Free-Body Diagram: The tension developed in the wire can be obtained by considering the equilibrium of the free-body diagram shown in Fig. *a*.

$$+\uparrow \Sigma F_y = 0; \qquad 2T\sin\theta - 2w = 0 \qquad T = \frac{w}{\sin\theta}$$



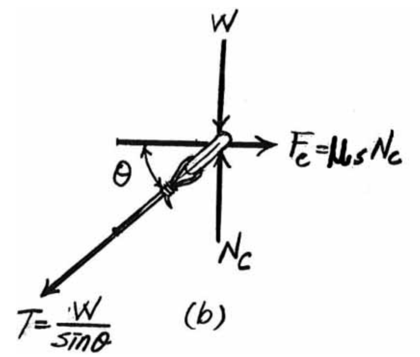
Due to the symmetrical loading and system, rings A and C will slip simultaneously. Thus, it's sufficient to consider the equilibrium of either ring. Here, the equilibrium of ring C will be considered. Since ring C is required to be on the verge of sliding to the left, the friction force $\mathbf{F}_C$ must act to the right such that $F_C = \mu_s N_C$ as indicated on the free-body diagram of the ring shown in Fig. *b*.

Equations of Equilibrium: Using the result of **T** and referring to Fig. *b*, we have

$$+\uparrow \Sigma F_y = 0; \qquad N_C - w - \left[\frac{W}{\sin\theta}\right]\sin\theta = 0 \qquad N_C = 2w$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad \mu_s(2w) - \left[\frac{W}{\sin\theta}\right]\cos\theta = 0$$

$$\tan\theta = \frac{1}{2\mu_s}$$

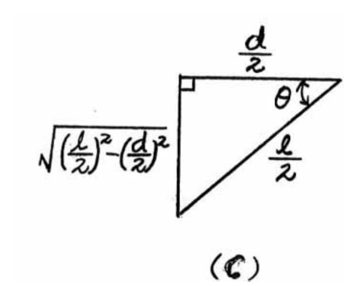


From the geometry of Fig. *c*, we find that $\tan\theta = \dfrac{\sqrt{\left(\frac{l}{2}\right)^2 - \left(\frac{d}{2}\right)^2}}{\frac{d}{2}} = \dfrac{\sqrt{l^2 - d^2}}{d}$.

Thus,

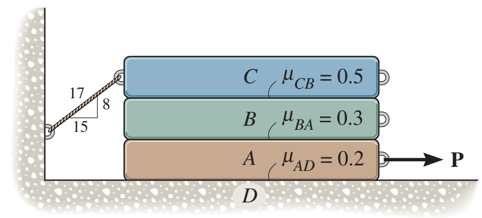
$$\frac{\sqrt{l^2 - d^2}}{d} = \frac{1}{2\mu_s}$$

$$d = \frac{2\mu_s l}{\sqrt{1 + 4\mu_s^{\,2}}} \qquad \textbf{Ans.}$$



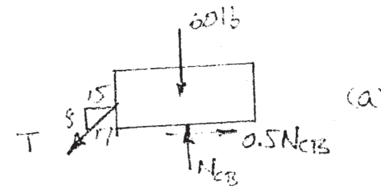
8–45.

The three bars have a weight of $W_A = 20$ lb, $W_B = 40$ lb, and $W_C = 60$ lb, respectively. If the coefficients of static friction at the surfaces of contact are as shown, determine the smallest horizontal force P needed to move block A.



SOLUTION

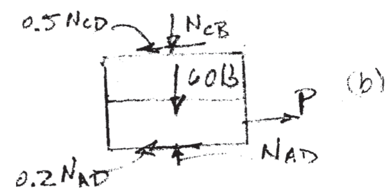
Equations of Equilibrium and Friction: If blocks A and B move together, then slipping will have to occur at the contact surfaces CB and AD. Hence, $F_{CB} = \mu_{s\,CB} N_{CB} = 0.5N_{CB}$ and $F_{AD} = \mu_{s\,AD} N_{AD} = 0.2N_{AD}$. From FBD (a)



$$+\uparrow \Sigma F_y = 0; \qquad N_{CB} - T\left(\frac{8}{17}\right) - 60 = 0 \qquad \textbf{(1)}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 0.5N_{CB} - T\left(\frac{15}{17}\right) = 0 \qquad \textbf{(2)}$$

and FBD (b)



$$+\uparrow \Sigma F_y = 0; \qquad N_{AD} - N_{CB} - 60 = 0 \qquad \textbf{(3)}$$

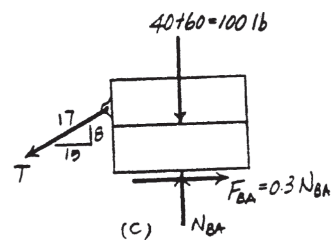
$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad P - 0.5N_{CB} - 0.2N_{AD} = 0 \qquad \textbf{(4)}$$

Solving Eqs. (1), (2), (3), and (4) yields

$$T = 46.36 \text{ lb} \qquad N_{CB} = 81.82 \text{ lb} \qquad N_{AD} = 141.82 \text{ lb}$$

$$P = 69.27 \text{ lb}$$

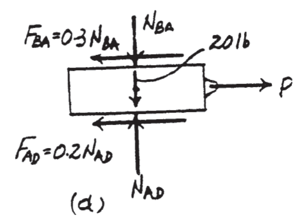
If only block A moves, then slipping will have to occur at contact surfaces BA and AD. Hence, $F_{BA} = \mu_{s\,BA} N_{BA} = 0.3N_{BA}$ and $F_{AD} = \mu_{s\,AD} N_{AD} = 0.2N_{AD}$. From FBD (c)



$$+\uparrow \Sigma F_y = 0; \qquad N_{BA} - T\left(\frac{8}{17}\right) - 100 = 0 \qquad \textbf{(5)}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 0.3N_{BA} - T\left(\frac{15}{17}\right) = 0 \qquad \textbf{(6)}$$

and FBD (d)



$$+\uparrow \Sigma F_y = 0; \qquad N_{AD} - N_{BA} - 20 = 0 \qquad \textbf{(7)}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad P - 0.3N_{BA} - 0.2N_{AD} = 0 \qquad \textbf{(8)}$$

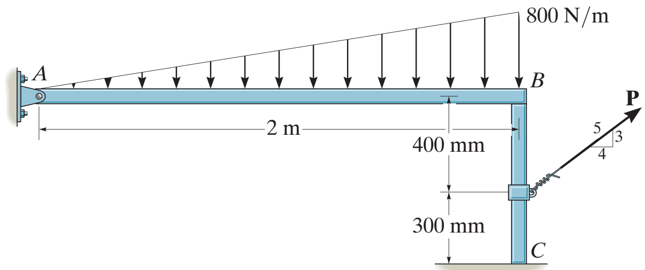
Solving Eqs. (5),(6),(7), and (8) yields

$$T = 40.48 \text{ lb} \qquad N_{BA} = 119.05 \text{ lb} \qquad N_{AD} = 139.05 \text{ lb}$$

$$P = 63.52 \text{ lb} = 63.5 \text{ lb} \quad (\textbf{\textit{Control!}})$$ **Ans.**

8–46.

The beam AB has a negligible mass and thickness and is subjected to a triangular distributed loading. It is supported at one end by a pin and at the other end by a post having a mass of 50 kg and negligible thickness. Determine the minimum force P needed to move the post. The coefficients of static friction at B and C are $\mu_B = 0.4$ and $\mu_C = 0.2$, respectively.

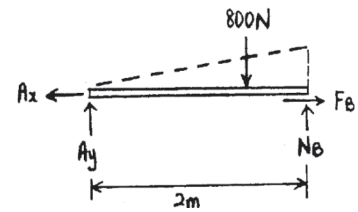


SOLUTION

Member AB:

$$\circlearrowleft + \Sigma M_A = 0; \qquad -800\left(\frac{4}{3}\right) + N_B\,(2) = 0$$

$$N_B = 533.3 \text{ N}$$



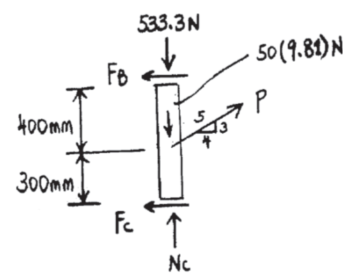
Post:

Assume slipping occurs at C; $F_C = 0.2\,N_C$

$$\circlearrowleft + \Sigma M_C = 0; \qquad -\frac{4}{5}P(0.3) + F_B(0.7) = 0$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad \frac{4}{5}P - F_B - 0.2N_C = 0$$

$$+\uparrow \Sigma F_y = 0; \qquad \frac{3}{5}P + N_C - 533.3 - 50(9.81) = 0$$



$P = 355 \text{ N}$ **Ans.**

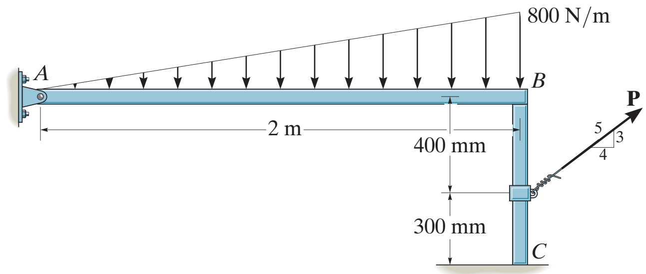
$N_C = 811.0 \text{ N}$

$F_B = 121.6 \text{ N}$

$(F_B)_{max} = 0.4(533.3) = 213.3 \text{ N} > 121.6 \text{ N}$ **(O.K.!)**

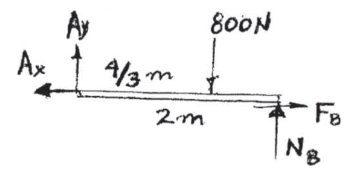
8–47.

The beam AB has a negligible mass and thickness and is subjected to a triangular distributed loading. It is supported at one end by a pin and at the other end by a post having a mass of 50 kg and negligible thickness. Determine the two coefficients of static friction at B and at C so that when the magnitude of the applied force is increased to $P = 150$ N, the post slips at both B and C simultaneously.



SOLUTION

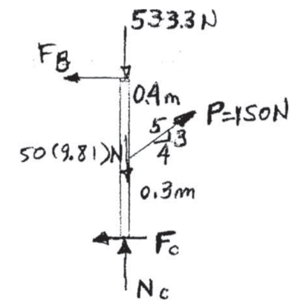
Member AB:



$\zeta + \Sigma M_A = 0; \qquad -800\left(\frac{4}{3}\right) + N_B(2) = 0$

$N_B = 533.3 \text{ N}$

Post:



$+\uparrow \Sigma F_y = 0; \qquad N_C - 533.3 + 150\left(\frac{3}{5}\right) - 50(9.81) = 0$

$N_C = 933.83 \text{ N}$

$\zeta + \Sigma M_C = 0; \qquad -\frac{4}{5}(150)(0.3) + F_B(0.7) = 0$

$F_B = 51.429 \text{ N}$

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad \frac{4}{5}(150) - F_C - 51.429 = 0$

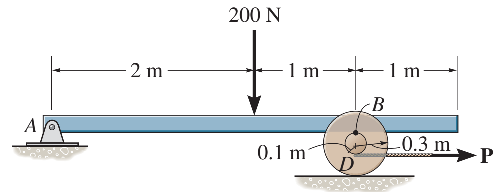
$F_C = 68.571 \text{ N}$

$\mu_C = \frac{F_C}{N_C} = \frac{68.571}{933.83} = 0.0734$ **Ans.**

$\mu_B = \frac{F_B}{N_B} = \frac{51.429}{533.3} = 0.0964$ **Ans.**

***8–48.**

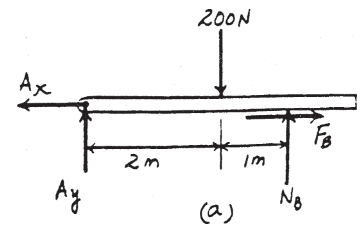
The beam AB has a negligible mass and thickness and is subjected to a force of 200 N. It is supported at one end by a pin and at the other end by a spool having a mass of 40 kg. If a cable is wrapped around the inner core of the spool, determine the minimum cable force P needed to move the spool. The coefficients of static friction at B and D are $\mu_B = 0.4$ and $\mu_D = 0.2$, respectively.



SOLUTION

Equations of Equilibrium: From FBD (a),

$\circlearrowleft + \Sigma M_A = 0; \qquad N_B(3) - 200(2) = 0 \qquad N_B = 133.33\text{ N}$

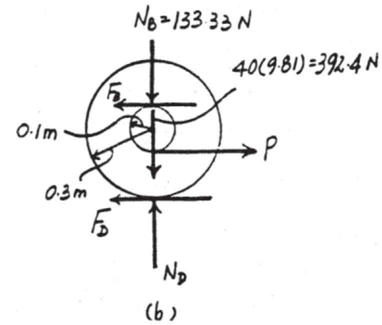


From FBD (b),

$+\uparrow \Sigma F_y = 0 \qquad N_D - 133.33 - 392.4 = 0 \qquad N_D = 525.73\text{ N}$

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad P - F_B - F_D = 0 \qquad$ **(1)**

$\circlearrowleft + \Sigma M_D = 0; \qquad F_B(0.4) - P(0.2) = 0 \qquad$ **(2)**



Friction: Assuming the spool slips at point B, then $F_B = \mu_{s\,B} N_B = 0.4(133.33) = 53.33$ N. Substituting this value into Eqs. (1) and (2) and solving, we have

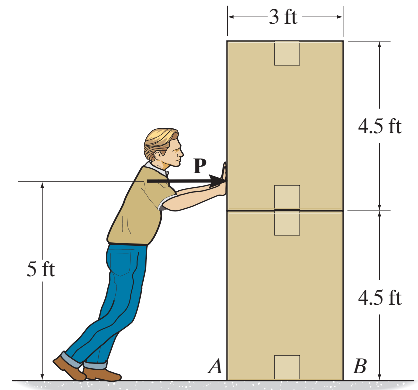
$$F_D = 53.33\text{ N}$$

$$P = 106.67\text{ N} = 107\text{ N} \qquad \textbf{Ans.}$$

Since $(F_D)_{max} = \mu_{s\,D} N_D = 0.2(525.73) = 105.15\text{ N} > F_D$, the spool does not slip at point D. Therefore the above assumption is correct.

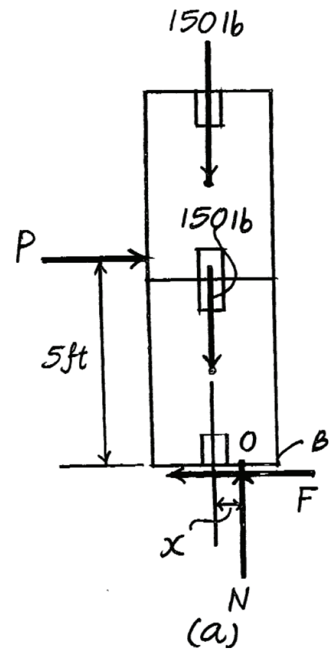
8–49.

If each box weighs 150 lb, determine the least horizontal force P that the man must exert on the top box in order to cause motion. The coefficient of static friction between the boxes is $\mu_s = 0.5$, and the coefficient of static friction between the box and the floor is $\mu_s' = 0.2$.



SOLUTION

Free-Body Diagram: There are three possible motions, namely (1) the top box slides, (2) both boxes slide together as a single unit on the ground, and (3) both boxes tip as a single unit about point B. We will assume that both boxes slide together as a single unit such that $F = \mu_s' N = 0.2N$ as indicated on the free-body diagram shown in Fig. *a*.



Equations of Equilibrium:

$+\uparrow \Sigma F_y = 0;\qquad N - 150 - 150 = 0$

$\overset{+}{\rightarrow} \Sigma F_x = 0;\qquad P - 0.2N = 0$

$\circlearrowleft + \Sigma M_O = 0;\qquad 150(x) + 150(x) - P(5) = 0$

Solving,

$$N = 300 \qquad x = 1 \text{ ft}$$

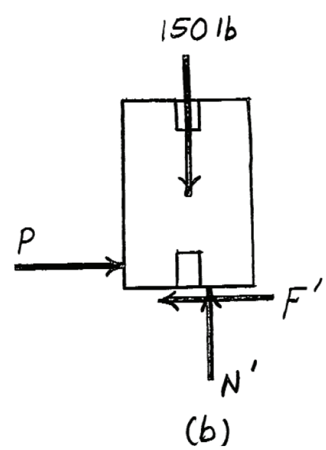
$$P = 60 \text{ lb}$$ **Ans.**

Since $x < 1.5$ ft, both boxes will not tip about point B. Using the result of **P** and considering the equilibrium of the free-body diagram shown in Fig. *b*, we have

$+\uparrow \Sigma F_y = 0;\qquad N' - 150 = 0 \qquad N' = 150 \text{ lb}$

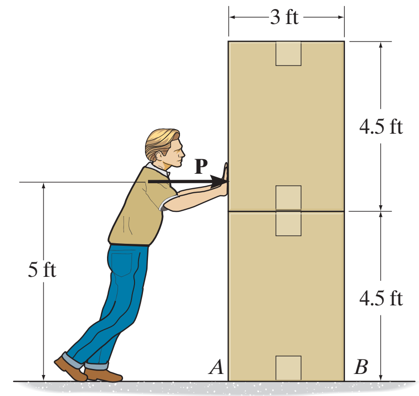
$\overset{+}{\rightarrow} \Sigma F_x = 0;\qquad 60 - F' = 0 \qquad F' = 60 \text{ lb}$

Since $F' < F_{max} = \mu_s N' = 0.5(150) = 75$ lb, the top box will not slide. Thus, the above assumption is correct.



8–50.

If each box weighs 150 lb, determine the least horizontal force P that the man must exert on the top box in order to cause motion. The coefficient of static friction between the boxes is $\mu_s = 0.65$, and the coefficient of static friction between the box and the floor is $\mu_s' = 0.35$.



SOLUTION

Free-Body Diagram: There are three possible motions, namely (1) the top box slides, (2) both boxes slide together as a single unit on the ground, and (3) both boxes tip as a single unit about point B. We will assume that both boxes tip as a single unit about point B. Thus, $x = 1.5$ ft.

Equations of Equilibrium: Referring to Fig. *a*,

$+\uparrow \Sigma F_y = 0; \qquad N - 150 - 150 = 0$

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad P - F = 0$

$\circlearrowleft + \Sigma M_B = 0; \qquad 150(1.5) + 150(1.5) - P(5) = 0$

Solving,

$$P = 90 \text{ lb} \qquad \textbf{Ans.}$$

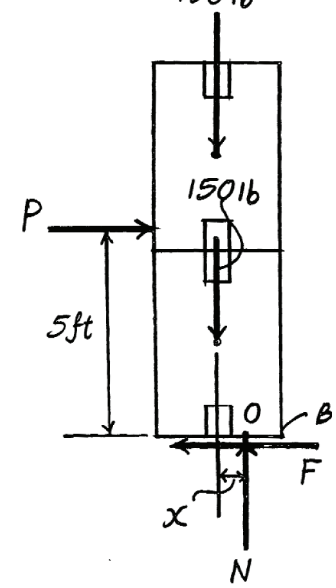
$$N = 300 \text{ lb} \quad F = 90 \text{ lb}$$

Since $F < F_{max} = \mu_s N' = 0.35(300) = 105$ lb, both boxes will not slide as a single unit on the floor. Using the result of **P** and considering the equilibrium of the free-body diagram shown in Fig. *b*,

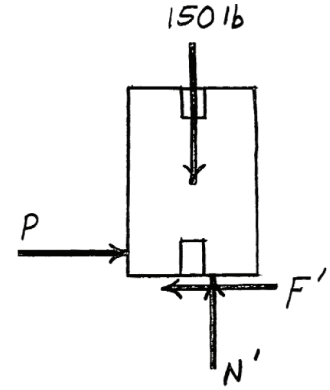
$+\uparrow \Sigma F_y = 0; \qquad N' - 150 = 0 \qquad N' = 150 \text{ lb}$

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 90 - F' = 0 \qquad F' = 90 \text{ lb}$

Since $F' < F_{max} = \mu_s' N' = 0.65(150) = 97.5$ lb, the top box will not slide. Thus, the above assumption is correct.



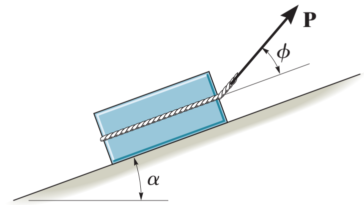
(a)



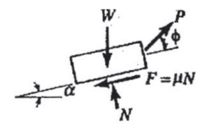
(b)

8–51.

The block of weight W is being pulled up the inclined plane of slope α using a force $\mathbf{P}$. If $\mathbf{P}$ acts at the angle ϕ as shown, show that for slipping to occur, $P = W\sin(\alpha + \theta)/\cos(\phi - \theta)$, where θ is the angle of friction; $\theta = \tan^{-1}\mu$.



SOLUTION



$\nearrow + \Sigma F_x = 0;\qquad P\cos\phi - W\sin\alpha - \mu N = 0$

$+\nwarrow \Sigma F_y = 0;\qquad N - W\cos\alpha + P\sin\phi = 0$

$$P\cos\phi - W\sin\alpha - \mu(W\cos\alpha - P\sin\phi) = 0$$

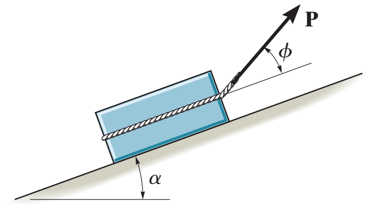
$$P = W\left(\frac{\sin\alpha + \mu\cos\alpha}{\cos\phi + \mu\sin\phi}\right)$$

Let $\mu = \tan\theta$

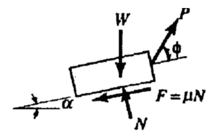
$$P = W\left(\frac{\sin(\alpha + \theta)}{\cos(\phi - \theta)}\right)$$ **(QED)**

***8–52.**

Determine the angle ϕ at which **P** should act on the block so that the magnitude of **P** is as small as possible to begin pushing the block up the incline. What is the corresponding value of P? The block weighs W and the slope α is known.



SOLUTION



Slipping occurs when $P = W\left(\dfrac{\sin(\alpha + \theta)}{\cos(\phi - \theta)}\right)$ where θ is the angle of friction $\theta = \tan^{-1}\theta$.

$$\frac{dP}{d\phi} = W\left(\frac{\sin(\alpha + \theta)\sin(\phi - \theta)}{\cos^2(\phi - \theta)}\right) = 0$$

$$\sin(\alpha + \theta)\sin(\phi - \theta) = 0$$

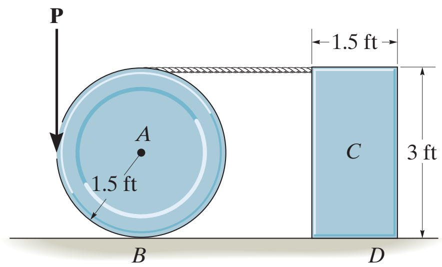
$$\sin(\alpha + \theta) = 0 \qquad \text{or} \qquad \sin(\phi - \theta) = 0$$

$$\alpha = -\theta \qquad\qquad \phi = \theta \quad \textbf{Ans.}$$

$$P = W\sin(\alpha + \phi) \qquad \textbf{Ans.}$$

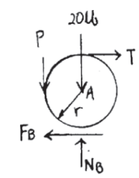
8–53.

The wheel weighs 20 lb and rests on a surface for which $\mu_B = 0.2$. A cord wrapped around it is attached to the top of the 30-lb homogeneous block. If the coefficient of static friction at D is $\mu_D = 0.3$, determine the smallest vertical force that can be applied tangentially to the wheel which will cause motion to impend.



SOLUTION

Cylinder A:



Assume slipping at B, $F_B = 0.2\,N_B$

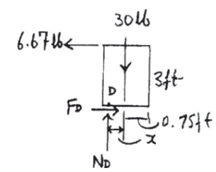
$\zeta + \Sigma M_A = 0;\qquad F_B + T = P$

$\overset{+}{\rightarrow} \Sigma F_x = 0;\qquad F_B = T$

$+\uparrow \Sigma F_y = 0;\qquad N_B = 20 + P$

$N_B = 20 + 2(0.2N_B)$

$N_B = 33.33\text{ lb}$



$F_B = 6.67\text{ lb}$

$T = 6.67\text{ lb}$

$P = 13.3\text{ lb}$ **Ans.**

$\overset{+}{\rightarrow} \Sigma F_x = 0;\qquad F_D = 6.67\text{ lb}$

$+\uparrow \Sigma F_y = 0;\qquad N_D = 30\text{ lb}$

$(F_D)_{\max} = 0.3(30) = 9\text{ lb} > 6.67\text{ lb}$ **(O.K.!)**

No slipping occurs.

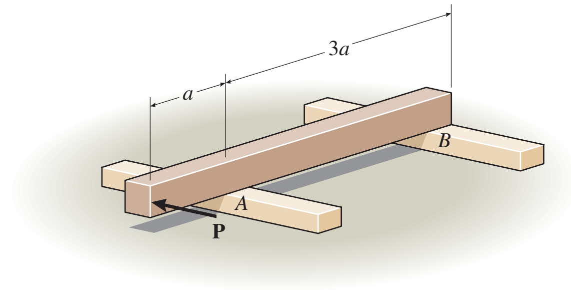
$\zeta + \Sigma M_D = 0;\qquad -30(x) + 6.67(3) = 0$

$x = 0.667\text{ ft} < \dfrac{1.5}{2} = 0.75\text{ ft}$ **(O.K.!)**

No tipping occurs.

8–54.

The uniform beam has a weight W and length $4a$. It rests on the fixed rails at A and B. If the coefficient of static friction at the rails is μ_s, determine the horizontal force P, applied perpendicular to the face of the beam, which will cause the beam to move.



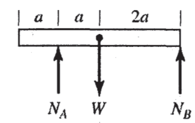
SOLUTION

From FBD (a),

$+\uparrow \Sigma F = 0;\qquad N_A + N_B - W = 0$

$\circlearrowleft + \Sigma M_B = 0;\qquad -N_A(3a) + W(2a) = 0$

$$N_A = \frac{2}{3}W \qquad N_B = \frac{1}{3}W$$



(a) Side view

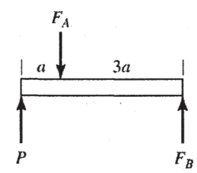
Support A can sustain twice as much static frictional force as support B.

From FBD (b),

$+\uparrow \Sigma F = 0;\qquad P + F_B - F_A = 0$

$\circlearrowleft + \Sigma M_B = 0;\qquad -P(4a) + F_A(3a) = 0$

$$F_A = \frac{4}{3}P \qquad F_B = \frac{1}{3}P$$



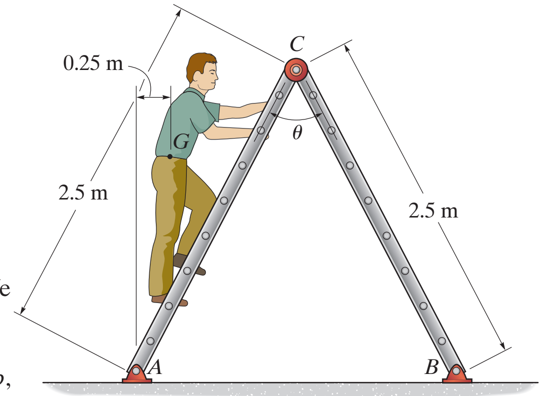
(b) Top view

The frictional load at A is 4 times as great as at B. The beam will slip at A first.

$$P = \frac{3}{4}(F_A)_{\max} = \frac{3}{4}(\mu_s N_A) = \frac{1}{2}\mu_s W \qquad \textbf{Ans.}$$

8–55.

Determine the greatest angle u so that the ladder does not slip when it supports the 75-kg man in the position shown. The surface is rather slippery, where the coefficient of static friction at A and B is $\mu_s = 0.3$.



SOLUTION

Free-Body Diagram: The slipping could occur at either end A or B of the ladder. We will assume that slipping occurs at end B. Thus, $F_B = \mu_s N_B = 0.3N_B$.

Equations of Equilibrium: Referring to the free-body diagram shown in Fig. b, we have

$\xrightarrow{+} \Sigma F_x = 0;\qquad F_{BC}\sin\theta/2 - 0.3N_B = 0$

$$F_{BC}\sin\theta/2 = 0.3N_B \qquad \textbf{(1)}$$

$+\uparrow \Sigma F_y = 0;\qquad N_B - F_{BC}\cos\theta/2 = 0$

$$F_{BC}\cos\theta/2 = N_B \textbf{(2)}$$

Dividing Eq. (1) by Eq. (2) yields

$$\tan\theta/2 = 0.3$$

$$\theta = 33.40° = 33.4° \qquad \textbf{Ans.}$$

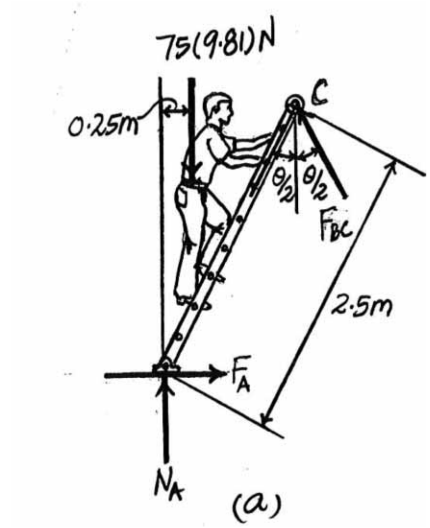
Using this result and referring to the free-body diagram of member AC shown in Fig. a, we have

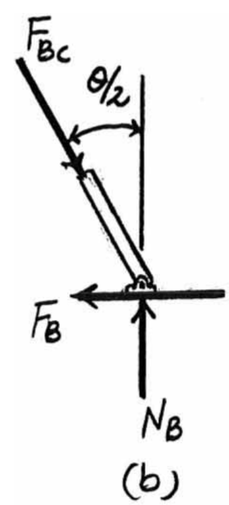
$\circlearrowleft + \Sigma M_A = 0;\qquad F_{BC}\sin 33.40°(2.5) - 75(9.81)(0.25) = 0 \qquad F_{BC} = 133.66\text{ N}$

$\xrightarrow{+} \Sigma F_x = 0;\qquad F_A - 133.66\sin\left(\frac{33.40°}{2}\right) = 0 \qquad F_A = 38.40\text{ N}$

$+\uparrow \Sigma F_y = 0;\qquad N_A + 133.66\cos\left(\frac{33.40°}{2}\right) - 75(9.81) = 0 \qquad N_A = 607.73\text{ N}$

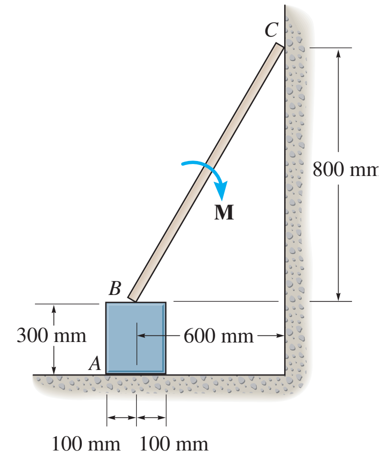
Since $F_A < (F_A)_{\max} = \mu_s N_A = 0.3(607.73) = 182.32$ N, end A will not slip. Thus, the above assumption is correct.





***8–56.**

The uniform 6-kg slender rod rests on the top center of the 3-kg block. If the coefficients of static friction at the points of contact are $\mu_A = 0.4$, $\mu_B = 0.6$, and $\mu_C = 0.3$, determine the largest couple moment M which can be applied to the rod without causing motion of the rod.



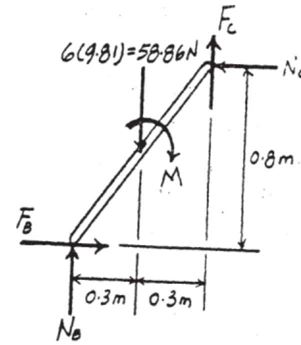
SOLUTION

Equations of Equilibrium: From FBD (a),

$\xrightarrow{+} \Sigma F_x = 0; \qquad F_B - N_C = 0 \qquad$ **(1)**

$+\uparrow \Sigma F_y = 0; \qquad N_B + F_C - 58.86 = 0 \qquad$ **(2)**

$\circlearrowleft + \Sigma M_B = 0; \qquad F_C(0.6) + N_C(0.8) - M - 58.86(0.3) = 0 \qquad$ **(3)**



(a)

From FBD (b),

$+\uparrow \Sigma F_y = 0; \qquad N_A - N_B - 29.43 = 0 \qquad$ **(4)**

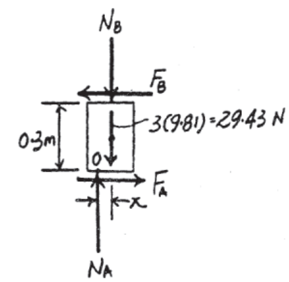
$\xrightarrow{+} \Sigma F_x = 0; \qquad F_A - F_B = 0 \qquad$ **(5)**

$\circlearrowleft + \Sigma M_O = 0; \qquad F_B\,(0.3) - N_B\,(x) - 29.43(x) = 0 \qquad$ **(6)**

Friction: Assume slipping occurs at point C and the block tips, then $F_C = \mu_{sC} N_C = 0.3 N_C$ and $x = 0.1$ m. Substituting these values into Eqs. (1), (2), (3), (4), (5), and (6) and solving, we have

$$M = 8.561\ \text{N}\cdot\text{m} = 8.56\ \text{N}\cdot\text{m} \qquad \textbf{Ans.}$$

$$N_B = 50.83\ \text{N} \qquad N_A = 80.26\ \text{N} \qquad F_A = F_B = N_C = 26.75\ \text{N}$$

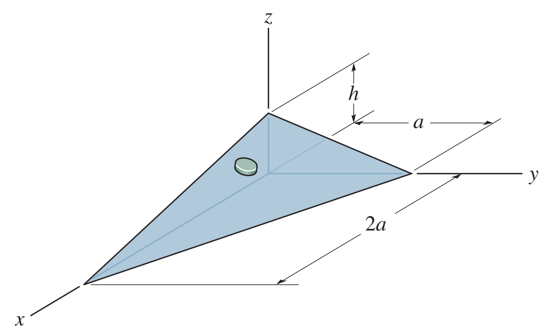


(b)

Since $(F_A)_{max} = \mu_{s\,A} N_A = 0.4(80.26) = 32.11\ \text{N} > F_A$, the block does not slip. Also, $(F_B)_{max} = \mu_{s\,B} N_B = 0.6(50.83) = 30.50\ \text{N} > F_B$, then slipping does not occur at point B. Therefore, the above assumption is correct.

8–57.

The disk has a weight W and lies on a plane which has a coefficient of static friction μ. Determine the maximum height h to which the plane can be lifted without causing the disk to slip.

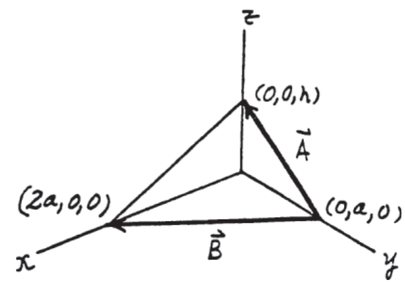


SOLUTION

Unit Vector: The unit vector perpendicular to the inclined plane can be determined using cross product.

$$\mathbf{A} = (0 - 0)\mathbf{i} + (0 - a)\mathbf{j} + (h - 0)\mathbf{k} = -a\mathbf{j} + h\mathbf{k}$$

$$\mathbf{B} = (2a - 0)\mathbf{i} + (0 - a)\mathbf{j} + (0 - 0)\mathbf{k} = 2a\mathbf{i} - a\mathbf{j}$$



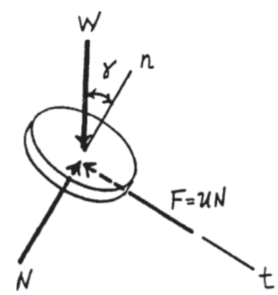
Then

$$\mathbf{N} = \mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -a & h \\ 2a & -a & 0 \end{vmatrix} = ah\mathbf{i} + 2ah\mathbf{j} + 2a^2\mathbf{k}$$

$$n = \frac{\mathbf{N}}{N} = \frac{ah\mathbf{i} + 2ah\mathbf{j} + 2a^2\mathbf{k}}{a\sqrt{5h^2 + 4a^2}}$$

Thus

$$\cos\gamma = \frac{2a}{\sqrt{5h^2 + 4a^2}} \qquad \text{hence} \qquad \sin\gamma = \frac{\sqrt{5}h}{\sqrt{5h^2 + 4a^2}}$$



Equations of Equilibrium and Friction: When the disk is on the verge of sliding down the plane, $F = \mu N$.

$$\Sigma F_n = 0; \qquad N - W\cos\gamma = 0 \qquad N = W\cos\gamma \qquad \textbf{(1)}$$

$$\Sigma F_t = 0; \qquad W\sin\gamma - \mu N = 0 \qquad N = \frac{W\sin\gamma}{\mu} \qquad \textbf{(2)}$$

Divide Eq. (2) by (1) yields

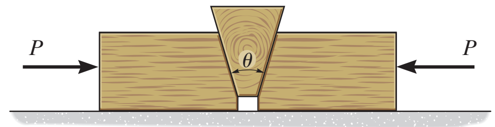
$$\frac{\sin\gamma}{\mu\cos\gamma} = 1$$

$$\frac{\dfrac{\sqrt{5}h}{\sqrt{5h^2 + 4a^2}}}{\mu\left(\dfrac{2a}{\sqrt{5h^2 + 4a^2}}\right)} = 1$$

$$h = \frac{2}{\sqrt{5}}a\mu \qquad \textbf{Ans.}$$

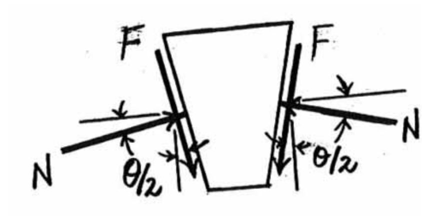
8–58.

Determine the largest angle u that will cause the wedge to be self-locking regardless of the magnitude of horizontal force P applied to the blocks. The coefficient of static friction between the wedge and the blocks is $\mu_s = 0.3$. Neglect the weight of the wedge.



SOLUTION

Free-Body Diagram: For the wedge to be self-locking, the frictional force F indicated on the free-body diagram of the wedge shown in Fig. *a* must act downward and its magnitude must be $F \leq \mu_s N = 0.3N$.



(a)

Equations of Equilibrium: Referring to Fig. *a,* we have

$+\uparrow \Sigma F_y = 0; \qquad 2N \sin\theta/2 - 2F\cos\theta/2 = 0$

$$F = N\tan\theta/2$$

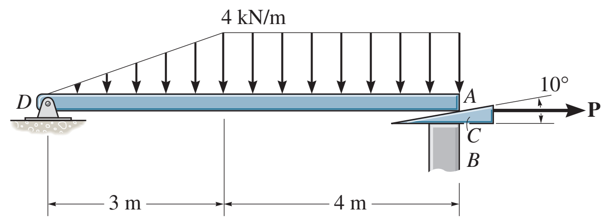
Using the requirement $F \leq 0.3N$, we obtain

$$N\tan\theta/2 \leq 0.3N$$

$$\theta = 33.4^\circ$$ **Ans.**

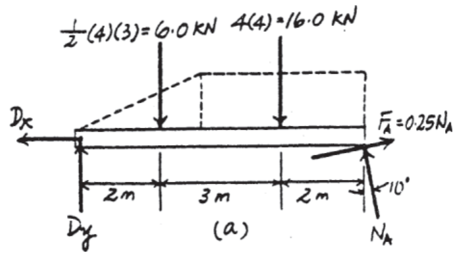
8–59.

If the beam AD is loaded as shown, determine the horizontal force P which must be applied to the wedge in order to remove it from under the beam. The coefficients of static friction at the wedge's top and bottom surfaces are $\mu_{CA} = 0.25$ and $\mu_{CB} = 0.35$, respectively. If $P = 0$, is the wedge self-locking? Neglect the weight and size of the wedge and the thickness of the beam.



SOLUTION

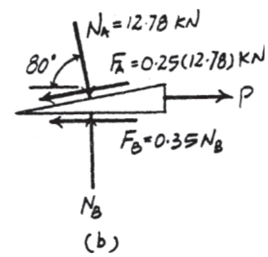
Equations of Equilibrium and Friction: If the wedge is on the verge of moving to the right, then slipping will have to occur at both contact surfaces. Thus, $F_A = \mu_{s\,A} N_A = 0.25N_A$ and $F_B = \mu_{s\,B} N_B = 0.35N_B$. From FBD (a),



$$\circlearrowleft + \Sigma M_D = 0; \qquad N_A \cos 10°(7) + 0.25N_A \sin 10°(7) - 6.00(2) - 16.0(5) = 0$$

$$N_A = 12.78 \text{ kN}$$

From FBD (b),



$$+\uparrow \Sigma F_y = 0; \qquad N_B - 12.78 \sin 80° - 0.25(12.78) \sin 10° = 0$$

$$N_B = 13.14 \text{ kN}$$

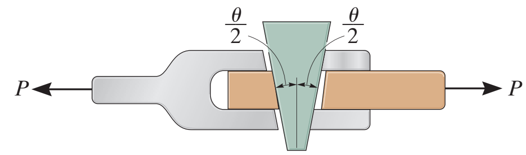
$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad P + 12.78 \cos 80° - 0.25(12.78) \cos 10° - 0.35(13.14) = 0$$

$$P = 5.53 \text{ kN} \qquad \textbf{Ans.}$$

Since a force P (> 0) is required to pull out the wedge, **the wedge will be self-locking when** $P = 0$. **Ans.**

***8–60.**

The wedge has a negligible weight and a coefficient of static friction $\mu_s = 0.35$ with all contacting surfaces. Determine the largest angle θ so that it is "self-locking." This requires no slipping for any magnitude of the force **P** applied to the joint.

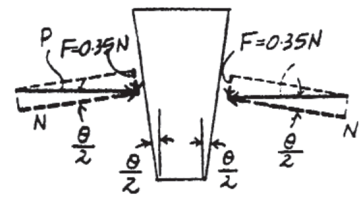


SOLUTION

Friction: When the wedge is on the verge of slipping, then $F = \mu N = 0.35N$. From the force diagram (P is the 'locking' force.),

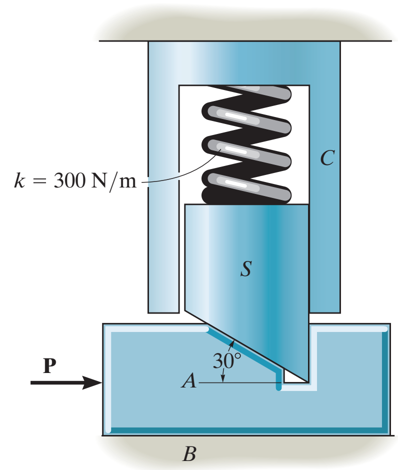
$$\tan\frac{\theta}{2} = \frac{0.35N}{N} = 0.35$$

$$\theta = 38.6° \qquad \textbf{Ans.}$$



8–61.

If the spring is compressed 60 mm and the coefficient of static friction between the tapered stub S and the slider A is $\mu_{SA} = 0.5$, determine the horizontal force $\mathbf{P}$ needed to move the slider forward. The stub is free to move without friction within the fixed collar C. The coefficient of static friction between A and surface B is $\mu_{AB} = 0.4$. Neglect the weights of the slider and stub.



SOLUTION

Stub:

$+\uparrow \Sigma F_y = 0; \qquad N_A \cos 30° - 0.5N_A \sin 30° - 300(0.06) = 0$

$N_A = 29.22 \text{ N}$

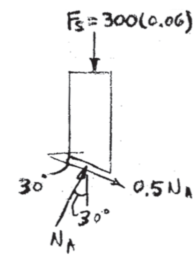
Slider:

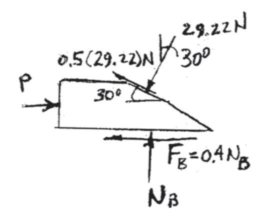
$+\uparrow \Sigma F_y = 0; \qquad N_B - 29.22 \cos 30° + 0.5(29.22) \sin 30° = 0$

$N_B = 18 \text{ N}$

$\xrightarrow{+} \Sigma F_x = 0; \qquad P - 0.4(18) - 29.22 \sin 30° - 0.5(29.22) \cos 30° = 0$

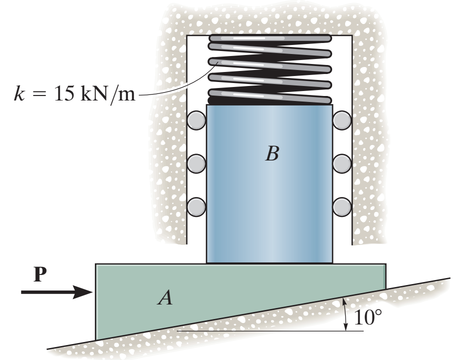
$P = 34.5 \text{ N}$ **Ans.**





8–62.

If $P = 250$ N, determine the required minimum compression in the spring so that the wedge will not move to the right. Neglect the weight of A and B. The coefficient of static friction for all contacting surfaces is $\mu_s = 0.35$. Neglect friction at the rollers.



SOLUTION

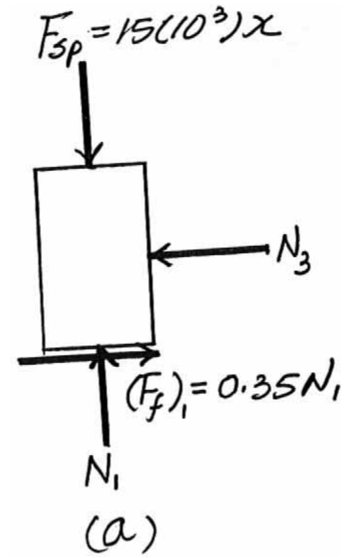
Free-Body Diagram: The spring force acting on the cylinder is $F_{sp} = kx = 15(10^3)x$. Since it is required that the wedge is on the verge to slide to the right, the frictional force must act to the left on the top and bottom surfaces of the wedge and their magnitude can be determined using friction formula.

$(F_f)_1 = \mu N_1 = 0.35N_1 \qquad (F_f)_2 = 0.35N_2$

Equations of Equilibrium: Referring to the FBD of the cylinder, Fig. a,

$+\uparrow \Sigma F_y = 0; \qquad N_1 - 15(10^3)x = 0 \qquad N_1 = 15(10^3)x$

Thus, $(F_f)_1 = 0.35[15(10^3)x] = 5.25(10^3)x$



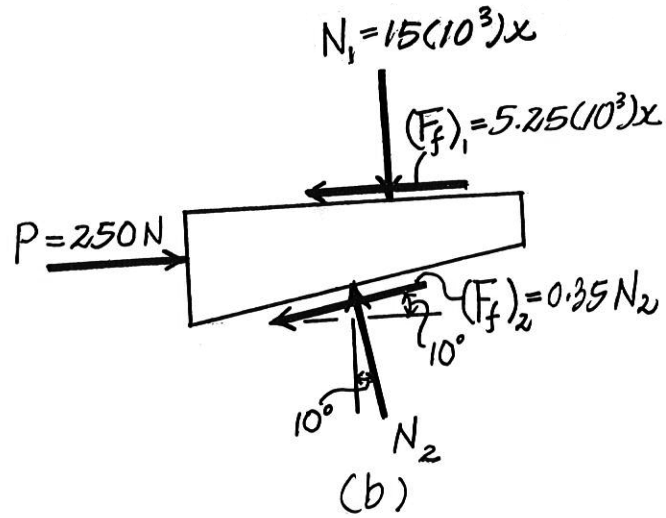
Referring to the FBD of the wedge shown in Fig. b,

$+\uparrow \Sigma F_y = 0; \qquad N_2 \cos 10° - 0.35N_2 \sin 10° - 15(10^3)x = 0$

$$N_2 = 16.233(10^3)x$$

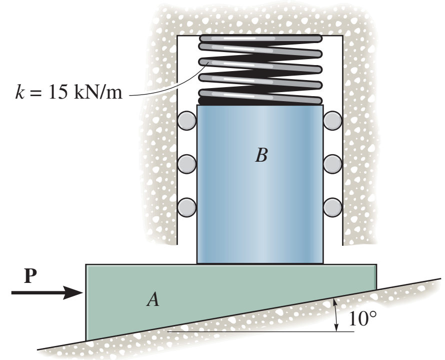
$\xrightarrow{+} \Sigma F_x = 0; \qquad 250 - 5.25(10^3)x - 0.35[16.233(10^3)x]\cos 10° - [16.233(10^3)x]\sin 10° = 0$

$$x = 0.01830\,m = 18.3\,mm$$ **Ans.**



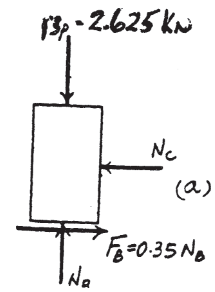
8–63.

Determine the minimum applied force **P** required to move wedge A to the right. The spring is compressed a distance of 175 mm. Neglect the weight of A and B. The coefficient of static friction for all contacting surfaces is $\mu_s = 0.35$. Neglect friction at the rollers.



SOLUTION

Equations of Equilibrium and Friction: Using the spring formula, $F_{sp} = kx = 15(0.175) = 2.625$ kN. If the wedge is on the verge of moving to the right, then slipping will have to occur at both contact surfaces. Thus, $F_A = \mu_s N_A = 0.35 N_A$ and $F_B = \mu_s N_B = 0.35 N_B$. From FBD (a),



$$+\uparrow \Sigma F_y = 0; \qquad N_B - 2.625 = 0 \qquad N_B = 2.625 \text{ kN}$$

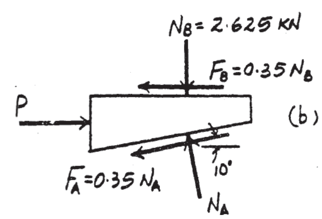
From FBD (b),

$$+\uparrow \Sigma F_y = 0; \qquad N_A \cos 10^\circ - 0.35 N_A \sin 10^\circ - 2.625 = 0$$

$$N_A = 2.841 \text{ kN}$$

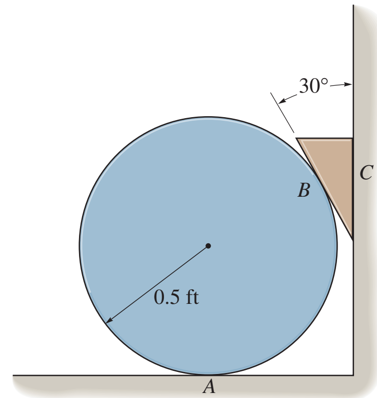
$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad P - 0.35(2.625) - 0.35(2.841)\cos 10^\circ - 2.841 \sin 10^\circ = 0$$

$$P = 2.39 \text{ kN} \qquad \textbf{Ans.}$$



***8–64.**

Determine the largest weight of the wedge that can be placed between the 8-lb cylinder and the wall without upsetting equilibrium. The coefficient of static friction at A and C is $\mu_s = 0.5$ and at B, $\mu_s' = 0.6$.

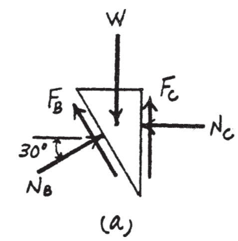


SOLUTION

Equations of Equilibrium: From FBD (a),

$\xrightarrow{+} \Sigma F_x = 0; \qquad N_B \cos 30° - F_B \cos 60° - N_C = 0 \qquad$ **(1)**

$+\uparrow \Sigma F_y = 0; \qquad N_B \sin 30° + F_B \sin 60° + F_C - W = 0 \qquad$ **(2)**



From FBD (b),

$+\uparrow \Sigma F_y = 0; \qquad N_A - N_B \sin 30° - F_B \sin 60° - 8 = 0 \qquad$ **(3)**

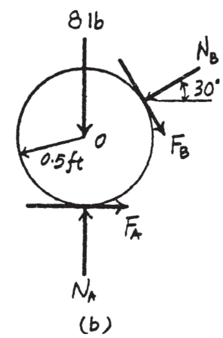
$\xrightarrow{+} \Sigma F_x = 0; \qquad F_A + F_B \cos 60° - N_B \cos 30° = 0 \qquad$ **(4)**

$\circlearrowleft + \Sigma M_O = 0; \qquad F_A (0.5) - F_B (0.5) = 0 \qquad$ **(5)**

Friction: Assume slipping occurs at points C and A, then $F_C = \mu_s N_C = 0.5 N_C$ and $F_A = \mu_s N_A = 0.5 N_A$. Substituting these values into Eqs. (1), (2), (3), (4), and (5) and solving, we have

$$W = 66.64 \text{ lb} = 66.6 \text{ lb} \qquad \textbf{Ans.}$$

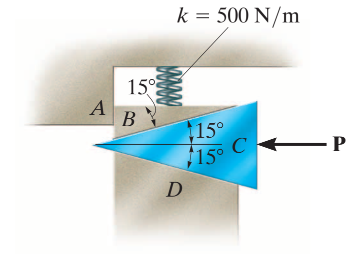
$$N_B = 51.71 \text{ lb} \qquad N_A = 59.71 \text{ lb} \qquad F_B = N_C = 29.86 \text{ lb}$$



Since $(F_B)_{max} = \mu_s' N_B = 0.6(51.71) = 31.03 \text{ lb} > F_B$, slipping does not occur at point B. Therefore, the above assumption is correct.

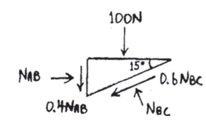
8–65.

The coefficient of static friction between wedges B and C is $\mu_s = 0.6$ and between the surfaces of contact B and A and C and D, $\mu_s' = 0.4$. If the spring is compressed 200 mm when in the position shown, determine the smallest force P needed to move wedge C to the left. Neglect the weight of the wedges.



SOLUTION

Wedge B:



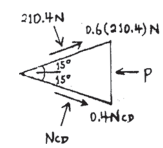
$\xrightarrow{+} \Sigma F_x = 0; \qquad N_{AB} - 0.6N_{BC} \cos 15° - N_{BC} \sin 15° = 0$

$+\uparrow \Sigma F_y = 0; \qquad N_{BC} \cos 15° - 0.6N_{BC} \sin 15° - 0.4N_{AB} - 100 = 0$

$N_{BC} = 210.4 \text{ N}$

$N_{AB} = 176.4 \text{ N}$

Wedge C:



$+\uparrow \Sigma F_y = 0; \qquad N_{CD} \cos 15° - 0.4N_{CD} \sin 15° + 0.6(210.4) \sin 15° - 210.4 \cos 15° = 0$

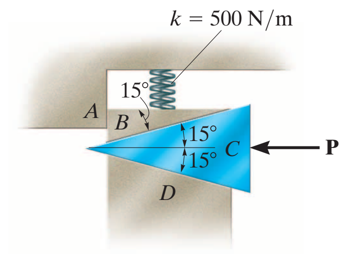
$N_{CD} = 197.8 \text{ N}$

$\xrightarrow{+} \Sigma F_x = 0; \qquad 197.8 \sin 15° + 0.4(197.8) \cos 15° + 210.4 \sin 15° + 0.6(210.4) \cos 15° - P = 0$

$P = 304 \text{ N}$ **Ans.**

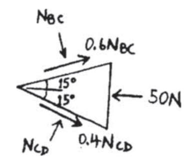
8–66.

The coefficient of static friction between the wedges B and C is $\mu_s = 0.6$ and between the surfaces of contact B and A and C and D, $\mu_s' = 0.4$. If $P = 50$ N, determine the largest allowable compression of the spring without causing wedge C to move to the left. Neglect the weight of the wedges.



SOLUTION

Wedge C:



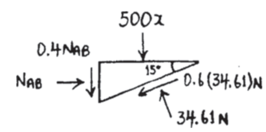
$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad (N_{CD} + N_{BC}) \sin 15° + (0.4N_{CD} + 0.6N_{BC}) \cos 15° - 50 = 0$

$\uparrow + \Sigma F_y = 0; \quad (N_{CD} - N_{BC}) \cos 15° + (-0.4N_{CD} + 0.6N_{BC}) \sin 15° = 0$

$N_{BC} = 34.61$ N

$N_{CD} = 32.53$ N

Wedge B:



$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad N_{AB} - 0.6(34.61) \cos 15° - 34.61 \sin 15° = 0$

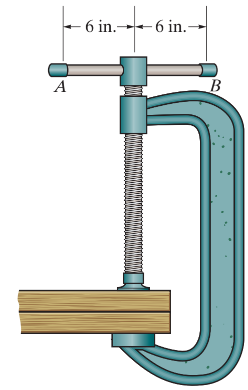
$N_{AB} = 29.01$ N

$\uparrow + \Sigma F_y = 0; \quad 34.61 \cos 15° - 0.6(34.61) \sin 15° - 0.4(29.01) - 500x = 0$

$x = 0.03290 \text{ m} = 32.9 \text{ mm}$ **Ans.**

8–67.

If couple forces of $F = 10$ lb are applied perpendicular to the lever of the clamp at A and B, determine the clamping force on the boards. The single square-threaded screw of the clamp has a mean diameter of 1 in. and a lead of 0.25 in. The coefficient of static friction is $\mu_s = 0.3$.



SOLUTION

Since the screw is being tightened, Eq. 8–3 should be used. Here,

$$M = 10(12) = 120 \text{ lb} \cdot \text{in}; \theta = \tan^{-1}\left(\frac{L}{2\pi r}\right) = \tan^{-1}\left[\frac{0.25}{2\pi(0.5)}\right] = 4.550°;$$

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}(0.3) = 16.699°. \text{ Thus}$$

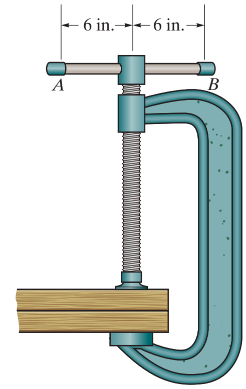
$$M = Wr \tan(\phi_s + \theta)$$

$$120 = P(0.5)\tan(16.699° + 4.550°)$$

$$P = 617 \text{ lb}$$ **Ans.**

Note: Since $\phi_s > \theta$, the screw is self-locking.

***8–68.**



If the clamping force on the boards is 600 lb, determine the required magnitude of the couple forces that must be applied perpendicular to the lever AB of the clamp at A and B in order to loosen the screw. The single square-threaded screw has a mean diameter of 1 in. and a lead of 0.25 in. The coefficient of static friction is $\mu_s = 0.3$.

SOLUTION

Since the screw is being loosened, Eq. 8–5 should be used. Here,

$$M = F(12);\ \theta = \tan^{-1}\left(\frac{L}{2\pi r}\right) = \tan^{-1}\left[\frac{0.25}{2\pi(0.5)}\right] = 4.550°;$$

$\phi_s = \tan^{-1}\mu_s = \tan^{-1}(0.3) = 16.699°$; and $W = 600$ lb. Thus

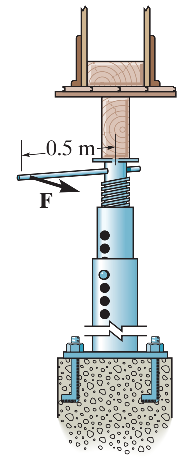
$$M = Wr\tan(\phi_s - \theta)$$

$$F(12) = 600(0.5)\tan(16.699° - 4.550°)$$

$$F = 5.38 \text{ lb}$$ **Ans.**

8–69.

The column is used to support the upper floor. If a force $F = 80$ N is applied perpendicular to the handle to tighten the screw, determine the compressive force in the column. The square-threaded screw on the jack has a coefficient of static friction of $\mu_s = 0.4$, mean diameter of 25 mm, and a lead of 3 mm.



SOLUTION

$$M = W(r)\tan(\phi_s + \theta_p)$$

$$\phi_s = \tan^{-1}(0.4) = 21.80°$$

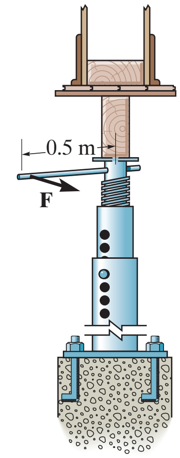
$$\theta_p = \tan^{-1}\left[\frac{3}{2\pi(12.5)}\right] = 2.188°$$

$$80(0.5) = W(0.0125)\tan(21.80° + 2.188°)$$

$$W = 7.19 \text{ kN}$$ **Ans.**

8–70.

If the force **F** is removed from the handle of the jack in Prob. 8-69, determine if the screw is self-locking.



SOLUTION

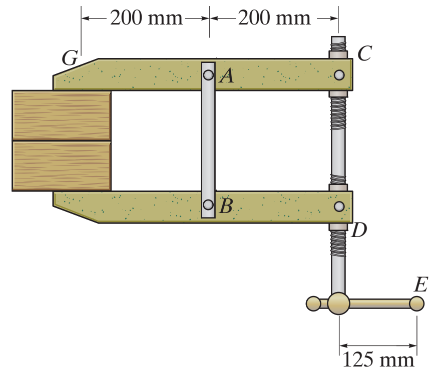
$$\phi_s = \tan^{-1}(0.4) = 21.80^\circ$$

$$\theta_p = \tan^{-1}\left[\frac{3}{2\pi(12.5)}\right] = 2.188^\circ$$

Since $\phi_s > \theta_p$, the screw is self locking. **Ans.**

8–71.

If the clamping force at G is 900 N, determine the horizontal force **F** that must be applied perpendicular to the handle of the lever at E. The mean diameter and lead of both single square-threaded screws at C and D are 25 mm and 5 mm, respectively. The coefficient of static friction is $\mu_s = 0.3$.



SOLUTION

Referring to the free-body diagram of member GAC shown in Fig. a, we have $\Sigma M_A = 0; F_{CD}(0.2) - 900(0.2) = 0 \qquad F_{CD} = 900\,\text{N}$

Since the screw is being tightened, Eq. 8–3 should be used. Here, $\theta = \tan^{-1}\left(\frac{L}{2\pi r}\right) = \tan^{-1}\left[\frac{5}{2\pi(12.5)}\right] = 3.643°$;

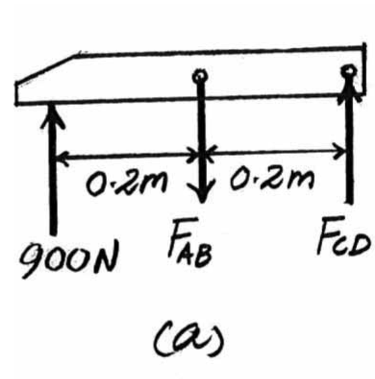
$\phi_s = \tan^{-1}\mu_s = \tan^{-1}(0.3) = 16.699°$; and $M = F(0.125)$. Since **M** must overcome the friction of two screws,

$$M = 2[Wr\tan(\phi_s + \theta)]$$

$$F(0.125) = 2\,[900(0.0125)\tan(16.699° + 3.643°)]$$

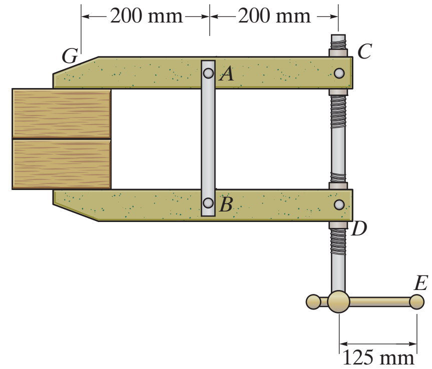
$$F = 66.7\,\text{N}$$ **Ans.**

Note: Since $\phi_s > \theta$, the screw is self-locking.



***8–72.**

If a horizontal force of $F = 50$ N is applied perpendicular to the handle of the lever at E, determine the clamping force developed at G. The mean diameter and lead of the single square-threaded screw at C and D are 25 mm and 5 mm, respectively. The coefficient of static friction is $\mu_s = 0.3$.



SOLUTION

Since the screw is being tightened, Eq. 8–3 should be used. Here, $\theta = \tan^{-1}\left(\frac{L}{2\pi r}\right) = \tan^{-1}\left[\frac{5}{2\pi(12.5)}\right] = 3.643°$;

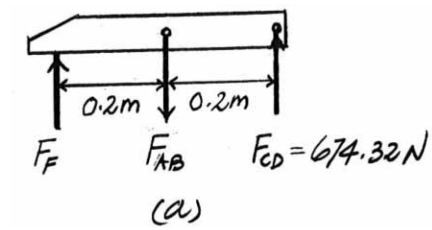
$\phi_s = \tan^{-1}\mu_s = \tan^{-1}(0.3) = 16.699°$; and $M = 50(0.125)$. Since **M** must overcome the friction of two screws,

$$M = 2[Wr\tan(\phi_s + \theta)]$$

$$50(0.125) = 2[F_{CD}(0.0125)\tan(16.699° + 3.643°)]$$

$$F_{CD} = 674.32\,\text{N}$$ **Ans.**

Using the result of F_{CD} and referring to the free-body diagram of member GAC shown in Fig. a, we have



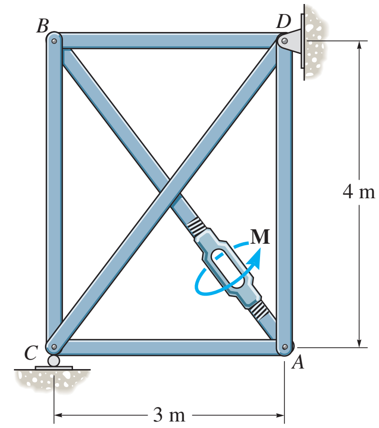
$$\Sigma M_A = 0;\ 674.32(0.2) - F_G(0.2) = 0$$

$$F_G = 674\,\text{N}$$ **Ans.**

Note: Since $\phi_s > \theta$, the screws are self-locking.

8–73.

A turnbuckle, similar to that shown in Fig. 8–17, is used to tension member AB of the truss. The coefficient of the static friction between the square threaded screws and the turnbuckle is $\mu_s = 0.5$. The screws have a mean radius of 6 mm and a lead of 3 mm. If a torque of $M = 10\ \text{N}\cdot\text{m}$ is applied to the turnbuckle, to draw the screws closer together, determine the force in each member of the truss. No external forces act on the truss.



SOLUTION

Frictional Forces on Screw: Here, $\theta = \tan^{-1}\left(\dfrac{l}{2\pi r}\right) = \tan^{-1}\left[\dfrac{3}{2\pi(6)}\right] = 4.550°$, $M = 10\ \text{N}\cdot\text{m}$ and $\phi_s = \tan^{-1}\mu_s = \tan^{-1}(0.5) = 26.565°$. Since friction at two screws must be overcome, then, $W = 2F_{AB}$. Applying Eq. 8–3, we have

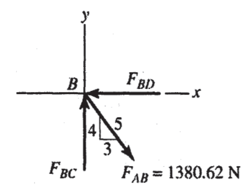
$$M = Wr\tan(\theta + \phi_S)$$

$$10 = 2F_{AB}(0.006)\tan(4.550° + 26.565°)$$

$$F_{AB} = 1380.62\ \text{N (T)} = 1.38\ \text{kN (T)} \qquad \textbf{Ans.}$$

Note: Since $\phi_s > \theta$, the screw is self-locking. It will not unscrew even if moment **M** is removed.

Method of Joints:



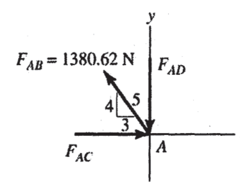
Joint B:

$\overset{+}{\rightarrow}\Sigma F_x = 0;$ $\quad 1380.62\left(\dfrac{3}{5}\right) - F_{BD} = 0$

$$F_{BD} = 828.37\ \text{N}(\textbf{C}) = 828\ \text{N}\ (\textbf{C}) \qquad \textbf{Ans.}$$

$+\uparrow\Sigma F_y = 0;$ $\quad F_{BC} - 1380.62\left(\dfrac{4}{5}\right) = 0$

$$F_{BC} = 1104.50\ \text{N}\ (\textbf{C}) = 1.10\ \text{kN}\ (\textbf{C}) \qquad \textbf{Ans.}$$



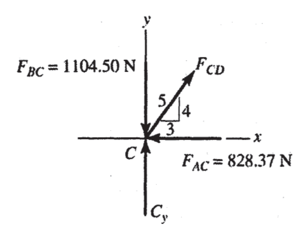
Joint A:

$\overset{+}{\rightarrow}\Sigma F_x = 0;$ $\quad F_{AC} - 1380.62\left(\dfrac{3}{5}\right) = 0$

$$F_{AC} = 828.37\ \text{N}\ (\textbf{C}) = 828\ \text{N}\ (\textbf{C}) \qquad \textbf{Ans.}$$

$+\uparrow\Sigma F_y = 0;$ $\quad 1380.62\left(\dfrac{4}{5}\right) - F_{AD} = 0$

$$F_{AD} = 1104.50\ \text{N}\ (\textbf{C}) = 1.10\ \text{kN}\ (\textbf{C}) \qquad \textbf{Ans.}$$



Joint C:

$\overset{+}{\rightarrow}\Sigma F_x = 0;$ $\quad F_{CD}\left(\dfrac{3}{5}\right) - 828.37 = 0$

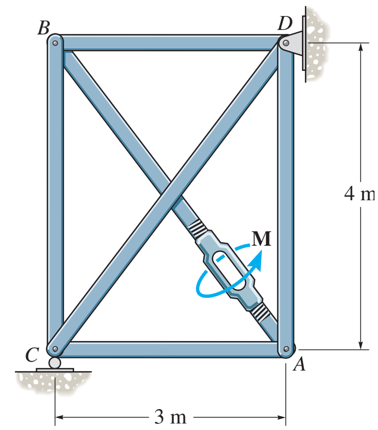
$$F_{CD} = 1380.62\ \text{N}\ (\textbf{T}) = 1.38\ \text{kN}\ (\textbf{T}) \qquad \textbf{Ans.}$$

$+\uparrow\Sigma F_y = 0;$ $\quad C_y + 1380.62\left(\dfrac{4}{5}\right) - 1104.50 = 0$

$C_y = 0$ ***(No external applied load. check!)***

8–74.

A turnbuckle, similar to that shown in Fig. 8–17, is used to tension member AB of the truss. The coefficient of the static friction between the square-threaded screws and the turnbuckle is $\mu_s = 0.5$. The screws have a mean radius of 6 mm and a lead of 3 mm. Determine the torque M which must be applied to the turnbuckle to draw the screws closer together, so that the compressive force of 500 N is developed in member BC.

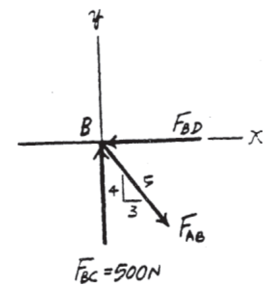


SOLUTION

Method of Joints:

Joint B:

$+\uparrow \Sigma F_y = 0; \qquad 500 - F_{AB}\left(\frac{4}{5}\right) = 0 \qquad F_{AB} = 625 \text{ N } (\mathbf{C})$



Frictional Forces on Screws: Here, $\theta = \tan^{-1}\left(\frac{l}{2\pi r}\right) = \tan^{-1}\left[\frac{3}{2\pi(6)}\right] = 4.550°$ and $\phi_s = \tan^{-1}\mu_s = \tan^{-1}(0.5) = 26.565°$. Since friction at two screws must be overcome, then, $W = 2F_{AB} = 2(625) = 1250$ N. Applying Eq. 8–3, we have

$$M = Wr\tan(\theta + \phi)$$

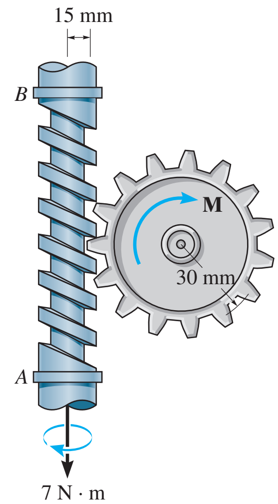
$$= 1250(0.006)\tan(4.550° + 26.565°)$$

$$= 4.53 \text{ N} \cdot \text{m}$$ **Ans.**

Note: Since $\phi_s > \theta$, the screw is self-locking. It will not unscrew even if moment **M** is removed.

8–75.

The shaft has a square-threaded screw with a lead of 8 mm and a mean radius of 15 mm. If it is in contact with a plate gear having a mean radius of 30 mm, determine the resisting torque **M** on the plate gear which can be overcome if a torque of 7 N·m is applied to the shaft. The coefficient of static friction at the screw is $\mu_B = 0.2$. Neglect friction of the bearings located at A and B.



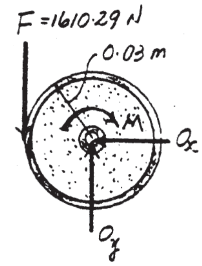
SOLUTION

Frictional Forces on Screw: Here, $\theta = \tan^{-1}\left(\frac{l}{2\pi r}\right) = \tan^{-1}\left[\frac{8}{2\pi(15)}\right] = 4.852°$, $W = F$, $M = 7\ \text{N}\cdot\text{m}$ and $\phi_s = \tan^{-1}\mu_s = \tan^{-1}(0.2) = 11.310°$. Applying Eq. 8–3, we have

$$M = Wr\tan(\theta + \phi)$$

$$7 = F(0.015)\tan(4.852° + 11.310°)$$

$$F = 1610.29\ \text{N}$$



Note: Since $\phi_s > \theta$, the screw is self-locking. It will not unscrew even if force **F** is removed.

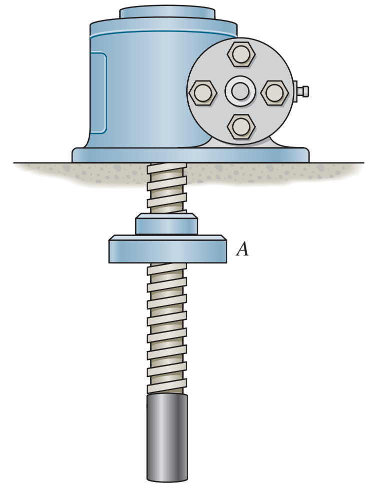
Equations of Equilibrium:

$\circlearrowleft + \Sigma M_O = 0;\qquad 1610.29(0.03) - M = 0$

$$M = 48.3\ \text{N}\cdot\text{m} \qquad \textbf{Ans.}$$

***8–76.**

The square-threaded screw has a mean diameter of 20 mm and a lead of 4 mm. If the weight of the plate A is 5 lb, determine the smallest coefficient of static friction between the screw and the plate so that the plate does not travel down the screw when the plate is suspended as shown.



SOLUTION

Frictional Forces on Screw: This requires a "self-locking" screw where $\phi_s \geq \theta$. Here, $\theta = \tan^{-1}\left(\frac{l}{2\pi r}\right) = \tan^{-1}\left[\frac{4}{2\pi(10)}\right] = 3.643°$.

$$\phi_s = \tan^{-1}\mu_s$$

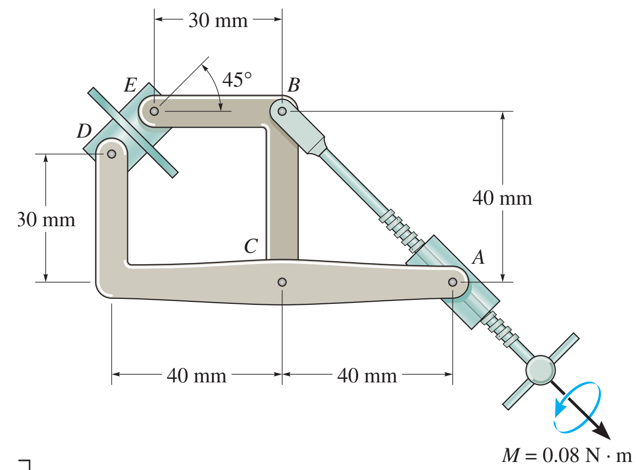
$$\mu_s = \tan\phi_s \qquad \text{where } \phi_s = \theta = 3.643°$$

$$= 0.0637$$

Ans.

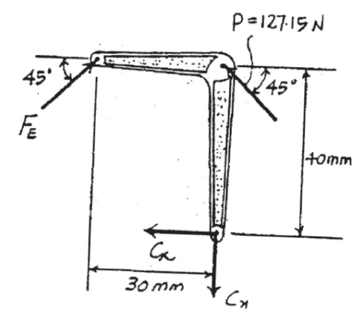
8–77.

The fixture clamp consist of a square-threaded screw having a coefficient of static friction of $\mu_s = 0.3$, mean diameter of 3 mm, and a lead of 1 mm. The five points indicated are pin connections. Determine the clamping force at the smooth blocks D and E when a torque of $M = 0.08\ \text{N}\cdot\text{m}$ is applied to the handle of the screw.



SOLUTION

Frictional Forces on Screw: Here, $\theta = \tan^{-1}\left(\frac{l}{2\pi r}\right) = \tan^{-1}\left[\frac{1}{2\pi(1.5)}\right] = 6.057°$, $W = P$, $M = 0.08\ \text{N}\cdot\text{m}$ and $\phi_s = \tan^{-1}\mu_s = \tan^{-1}(0.3) = 16.699°$. Applying Eq. 8–3, we have



$$M = Wr\tan(\theta + \phi)$$

$$0.08 = P(0.0015)\tan(6.057° + 16.699°)$$

$$P = 127.15\ \text{N}$$

Note: Since $\phi_s > \theta$, the screw is self-locking. It will not unscrew even if moment **M** is removed.

Equation of Equilibrium:

$\circlearrowleft + \Sigma M_C = 0;\qquad 127.15\cos 45°(40) - F_E\cos 45°(40) - F_E\sin 45°(30) = 0$

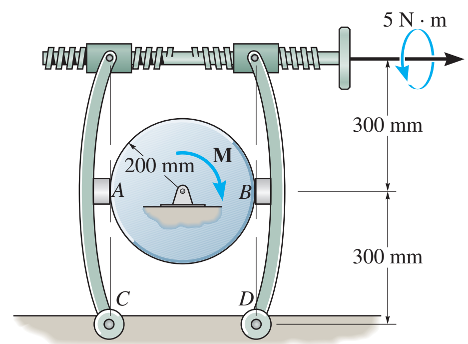
$$F_E = 72.66\ \text{N} = 72.7\ \text{N}$$ **Ans.**

The equilibrium of the clamped blocks requires that

$$F_D = F_E = 72.7\ \text{N}$$ **Ans.**

8–78.

The braking mechanism consists of two pinned arms and a square-threaded screw with left and righthand threads. Thus when turned, the screw draws the two arms together. If the lead of the screw is 4 mm, the mean diameter 12 mm, and the coefficient of static friction is $\mu_s = 0.35$, determine the tension in the screw when a torque of $5\ \mathrm{N\cdot m}$ is applied to tighten the screw. If the coefficient of static friction between the brake pads A and B and the circular shaft is $\mu_s' = 0.5$, determine the maximum torque M the brake can resist.



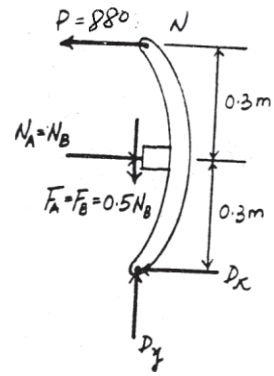
SOLUTION

Frictional Forces on Screw: Here, $\theta = \tan^{-1}\left(\frac{l}{2\pi r}\right) = \tan^{-1}\left[\frac{4}{2\pi(6)}\right] = 6.057°$, $M = 5\ \mathrm{N\cdot m}$ and $\phi_s = \tan^{-1}\mu_s = \tan^{-1}(0.35) = 19.290°$. Since friction at two screws must be overcome, then, $W = 2P$. Applying Eq. 8–3, we have

$$M = Wr\tan(\theta + \phi)$$

$$5 = 2P(0.006)\tan(6.057° + 19.290°)$$

$$P = 879.61\ \mathrm{N} = 880\ \mathrm{N}$$ **Ans.**



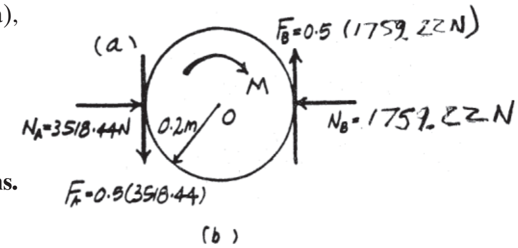
Note: Since $\phi_s > \theta$, the screw is self-locking. It will not unscrew even if moment **M** is removed.

Equations of Equilibrium and Friction: Since the shaft is on the verge to rotate about point O, then, $F_A = \mu_s' N_A = 0.5N_A$ and $F_B = \mu_s' N_B = 0.5N_B$. From FBD (a),

$\circlearrowleft + \Sigma M_D = 0;\qquad 879.61\,(0.6) - N_B\,(0.3) = 0 \qquad N_B = 1759.22\ \mathrm{N}$

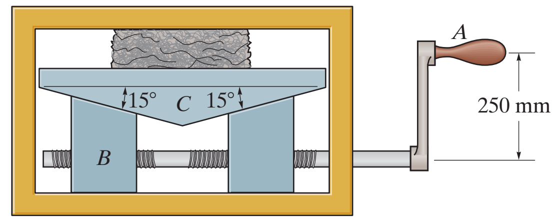
From FBD (b),

$\circlearrowleft + \Sigma M_O = 0;\qquad 2[0.5(1759.22)](0.2) - M = 0 \qquad M = 352\ \mathrm{N\cdot m}$ **Ans.**



8–79.

If a horizontal force of $P = 100$ N is applied perpendicular to the handle of the lever at A, determine the compressive force **F** exerted on the material. Each single square-threaded screw has a mean diameter of 25 mm and a lead of 7.5 mm. The coefficient of static friction at all contacting surfaces of the wedges is $\mu_s = 0.2$, and the coefficient of static friction at the screw is $\mu_s' = 0.15$.



SOLUTION

Since the screws are being tightened, Eq. 8–3 should be used. Here,

$$\theta = \tan^{-1}\left(\frac{L}{2\pi r}\right) = \tan^{-1}\left[\frac{7.5}{2\pi(12.5)}\right] = 5.455°;$$

$\phi_s = \tan^{-1}\mu_s = \tan^{-1}(0.15) = 8.531°$; $M = 100(0.25) = 25\ \text{N}\cdot\text{m}$; and $W = T$, where T is the tension in the screw shank. Since **M** must overcome the friction of two screws,

$$M = 2[Wr, \tan(\phi_s + \theta)]$$

$$25 = 2[T(0.0125)\tan(8.531° + 5.455°)]$$

$$T = 4015.09\ \text{N} = 4.02\ \text{kN}$$ **Ans.**

Referring to the free-body diagram of wedge B shown in Fig. a using the result of T, we have

$\xrightarrow{+} \Sigma F_x = 0;\quad 4015.09 - 0.2N' - 0.2N\cos 15° - N\sin 15° = 0$ **(1)**

$+\uparrow \Sigma F_y = 0;\quad N' + 0.2N\sin 15° - N\cos 15° = 0$ **(2)**

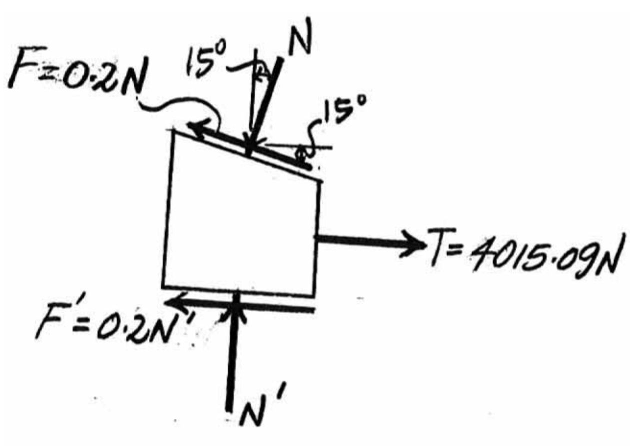
Solving,

$$N = 6324.60\ \text{N} \qquad N' = 5781.71\ \text{N}$$

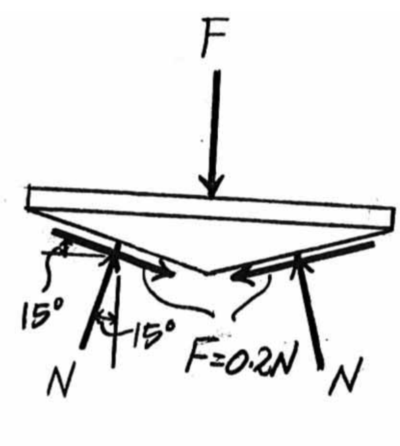
Using the result of N and referring to the free-body diagram of wedge C shown in Fig. b, we have

$+\uparrow \Sigma F_y = 0;\quad 2(6324.60)\cos 15° - 2[0.2(6324.60)\sin 15°] - F = 0$

$F = 11563.42\ \text{N} = 11.6\ \text{kN}$ **Ans.**



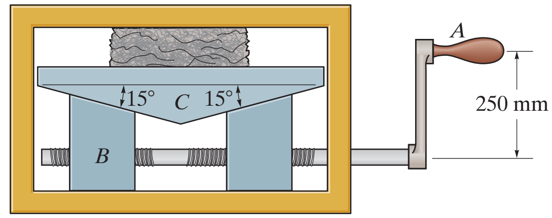
(a)



(b)

***8–80.**

Determine the horizontal force **P** that must be applied perpendicular to the handle of the lever at A in order to develop a compressive force of 12 kN on the material. Each single square-threaded screw has a mean diameter of 25 mm and a lead of 7.5 mm. The coefficient of static friction at all contacting surfaces of the wedges is $\mu_s = 0.2$, and the coefficient of static friction at the screw is $\mu_s' = 0.15$.



SOLUTION

Referring to the free-body diagram of wedge C shown in Fig. a, we have

$+\uparrow \Sigma F_y = 0;\qquad 2N\cos 15° - 2[0.2N\sin 15°] - 12000 = 0$

$N = 6563.39\ \text{N}$

Using the result of N and referring to the free-body diagram of wedge B shown in Fig. b, we have

$+\uparrow \Sigma F_y = 0;\qquad N' - 6563.39\cos 15° + 0.2(6563.39)\sin 15° = 0$

$N' = 6000\ \text{N}$

$\overset{+}{\rightarrow} \Sigma F_x = 0;\qquad T - 6563.39\sin 15° - 0.2(6563.39)\cos 15° - 0.2(6000) = 0$

$T = 4166.68\ \text{N}$

Since the screw is being tightened, Eq. 8–3 should be used. Here,

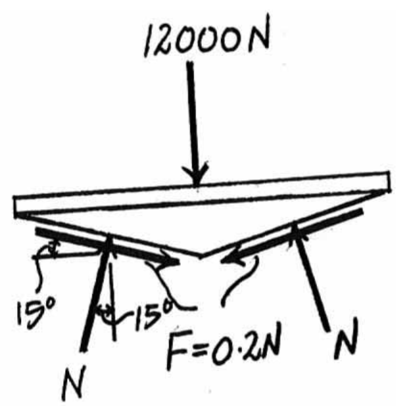
$$\theta = \tan^{-1}\left[\frac{L}{2\pi r}\right] = \tan^{-1}\left[\frac{7.5}{2\pi(12.5)}\right] = 5.455°;$$

$\phi_s = \tan^{-1}\mu_s = \tan^{-1}(0.15) = 8.531°$; $M = P(0.25)$; and $W = T = 4166.68\text{N}$. Since **M** must overcome the friction of two screws,

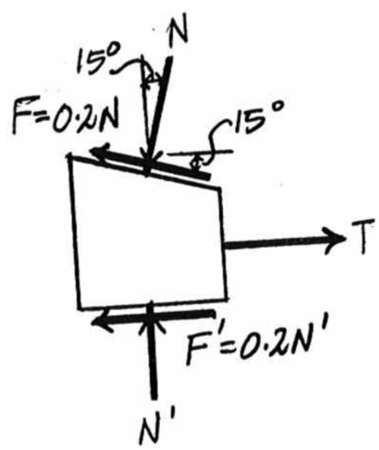
$$M = 2[Wr\tan(\phi_s + \theta)]$$

$$P(0.25) = 2[4166.68(0.0125)\tan(8.531° + 5.455°)]$$

$$P = 104\ \text{N}$$ **Ans.**



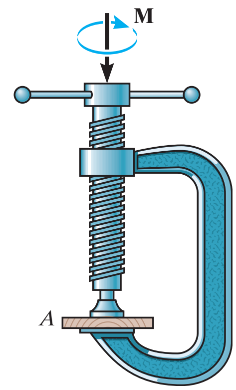
(a)



(b)

8–81.

Determine the clamping force on the board A if the screw of the "C" clamp is tightened with a twist of $M = 8\ \text{N}\cdot\text{m}$. The single square-threaded screw has a mean radius of 10 mm, a lead of 3 mm, and the coefficient of static friction is $\mu_s = 0.35$.



SOLUTION

$$\phi_s = \tan^{-1}(0.35) = 19.29^\circ$$

$$\theta_p = \tan^{-1}\left[\frac{3}{2\pi(10)}\right] = 2.734^\circ$$

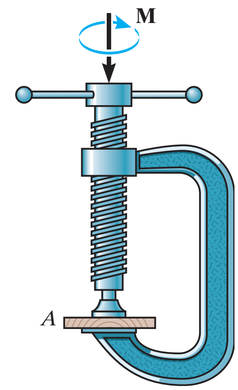
$$M = W(r)\ \tan(\phi_s + \theta_p)$$

$$8 = P(0.01)\tan(19.29^\circ + 2.734^\circ)$$

$$P = 1978\ \text{N} = 1.98\ \text{kN}$$ **Ans.**

8–82.

If the required clamping force at the board A is to be 50 N, determine the torque M that must be applied to the handle of the "C" clamp to tighten it down. The single square-threaded screw has a mean radius of 10 mm, a lead of 3 mm, and the coefficient of static friction is $\mu_s = 0.35$.



SOLUTION

$$\phi_s = \tan^{-1}(0.35) = 19.29^\circ$$

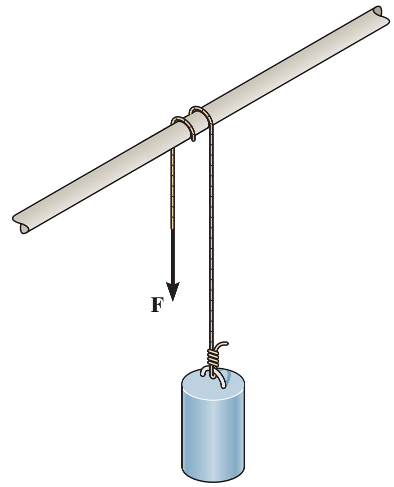
$$\theta_p = \tan^{-1}\left(\frac{l}{2\pi r}\right) = \tan^{-1}\left[\frac{3}{2\pi(10)}\right] = 2.734^\circ$$

$$M = W(r)\tan(\phi_s + \theta_p)$$

$$= 50(0.01)\tan(19.29^\circ + 2.734^\circ) = 0.202\ \text{N}\cdot\text{m}$$ **Ans.**

8–83.

A cylinder having a mass of 250 kg is to be supported by the cord which wraps over the pipe. Determine the smallest vertical force F needed to support the load if the cord passes (a) once over the pipe, $\beta = 180°$, and (b) two times over the pipe, $\beta = 540°$. Take $\mu_s = 0.2$.



SOLUTION

Frictional Force on Flat Belt: Here, $T_1 = F$ and $T_2 = 250(9.81) = 2452.5$ N. Applying Eq. 8–6, we have

a) If $\beta = 180° = \pi$ rad

$$T_2 = T_1 e^{\mu\beta}$$

$$2452.5 = Fe^{0.2\pi}$$

$$F = 1308.38 \text{ N} = 1.31 \text{ kN}$$ **Ans.**

b) If $\beta = 540° = 3\pi$ rad

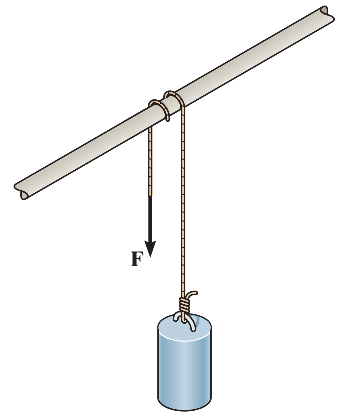
$$T_2 = T_1 e^{\mu\beta}$$

$$2452.5 = Fe^{0.2(3\pi)}$$

$$F = 372.38 \text{ N} = 372 \text{ N}$$ **Ans.**

***8–84.**

A cylinder having a mass of 250 kg is to be supported by the cord which wraps over the pipe. Determine the largest vertical force F that can be applied to the cord without moving the cylinder. The cord passes (a) once over the pipe, $\beta = 180°$, and (b) two times over the pipe, $\beta = 540°$. Take $\mu_s = 0.2$.



SOLUTION

Frictional Force on Flat Belt: Here, $T_1 = 250(9.81) = 2452.5\text{ N}$ and $T_2 = F$. Applying Eq. 8–6, we have

a) If $\beta = 180° = \pi$ rad

$$T_2 = T_1\, e^{\mu\beta}$$

$$F = 2452.5e^{0.2\,\pi}$$

$$F = 4597.10\text{ N} = 4.60\text{ kN}$$ **Ans.**

b) If $\beta = 540° = 3\,\pi$ rad

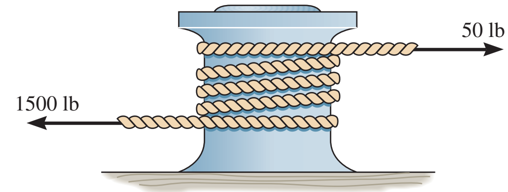
$$T_2 = T_1 e^{\mu\beta}$$

$$F = 2452.5e^{0.2(3\,\pi)}$$

$$F = 16\,152.32\text{ N} = 16.2\text{ kN}$$ **Ans.**

8–85.

A “hawser” is wrapped around a fixed “capstan” to secure a ship for docking. If the tension in the rope, caused by the ship, is 1500 lb, determine the least number of complete turns the rope must be wrapped around the capstan in order to prevent slipping of the rope. The greatest horizontal force that a longshoreman can exert on the rope is 50 lb. The coefficient of static friction is $\mu_s = 0.3$.



SOLUTION

Frictional Force on Flat Belt: Here, $T_1 = 50$ lb and $T_2 = 1500$ lb. Applying Eq. 8–6, we have

$$T_2 = T_1\,e^{\mu\beta}$$

$$1500 = 50e^{0.3\beta}$$

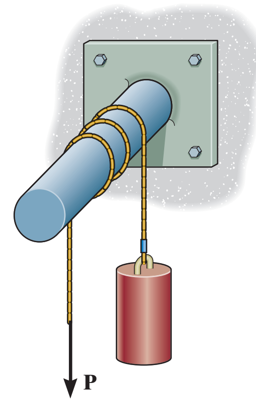
$$\beta = 11.337 \text{ rad}$$

The least number of turns of the rope required is $\dfrac{11.337}{2\pi} = 1.80$ turns. Thus

Use $\quad n = 2$ turns **Ans.**

8–86.

A force of $P = 25$ N is just sufficient to prevent the 20-kg cylinder from descending. Determine the required force **P** to begin lifting the cylinder. The rope passes over a rough peg with two and half turns.



SOLUTION

The coefficient of static friction μ_s between the rope and the peg when the cylinder is on the verge of descending requires $T_2 = 20(9.81)$ N, $T_1 = P = 25$ Nand $\beta = 2.5(2\pi) = 5\pi\,\text{rad}$. Thus,

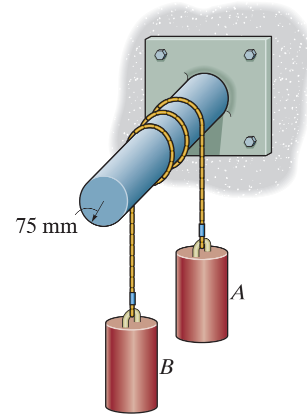
$$T_2 = T_1 e^{\mu_s \beta}$$
$$20(9.81) = 25e^{\mu_s(5\pi)}$$
$$\text{In}\,7.848 = 5\pi\mu_s$$
$$\mu_s = 0.1312$$

In the case of the cylinder ascending $T_2 = P$ and $T_1 = 20(9.81)$ N. Using the result of μ_s, we can write

$$T_2 = T_1 e^{\mu_s \beta}$$
$$P = 20(9.81)e^{0.1312(5\pi)}$$
$$= 1539.78 \text{ N}$$
$$= 1.54 \text{ kN}$$ **Ans.**

8–87.

The 20-kg cylinder A and 50-kg cylinder B are connected together using a rope that passes around a rough peg two and a half turns. If the cylinders are on the verge of moving, determine the coefficient of static friction between the rope and the peg.



SOLUTION

In this case, $T_1 = 50(9.81)\text{N}$, $T_2 = 20(9.81)\text{N}$ and $\beta, = 2.5(2\pi) = 5\pi$ rad. Thus,

$$T_1 = T_2 e^{\mu_s \beta}$$

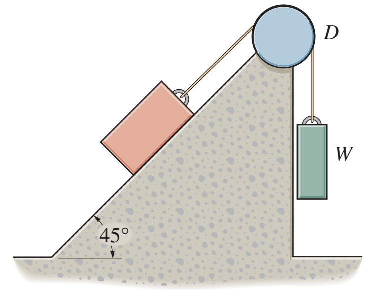
$$50(9.81) = 20(9.81)e^{\mu_s(5\pi)}$$

$$\text{In}\, 2.5 = \mu_s(5\pi)$$

$$\mu_s = 0.0583$$ **Ans.**

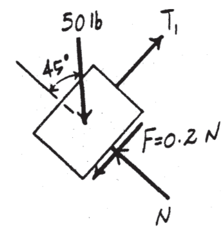
***8–88.**

Determine the maximum and the minimum values of weight W which may be applied without causing the 50-lb block to slip. The coefficient of static friction between the block and the plane is $\mu_s = 0.2$, and between the rope and the drum D $\mu_s' = 0.3$.



SOLUTION

Equations of Equilibrium and Friction: Since the block is on the verge of sliding up or down the plane, then, $F = \mu_s N = 0.2N$. If the block is on the verge of sliding up the plane [FBD (a)],

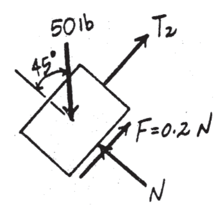


(a)

$\nwarrow + \Sigma F_{y'} = 0; \qquad N - 50\cos 45° = 0 \qquad N = 35.36 \text{ lb}$

$\nearrow + \Sigma F_{x'} = 0; \qquad T_1 - 0.2(35.36) - 50\sin 45° = 0 \qquad T_1 = 42.43 \text{ lb}$

If the block is on the verge of sliding down the plane [FBD (b)],



(b)

$\nwarrow + \Sigma F_{y'} = 0; \qquad N - 50\cos 45° = 0 \qquad N = 35.36 \text{ lb}$

$\nearrow + \Sigma F_{x'} = 0; \qquad T_2 + 0.2(35.36) - 50\sin 45° = 0 \qquad T_2 = 28.28 \text{ lb}$

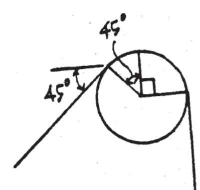
Frictional Force on Flat Belt: Here, $\beta = 45° + 90° = 135° = \dfrac{3\pi}{4}$ rad. If the block is on the verge of sliding up the plane, $T_1 = 42.43$ lb and $T_2 = W$.

$$T_2 = T_1 e^{\mu\beta}$$

$$W = 42.43e^{0.3\left(\frac{3\pi}{4}\right)}$$

$$= 86.02 \text{ lb} = 86.0 \text{ lb} \qquad \textbf{Ans.}$$

If the block is on the verge of sliding down the plane, $T_1 = W$ and $T_2 = 28.28$ lb.



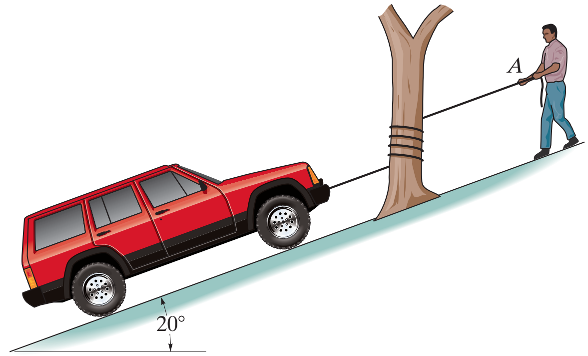
$$T_2 = T_1 e^{\mu\beta}$$

$$28.28 = We^{0.3\left(\frac{3\pi}{4}\right)}$$

$$W = 13.95 \text{ lb} = 13.9 \text{ lb} \qquad \textbf{Ans.}$$

8–89.

The truck, which has a mass of 3.4 Mg, is to be lowered down the slope by a rope that is wrapped around a tree. If the wheels are free to roll and the man at A can resist a pull of 300 N, determine the minimum number of turns the rope should be wrapped around the tree to lower the truck at a constant speed. The coefficient of kinetic friction between the tree and rope is $\mu_k = 0.3$.



SOLUTION

$\nearrow + \Sigma F_x = 0;\qquad T_2 - 33\,354 \sin 20° = 0$

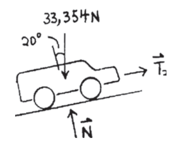
$T_2 = 11\,407.7$

$T_2 = T_1\, e^{\mu\beta}$

$11\,407.7 = 300\, e^{0.3\beta}$

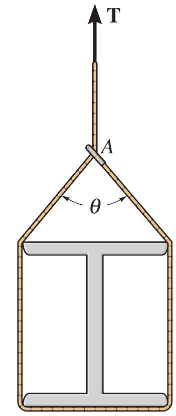
$\beta = 12.1275 \text{ rad}$

Approx. 2 turns (695°) **Ans.**



8–90.

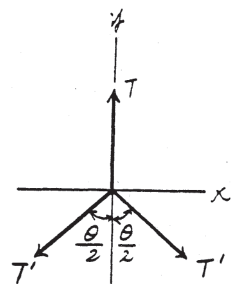
The smooth beam is being hoisted using a rope which is wrapped around the beam and passes through a ring at A as shown. If the end of the rope is subjected to a tension $\mathbf{T}$ and the coefficient of static friction between the rope and ring is $\mu_s = 0.3$, determine the angle of θ for equilibrium.



SOLUTION

Equation of Equilibrium:

$$+\uparrow \Sigma F_x = 0; \qquad T - 2T' \cos\frac{\theta}{2} = 0 \qquad T = 2T'\cos\frac{\theta}{2} \qquad \textbf{(1)}$$



Frictional Force on Flat Belt: Here, $\beta = \dfrac{\theta}{2}$, $T_2 = T$ and $T_1 = T'$. Applying Eq. 8–6 $T_2 = T_1 e^{\mu\beta}$, we have

$$T = T'e^{0.3(\theta/2)} = T'e^{0.15\,\theta} \qquad \textbf{(2)}$$

Substituting Eq. (1) into (2) yields

$$2T'\cos\frac{\theta}{2} = T'e^{0.15\,\theta}$$

$$e^{0.15\,\theta} = 2\cos\frac{\theta}{2}$$

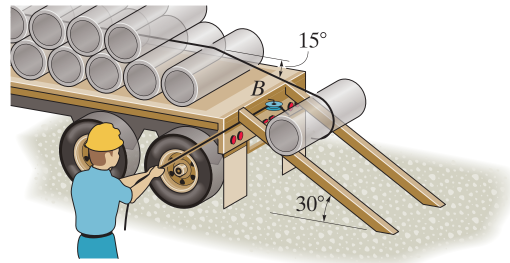
Solving by trial and error

$$\theta = 1.73104 \text{ rad} = 99.2° \qquad \textbf{Ans.}$$

The other solution, which starts with $T' = Te^{0.3(0/2)}$ based on cinching the ring tight, is 2.4326 rad = 139°. Any angle from 99.2° to 139° is equilibrium.

8–91.

The uniform concrete pipe has a weight of 800 lb and is unloaded slowly from the truck bed using the rope and skids shown. If the coefficient of kinetic friction between the rope and pipe is $\mu_k = 0.3$, determine the force the worker must exert on the rope to lower the pipe at constant speed. There is a pulley at B, and the pipe does not slip on the skids. The lower portion of the rope is parallel to the skids.



SOLUTION

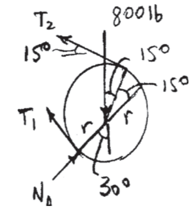
$\circlearrowleft + \Sigma M_A = 0; \qquad -800(r \sin 30°) + T_2 \cos 15°(r \cos 15° + r \cos 30°) + T_2 \sin 15°(r \sin 15° + r \sin 15°) = 0$

$T_2 = 203.466 \text{ lb}$

$\beta = 180° + 15° = 195°$

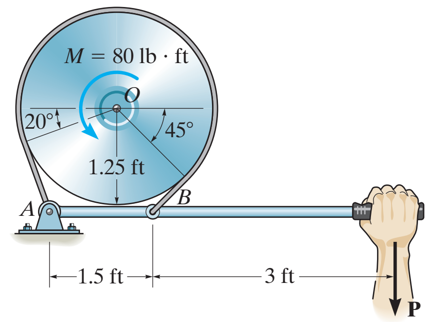
$T_2 = T_1 e^{\mu\beta}, \qquad 203.466 = T_1 e^{(0.3)(\frac{195°}{180°})(\pi)}$

$T_1 = 73.3 \text{ lb}$ **Ans.**



***8–92.**

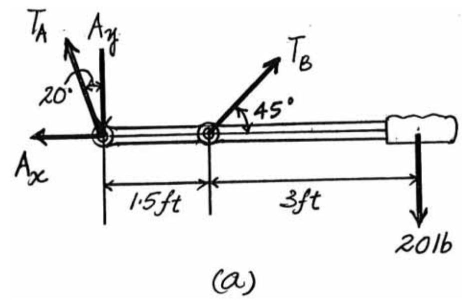
The simple band brake is constructed so that the ends of the friction strap are connected to the pin at A and the lever arm at B. If the wheel is subjected to a torque of $M = 80\ \text{lb}\cdot\text{ft}$, and the minimum force $P = 20$ lb is needed to apply to the lever to hold the wheel stationary, determine the coefficient of static friction between the wheel and the band.



SOLUTION

Equations of Equilibrium: Write the moment equation of equilibrium about point A by referring to the FBD of the lever shown in Fig. *a*,

$\circlearrowleft + \Sigma M_A = 0; \qquad T_B \sin 45°(1.5) - 20(4.5) = 0 \qquad T_B = 84.85\ \text{lb}$



Using this result to write the moment equation of equilibrium about point *0* by referring to the FBD of the wheel shown in Fig. *b*,

$\circlearrowleft + \Sigma M_O = 0; \qquad T_A(1.25) + 80 - 84.85(1.25) = 0 \qquad T_A = 20.85\ \text{lb}$

Frictional Force on Flat Belt: Here, $\beta = \left(\frac{245°}{180°}\right)\pi = \frac{49}{36}\pi$, $T_1 = T_A = 20.85$ lb and $T_2 = T_B = 84.85$ lb. Applying Eq. 8–6,

$$T_2 = T_1 e^{\mu\beta}$$

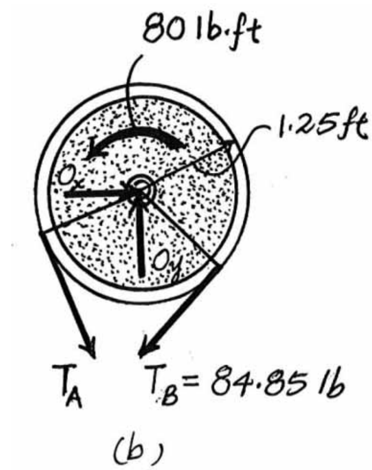
$$84.85 = 20.85 e^{\mu\left(\frac{49}{36}\right)\pi}$$

$$e^{\mu\left(\frac{49}{36}\right)\pi} = 4.069$$

$$In\, e^{\mu\left(\frac{49}{36}\right)\pi} = In\, 4.069$$

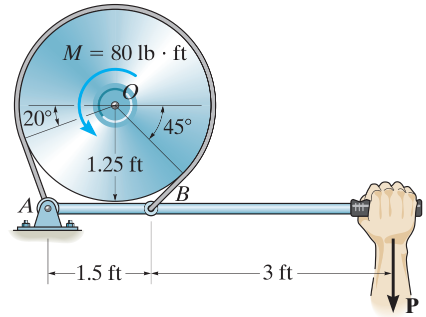
$$\mu\left(\frac{49}{36}\right)\pi = In\, 4.069$$

$$\mu = 0.328$$ **Ans.**



8–93.

The simple band brake is constructed so that the ends of the friction strap are connected to the pin at A and the lever arm at B. If the wheel is subjected to a torque of $M = 80\ \text{lb}\cdot\text{ft}$, determine the smallest force P applied to the lever that is required to hold the wheel stationary. The coefficient of static friction between the strap and wheel is $\mu_s = 0.5$.

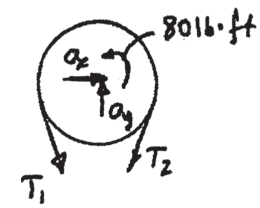


SOLUTION

$\beta = 20° + 180° + 45° = 245°$

$\circlearrowleft + \Sigma M_O = 0; \qquad T_1(1.25) + 80 - T_2(1.25) = 0$

$T_2 = T_1 e^{\mu\beta}; \qquad T_2 = T_1 e^{0.5(245°)(\frac{\pi}{180°})} = 8.4827T_1$



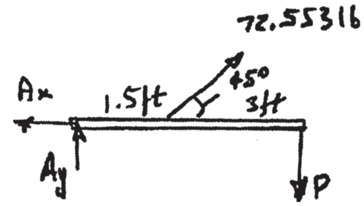
Solving;

$T_1 = 8.553\text{ lb}$

$T_2 = 72.553\text{ lb}$

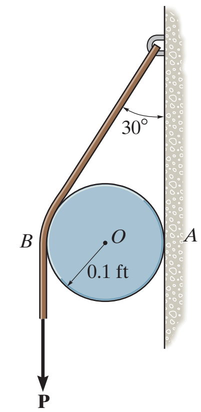
$\circlearrowleft + \Sigma M_A = 0; \qquad -72.553(\sin 45°)(1.5) - 4.5P = 0$

$P = 17.1\text{ lb}$ **Ans.**



8–94.

A minimum force of $P = 50$ lb is required to hold the cylinder from slipping against the belt and the wall. Determine the weight of the cylinder if the coefficient of friction between the belt and cylinder is $\mu_s = 0.3$ and slipping does not occur at the wall.



SOLUTION

Equations of Equilibrium: Write the moment equation of equilibrium about point A by referring to the FBD of the cylinder shown in Fig. *a*,

$$\zeta + \Sigma M_A = 0; \quad 50(0.2) + W(0.1) - T_2 \cos 30°(0.1 + 0.1 \cos 30°) - T_2 \sin 30°(0.1 \sin 30°) = 0 \qquad \textbf{(1)}$$

Frictional Force on Flat Belt: Here, $T_1 = 50$ lb,

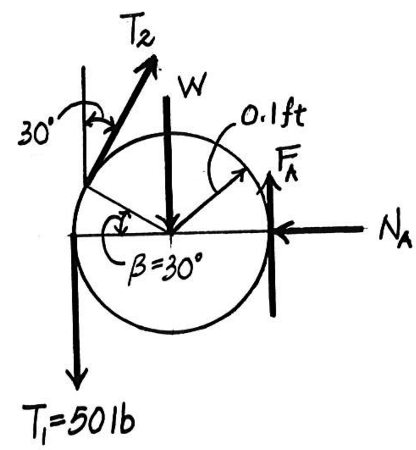
$\beta = \left(\frac{30°}{180°}\right)\pi = \frac{\pi}{6}$ rad. Applying Eq. 8–6

$$T_2 = T_1 e^{\mu\beta}$$

$$= 50 e^{0.3\left(\frac{\pi}{6}\right)} = 58.50 \text{ lb}$$

Substitute this result into Eq. (1),

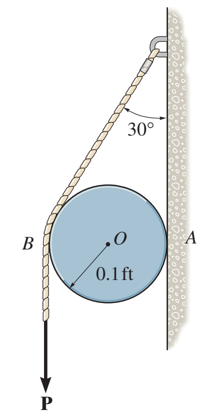
$W = 9.17$ lb **Ans.**



(a)

8–95.

The cylinder weighs 10 lb and is held in equilibrium by the belt and wall. If slipping does not occur at the wall, determine the minimum vertical force P which must be applied to the belt for equilibrium. The coefficient of static friction between the belt and the cylinder is $\mu_s = 0.25$.

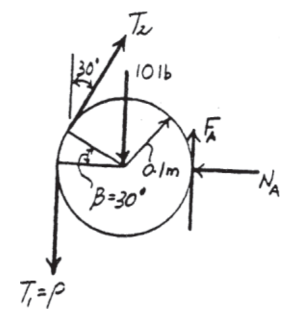


SOLUTION

Equations of Equilibrium:

$$\circlearrowleft + \Sigma M_A = 0; \qquad P(0.2) + 10(0.1) - T_2 \cos 30°(0.1 + 0.1 \cos 30°)$$

$$- T_2 \sin 30°(0.1 \sin 30°) = 0 \qquad \textbf{(1)}$$



Frictional Force on Flat Belt: Here, $\beta = 30° = \dfrac{\pi}{6}$ rad and $T_1 = P$. Applying Eq. 8–6, $T_2 = T_1 e^{\mu\beta}$, we have

$$T_2 = Pe^{0.25(\pi/6)} = 1.140P \qquad \textbf{(2)}$$

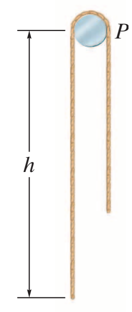
Solving Eqs. (1) and (2) yields

$$P = 78.7 \text{ lb} \qquad \textbf{Ans.}$$

$$T_2 = 89.76 \text{ lb}$$

***8–96.**

A cord having a weight of 0.5 lb/ft and a total length of 10 ft is suspended over a peg P as shown. If the coefficient of static friction between the peg and cord is $\mu_s = 0.5$, determine the longest length h which one side of the suspended cord can have without causing motion. Neglect the size of the peg and the length of cord draped over it.

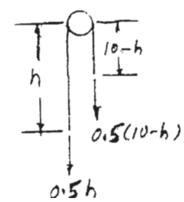


SOLUTION

$T_2 = T_1 e^{\mu\beta}$ Where $T_2 = 0.5h, T_1 = 0.5(10 - h), \beta = \pi$ rad

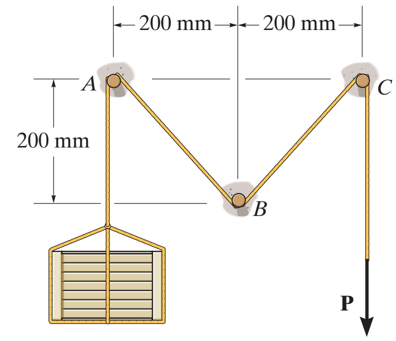
$$0.5h = 0.5(10 - h)e^{0.5(\pi)}$$

$$h = 8.28 \text{ ft}$$ **Ans.**



8–97.

Determine the smallest force **P** required to lift the 40-kg crate. The coefficient of static friction between the cable and each peg is $\mu_s = 0.1$.



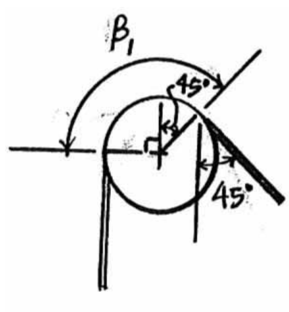
SOLUTION

Since the crate is on the verge of ascending, $T_1 = 40(9.81)$ N and $T_2 = P$. From the geometry shown in Figs. *a* and *b*, the total angle the rope makes when in contact with the peg is $\beta = 2\beta_1 + \beta_2 = 2\left(\frac{135°}{180°}\pi\right) + \left(\frac{90°}{180°}\pi\right) = 2\pi$ rad. Thus,

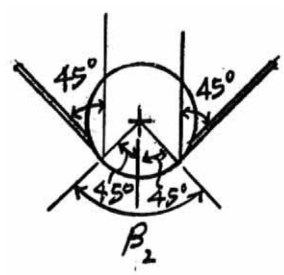
$$T_2 = T_1 e^{\mu_s \beta}$$

$$P = 40(9.81)e^{0.1(2\pi)}$$

$$= 736 \text{ N} \qquad \textbf{Ans.}$$



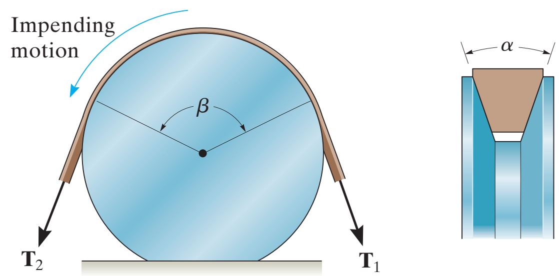
(a)



(b)

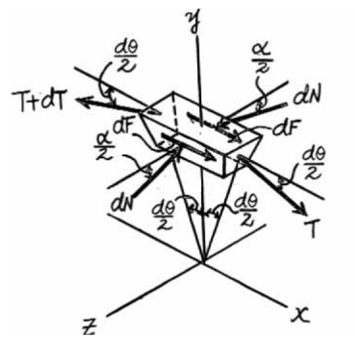
8–98.

Show that the frictional relationship between the belt tensions, the coefficient of friction μ, and the angular contacts α and β for the V-belt is $T_2 = T_1 e^{\mu\beta/\sin(\alpha/2)}$.



SOLUTION

FBD of a section of the belt is shown.



Proceeding in the general manner:

$\Sigma F_x = 0;$ $$-(T + dT)\cos\frac{d\theta}{2} + T\cos\frac{d\theta}{2} + 2\,dF = 0$$

$\Sigma F_y = 0;$ $$-(T + dT)\sin\frac{d\theta}{2} - T\sin\frac{d\theta}{2} + 2\,dN\sin\frac{\alpha}{2} = 0$$

Replace $\sin\frac{d\theta}{2}$ by $\frac{d\theta}{2}$,

$\cos\frac{d\theta}{2}$ by 1,

$dF = \mu\,dN$

Using this and $(dT)(d\theta) \rightarrow 0$, the above relations become

$$dT = 2\mu\,dN$$

$$T\,d\theta = 2\left(dN\sin\frac{\alpha}{2}\right)$$

Combine

$$\frac{dT}{T} = \mu\frac{d\theta}{\sin\frac{\alpha}{2}}$$

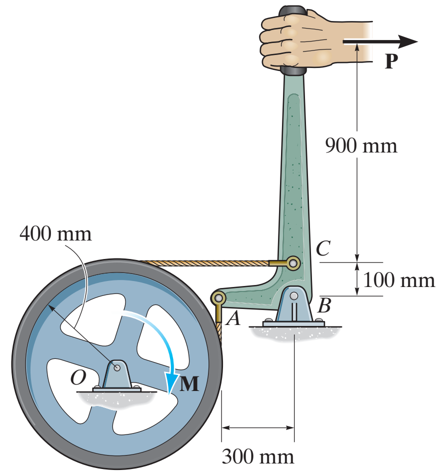
Integrate from $\theta = 0,\ T = T_1$
to $\theta = \beta, T = T_2$

we get,

$$T_2 = T_1\,e^{\left(\frac{\mu\beta}{\sin\frac{\alpha}{2}}\right)}$$ **Q.E.D**

8–99.

If a force of $P = 200\text{ N}$ is applied to the handle of the bell crank, determine the maximum torque **M** that can be resisted so that the flywheel does not rotate clockwise. The coefficient of static friction between the brake band and the rim of the wheel is $\mu_s = 0.3$.



SOLUTION

Referring to the free-body diagram of the bell crane shown in Fig. *a* and the flywheel shown in Fig. *b*, we have

$\circlearrowleft + \Sigma M_B = 0;\qquad T_A(0.3) + T_C(0.1) - 200(1) = 0$ **(1)**

$\circlearrowleft + \Sigma M_O = 0;\qquad T_A(0.4) - T_C(0.4) - M = 0$ **(2)**

By considering the friction between the brake band and the rim of the wheel where $\beta = \dfrac{270°}{180°}\pi = 1.5\pi$ rad and $T_A > T_C$, we can write

$$T_A = T_C e^{\mu_s \beta}$$

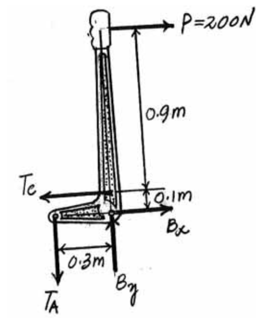
$$T_A = T_C e^{0.3(1.5\pi)}$$

$$T_A = 4.1112 T_C \qquad \textbf{(3)}$$

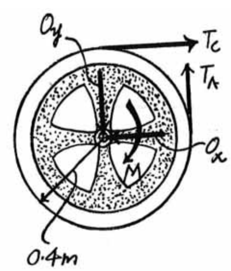
Solving Eqs. (1), (2), and (3) yields

$$M = 187\text{ N}\cdot\text{m}$$

$$T_A = 616.67\text{ N} \quad T_C = 150.00\text{ N}$$ **Ans.**



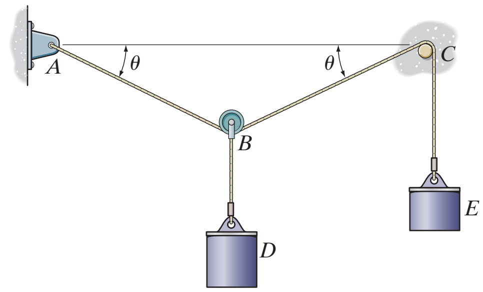
(a)



(b)

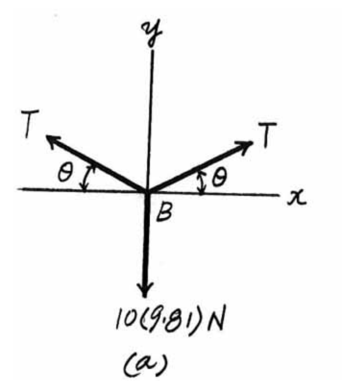
***8–100.**

A 10-kg cylinder D, which is attached to a small pulley B, is placed on the cord as shown. Determine the largest angle u so that the cord does not slip over the peg at C. The cylinder at E also has a mass of 10 kg, and the coefficient of static friction between the cord and the peg is $\mu_s = 0.1$.



SOLUTION

Since pully B is smooth, the tension in the cord between pegs A and C remains constant. Referring to the free-body diagram of the joint B shown in Fig. a, we have



$$+\uparrow \Sigma F_y = 0; \qquad 2T\sin\theta - 10(9.81) = 0 \qquad T = \frac{49.05}{\sin\theta}$$

In the case where cylinder E is on the verge of ascending, $T_2 = T = \dfrac{49.05}{\sin\theta}$ and $T_1 = 10(9.81)\,\text{N}$. Here, $\dfrac{\pi}{2} + \theta$, Fig. b. Thus,

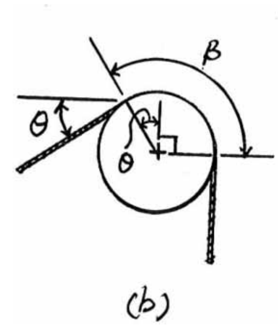
$$T_2 = T_1 e^{\mu_s \beta}$$

$$\frac{49.05}{\sin\theta} = 10(9.81)\,e^{0.1\left(\frac{\pi}{2} + \theta\right)}$$

$$\ln\frac{0.5}{\sin\theta} = 0.1\left(\frac{\pi}{2} + \theta\right)$$

Solving by trial and error, yields

$$\theta = 0.4221 \text{ rad} = 24.2^\circ$$



In the case where cylinder E is on the verge of descending, $T_2 = 10(9.81)$ N and $T_1 = \dfrac{49.05}{\sin\theta}$. Here, $\dfrac{\pi}{2} + \theta$. Thus,

$$T_2 = T_1 e^{\mu_s \beta}$$

$$10(9.81) = \frac{49.05}{\sin\theta}\,e^{0.1\left(\frac{\pi}{2} + \theta\right)}$$

$$\ln(2\sin\theta) = 0.1\left(\frac{\pi}{2} + \theta\right)$$

Solving by trial and error, yields

$$\theta = 0.6764 \text{ rad} = 38.8^\circ$$

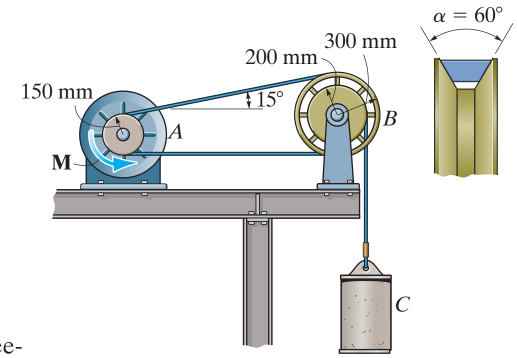
Thus, the range of θ at which the wire does not slip over peg C is

$$24.2^\circ < \theta < 38.8^\circ$$

$$\theta_{max} = 38.8^\circ \qquad \textbf{Ans.}$$

8–101.

A V-belt is used to connect the hub A of the motor to wheel B. If the belt can withstand a maximum tension of 1200 N, determine the largest mass of cylinder C that can be lifted and the corresponding torque **M** that must be supplied to A. The coefficient of static friction between the hub and the belt is $\mu_s = 0.3$, and between the wheel and the belt is $\mu_s' = 0.20$. *Hint:* See Prob. 8–98.



SOLUTION

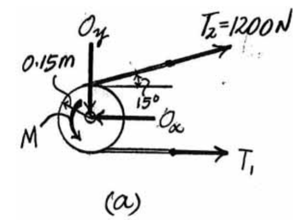
In this case, the maximum tension in the belt is $T_2 = 1200$ N. Referring to the free-body diagram of hub A, shown in Fig. a and the wheel B shown in Fig. b, we have

$\circlearrowleft + \Sigma M_O = 0; \qquad M + T_1(0.15) - 1200(0.15) = 0$

$$M = 0.15(1200 - T_1) \qquad \textbf{(1)}$$

$\circlearrowleft + \Sigma M_{O'} = 0; \qquad 1200(0.3) - T_1(0.3) - M_C(9.81)(0.2) = 0$

$$1200 - T_1 = 6.54 M_C \qquad \textbf{(2)}$$



If hub A is on the verge of slipping, then

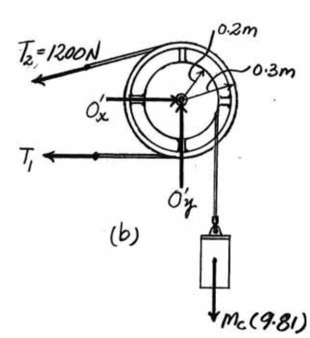
$$T_2 = T_1 e^{\mu_s \beta_1/\sin(\alpha/2)} \text{ where } \beta_1 = \left(\frac{90^\circ + 75^\circ}{180^\circ}\right)\pi = 0.9167\pi \text{ rad}$$

$$1200 = T_1 e^{0.3(0.9167\pi)/\sin 30^\circ}$$

$$T_1 = 213.19 \text{ N}$$

Substituting $T_1 = 213.19$ N into Eq. (2), yields

$$M_C = 150.89 \text{ kg}$$



If wheel B is on the verge of slipping, then

$$T_2 = T_1 e^{\mu_s' \beta_1/\sin(\alpha/2)} \text{ where } \beta_2 = \left(\frac{180^\circ + 15^\circ}{180^\circ}\right)\pi = 1.0833\pi \text{ rad}$$

$$1200 = T_1 e^{0.2(1.0833\pi)/\sin 30^\circ}$$

$$T_1 = 307.57 \text{ N}$$

Substituting $T_1 = 307.57$ N into Eq. (2), yields

$$M_C = 136.45 \text{ kg} = 136 \text{ kg (controls!)} \qquad \textbf{Ans.}$$

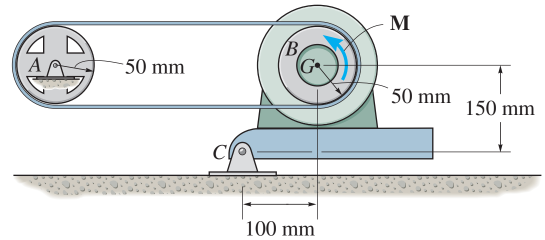
Substituting $T_1 = 307.57$ N into Eq. (1), we obtain

$$M = 0.15(1200 - 307.57)$$

$$= 134 \text{ N} \cdot \text{m} \qquad \textbf{Ans.}$$

8–102.

The 20-kg motor has a center of gravity at G and is pin-connected at C to maintain a tension in the drive belt. Determine the smallest counterclockwise twist or torque **M** that must be supplied by the motor to turn the disk B if wheel A locks and causes the belt to slip over the disk. No slipping occurs at A. The coefficient of static friction between the belt and the disk is $\mu_s = 0.3$.



SOLUTION

Equations of Equilibrium: From FBD (a),

$$\zeta + \Sigma M_C = 0; \quad T_2\,(100) + T_1\,(200) - 196.2(100) = 0 \qquad \textbf{(1)}$$

From FBD (b),

$$\zeta + \Sigma M_O = 0; \quad M + T_1\,(0.05) - T_2\,(0.05) = 0 \qquad \textbf{(2)}$$

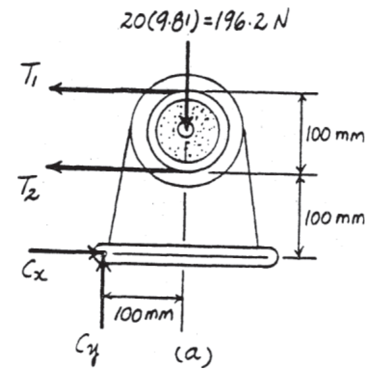
Frictional Force on Flat Belt: Here, $\beta = 180° = \pi$ rad. Applying Eq. 8–6, $T_2 = T_1\,e^{\mu\beta}$, we have

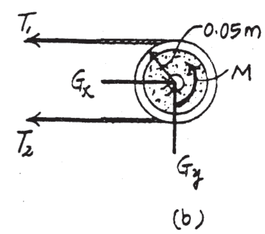
$$T_2 = T_1\,e^{0.3\pi} = 2.566T_1 \qquad \textbf{(3)}$$

Solving Eqs. (1), (2), and (3) yields

$$M = 3.37\ \text{N}\cdot\text{m} \qquad \textbf{Ans.}$$

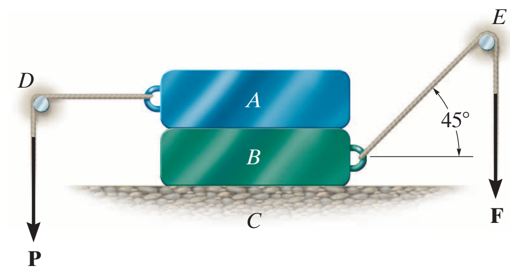
$$T_1 = 42.97\ \text{N} \qquad T_2 = 110.27\ \text{N}$$





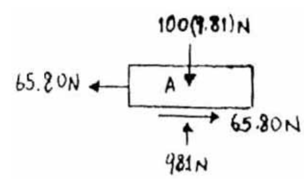
8–103.

Blocks A and B have a mass of 100 kg and 150 kg, respectively. If the coefficient of static friction between A and B and between B and C is $\mu_s = 0.25$ and between the ropes and the pegs D and E $\mu'_s = 0.5$ determine the smallest force F needed to cause motion of block B if $P = 30$ N.



SOLUTION

Assume no slipping between A and B.



Peg D:

$T_2 = T_1 e^{\mu\beta}; \quad F_{AD} = 30\, e^{0.5(\frac{\pi}{2})} = 65.80 \text{ N}$

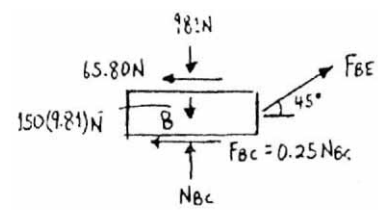
Block B:

$\xrightarrow{+} \Sigma F_x = 0; \qquad -65.80 - 0.25\, N_{BC} + F_{BE} \cos 45° = 0$

$+\uparrow \Sigma F_y = 0; \qquad N_{BC} - 981 + F_{BE} \sin 45° - 150\,(9.81) = 0$

$F_{BE} = 768.1 \text{ N}$

$N_{BC} = 1909.4 \text{ N}$



Peg E:

$T_2 = T_1 e^{\mu\beta}; \quad F = 768.1 e^{0.5(\frac{3\pi}{4})} = 2.49 \text{ kN}$ **Ans.**

Note: Since B moves to the right,

$(F_{AB})_{\max} = 0.25\,(981) = 245.25 \text{ N}$

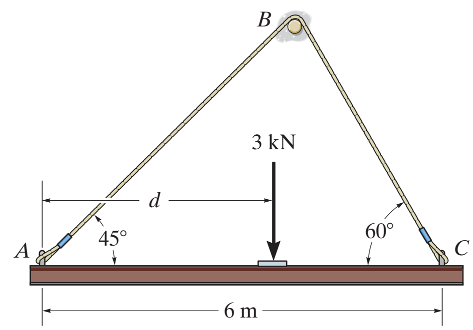
$245.25 = P_{\max}\, e^{0.5(\frac{\pi}{2})}$

$P_{\max} = 112 \text{ N} > 30 \text{ N}$

Hence, no slipping occurs between A and B as originally assumed.

***8–104.**

Determine the minimum coefficient of static friction μ_s between the cable and the peg and the placement d of the 3-kN force for the uniform 100-kg beam to maintain equilibrium.



SOLUTION

Referring to the free-body diagram of the beam shown in Fig. a, we have

$$\xrightarrow{+} \Sigma F_x = 0; \qquad T_{AB}\cos 45° - T_{BC}\cos 60° = 0$$

$$+\uparrow \Sigma F_y = 0; \qquad T_{AB}\sin 45° + T_{BC}\sin 60° - 3 - \frac{100(9.81)}{1000} = 0$$

$$\circlearrowleft + \Sigma M_A = 0; \qquad T_{BC}\sin 60°(6) - \frac{100(9.81)}{1000}(3) - 3d = 0$$

Solving,

$$d = 4.07\text{ m} \qquad \textbf{Ans.}$$

$$T_{BC} = 2.914\text{ kN} \quad T_{AB} = 2.061\text{ kN}$$

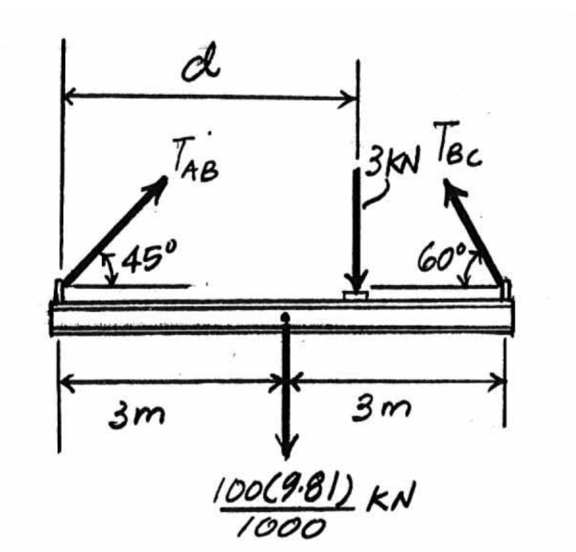
Using the results for T_{BC} and T_{AB} and considering the friction between the cable and the peg, where $\beta = \left[\left(\frac{45° + 60°}{180°}\right)\pi\right] = 0.5833\pi$ rad, we have

$$T_{BC} = T_{AB}\, e^{\mu_s \beta}$$

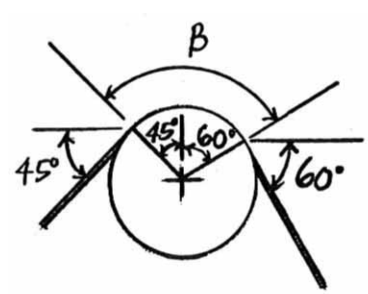
$$2.914 = 2.061\, e^{\mu_s(0.5833\pi)}$$

$$\ln 1.414 = \mu_s(0.5833\pi)$$

$$\mu_s = 0.189 \qquad \textbf{Ans.}$$

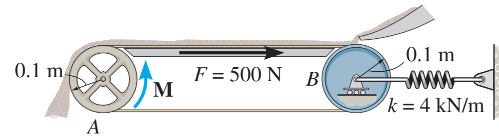


(a)



8–105.

A conveyer belt is used to transfer granular material and the frictional resistance on the top of the belt is $F = 500$ N. Determine the smallest stretch of the spring attached to the moveable axle of the idle pulley B so that the belt does not slip at the drive pulley A when the torque $\mathbf{M}$ is applied. What minimum torque $\mathbf{M}$ is required to keep the belt moving? The coefficient of static friction between the belt and the wheel at A is $\mu_s = 0.2$.



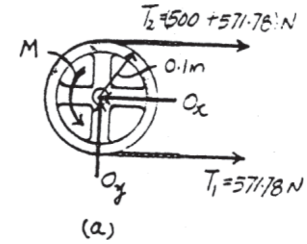
SOLUTION

Frictional Force on Flat Belt: Here, $\beta = 180° = \pi$ rad and $T_2 = 500 + T$ and $T_1 = T$. Applying Eq. 8–6, we have

$$T_2 = T_1 e^{\mu\beta}$$

$$500 + T = Te^{0.2\pi}$$

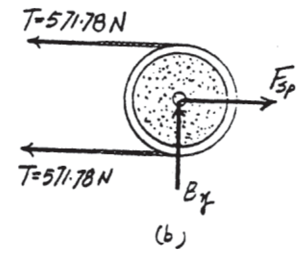
$$T = 571.78 \text{ N}$$



Equations of Equilibrium: From FBD (a),

$\circlearrowleft + \Sigma M_O = 0; \qquad M + 571.78(0.1) - (500 + 578.1)(0.1) = 0$

$$M = 50.0 \text{ N} \cdot \text{m}$$ **Ans.**



From FBD (b),

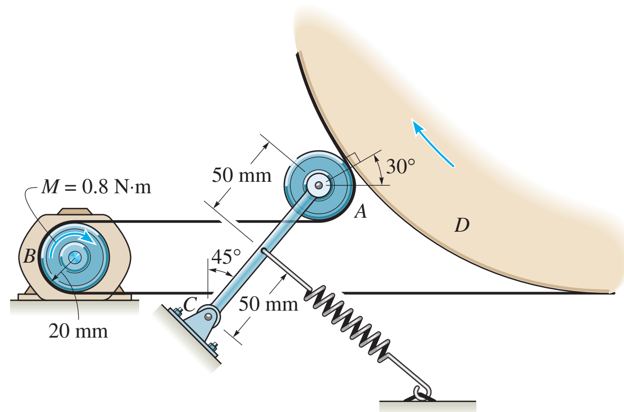
$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad F_{sp} - 2(578.71) = 0 \qquad F_{sp} = 1143.57 \text{ N}$

Thus, the spring stretch is

$$x = \frac{F_{sp}}{k} = \frac{1143.57}{4000} = 0.2859 \text{ m} = 286 \text{ mm}$$ **Ans.**

8–106.

The belt on the portable dryer wraps around the drum D, idler pulley A, and motor pulley B. If the motor can develop a maximum torque of $M = 0.80\ \text{N}\cdot\text{m}$, determine the smallest spring tension required to hold the belt from slipping. The coefficient of static friction between the belt and the drum and motor pulley is $\mu_s = 0.3$. Ignore the size of the idler pulley A.



SOLUTION

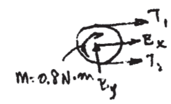
$\zeta + \Sigma M_B = 0; \qquad -T_1\,(0.02) + T_2\,(0.02) - 0.8 = 0$

$T_2 = T_1\,e^{\mu\beta}; \qquad T_2 = T_1\,e^{(0.3)(\pi)} = 2.5663T_1$

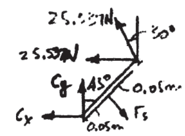
$T_1 = 25.537\ \text{N}$

$T_2 = 65.53\ \text{N}$

$\zeta + \Sigma M_C = 0; \qquad -F_s(0.05) + (25.537 + 25.537 \sin 30°)(0.1 \cos 45°) + 25.537 \cos 30°(0.1 \sin 45°) = 0$

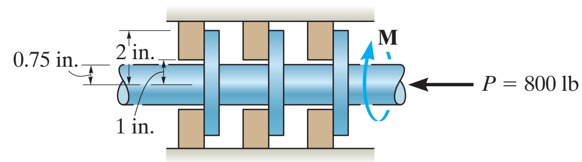


$F_s = 85.4\ \text{N}$ **Ans.**



8–107.

The annular ring bearing is subjected to a thrust of 800 lb. Determine the smallest required coefficient of static friction if a torque of $M = 15\ \text{lb}\cdot\text{ft}$ must be resisted to prevent the shaft from rotating.



SOLUTION

Bearing Friction. Applying Eq. 8–7 with $R_2 = 2$ in., $R_1 = 1$ in., $P = 800$ lb and $M = 15\ \text{lb}\cdot\text{ft}\left(\frac{12\ \text{in}}{1\ \text{ft}}\right) = 180\ \text{lb}\cdot\text{in}$,

$$M = \frac{2}{3}\mu_s P\left(\frac{R_2^3 - R_1^3}{R_2^2 - R_1^2}\right)$$

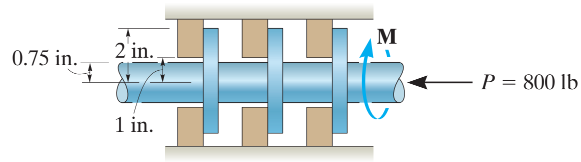
$$180 = \frac{2}{3}\mu_s(800)\left(\frac{2^3 - 1^3}{2^2 - 1^2}\right)$$

$$\mu_s = 0.145$$ **Ans.**

Note that each of the bearings will result $\frac{1}{3}M$ and the bond on each bearing is $\frac{1}{3}P$, which yields the same result.

***8–108.**

The annular ring bearing is subjected to a thrust of 800 lb. If $\mu_s = 0.35$, determine the torque M that must be applied to overcome friction.



SOLUTION

$$M = \frac{2}{3}\mu_s P\left(\frac{R_2^3 - R_1^3}{R_2^2 - R_1^2}\right)$$

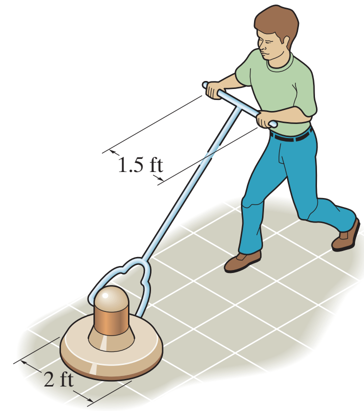
$$= \frac{2}{3}(0.35)(800)\left[\frac{(2)^3 - 1^3}{(2)^2 - 1^2}\right]$$

$$= 435.6 \text{ lb} \cdot \text{in.}$$

$$M = 36.3 \text{ lb} \cdot \text{ft}$$ **Ans.**

8–109.

The floor-polishing machine rotates at a constant angular velocity. If it has a weight of 80 lb. determine the couple forces F the operator must apply to the handles to hold the machine stationary. The coefficient of kinetic friction between the floor and brush is $\mu_k = 0.3$. Assume the brush exerts a uniform pressure on the floor.



SOLUTION

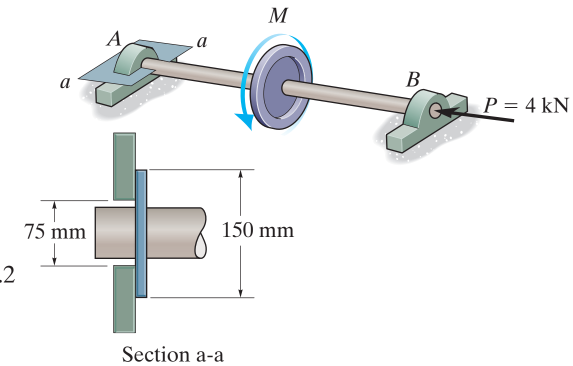
$$M = \frac{2}{3}\mu \, P \, R$$

$$F(1.5) = \frac{2}{3}(0.3)\,(80)(1)$$

$$F = 10.7 \text{ lb}$$ **Ans.**

8–110.

The shaft is supported by a thrust bearing A and a journal bearing B. Determine the torque **M** required to rotate the shaft at constant angular velocity. The coefficient of kinetic friction at the thrust bearing is $\mu_k = 0.2$. Neglect friction at B.



SOLUTION

Applying Eq. 8–7 with $R_1 = \dfrac{0.075}{2} = 0.0375\text{ m},\ R_2 = \dfrac{0.15}{2} = 0.075\text{ m},\ \mu_s = 0.2$ and $P = 4000$ N, we have

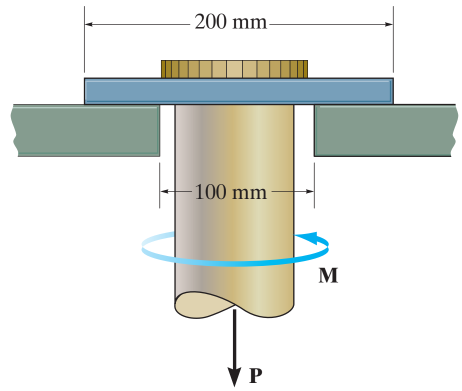
$$M = \frac{2}{3}\mu_s P\left(\frac{R_2^3 - R_1^3}{R_2^2 - R_1^2}\right)$$

$$= \frac{2}{3}(0.2)(4000)\left(\frac{0.075^3 - 0.0375^3}{0.075^2 - 0.0375^2}\right)$$

$$= 46.7\text{ N}\cdot\text{m}$$ **Ans.**

8–111.

The thrust bearing supports an axial load of $P = 6$ kN. If a torque of $M = 150$ N·m is required to rotate the shaft, determine the coefficient of static friction at the constant surface.



SOLUTION

Applying Eq. 8–7 with $R_1 = \dfrac{0.1\text{ m}}{2} = 0.05\text{ m}$, $R_2 = \dfrac{0.2\text{ m}}{2} = 0.1\text{ m}$, $M = 150\text{ N}\cdot\text{m}$ and $P = 6000$ N, we have

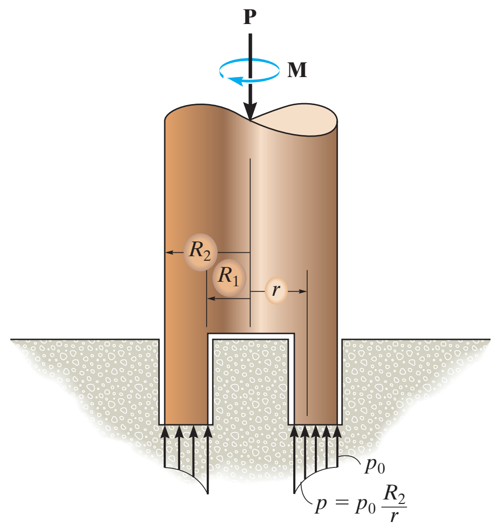
$$M = \frac{2}{3}\mu_s P\left(\frac{R_2^{\,3} - R_1^{\,3}}{R_2^{\,2} - R_1^{\,2}}\right)$$

$$150 = \frac{2}{3}\,\mu_s(6000)\left(\frac{0.1^3 - 0.05^3}{0.1^2 - 0.05^2}\right)$$

$$\mu_s = 0.321 \qquad \textbf{Ans.}$$

***8–112.**

Assuming that the variation of pressure at the bottom of the pivot bearing is defined as $p = p_0(R_2/r)$, determine the torque M needed to overcome friction if the shaft is subjected to an axial force **P**. The coefficient of static friction is μ_s. For the solution, it is necessary to determine p_0 in terms of P and the bearing dimensions R_1 and R_2.



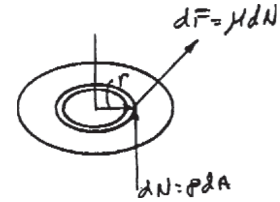
SOLUTION

$$\Sigma F_z = 0; \qquad P = \int_A dN = \int_0^{2\pi}\int_{R_1}^{R_2} pr\, dr\, d\theta$$

$$= \int_0^{2\pi}\int_{R_1}^{R_2} p_0\left(\frac{R_2}{r}\right) r\, dr\, d\theta$$

$$= 2\pi\, p_0\, R_2\,(R_2 - R_1)$$

Thus, $p_0 = \dfrac{P}{\left[2\pi R_2\,(R_2 - R_1)\right]}$



$$\Sigma M_z = 0; \qquad M = \int_A r\, dF = \int_0^{2\pi}\int_{R_1}^{R_2} \mu_s\, pr^2\, dr\, d\theta$$

$$= \int_0^{2\pi}\int_{R_1}^{R_2} \mu_s\, p_0\left(\frac{R_2}{r}\right) r^2\, dr\, d\theta$$

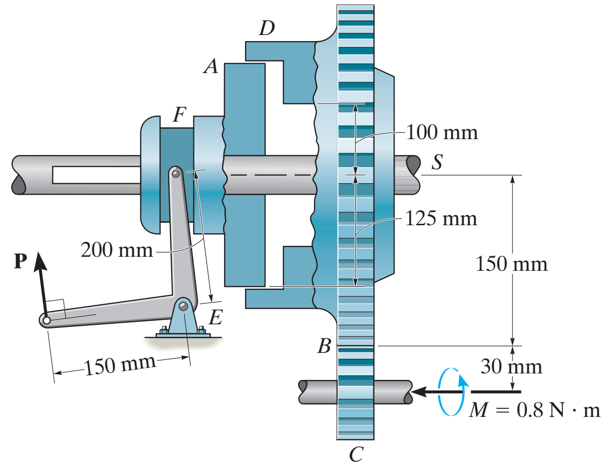
$$= \mu_s\,(2\pi\, p_0)\, R_2\, \frac{1}{2}\left(R_2^2 - R_1^2\right)$$

Using Eq. (1):

$$M = \frac{1}{2}\mu_s\, P\,(R_2 + R_1) \qquad \textbf{Ans.}$$

8–113.

The plate clutch consists of a flat plate A that slides over the rotating shaft S. The shaft is fixed to the driving plate gear B. If the gear C, which is in mesh with B, is subjected to a torque of $M = 0.8\text{N} \cdot \text{m}$, determine the smallest force P, that must be applied via the control arm, to stop the rotation. The coefficient of static friction between the plates A and D is $\mu_s = 0.4$. Assume the bearing pressure between A and D to be uniform.



SOLUTION

$$F = \frac{0.8}{0.03} = 26.667 \text{ N}$$

$$M = 26.667(0.150) = 4.00 \text{ N} \cdot \text{m}$$

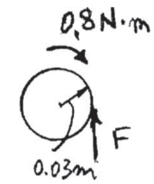
$$M = \frac{2}{3}\mu P'\left(\frac{R_2^3 - R_1^3}{R_2^2 - R_1^2}\right)$$

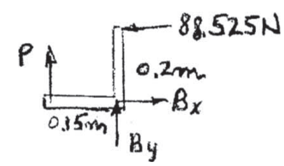
$$4.00 = \frac{2}{3}(0.4)(P')\left(\frac{(0.125)^3 - (0.1)^3}{(0.125)^2 - (0.1)^2}\right)$$

$$P' = 88.525 \text{ N}$$

$$\zeta + \Sigma M_F = 0; \qquad 88.525(0.2) - P(0.15) = 0$$

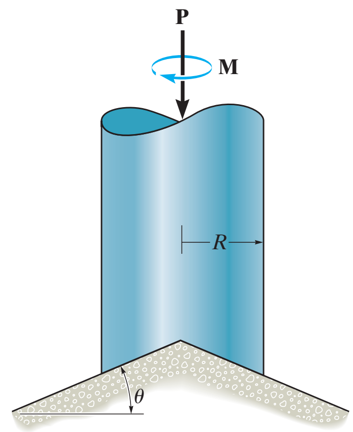
$$P = 118 \text{ N}$$ **Ans.**





8–114.

The conical bearing is subjected to a constant pressure distribution at its surface of contact. If the coefficient of static friction is μ_s, determine the torque M required to overcome friction if the shaft supports an axial force **P**.



SOLUTION

The differential area (shaded) $dA = 2\pi r\left(\dfrac{dr}{\cos\theta}\right) = \dfrac{2\pi r dr}{\cos\theta}$

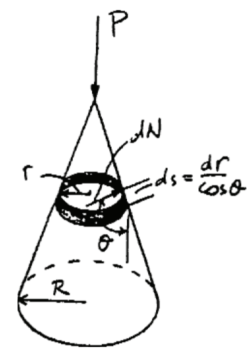
$$P = \int p\cos\theta\, dA = \int p\cos\theta\left(\frac{2\pi r dr}{\cos\theta}\right) = 2\pi p\int_0^R r dr$$

$$P = \pi p R^2 \qquad p = \frac{P}{\pi R^2}$$

$$dN = p dA = \frac{P}{\pi R^2}\left(\frac{2\pi r dr}{\cos\theta}\right) = \frac{2P}{R^2\cos\theta} r dr$$

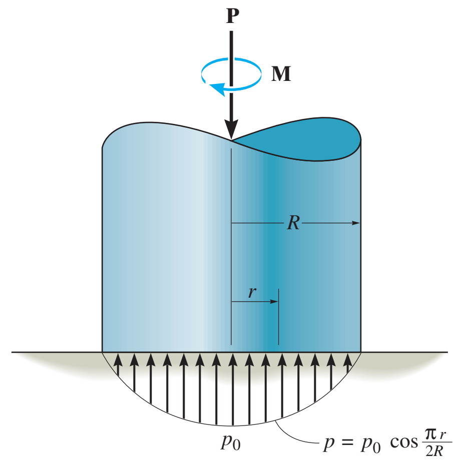
$$M = \int r dF = \int \mu_s r dN = \frac{2\mu_s P}{R^2\cos\theta}\int_0^R r^2\, dr$$

$$= \frac{2\mu_s P}{R^2\cos\theta}\frac{R^3}{3} = \frac{2\mu_s PR}{3\cos\theta} \qquad \textbf{Ans.}$$



8–115.

The pivot bearing is subjected to a pressure distribution at its surface of contact which varies as shown. If the coefficient of static friction is μ, determine the torque M required to overcome friction if the shaft supports an axial force **P.**



SOLUTION

$$dF = \mu\, dN = \mu\, p_0 \cos\left(\frac{\pi r}{2R}\right) dA$$

$$M = \int_A r\mu\, p_0 \cos\left(\frac{\pi r}{2R}\right) r\, dr\, d\theta$$

$$= \mu\, p_0 \int_0^R \left(r^2 \cos\left(\frac{\pi r}{2R}\right) dr\right) \int_0^{2\pi} d\theta$$

$$= \mu\, p_0 \left[\frac{2r}{\left(\frac{\pi}{2R}\right)^2} \cos\left(\frac{\pi r}{2R}\right) + \frac{\left(\frac{\pi}{2R}\right)^2 r^2 - 2}{\left(\frac{\pi}{2R}\right)^3} \sin\left(\frac{\pi r}{2R}\right)\right]_0^R (2\pi)$$

$$= \mu p_0 \left(\frac{16R^3}{\pi^2}\right)\left[\left(\frac{\pi}{2}\right)^2 - 2\right]$$

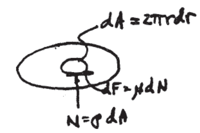
$$= 0.7577\mu\, p_0\, R^3$$

$$P = \int_A dN = \int_0^R p_0 \left(\cos\left(\frac{\pi r}{2R}\right) r dr\right) \int_0^{2\pi} d\theta$$

$$= p_0 \left[\frac{1}{\left(\frac{\pi}{2R}\right)^2} \cos\left(\frac{\pi r}{2R}\right) + \frac{r}{\left(\frac{\pi}{2R}\right)} \sin\left(\frac{\pi r}{2R}\right)\right]_0^R (2\pi)$$

$$= 4p_0\, R^2\left(1 - \frac{2}{\pi}\right)$$

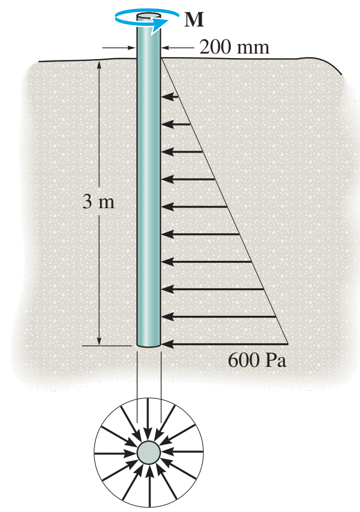
$$= 1.454 p_0\, R^2$$



Thus, $M = 0.521\, P\mu R$ **Ans.**

***8–116.**

A 200-mm diameter post is driven 3 m into sand for which $\mu_s = 0.3$. If the normal pressure acting *completely around the post* varies linearly with depth as shown, determine the frictional torque **M** that must be overcome to rotate the post.

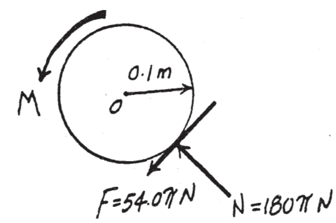


SOLUTION

Equations of Equilibrium and Friction: The resultant normal force on the post is $N = \frac{1}{2}(600 + 0)(3)(\pi)(0.2) = 180\pi$ N. Since the post is on the verge of rotating, $F = \mu_s N = 0.3(180\pi) = 54.0\pi$ N.

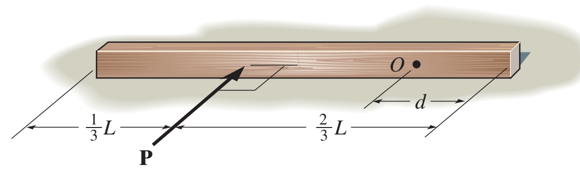
$\circlearrowleft + \Sigma M_O = 0; \qquad M - 54.0\pi(0.1) = 0$

$$M = 17.0 \text{ N} \cdot \text{m}$$ **Ans.**

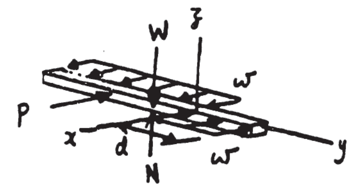


8–117.

A beam having a uniform weight W rests on the rough horizontal surface having a coefficient of static friction μ_s. If the horizontal force **P** is applied perpendicular to the beam's length, determine the location d of the point O about which the beam begins to rotate.



SOLUTION



$$w = \frac{\mu_s N}{L}$$

$\Sigma F_z = 0;\qquad N = W$

$\Sigma F_x = 0;\qquad P + \frac{\mu_s N d}{L} - \frac{\mu_s N(L - d)}{L} = 0$

$\Sigma M_{Oz} = 0;\qquad \frac{\mu_s N(L - d)^2}{2L} + \frac{\mu_s N d^2}{2L} - P\left(\frac{2L}{3} - d\right) = 0$

$$\frac{\mu_s W(L - d)^2}{2L} + \frac{\mu_s W d^2}{2L} - \left(\frac{2L}{3} - d\right)\left(\frac{\mu_s W(L - d)}{L} - \frac{\mu_s W d}{L}\right) = 0$$

$$3(L - d)^2 + 3d^2 - 2(2L - 3d)(L - 2d) = 0$$

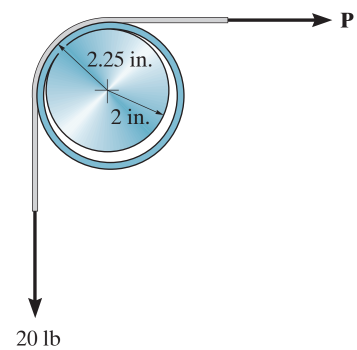
$$6d^2 - 8Ld + L^2 = 0$$

Choose the root $< L$.

$$d = 0.140\ L \qquad \textbf{Ans.}$$

8–118.

The collar fits *loosely* around a fixed shaft that has a radius of 2 in. If the coefficient of kinetic friction between the shaft and the collar is $\mu_k = 0.3$, determine the force P on the horizontal segment of the belt so that the collar rotates *counterclockwise* with a constant angular velocity. Assume that the belt does not slip on the collar; rather, the collar slips on the shaft. Neglect the weight and thickness of the belt and collar. The radius, measured from the center of the collar to the mean thickness of the belt, is 2.25 in.

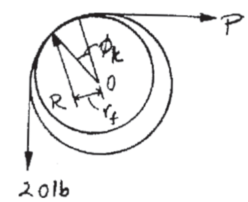


SOLUTION

$\phi_k = \tan^{-1}\mu_k = \tan^{-1}0.3 = 16.699°$

$r_f = 2 \sin 16.699° = 0.5747$ in.

Equilibrium:



$+\uparrow \Sigma F_y = 0; \qquad R_y - 20 = 0 \qquad R_y = 20$ lb

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad P - R_x = 0 \qquad R_x = P$

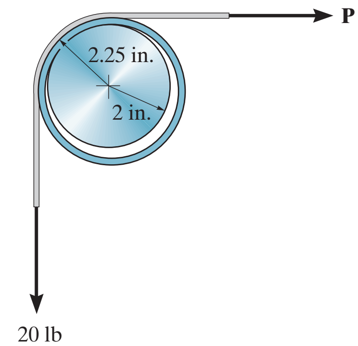
Hence $R = \sqrt{R_x^2 + R_y^2} = \sqrt{P^2 + 20^2}$

$\circlearrowleft + \Sigma M_O = 0; \qquad -\left(\sqrt{P^2 + 20^2}\right)(0.5747) + 20(2.25) - P(2.25) = 0$

$P = 13.8$ lb **Ans.**

8–119.

The collar fits *loosely* around a fixed shaft that has a radius of 2 in. If the coefficient of kinetic friction between the shaft and the collar is $\mu_k = 0.3$, determine the force P on the horizontal segment of the belt so that the collar rotates *clockwise* with a constant angular velocity. Assume that the belt does not slip on the collar; rather, the collar slips on the shaft. Neglect the weight and thickness of the belt and collar. The radius, measured from the center of the collar to the mean thickness of the belt, is 2.25 in.

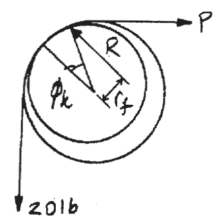


SOLUTION

$\phi_k = \tan^{-1}\mu_k = \tan^{-1} 0.3 = 16.699°$

$r_f = 2 \sin 16.699° = 0.5747$ in.

Equilibrium:



$+\uparrow \Sigma F_y = 0; \quad R_y - 20 = 0 \quad R_y = 20$ lb

$\xrightarrow{+} \Sigma F_x = 0; \quad P - R_x = 0 \quad R_x = P$

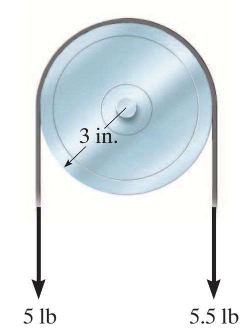
Hence $R = \sqrt{R_x^2 + R_y^2} = \sqrt{P^2 + 20^2}$

$\circlearrowleft + \Sigma M_O = 0; \quad \left(\sqrt{P^2 + 20^2}\right)(0.5747) + 20(2.25) - P(2.25) = 0$

$P = 29.0$ lb **Ans.**

***8–120.**

The pulley has a radius of 3 in. and fits loosely on the 0.5-in.-diameter shaft. If the loadings acting on the belt cause the pulley to rotate with constant angular velocity, determine the frictional force between the shaft and the pulley and compute the coefficient of kinetic friction. The pulley weighs 18 lb.



SOLUTION

$+\uparrow \Sigma F_y = 0;\qquad R - 18 - 10.5 = 0$

$R = 28.5 \text{ lb}$

$\circlearrowleft + \Sigma M_O = 0;\qquad -5.5(3) + 5(3) + 28.5\, r_f = 0$

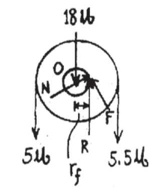
$r_f = 0.05263 \text{ in.}$

$r_f = r \sin \phi_k$

$$0.05263 = \frac{0.5}{2} \sin\phi_k$$

$\phi_k = 12.15°$

$\mu_k = \tan \phi_k = \tan 12.15° = 0.215$ **Ans.**



Note also by approximation,

$r_f = r\,\mu$

$$0.05263 = \frac{0.5}{2}\mu$$

$\mu = 0.211$ (approx.)

Also,

$$\circlearrowleft + \Sigma M_O = 0;\qquad -5.5(3) + 5(3) + F\left(\frac{0.5}{2}\right) = 0$$

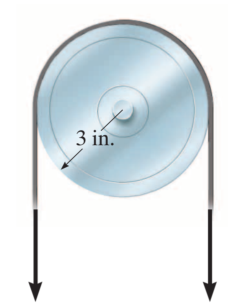
$F = 6 \text{ lb}$ **Ans.**

$N = \sqrt{R^2 - F^2} = \sqrt{(28.5)^2 - 6^2} = 27.86 \text{ lb}$

$$\mu_k = \frac{F}{N} = \frac{6}{27.86} = 0.215$$ **Ans.**

8–121.

The pulley has a radius of 3 in. and fits loosely on the 0.5-in.-diameter shaft. If the loadings acting on the belt cause the pulley to rotate with constant angular velocity, determine the frictional force between the shaft and the pulley and compute the coefficient of kinetic friction. Neglect the weight of the pulley.



SOLUTION

$+\uparrow \Sigma F_y = 0;\qquad R - 5 - 5.5 = 0$

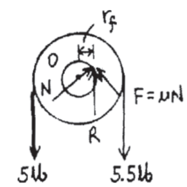
$R = 10.5$ lb

$\circlearrowleft + \Sigma M_O = 0;\qquad -5.5(3) + 5(3) + F(0.25) = 0$

$F = 6$ lb **Ans.**

$N = \sqrt{(10.5)^2 - 6^2} = 8.617$ lb

$\mu_k = \frac{F}{N} = \frac{6}{8.617} = 0.696$ **Ans.**



Also,

$\circlearrowleft + \Sigma M_O = 0;\qquad -5.5(3) + 5(3) + 10.5(r_f) = 0$

$r_f = 0.1429$ in.

$0.1429 = \frac{0.5}{2}\sin\phi_k$

$\phi_k = 34.85°$

$\mu_k = \tan 34.85° = 0.696$ **Ans.**

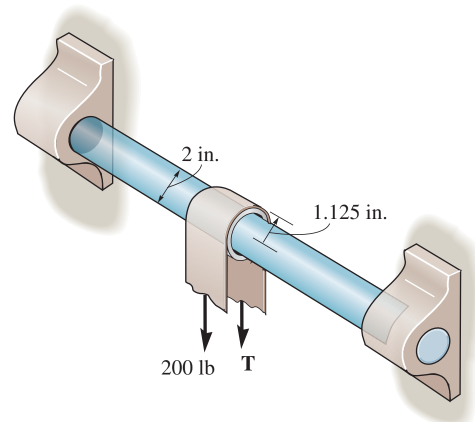
By approximation,

$r_f = r\mu_k$

$\mu_k = \frac{0.1429}{0.25} = 0.571 \qquad \text{(approx.)}$

8–122.

Determine the tension **T** in the belt needed to overcome the tension of 200 lb created on the other side. Also, what are the normal and frictional components of force developed on the collar bushing? The coefficient of static friction is $\mu_s = 0.21$.



SOLUTION

Frictional Force on Journal Bearing: Here, $\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.21 = 11.86°$. Then the radius of friction circle is

$$r_f = r \sin \phi_k = 1 \sin 11.86° = 0.2055 \text{ in.}$$

Equations of Equilibrium:

$$\circlearrowleft + \Sigma M_P = 0; \quad 200(1.125 + 0.2055) - T(1.125 - 0.2055) = 0$$

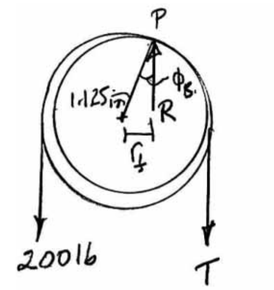
$$T = 289.41 \text{ lb} = 289 \text{ lb} \qquad \textbf{Ans.}$$

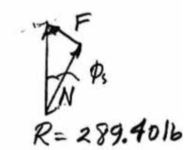
$$+\uparrow F_y = 0; \qquad R - 200 - 289.41 = 0 \qquad R = 489.41 \text{ lb}$$

Thus, the normal and friction force are

$$N = R \cos \phi_s = 489.41 \cos 11.86° = 479 \text{ lb} \qquad \textbf{Ans.}$$

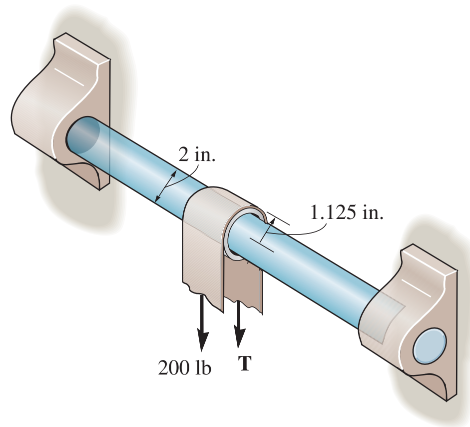
$$F = R \sin \phi_s = 489.41 \sin 11.86° = 101 \text{ lb} \qquad \textbf{Ans.}$$





8–123.

If a tension force $T = 215$ lb is required to pull the 200-lb force around the collar bushing, determine the coefficient of static friction at the contacting surface. The belt does not slip on the collar.



SOLUTION

Equation of Equilibrium:

$$\zeta + \Sigma M_P = 0; \quad 200(1.125 + r_f) - 215(1.125 - r_f) = 0$$

$$r_f = 0.04066 \text{ in.}$$

Frictional Force on Journal Bearing: The radius of friction circle is

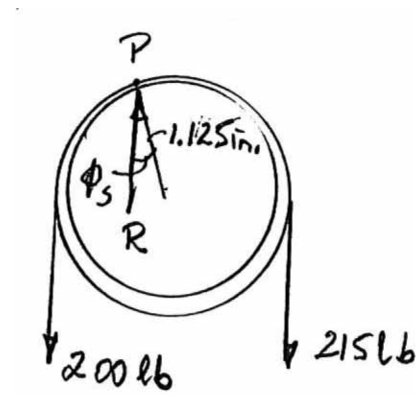
$$r_f = r \sin \phi_k$$

$$0.04066 = 1 \sin \phi_k$$

$$\phi_k = 2.330^\circ$$

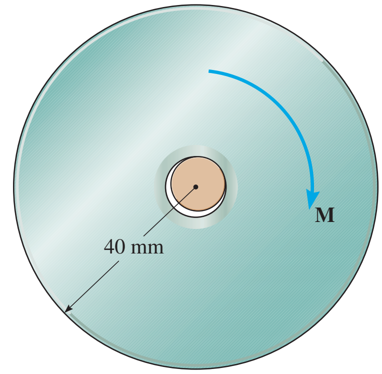
and the coefficient of static friction is

$$\mu_s = \tan \phi_s = \tan 2.330^\circ = 0.0407 \qquad \textbf{Ans.}$$



***8–124.**

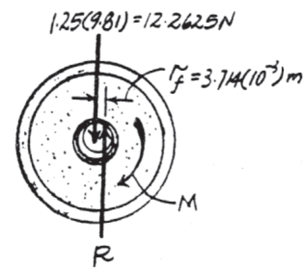
A pulley having a diameter of 80 mm and mass of 1.25 kg is supported loosely on a shaft having a diameter of 20 mm. Determine the torque M that must be applied to the pulley to cause it to rotate with constant motion. The coefficient of kinetic friction between the shaft and pulley is $\mu_k = 0.4$. Also calculate the angle θ which the normal force at the point of contact makes with the horizontal. The shaft itself cannot rotate.



SOLUTION

Frictional Force on Journal Bearing: Here, $\phi_k = \tan^{-1}\mu_k = \tan^{-1}0.4 = 21.80°$. Then the radius of friction circle is $r_f = r \sin\phi_k = 0.01 \sin 21.80° = 3.714(10^{-3})$ m. The angle which the normal force makes with horizontal is

$$\theta = 90° - \phi_k = 68.2° \qquad \textbf{Ans.}$$

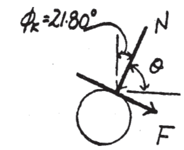


Equations of Equilibrium:

$+\uparrow \Sigma F_y = 0;\qquad R - 12.2625 = 0 \qquad R = 12.2625\text{ N}$

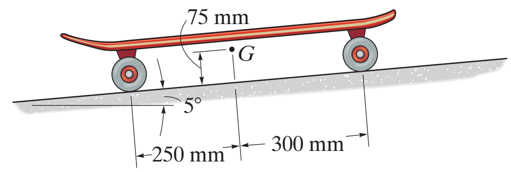
$\circlearrowleft + \Sigma M_O = 0;\qquad 12.2625(3.714)(10^{-3}) - M = 0$

$$M = 0.0455\text{ N}\cdot\text{m} \qquad \textbf{Ans.}$$



8–125.

The 5-kg skateboard rolls down the 5° slope at constant speed. If the coefficient of kinetic friction between the 12.5 mm diameter axles and the wheels is $\mu_k = 0.3$, determine the radius of the wheels. Neglect rolling resistance of the wheels on the surface. The center of mass for the skateboard is at G.

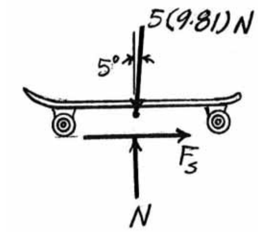


SOLUTION

Referring to the free-body diagram of the skateboard shown in Fig. *a*, we have

$\Sigma F_{x'} = 0; \qquad F_s - 5(9.81)\sin 5° = 0 \qquad F_s = 4.275\text{ N}$

$\Sigma F_{y'} = 0; \qquad N - 5(9.81)\cos 5° = 0 \qquad N = 48.86\text{ N}$



(a)

The effect of the forces acting on the wheels can be represented as if these forces are acting on a single wheel as indicated on the free-body diagram shown in Fig. *b*. We have

$\Sigma F_{x'} = 0; \qquad R_{x'} - 4.275 = 0 \qquad R_{x'} = 4.275\text{ N}$

$\Sigma F_{y'} = 0; \qquad 48.86 - R_{y'} = 0 \qquad R_{y'} = 48.86\text{ N}$

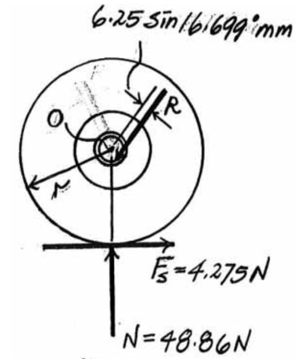
Thus, the magnitude of **R** is

$$R = \sqrt{R_{x'}^{\,2} + R_{y'}^{\,2}} = \sqrt{4.275^2 + 48.86^2} = 49.05\text{ N}$$

$\phi_s = \tan^{-1}\mu_s = \tan^{-1}(0.3) = 16.699°$. Thus, the moment arm of **R** from point O is $(6.25 \sin 16.699°)$ mm. Using these results and writing the moment equation about point O, Fig. *b*, we have

$\circlearrowleft + \Sigma M_O = 0; \qquad 4.275(r) - 49.05(6.25 \sin 16.699° = 0)$

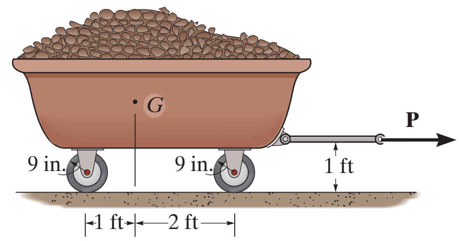
$r = 20.6\text{ mm}$ **Ans.**



(b)

8–126.

The cart together with the load weighs 150 lb and has a center of gravity at G. If the wheels fit loosely on the 1.5-in. diameter axles, determine the horizontal force **P** required to pull the cart with constant velocity. The coefficient of kinetic friction between the axles and the wheels is $\mu_k = 0.2$. Neglect rolling resistance of the wheels on the ground.



SOLUTION

Here, the total frictional force and normal force acting on the wheels of the wagon are $F_s = p$ and $N = 150$ lb, respectively. The effect of the forces acting on the wheels can be represented as if these forces are acting on a single wheel as indicated on the free-body diagram shown in Fig. *a*. We have

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad R_x - p = 0 \quad R_x = p$

$+\uparrow \Sigma F_y = 0;$ $\quad 150 - R_y = 0 \quad R_y = 150$ lb

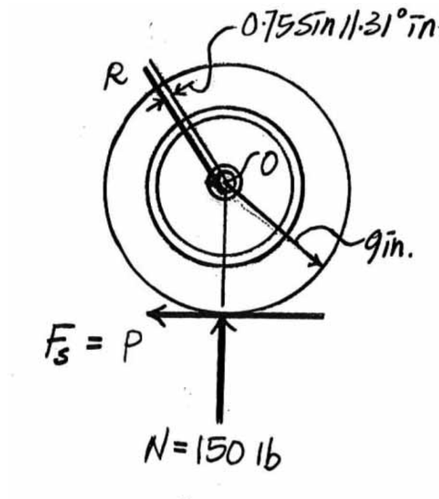
Thus, the magnitude of **R** is

$$R = \sqrt{R_x^{\,2} + R_y^{\,2}} = \sqrt{p^2 + 150^2}$$

$\phi_s = \tan^{-1} \mu_s = \tan^{-1}(0.2) = 11.31°$. Thus, the moment arm of **R** from point O is $(0.75 \sin 11.31°)$ in. Using these results and writing the moment equation about point O, Fig. *a*, we have

$\circlearrowleft + \Sigma M_O = 0;$ $\quad \left(\sqrt{P^2 + 150^2}\right)(0.75 \sin 11.31°) - p(9) = 0$

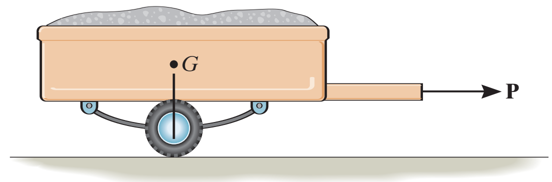
$P = 2.45$ lb **Ans.**



(a)

8–127.

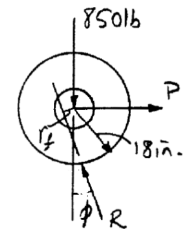
The trailer has a total weight of 850 lb and center of gravity at G which is directly over its axle. If the axle has a diameter of 1 in., the radius of the wheel is $r = 1.5$ ft, and the coefficient of kinetic friction at the bearing is $\mu_k = 0.08$, determine the horizontal force P needed to pull the trailer.



SOLUTION

$\xrightarrow{+} \Sigma F_x = 0; \qquad R \sin \phi = P$

$+\uparrow \Sigma F_y = 0; \qquad R \cos \phi = 850$



Thus,

$$P = 850 \tan \phi$$

$$\phi_k = \tan^{-1}(0.08) = 4.574^\circ$$

$$r_f = r \sin \phi_k = 0.5 \sin 4.574^\circ = 0.03987 \text{ in.}$$

$$\phi = \sin^{-1}\left(\frac{r_f}{18}\right) = \sin^{-1}\left(\frac{0.03987}{18}\right) = 0.1269^\circ$$

Thus,

$$P = 850 \tan 0.1269^\circ = 1.88 \text{ lb}$$ **Ans.**

Note that this is equivalent to an overall coefficient of kinetic friction μ_k

$$\mu_k = \frac{1.88}{850} = 0.00222$$

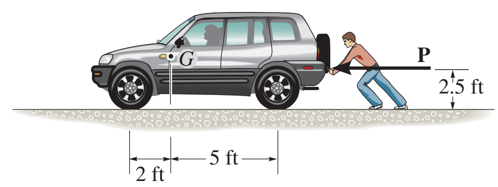
Obviously, it is eaiser to pull the load on the trailer than push it.

If the approximate value of $r_f = r\mu_k = 0.5\,(0.08) = 0.04$ in. is used, then

$$P = 1.89 \text{ lb} \qquad \text{(approx.)}$$ **Ans.**

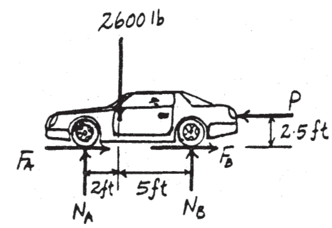
***8–128.**

The vehicle has a weight of 2600 lb and center of gravity at G. Determine the horizontal force **P** that must be applied to overcome the rolling resistance of the wheels. The coefficient of rolling resistance is 0.5 in. The tires have a diameter of 2.75 ft.



SOLUTION

Rolling Resistance: Here, $W = N_A + N_B = \dfrac{5200 - 2.5P}{7} + \dfrac{13000 + 2.5P}{7} = 2600$ lb, $a = 0.5$ in. and $r = \left(\dfrac{2.75}{2}\right)(12) = 16.5$ in. Applying Eq. 8–11, we have



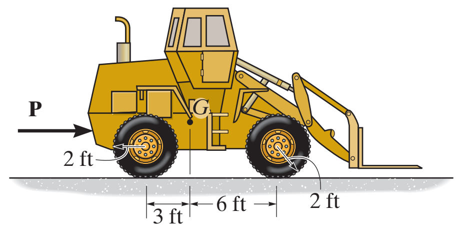
$$P \approx \frac{Wa}{r}$$

$$\approx \frac{2600(0.5)}{16.5}$$

$$\approx 78.8 \text{ lb}$$ **Ans.**

8–129.

The tractor has a weight of 16 000 lb and the coefficient of rolling resistance is $a = 2$ in. Determine the force **P** needed to overcome rolling resistance at all four wheels and push it forward.



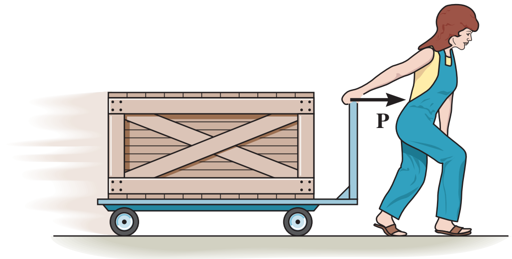
SOLUTION

Applying Eq. 8–11 with $W = 16\,000$ lb, $a = \left(\frac{2}{12}\right)$ ft and $r = 2$ ft, we have

$$P \approx \frac{Wa}{r} = \frac{16000\left(\frac{2}{12}\right)}{2} = 1333 \text{ lb}$$ **Ans.**

8–130.

The hand cart has wheels with a diameter of 80 mm. If a crate having a mass of 500 kg is placed on the cart so that each wheel carries an equal load, determine the horizontal force P that must be applied to the handle to overcome the rolling resistance. The coefficient of rolling resistance is 2 mm. Neglect the mass of the cart.



SOLUTION

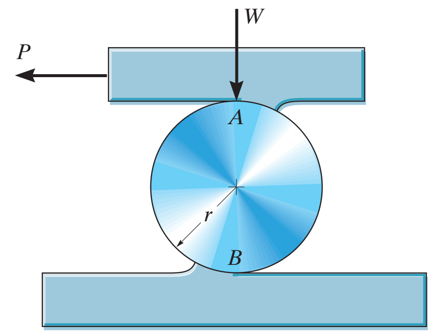
$$P \approx \frac{Wa}{r}$$

$$= 500(9.81)\left(\frac{2}{40}\right)$$

$$P = 245\text{ N}$$ **Ans.**

8–131.

The cylinder is subjected to a load that has a weight W. If the coefficients of rolling resistance for the cylinder's top and bottom surfaces are a_A and a_B, respectively, show that a horizontal force having a magnitude of $P = [W(a_A + a_B)]/2r$ is required to move the load and thereby roll the cylinder forward. Neglect the weight of the cylinder.



SOLUTION

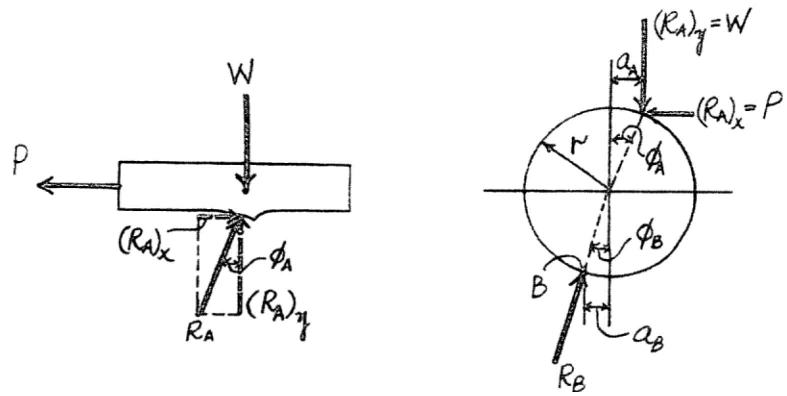
$\xrightarrow{+} \Sigma F_x = 0; \qquad (R_A)_x - P = 0 \qquad (R_A)_x = P$

$+\uparrow \Sigma F_y = 0; \qquad (R_A)_y - W = 0 \qquad (R_A)_y = W$

$\circlearrowleft + \Sigma M_B = 0; \qquad P(r \cos \phi_A + r \cos \phi_B) - W(a_A + a_B) = 0 \qquad$ **(1)**

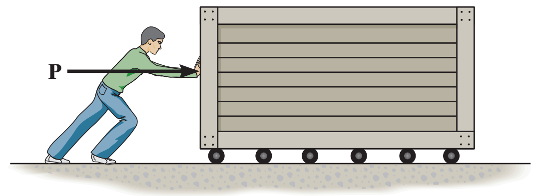
Since ϕ_A and ϕ_B are very small, $\cos \phi_A - \cos \phi_B = 1$. Hence, from Eq. (1)

$$P = \frac{W(a_A + a_B)}{2r} \qquad \textbf{(QED)}$$



***8–132.**

A large crate having a mass of 200 kg is moved along the floor using a series of 150-mm-diameter rollers for which the coefficient of rolling resistance is 3 mm at the ground and 7 mm at the bottom surface of the crate. Determine the horizontal force **P** needed to push the crate forward at a constant speed. *Hint:* Use the result of Prob. 8–131.



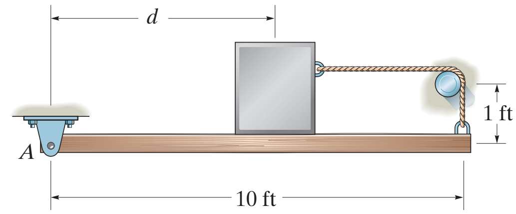
SOLUTION

Rolling Resistance: Applying the result obtained in Prob. 8–131. $P = \dfrac{W(a_A + a_B)}{2r}$, with $a_A = 7$ mm, $a_B = 3$ mm, $W = 200(9.81) = 1962$ N, and $r = 75$ mm, we have

$$P = \frac{1962(7 + 3)}{2(75)} = 130.8 \text{ N} = 131 \text{ N}$$ **Ans.**

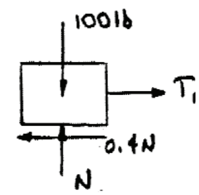
8–133.

The uniform 50-lb beam is supported by the rope which is attached to the end of the beam, wraps over the rough peg, and is then connected to the 100-lb block. If the coefficient of static friction between the beam and the block, and between the rope and the peg, is $\mu_s = 0.4$, determine the maximum distance that the block can be placed from A and still remain in equilibrium. Assume the block will not tip.



SOLUTION

Block:



$+\uparrow \Sigma F_y = 0; \qquad N - 100 = 0$

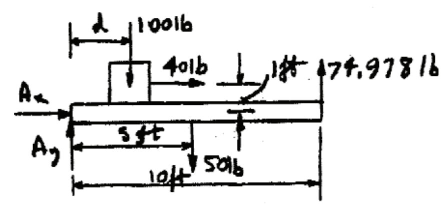
$N = 100 \text{ lb}$

$\xrightarrow{+} \Sigma F_x = 0; \qquad T_1 - 0.4(100) = 0$

$T_1 = 40 \text{ lb}$

$T_2 = T_1 e^{\mu\beta}; \qquad T_2 = 40e^{0.4\left(\frac{\pi}{2}\right)} = 74.978 \text{ lb}$

System:

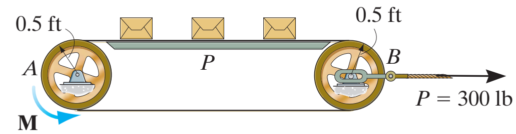


$\circlearrowleft + \Sigma M_A = 0; \qquad -100(d) - 40(1) - 50(5) + 74.978(10) = 0$

$d = 4.60 \text{ ft}$ **Ans.**

8–134.

Determine the maximum number of 50-lb packages that can be placed on the belt without causing the belt to slip at the drive wheel A which is rotating with a constant angular velocity. Wheel B is free to rotate. Also, find the corresponding torsional moment **M** that must be supplied to wheel A. The conveyor belt is pre-tensioned with the 300-lb horizontal force. The coefficient of kinetic friction between the belt and platform P is $\mu_k = 0.2$, and the coefficient of static friction between the belt and the rim of each wheel is $\mu_s = 0.35$.



SOLUTION

The maximum tension $\mathbf{T}_2$ of the conveyor belt can be obtained by considering the equilibrium of the free-body diagram of the top belt shown in Fig. *a*.

$+\uparrow \Sigma F_y = 0; \qquad n(50) - N = 0 \qquad N = 50n \qquad$ **(1)**

$\xrightarrow{+} \Sigma F_x = 0; \qquad 150 + 0.2(50n) - T_2 = 0 \qquad T_2 = 150 + 10n \qquad$ **(2)**

By considering the case when the drive wheel A is on the verge of slipping, where $\beta = \pi$ rad, $T_2 = 150 + 10n$ and $T_1 = 150$ lb,

$$T_2 = T_1 e^{\mu\beta}$$

$$150 + 10n = 150e^{0.35(\pi)}$$

$$n = 30.04$$

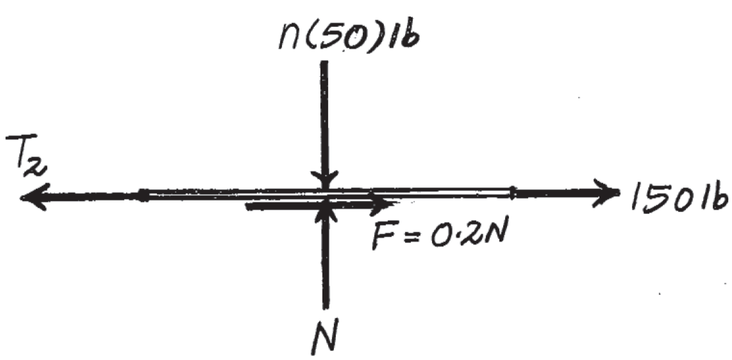
Thus, the maximum allowable number of boxes on the belt is

$$n = 30$$ **Ans.**

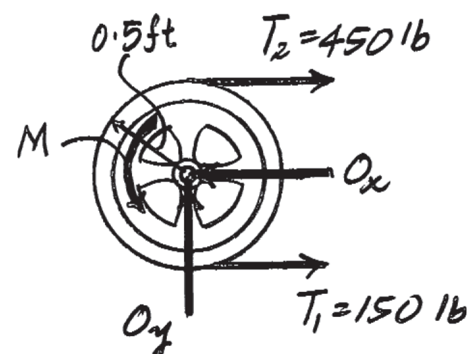
Substituting $n = 30$ into Eq. (2) gives $T_2 = 450$ lb. Referring to the free-body diagram of the wheel A shown in Fig. b,

$\circlearrowleft + \Sigma M_O = 0; \qquad M + 150(0.5) - 450(0.5) = 0$

$M = 150 \text{ lb} \cdot \text{ft}$ **Ans.**



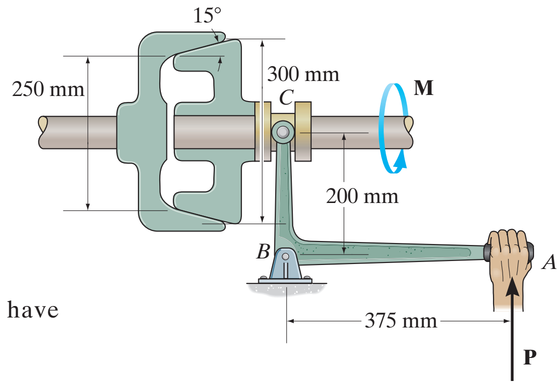
(a)



(b)

8–135.

If $P = 900$ N is applied to the handle of the bell crank, determine the maximum torque M the cone clutch can transmit. The coefficient of static friction at the contacting surface is $\mu_s = 0.3$.



SOLUTION

Referring to the free-body diagram of the bellcrank shown in Fig. *a*,we have

$$\circlearrowleft + \Sigma M_B = 0; \quad 900(0.375) - F_C(0.2) = 0 \qquad F_C = 1687.5 \text{ N}$$

Using this result and referring to the free-body diagram of the cone clutch shown in Fig. *b*,

$$\xrightarrow{+} \Sigma F_x = 0; \quad 2\left(\frac{N}{2}\sin 15°\right) - 1687.5 = 0 \qquad N = 6520.00 \text{ N}$$

The area of the differential element shown shaded in Fig. *c* is
$dA = 2\pi r\, ds = 2\pi r \dfrac{dr}{\sin 15°} = \dfrac{2\pi}{\sin 15°} r\, dr$. Thus,

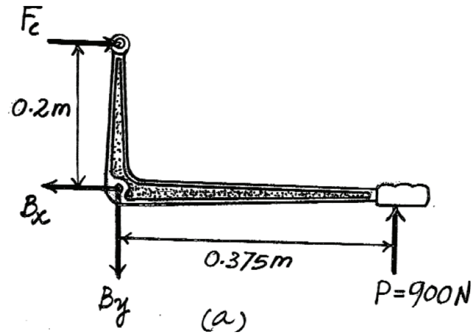
$A = \displaystyle\int_A dA = \int_{0.125 \text{ m}}^{0.15 \text{ m}} \frac{2\pi}{\sin 15°} r\, dr = 0.08345 \text{ m}^2$. The pressure acting on the cone surface is $p = \dfrac{N}{A} = \dfrac{6520.00}{0.08345} = 78.13(10^3) \text{ N}/\text{m}^2$

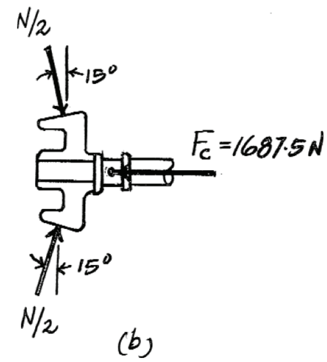
The normal force acting on the differential element dA is
$dN = p\, dA = 78.13(10^3)\left[\dfrac{2\pi}{\sin 15°}\right] r\, dr = 1896.73(10^3) r\, dr$.
Thus, the frictional force acting on this differential element is given by $dF = \mu_s dN = 0.3(1896.73)(10^3) r\, dr = 569.02(10^3) r\, dr$. The moment equation about the axle of the cone clutch gives

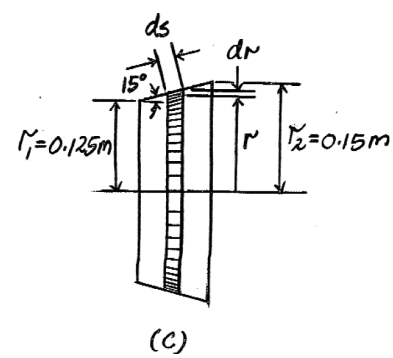
$$\Sigma M = 0; \quad M - \int r\, dF = 0$$

$$M = \int r\, dF = 569.02(10^3) \int_{0.125 \text{ m}}^{0.15 \text{ m}} r^2\, dr$$

$$M = 270 \text{ N} \cdot \text{m}$$ **Ans.**







***8–136.**

The lawn roller has a mass of 80 kg. If the arm BA is held at an angle of 30° from the horizontal and the coefficient of rolling resistance for the roller is 25 mm, determine the force P needed to push the roller at constant speed. Neglect friction developed at the axle, A, and assume that the resultant force $\mathbf{P}$ acting on the handle is applied along arm BA.

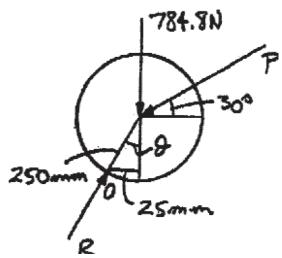
SOLUTION

$$\theta = \sin^{-1}\left(\frac{25}{250}\right) = 5.74°$$

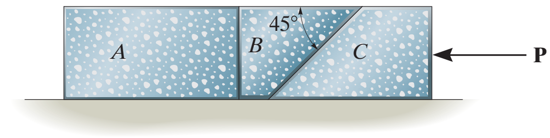
$$\circlearrowleft + \Sigma M_O = 0; \qquad -25(784.8) - P \sin 30°(25) + P \cos 30°(250 \cos 5.74°) = 0$$

Solving,

$$P = 96.7 \text{ N}$$ **Ans.**

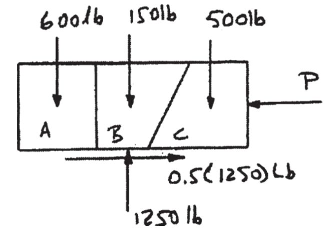


8–137.



The three stone blocks have weights of $W_A = 600$ lb, $W_B = 150$ lb, and $W_C = 500$ lb. Determine the smallest horizontal force P that must be applied to block C in order to move this block. The coefficient of static friction between the blocks is $\mu_s = 0.3$, and between the floor and each block $\mu_s' = 0.5$.

SOLUTION

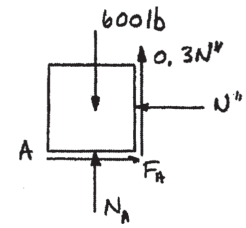


$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad -P + 0.5\,(1250) = 0$

$P = 625 \text{ lb}$

Assume block B slips up, block A does not move.

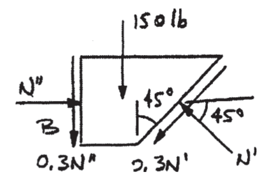
Block A:



$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad F_A - N'' = 0$

$+\uparrow \Sigma F_y = 0; \qquad N_A - 600 + 0.3N'' = 0$

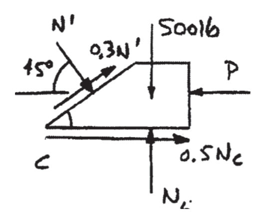
Block B:



$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad N'' - N' \cos 45° - 0.3\,N' \sin 45° = 0$

$+\uparrow \Sigma F_y = 0; \qquad N' \sin 45° - 0.3\,N' \cos 45° - 150 - 0.3\,N'' = 0$

Block C:



$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 0.3\,N' \cos 45° + N' \cos 45° + 0.5\,N_C - P = 0$

$+\uparrow \Sigma F_y = 0; \qquad N_C - N' \sin 45° + 0.3\,N' \sin 45° - 500 = 0$

Solving,

$N'' = 629.0 \text{ lb}, \qquad N' = 684.3 \text{ lb}, \qquad N_C = 838.7 \text{ lb}, \qquad P = 1048 \text{ lb},$

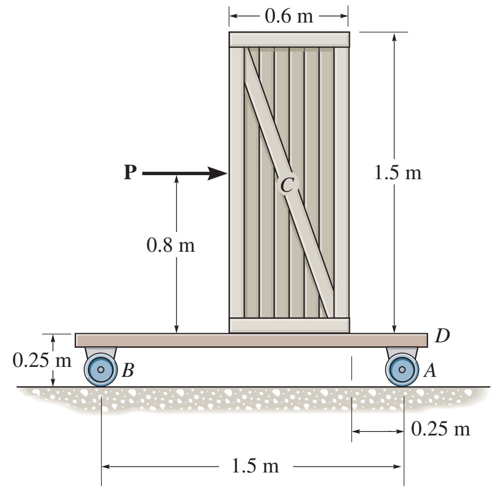
$N_A = 411.3 \text{ lb}$

$F_A = 629.0 \text{ lb} > 0.5\,(411.3) = 205.6 \text{ lb}$ **No good**

All blocks slip at the same time; $\qquad P = 625 \text{ lb}$ **Ans.**

8–138.

The uniform 60-kg crate C rests uniformly on a 10-kg dolly D. If the front casters of the dolly at A are locked to prevent rolling while the casters at B are free to roll, determine the maximum force $\mathbf{P}$ that may be applied without causing motion of the crate. The coefficient of static friction between the casters and the floor is $\mu_f = 0.35$ and between the dolly and the crate, $\mu_d = 0.5$.



SOLUTION

Equations of Equilibrium: From FBD (a),

$+\uparrow \Sigma F_y = 0; \qquad N_d - 588.6 = 0 \qquad N_d = 588.6 \text{ N}$

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad P - F_d = 0 \qquad$ **(1)**

$\circlearrowleft + \Sigma M_A = 0; \qquad 588.6(x) - P(0.8) = 0 \qquad$ **(2)**

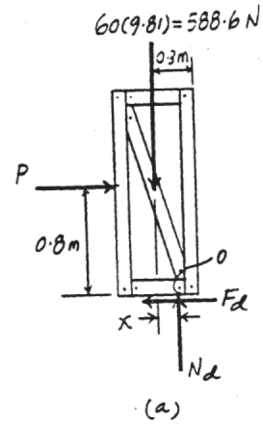
From FBD (b),

$+\uparrow \Sigma F_y = 0 \qquad N_B + N_A - 588.6 - 98.1 = 0 \qquad$ **(3)**

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad P - F_A = 0 \qquad$ **(4)**

$\circlearrowleft + \Sigma M_B = 0; \qquad N_A(1.5) - P(1.05)$

$\qquad - 588.6(0.95) - 98.1(0.75) = 0 \qquad$ **(5)**



Friction: Assuming the crate slips on dolly, then $F_d = \mu_{s\,d} N_d = 0.5(588.6) = 294.3$ N. Substituting this value into Eqs. (1) and (2) and solving, we have

$$P = 294.3 \text{ N} \qquad x = 0.400 \text{ m}$$

Since $x > 0.3$ m, the crate tips on the dolly. If this is the case $x = 0.3$ m. Solving Eqs. (1) and (2) with $x = 0.3$ m yields

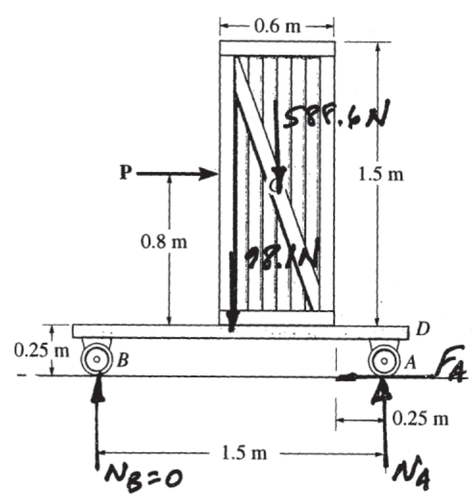
$$P = 220.725 \text{ N}$$

$$F_d = 220.725 \text{ N}$$

Assuming the dolly slips at A, then $F_A = \mu_{s\,f} N_A = 0.35 N_A$. Substituting this value into Eqs. (3), (4), and (5) and solving, we have

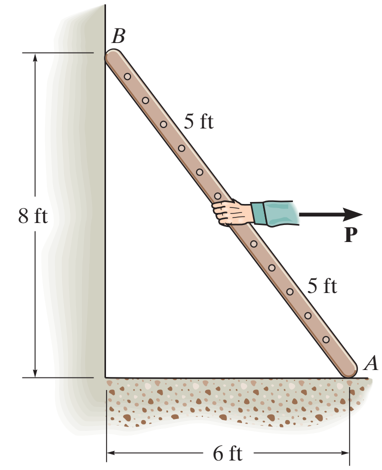
$$N_A = 559 \text{ N} \qquad N_B = 128 \text{ N}$$

$$P = 195.6 \text{ N} = 196 \text{ N} \quad (\textit{Control!})$$ **Ans.**



8–139.

The uniform 20-lb ladder rests on the rough floor for which the coefficient of static friction is $\mu_s = 0.8$ and against the smooth wall at B. Determine the horizontal force P the man must exert on the ladder in order to cause it to move.



SOLUTION

Assume that the ladder tips about A:

$N_B = 0;$

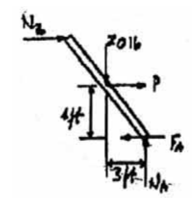
$\xrightarrow{+} \Sigma F_x = 0; \qquad P - F_A = 0$

$+\uparrow \Sigma F_y = 0; \qquad -20 + N_A = 0$

$N_A = 20 \text{ lb}$

$\circlearrowleft + \Sigma M_A = 0; \qquad 20\,(3) - P\,(4) = 0$

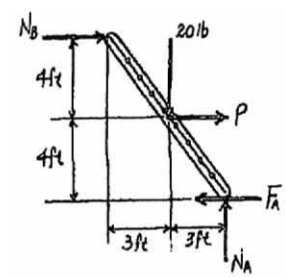
$P = 15 \text{ lb}$



Thus

$$F_A = 15 \text{ lb}$$

$$(F_A)_{\max} = 0{:}8\,(20) = 16 \text{ lb} > 15 \text{ lb} \qquad \textbf{OK}$$

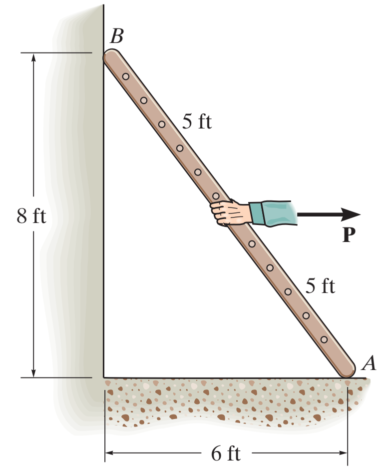


Ladder tips as assumed.

$$P = 15 \text{ lb} \qquad \textbf{Ans.}$$

***8–140.**

The uniform 20-lb ladder rests on the rough floor for which the coefficient of static friction is $\mu_s = 0.4$ and against the smooth wall at B. Determine the horizontal force P the man must exert on the ladder in order to cause it to move.



SOLUTION

Assume that the ladder slips at A:

$F_A = 0.4\,N_A$

$+\uparrow \Sigma F_y = 0; \qquad N_A - 20 = 0$

$N_A = 20 \text{ lb}$

$F_A = 0.4(20) = 8 \text{ lb}$

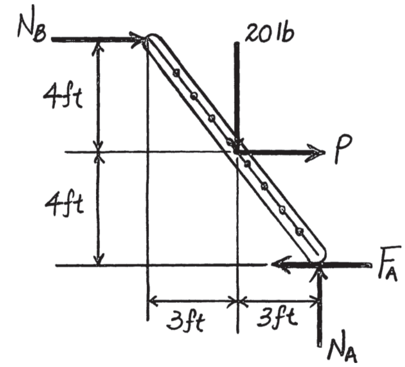
$\circlearrowleft + \Sigma M_B = 0; \qquad P(4) - 20(3) + 20(6) - 8(8) = 0$

$P = 1 \text{ lb}$ **Ans.**

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad N_B + 1 - 8 = 0$

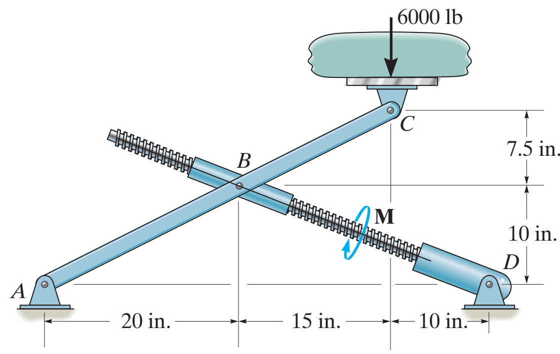
$N_B = 7 \text{ lb} > 0$ **OK**

The ladder will remain in contact with the wall.



8–141.

The jacking mechanism consists of a link that has a square-threaded screw with a mean diameter of 0.5 in. and a lead of 0.20 in., and the coefficient of static friction is $\mu_s = 0.4$. Determine the torque M that should be applied to the screw to start lifting the 6000-lb load acting at the end of member ABC.



SOLUTION

$$\alpha = \tan^{-1}\left(\frac{10}{25}\right) = 21.80^\circ$$

$\zeta + \Sigma M_A = 0;$ $\quad -6000(35) + F_{BD}\cos 21.80^\circ(10) + F_{BD}\sin 21.80^\circ(20) = 0$

$$F_{BD} = 12\,565 \text{ lb}$$

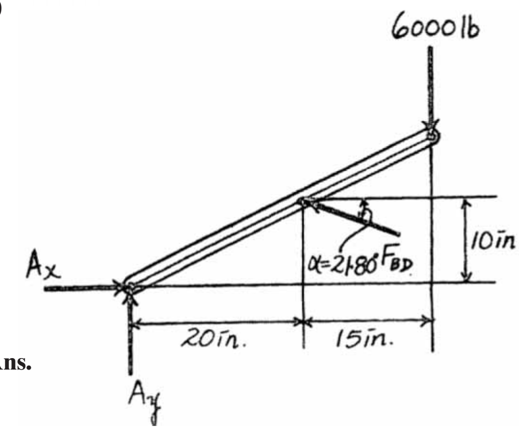
$$\phi_s = \tan^{-1}(0.4) = 21.80^\circ$$

$$\theta = \tan^{-1}\left(\frac{0.2}{2\pi(0.25)}\right) = 7.256^\circ$$

$$M = Wr\tan(\theta + \phi)$$

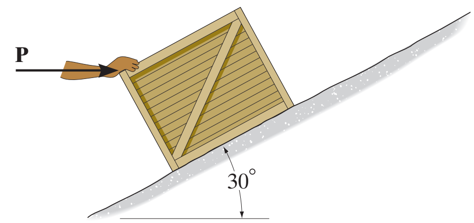
$$M = 12\,565(0.25)\tan(7.256^\circ + 21.80^\circ)$$

$$M = 1745 \text{ lb}\cdot\text{in} = 145 \text{ lb}\cdot\text{ft}$$ **Ans.**



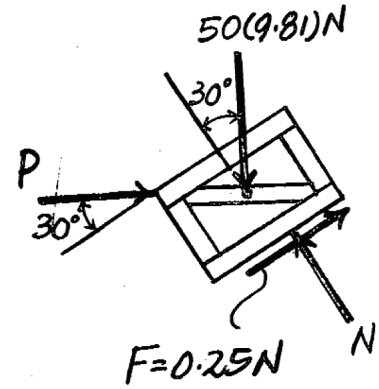
8–142.

Determine the minimum horizontal force P required to hold the crate from sliding down the plane. The crate has a mass of 50 kg and the coefficient of static friction between the crate and the plane is $\mu_s = 0.25$.



SOLUTION

Free-Body Diagram: When the crate is on the verge of sliding down the plane, the frictional force **F** will act up the plane as indicated on the free-body diagram of the crate shown in Fig. *a*.



(a)

Equations of Equilibrium:

$\nwarrow + \Sigma F_{y'} = 0; \quad N - P\sin 30° - 50(9.81)\cos 30° = 0$

$\nearrow + \Sigma F_{x'} = 0; \quad P\cos 30° + 0.25N - 50(9.81)\sin 30° = 0$

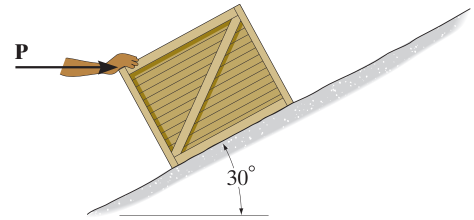
Solving

$$P = 140 \text{ N}$$ **Ans.**

$$N = 494.94 \text{ N}$$

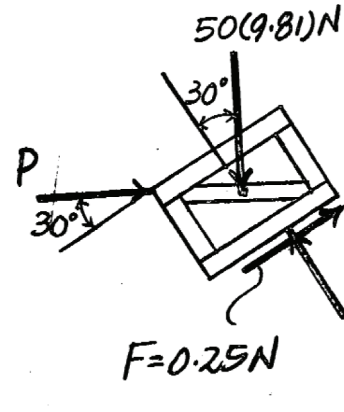
8–143.

Determine the minimum force P required to push the crate up the plane. The crate has a mass of 50 kg and the coefficient of static friction between the crate and the plane is $\mu_s = 0.25$.



SOLUTION

When the crate is on the verge of sliding up the plane, the frictional force F' will act down the plane as indicated on the free-body diagram of the crate shown in Fig.*b*.



(a)

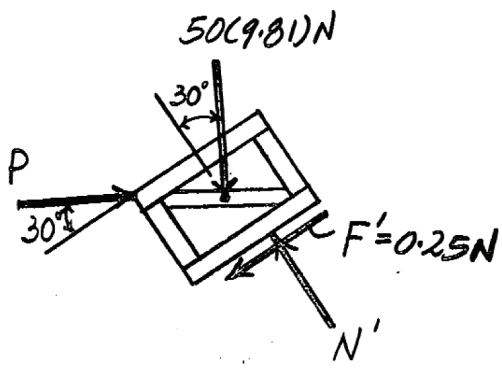
$$\nwarrow + \Sigma F_{y'} = 0; \quad N' - P\sin 30^\circ - 50(9.81)\cos 30^\circ = 0$$

$$\nearrow + \Sigma F_{x'} = 0; \quad P\cos 30^\circ - 0.25N' - 50(9.81)\sin 30^\circ = 0$$

Solving,

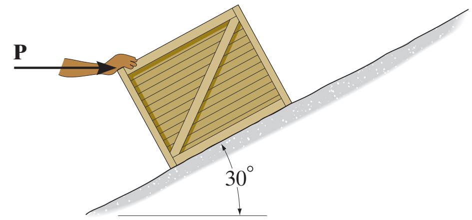
$$P = 474 \text{ N} \qquad \textbf{Ans.}$$

$$N' = 661.92 \text{ N}$$



(b)

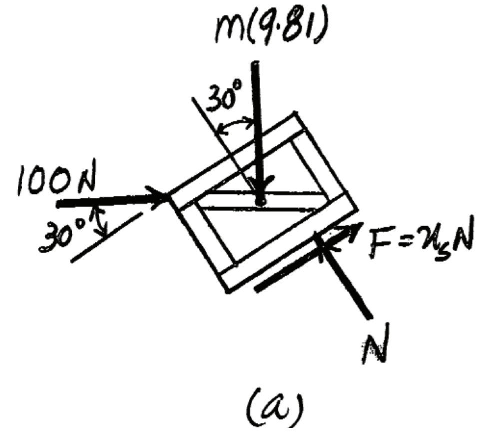
***8–144.**



A horizontal force of $P = 100$ N is just sufficient to hold the crate from sliding down the plane, and a horizontal force of $P = 350$ N is required to just push the crate up the plane. Determine the coefficient of static friction between the plane and the crate, and find the mass of the crate.

SOLUTION

Free-Body Diagram: When the crate is subjected to a force of $P = 100$ N, it is on the verge of slipping down the plane. Thus, the frictional force F will act up the plane as indicated on the free-body diagram of the crate shown in Fig. *a*. When $P = 350$ N, it will cause the crate to be on the verge of slipping up the plane, and so the frictional force F' acts down the plane as indicated on the free-body diagram of the crate shown in Fig. *b*. Thus, $F = \mu_s N$ and $F' = \mu_s N'$.



(a)

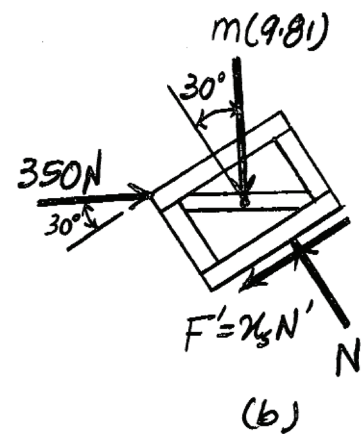
Equations of Equilibrium:

$+\nwarrow \Sigma F_{y'} = 0;\; N - 100 \sin 30° - m(9.81) \cos 30° = 0$

$+\nearrow \Sigma F_{x'} = 0;\; \mu_s N + 100 \cos 30° - m(9.81) \sin 30° = 0$

Eliminating N,

$$\mu_s = \frac{4.905m - 86.603}{8.496m + 50} \qquad \textbf{(1)}$$



(b)

Also by referring to Fig, *b*, we can write

$+\nwarrow \Sigma F_{y'} = 0;\; N' - m(9.81) \cos 30° - 350 \sin 30° = 0$

$+\nearrow \Sigma F_{x'} = 0;\; 350 \cos 30° - m(9.81) \sin 30° - \mu_s N' = 0$

Eliminating N',

$$\mu_s = \frac{303.11 - 4.905m}{175 + 8.496m} \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

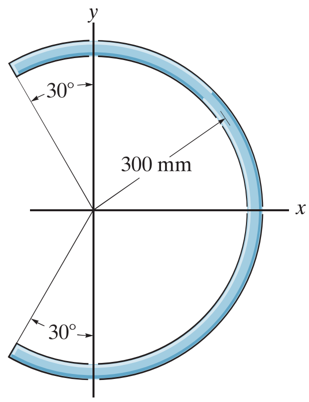
$m = 36.5$ kg **Ans.**

$\mu_s = 0.256$ **Ans.**

MISSING

9–1.

Locate the center of mass of the homogeneous rod bent into the shape of a circular arc.



SOLUTION

$$dL = 300\, d\theta$$

$$\tilde{x} = 300 \cos\theta$$

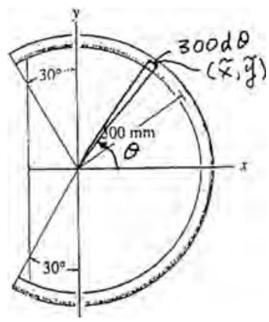
$$\tilde{y} = 300 \sin\theta$$

$$\bar{x} = \frac{\int \tilde{x}\, dL}{\int dL} = \frac{\int_{-\frac{2\pi}{3}}^{\frac{2\pi}{3}} 300 \cos\theta\,(300 d\theta)}{\int_{-\frac{2\pi}{3}}^{\frac{2\pi}{3}} 300 d\theta}$$

$$= \frac{(300)^2 \left[\sin\theta\right]_{-\frac{2\pi}{3}}^{\frac{2\pi}{3}}}{300\left(\frac{4}{3}\pi\right)}$$

$$= 124 \text{ mm}$$ **Ans.**

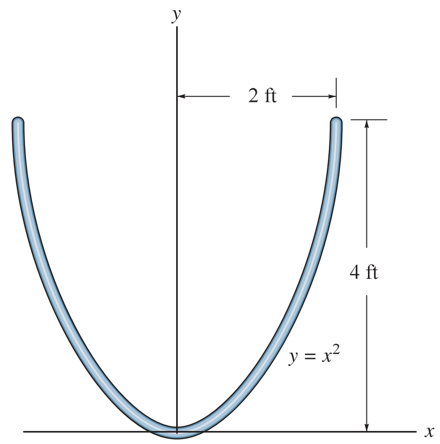
$\bar{y} = 0$ (By symmetry) **Ans.**



Ans:
$\bar{x} = 124$ mm
$\bar{y} = 0$

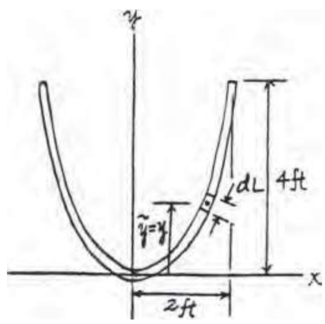
9–2.

Determine the location $(\bar{x}, \bar{y})$ of the centroid of the wire.



SOLUTION

Length and Moment Arm: The length of the differential element is $dL = \sqrt{dx^2 + dy^2} = \left(\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\right)dx$ and its centroid is $\tilde{y} = y = x^2$. Here, $\frac{dy}{dx} = 2x$.



Centroid: Due to symmetry

$$\bar{x} = 0$$ **Ans.**

Applying Eq. 9–7 and performing the integration, we have

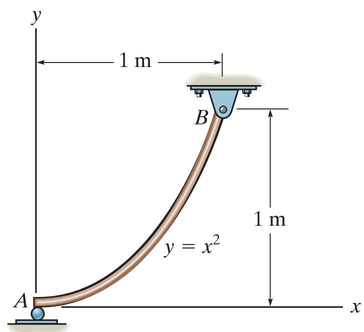
$$\bar{y} = \frac{\int_L \tilde{y}\,dL}{\int_L dL} = \frac{\int_{-2\text{ ft}}^{2\text{ ft}} x^2\sqrt{1 + 4x^2}\,dx}{\int_{-2\text{ ft}}^{2\text{ ft}} \sqrt{1 + 4x^2}\,dx}$$

$$= \frac{16.9423}{9.2936} = 1.82 \text{ ft}$$ **Ans.**

Ans:
$\bar{x} = 0$
$\bar{y} = 1.82$ ft

9–3.

Locate the center of gravity $\bar{x}$ of the homogeneous rod. If the rod has a weight per unit length of 100 N/m, determine the vertical reaction at A and the x and y components of reaction at the pin B.



SOLUTION

Length And Moment Arm. The length of the differential element is $dL = \sqrt{dx^2 + dy^2} = \left[\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\right]dx$ and its centroid is $\tilde{x} = x$. Here $\frac{dy}{dx} = 2x$.

Perform the integration

$$L = \int_L dL = \int_0^{1\text{ m}} \sqrt{1 + 4x^2}\,dx$$

$$= 2\int_0^{1\text{ m}} \sqrt{x^2 + \frac{1}{4}}\,dx$$

$$= \left[x\sqrt{x^2 + \frac{1}{4}} + \frac{1}{4}\ln\left(x + \sqrt{x^2 + \frac{1}{4}}\right)\right]_0^{1\text{ m}}$$

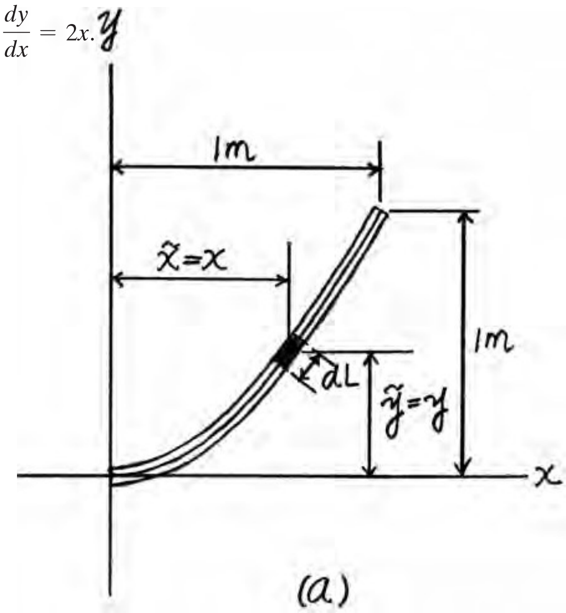
$$= 1.4789 \text{ m}$$

$$\int_L \tilde{x}\,dL = \int_0^{1\text{ m}} x\sqrt{1 + 4x^2}\,dx$$

$$= 2\int_0^{1\text{ m}} x\sqrt{x^2 + \frac{1}{4}}\,dx$$

$$= \left[\frac{2}{3}\left(x^2 + \frac{1}{4}\right)^{3/2}\right]_0^{1\text{ m}}$$

$$= 0.8484 \text{ m}^2$$



Centroid.

$$\bar{x} = \frac{\int_L \tilde{x}\,dL}{\int_L dL} = \frac{0.8484 \text{ m}^2}{1.4789 \text{ m}} = 0.5736 \text{ m} = 0.574 \text{ m}$$ **Ans.**

Equations of Equilibrium. Refering to the *FBD* of the rod shown in Fig. *a*

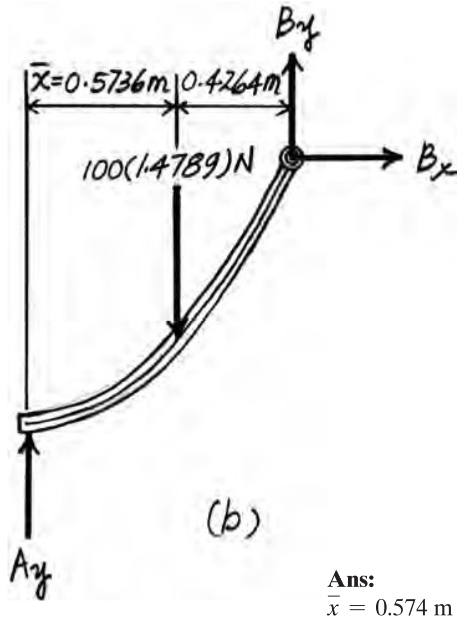
$\xrightarrow{+}\Sigma F_x = 0;\quad B_x = 0$ **Ans.**

$\circlearrowleft+\Sigma M_B = 0;\quad 100(1.4789)\,(0.4264) - A_y(1) = 0$

$A_y = 63.06 \text{ N} = 63.1 \text{ N}$ **Ans.**

$\circlearrowleft+\Sigma M_A = 0;\quad B_y(1) - 100(1.4789)\,(0.5736) = 0$

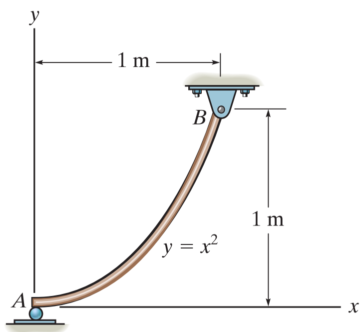
$B_y = 84.84 \text{ N} = 84.8 \text{ N}$ **Ans.**



Ans:
$\bar{x} = 0.574$ m
$B_x = 0$
$A_y = 63.1$ N
$B_y = 84.8$ N

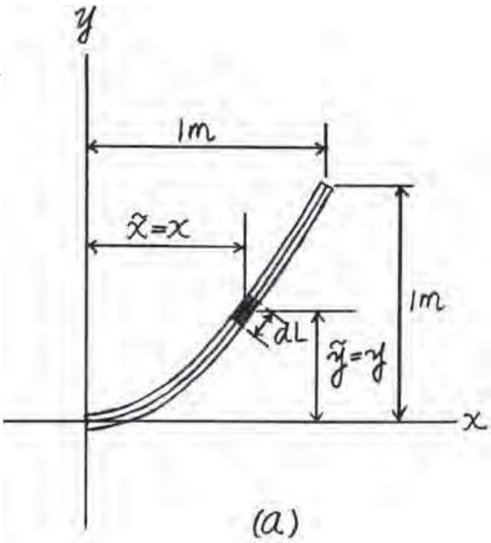
***9–4.**

Locate the center of gravity $\bar{y}$ of the homogeneous rod.



SOLUTION

Length And Moment Arm. The length of the differential element is $dL = \sqrt{dx^2 + dy^2} = \left[\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\right]dx$ and its centroid is $\tilde{y} = y$. Here $\frac{dy}{dx} = 2x$. Perform the integration,



$$L = \int_L dL = \int_0^{1\text{ m}} \sqrt{1 + 4x^2}\, dx$$

$$= 2\int_0^{1\text{ m}} \sqrt{x^2 + \frac{1}{4}}\, dx$$

$$= \left[x\sqrt{x^2 + \frac{1}{4}} + \frac{1}{4}\ln\left(x + \sqrt{x^2 + \frac{1}{4}}\right)\right]_0^{1\text{ m}}$$

$$= 1.4789 \text{ m}$$

$$\int_L \tilde{y}\, dL = \int_0^{1\text{ m}} x^2\sqrt{1 + 4x^2}\, dx$$

$$= 2\int_0^{1\text{ m}} x^2\sqrt{x^2 + \frac{1}{4}}\, dx$$

$$= 2\left[\frac{x}{4}\sqrt{\left(x^2 + \frac{1}{4}\right)^3} - \frac{1}{32}x\sqrt{x^2 + \frac{1}{4}} - \frac{1}{128}\ln\left(x + \sqrt{x^2 + \frac{1}{4}}\right)\right]_0^{1\text{ m}}$$

$$= 0.6063 \text{ m}^2$$

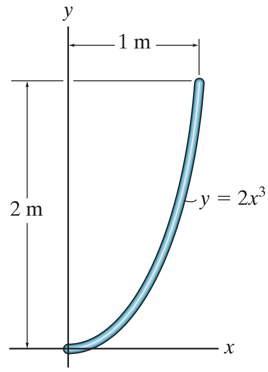
Centroid.

$$\bar{y} = \frac{\int_L \tilde{y}\, dL}{\int_L dL} = \frac{0.6063 \text{ m}^2}{1.4789 \text{ m}} = 0.40998 \text{ m} = 0.410 \text{ m}$$ **Ans.**

Ans:
$\bar{y} = 0.410$ m

9–5.

Determine the distance $\bar{y}$ to the center of gravity of the homogeneous rod.



SOLUTION

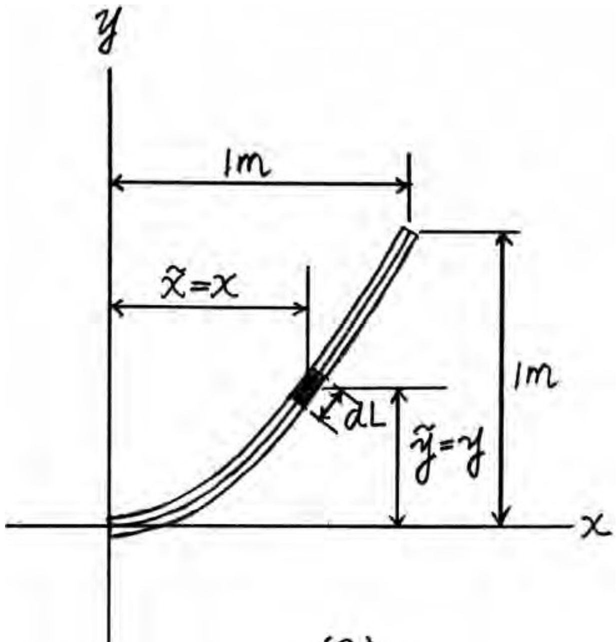
Length And Moment Arm. The length of the differential element is $dL = \sqrt{dx^2 + dy^2} = \left(\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\right) dx$ and its centroid is at $\tilde{y} = y$. Here $\frac{dy}{dx} = 6x^2$. Evaluate the integral numerically,

$$L = \int_L dL = \int_0^{1\text{ m}} \sqrt{1 + 36x^4}\, dx = 2.4214 \text{ m}$$

$$\int_L \tilde{y}\, dL = \int_0^{1\text{ m}} 2x^3\sqrt{1 + 36x^4}\, dx = 2.0747 \text{ m}^2$$

Centroid. Applying Eq. 9–7,

$$\bar{y} = \frac{\int_L \tilde{y}\, dL}{\int_L dL} = \frac{2.0747 \text{ m}^2}{2.4214 \text{ m}} = 0.8568 = 0.857 \text{ m} \qquad \textbf{Ans.}$$

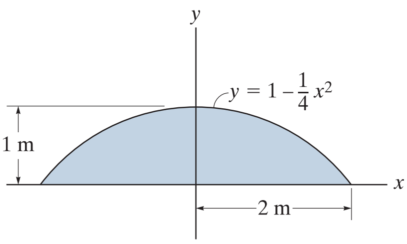


(a)

Ans:
$\bar{y} = 0.857$ m

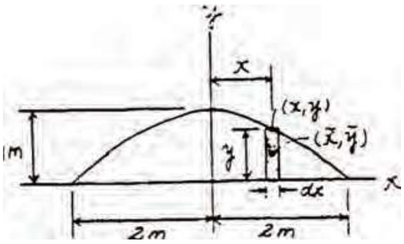
9–6.

Locate the centroid $\bar{y}$ of the area.



SOLUTION

Area and Moment Arm: The area of the differential element is $dA = y\,dx = \left(1 - \frac{1}{4}x^2\right)dx$ and its centroid is $\tilde{y} = \frac{y}{2} = \frac{1}{2}\left(1 - \frac{1}{4}x^2\right)$.



Centroid: Due to symmetry

$$\bar{x} = 0$$ **Ans.**

Applying Eq. 9–4 and performing the integration, we have

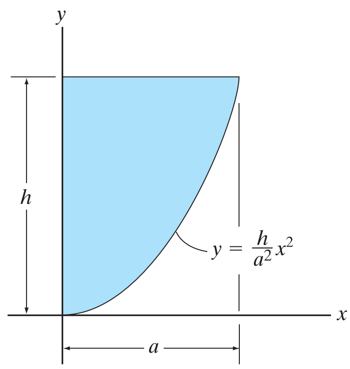
$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\int_{-2\text{m}}^{2\text{m}} \frac{1}{2}\left(1 - \frac{1}{4}x^2\right)\left(1 - \frac{1}{4}x^2\right)dx}{\int_{-2\text{m}}^{2\text{m}}\left(1 - \frac{1}{4}x^2\right)dx}$$

$$= \frac{\left(\frac{x}{2} - \frac{x^3}{12} + \frac{x^5}{160}\right)\Big|_{-2\text{m}}^{2\text{m}}}{\left(x - \frac{x^3}{12}\right)\Big|_{-2\text{m}}^{2\text{m}}} = \frac{2}{5}\text{ m}$$ **Ans.**

Ans:
$\bar{y} = \frac{2}{5}$ m

9–7.

Determine the area and the centroid $\bar{x}$ of the parabolic area.



SOLUTION

Differential Element: The area element parallel to the x axis shown shaded in Fig. a will be considered. The area of the element is

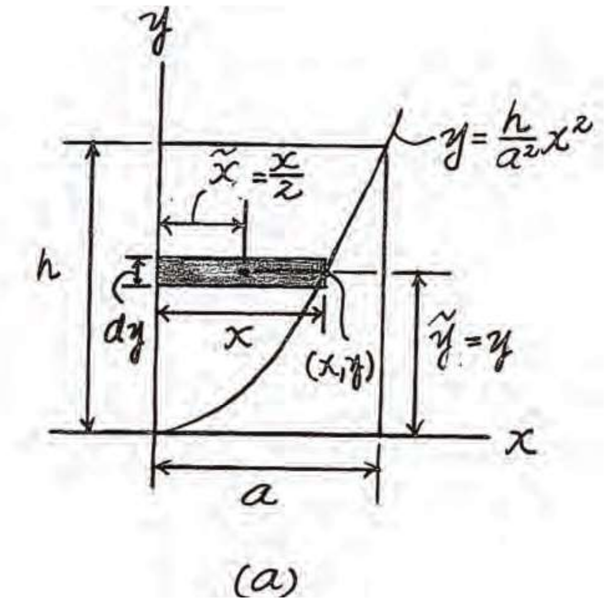
$$dA = x\,dy = \frac{a}{h^{1/2}}\,y^{1/2}\,dy$$

Centroid: The centroid of the element is located at $\tilde{x} = \frac{x}{2} = \frac{a}{2h^{1/2}}\,y^{1/2}$ and $\tilde{y} = y$.

Area: Integrating,

$$A = \int_A dA = \int_0^h \frac{a}{h^{1/2}}\,y^{1/2}\,dy = \frac{2a}{3h^{1/2}}\left(y^{3/2}\right)\Big|_0^h = \frac{2}{3}ah \qquad \textbf{Ans.}$$

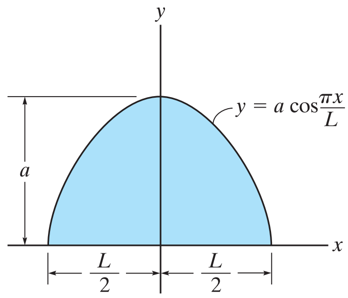
$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_0^h \left(\frac{a}{2h^{1/2}}y^{1/2}\right)\left(\frac{a}{h^{1/2}}\,y^{1/2}\,dy\right)}{\frac{2}{3}ah} = \frac{\int_0^h \frac{a^2}{2h}\,y\,dy}{\frac{2}{3}\,ah} = \frac{\frac{a^2}{2h}\left(\frac{y^2}{2}\right)\Big|_0^h}{\frac{2}{3}\,ah} = \frac{3}{8}\,a \qquad \textbf{Ans.}$$



Ans:
$\bar{x} = \frac{3}{8}a$

***9–8.**

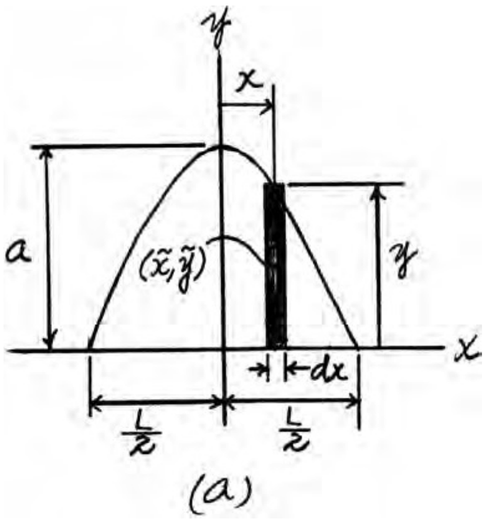
Locate the centroid of the shaded area.



SOLUTION

Area And Moment Arm. The area of the differential element shown shaded in Fig. *a* is $dA = ydx = a\cos\frac{\pi}{L}x\,dx$ and its centroid is at $\tilde{y} = \frac{y}{2} = \frac{a}{2}\cos\frac{\pi}{2}x$.

Centroid. Perform the integration



$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\int_{-L/2}^{L/2}\left(\frac{a}{2}\cos\frac{\pi}{L}x\right)\left(a\cos\frac{\pi}{L}x\,dx\right)}{\int_{-L/2}^{L/2} a\cos\frac{\pi}{L}x\,dx}$$

$$= \frac{\int_{-L/2}^{L/2}\frac{a^2}{4}\left(\cos\frac{2\pi}{L}x + 1\right)dx}{\int_{-L/2}^{L/2} a\cos\frac{\pi}{L}x\,dx}$$

$$= \frac{\frac{a^2}{4}\left(\frac{L}{2\pi}\sin\frac{2\pi}{L}x + x\right)\Big|_{-L/2}^{L/2}}{\left(\frac{aL}{\pi}\sin\frac{\pi}{L}x\right)\Big|_{-L/2}^{L/2}}$$

$$= \frac{a^2 L/4}{2aL/\pi} = \frac{\pi}{8}a \qquad \textbf{Ans.}$$

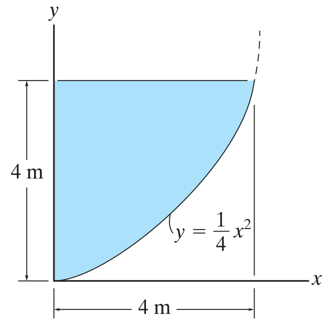
Due to Symmetry,

$$\bar{x} = 0 \qquad \textbf{Ans.}$$

Ans:
$\bar{y} = \frac{\pi}{8}a$
$\bar{x} = 0$

9–9.

Locate the centroid $\bar{x}$ of the shaded area.



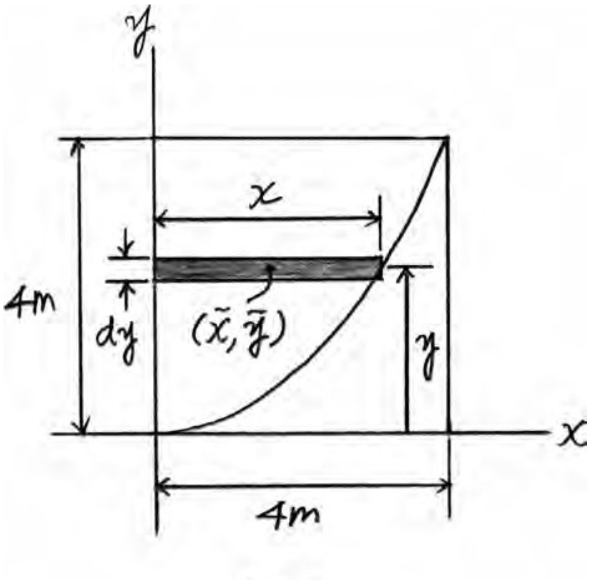
SOLUTION

Area And Moment Arm. The area of the differential element shown shaded in Fig. *a* is $dA = x\,dy$ and its centroid is at $\tilde{x} = \frac{1}{2}x$. Here, $x = 2y^{1/2}$

Centroid. Perform the integration

$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_0^{4\text{ m}} \frac{1}{2}\left(2y^{1/2}\right)\left(2y^{1/2}\,dy\right)}{\int_0^{4\text{ m}} 2y^{1/2}\,dy}$$

$$= \frac{3}{2}\text{ m}$$ **Ans.**

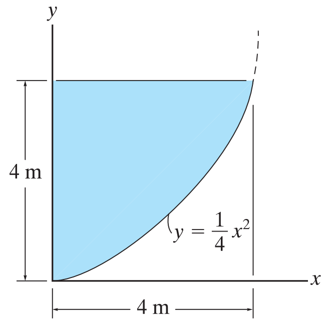


(a)

Ans:
$\bar{x} = \frac{3}{2}\text{ m}$

9–10.

Locate the centroid $\bar{y}$ of the shaded area.



SOLUTION

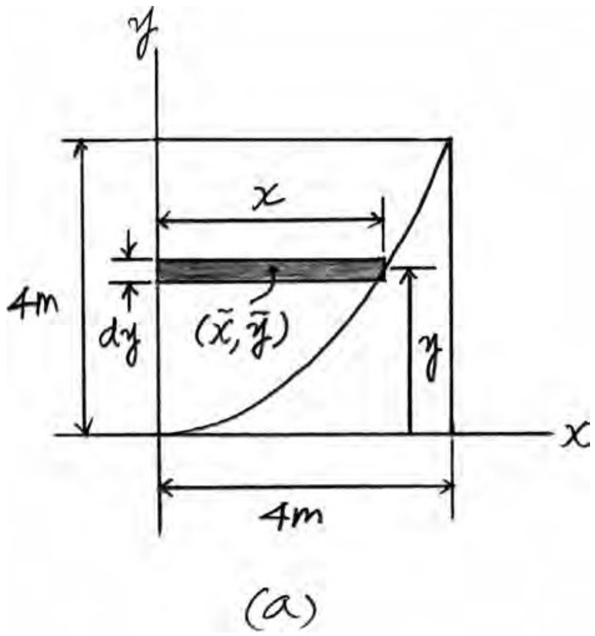
Area And Moment Arm. The area of the differential element shown shaded in Fig. *a* is $dA = x\,dy$ and its centroid is at $\tilde{y} = y$. Here, $x = 2y^{1/2}$.

Centroid. Perform the integration

$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\int_0^{4\text{ m}} y\left(2y^{1/2}\,dy\right)}{\int_0^{4\text{ m}} 2y^{1/2}\,dy}$$

$$= \frac{\left(\frac{4}{5}y^{5/2}\right)\Big|_0^{4\text{ m}}}{\left(\frac{4}{3}y^{3/2}\right)\Big|_0^{4\text{ m}}}$$

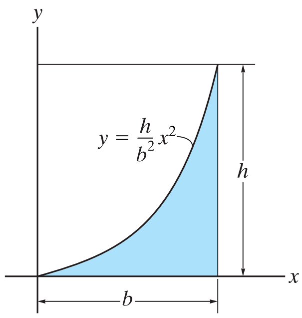
$$= \frac{12}{5}\text{ m}$$ **Ans.**



Ans:
$\bar{y} = \frac{12}{5}$ m

9–11.

Locate the centroid $\bar{x}$ of the area.



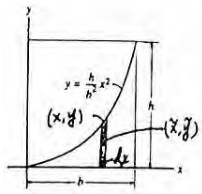
SOLUTION

$dA = y\,dx$

$\tilde{x} = x$

$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_0^b \frac{h}{b^2}x^3\,dx}{\int_0^b \frac{h}{b^2}x^2\,dx} = \frac{\left[\frac{h}{4b^2}x^4\right]_0^b}{\left[\frac{h}{3b^2}x^3\right]_0^b} = \frac{3}{4}b$$

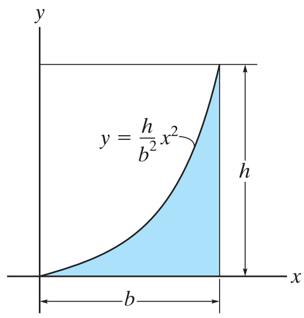
Ans.



Ans:
$\bar{x} = \frac{3}{4}b$

***9–12.**

Locate the centroid $\bar{y}$ of the shaded area.

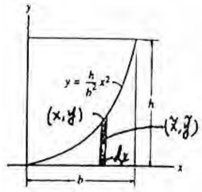


SOLUTION

$$dA = y\,dx$$

$$\tilde{y} = \frac{y}{2}$$

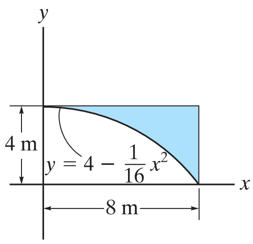
$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\int_0^b \frac{h^2}{2b^4}x^4\,dx}{\int_0^b \frac{h}{b^2}x^2\,dx} = \frac{\left[\frac{h^2}{10b^4}x^5\right]_0^b}{\left[\frac{h}{3b^2}x^3\right]_0^b} = \frac{3}{10}h$$ **Ans.**



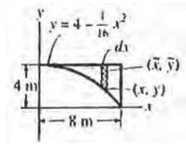
Ans:
$\bar{y} = \frac{3}{10}h$

9–13.

Locate the centroid $\overline{x}$ of the shaded area.



SOLUTION



$$dA = (4 - y)dx = \left(\frac{1}{16}x^2\right)dx$$

$$\tilde{x} = x$$

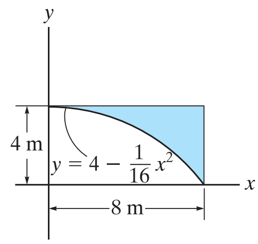
$$\overline{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_0^8 x\left(\frac{x^2}{16}\right)dx}{\int_0^8 \left(\frac{1}{16}x^2\right)dx}$$

$$\overline{x} = 6 \text{ m}$$ **Ans.**

Ans:
$\overline{x} = 6$ m

9–14.

Locate the centroid $\overline{y}$ of the shaded area.



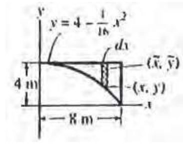
SOLUTION

$$dA = (4 - y)dx = \left(\frac{1}{16}x^2\right)dx$$

$$\overline{y} = \frac{4 + y}{2}$$

$$\overline{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\frac{1}{2}\int_0^8 \left(8 - \frac{x^2}{16}\right)\left(\frac{x^2}{16}\right)dx}{\int_0^8 \left(\frac{1}{16}x^2\right)dx}$$

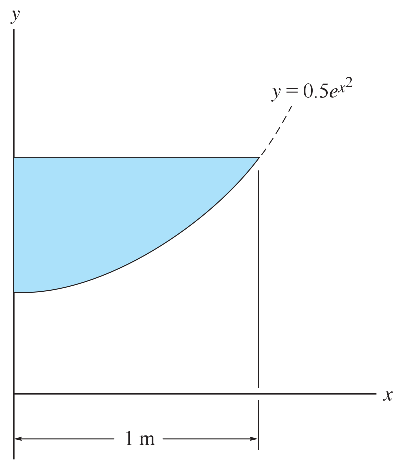
$\overline{y} = 2.8$ m **Ans.**



Ans:
$\overline{y} = 2.8$ m

9–15.

Locate the centroid $\bar{x}$ of the shaded area. Solve the problem by evaluating the integrals using Simpson's rule.



SOLUTION

At $x = 1$ m

$$y = 0.5e^{1^2} = 1.359 \text{ m}$$

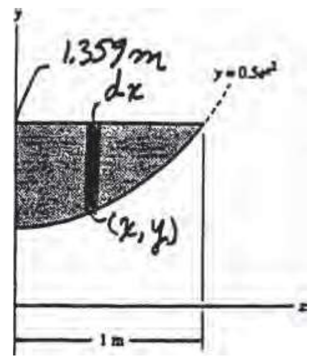
$$\int_A dA = \int_0^1 (1.359 - y)\,dx = \int_0^1 \left(1.359 = 0.5\,e^{x^2}\right) dx = 0.6278 \text{ m}^2$$

$$\bar{x} = x$$

$$\int_A \bar{x}\,dA = \int_0^1 x\left(1.359 - 0.5\,e^{x^2}\right) dx$$

$$= 0.25 \text{ m}^3$$

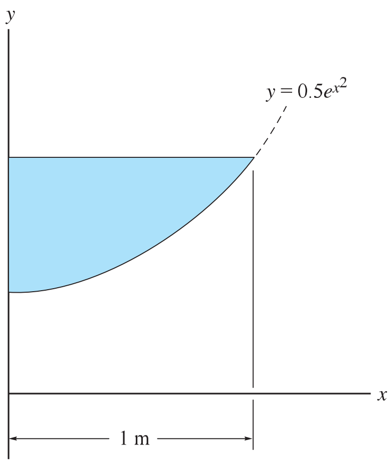
$$\bar{x} = \frac{\int_A \bar{x}\,dA}{\int_A dA} = \frac{0.25}{0.6278} = 0.398 \text{ m}$$ **Ans.**



Ans:
$\bar{x} = 0.398$ m

***9–16.**

Locate the centroid $\overline{y}$ of the shaded area. Solve the problem by evaluating the integrals using Simpson's rule.



SOLUTION

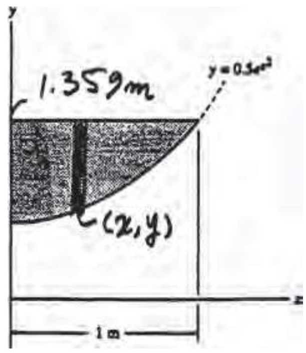
$$\int_A dA = \int_0^1 (1.359 - y)\,dx = \int_0^1 \left(1.359 - 0.5e^{x^2}\right) dx = 0.6278 \text{ m}^2$$

$$\overline{y} = \frac{1.359 + y}{2}$$

$$\int_A \overline{y}\,dA = \int_0^1 \left(\frac{1.359 + 0.5\,e^{x^2}}{2}\right)\left(1.359 - 0.5\,e^{x^2}\right) dx$$

$$= \frac{1}{2}\int_0^1 \left(1.847 - 0.25\,e^{2x^2}\right) dx = 0.6278 \text{ m}^3$$

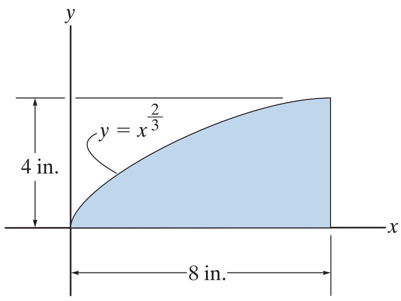
$$\overline{y} = \frac{\int_A \overline{y}\,dA}{\int_A dA} = \frac{0.6278}{0.6278} = 1.00 \text{ m}$$ **Ans.**



Ans:
$\overline{y} = 1.00$ m

9–17.

Locate the centroid $\bar{y}$ of the area.



SOLUTION

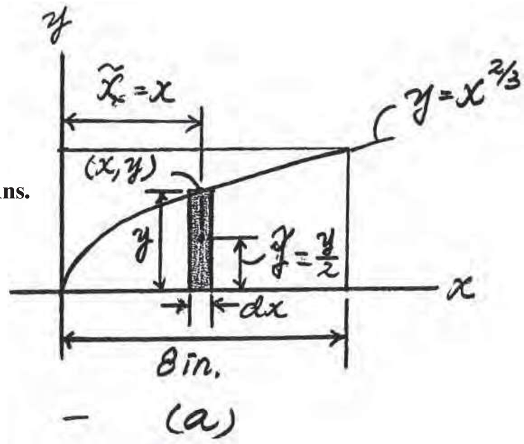
Area: Integrating the area of the differential element gives

$$A = \int_A dA = \int_0^{8\text{ in.}} x^{2/3}\,dx = \left[\frac{3}{5}x^{5/3}\right]\Bigg|_0^{8\text{ in.}} = 19.2\text{ in.}^2$$

Centroid: The centroid of the element is located at $\tilde{y} = y/2 = \frac{1}{2}x^{2/3}$. Applying Eq. 9–4, we have

$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\int_0^{8\text{ in.}} \frac{1}{2}x^{2/3}\left(x^{2/3}\right)dx}{19.2} = \frac{\int_0^{8\text{ in.}} \frac{1}{2}x^{4/3}dx}{19.2}$$

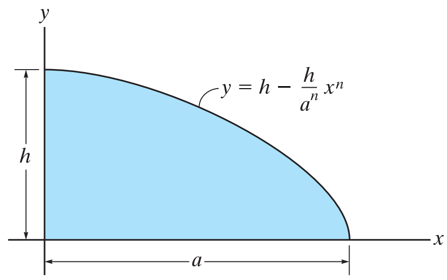
$$= \frac{\left[\frac{3}{14}x^{7/3}\right]\Bigg|_0^{8\text{ in.}}}{19.2} = 1.43\text{ in.}$$ **Ans.**



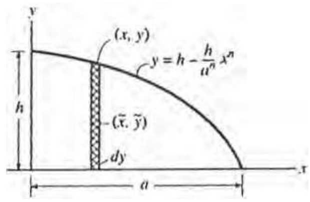
Ans:
$\bar{y} = 1.43$ in.

9–18.

Locate the centroid $\bar{x}$ of the area.



SOLUTION



$$dA = y\,dx$$

$$\tilde{x} = x$$

$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_0^a \left(hx - \frac{h}{a^n}x^{n+1}\right)dx}{\int_0^a \left(h - \frac{h}{a^n}x^n\right)dx}$$

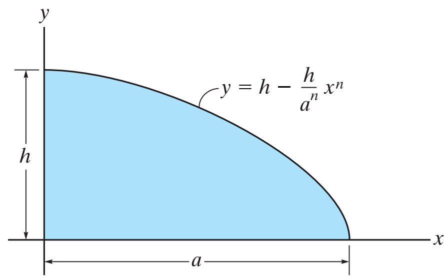
$$= \frac{\left[\frac{h}{2}x^2 - \frac{h(x^{n+2})}{a^n(n+2)}\right]_0^a}{\left[hx - \frac{h(x^{n+1})}{a^n(n+1)}\right]_0^a}$$

$$\bar{x} = \frac{\left(\frac{h}{2} - \frac{h}{n+2}\right)a^2}{\left(h - \frac{h}{n+1}\right)a} = \frac{a(1+n)}{2(2+n)}$$ **Ans.**

Ans:
$\bar{x} = \frac{a(1+n)}{2(2+n)}$

9–19.

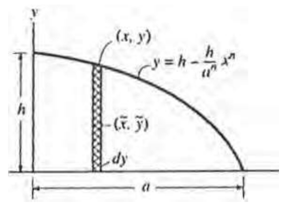
Locate the centroid $\overline{y}$ of the area.



SOLUTION

$$dA = y\,dx$$

$$\tilde{y} = \frac{y}{2}$$



$$\overline{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\frac{1}{2}\int_0^a \left(h^2 - 2\frac{h^2}{a^n}x^n + \frac{h^2}{a^{2n}}x^{2n}\right)dx}{\int_0^a \left(h - \frac{h}{a^n}x^n\right)dx}$$

$$= \frac{\frac{1}{2}\left[h^2x - \frac{2h^2(x^{n+1})}{a^n(n+1)} + \frac{h^2(x^{2n+1})}{a^{2n}(2n+1)}\right]_0^a}{\left[hx - \frac{h(x^{n+1})}{a^n(n+1)}\right]_0^a}$$

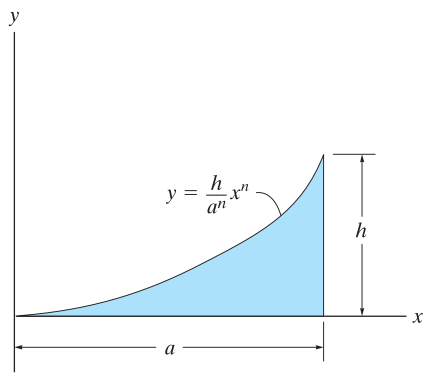
$$\overline{y} = \frac{\frac{2n^2}{2(n+1)(2n+1)}h}{\frac{n}{n+1}} = \frac{hn}{2n+1}$$ **Ans.**

Ans:

$$\overline{y} = \frac{hn}{2n+1}$$

***9–20.**

Locate the centroid $\overline{y}$ of the shaded area.

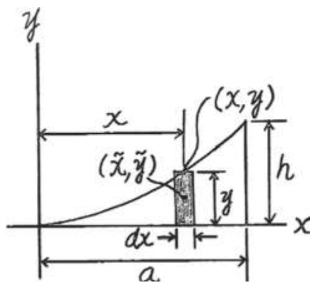


SOLUTION

$$dA = y\,dx$$

$$\tilde{y} = \frac{y}{2}$$

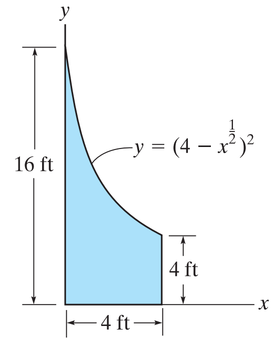
$$\overline{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\frac{1}{2}\int_0^a \frac{h^2}{a^{2n}}\,x^{2n}\,dx}{\int_0^a \frac{h}{a^n}\,x^n\,dx} = \frac{\frac{h^2(a^{2n+1})}{2a^{2n}(2n+1)}}{\frac{h(a^{n+1})}{a^n(n+1)}} = \frac{hn+1}{2(2n+1)}$$ **Ans.**



Ans:
$\overline{y} = \frac{hn+1}{2(2n+1)}$

9–21.

Locate the centroid $\bar{x}$ of the shaded area.



SOLUTION

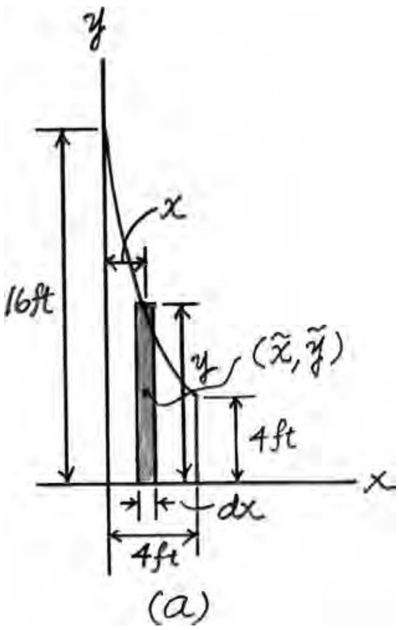
Area And Moment Arm. The area of the differential element shown shaded in Fig. *a* is $dA = y\,dx = \left(4 - x^{1/2}\right)^2 dx = (x - 8x^{1/2} + 16)dx$ and its centroid is at $\tilde{x} = x$.

Centroid. Perform the integration

$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_0^{4\text{ ft}} x(x - 8x^{1/2} + 16)dx}{\int_0^{4\text{ ft}} (x - 8x^{1/2} + 16)\,dx}$$

$$= \frac{\left(\frac{x^3}{3} - \frac{16}{5}x^{5/2} + 8x^2\right)\Big|_0^{4\text{ ft}}}{\left(\frac{x^2}{2} - \frac{16}{3}x^{3/2} + 16x\right)\Big|_0^{4\text{ ft}}}$$

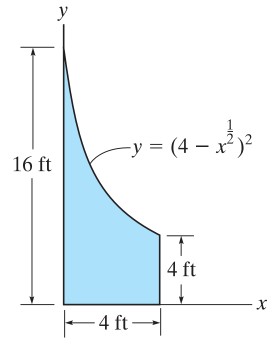
$$= 1\frac{3}{5}\text{ ft}$$ **Ans.**



Ans:
$\bar{x} = 1\frac{3}{5}$ ft

9–22.

Locate the centroid $\bar{y}$ of the shaded area.



SOLUTION

Area And Moment Arm. The area of the differential element shown shaded in Fig. *a* is $dA = y\,dx = \left(4 - x^{\frac{1}{2}}\right)^2 dx = (x - 8x^{1/2} + 16)dx$ and its centroid is at $\tilde{y} = \frac{y}{2} = \frac{1}{2}\left(4 - x^{1/2}\right)^2$.

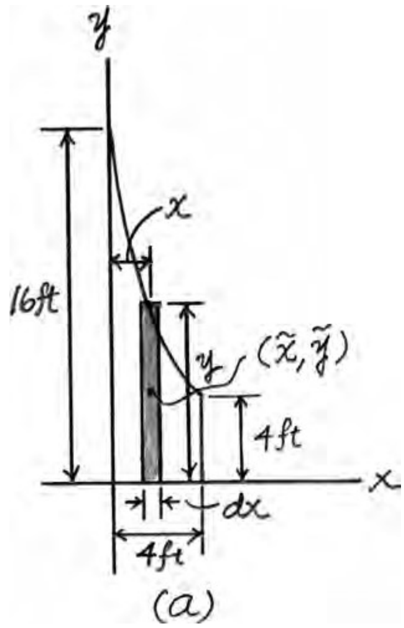
Centroid. Perform the integration,

$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\int_0^{4\text{ ft}} \frac{1}{2}\left(4 - x^{1/2}\right)^2\left(x - 8x^{1/2} + 16\right)dx}{\int_0^{4\text{ ft}} (x - 8x^{1/2} + 16)dx}$$

$$= \frac{\int_0^{4\text{ ft}} \left(\frac{1}{2}x^2 - 8x^{3/2} + 48x - 128x^{1/2} + 128\right)dx}{\int_0^{4\text{ ft}} \left(x - 8x^{1/2} + 16\right)dx}$$

$$= \frac{\left.\left(\frac{x^3}{6} - \frac{16}{5}x^{5/2} + 24x^2 - \frac{256}{3}x^{3/2} + 128x\right)\right|_0^{4\text{ ft}}}{\left.\left(\frac{x^2}{2} - \frac{16}{3}x^{3/2} + 16x\right)\right|_0^{4\text{ ft}}}$$

$$= 4\,\frac{8}{55}\text{ ft}$$ **Ans.**

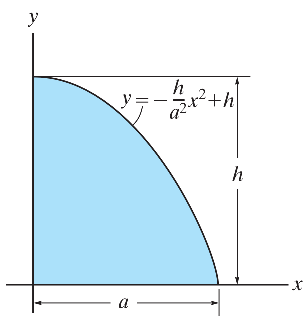


Ans:

$\bar{y} = 4\,\frac{8}{55}$ ft

9–23.

Locate the centroid $\bar{x}$ of the shaded area.



SOLUTION

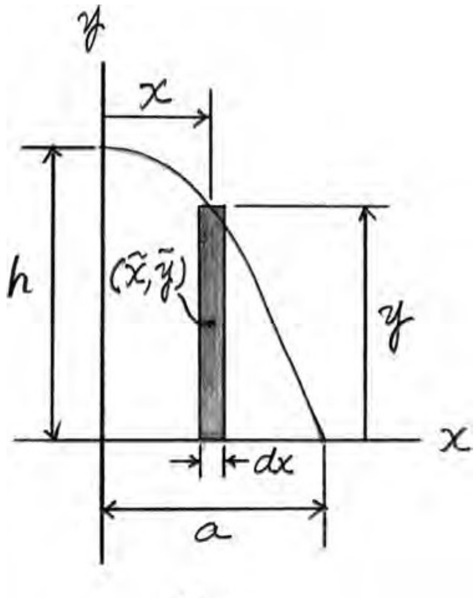
Area And Moment Arm. The area of the differential element shown shaded in Fig. *a* is $dA = y\,dx = \left(-\frac{h}{a^2}x^2 + h\right)dx$ and its centroid is at $\tilde{x} = x$.

Centroid. Perform the integration,

$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_0^a x\left(-\frac{h}{a^2}x^2 + h\right)dx}{\int_0^a \left(-\frac{h}{a^2}x^2 + h\right)dx}$$

$$= \frac{\left(-\frac{h}{4a^2}x^4 + \frac{h}{2}x^2\right)\Big|_0^a}{\left(-\frac{h}{3a^2}x^3 + hx\right)\Big|_0^a}$$

$$= \frac{3}{8}a \qquad \textbf{Ans.}$$

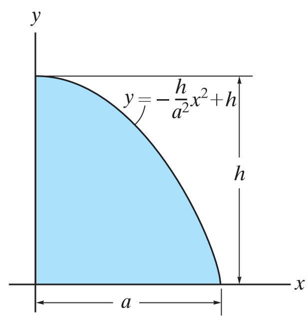


(a)

Ans:
$\bar{x} = \frac{3}{8}a$

***9–24.**

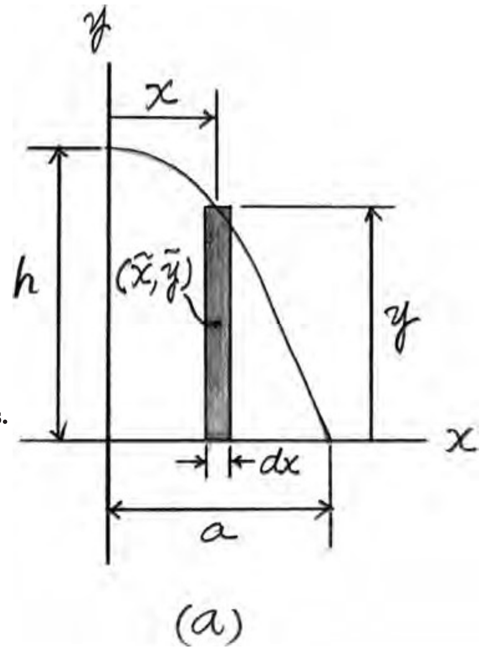
Locate the centroid $\bar{y}$ of the shaded area.



SOLUTION

Area And Moment Arm. The area of the differential element shown shaded in Fig. *a* is $dA = y\,dx = \left(-\frac{h}{a^2}x^2 + h\right)dx$ and its centroid is at $\tilde{y} = \frac{y}{2} = \frac{1}{2}\left(-\frac{h^2}{a}x^2 + h\right)$.

Centroid. Perform the integration,



$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\int_0^a \frac{1}{2}\left(-\frac{h}{a^2}x^2 + h\right)\left(-\frac{h}{a^2}x^2 + h\right)dx}{\int_0^a \left(-\frac{h}{a^2}x^2 + h\right)dx}$$

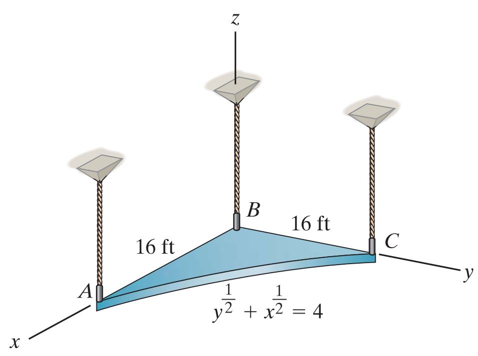
$$= \frac{\frac{1}{2}\left(\frac{h^2}{5a^4}x^5 - \frac{2h^2}{3a^2}x^3 + h^2x\right)\Big|_0^a}{\left(-\frac{h}{3a^2}x^3 + hx\right)\Big|_0^a}$$

$$= \frac{2}{5}h$$ **Ans.**

Ans:
$\bar{y} = \frac{2}{5}h$

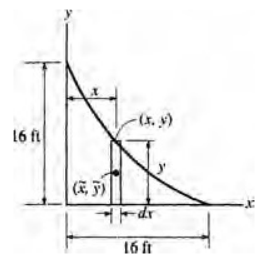
9–25.

The plate has a thickness of 0.25 ft and a specific weight of $\gamma = 180\ \text{lb/ft}^3$. Determine the location of its center of gravity. Also, find the tension in each of the cords used to support it.



SOLUTION

Area and Moment Arm: Here, $y = x - 8x^{\frac{1}{2}} + 16$. The area of the differential element is $dA = y\,dx = (x - 8x^{\frac{1}{2}} + 16)dx$ and its centroid is $\tilde{x} = x$ and $\tilde{y} = \frac{1}{2}(x - 8x^{\frac{1}{2}} + 16)$. Evaluating the integrals, we have



$$A = \int_A dA = \int_0^{16\text{ ft}} (x - 8x^{\frac{1}{2}} + 16)dx$$

$$= \left(\frac{1}{2}x^2 - \frac{16}{3}x^{\frac{3}{2}} + 16x\right)\Bigg|_0^{16\text{ ft}} = 42.67\ \text{ft}^2$$

$$\int_A \tilde{x}\,dA = \int_0^{16\text{ ft}} x[(x - 8x^{\frac{1}{2}} + 16)dx]$$

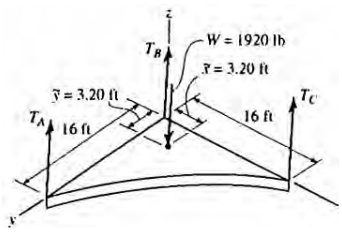
$$= \left(\frac{1}{3}x^3 - \frac{16}{5}x^{\frac{5}{2}} + 8x^2\right)\Bigg|_0^{16\text{ ft}} = 136.53\ \text{ft}^3$$

$$\int_A \tilde{y}\,dA = \int_0^{16\text{ ft}} \frac{1}{2}(x - 8x^{\frac{1}{2}} + 16)[(x - 8x^{\frac{1}{2}} + 16)dx]$$

$$= \frac{1}{2}\left(\frac{1}{3}x^3 - \frac{32}{5}x^{\frac{5}{2}} + 48x^2 - \frac{512}{3}x^{\frac{3}{2}} + 256x\right)\Bigg|_0^{16\text{ ft}}$$

$$= 136.53\ \text{ft}^3$$

Centroid: Applying Eq. 9–6, we have



$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{136.53}{42.67} = 3.20\ \text{ft} \qquad \textbf{Ans.}$$

$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{136.53}{42.67} = 3.20\ \text{ft} \qquad \textbf{Ans.}$$

Equations of Equilibrium: The weight of the plate is $W = 42.67(0.25)(180) = 1920$ lb.

$\Sigma M_x = 0;\quad 1920(3.20) - T_A(16) = 0 \quad T_A = 384\ \text{lb}$ **Ans.**

$\Sigma M_y = 0;\quad T_C(16) - 1920(3.20) = 0 \quad T_C = 384\ \text{lb}$ **Ans.**

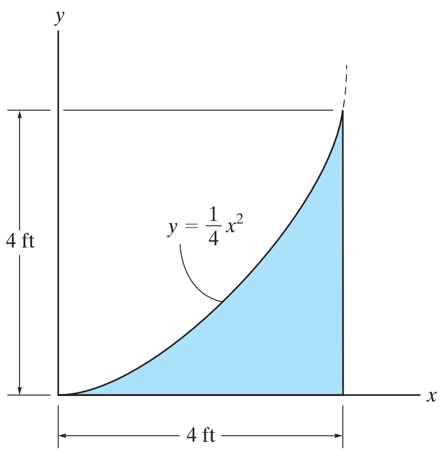
$\Sigma F_z = 0;\quad T_B + 384 + 384 - 1920 = 0$

$T_B = 1152\ \text{lb} = 1.15\ \text{kip}$ **Ans.**

Ans:
$\bar{x} = 3.20$ ft
$\bar{y} = 3.20$ ft
$T_A = 384$ lb
$T_C = 384$ lb
$T_B = 1.15$ kip

9–26.

Locate the centroid $\bar{x}$ of the shaded area.



SOLUTION

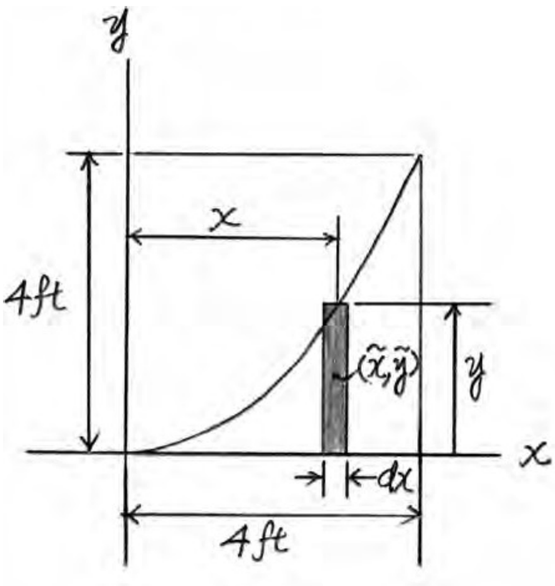
Area And Moment Arm. The area of the differential element shown shaded in Fig. *a* is $dA = y\,dx = \frac{1}{4}x^2\,dx$ and its centroid is at $\tilde{x} = x$.

Centroid. Perform the integration

$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\displaystyle\int_0^{4\text{ ft}} x\left(\frac{1}{4}x^2\,dx\right)}{\displaystyle\int_0^{4\text{ ft}} \frac{1}{4}x^2 dx}$$

$$= \frac{\left(\frac{1}{16}x^4\right)\Big|_0^{4\text{ ft}}}{\left(\frac{1}{12}x^3\right)\Big|_0^{4\text{ ft}}}$$

$$= 3\text{ ft}$$ **Ans.**

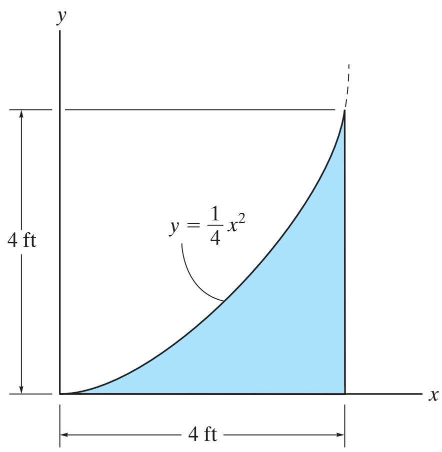


(a)

Ans:
$\bar{x} = 3$ ft

9–27.

Locate the centroid $\bar{y}$ of the shaded area.



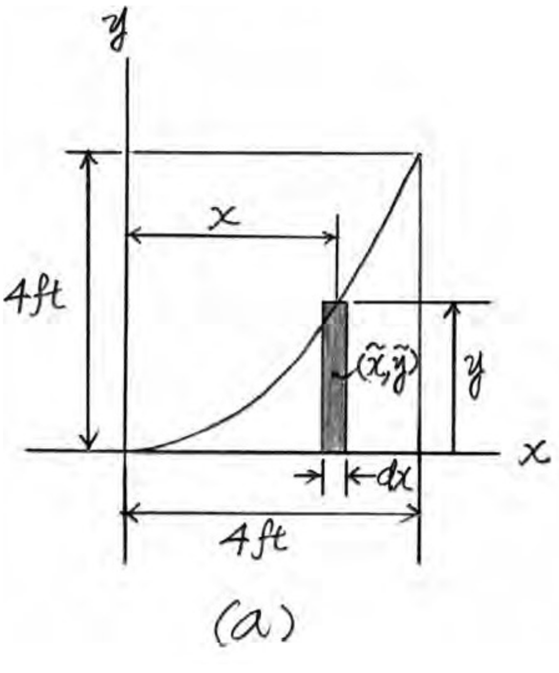
SOLUTION

Area And Moment Arm. The area of the differential element shown shaded in Fig. *a* is $dA = y\,dx = \frac{1}{4}x^2\,dx$ and its centroid is located at $\tilde{y} = \frac{y}{2} = \frac{1}{2}\left(\frac{1}{4}x^2\right) = \frac{1}{8}x^2$.

Centroid. Perform the integration,

$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\int_0^{4\text{ ft}} \frac{1}{8}x^2\left(\frac{1}{4}x^2\,dx\right)}{\int_0^{4\text{ ft}} \frac{1}{4}x^2 dx}$$

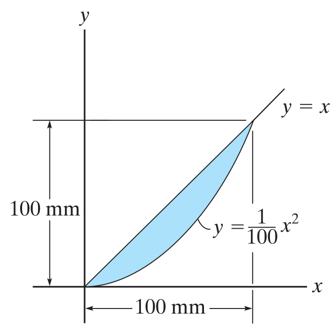
$$= \frac{6}{5}\text{ ft}$$ **Ans.**



Ans:
$\bar{y} = \frac{6}{5}$ ft

***9–28.**

Locate the centroid $\bar{x}$ of the shaded area.



SOLUTION

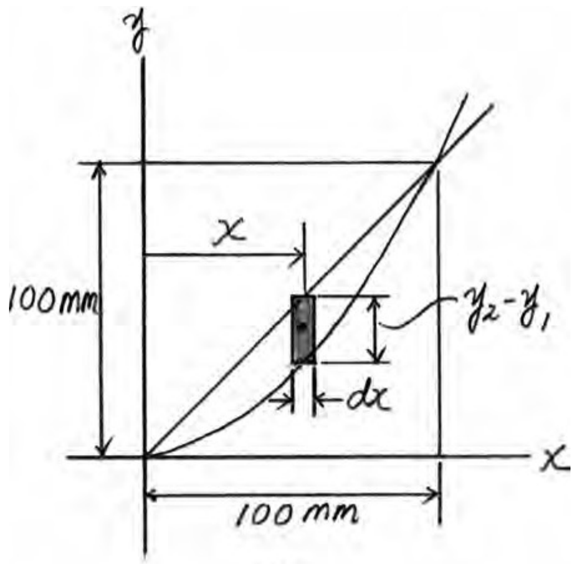
Area And Moment Arm. Here, $y_2 = x$ and $y_1 = \frac{1}{100}x^2$. Thus the area of the differential element shown shaded in Fig. *a* is $dA = (y_2 - y_1)\,dx = \left(x - \frac{1}{100}x^2\right)dx$ and its centroid is at $\tilde{x} = x$.

Centroid. Perform the integration

$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_0^{100\text{ mm}} x\left(x - \frac{1}{100}x^2\right)dx}{\int_0^{100\text{ mm}}\left(x - \frac{1}{100}x^2\right)dx}$$

$$= \frac{\left(\frac{x^3}{3} - \frac{1}{400}x^4\right)\Big|_0^{100\text{ mm}}}{\left(\frac{x^2}{2} - \frac{1}{300}x^3\right)\Big|_0^{100\text{ mm}}}$$

$$= 50.0\text{ mm}$$ **Ans.**

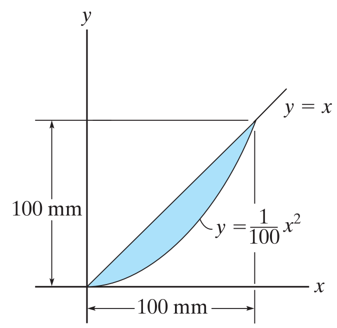


(a)

Ans:
$\bar{x} = 50.0$ mm

9–29.

Locate the centroid $\bar{y}$ of the shaded area.



SOLUTION

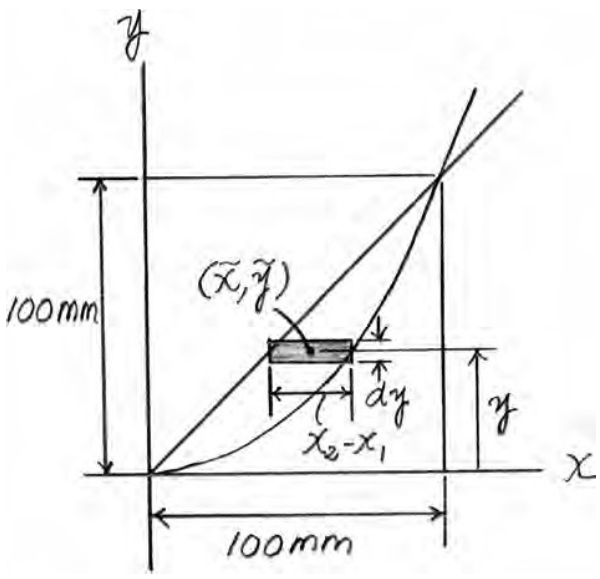
Area And Moment Arm. Here, $x_2 = 10y^{1/2}$ and $x_1 = y$. Thus, the area of the differential element shown shaded in Fig. *a* is $dA = (x_2 - x_1)\,dy = (10y^{1/2} - y)dy$ and its centroid is at $\tilde{y} = y$.

Centroid. Perform the integration,

$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\int_0^{100\text{ mm}} y\left(10y^{1/2} - y\right)dy}{\int_0^{100\text{ mm}} \left(10y^{1/2} - y\right)dy}$$

$$= \frac{\left(4y^{5/2} - \dfrac{y^3}{3}\right)\Big|_0^{100\text{ mm}}}{\left(\dfrac{20}{3}y^{3/2} - \dfrac{y^2}{2}\right)\Big|_0^{100\text{ mm}}}$$

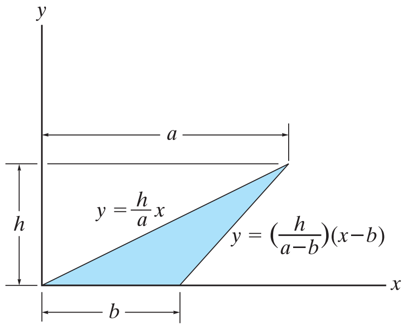
$$= 40.0\text{ mm}$$ **Ans.**



Ans:
$\bar{y} = 40.0$ mm

9–30.

Locate the centroid $\bar{x}$ of the shaded area.



SOLUTION

Area And Moment Arm. Here $x_1 = \frac{a}{h}y$ and $x_2 = \left(\frac{a-b}{h}\right)y + b$. Thus the area of the differential element is $dA = (x_2 - x_1)\,dy = \left[\left(\frac{a-b}{h}\right)y + b - \frac{a}{h}y\right]dy = \left(b - \frac{b}{h}y\right)dy$ and its centroid is at $\tilde{x} = x_1 + \frac{x_2 - x_1}{2} = \frac{1}{2}(x_2 + x_1) = \frac{a}{h}y - \frac{b}{2h}y + \frac{b}{2}$.

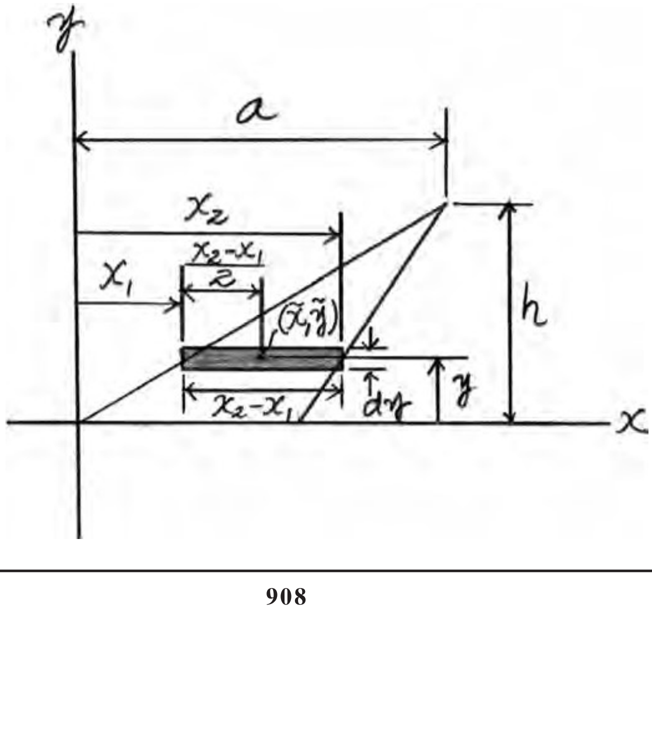
Centroid. Perform the integration,

$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_0^h \left(\frac{a}{h}y - \frac{b}{2h}y + \frac{b}{2}\right)\left[\left(b - \frac{b}{h}y\right)dy\right]}{\int_0^h \left(b - \frac{b}{h}y\right)dy}$$

$$= \frac{\left[\frac{b}{2h}(a-b)y^2 + \frac{b}{6h^2}(b-2a)y^3 + \frac{b^2}{2}y\right]\Big|_0^h}{\left(by - \frac{b}{2h}y^2\right)\Big|_0^h}$$

$$= \frac{\frac{bh}{6}(a+b)}{\frac{bh}{2}}$$

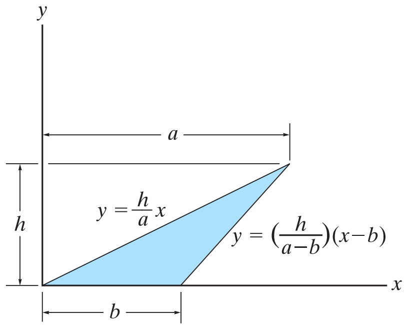
$$= \frac{1}{3}(a+b) \qquad \textbf{Ans.}$$



Ans:
$\bar{x} = \frac{1}{3}(a+b)$

9–31.

Locate the centroid $\bar{y}$ of the shaded area.



SOLUTION

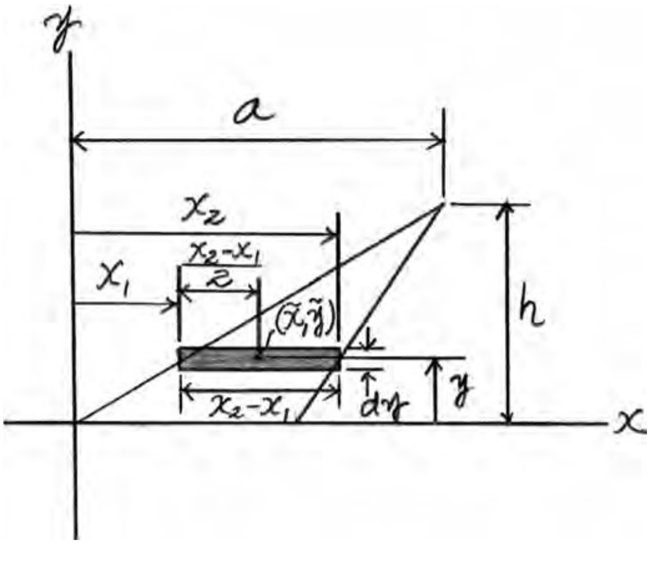
Area And Moment Arm. Here, $x_1 = \frac{a}{h}y$ and $x_2 = \left(\frac{a-b}{h}\right)y + b$. Thus the area of the differential element is $dA = (x_2 - x_1)\,dy = \left[\left(\frac{a-b}{h}\right)y + b - \frac{a}{h}y\right]dy = \left(b - \frac{b}{h}y\right)dy$ and its centroid is at $\tilde{y} = y$.

Centroid. Perform the integration,

$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\int_0^h y\left(b - \frac{b}{h}y\right)dy}{\int_0^h \left(b - \frac{b}{h}y\right)dy}$$

$$= \frac{\left(\frac{b}{2}y^2 - \frac{b}{3h}y^3\right)\Big|_0^h}{\left(by - \frac{b}{2h}y^2\right)\Big|_0^h}$$

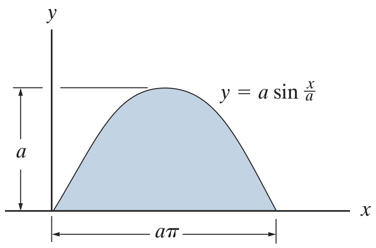
$$= \frac{\frac{1}{6}bh^2}{\frac{1}{2}bh} = \frac{h}{3}$$ **Ans.**



Ans:
$\bar{y} = \frac{h}{3}$

***9–32.**

Locate the centroid $\bar{x}$ of the shaded area.



SOLUTION

Area and Moment Arm: The area of the differential element is $dA = ydx = a\sin\frac{x}{a}dx$ and its centroid are $\tilde{x} = x$

$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_0^{\pi a} x\left(a\sin\frac{x}{a}dx\right)}{\int_0^{\pi a} a\sin\frac{x}{a}dx}$$

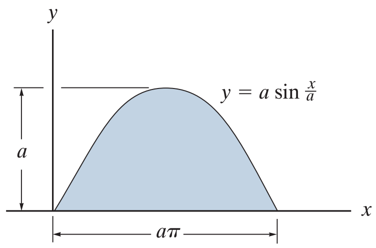
$$= \frac{\left[a^3\sin\frac{x}{a} - x\left(a^2\cos\frac{x}{a}\right)\right]\Big|_0^{\pi a}}{\left(-a^2\cos\frac{x}{a}\right)\Big|_0^{\pi a}}$$

$$= \frac{\pi}{2}a$$ **Ans.**

Ans:
$\bar{x} = \frac{\pi}{2}a$

9–33.

Locate the centroid $\overline{y}$ of the shaded area.



SOLUTION

Area and Moment Arm: The area of the differential element is $dA = y\,dx = a\sin\frac{x}{a}dx$ and its centroid are $\overline{y} = \frac{y}{2} = \frac{a}{2}\sin\frac{x}{a}$.

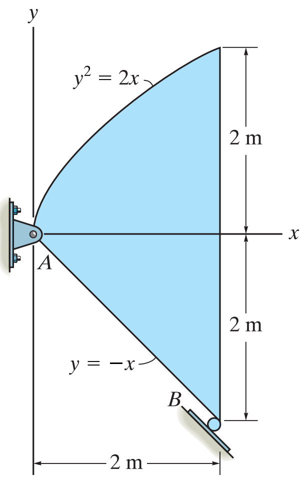
$$\overline{y} = \frac{\int_A \overline{y}\,dA}{\int_A dA} = \frac{\int_0^{\pi a} \frac{a}{2}\sin\frac{x}{a}\left(a\sin\frac{x}{a}dx\right)}{\int_0^{\pi a} a\sin\frac{x}{a}dx} = \frac{\left[\frac{1}{4}a^2\left(x - \frac{1}{2}a\sin\frac{2x}{a}\right)\right]\Big|_0^{\pi a}}{\left(-a^2\cos\frac{x}{a}\right)\Big|_0^{\pi a}} = \frac{\pi a}{8} \quad \textbf{Ans.}$$

Ans:

$\overline{y} = \frac{\pi a}{8}$

9–34.

The steel plate is 0.3 m thick and has a density of $7850\ \text{kg/m}^3$. Determine the location of its center of mass. Also compute the reactions at the pin and roller support.



SOLUTION

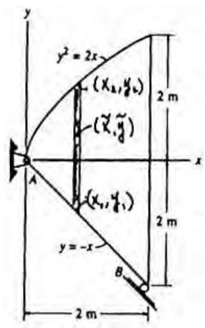
$y_1 = -x_1$

$y_2^2 = 2x_2$

$dA = (y_2 - y_1)\,dx = \left(\sqrt{2x} + x\right)dx$

$\widetilde{x} = x$

$\widetilde{y} = \dfrac{y_2 + y_1}{2} = \dfrac{\sqrt{2x} - x}{2}$

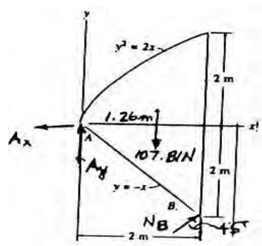


$$\overline{x} = \frac{\int_A \widetilde{x}\,dA}{\int_A dA} = \frac{\int_0^2 x\left(\sqrt{2x} + x\right)dx}{\int_0^2 \left(\sqrt{2x} + x\right)dx} = \frac{\left[\frac{2\sqrt{2}}{5}x^{5/2} + \frac{1}{3}x^3\right]_0^2}{\left[\frac{2\sqrt{2}}{3}x^{3/2} + \frac{1}{2}x^2\right]_0^2} = 1.2571 = 1.26\ \text{m} \quad \textbf{Ans.}$$

$$\overline{y} = \frac{\int_A \widetilde{y}\,dA}{\int_A dA} = \frac{\int_0^2 \frac{\sqrt{2x} - x}{2}\left(\sqrt{2x} + x\right)dx}{\int_0^2 \left(\sqrt{2x} + x\right)dx} = \frac{\left[\frac{x^2}{2} - \frac{1}{6}x^3\right]_0^2}{\left[\frac{2\sqrt{2}}{3}x^{3/2} + \frac{1}{2}x^2\right]_0^2} = 0.143\ \text{m} \quad \textbf{Ans.}$$

$A = 4.667\ \text{m}^2$

$W = 7850(9.81)(4.667)(0.3) = 107.81\ \text{kN}$



$\circlearrowleft + \Sigma M_A = 0; \qquad -1.2571(107.81) + N_B\left(2\sqrt{2}\right) = 0$

$N_B = 47.92 = 47.9\ \text{kN}$ **Ans.**

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad -A_x + 47.92 \sin 45° = 0$

$A_x = 33.9\ \text{kN}$ **Ans.**

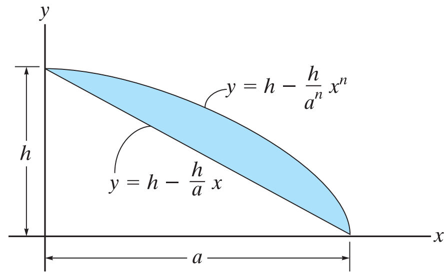
$+\uparrow \Sigma F_y = 0; \qquad A_y + 47.92 \cos 45° - 107.81 = 0$

$A_y = 73.9\ \text{kN}$ **Ans.**

Ans:
$\overline{x} = 1.26\ \text{m}$
$\overline{y} = 0.143\ \text{m}$
$N_B = 47.9\ \text{kN}$
$A_x = 33.9\ \text{kN}$
$A_y = 73.9\ \text{kN}$

9–35.

Locate the centroid $\bar{x}$ of the shaded area.



SOLUTION

Area And Moment Arm. Here, $y_2 = h - \frac{h}{a^n}x^n$ and $y_1 = h - \frac{h}{a}x$. Thus, the area of the differential element shown shaded in Fig. *a* is $dA = (y_2 - y_1)\,dx = \left[h - \frac{h}{a^n}x^n - \left(h - \frac{h}{a}x\right)\right]dx = \left(\frac{h}{a}x - \frac{h}{a^n}x^n\right)dx$ and its centroid is $\tilde{x} = x$.

Centroid. Perform the integration

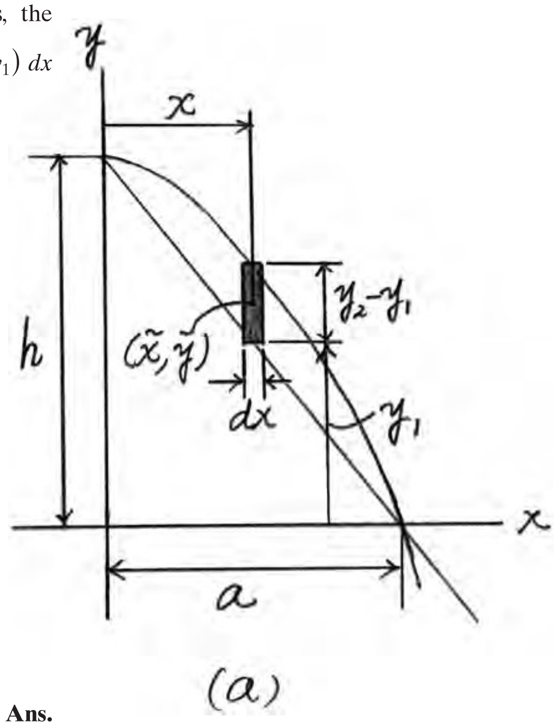
$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_0^a x\left(\frac{h}{a}x - \frac{h}{a^n}x^n\right)dx}{\int_0^a \left(\frac{h}{a}x - \frac{h}{a^n}x^n\right)dx}$$

$$= \frac{\left[\frac{h}{3a}x^3 - \frac{h}{a^n(n+2)}x^{n+2}\right]\Big|_0^a}{\left[\frac{h}{2a}x^2 - \frac{h}{a^n(n+1)}x^{n+1}\right)\Big|_0^a}$$

$$= \frac{\dfrac{ha^2(n-1)}{3(n+2)}}{\dfrac{ha(n-1)}{2(n+1)}}$$

$$= \left[\frac{2(n+1)}{3(n+2)}\right]a$$

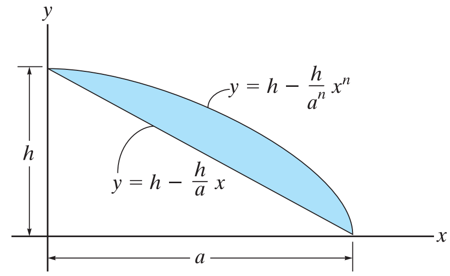
Ans.



Ans:
$\bar{x} = \left[\frac{2(n+1)}{3(n+2)}\right]a$

***9–36.**

Locate the centroid $\bar{y}$ of the shaded area.



SOLUTION

Area And Moment Arm. Here, $y_2 = h - \frac{h}{a^n}x^n$ and $y_1 = h - \frac{h}{a}x$. Thus, the area of the differential element shown shaded in Fig. *a* is $dA = (y_2 - y_1)\,dx = \left[h - \frac{h}{a^n}x^n - \left(h - \frac{h}{a}x\right)\right]dx = \left(\frac{h}{a}x - \frac{h}{a^n}x^n\right)dx$ and its centroid is at $\tilde{y} = y_1 + \left(\frac{y_2 - y_1}{2}\right) = \frac{1}{2}(y_2 + y_1) = \frac{1}{2}\left(h - \frac{h}{a^n}x^n + h - \frac{h}{a}x\right) = \frac{1}{2}\left(2h - \frac{h}{a^n}x^n - \frac{h}{a}x\right).$

Centroid. Perform the integration

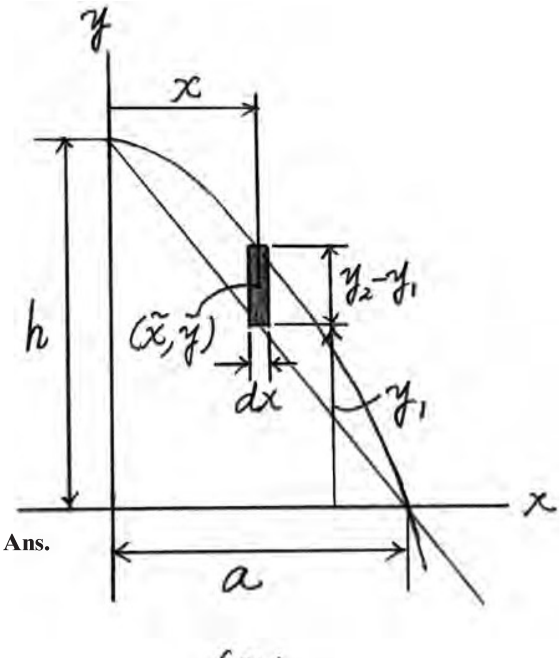
$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\int_0^a \frac{1}{2}\left(2h - \frac{h}{a^n}x^n - \frac{h}{a}x\right)\left(\frac{h}{a}x - \frac{h}{a^n}x^n\right)dx}{\int_0^a \left(\frac{h}{a}x - \frac{h}{a^n}x^n\right)dx}$$

$$= \frac{\frac{1}{2}\left[\frac{h^2}{a}x^2 - \frac{h^2}{3a^2}x^3 - \frac{2h^2}{a^n(n+1)}x^{n+1} + \frac{h^2}{a^{2n}(2n+1)}x^{2n+1}\right]\Bigg|_0^a}{\left[\frac{h}{2a}x^2 - \frac{h}{a^n(n+1)}x^{n+1}\right)\Bigg|_0^a}$$

$$= \frac{h^2 a\left[\frac{(4n+1)(n-1)}{6(n+1)(2n+1)}\right]}{ha\left[\frac{n-1}{2(n+1)}\right]}$$

$$= \left[\frac{(4n+1)}{3(2n+1)}\right]h$$

Ans.

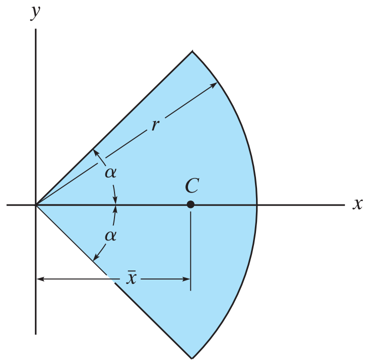


(a)

Ans:
$\bar{y} = \left[\frac{(4n+1)}{3(2n+1)}\right]h$

9–37.

Locate the centroid $\bar{x}$ of the circular sector.



SOLUTION

Area And Moment Arm. The area of the differential element shown in Fig. *a* is $dA = \frac{1}{2}r^2\,d\theta$ and its centroid is at $\tilde{x} = \frac{2}{3}r\cos\theta$.

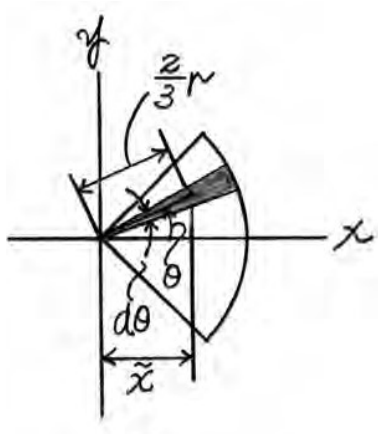
Centroid. Perform the integration

$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_{-\alpha}^{\alpha}\left(\frac{2}{3}r\cos\theta\right)\left(\frac{1}{2}r^2\,d\theta\right)}{\int_{-\alpha}^{\alpha}\frac{1}{2}r^2\,d\theta}$$

$$= \frac{\left(\frac{1}{3}r^3\sin\theta\right)\Big|_{-\alpha}^{\alpha}}{\left(\frac{1}{2}r^2\,\theta\right)\Big|_{-\alpha}^{\alpha}}$$

$$= \frac{\frac{2}{3}r^3\sin\alpha}{r^2\,\alpha}$$

$$= \frac{2}{3}\left(\frac{r\sin\alpha}{\alpha}\right)$$ **Ans.**

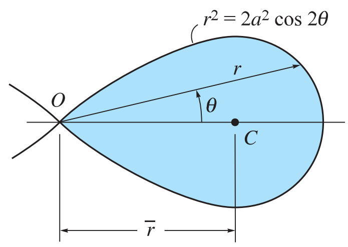


(a)

Ans:
$\bar{x} = \frac{2}{3}\left(\frac{r\sin\alpha}{\alpha}\right)$

9–38.

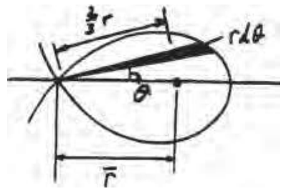
Determine the location $\bar{r}$ of the centroid C for the loop of the lemniscate, $r^2 = 2a^2 \cos 2\theta, (-45° \leq \theta \leq 45°)$.



SOLUTION

$$dA = \frac{1}{2}(r)\, r\, d\theta = \frac{1}{2} r^2\, d\theta$$

$$A = 2\int_0^{45°} \frac{1}{2}(2a^2 \cos 2\theta) d\theta = a^2\big[\sin 2\theta\big]_0^{45°} = a^2$$



$$\bar{x} = \frac{\int_A \bar{x}\, dA}{\int_A dA} = \frac{2\int_0^{45°}\left(\frac{2}{3} r \cos\theta\right)\left(\frac{1}{2} r^2 d\theta\right)}{a^2} = \frac{\frac{2}{3}\int_0^{45°} r^3 \cos\theta\, d\theta}{a^2}$$

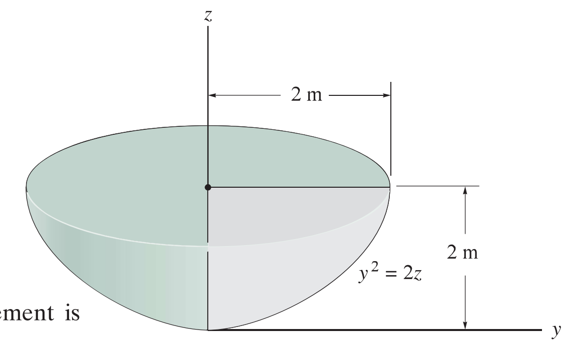
$$\int_A \bar{x}\, dA = \frac{2}{3}\int_0^{45°} r^3 \cos\theta\, d\theta = \frac{2}{3}\int_0^{45°} \left(2a^2\right)^{3/2} \cos\theta\, (\cos 2\theta)^{3/2} d\theta = 0.7854\, a^3$$

$$\bar{x} = \frac{0.7854\, a^3}{a^2} = 0.785\, a$$ **Ans.**

Ans:
$\bar{x} = 0.785\, a$

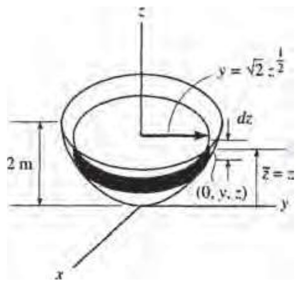
9–39.

Locate the center of gravity of the volume. The material is homogeneous.



SOLUTION

Volume and Moment Arm: The volume of the thin disk differential element is $dV = \pi y^2 dz = \pi(2z)dz = 2\pi z dz$ and its centroid $\tilde{z} = z$.



Centroid: Due to symmetry about z axis

$$\bar{x} = \bar{y} = 0 \qquad \textbf{Ans.}$$

Applying Eq. 9–3 and performing the integration, we have

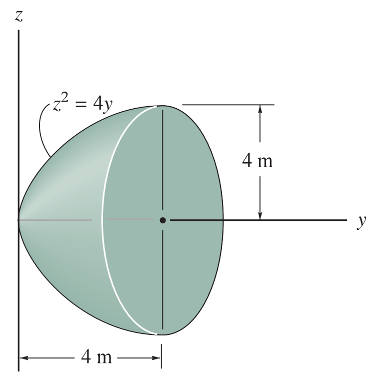
$$\bar{z} = \frac{\int_v \tilde{z}\,dV}{\int_v dV} = \frac{\int_0^{2\text{ m}} z(2\pi z dz)}{\int_0^{2\text{ m}} 2\pi z dz}$$

$$= \frac{2\pi\left(\frac{z^3}{3}\right)\Big|_0^{2\text{ m}}}{2\pi\left(\frac{z^2}{2}\right)\Big|_0^{2\text{ m}}} = \frac{4}{3}\text{ m} \qquad \textbf{Ans.}$$

Ans:
$\bar{x} = \bar{y} = 0$
$\bar{z} = \frac{4}{3}$ m

***9–40.**

Locate the centroid $\overline{y}$ of the paraboloid.



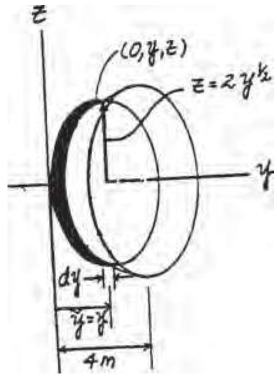
SOLUTION

Volume and Moment Arm: The volume of the thin disk differential element is $dV = \pi z^2 dy = \pi(4y)dy$ and its centroid $\widetilde{y} = y$.

Centroid: Applying Eq. 9–3 and performing the integration, we have

$$\overline{y} = \frac{\int_V \widetilde{y}\,dV}{\int_V dV} = \frac{\int_0^{4\text{ m}} y[\pi(4y)dy]}{\int_0^{4\text{ m}} \pi(4y)dy}$$

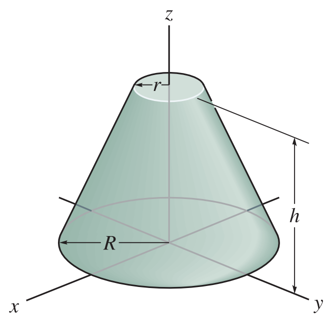
$$= \frac{4\pi\left(\frac{y^3}{3}\right)\Big|_0^{4\text{ m}}}{4\pi\left(\frac{y^2}{2}\right)\Big|_0^{4\text{ m}}} = 2.67\text{ m} \qquad \textbf{Ans.}$$



Ans:
$\overline{y} = 2.67$ m

9–41.

Locate the centroid $\bar{z}$ of the frustum of the right-circular cone.



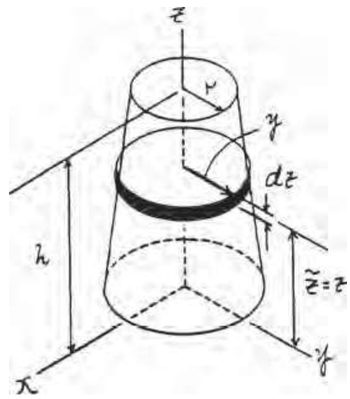
SOLUTION

Volume and Moment Arm: From the geometry, $\dfrac{y - r}{R - r} = \dfrac{h - z}{h}$, $y = \dfrac{(r - R)z + Rh}{h}$. The volume of the thin disk differential element is

$$dV = \pi y^2 dz = \pi\left[\left(\frac{(r - R)z + Rh}{h}\right)^2\right]dz$$

$$= \frac{\pi}{h^2}\left[(r - R)^2 z^2 + 2Rh(r - R)z + R^2h^2\right]dz$$

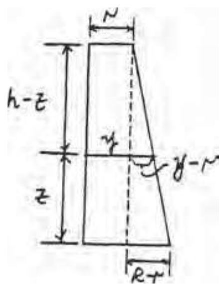
and its centroid $\tilde{z} = z$.



Centroid: Applying Eq. 9–5 and performing the integration, we have

$$\bar{z} = \frac{\int_V \tilde{z}\,dV}{\int_V dV} = \frac{\int_0^h z\left\{\frac{\pi}{h^2}[(r - R)^2z^2 + 2Rh(r - R)z + R^2h^2]dz\right\}}{\int_0^h \frac{\pi}{h^2}[r - R)^2z^2 + 2Rh(r - R)z + R^2h^2]dz}$$

$$= \frac{\frac{\pi}{h^2}\left[(r - R)^2\left(\frac{z^4}{4}\right) + 2Rh(r - R)\left(\frac{z^3}{3}\right) + R^2h^2\left(\frac{z^2}{2}\right)\right]\Bigg|_0^h}{\frac{\pi}{h^2}\left[(r - R)^2\left(\frac{z^3}{3}\right) + 2Rh(r - R)\left(\frac{z^2}{2}\right) + R^2h^2(z)\right]\Bigg|_0^h}$$

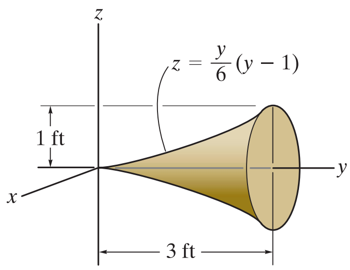


$$= \frac{R^2 + 3r^2 + 2rR}{4(R^2 + r^2 + rR)}h \qquad \textbf{Ans.}$$

Ans:
$\bar{z} = \dfrac{R^2 + 3r^2 + 2rR}{4(R^2 + r^2 + rR)}h$

9–42.

Determine the centroid $\overline{y}$ of the solid.



SOLUTION

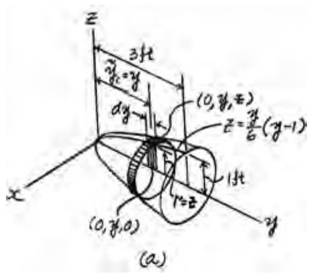
Differential Element: The thin disk element shown shaded in Fig. *a* will be considered. The volume of the element is

$$dV = \pi z^2\,dy = \pi\left[\frac{y}{6}(y-1)\right]^2 dy = \frac{\pi}{36}(y^4 - 2y^3 + y^2)dy$$

Centroid: The centroid of the element is located at $y_c = y$. We have

$$\overline{y} = \frac{\int_V \tilde{y}\,dV}{\int_V dV} = \frac{\int_0^{3\text{ ft}} y\left[\frac{\pi}{36}\left(y^4 - 2y^3 + y^2\right)dy\right]}{\int_0^{3\text{ ft}} \frac{\pi}{36}\left(y^4 - 2y^3 + y^2\right)dy} = \frac{\int_0^{3\text{ ft}} \frac{\pi}{36}\left(y^5 - 2y^4 + y^3\right)dy}{\int_0^{3\text{ ft}} \frac{\pi}{36}\left(y^4 - 2y^3 + y^2\right)dy} = \frac{\frac{\pi}{36}\left[\frac{y^6}{6} - \frac{2}{5}y^5 + \frac{y^4}{4}\right]\Bigg|_0^{3\text{ ft}}}{\frac{\pi}{36}\left[\frac{y^5}{5} - \frac{y^4}{2} + \frac{y^3}{3}\right]\Bigg|_0^{3\text{ ft}}}$$

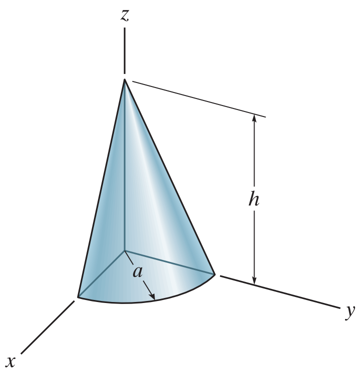
$= 2.61$ ft **Ans.**



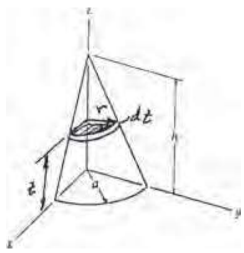
Ans:
$\overline{y} = 2.61$ ft

9–43.

Locate the centroid of the quarter-cone.



SOLUTION



$$\tilde{z} = z$$

$$r = \frac{a}{h}(h - z)$$

$$dV = \frac{\pi}{4} r^2\, dz = \frac{\pi\, a^2}{4\, h^2}(h - z)^2\, dz$$

$$\int dV = \frac{\pi\, a^2}{4\, h^2}\int_0^h (h^2 - 2hz + z^2)\, dz = \frac{\pi\, a^2}{4\, h^2}\left[h^2 z - hz^2 + \frac{z^3}{3}\right]_0^h$$

$$= \frac{\pi\, a^2}{4\, h^2}\left(\frac{h^3}{3}\right) = \frac{\pi a^2\, h}{12}$$

$$\int \tilde{z} dV = \frac{\pi\, a^2}{4\, h^2}\int_0^h (h^2 - 2hz + z^2)\, z\, dz = \frac{\pi\, a^2}{4\, h^2}\left[h^2\frac{z^2}{2} - 2h\frac{z^3}{3} + \frac{z^4}{4}\right]_0^h$$

$$= \frac{\pi\, a^2}{4\, h^2}\left(\frac{h^4}{12}\right) = \frac{\pi a^2 h^2}{48}$$

$$\bar{z} = \frac{\int \tilde{z} dV}{\int dV} = \frac{\frac{\pi\, a^2\, h^2}{48}}{\frac{\pi\, a^2 h}{12}} = \frac{h}{4}$$ **Ans.**

$$\int \tilde{x} dV = \frac{\pi a^2}{4\, h^2}\int_0^h \frac{4\, r}{3\pi}(h - z)^2\, dz = \frac{\pi a^2}{4\, h^2}\int_0^h \frac{4a}{3\,\pi\, h}(h^3 - 3h^2\, z + 3hz^2 - z^3)\, dz$$

$$= \frac{\pi\, a^2}{4\, h^2}\,\frac{4a}{3\pi h}\left(h^4 - \frac{3h^4}{2} + h^4 - \frac{h^4}{4}\right)$$

$$= \frac{\pi\, a^2}{4\, h^2}\left(\frac{a\, h^3}{3\pi}\right) = \frac{a^3\, h}{12}$$

$$\bar{x} = \bar{y} = \frac{\int \tilde{x} dV}{\int dV} = \frac{\frac{a^3\, h}{12}}{\frac{\pi\, a^2 h}{12}} = \frac{a}{\pi}$$ **Ans.**

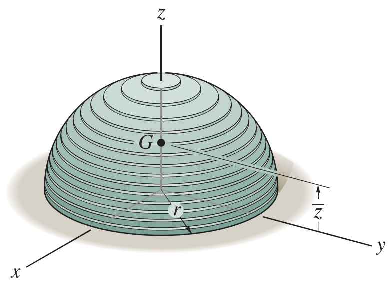
Ans:

$\bar{z} = \frac{h}{4}$

$\bar{x} = \bar{y} = \frac{a}{\pi}$

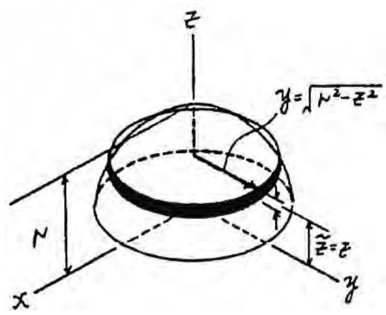
***9–44.**

The hemisphere of radius r is made from a stack of very thin plates such that the density varies with height $\rho = kz$, where k is a constant. Determine its mass and the distance to the center of mass G.



SOLUTION

Mass and Moment Arm: The density of the material is $\rho = kz$. The mass of the thin disk differential element is $dm = \rho dV = \rho\pi y^2 dz = kz[\pi(r^2 - z^2)\,dz]$ and its centroid $\tilde{z} = z$. Evaluating the integrals, we have



$$m = \int_m dm = \int_0^r kz[\pi(r^2 - z^2)\,dz]$$

$$= \pi k\left(\frac{r^2z^2}{2} - \frac{z^4}{4}\right)\Big|_0^r = \frac{\pi k r^4}{4} \qquad \textbf{Ans.}$$

$$\int_m \tilde{z}dm = \int_0^r z\{kz[\pi(r^2 - z^2)\,dz]\}$$

$$= \pi k\left(\frac{r^2z^3}{3} - \frac{z^5}{5}\right)\Big|_0^r = \frac{2\pi k r^5}{15}$$

Centroid: Applying Eq. 9–3, we have

$$\bar{z} = \frac{\int_m \tilde{z}dm}{\int_m dm} = \frac{2\pi k r^5/15}{\pi k r^4/4} = \frac{8}{15}r \qquad \textbf{Ans.}$$

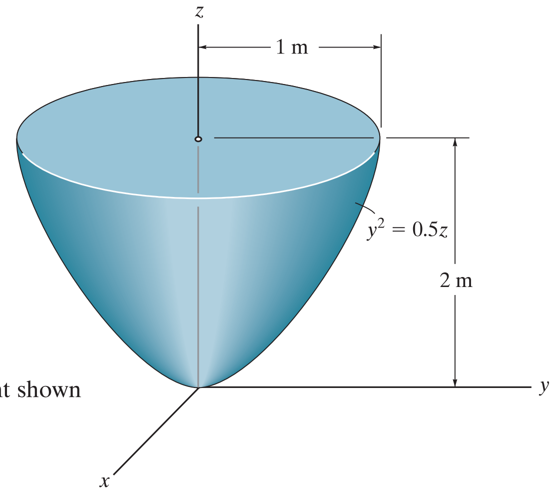
Ans:

$m = \dfrac{\pi k r^4}{4}$

$\bar{z} = \dfrac{8}{15}r$

9–45.

Locate the centroid $\bar{z}$ of the volume.



SOLUTION

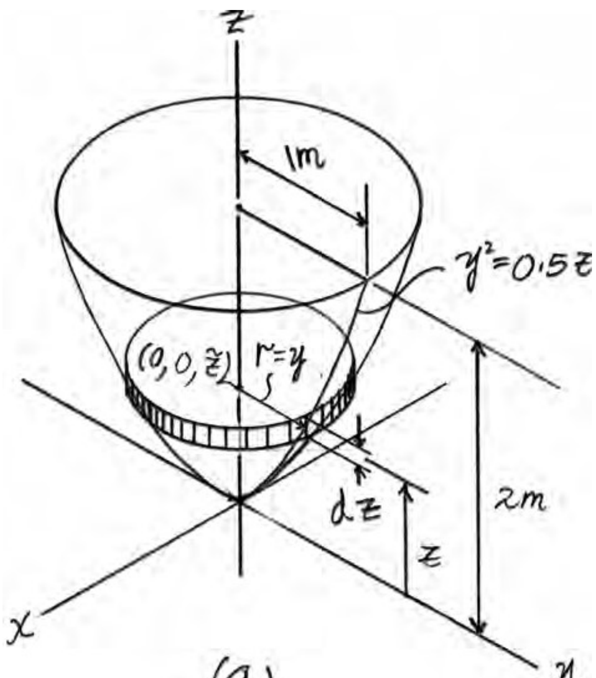
Volume And Moment Arm. The volume of the thin disk differential element shown shaded in Fig. *a* is $dV = \pi y^2\,dz = \pi(0.5z)dz$ and its centroid is at $\tilde{z} = z$.

Centroid. Perform the integration

$$\bar{z} = \frac{\int_V \tilde{z}\,dV}{\int_V dV} = \frac{\int_0^{2\text{ m}} z[\pi(0.5z)dz]}{\int_0^{2\text{ m}} \pi(0.5z)dz}$$

$$= \frac{\left.\frac{0.5\pi}{3} z^3\right|_0^{2\text{ m}}}{\left.\frac{0.5\pi}{2} z^2\right|_0^{2\text{ m}}}$$

$$= \frac{4}{3}\text{ m}$$ **Ans.**

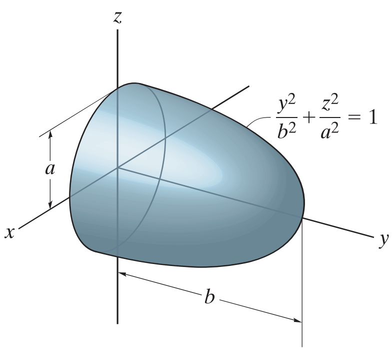


(a)

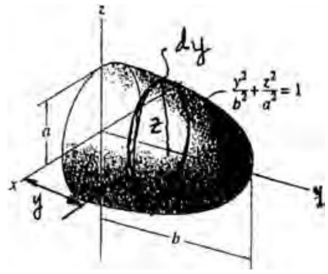
Ans:
$\bar{z} = \frac{4}{3}$ m

9–46.

Locate the centroid of the ellipsoid of revolution.



SOLUTION



$dV = \pi z^2 \, dy$

$$\int dV = \int_0^b \pi a^2 \left(1 - \frac{y^2}{b^2}\right) dy = \pi a^2 \left[y - \frac{y^3}{3b^2} \right]_0^b = \frac{2\pi a^2 b}{3}$$

$$\int \tilde{y} dV = \int_0^b \pi a^2 y \left(1 - \frac{y^2}{b^2}\right) dy = \pi a^2 \left[\frac{y^2}{2} - \frac{y^4}{4b^2} \right]_0^b = \frac{\pi a^2 b^2}{4}$$

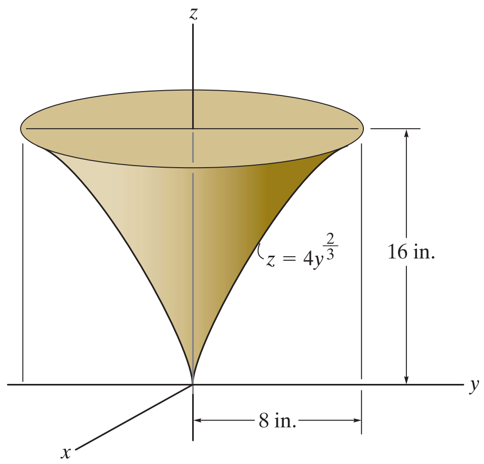
$$\bar{y} = \frac{\int_V \tilde{y} dV}{\int_V dV} = \frac{\frac{\pi a^2 b^2}{4}}{\frac{2\pi a^2 b}{3}} = \frac{3}{8} b$$ **Ans.**

$\bar{x} = \bar{z} = 0$ (By symmetry) **Ans.**

Ans:
$\bar{y} = \frac{3}{8} b$
$\bar{x} = \bar{z} = 0$

9–47.

Locate the center of gravity $\bar{z}$ of the solid.



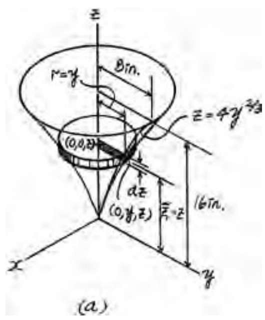
SOLUTION

Differential Element: The thin disk element shown shaded in Fig. *a* will be considered. The volume of the element is

$$dV = \pi y^2\, dz = \pi\left[\frac{1}{8}z^{3/2}\right]^2 dz = \frac{\pi}{64}z^3\, dz$$

Centroid: The centroid of the element is located at $z_c = z$. We have

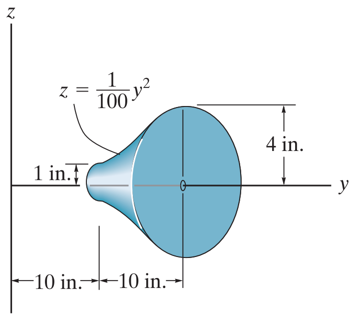
$$\bar{z} = \frac{\int_V \tilde{z}\, dV}{\int_V dV} = \frac{\int_0^{16\text{ in.}} z\left[\frac{\pi}{64}z^3\, dz\right]}{\int_0^{16\text{ in.}} \frac{\pi}{64}z^3\, dz} = \frac{\int_0^{16\text{ in.}} \frac{\pi}{64}z^4\, dz}{\int_0^{16\text{ in.}} \frac{\pi}{64}z^3\, dz} = \frac{\frac{\pi}{64}\left(\frac{z^5}{5}\right)\Big|_0^{16\text{ in.}}}{\frac{\pi}{64}\left(\frac{z^4}{4}\right)\Big|_0^{16\text{ in.}}} = 12.8 \text{ in.} \quad \textbf{Ans.}$$



Ans:
$\bar{z} = 12.8$ in.

***9–48.**

Locate the center of gravity $\bar{y}$ of the volume. The material is homogeneous.



SOLUTION

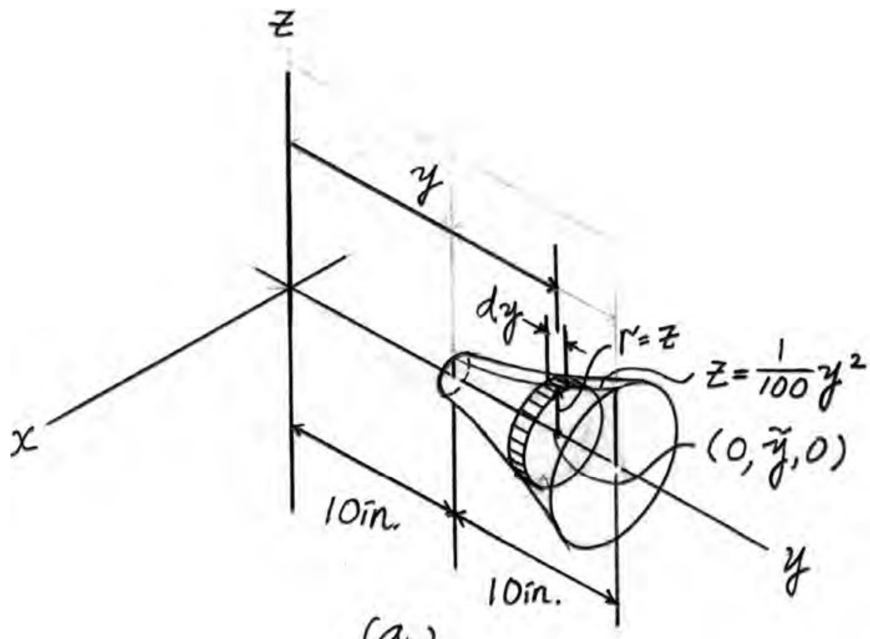
Volume And Moment Arm. The volume of the thin disk differential element shown shaded in Fig. *a* is $dV = \pi z^2\,dy = \pi\left(\frac{1}{100}y^2\right)^2 dy = \frac{\pi}{10000}y^4\,dy$ and its centroid is at $\tilde{y} = y$.

Centroid. Perform the integration

$$\bar{y} = \frac{\int_V \tilde{y}\,dV}{\int_V dV} = \frac{\int_{10\text{ in.}}^{20\text{ in.}} y\left(\frac{\pi}{10000}y^4\,dy\right)}{\int_{10\text{ in.}}^{20\text{ in.}} \frac{\pi}{10000}y^4\,dy}$$

$$= \frac{\left(\frac{\pi}{60000}y^6\right)\Big|_{10\text{ in.}}^{20\text{ in.}}}{\left(\frac{\pi}{50000}y^5\right)\Big|_{10\text{ in.}}^{20\text{ in.}}}$$

$$= 16.94\text{ in.} = 16.9\text{ in.}$$ **Ans.**

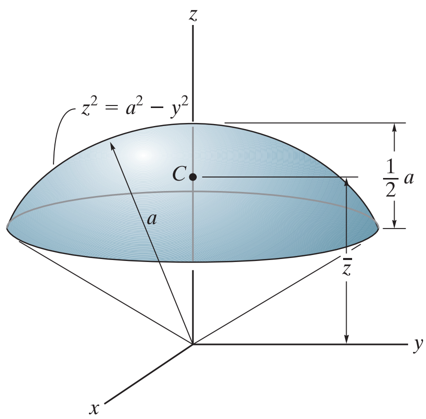


(a)

Ans:
$\bar{y} = 16.9$ in.

9–49.

Locate the centroid $\bar{z}$ of the spherical segment.



SOLUTION

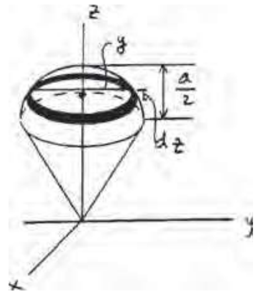
$dV = \pi y^2 dz = \pi(a^2 - z^2)\,dz$

$\tilde{z} = z$

$$\bar{z} = \frac{\int_V \tilde{z}\,dV}{\int_V dV} = \frac{\pi \int_{\frac{a}{2}}^{a} z\,(a^2 - z^2)dz}{\pi \int_{\frac{a}{2}}^{a} (a^2 - z^2)dz}$$

$$= \frac{\pi\left[a^2\left(\frac{z^2}{2}\right) - \left(\frac{z^4}{4}\right)\right]_{\frac{a}{2}}^{a}}{\pi\left[a^2(z) - \left(\frac{z^3}{3}\right)\right]_{\frac{a}{2}}^{a}} = \frac{\pi\left[\frac{a^4}{2} - \frac{a^4}{4} - \frac{a^4}{8} + \frac{a^4}{64}\right]}{\pi\left[a^3 - \frac{a^3}{3} - \frac{a^3}{2} + \frac{a^3}{24}\right]} = \frac{\pi\left[\frac{9a^4}{64}\right]}{\pi\left[\frac{5a^3}{24}\right]}$$

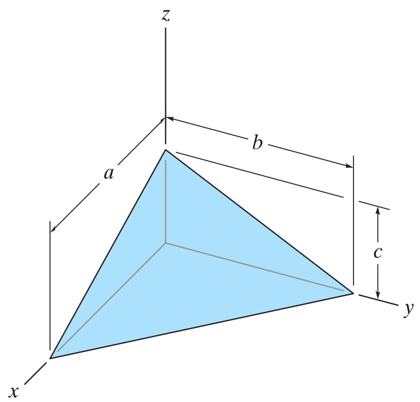
$\bar{z} = 0.675\,a$ **Ans.**



Ans:
$\bar{z} = 0.675a$

9–50.

Determine the location $\bar{z}$ of the centroid for the tetrahedron. *Hint:* Use a triangular "plate" element parallel to the x–y plane and of thickness dz.



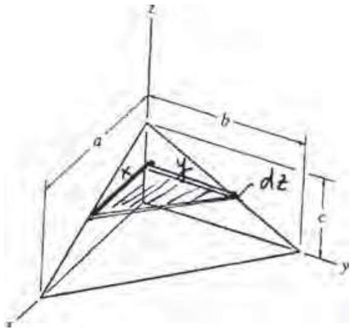
SOLUTION

$$z = c\left(1 - \frac{1}{b}y\right) = c\left(1 - \frac{1}{a}x\right)$$

$$\int dV = \int_0^c \frac{1}{2}(x)(y)dz = \frac{1}{2}\int_0^c a\left(1 - \frac{z}{c}\right)b\left(1 - \frac{z}{c}\right)dz = \frac{abc}{6}$$

$$\int \tilde{z}dV = \frac{1}{2}\int_0^c z\,a\left(1 - \frac{z}{c}\right)b\left(1 - \frac{z}{c}\right)dz = \frac{abc^2}{24}$$

$$\bar{z} = \frac{\int \tilde{z}dV}{\int dV} = \frac{\frac{abc^2}{24}}{\frac{abc}{6}} = \frac{c}{4}$$ **Ans.**

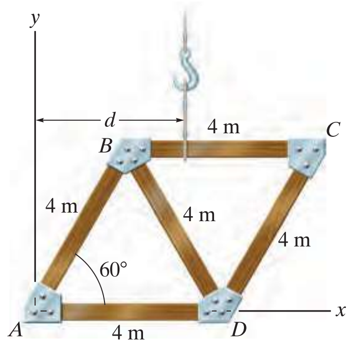


Ans:

$\bar{z} = \frac{c}{4}$

9–51.

The truss is made from five members, each having a length of 4 m and a mass of 7 kg/m. If the mass of the gusset plates at the joints and the thickness of the members can be neglected, determine the distance d to where the hoisting cable must be attached, so that the truss does not tip (rotate) when it is lifted.



SOLUTION

$$\Sigma \tilde{x} M = 4(7)(1+4+2+3+5) = 420 \text{ kg} \cdot \text{m}$$

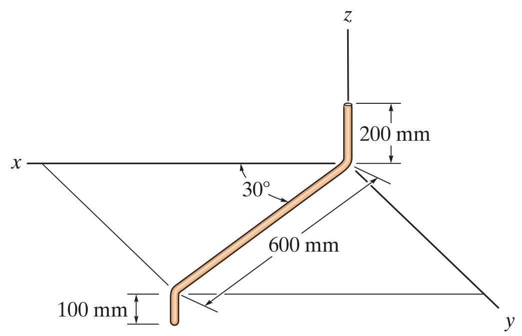
$$\Sigma M = 4(7)(5) = 140 \text{ kg}$$

$$d = \bar{x} = \frac{\Sigma \tilde{x} M}{\Sigma M} = \frac{420}{140} = 3 \text{ m}$$ **Ans.**

Ans:
$d = 3$ m

***9–52.**

Determine the location $(\bar{x}, \bar{y}, \bar{z})$ of the centroid of the homogeneous rod.



SOLUTION

Centroid. Referring to Fig. *a*, the length of the segments and the locations of their respective centroids are tabulated below

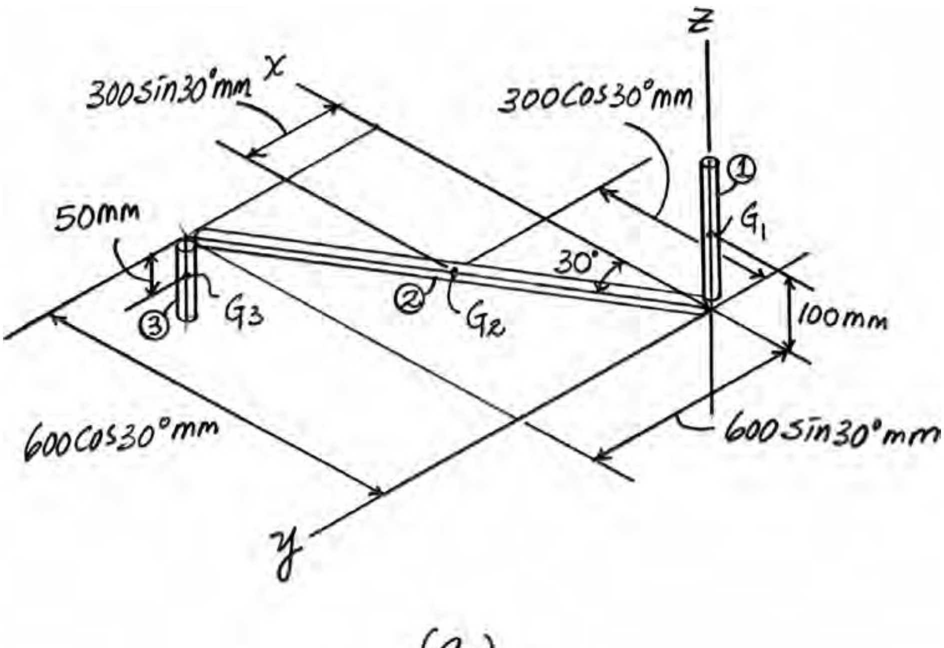
Segment	L(mm)	$\tilde{x}$(mm)	$\tilde{y}$(mm)	$\tilde{z}$(mm)	$\tilde{x}L(\text{mm}^2)$	$\tilde{y}L(\text{mm}^2)$	$\tilde{z}L(\text{mm}^2)$
1	200	0	0	100	0	0	$20.0(10^3)$
2	600	300 cos 30°	300 sin 30°	0	$155.88(10^3)$	$90.0(10^3)$	0
3	100	600 cos 30°	600 sin 30°	–50	$51.96(10^3)$	$30.0(10^3)$	$-5.0(10^3)$
Σ	900				$207.85(10^3)$	$120.0(10^3)$	$15.0(10^3)$

Thus,

$$\bar{x} = \frac{\Sigma \tilde{x}L}{\Sigma L} = \frac{207.85(10^3)\text{mm}^2}{900\text{ mm}} = 230.94\text{ mm} = 231\text{ mm} \qquad \textbf{Ans.}$$

$$\bar{y} = \frac{\Sigma \tilde{y}L}{\Sigma L} = \frac{120.0(10^3)\text{mm}^2}{900\text{ mm}} = 133.33\text{ mm} = 133\text{ mm} \qquad \textbf{Ans.}$$

$$\bar{z} = \frac{\Sigma \tilde{z}L}{\Sigma L} = \frac{15.0(10^3)\text{mm}^2}{900\text{ mm}^2} = 16.67\text{ mm} = 16.7\text{ mm} \qquad \textbf{Ans.}$$

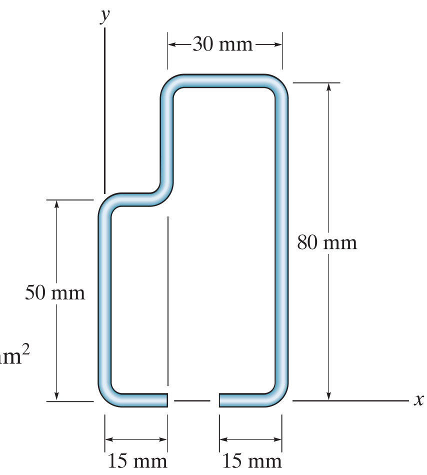


(a)

Ans:
$\bar{x} = 231$ mm
$\bar{y} = 133$ mm
$\bar{z} = 16.7$ mm

9–53.

A rack is made from roll-formed sheet steel and has the cross section shown. Determine the location $(\bar{x}, \bar{y})$ of the centroid of the cross section. The dimensions are indicated at the center thickness of each segment.



SOLUTION

$$\Sigma L = 15 + 50 + 15 + 30 + 30 + 80 + 15 = 235 \text{ mm}$$

$$\Sigma \tilde{x} L = 7.5(15) + 0(50) + 7.5(15) + 15(30) + 30(30) + 45(80) + 37.5(15) = 5737.50 \text{ mm}^2$$

$$\Sigma \tilde{y} L = 0(15) + 25(50) + 50(15) + 65(30) + 80(30) + 40(80) + 0(15) = 9550 \text{ mm}^2$$

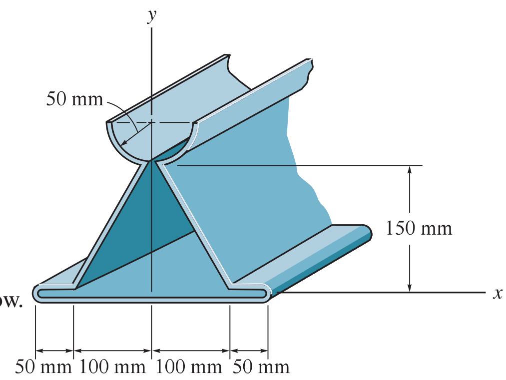
$$\bar{x} = \frac{\Sigma \tilde{x} L}{\Sigma L} = \frac{5737.50}{235} = 24.4 \text{ mm}$$ **Ans.**

$$\bar{y} = \frac{\Sigma \tilde{y} L}{\Sigma L} = \frac{9550}{235} = 40.6 \text{ mm}$$ **Ans.**

Ans:
$\bar{x} = 24.4$ mm
$\bar{y} = 40.6$ mm

9–54.

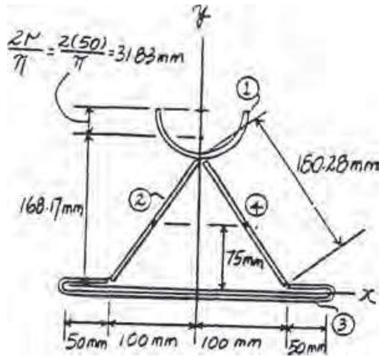
Locate the centroid $(\bar{x}, \bar{y})$ of the metal cross section. Neglect the thickness of the material and slight bends at the corners.



SOLUTION

Centroid: The length of each segment and its respective centroid are tabulated below.

Segment	L (mm)	$\tilde{y}$ (mm)	$\tilde{y}L$ (mm^2)
1	50π	168.17	26415.93
2	180.28	75	13520.82
3	400	0	0
4	180.28	75	13520.82
Σ	917.63		53457.56



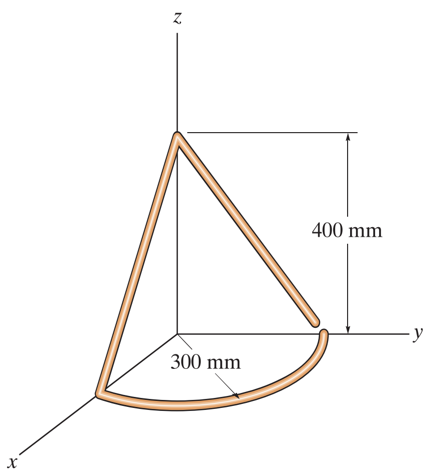
Due to symmetry about y axis, $\bar{x} = 0$ **Ans.**

$$\bar{y} = \frac{\Sigma \tilde{y}L}{\Sigma L} = \frac{53457.56}{917.63} = 58.26 \text{ mm} = 58.3 \text{ mm}$$ **Ans.**

Ans:
$\bar{x} = 0$
$\bar{y} = 58.3$ mm

9–55.

Locate the center of gravity $(\bar{x}, \bar{y}, \bar{z})$ of the homogeneous wire.



SOLUTION

$$\Sigma \tilde{x}L = 150(500) + 0(500) + \frac{2\,(300)}{\pi}\left(\frac{\pi}{2}\right)(300) = 165\,000\text{ mm}^2$$

$$\Sigma L = 500 + 500 + \left(\frac{\pi}{2}\right)(300) = 1471.24\text{ mm}$$

$$\bar{x} = \frac{\Sigma \tilde{x}L}{\Sigma L} = \frac{165\,000}{1471.24} = 112\text{ mm}$$ **Ans.**

Due to symmetry,

$\bar{y} = 112\text{ mm}$ **Ans.**

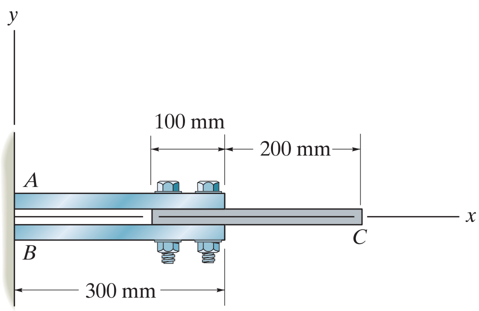
$$\Sigma \tilde{z}L = 200(500) + 200(500) + 0\left(\frac{\pi}{2}\right)(300) = 200\,000\text{ mm}^2$$

$$\bar{z} = \frac{\Sigma \tilde{z}L}{\Sigma L} = \frac{200\,000}{1471.24} = 136\text{ mm}$$ **Ans.**

Ans:
$\bar{x} = 112\text{ mm}$
$\bar{y} = 112\text{ mm}$
$\bar{z} = 136\text{ mm}$

***9–56.**

The steel and aluminum plate assembly is bolted together and fastened to the wall. Each plate has a constant width in the z direction of 200 mm and thickness of 20 mm. If the density of A and B is $\rho_s = 7.85\ \text{Mg/m}^3$, and for C, $\rho_{al} = 2.71\ \text{Mg/m}^3$, determine the location $\bar{x}$ of the center of mass. Neglect the size of the bolts.



SOLUTION

$$\Sigma m = 2\left[7.85(10)^3(0.3)(0.2)(0.02)\right] + 2.71(10)^3(0.3)(0.2)(0.02) = 22.092\text{ kg}$$

$$\Sigma \tilde{x} m = 150\{2\left[7.85(10)^3(0.3)(0.2)(0.02)\right]\} + 350\left[2.71(10)^3(0.3)(0.2)(0.02)\right]$$

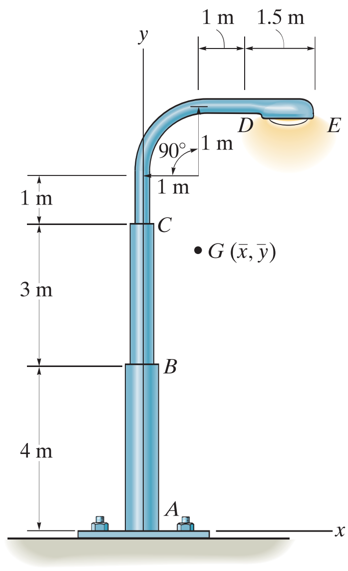
$$= 3964.2\text{ kg.mm}$$

$$\bar{x} = \frac{\Sigma \tilde{x} m}{\Sigma m} = \frac{3964.2}{22.092} = 179\text{ mm}$$ **Ans.**

Ans:
$\bar{x} = 179\text{ mm}$

9–57.

Locate the center of gravity $G(\bar{x}, \bar{y})$ of the streetlight. Neglect the thickness of each segment. The mass per unit length of each segment is as follows: $\rho_{AB} = 12$ kg/m, $\rho_{BC} = 8$ kg/m, $\rho_{CD} = 5$ kg/m, and $\rho_{DE} = 2$ kg/m.



SOLUTION

$$\Sigma\tilde{x}m = 0(4)(12) + 0(3)(8) + 0(1)(5) + \left(1 - \frac{2(1)}{\pi}\right)\left(\frac{\pi}{2}\right)(5)$$

$$+ 1.5\,(1)\,(5) + 2.75\,(1.5)\,(2) = 18.604 \text{ kg} \cdot \text{m}$$

$$\Sigma m = 4\,(12) + 3\,(8) + 1(5) + \frac{\pi}{2}\,(5) + 1(5) + 1.5\,(2) = 92.854 \text{ kg}$$

$$\bar{x} = \frac{\Sigma\tilde{x}m}{\Sigma m} = \frac{18.604}{92.854} = 0.200 \text{ m}$$ **Ans.**

$$\Sigma\tilde{y}m = 2\,(4)\,(12) + 5.5\,(3)(8) + 7.5(1)\,(5) + \left(8 + \frac{2(1)}{\pi}\right)\left(\frac{\pi}{2}\right)(5)$$

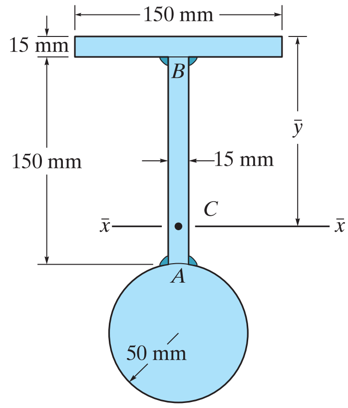
$$+ 9\,(1)\,(5) + 9(1.5)\,(2) = 405.332 \text{ kg} \cdot \text{m}$$

$$\bar{y} = \frac{\Sigma\tilde{y}m}{\Sigma m} = \frac{405.332}{92.854} = 4.37 \text{ m}$$ **Ans.**

Ans:
$\bar{x} = 0.200$ m
$\bar{y} = 4.37$ m

9–58.

Determine the location $\bar{y}$ of the centroidal axis $\bar{x}-\bar{x}$ of the beam's cross-sectional area. Neglect the size of the corner welds at A and B for the calculation.



SOLUTION

$$\Sigma \tilde{y}A = 7.5(15)(150) + 90(150)(15) + 215(\pi)(50)^2$$

$$= 1\,907\,981.05 \text{ mm}^2$$

$$\Sigma A = 15(150) + 150(15) + \pi(50)^2$$

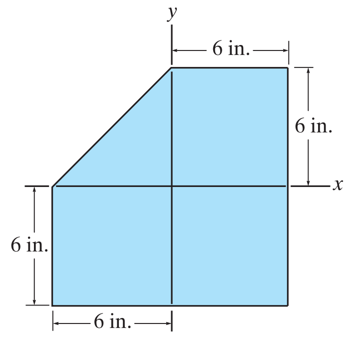
$$= 12\,353.98 \text{ mm}^2$$

$$\bar{y} = \frac{\Sigma \tilde{y}A}{\Sigma A} = \frac{1\,907\,981.05}{12\,353.98} = 154 \text{ mm}$$ **Ans.**

Ans:
$\bar{y} = 154$ mm

9–59.

Locate the centroid ($\bar{x}$, $\bar{y}$) of the shaded area.



SOLUTION

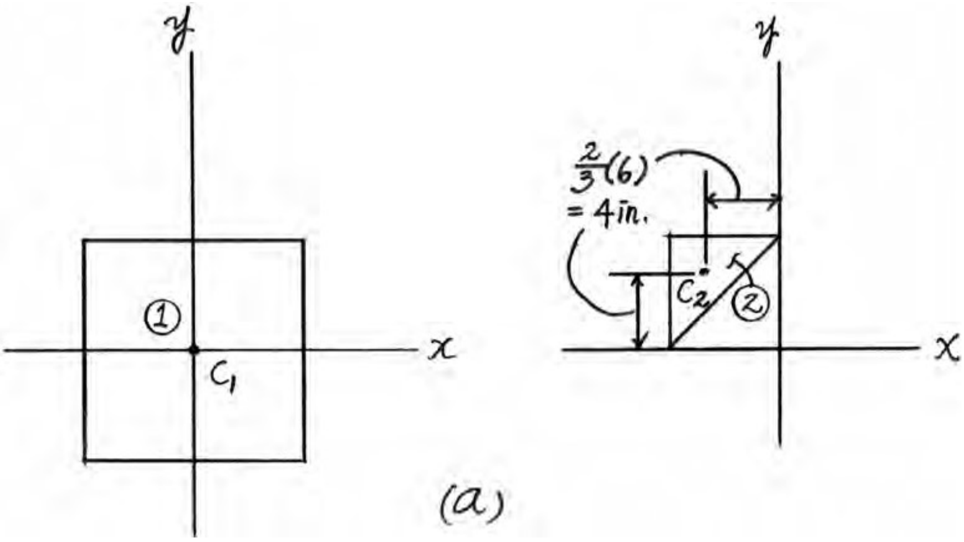
Centroid. Referring to Fiq. *a*, the areas of the segments and the locations of their respective centroids are tabulated below

Segment	A (in.^{2})	$\tilde{x}$ (in.)	$\tilde{y}$ (in.)	$\tilde{x}A$ (in.^{3})	$\tilde{y}A$ (in.^{3})
1	12(12)	0	0	0	0
2	$-\frac{1}{2}(6)(6)$	-4	4	72.0	-72.0
Σ	126			72.0	-72.0

Thus,

$$\bar{x} = \frac{\Sigma\tilde{x}A}{\Sigma A} = \frac{72.0 \text{ in.}^3}{126 \text{ in.}^2} = 0.5714 \text{ in.} = 0.571 \text{ in.} \qquad \textbf{Ans.}$$

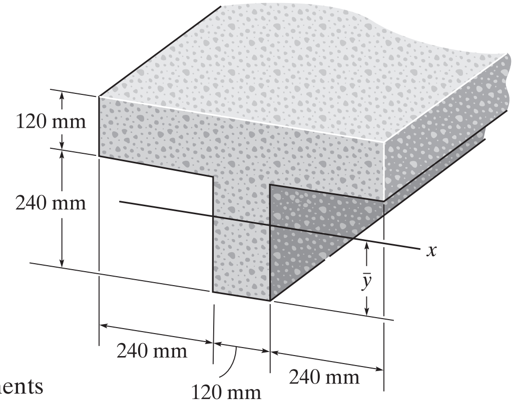
$$\bar{y} = \frac{\Sigma\tilde{y}A}{\Sigma A} = \frac{-72.0 \text{ in.}^3}{126 \text{ in.}^2} = -0.5714 \text{ in.} = -0.571 \text{ in.} \qquad \textbf{Ans.}$$



Ans:
$\bar{x} = 0.571$ in.
$\bar{y} = -0.571$ in.

***9–60.**

Locate the centroid $\bar{y}$ for the beam's cross-sectional area.

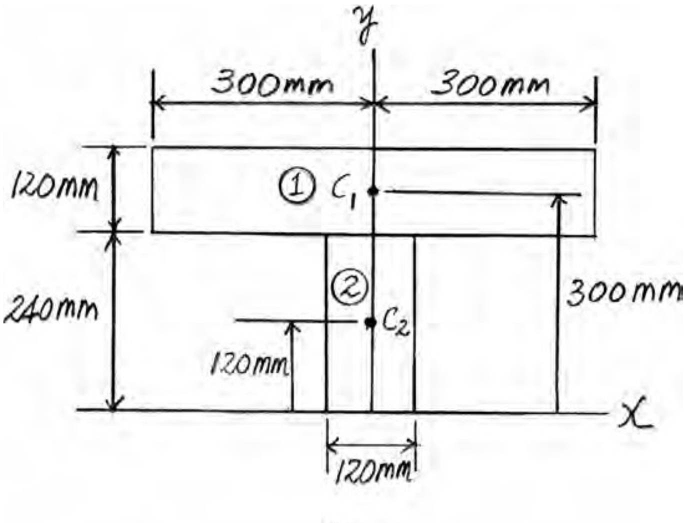


SOLUTION

Centroid. The locations of the centroids measuring from the x axis for segments ①and②are indicated in Fig. a. Thus

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{300(120)(600) + 120(240)(120)}{120(600) + 240(120)}$$

$$= 248.57 \text{ mm} = 249 \text{ mm}$$

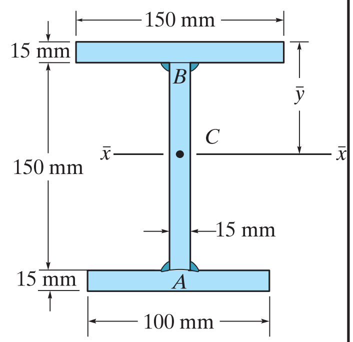


(a)

Ans:
$\bar{y} = 249$ mm

9–61.

Determine the location $\bar{y}$ of the centroid C of the beam having the cross-sectional area shown.

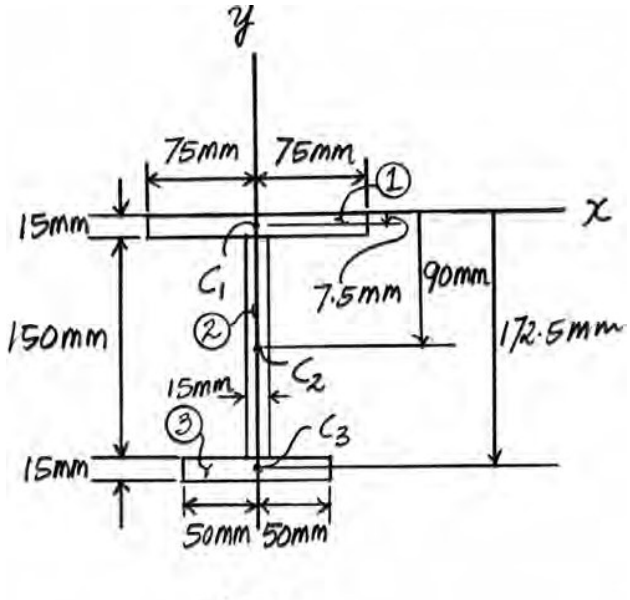


SOLUTION

Centroid. The locations of the centroids measuring from the x axis for segments ①, ② and ③ are indicated in Fig. a. Thus

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{7.5(15)(150) + 90(150)(15) + 172.5(15)(100)}{15(150) + 150(15) + 15(100)}$$

$$= 79.6875 \text{ mm} = 79.7 \text{ mm} \qquad \textbf{Ans.}$$

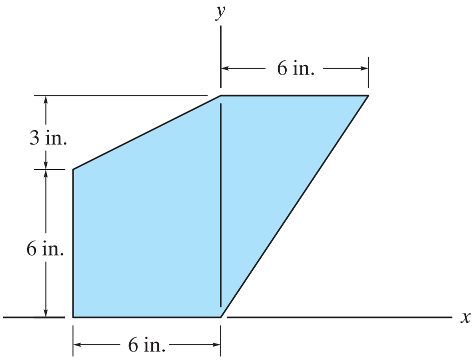


(a)

Ans:
$\bar{y} = 79.7$ mm

9–62.

Locate the centroid $(\bar{x}, \bar{y})$ of the shaded area.



SOLUTION

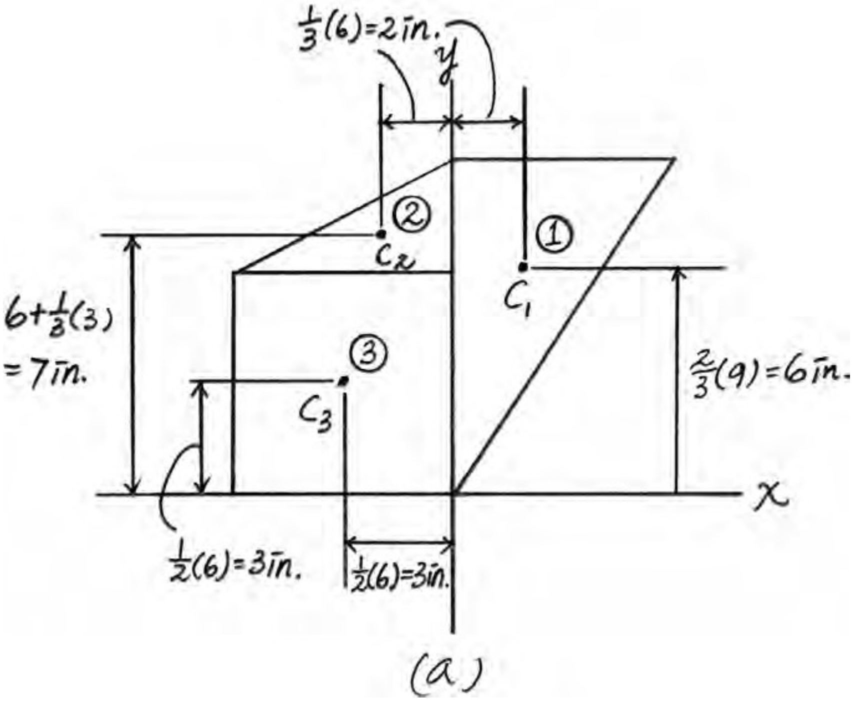
Centroid. Referring to Fig. *a*, the areas of the segments and the locations of their respective centroids are tabulated below

Segment	A (in.^{2})	$\tilde{x}$ (in.)	$\tilde{y}$ (in.)	$\tilde{x}A$ (in.^{3})	$\tilde{y}A$ (in.^{3})
1	$\frac{1}{2}(6)(9)$	2	6	54.0	162.0
2	$\frac{1}{2}(6)(3)$	−2	7	−18.0	63.0
3	6(6)	−3	3	−108.0	108.00
Σ	72.0			−72.0	333.0

Thus,

$$\bar{x} = \frac{\Sigma \tilde{x}A}{\Sigma A} = \frac{-72.0 \text{ in.}^3}{72.0 \text{ in.}^2} = -1.00 \text{ in.} \qquad \textbf{Ans.}$$

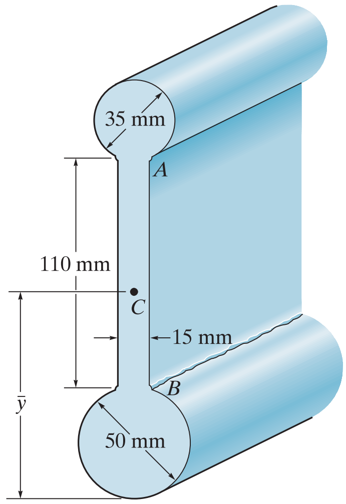
$$\bar{y} = \frac{\Sigma \tilde{y}A}{\Sigma A} = \frac{333.0 \text{ in.}^3}{72.0 \text{ in.}^2} = 4.625 \text{ in.} \qquad \textbf{Ans.}$$



Ans:
$\bar{x} = -1.00$ in.
$\bar{y} = 4.625$ in.

9–63.

Determine the location $\bar{y}$ of the centroid of the beam's cross-sectional area. Neglect the size of the corner welds at A and B for the calculation.



SOLUTION

$$\Sigma \tilde{y}A = \pi(25)^2(25) + 15(110)(50 + 55) + \pi\left(\frac{35}{2}\right)^2\left(50 + 110 + \frac{35}{2}\right) = 393\ 112\ \text{mm}^3$$

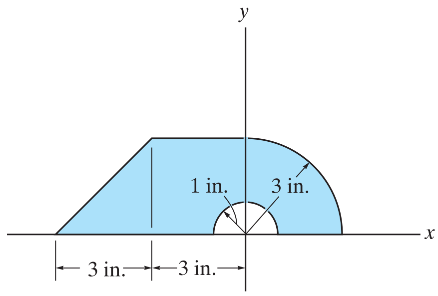
$$\Sigma A = \pi(25)^2 + 15(110) + \pi\left(\frac{35}{2}\right)^2 = 4575.6\ \text{mm}^2$$

$$\bar{y} = \frac{\Sigma \tilde{y}A}{\Sigma A} = \frac{393\ 112}{4575.6} = 85.9\ \text{mm}$$ **Ans.**

Ans:
$\bar{y} = 85.9$ mm

***9–64.**

Locate the centroid $(\bar{x}, \bar{y})$ of the shaded area.



SOLUTION

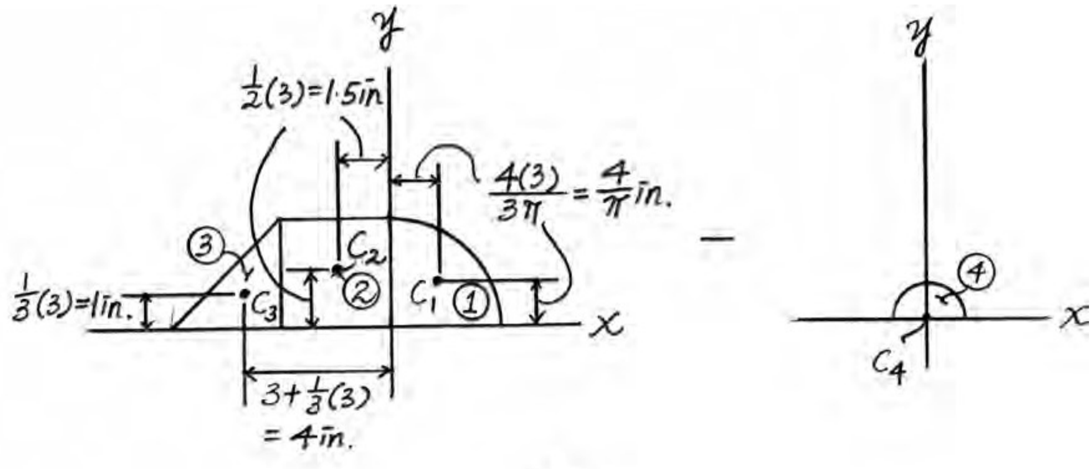
Centroid. Referring to Fig. *a*, the areas of the segments and the locations of their respective centroids are tabulated below

Segment	A (in.^{2})	$\tilde{x}$ (in.)	$\tilde{y}$ (in.)	$\tilde{x}A$ (in.^{3})	$\tilde{y}A$ (in.^{3})
1	$\frac{\pi}{4}(3^2)$	$\frac{4}{\pi}$	$\frac{4}{\pi}$	9.00	9.00
2	3(3)	−1.5	1.5	−13.50	13.50
3	$\frac{1}{2}(3)(3)$	−4	1	−18.00	4.50
4	$-\frac{\pi}{2}(1^2)$	0	$\frac{4}{3\pi}$	0	−0.67
Σ	18.9978			−22.50	26.33

Thus,

$$\bar{x} = \frac{\Sigma\tilde{x}A}{\Sigma A} = \frac{-22.50 \text{ in.}^3}{18.9978 \text{ in.}^2} = -1.1843 \text{ in.} = -1.18 \text{ in.} \qquad \textbf{Ans.}$$

$$\bar{y} = \frac{\Sigma\tilde{y}A}{\Sigma A} = \frac{26.33 \text{ in.}^3}{18.9978 \text{ in.}^2} = 1.3861 \text{ in.} = 1.39 \text{ in.} \qquad \textbf{Ans.}$$

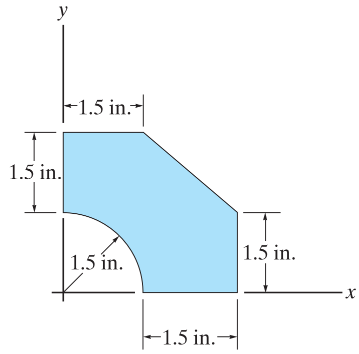


(a)

Ans:
$\bar{x} = -1.18$ in.
$\bar{y} = 1.39$ in.

9–65.

Determine the location $(\bar{x}, \bar{y})$ of the centroid C of the area.



SOLUTION

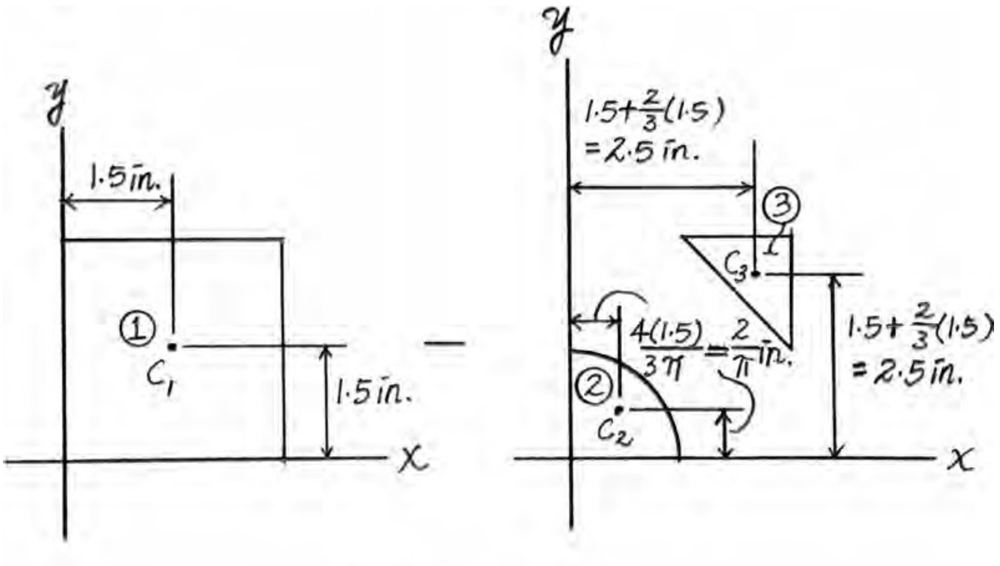
Centroid. Referring to Fig. *a*, the areas of the segments and the locations of their respective centroids are tabulated below.

Segment	A(in.^{2})	$\tilde{x}$(in.)	$\tilde{y}$(in.)	$\tilde{x}A$(in.^{3})	$\tilde{y}A$(in.^{3})
1	3(3)	1.5	1.5	13.5	13.5
2	$-\frac{\pi}{4}(1.5^2)$	$\frac{2}{\pi}$	$\frac{2}{\pi}$	−1.125	−1.125
3	$-\frac{1}{2}(1.5)(1.5)$	2.5	2.5	−2.8125	−2.8125
Σ	6.1079			9.5625	9.5625

Thus

$$\bar{x} = \frac{\Sigma \tilde{x}A}{\Sigma A} = \frac{9.5625 \text{ in.}^3}{6.1079 \text{ in.}^2} = 1.5656 \text{ in.} = 1.57 \text{ in.} \qquad \textbf{Ans.}$$

$$\bar{y} = \frac{\Sigma \tilde{y}A}{\Sigma A} = \frac{9.5625 \text{ in.}^3}{6.1079 \text{ in.}^2} = 1.5656 \text{ in.} = 1.57 \text{ in.} \qquad \textbf{Ans.}$$

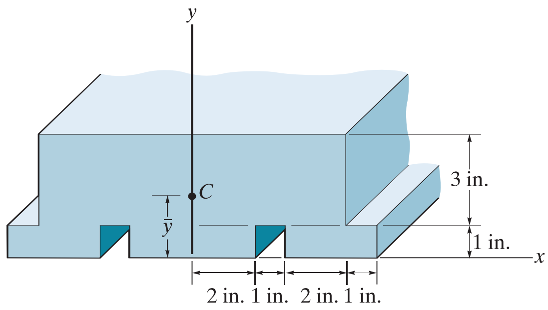


(a)

Ans:
$\bar{x} = 1.57$ in.
$\bar{y} = 1.57$ in.

9–66.

Determine the location $\overline{y}$ of the centroid C for a beam having the cross-sectional area shown. The beam is symmetric with respect to the y axis.



SOLUTION

$$\Sigma \tilde{y}A = 6(4)(2) - 1(1)(0.5) - 3(1)(2.5) = 40 \text{ in}^3$$

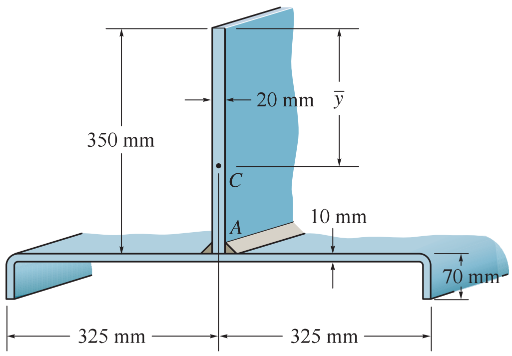
$$\Sigma A = 6(4) - 1(1) - 3(1) = 20 \text{ in}^2$$

$$\overline{y} = \frac{\Sigma \tilde{y}A}{\Sigma A} = \frac{40}{20} = 2 \text{ in.} \qquad \textbf{Ans.}$$

Ans:
$\overline{y} = 2$ in.

9–67.

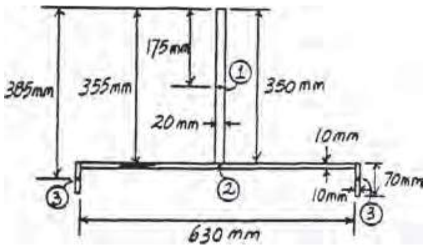
Locate the centroid $\bar{y}$ of the cross-sectional area of the beam constructed from a channel and a plate. Assume all corners are square and neglect the size of the weld at A.



SOLUTION

Centroid: The area of each segment and its respective centroid are tabulated below.

Segment	A (mm^2)	$\tilde{y}$ (mm)	$\tilde{y}A$ (mm^3)
1	350(20)	175	1 225 000
2	630(10)	355	2 236 500
3	70(20)	385	539 000
Σ	14 700		4 000 500



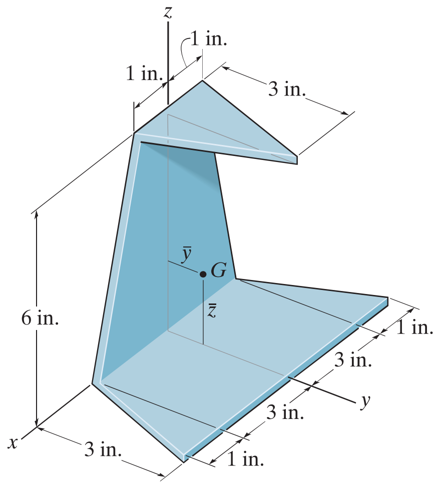
Thus,

$$\tilde{y} = \frac{\Sigma\tilde{y}A}{\Sigma A} = \frac{4\,000\,500}{14\,700} = 272.14 \text{ mm} = 272 \text{ mm}$$ **Ans.**

Ans:
$\bar{y} = 272\,\text{mm}$

***9–68.**

A triangular plate made of homogeneous material has a constant thickness which is very small. If it is folded over as shown, determine the location $\bar{y}$ of the plate's center of gravity G.



SOLUTION

$$\Sigma A = \frac{1}{2}(8)(12) = 48 \text{ in}^2$$

$$\Sigma \tilde{y}A = 2(1)\left(\frac{1}{2}\right)(1)(3) + 1.5(6)(3) + 2(2)\left(\frac{1}{2}\right)(1)(3)$$

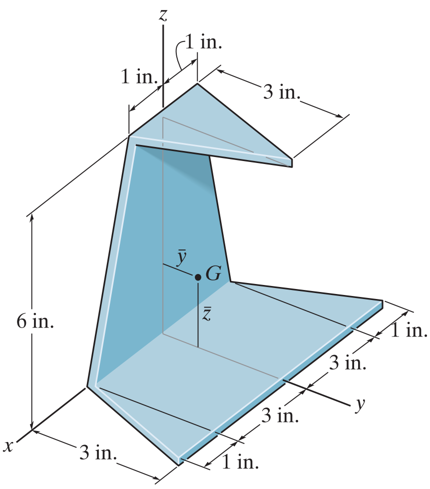
$$= 36 \text{ in}^3$$

$$\bar{y} = \frac{\Sigma \tilde{y}A}{\Sigma A} = \frac{36}{48} = 0.75 \text{ in.}$$ **Ans.**

Ans:
$\bar{y} = 0.75$ in.

9–69.

A triangular plate made of homogeneous material has a constant thickness which is very small. If it is folded over as shown, determine the location $\bar{z}$ of the plate's center of gravity *G*.



SOLUTION

$$\Sigma A = \frac{1}{2}(8)(12) = 48 \text{ in}^2$$

$$\Sigma \tilde{z} A = 2(2)\left(\frac{1}{2}\right)(2)(6) + 3(6)(2) + 6\left(\frac{1}{2}\right)(2)(3)$$

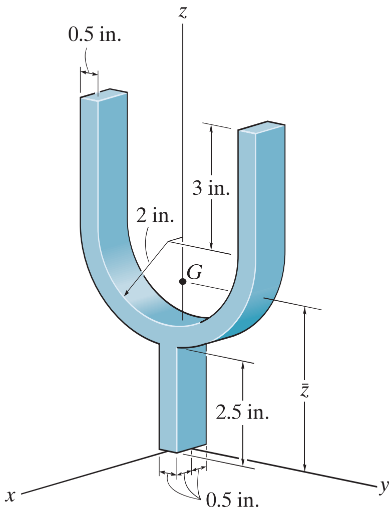
$$= 78 \text{ in}^3$$

$$\bar{z} = \frac{\Sigma \tilde{z} A}{\Sigma A} = \frac{78}{48} = 1.625 \text{ in.}$$ **Ans.**

Ans:
$\bar{z} = 1.625$ in.

9–70.

Locate the center of mass $\bar{z}$ of the forked lever, which is made from a homogeneous material and has the dimensions shown.



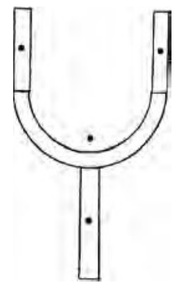
SOLUTION

$$\Sigma A = 2.5(0.5) + \left[\frac{1}{2}\pi(2.5)^2 - \frac{1}{2}\pi(2)^2\right] + 2[(3)(0.5)] = 7.7843 \text{ in}^2$$

$$\Sigma \tilde{z} A = \frac{2.5}{2}(2.5)(0.5) + \left(5 - \frac{4(2.5)}{3\pi}\right)\left(\frac{1}{2}\pi(2.5)^2\right)$$

$$- \left(5 - \frac{4(2)}{3\pi}\right)\left(\frac{1}{2}\pi(2)^2\right) + 6.5(2)(3)(0.5) = 33.651 \text{ in}^3$$

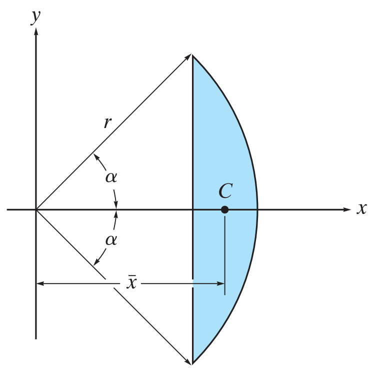
$$\bar{z} = \frac{\Sigma \tilde{z} A}{\Sigma A} = \frac{33.651}{7.7843} = 4.32 \text{ in.}$$ **Ans.**



Ans:
$\bar{z} = 4.32$ in.

9–71.

Determine the location $\overline{x}$ of the centroid C of the shaded area which is part of a circle having a radius r.



SOLUTION

Using symmetry, to simplify, consider just the top half:

$$\Sigma\widetilde{x}A = \frac{1}{2}r^2\alpha\left(\frac{2r}{3\alpha}\sin\alpha\right) - \frac{1}{2}(r\sin\alpha)(r\cos\alpha)\left(\frac{2}{3}r\cos\alpha\right)$$

$$= \frac{r^3}{3}\sin\alpha - \frac{r^3}{3}\sin\alpha\cos^2\alpha$$

$$= \frac{r^3}{3}\sin^3\alpha$$

$$\Sigma A = \frac{1}{2}r^2\alpha - \frac{1}{2}(r\sin\alpha)(r\cos\alpha)$$

$$= \frac{1}{2}r^2\left(\alpha - \frac{\sin 2\alpha}{2}\right)$$

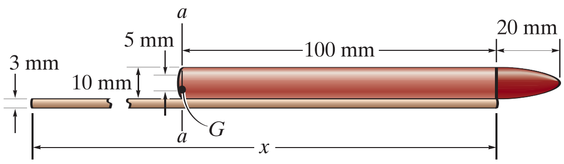
$$\overline{x} = \frac{\Sigma\widetilde{x}A}{\Sigma A} = \frac{\frac{r^3}{3}\sin^3\alpha}{\frac{1}{2}r^2\left(\alpha - \frac{\sin 2\alpha}{2}\right)} = \frac{\frac{2}{3}r\sin^3\alpha}{\alpha - \frac{\sin 2\alpha}{2}}$$ **Ans.**

Ans:

$$\overline{x} = \frac{\frac{2}{3}r\sin^3\alpha}{\alpha - \frac{\sin 2\alpha}{2}}$$

***9–72.**

A toy skyrocket consists of a solid conical top, $\rho_t = 600\ \text{kg/m}^3$, a hollow cylinder, $\rho_c = 400\ \text{kg/m}^3$, and a stick having a circular cross section, $\rho_s = 300\ \text{kg/m}^3$. Determine the length of the stick, x, so that the center of gravity G of the skyrocket is located along line aa.



SOLUTION

$$\Sigma\tilde{x}m = \left(\frac{20}{4}\right)\left[\left(\frac{1}{3}\right)\pi\,(5)^2(20)\right](600) - 50\left[\pi\left(5^2 - 2.5^2\right)(100)\right](400) - \frac{x}{2}\left[(x)\,\pi\,(1.5)^2\right](300)$$

$$= -116.24\left(10^6\right) - x^2(1060.29)\ \text{kg}\cdot\text{mm}^4/\text{m}^3$$

$$\Sigma m = \left[\frac{1}{3}\pi\,(5)^2\,(20)\right](600) + \pi\left(5^2 - 2.5^2\right)(100)(400) + \left[x\pi\,(1.5)^2\right](300)$$

$$= 2.670\left(10^6\right) + 2120.58x\ \text{kg}\cdot\text{mm}^3/\text{m}^3$$

$$\bar{x} = \frac{\Sigma\tilde{x}m}{\Sigma m} = \frac{-116.24(10^6) - x^2(1060.29)}{2.670(10^6) + 2120.58x} = -100$$

$$-116.24\left(10^6\right) - x^2(1060.29) = -267.0\left(10^6\right) - 212.058\left(10^3\right)x$$

$$1060.29x^2 - 212.058\left(10^3\right)x - 150.80\left(10^6\right) = 0$$

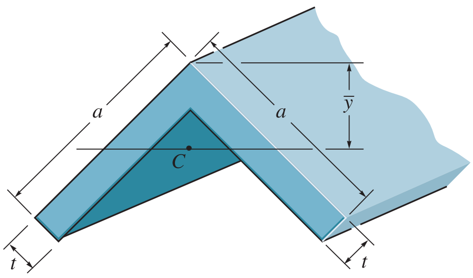
Solving for the positive root gives

$$x = 490\ \text{mm} \qquad \textbf{Ans.}$$

Ans:
$x = 490$ mm

9–73.

Locate the centroid $\bar{y}$ for the cross-sectional area of the angle.



SOLUTION

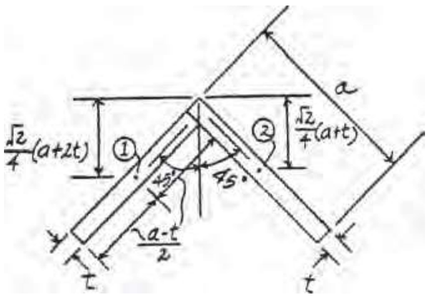
Centroid: The area and the centroid for segments 1 and 2 are

$$A_1 = t(a - t)$$

$$\tilde{y}_1 = \left(\frac{a-t}{2} + \frac{t}{2}\right)\cos 45^\circ + \frac{t}{2\cos 45^\circ} = \frac{\sqrt{2}}{4}(a + 2t)$$

$$A_2 = at$$

$$\tilde{y}_2 = \left(\frac{a}{2} - \frac{t}{2}\right)\cos 45^\circ + \frac{t}{2\cos 45^\circ} = \frac{\sqrt{2}}{4}(a + t)$$



Listed in a tabular form, we have

Segment	A	$\tilde{y}$	$\tilde{y}A$
1	$t(a - t)$	$\frac{\sqrt{2}}{4}(a + 2t)$	$\frac{\sqrt{2}t}{4}(a^2 + at - 2t^2)$
2	at	$\frac{\sqrt{2}}{4}(a + t)$	$\frac{\sqrt{2}t}{4}(a^2 + at)$
Σ	$t(2a - t)$		$\frac{\sqrt{2}t}{2}(a^2 + at - t^2)$

Thus,

$$\bar{y} = \frac{\Sigma\tilde{y}A}{\Sigma A} = \frac{\frac{\sqrt{2}t}{2}(a^2 + at - t^2)}{t(2a - t)}$$

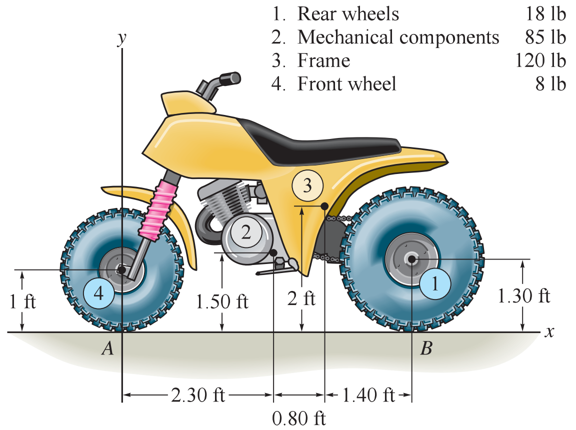
$$= \frac{\sqrt{2}(a^2 + at - t^2)}{2(2a - t)} \qquad \textbf{Ans.}$$

Ans:

$$\bar{y} = \frac{\sqrt{2}(a^2 + at - t^2)}{2(2a - t)}$$

9–74.

Determine the location $(\bar{x}, \bar{y})$ of the center of gravity of the three-wheeler. The location of the center of gravity of each component and its weight are tabulated in the figure. If the three-wheeler is symmetrical with respect to the x–y plane, determine the normal reaction each of its wheels exerts on the ground.



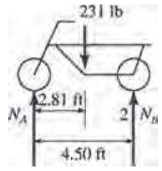
SOLUTION

$$\Sigma\tilde{x}W = 4.5(18) + 2.3(85) + 3.1(120)$$

$$= 648.5 \text{ lb}\cdot\text{ft}$$

$$\Sigma W = 18 + 85 + 120 + 8 = 231 \text{ lb}$$

$$\bar{x} = \frac{\Sigma\tilde{x}W}{\Sigma W} = \frac{648.5}{231} = 2.81 \text{ ft}$$ **Ans.**



$$\Sigma\tilde{y}W = 1.30(18) + 1.5(85) + 2(120) + 1(8)$$

$$= 398.9 \text{ lb}\cdot\text{ft}$$

$$\bar{y} = \frac{\Sigma\tilde{y}W}{\Sigma W} = \frac{398.9}{231} = 1.73 \text{ ft}$$ **Ans.**

$\zeta + \Sigma M_A = 0;$ $\quad 2(N_B)(4.5) - 231(2.81) = 0$

$$N_B = 72.1 \text{ lb}$$ **Ans.**

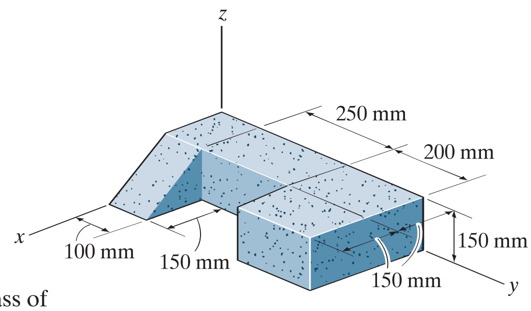
$+\uparrow \Sigma F_y = 0;$ $\quad N_A + 2(72.1) - 231 = 0$

$$N_A = 86.9 \text{ lb}$$ **Ans.**

Ans:
$\bar{x} = 2.81$ ft
$\bar{y} = 1.73$ ft
$N_B = 72.1$ lb
$N_A = 86.9$ lb

9–75.

Locate the center of mass $(\bar{x}, \bar{y}, \bar{z})$ of the homogeneous block assembly.



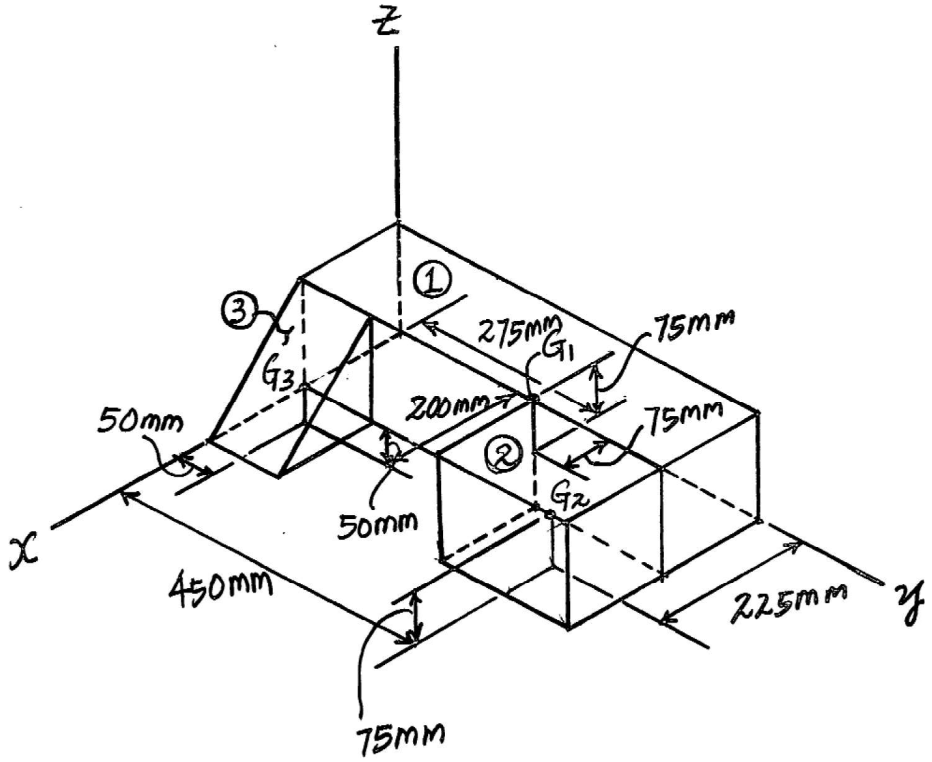
SOLUTION

Centroid: Since the block is made of a homogeneous material, the center of mass of the block coincides with the centroid of its volume. The centroid of each composite segment is shown in Fig. *a*.

$$\bar{x} = \frac{\Sigma \tilde{x}V}{\Sigma V} = \frac{(75)(150)(150)(550) + (225)(150)(150)(200) + (200)\left(\frac{1}{2}\right)(150)(150)(100)}{(150)(150)(550) + (150)(150)(200) + \frac{1}{2}(150)(150)(100)} = \frac{2.165625(10^9)}{18(10^6)} = 120 \text{ mm} \qquad \textbf{Ans.}$$

$$\bar{y} = \frac{\Sigma \tilde{y}V}{\Sigma V} = \frac{(275)(150)(150)(550) + (450)(150)(150)(200) + (50)\left(\frac{1}{2}\right)(150)(150)(100)}{(150)(150)(550) + (150)(150)(200) + \frac{1}{2}(150)(150)(100)} = \frac{5.484375(10^9)}{18(10^6)} = 305 \text{ mm} \qquad \textbf{Ans.}$$

$$\bar{z} = \frac{\Sigma \tilde{z}V}{\Sigma V} = \frac{(75)(150)(150)(550) + (75)(150)(150)(200) + (50)\left(\frac{1}{2}\right)(150)(150)(100)}{(150)(150)(550) + (150)(150)(200) + \frac{1}{2}(150)(150)(100)} = \frac{1.321875(10^9)}{18(10^6)} = 73.4 \text{ mm} \qquad \textbf{Ans.}$$

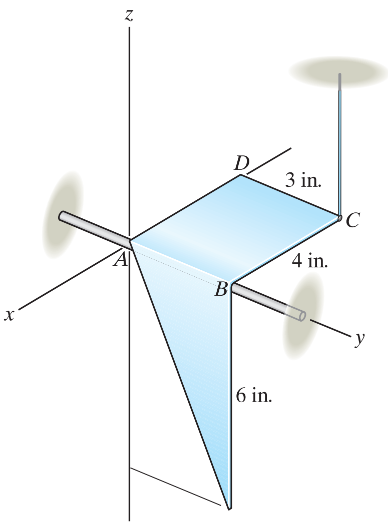


(a)

Ans:
$\bar{x} = 120$ mm
$\bar{y} = 305$ mm
$\bar{z} = 73.4$ mm

***9–76.**

The sheet metal part has the dimensions shown. Determine the location $(\bar{x}, \bar{y}, \bar{z})$ of its centroid.



SOLUTION

$$\Sigma A = 4(3) + \frac{1}{2}(3)(6) = 21 \text{ in}^2$$

$$\Sigma \tilde{x} A = -2(4)(3) + 0\left(\frac{1}{2}\right)(3)(6) = -24 \text{ in}^3$$

$$\Sigma \tilde{y} A = 1.5(4)(3) + \frac{2}{3}(3)\left(\frac{1}{2}\right)(3)(6) = 36 \text{ in}^3$$

$$\Sigma \tilde{z} A = 0(4)(3) - \frac{1}{3}(6)\left(\frac{1}{2}\right)(3)(6) = -18 \text{ in}^3$$

$$\bar{x} = \frac{\Sigma \tilde{x} A}{\Sigma A} = \frac{-24}{21} = -1.14 \text{ in.}$$ **Ans.**

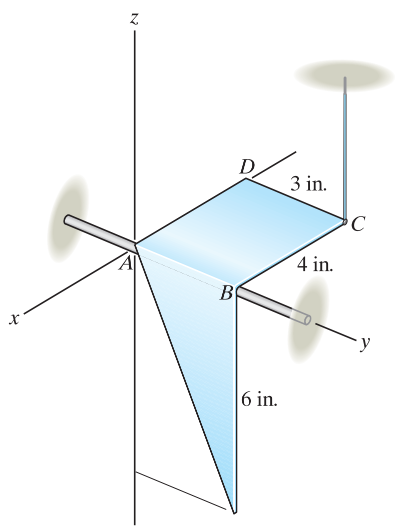
$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{36}{21} = 1.71 \text{ in.}$$ **Ans.**

$$\bar{z} = \frac{\Sigma \tilde{z} A}{\Sigma A} = \frac{-18}{21} = -0.857 \text{ in.}$$ **Ans.**

Ans:
$\bar{x} = -1.14$ in.
$\bar{y} = 1.71$ in.
$\bar{z} = -0.857$ in.

9–77.

The sheet metal part has a weight per unit area of 2 lb/ft^2 and is supported by the smooth rod and at C. If the cord is cut, the part will rotate about the y axis until it reaches equilibrium. Determine the equilibrium angle of tilt, measured downward from the negative x axis, that AD makes with the $-x$ axis.

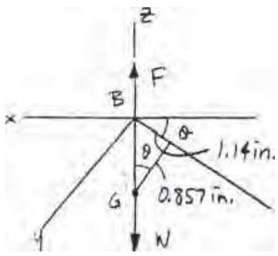


SOLUTION

Since the material is homogeneous, the center of gravity coincides with the centroid.

See solution to Prob. 9–74.

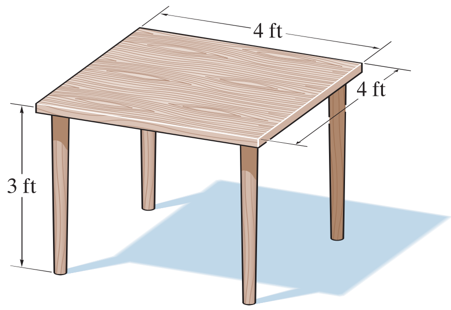
$$\theta = \tan^{-1}\left(\frac{1.14}{0.857}\right) = 53.1^\circ \qquad \textbf{Ans.}$$



Ans:
$\theta = 53.1^\circ$

9–78.

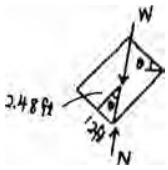
The wooden table is made from a square board having a weight of 15 lb. Each of the legs weighs 2 lb and is 3 ft long. Determine how high its center of gravity is from the floor. Also, what is the angle, measured from the horizontal, through which its top surface can be tilted on two of its legs before it begins to overturn? Neglect the thickness of each leg.



SOLUTION

$$\bar{z} = \frac{\Sigma\tilde{z}W}{\Sigma W} = \frac{15(3) + 4(2)(1.5)}{15 + 4(2)} = 2.48 \text{ ft}$$ **Ans.**

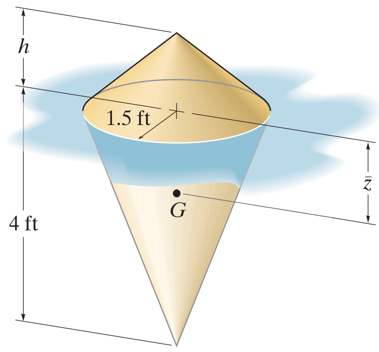
$$\theta = \tan^{-1}\left(\frac{2}{2.48}\right) = 38.9°$$ **Ans.**



Ans:
$\bar{z} = 2.48$ ft
$\theta = 38.9°$

9–79.

The buoy is made from two homogeneous cones each having a radius of 1.5 ft. If $h = 1.2$ ft, find the distance $\bar{z}$ to the buoy's center of gravity G.



SOLUTION

$$\Sigma \tilde{z}\, V = \frac{1}{3}\pi\,(1.5)^2(1.2)\left(-\frac{1.2}{4}\right) + \frac{1}{3}\pi(1.5)^2(4)\left(\frac{4}{4}\right)$$

$$= 8.577 \text{ ft}^4$$

$$\Sigma V = \frac{1}{3}\pi\,(1.5)^2(1.2) + \frac{1}{3}\pi(1.5)^2(4)$$

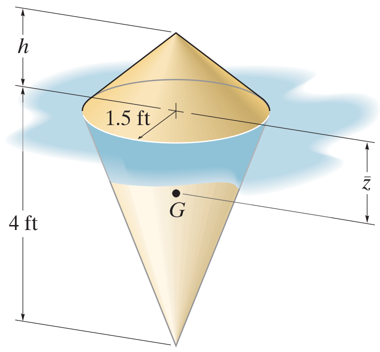
$$= 12.25 \text{ ft}^3$$

$$\bar{z} = \frac{\Sigma \tilde{z} V}{\Sigma V} = \frac{8.577}{12.25} = 0.70 \text{ ft}$$ **Ans.**

Ans:
$\bar{z} = 0.70$ ft

***9–80.**

The buoy is made from two homogeneous cones each having a radius of 1.5 ft. If it is required that the buoy's center of gravity G be located at $\bar{z} = 0.5$ ft, determine the height h of the top cone.



SOLUTION

$$\Sigma\tilde{z}V = \frac{1}{3}\pi(1.5)^2(h)\left(-\frac{h}{4}\right) + \frac{1}{3}\pi(1.5)^2(4)\left(\frac{4}{4}\right)$$

$$= -0.5890\,h^2 + 9.4248$$

$$\Sigma V = \frac{1}{3}\pi(1.5)^2(h) + \frac{1}{3}\pi(1.5)^2(4)$$

$$= 2.3562\,h + 9.4248$$

$$\tilde{z} = \frac{\Sigma\tilde{z}V}{\Sigma V} = \frac{-0.5890\,h^2 + 9.4248}{2.3562\,h + 9.4248} = 0.5$$

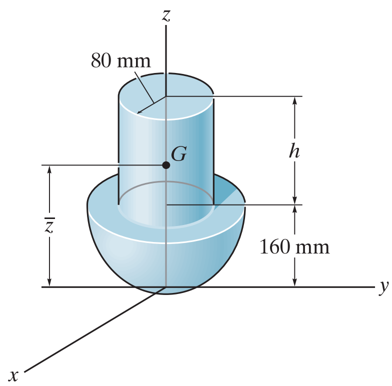
$$-0.5890\,h^2 + 9.4248 = 1.1781\,h + 4.7124$$

$h = 2.00$ ft **Ans.**

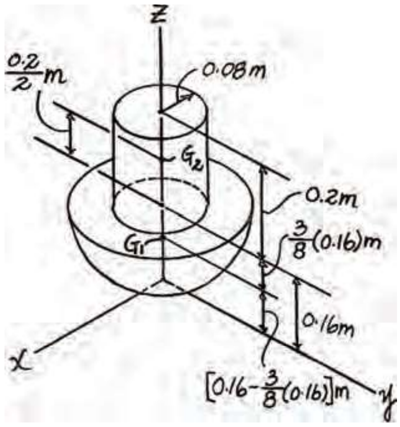
Ans:
$h = 2.00$ ft

9–81.

The assembly is made from a steel hemisphere, $\rho_{st} = 7.80\ \text{Mg/m}^3$, and an aluminum cylinder, $\rho_{al} = 2.70\ \text{Mg/m}^3$. Determine the mass center of the assembly if the height of the cylinder is $h = 200$ mm.



SOLUTION



$$\Sigma\,\tilde{z}m = \left[0.160 - \tfrac{3}{8}(0.160)\right]\left(\tfrac{2}{3}\right)\pi(0.160)^3(7.80) + \left(0.160 + \tfrac{0.2}{2}\right)\pi(0.2)(0.08)^2(2.70)$$

$$= 9.51425(10^{-3})\ \text{Mg}\cdot\text{m}$$

$$\Sigma m = \left(\tfrac{2}{3}\right)\pi(0.160)^3(7.80) + \pi(0.2)(0.08)^2(2.70)$$

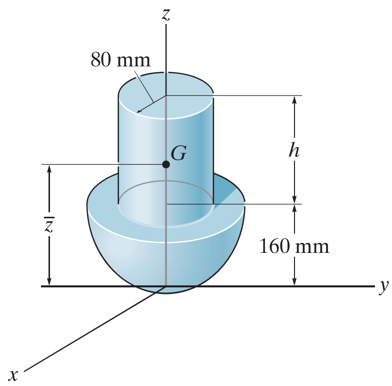
$$= 77.7706(10^{-3})\ \text{Mg}$$

$$\bar{z} = \frac{\Sigma\,\tilde{z}m}{\Sigma m} = \frac{9.51425(10^{-3})}{77.7706(10^{-3})} = 0.122\ \text{m} = 122\ \text{mm} \qquad \textbf{Ans.}$$

Ans:
$\bar{z} = 122$ mm

9–82.

The assembly is made from a steel hemisphere, $\rho_{st} = 7.80\ \text{Mg/m}^3$, and an aluminum cylinder, $\rho_{al} = 2.70\ \text{Mg/m}^3$. Determine the height h of the cylinder so that the mass center of the assembly is located at $\bar{z} = 160$ mm.



SOLUTION

$$\Sigma\,\bar{z}m = \left[0.160 - \tfrac{3}{8}(0.160)\right]\left(\tfrac{2}{3}\right)\pi(0.160)^3(7.80) + \left(0.160 + \tfrac{h}{2}\right)\pi(h)(0.08)^2(2.70)$$

$$= 6.691(10^{-3}) + 8.686(10^{-3})\,h + 27.143(10^{-3})\,h^2$$

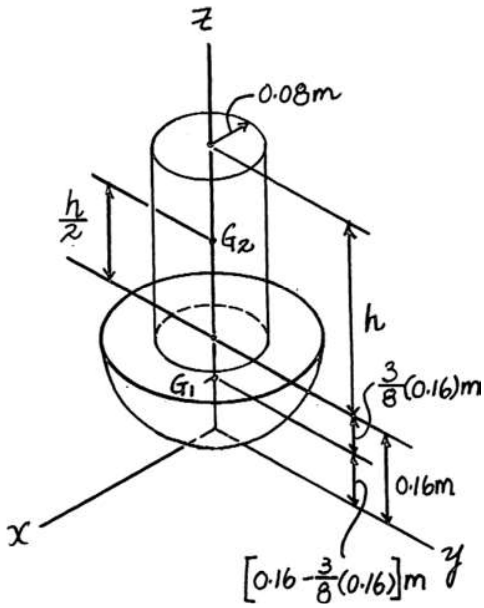
$$\Sigma m = \left(\tfrac{2}{3}\right)\pi(0.160)^3(7.80) + \pi(h)(0.08)^2(2.70)$$

$$= 66.91(10^{-3}) + 54.29(10^{-3})\,h$$

$$\bar{z} = \frac{\Sigma\,\bar{z}m}{\Sigma m} = \frac{6.691(10^{-3}) + 8.686(10^{-3})h + 27.143(10^{-3})h^2}{66.91(10^{-3}) + 54.29(10^{-3})h} = 0.160$$

Solving

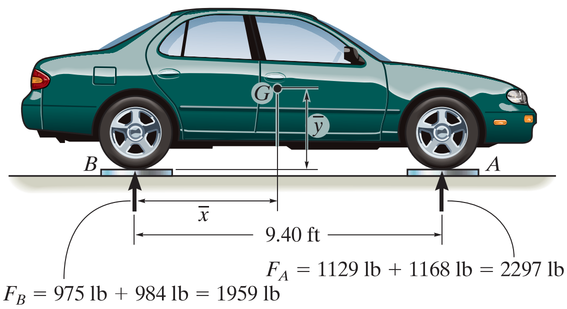
$$h = 0.385\ \text{m} = 385\ \text{mm}$$ **Ans.**

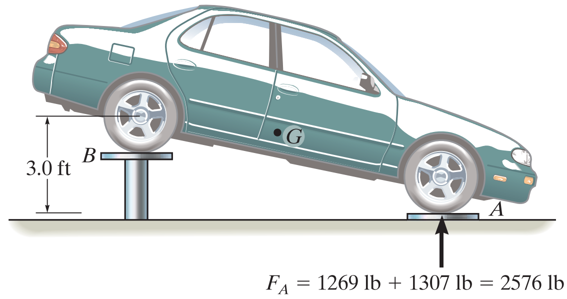


Ans:
$h = 385$ mm

9–83.

The car rests on four scales and in this position the scale readings of both the front and rear tires are shown by F_A and F_B. When the rear wheels are elevated to a height of 3 ft above the front scales, the new readings of the front wheels are also recorded. Use this data to compute the location $\bar{x}$ and $\bar{y}$ to the center of gravity G of the car. The tires each have a diameter of 1.98 ft.





SOLUTION

In horizontal position

$$W = 1959 + 2297 = 4256 \text{ lb}$$

$$\zeta + \Sigma M_B = 0; \qquad 2297(9.40) - 4256\bar{x} = 0$$

$$\bar{x} = 5.0733 = 5.07 \text{ ft} \qquad \textbf{Ans.}$$

$$\theta = \sin^{-1}\left(\frac{3 - 0.990}{9.40}\right) = 12.347°$$

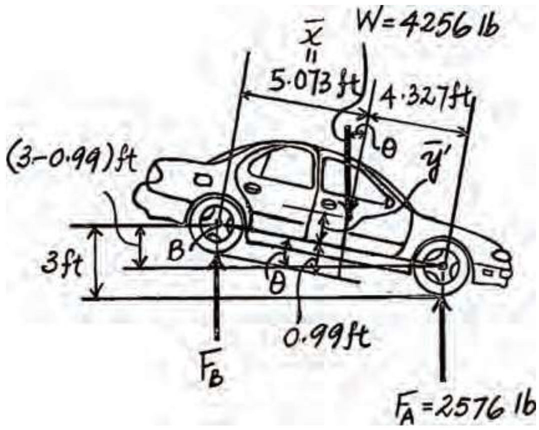
With rear whells elevated

$$\zeta + \Sigma M_B = 0; \qquad 2576(9.40 \cos 12.347°) - 4256 \cos 12.347°(5.0733)$$

$$- 4256 \sin 12.347° \bar{y}' = 0$$

$$\bar{y}' = 2.86 \text{ ft}$$

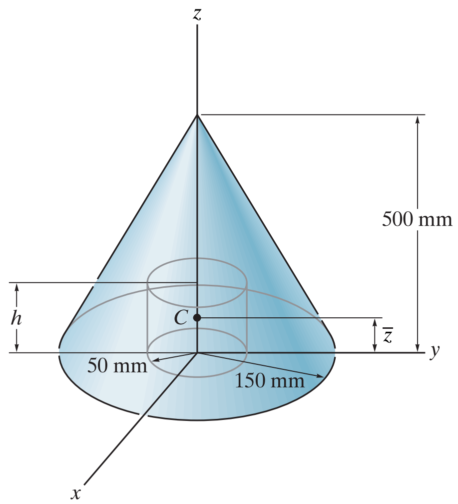
$$\bar{y} = 2.815 + 0.990 = 3.80 \text{ ft} \qquad \textbf{Ans.}$$



Ans:
$\bar{x} = 5.07$ ft
$\bar{y} = 3.80$ ft

***9–84.**

Determine the distance h to which a 100-mm diameter hole must be bored into the base of the cone so that the center of mass of the resulting shape is located at $\bar{z} = 115$ mm. The material has a density of 8 Mg/m^3.



$$\frac{\frac{1}{3}\pi(0.15)^2(0.5)\left(\frac{0.5}{4}\right) - \pi(0.05)^2(h)\left(\frac{h}{2}\right)}{\frac{1}{3}\pi(0.15)^2(0.5) - \pi(0.05)^2(h)} = 0.115$$

$$0.4313 - 0.2875\,h = 0.4688 - 1.25\,h^2$$

$$h^2 - 0.230\,h - 0.0300 = 0$$

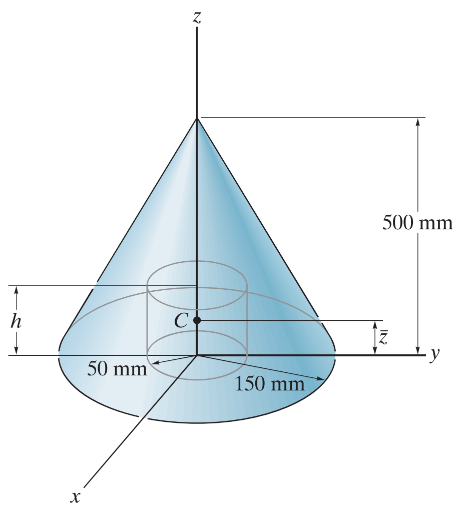
Choosing the positive root,

$$h = 323 \text{ mm}$$ **Ans.**

Ans:
$h = 323$ mm

9–85.

Determine the distance $\bar{z}$ to the centroid of the shape which consists of a cone with a hole of height $h = 50$ mm bored into its base.



SOLUTION

$$\Sigma\tilde{z}V = \frac{1}{3}\pi\,(0.15)^2(0.5)\left(\frac{0.5}{4}\right) - \pi\,(0.05)^2(0.05)\left(\frac{0.05}{2}\right)$$

$$= 1.463(10^{-3})\text{ m}^4$$

$$\Sigma V = \frac{1}{3}\pi\,(0.15)^2(0.5) - \pi\,(0.05)^2(0.05)$$

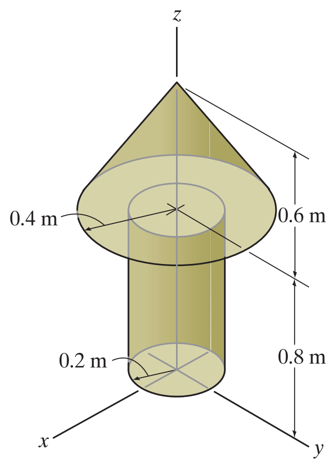
$$= 0.01139\text{ m}^3$$

$$\bar{z} = \frac{\Sigma\tilde{z}V}{\Sigma V} = \frac{1.463\,(10^{-3})}{0.01139} = 0.12845\text{ m} = 128\text{ mm}$$ **Ans.**

Ans:
$\bar{z} = 128$ mm

9–86.

Locate the center of mass $\bar{z}$ of the assembly. The cylinder and the cone are made from materials having densities of 5 Mg/m^3 and 9 Mg/m^3, respectively.

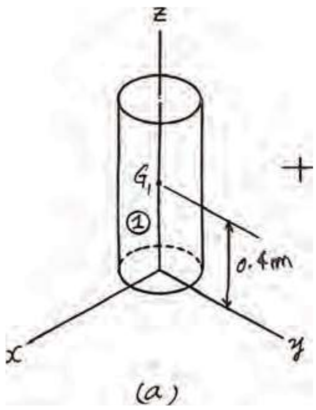


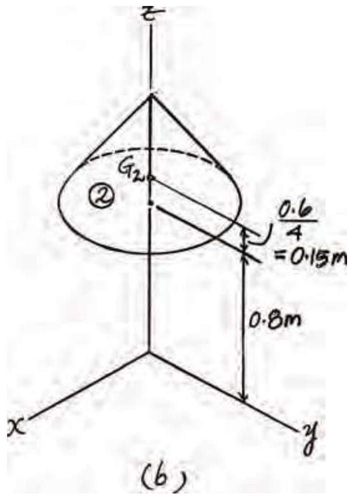
SOLUTION

Center of mass: The assembly is broken into two composite segments, as shown in Figs. *a* and *b*.

$$\bar{z} = \frac{\Sigma \tilde{z} m}{\Sigma m} = \frac{5000(0.4)\left[\pi(0.2^2)(0.8)\right] + 9000(0.8 + 0.15)\left[\frac{1}{3}\pi(0.4^2)(0.6)\right]}{5000\left[\pi(0.2^2)(0.8)\right] + 9000\left[\frac{1}{3}\pi(0.4^2)(0.6)\right]}$$

$$= \frac{1060.60}{1407.4} = 0.754 \text{ m} = 754 \text{ mm} \qquad \textbf{Ans.}$$

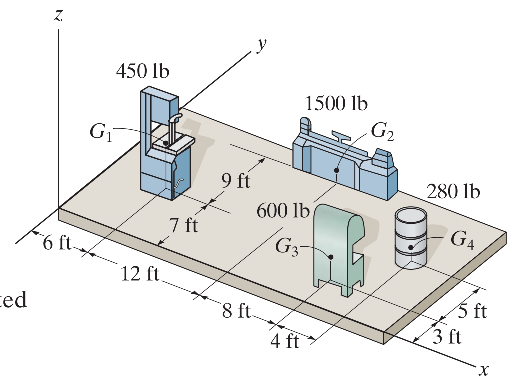




Ans:
$\bar{z} = 754 \text{ mm}$

9–87.

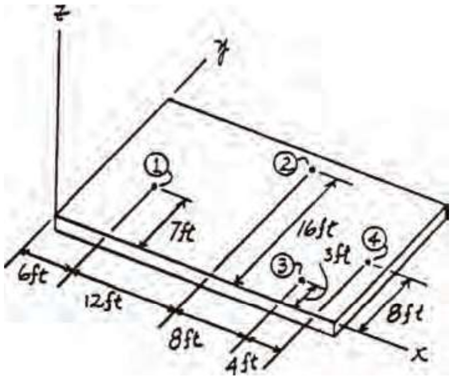
Major floor loadings in a shop are caused by the weights of the objects shown. Each force acts through its respective center of gravity G. Locate the center of gravity $(\bar{x}, \bar{y})$ of all these components.



SOLUTION

Centroid: The floor loadings on the floor and its respective centroid are tabulated below.

Loading	**W (lb)**	**$\bar{x}$ (ft)**	**$\bar{y}$ (ft)**	**$\bar{x}W$(lb · ft)**	**$\bar{y}W$(lb · ft)**
1	450	6	7	2700	3150
2	1500	18	16	27000	24000
3	600	26	3	15600	1800
4	280	30	8	8400	2240
Σ	2830			53700	31190



Thus,

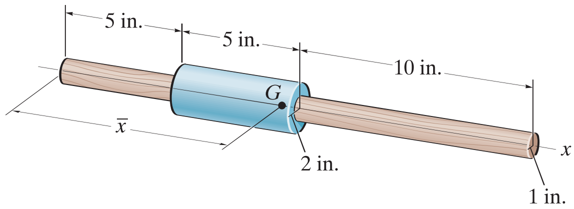
$$\bar{x} = \frac{\Sigma \bar{x}W}{\Sigma W} = \frac{53700}{2830} = 18.98 \text{ ft} = 19.0 \text{ ft} \qquad \textbf{Ans.}$$

$$\bar{y} = \frac{\Sigma \bar{y}W}{\Sigma W} = \frac{31190}{2830} = 11.02 \text{ ft} = 11.0 \text{ ft} \qquad \textbf{Ans.}$$

Ans:
$\bar{x} = 19.0$ ft
$\bar{y} = 11.0$ ft

***9–88.**

The assembly consists of a 20-in. wooden dowel rod and a tight-fitting steel collar. Determine the distance $\bar{x}$ to its center of gravity if the specific weights of the materials are $\gamma_w = 150 \text{ lb/ft}^3$ and $\gamma_{st} = 490 \text{ lb/ft}^3$. The radii of the dowel and collar are shown.



SOLUTION

$$\Sigma \bar{x} W = \left\{10\pi(1)^2(20)(150) + 7.5\pi(5)(2^2 - 1^2)(490)\right\}\frac{1}{(12)^3}$$

$$= 154.8 \text{ lb} \cdot \text{in.}$$

$$\Sigma W = \left\{\pi(1)^2(20)(150) + \pi(5)(2^2 - 1^2)(490)\right\}\frac{1}{(12)^3}$$

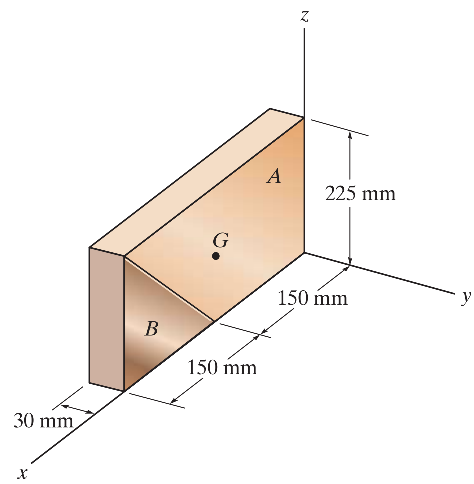
$$= 18.82 \text{ lb}$$

$$\bar{x} = \frac{\Sigma \bar{x} W}{\Sigma W} = \frac{154.8}{18.82} = 8.22 \text{ in.}$$ **Ans.**

Ans:
$\bar{x} = 8.22$ in.

9–89.

The composite plate is made from both steel (A) and brass (B) segments. Determine the mass and location $(\bar{x}, \bar{y}, \bar{z})$ of its mass center G. Take $\rho_{st} = 7.85\ \text{Mg/m}^3$ and $\rho_{br} = 8.74\ \text{Mg/m}^3$.



SOLUTION

$$\Sigma m = \Sigma \rho V = \left[8.74\left(\frac{1}{2}(0.15)(0.225)(0.03)\right)\right] + \left[7.85\left(\frac{1}{2}(0.15)(0.225)(0.03)\right)\right]$$

$$+ [7.85(0.15)(0.225)(0.03)]$$

$$= \left[4.4246\left(10^{-3}\right)\right] + \left[3.9741\left(10^{-3}\right)\right] + \left[7.9481\left(10^{-3}\right)\right]$$

$$= 16.347\left(10^{-3}\right) = 16.4\ \text{kg} \qquad \textbf{Ans.}$$

$$\Sigma \bar{x} m = \left(0.150 + \frac{2}{3}(0.150)\right)(4.4246)\left(10^{-3}\right) + \left(0.150 + \frac{1}{3}(0.150)\right)(3.9741)\left(10^{-3}\right)$$

$$+ \frac{1}{2}(0.150)(7.9481)(10^{-3}) = 2.4971(10^{-3})\ \text{kg}\cdot\text{m}$$

$$\Sigma \bar{z} m = \left(\frac{1}{3}(0.225)\right)(4.4246)\left(10^{-3}\right) + \left(\frac{2}{3}(0.225)\right)(3.9471)\left(10^{-3}\right) + \left(\frac{0.225}{2}\right)(7.9481)\left(10^{-3}\right)$$

$$= 1.8221\left(10^{-3}\right)\text{kg}\cdot\ \text{m}$$

$$\bar{x} = \frac{\Sigma \bar{x} m}{\Sigma m} = \frac{2.4971(10^{-3})}{16.347(10^{-3})} = 0.153\ \text{m} = 153\ \text{mm} \qquad \textbf{Ans.}$$

Due to symmetry:

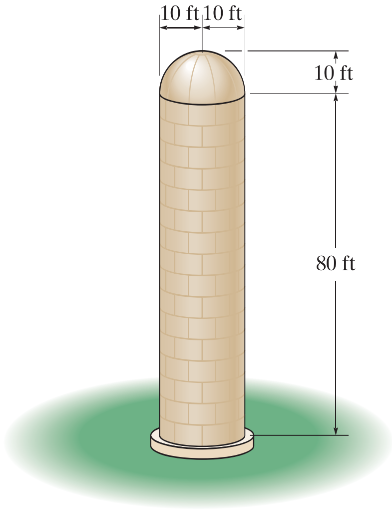
$$\bar{y} = -15\ \text{mm} \qquad \textbf{Ans.}$$

$$\bar{z} = \frac{\Sigma \bar{z} m}{\Sigma m} = \frac{1.8221(10^{-3})}{16.347(10^{-3})} = 0.1115\ \text{m} = 111\ \text{mm} \qquad \textbf{Ans.}$$

Ans:
$\Sigma m = 16.4$ kg
$\bar{x} = 153$ mm
$\bar{y} = -15$ mm
$\bar{z} = 111$ mm

9–90.

Determine the volume of the silo which consists of a cylinder and hemispherical cap. Neglect the thickness of the plates.



SOLUTION

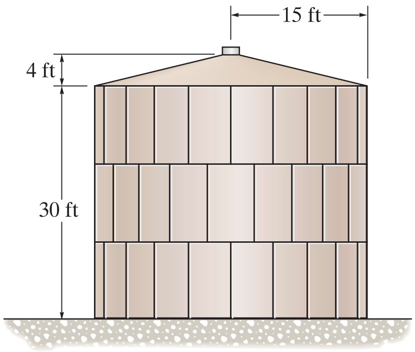
$$V = \Sigma\theta\,\bar{r}\,A = 2\pi\left[\frac{4(10)}{3\pi}\left(\frac{1}{4}\right)\pi\,(10)^2 + 5(80)(10)\right]$$

$$= 27.2\left(10^3\right)\text{ft}^3$$ **Ans.**

Ans:
$V = 27.2(10^3)\text{ ft}^3$

9–91.

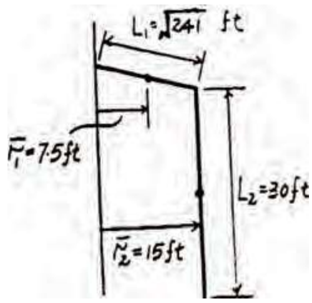
Determine the outside surface area of the storage tank.



SOLUTION

Surface Area: Applying the theorem of Pappus and Guldinus, Eq.9–7. with $\theta = 2\pi$, $L_1 = \sqrt{15^2 + 4^2} = \sqrt{241}$ ft, $L_2 = 30$ ft, $\bar{r}_1 = 7.5$ ft and $\bar{r}_2 = 15$ ft, we have

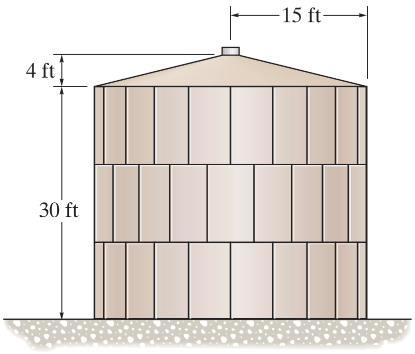
$$A = \theta \Sigma \tilde{r} L = 2\pi\left[7.5\left(\sqrt{241}\right) + 15(30)\right] = 3.56\left(10^3\right)\ \text{ft}^2$$ **Ans.**



Ans:
$A = 3.56\left(10^3\right)\ \text{ft}^2$

***9–92.**

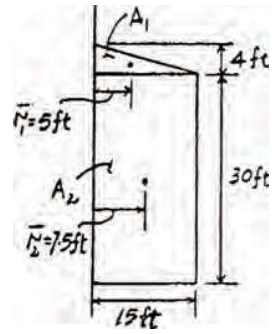
Determine the volume of the storage tank.



SOLUTION

Volume: Applying the theorem of Pappus and Guldinus, Eq. 9–8 with $\theta = 2\pi$, $\bar{r}_1 = 5\text{ ft}, \bar{r}_2 = 7.5\text{ ft}, A_1 = \frac{1}{2}(15)(4) = 30.0\text{ ft}^2$ and $A_2 = 30(15) = 450\text{ ft}^2$, we have

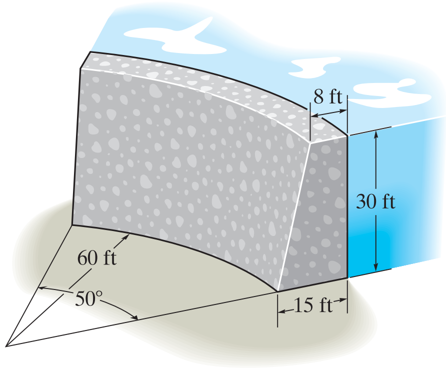
$$V = \theta\Sigma\bar{r}A = 2\pi[5(30.0) + 7.5(450)] = 22.1\left(10^3\right)\text{ ft}^3 \qquad \textbf{Ans.}$$



Ans:
$V = 22.1\left(10^3\right)\text{ ft}^3$

9–93.

Determine the surface area of the concrete sea wall, excluding its bottom.



SOLUTION

Surface Area: Applying Theorem of Pappus and Guldinus, Eq. 9–9 with $\theta = \left(\frac{50}{180}\right)\pi = \frac{5}{18}\pi$ rad, $L_1 = 30$ ft, $L_2 = 8$ ft, $L_3 = \sqrt{7^2 + 30^2} = \sqrt{949}$ ft, $\overline{N}_1 = 75$ ft, $\overline{N}_2 = 71$ ft and $\overline{N}_3 = 63.5$ ft as indicated in Fig. *a*,

$$A_1 = \theta\Sigma\overline{N}L = \frac{5}{18}\pi\left[75(30) + 71(8) + 63.5(\sqrt{949})\right]$$

$$= 4166.25 \text{ ft}^2$$

The surface area of two sides of the wall is

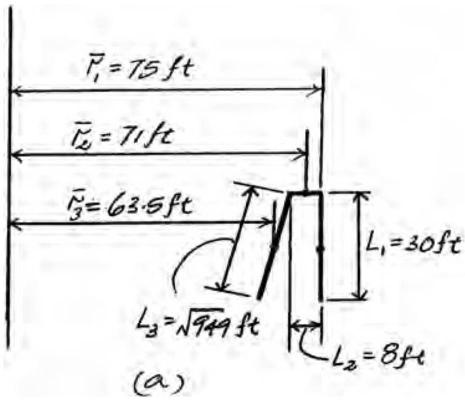
$$A_2 = 2\left[\frac{1}{2}(8 + 15)(30)\right] = 690 \text{ ft}^2$$

Thus the total surface area is

$$A = A_1 + A_2 = 4166.25 + 690$$

$$= 4856.25 \text{ ft}^2$$

$$= 4856 \text{ ft}^2$$ **Ans.**

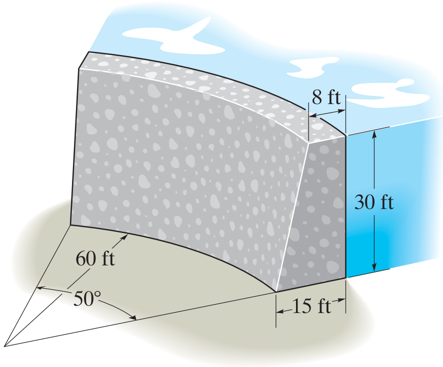


(a)

Ans:
$A = 4856 \text{ ft}^2$

9–94.

A circular sea wall is made of concrete. Determine the total weight of the wall if the concrete has a specific weight of $\gamma_c = 150\ \text{lb/ft}^3$.

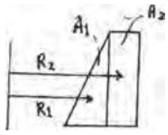


SOLUTION

$$V = \Sigma\theta\tilde{r}A = \left(\frac{50°}{180°}\right)\pi\left[\left(60 + \frac{2}{3}(7)\right)\left(\frac{1}{2}\right)(30)(7) + 71(30)(8)\right]$$

$$= 20\,795.6\ \text{ft}^3$$

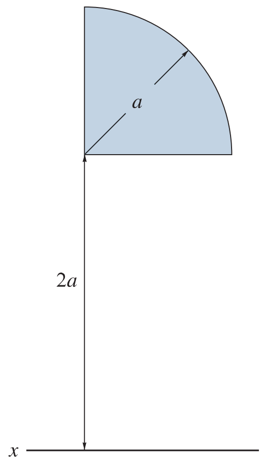
$$W = \gamma V = 150(20\,795.6) = 3.12(10^6)\ \text{lb}$$ **Ans.**



Ans:
$W = 3.12(10^6)$ lb

9–95.

A ring is generated by rotating the quartercircular area about the x axis. Determine its volume.



SOLUTION

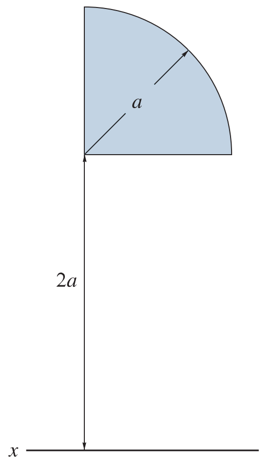
Volume: Applying the theorem of Pappus and Guldinus, Eq. 9–10, with $\theta = 2\pi$, $\bar{r} = 2a + \frac{4a}{3\pi} = \frac{6\pi + 4}{3\pi}a$ and $A = \frac{\pi}{4}a^2$, we have

$$V = \theta \bar{r} A = 2\pi\left(\frac{6\pi + 4}{3\pi}a\right)\left(\frac{\pi}{4}a^2\right) = \frac{\pi(6\pi + 4)}{6}a^3$$ **Ans.**

Ans:
$V = \frac{\pi(6\pi + 4)}{6}a^3$

***9–96.**

A ring is generated by rotating the quartercircular area about the x axis. Determine its surface area.



SOLUTION

Surface Area: Applying the theorem of Pappus and Guldinus, Eq. 9–11, with $\theta = 2\pi$, $L_1 = L_3 = a, L_2 = \dfrac{\pi a}{2}, \bar{r}_1 = 2a, \bar{r}_2 = \dfrac{2(\pi + 1)}{\pi}a$ and $\bar{r}_3 = \dfrac{5}{2}a$, we have

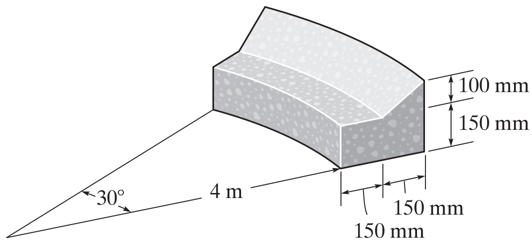
$$A = \theta \Sigma \bar{r} L = 2\pi\left[2a(a) + \left(\frac{2(\pi + 1)}{\pi}a\right)\left(\frac{\pi a}{2}\right) + \frac{5}{2}a\,(a)\right]$$

$$= \pi(2\pi + 11)a^2 \qquad \textbf{Ans.}$$

Ans:
$A = \pi(2\pi + 11)a^2$

9–97.

Determine the volume of concrete needed to construct the curb.



SOLUTION

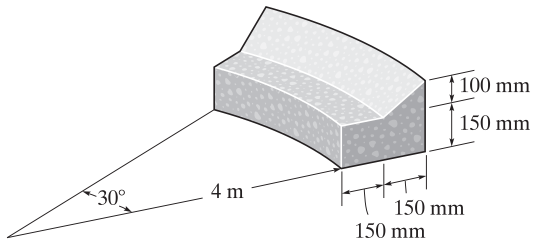
$$V = \Sigma\theta\, A\, \bar{r} = \left(\frac{\pi}{6}\right)[(0.15)(0.3)(4.15)] + \left(\frac{\pi}{6}\right)\left[\left(\frac{1}{2}\right)(0.15)(0.1)(4.25)\right]$$

$V = 0.114\ \text{m}^3$ **Ans.**

Ans:
$V = 0.114\ \text{m}^3$

9–98.

Determine the surface area of the curb. Do not include the area of the ends in the calculation.



SOLUTION

$$A = \Sigma\,\theta\,\bar{r}\,L = \frac{\pi}{6}\{4(0.15) + 4.075(0.15) + (4.15 + 0.075)\left(\sqrt{0.15^2 + 0.1^2}\right)$$

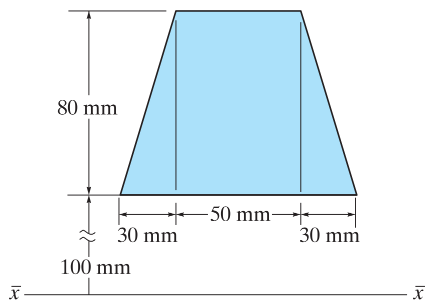
$$+ 4.3(0.25) + 4.15(0.3)\}$$

$$A = 2.25\text{ m}^2$$ **Ans.**

Ans:
$A = 2.25\text{ m}^2$

9–99.

A ring is formed by rotating the area 360° about the $\bar{x}-\bar{x}$ axes. Determine its surface area.



SOLUTION

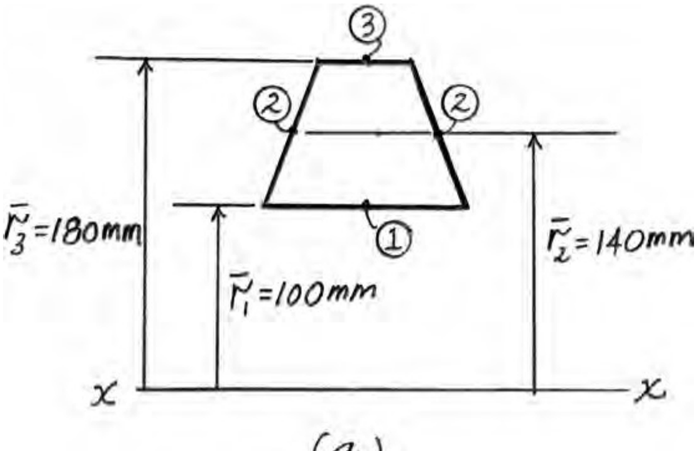
Surface Area. Referring to Fig. *a*, $L_1 = 110$ mm, $L_2 = \sqrt{30^2 + 80^2} = \sqrt{7300}$ mm $L_3 = 50$ mm, $\bar{r}_1 = 100$ mm, $\bar{r}_2 = 140$ mm and $\bar{r}_3 = 180$ mm. Applying the theorem of Pappus and Guldinus, with $\theta = 2\pi$ rad,

$$A = \theta \Sigma \bar{r} L$$

$$= 2\pi\left[100(110) + 2(140)\left(\sqrt{7300}\right) + 180(50)\right]$$

$$= 275.98\left(10^3\right) \text{ mm}^2$$

$$= 276\left(10^3\right) \text{ mm}^2$$ **Ans.**

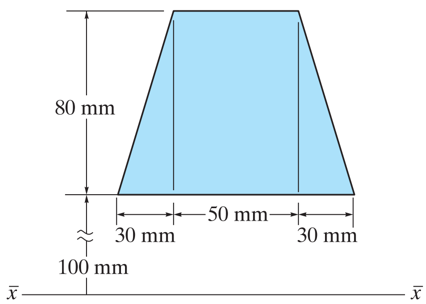


(a)

Ans:
$A = 276(10^3)$ mm^2

***9–100.**

A ring is formed by rotating the area 360° about the $\bar{x}-\bar{x}$ axes. Determine its volume.



SOLUTION

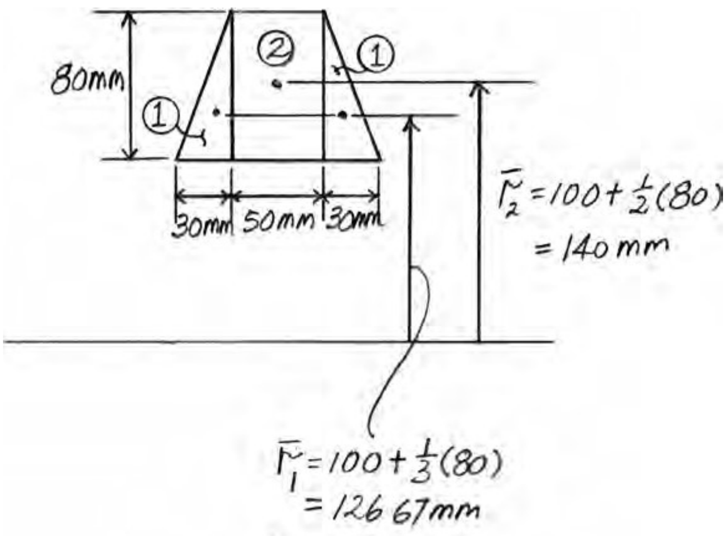
Volume. Referring to Fig. *a*, $A_1 = \frac{1}{2}(60)(80) = 2400\text{ mm}^2$, $A_2 = 50(80) = 4000\text{ mm}^2$, $\bar{r}_1 = 126.67$ mm and $\bar{r}_2 = 140$ mm. Applying the theorem of Pappus and Guldinus, with $\theta = 2\pi$ rad,

$$V = \theta\Sigma\bar{r}A$$

$$= 2\pi\left[126.67(2400) + 140(4000)\right]$$

$$= 5.429\left(10^6\right)\text{ mm}^3$$

$$= 5.43\left(10^6\right)\text{ mm}^3 \qquad \textbf{Ans.}$$

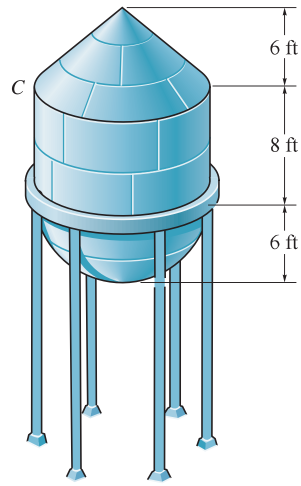


(a)

Ans:
$V = 5.43\left(10^6\right)\text{ mm}^3$

9–101.

The water-supply tank has a hemispherical bottom and cylindrical sides. Determine the weight of water in the tank when it is filled to the top at C. Take $\gamma_w = 62.4\ \text{lb/ft}^3$.

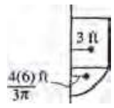


SOLUTION

$$V = \Sigma\theta\tilde{r}A = 2\pi\left\{3(8)(6) + \frac{4(6)}{3\pi}\left(\frac{1}{4}\right)(\pi)(6)^2\right\}$$

$$V = 1357.17\ \text{ft}^3$$

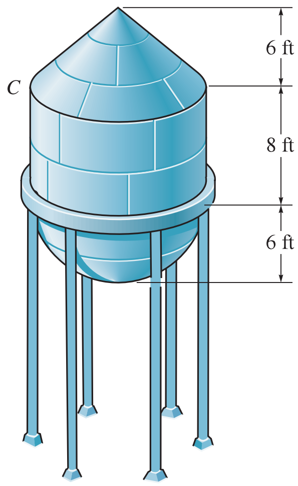
$$W = \gamma V = 62.4(1357.17) = 84.7\ \text{kip}$$ **Ans.**



Ans:
$W = 84.7$ kip

9–102.

Determine the number of gallons of paint needed to paint the outside surface of the water-supply tank, which consists of a hemispherical bottom, cylindrical sides, and conical top. Each gallon of paint can cover 250 ft^2.

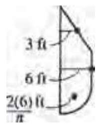


SOLUTION

$$A = \Sigma\theta\tilde{r}L = 2\pi\left\{3\left(6\sqrt{2}\right) + 6(8) + \frac{2(6)}{\pi}\left(\frac{2(6)\pi}{4}\right)\right\}$$

$$= 687.73 \text{ ft}^2$$

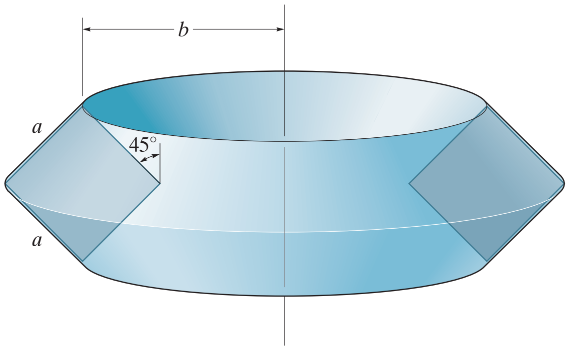
$$\text{Number of gal.} = \frac{687.73 \text{ ft}^2}{250 \text{ ft}^2/\text{gal.}} = 2.75 \text{ gal.}$$ **Ans.**



Ans:
Number of gal. $= 2.75$ gal

9–103.

Determine the surface area and the volume of the ring formed by rotating the square about the vertical axis.

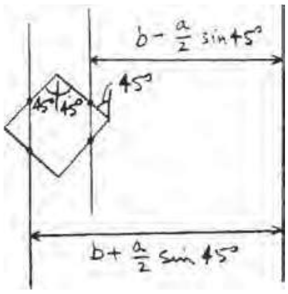


SOLUTION

$$A = \Sigma\theta\widetilde{r}L = 2\left[2\pi\left(b - \frac{a}{2}\sin 45°\right)(a)\right] + 2\left[2\pi\left(b + \frac{a}{2}\sin 45°\right)(a)\right]$$

$$= 4\pi\left[ba - \frac{a^2}{2}\sin 45° + ba + \frac{a^2}{2}\sin 45°\right]$$

$$= 8\pi ba$$ **Ans.**



Also

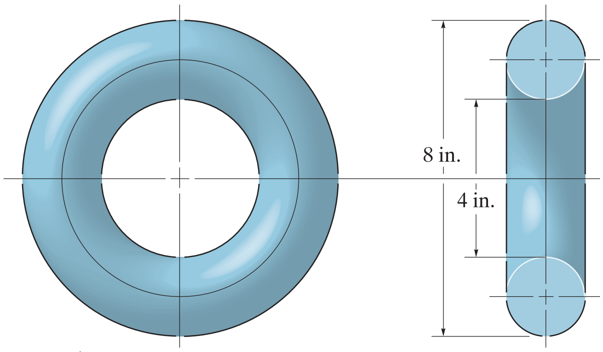
$$A = \Sigma\theta\bar{r}L = 2\pi(b)(4a) = 8\pi ba$$

$$V = \Sigma\theta\widetilde{r}A = 2\pi(b)(a)^2 = 2\pi ba^2$$ **Ans.**

Ans:
$A = 8\pi ba$
$V = 2\pi ba^2$

***9–104.**

Determine the surface area of the ring. The cross section is circular as shown.



SOLUTION

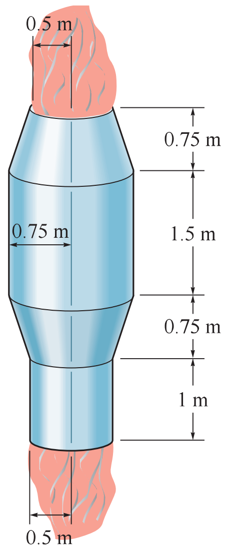
$$A = \theta \tilde{r} L = 2\pi(3)2\pi(1)$$

$$= 118 \text{ in.}^2 \qquad \textbf{Ans.}$$

Ans:
$A = 118 \text{ in.}^2$

9–105.

The heat exchanger radiates thermal energy at the rate of 2500 kJ/h for each square meter of its surface area. Determine how many joules (J) are radiated within a 5-hour period.



SOLUTION

$$A = \Sigma\theta\,\bar{r}\,L = (2\pi)\left[2\left(\frac{0.75 + 0.5}{2}\right)\sqrt{(0.75)^2 + (0.25)^2} + (0.75)(1.5) + (0.5)(1)\right]$$

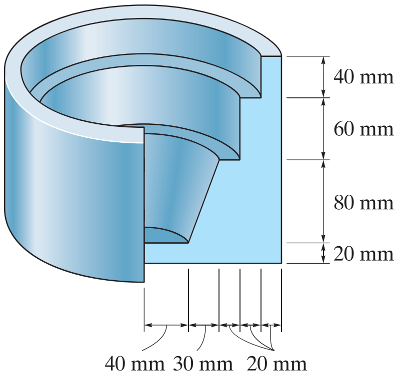
$$= 16.419\ \text{m}^2$$

$$Q = 2500\left(10^3\right)\left(\frac{\text{J}}{\text{h}\cdot\text{m}^2}\right)\left(16.416\ \text{m}^2\right)(5\ \text{h}) = 205\ \text{MJ}$$ **Ans.**

Ans:
$Q = 205$ MJ

9–106.

Determine the interior surface area of the brake piston. It consists of a full circular part. Its cross section is shown in the figure.



SOLUTION

$$A = \Sigma\, \theta\, \bar{r}\, L = 2\,\pi\,[20(40) + 55\sqrt{(30)^2 + (80)^2} + 80(20)$$

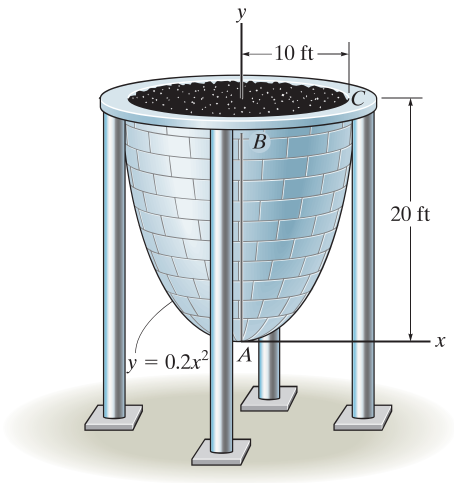
$$+ 90(60) + 100(20) + 110(40)]$$

$A = 119(10^3)\ \text{mm}^2$ **Ans.**

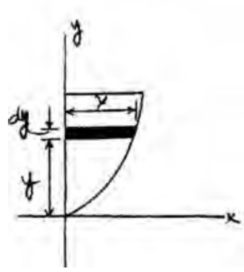
Ans:
$A = 119\left(10^3\right)\ \text{mm}^2$

9–107.

The suspension bunker is made from plates which are curved to the natural shape which a completely flexible membrane would take if subjected to a full load of coal. This curve may be approximated by a parabola, $y = 0.2x^2$. Determine the weight of coal which the bunker would contain when completely filled. Coal has a specific weight of $\gamma = 50\ \text{lb/ft}^3$, and assume there is a 20% loss in volume due to air voids. Solve the problem by integration to determine the cross-sectional area of ABC; then use the second theorem of Pappus–Guldinus to find the volume.



SOLUTION



$$\tilde{x} = \frac{x}{2}$$

$$\tilde{y} = y$$

$$dA = x\,dy$$

$$\int_A dA = \int_0^{20} \sqrt{\frac{y}{0.2}}\,dy = \frac{2}{3\sqrt{0.2}}\,y^{\frac{3}{2}}\Big|_0^{20} = 133.3\ \text{ft}^2$$

$$\int_A \tilde{x}\,dA = \int_0^{20} \frac{y}{0.4}\,dy = \frac{y^2}{0.8}\Big|_0^{20} = 500\ \text{ft}^3$$

$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{500}{133.3} = 3.75\ \text{ft}$$

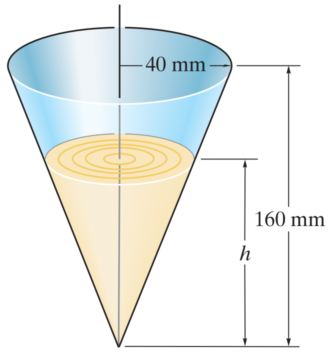
$$V = \theta\,\bar{r}\,A = 2\pi\,(3.75)\,(133.3) = 3142\ \text{ft}^3$$

$$W = 0.8\,\gamma\,V = 0.8(50)(3142) = 125\,664\ \text{lb} = 126\ \text{kip}$$ **Ans.**

Ans:
$W = 126$ kip

***9–108.**

Determine the height h to which liquid should be poured into the cup so that it contacts three-fourths the surface area on the inside of the cup. Neglect the cup's thickness for the calculation.



SOLUTION

Surface Area. From the geometry shown in Fig. *a*,

$$\frac{r}{h} = \frac{40}{160}; \qquad r = \frac{1}{4}h$$

Thus, $\bar{r} = \frac{1}{8}h$ and $L = \sqrt{\left(\frac{1}{4}h\right)^2 + h^2} = \frac{\sqrt{17}}{4}h$, Fig. *b*. Applying the theorem of Pappus and Guldinus, with $\theta = 2\pi$ rad,

$$A = \theta\,\Sigma\bar{r}\,L = 2\pi\left(\frac{1}{8}h\right)\left(\frac{\sqrt{17}}{4}h\right) = \frac{\pi\sqrt{17}}{16}h^2$$

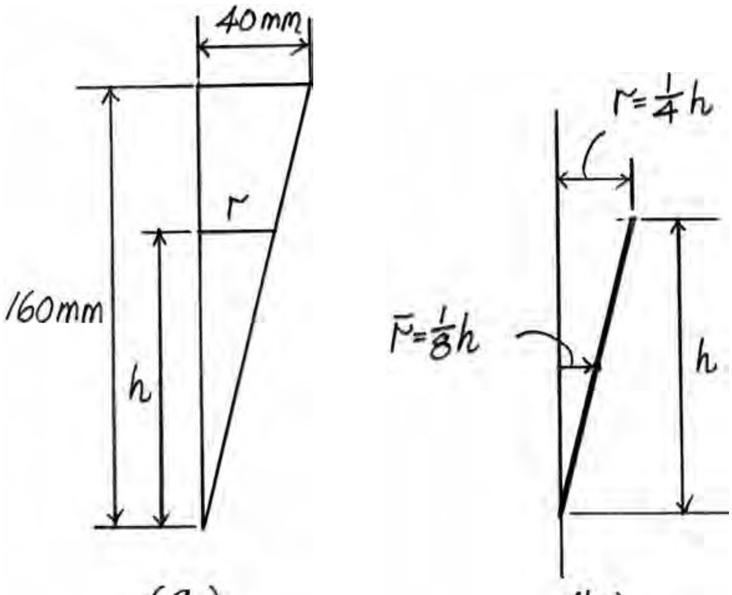
For the whole cup, $h = 160$ mm. Thus

$$A_o = \left(\frac{\pi\sqrt{17}}{16}\right)\left(160^2\right) = 1600\pi\sqrt{17}\text{ mm}^2$$

It is required that $A = \frac{3}{4}A_o = \frac{3}{4}\left(1600\pi\sqrt{17}\right) = 1200\pi\sqrt{17}\text{ mm}^2$. Thus

$$1200\pi\sqrt{17} = \frac{\pi\sqrt{17}}{16}h^2$$

$$h = 138.56\text{ mm} = 139\text{ mm}$$ **Ans.**

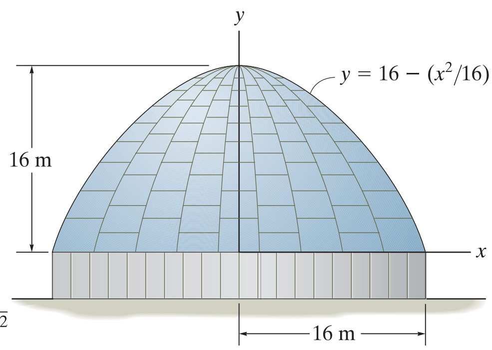


(a) (b)

Ans:
$h = 139$ mm

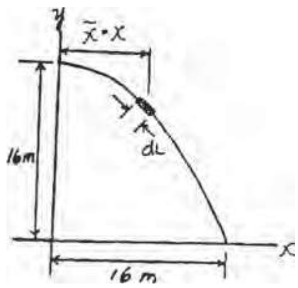
9–109.

Determine the surface area of the roof of the structure if it is formed by rotating the parabola about the y axis.



SOLUTION

Centroid: The length of the differential element is $dL = \sqrt{dx^2 + dy^2} = \left(\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\right)dx$ and its centroid is $x = x$. Here, $\frac{dy}{dx} = -\frac{x}{8}$. Evaluating the integrals, we nave



$$L = \int dL = \int_0^{16\text{ m}} \left(\sqrt{1 + \frac{x^2}{64}}\right)dx = 23.663 \text{ m}$$

$$\int_L \tilde{x}dL = \int_0^{16\text{m}} \tilde{x}\left(\sqrt{1 + \frac{x^2}{64}}\right)dx = 217.181 \text{ m}^2$$

Applying Eq. 9–5, we have

$$\bar{x} = \frac{\int_L \tilde{x}dL}{\int_L dL} = \frac{217.181}{23.663} = 9.178 \text{ m}$$

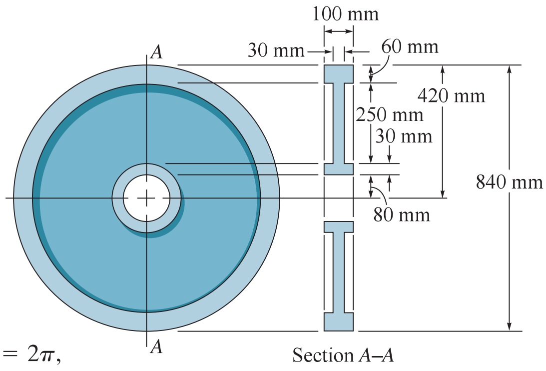
Surface Area: Applying the theorem of Pappus and Guldinus, Eq. 9–7, with $\theta = 2\pi$, $L = 23.663$ m, $\bar{r} = \bar{x} = 9.178$, we have

$$A = \theta\bar{r}L = 2\pi(9.178)\,(23.663) = 1365 \text{ m}^2$$ **Ans.**

Ans:
$A = 1365 \text{ m}^2$

9–110.

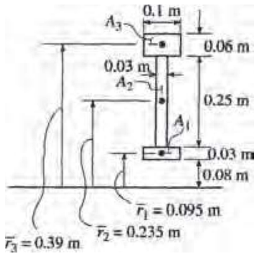
A steel wheel has a diameter of 840 mm and a cross section as shown in the figure. Determine the total mass of the wheel if $\rho = 5\ \text{Mg/m}^3$.



Section A–A

SOLUTION

Volume: Applying the theorem of Pappus and Guldinus, Eq. 9–12, with $\theta = 2\pi$, $\bar{r}_1 = 0.095$ m, $\bar{r}_2 = 0.235$ m, $\bar{r}_3 = 0.39$ m, $A_1 = 0.1(0.03) = 0.003\ \text{m}^2$, $A_2 = 0.25(0.03) = 0.0075\ \text{m}^2$ and $A_3 = (0.1)(0.06) = 0.006\ \text{m}^2$, we have



$$V = \theta \Sigma \bar{r} A = 2\pi[0.095(0.003) + 0.235(0.0075) + 0.39(0.006)]$$

$$= 8.775\pi(10^{-3})\text{m}^3$$

The mass of the wheel is

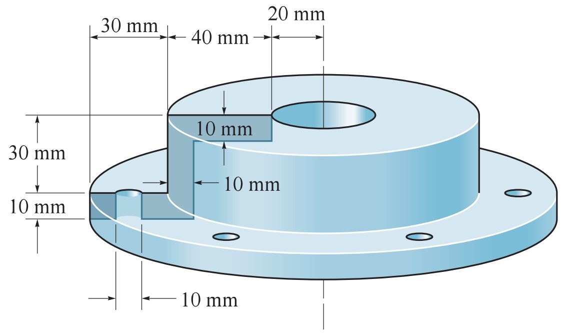
$$m = \rho V = 5(10^3)[8.775(10^{-3})\pi]$$

$$= 138\ \text{kg} \qquad \textbf{Ans.}$$

Ans:
$m = 138$ kg

9–111.

Half the cross section of the steel housing is shown in the figure. There are six 10-mm-diameter bolt holes around its rim. Determine its mass. The density of steel is 7.85 Mg/m^3. The housing is a full circular part.

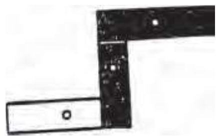


SOLUTION

$$V = 2\pi[(40)(40)(10) + (55)(30)(10) + (75)(30)(10)] - 6\left[\pi(5)^2(10)\right] = 340.9\left(10^3\right)\text{ mm}^3$$

$$m = \rho V = \left(7850\,\frac{\text{kg}}{\text{m}^3}\right)(340.9)\left(10^3\right)\left(10^{-9}\right)\text{ m}^3$$

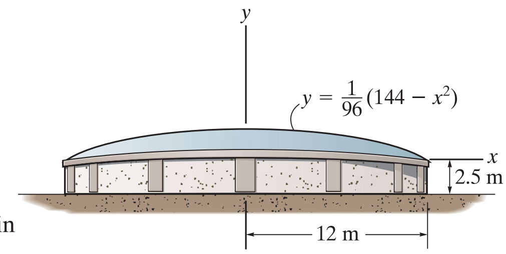
$$= 2.68 \text{ kg}$$ **Ans.**



Ans:
$m = 2.68$ kg

***9–112.**

The water tank has a paraboloid-shaped roof. If one liter of paint can cover 3 m^2 of the tank, determine the number of liters required to coat the roof.



SOLUTION

Length and Centroid: The length of the differential element shown shaded in Fig. *a* is

$$dL = \sqrt{dx^2 + dy^2} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx$$

where $\frac{dy}{dx} = -\frac{1}{48}x$. Thus,

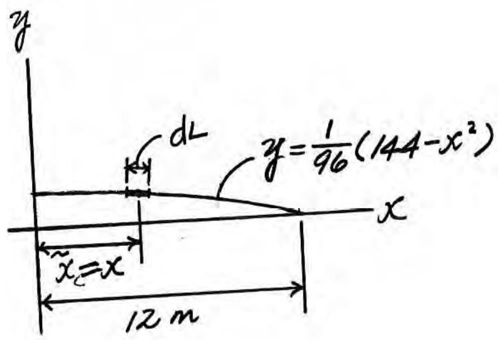
$$dL = \sqrt{1 + \left(-\frac{1}{48}x\right)^2}\,dx = \sqrt{1 + \frac{x^2}{48^2}}\,dx = \frac{1}{48}\sqrt{48^2 + x^2}\,dx$$

Integrating,

$$L = \int_L dL = \int_0^{12\text{ m}} \frac{1}{48}\sqrt{48^2 + x^2}\,dx = 12.124\text{ m}$$

The centroid $\bar{x}$ of the line can be obtained by applying Eq. 9–5 with $x_c = x$.

$$\bar{x} = \frac{\int_L \tilde{x}\,dL}{\int_L dL} = \frac{\int_0^{12\text{ m}} x\left[\frac{1}{48}\sqrt{48^2 + x^2\,dx}\right]}{12.124} = \frac{73.114}{12.124} = 6.031\text{ m}$$



(a)

Surface Area: Applying the first theorem of Pappus and Guldinus and using the results obtained above with $\bar{r} = \bar{x} = 6.031$ m, we have

$$A = 2\pi\bar{r}L = 2\pi(6.031)(12.124) = 459.39\text{ m}^2$$

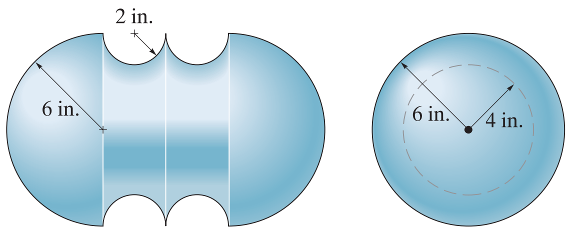
Thus, the amount of paint required is

$$\#\text{ of liters} = \frac{459.39}{3} = 153\text{ liters}$$ **Ans.**

Ans:
153 liters

9–113.

Determine the volume of material needed to make the casting.



SOLUTION

$V = \Sigma\, \theta\, A\, \overline{y}$

$$= 2\,\pi\left[2\left(\frac{1}{4}\,\pi\right)(6)^2\left(\frac{4(6)}{3\,\pi}\right) + 2(6)(4)(3) - 2\left(\frac{1}{2}\,\pi\right)(2)^2\left(6 - \frac{4(2)}{3\pi}\right)\right]$$

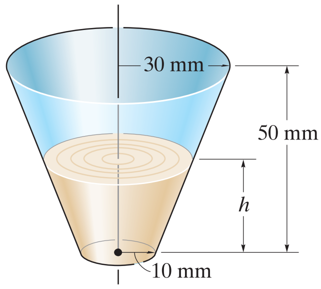
$= 1402.8 \text{ in}^3$

$V = 1.40(10^3) \text{ in}^3$ **Ans.**

Ans:
$V = 1.40\left(10^3\right) \text{ in}^3$

9–114.

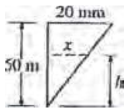
Determine the height h to which liquid should be poured into the cup so that it contacts half the surface area on the inside of the cup. Neglect the cup's thickness for the calculation.



SOLUTION

$$A = \theta \bar{z} \tilde{r} L = 2\pi\{20\sqrt{(20)^2 + (50)^2} + 5(10)\}$$

$$= 2\pi(1127.03)\ \text{mm}^2$$



$$x = \frac{20h}{50} = \frac{2h}{5}$$

$$2\pi\left\{5(10) + \left(10 + \frac{h}{5}\right)\sqrt{\left(\frac{2h}{5}\right)^2 + h^2}\right\} = \frac{1}{2}(2\pi)(1127.03)$$

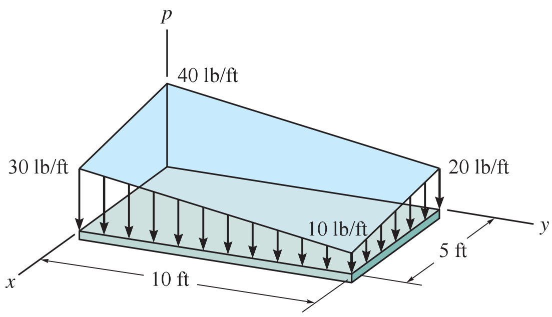
$$10.77h + 0.2154h^2 = 513.5$$

$$h = 29.9\ \text{mm}$$ **Ans.**

Ans:
$h = 29.9$ mm

9–115.

The pressure loading on the plate varies uniformly along each of its edges. Determine the magnitude of the resultant force and the coordinates $(\bar{x}, \bar{y})$ of the point where the line of action of the force intersects the plate. *Hint:* The equation defining the boundary of the load has the form $p = ax + by + c$, where the constants a, b, and c have to be determined.



SOLUTION

$p = ax + by + c$

At $x = 0, \quad y = 0; \qquad p = 40$

$40 = 0 + 0 + c; \qquad c = 40$

At $x = 5, \quad y = 0, \qquad p = 30$

$30 = a(5) + 0 + 40; \qquad a = -2$

At $x = 0; \quad y = 10, \qquad p = 20$

$20 = 0 + b(10) + 40; \qquad b = -2$

Thus,

$p = -2x - 2y + 40$

$$F_R = \int_A p(x,y)dA = \int_0^5 \int_0^{10} (-2x - 2y + 40)\,dy\,dx$$

$$= -2\left(\tfrac{1}{2}(5)^2\right)(10) - 2\left(\tfrac{1}{2}(10)^2\right)5 + 40(5)(10)$$

$$= 1250 \text{ lb}$$ **Ans.**

$$\int_A xp(x,y)\,dA = \int_0^5 \int_0^{10} (-2x^2 - 2\,yx + 40\,x)\,dy\,dx$$

$$= -2\left(\tfrac{1}{3}(5)^2\right)(10) - 2\left(\tfrac{1}{2}(10)^2\right)\left(\tfrac{1}{2}(5)^2\right) + 40\left(\tfrac{1}{2}(5)^2\right)(10)$$

$$= 2916.67 \text{ lb}\cdot\text{ft}$$

$$\bar{x} = \frac{\int_A xp(x,y)\,dA}{\int_A p(x,y)dA} = \frac{2916.67}{1250} = 2.33 \text{ ft}$$ **Ans.**

$$\int_A yp(x,y)\,dA = \int_0^5 \int_0^{10} (-2x\,y - 2y^2 + 40y)\,dy\,dx$$

$$= -2\left(\tfrac{1}{2}(5)^2\right)\left(\tfrac{1}{2}(10)^2\right) - 2\left(\tfrac{1}{3}(10)^3\right)(5) + 40(5)\left(\tfrac{1}{2}(10)^2\right)$$

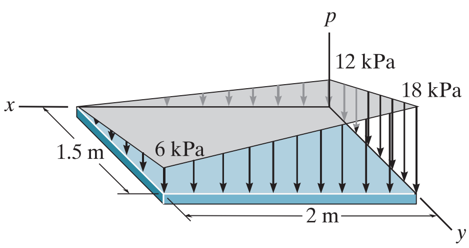
$$= 5416.67 \text{ lb}\cdot\text{ft}$$

$$\bar{y} = \frac{\int_A yp(x,y)\,dA}{\int_A p(x,y)\,dA} = \frac{5416.67}{1250} = 4.33 \text{ ft}$$ **Ans.**

Ans:
$F_R = 1250$ lb
$\bar{x} = 2.33$ ft
$\bar{y} = 4.33$ ft

***9–116.**

The load over the plate varies linearly along the sides of the plate such that $p = (12 - 6x + 4y)$ kPa. Determine the magnitude of the resultant force and the coordinates $(\bar{x}, \bar{y})$ of the point where the line of action of the force intersects the plate.



SOLUTION

Centroid. Perform the double integration.

$$F_R = \int_A \rho(x, y)dA = \int_0^{1.5\text{ m}}\int_0^{2\text{ m}} (12 - 6x + 4y)dxdy$$

$$= \int_0^{1.5\text{ m}} \left(12x - 3x^2 + 4xy\right)\Big|_0^{2\text{ m}} dy$$

$$= \int_0^{1.5\text{ m}} (8y + 12)dy$$

$$= (4y^2 + 12y)\Big|_0^{1.5\text{ m}}$$

$$= 27.0\text{ kN}$$ **Ans.**

$$\int_A x\rho(x, y)dA = \int_0^{1.5\text{ m}}\int_0^{2\text{ m}} \left(12x - 6x^2 + 4xy\right)dx\,dy$$

$$= \int_0^{1.5\text{ m}} (6x^2 - 2x^3 + 2x^2y)\Big|_0^{2\text{ m}} dy$$

$$= \int_0^{1.5\text{ m}} (8y + 8)dy$$

$$= \left(4y^2 + 8y\right)\Big|_0^{1.5\text{ m}}$$

$$= 21.0\text{ kN}\cdot\text{m}$$

$$\int_A y\rho(x, y)dA = \int_0^{1.5\text{ m}}\int_0^{2\text{ m}} \left(12y - 6xy + 4y^2\right)dx\,dy$$

$$= \int_0^{1.5\text{ m}} \left(12xy - 3x^2y + 4xy^2\right)\Big|_0^{2\text{ m}} dy$$

$$= \int_0^{1.5\text{ m}} (8y^2 + 12y)dy$$

$$= \left(\frac{8}{3}y^3 + 6y^2\right)\Big|_0^{1.5\text{ m}}$$

$$= 22.5\text{ kN}\cdot\text{m}$$

***9–116. Continued**

Thus,

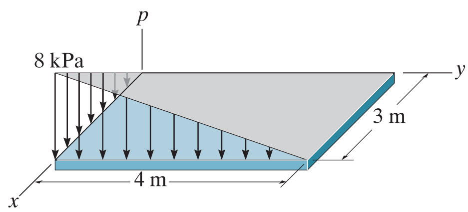
$$\bar{x} = \frac{\int_A xp(x, y)dA}{\int_A p(x, y)dA} = \frac{21.0\text{ kN}\cdot\text{m}}{27.0\text{ kN}} = \frac{7}{9}\text{ m} = 0.778\text{ m} \qquad \textbf{Ans.}$$

$$\bar{y} = \frac{\int_A yp(x, y)dA}{\int_A p(x, y)dA} = \frac{22.5\text{ kN}\cdot\text{m}}{27.0\text{ kN}} = 0.833\text{ m} \qquad \textbf{Ans.}$$

Ans:
$F_R = 27.0$ kN
$\bar{x} = 0.778$ m
$\bar{y} = 0.833$ m

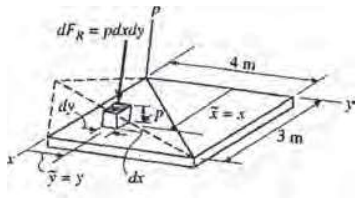
9–117.

The load over the plate varies linearly along the sides of the plate such that $p = \frac{2}{3}[x(4 - y)]$ kPa. Determine the resultant force and its position $(\bar{x}, \bar{y})$ on the plate.



SOLUTION

Resultant Force and its Location: The volume of the differential element is $dV = d\,F_R = p dx dy = \frac{2}{3}(xdx)[(4 - y)dy]$ and its centroid is at $\tilde{x} = x$ and $\tilde{y} = y$.



$$F_R = \int_{F_k} dF_R = \int_0^{3\text{ m}} \frac{2}{3}(xdx)\int_0^{4\text{ m}} (4 - y)dy$$

$$= \frac{2}{3}\left[\left(\frac{x^2}{2}\right)\Big|_0^{3\text{ m}}\left(4y - \frac{y^2}{2}\right)\Big|_0^{4\text{ m}}\right] = 24.0\text{ kN} \qquad \textbf{Ans.}$$

$$\int_{F_R} \tilde{x} dF_R = \int_0^{3\text{ m}} \frac{2}{3}(x^2 dx)\int_0^{4\text{ m}} (4 - y)dy$$

$$= \frac{2}{3}\left[\left(\frac{x^3}{3}\right)\Big|_0^{3\text{ m}}\left(4y - \frac{y^2}{2}\right)\Big|_0^{4\text{ m}}\right] = 48.0\text{ kN}\cdot\text{m}$$

$$\int_{F_R} \tilde{y} dF_R = \int_0^{3\text{ m}} \frac{2}{3}(xdx)\int_0^{4\text{ m}} y(4 - y)dy$$

$$= \frac{2}{3}\left[\left(\frac{x^2}{2}\right)\Big|_0^{3\text{m}}\left(2y^2 - \frac{y^3}{3}\right)\Big|_0^{4\text{ m}}\right] = 32.0\text{ kN}\cdot\text{m}$$

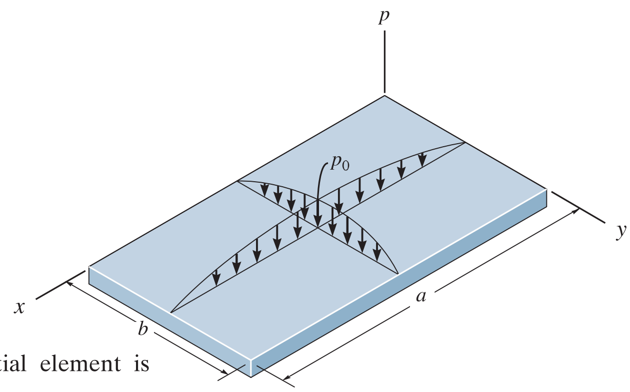
$$\bar{x} = \frac{\int_{FR} \tilde{x} dF_R}{\int_{FR} dF_R} = \frac{48.0}{24.0} = 2.00\text{ m} \qquad \textbf{Ans.}$$

$$\bar{y} = \frac{\int_{FR} \tilde{y} dF_R}{\int_{FR} dF_R} = \frac{32.0}{24.0} = 1.33\text{ m} \qquad \textbf{Ans.}$$

Ans:
$F_R = 24.0$ kN
$\bar{x} = 2.00$ m
$\bar{y} = 1.33$ m

9–118.

The rectangular plate is subjected to a distributed load over its *entire surface.* The load is defined by the expression $p = p_0 \sin(\pi x/a)\sin(\pi y/b)$, where p_0 represents the pressure acting at the center of the plate. Determine the magnitude and location of the resultant force acting on the plate.



SOLUTION

Resultant Force and its Location: The volume of the differential element is $dV = dF_R = p\,dx\,dy = p_0\left(\sin\frac{\pi x}{a}dx\right)\left(\sin\frac{\pi y}{b}dy\right)$.

$$F_R = \int_{F_R} dF_R = p_0\int_0^a\left(\sin\frac{\pi x}{a}dx\right)\int_0^b\left(\sin\frac{\pi y}{b}dy\right)$$

$$= p_0\left[\left(-\frac{a}{\pi}\cos\frac{\pi x}{a}\right)\Big|_0^a\left(-\frac{b}{\pi}\cos\frac{\pi x}{b}\right)\Big|_0^b\right]$$

$$= \frac{4ab}{\pi^2}p_0 \qquad \textbf{Ans.}$$

Since the loading is symmetric, the location of the resultant force is at the center of the plate. Hence,

$$\bar{x} = \frac{a}{2} \qquad \bar{y} = \frac{b}{2} \qquad \textbf{Ans.}$$

Ans:

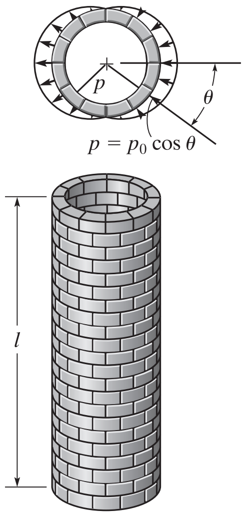
$F_R = \frac{4ab}{\pi^2}p_0$

$\bar{x} = \frac{a}{2}$

$\bar{y} = \frac{b}{2}$

9–119.

A wind loading creates a positive pressure on one side of the chimney and a negative (suction) pressure on the other side, as shown. If this pressure loading acts uniformly along the chimney's length, determine the magnitude of the resultant force created by the wind.



SOLUTION

$$F_{Rx} = 2l\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (p_0 \cos\theta)\cos\theta\, r\, d\theta = 2rlp_0 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^2\theta\, d\theta$$

$$= 2rlp_0\left(\frac{\pi}{2}\right)$$ **Ans.**

$$F_{Rx} = \pi l r p_0$$

$$F_{Ry} = 2l\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (p_0 \cos\theta)\sin\theta\, r\, d\theta = 0$$

Thus,

$$F_R = \pi l r p_0$$ **Ans.**

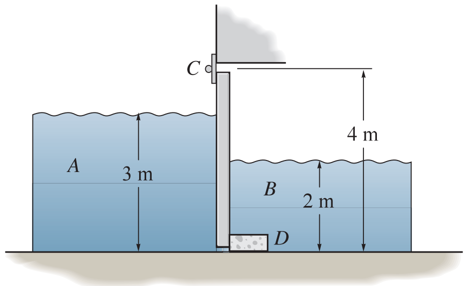
Ans:

$F_{Rx} = 2rlp_0\left(\frac{\pi}{2}\right)$

$F_R = \pi l r p_0$

***9–120.**

When the tide water A subsides, the tide gate automatically swings open to drain the marsh B. For the condition of high tide shown, determine the horizontal reactions developed at the hinge C and stop block D. The length of the gate is 6 m and its height is 4 m. $\rho_w = 1.0\ \text{Mg/m}^3$.

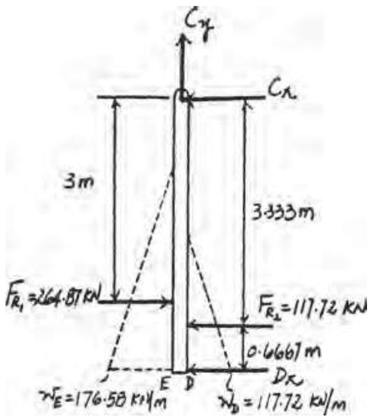


SOLUTION

Fluid Pressure: The fluid pressure at points D and E can be determined using Eq. 9–13, $p = \rho g z$.

$$p_D = 1.0(10^3)(9.81)(2) = 19\,620\ \text{N/m}^2 = 19.62\ \text{kN/m}^2$$

$$p_E = 1.0(10^3)(9.81)(3) = 29\,430\ \text{N/m}^2 = 29.43\ \text{kN/m}^2$$



Thus,

$$w_D = 19.62(6) = 117.72\ \text{kN/m}$$

$$w_E = 29.43(6) = 176.58\ \text{kN/m}$$

Resultant Forces:

$$F_{R_1} = \frac{1}{2}(176.58)(3) = 264.87\ \text{kN}$$

$$F_{R_2} = \frac{1}{2}(117.72)(2) = 117.72\ \text{kN}$$

Equations of Equilibrium:

$\circlearrowleft + \Sigma M_C = 0;\qquad 264.87(3) - 117.72(3.333) - D_x(4) = 0$

$$D_x = 100.55\ \text{kN} = 101\ \text{kN} \qquad \textbf{Ans.}$$

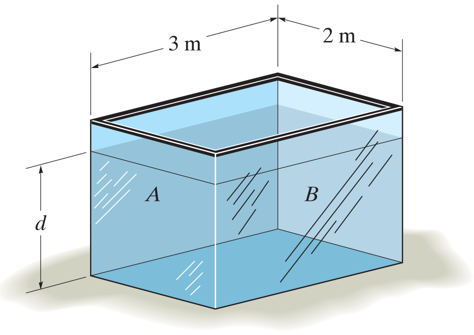
$\overset{+}{\rightarrow} \Sigma F_x = 0;\qquad 264.87 - 117.72 - 100.55 - C_x = 0$

$$C_x = 46.6\ \text{kN} \qquad \textbf{Ans.}$$

Ans:
$D_x = 101\ \text{kN}$
$C_x = 46.6\ \text{kN}$

9–121.

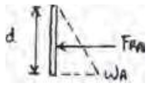
The tank is filled with water to a depth of $d = 4$ m. Determine the resultant force the water exerts on side A and side B of the tank. If oil instead of water is placed in the tank, to what depth d should it reach so that it creates the same resultant forces? $\rho_o = 900\ \text{kg/m}^3$ and $\rho_w = 1000\ \text{kg/m}^3$.



SOLUTION

For water

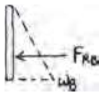
At side A:



$$w_A = b\,\rho_w\,g\,d$$
$$= 2(1000)(9.81)\,(4)$$
$$= 78\,480\ \text{N/m}$$

$$F_{R_A} = \frac{1}{2}(78\,480)(4) = 156\,960\ \text{N} = 157\ \text{kN} \qquad \textbf{Ans.}$$

At side B:



$$w_B = b\,\rho_w\,g\,d$$
$$= 3(1000)(9.81)(4)$$
$$= 117\,720\ \text{N/m}$$

$$F_{R_B} = \frac{1}{2}(117\,720)(4) = 235\,440\ \text{N} = 235\ \text{kN} \qquad \textbf{Ans.}$$

For oil

At side A:

$$w_A = b\,\rho_o\,g\,d$$
$$= 2(900)(9.81)d$$
$$= 17\,658\,d$$

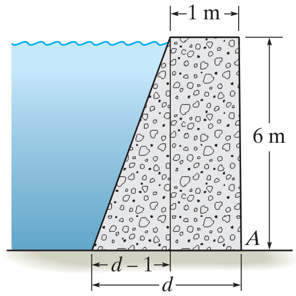
$$F_{R_A} = \frac{1}{2}(17\,658\,d)(d) = 156\,960\ \text{N}$$

$$d = 4.22\ \text{m} \qquad \textbf{Ans.}$$

Ans:
For water: $F_{R_A} = 157$ kN
$F_{R_B} = 235$ kN
For oil: $d = 4.22$ m

9–122.

The concrete "gravity" dam is held in place by its own weight. If the density of concrete is $\rho_c = 2.5\ \text{Mg/m}^3$, and water has a density of $\rho_w = 1.0\ \text{Mg/m}^3$, determine the smallest dimension d that will prevent the dam from overturning about its end A.



SOLUTION

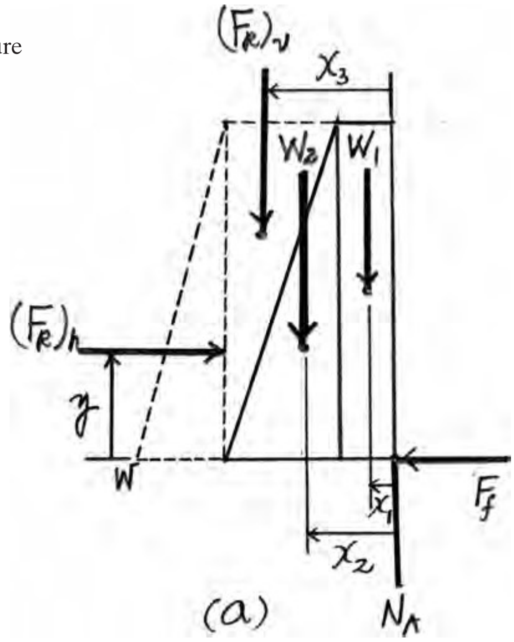
Loadings. The computation will be based on $b = 1$ m width of the dam. The pressure at the base of the dam is.

$$P = \rho g h = 1000(9.81)(6) = 58.86\left(10^3\right) pa = 58.86\ \text{kPa}$$

Thus

$$w = pb = 58.86(1) = 58.86\ \text{kN/m}$$

The forces that act on the dam and their respective points of application, shown in Fig. *a*, are



$$W_1 = 2500\left[1(6)(1)\right](9.81) = 147.15\left(10^3\right)\ \text{N} = 147.15\ \text{kN}$$

$$W_2 = 2500\left[\frac{1}{2}(d-1)(6)(1)\right](9.81) = 73.575(d-1)\left(10^3\right) = 73.575(d-1)\ \text{kN}$$

$$(F_R)_v = 1000\left[\frac{1}{2}(d-1)(6)(1)\right](9.81) = 29.43(d-1)\left(10^3\right) = 29.43(d-1)\ \text{kN}$$

$$(F_R)_h = \frac{1}{2}(58.86)(6) = 176.58\ \text{kN}$$

$$x_1 = 0.5 \quad x_2 = 1 + \frac{1}{3}(d-1) = \frac{1}{3}(d+2) \quad x_3 = 1 + \frac{2}{3}(d-1) = \frac{1}{3}(2d+1)$$

$$y = \frac{1}{3}(6) = 2\ \text{m}$$

Equation of Equilibrium. Write the moment equation of equilibrium about A by referring to the *FBD* of the dam, Fig. *a*,

$$\circlearrowleft + \Sigma M_A = 0; \quad 147.15(0.5) + [73.575(d-1)]\left[\frac{1}{3}(d+2)\right]$$

$$+ [29.43(d-1)]\left[\frac{1}{3}(2d+1)\right] - 176.58(2) = 0$$

$$44.145d^2 + 14.715d - 338.445 = 0$$

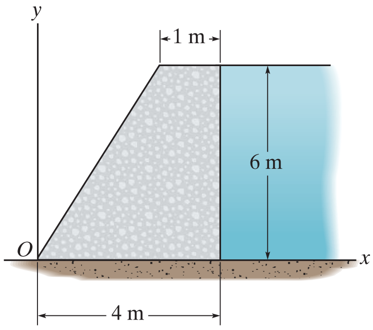
Solving and chose the positive root

$$d = 2.607\ \text{m} = 2.61\ \text{m}$$ **Ans.**

Ans:
$d = 2.61$ m

9–123.

The factor of safety for tipping of the concrete dam is defined as the ratio of the stabilizing moment due to the dam's weight divided by the overturning moment about O due to the water pressure. Determine this factor if the concrete has a density of $\rho_{conc} = 2.5\ \text{Mg/m}^3$ and for water $\rho_w = 1\ \text{Mg/m}^3$.



SOLUTION

Loadings. The computation will be based on $b = 1\text{m}$ width of the dam. The pressure at the base of the dam is

$$P = p_{wgh} = 1000(9.81)(6) = 58.86(10^3)p_a = 58.86\ \text{kPa}$$

Thus,

$$w = Pb = 58.86(1) = 58.86\ \text{kN/m}$$

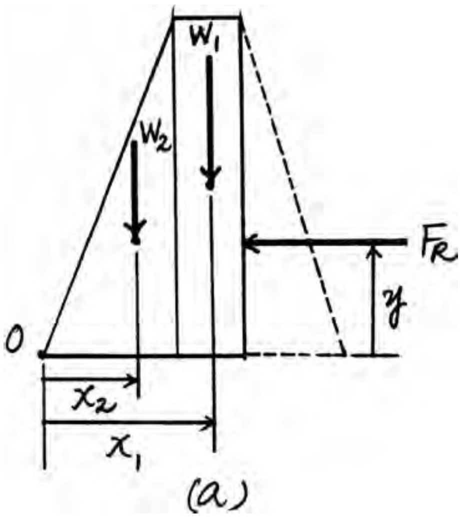
The forces that act on the dam and their respective points of application, shown in Fig. *a*, are

$$W_1 = (2500)[(1(6)(1)](9.81) = 147.15(10^3)]\ \text{N} = 147.15\ \text{kN}$$

$$W_2 = (2500)\left[\frac{1}{2}(3)(6)(1)\right](9.81) = 220.725(10^3)\ \text{N} = 220.725\ \text{kN}$$

$$F_R = \frac{1}{2}(58.86)(6) = 176.58\ \text{kN}$$

$$x_1 = 3 + \frac{1}{2}(1) = 3.5\ \text{ft} \qquad x_2 = \frac{2}{3}(3) = 2\text{m} \qquad y = \frac{1}{3}(6) = 2\ \text{m}$$



Thus, the overturning moment about O is

$$M_{OT} = 176.58(2) = 353.16\ \text{kN}\cdot\text{m}$$

And the stabilizing moment about O is

$$M_s = 147.15(3.5) + 220.725(2) = 956.475\ \text{kN}\cdot\text{m}$$

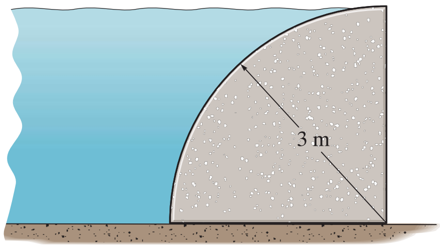
Thus, the factor of safety is

$$\text{F.S.} = \frac{M_s}{M_{OT}} = \frac{956.475}{353.16} = 2.7083 = 2.71$$ **Ans.**

Ans:
F.S. = 2.71

***9–124.**

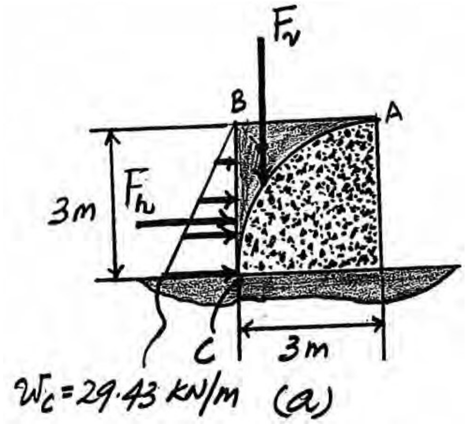
The concrete dam in the shape of a quarter circle. Determine the magnitude of the resultant hydrostatic force that acts on the dam per meter of length. The density of water is $\rho_w = 1 \text{ Mg/m}^3$.



SOLUTION

Loading: The hydrostatic force acting on the circular surface of the dam consists of the vertical component $\mathbf{F}_v$ and the horizontal component $\mathbf{F}_h$ as shown in Fig. *a*.

Resultant Force Component: The vertical component $\mathbf{F}_v$ consists of the weight of water contained in the shaded area shown in Fig. *a*. For a 1-m length of dam, we have



$$F_v = \rho g A_{ABC} b = (1000)(9.81)\left[(3)(3) - \frac{\pi}{4}(3^2)\right](1) = 18947.20 \text{ N} = 18.95 \text{ kN}$$

The horizontal component $\mathbf{F}_h$ consists of the horizontal hydrostatic pressure. Since the width of the dam is constant (1 m), this loading can be represented by a triangular distributed loading with an intensity of $w_C = \rho g h_C b = 1000(9.81)(3)(1) = 29.43 \text{ kN/m}$ at point C, Fig. *a*.

$$F_h = \frac{1}{2}(29.43)(3) = 44.145 \text{ kN}$$

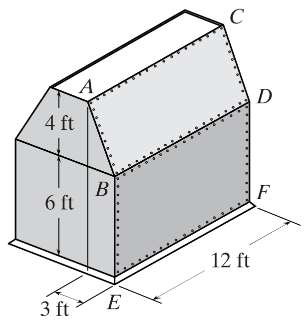
Thus, the magnitude of the resultant hydrostatic force acting on the dam is

$$F_R = \sqrt{F_h{}^2 + F_v{}^2} = \sqrt{44.145^2 + 18.95^2} = 48.0 \text{ kN} \qquad \textbf{Ans.}$$

Ans:
$F_R = 48.0 \text{ kN}$

9–125.

The tank is used to store a liquid having a density of 80 lb/ft^3. If it is filled to the top, determine the magnitude of force the liquid exerts on each of its two sides *ABDC* and *BDFE*.



SOLUTION

$$w_1 = 80(4)(12) = 3840 \text{ lb/ft}$$

$$w_2 = 80(10)(12) = 9600 \text{ lb/ft}$$

ABDC:

$$F_1 = \frac{1}{2}(3840)(5) = 9.60 \text{ kip}$$ **Ans.**

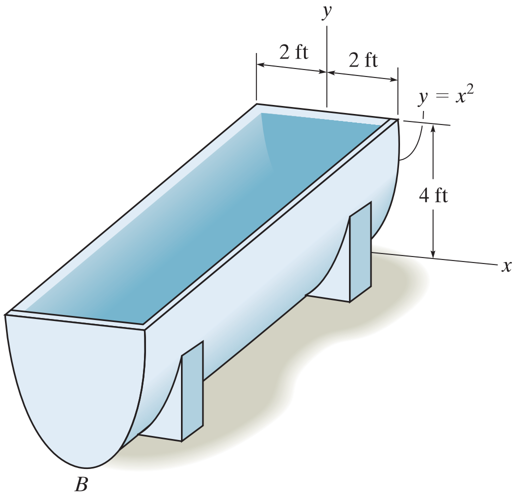
BDEF:

$$F_2 = \frac{1}{2}(9600 - 3840)(6) + 3840(6) = 40.3 \text{ kip}$$ **Ans.**

Ans:
$F_1 = 9.60$ kip
$F_2 = 40.3$ kip

9–126.

The parabolic plate is subjected to a fluid pressure that varies linearly from 0 at its top to 100 lb/ft at its bottom B. Determine the magnitude of the resultant force and its location on the plate.



SOLUTION

$$F_R = \int_A p\,dA = \int_0^4 (100 - 25y)(2x\,dy)$$

$$= 2\int_0^4 (100 - 25y)\left(y^{\frac{1}{2}}\,dy\right)$$

$$= 2\left[100\left(\frac{2}{3}\right)y^{\frac{3}{2}} - 25\left(\frac{2}{5}\right)y^{\frac{5}{2}}\right]_0^4 = 426.7\text{ lb} = 427\text{ lb} \qquad \textbf{Ans.}$$

$$F_R\bar{y} = \int_A yp\,dA; \qquad 426.7\,\bar{y} = 2\int_0^4 y(100 - 25y)y^{\frac{1}{2}}\,dy$$

$$426.7\,\bar{y} = 2\left[100\left(\frac{2}{5}\right)y^{\frac{5}{2}} - 25\left(\frac{2}{7}\right)y^{\frac{7}{2}}\right]_0^4$$

$$426.7\,\bar{y} = 731.4$$

$\bar{y} = 1.71$ ft **Ans.**

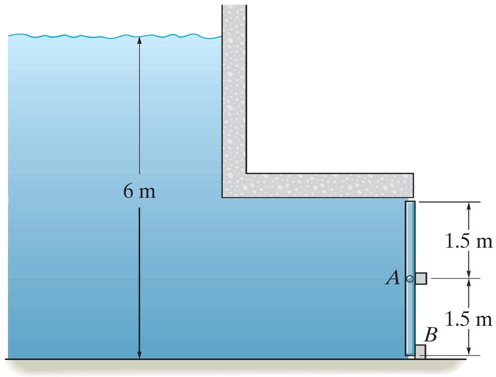
Due to symmetry,

$\bar{x} = 0$ **Ans.**

Ans:
$F_R = 427$ lb
$\bar{y} = 1.71$ ft
$\bar{x} = 0$

9–127.

The 2-m-wide rectangular gate is pinned at its center A and is prevented from rotating by the block at B. Determine the reactions at these supports due to hydrostatic pressure. $\rho_w = 1.0\ \text{Mg/m}^3$.



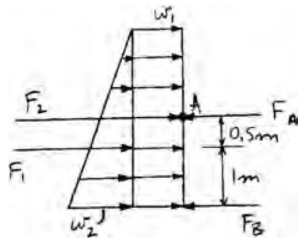
SOLUTION

$w_1 = 1000(9.81)(3)(2) = 58\,860\ \text{N/m}$

$w_2 = 1000(9.81)(3)(2) = 58\,860\ \text{N/m}$

$F_1 = \frac{1}{2}(3)(58\,860) = 88\,290$

$F_2 = (58\,860)(3) = 176\,580$



$\circlearrowleft + \Sigma M_A = 0;\qquad 88\,290(0.5) - F_B(1.5) = 0$

$F_B = 29\,430\ \text{N} = 29.4\ \text{kN}$ **Ans.**

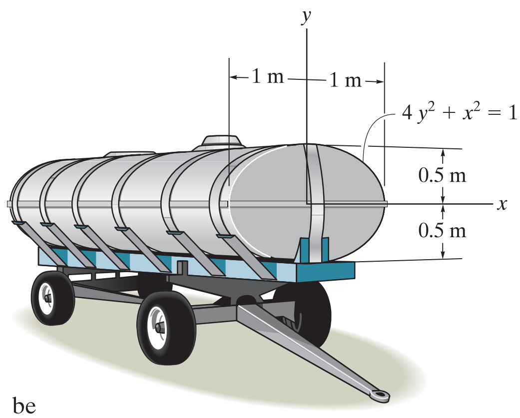
$\overset{+}{\rightarrow} \Sigma F_x = 0;\qquad 88\,290 + 176\,580 - 29\,430 - F_A = 0$

$F_A = 235\,440\ \text{N} = 235\ \text{kN}$ **Ans.**

Ans:
$F_B = 29.4\ \text{kN}$
$F_A = 235\ \text{kN}$

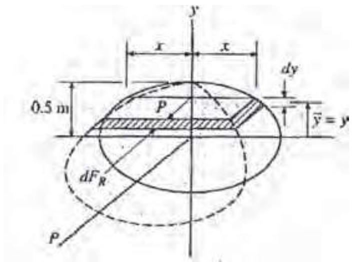
***9–128.**

The tank is filled with a liquid which has a density of 900 kg/m^3. Determine the resultant force that it exerts on the elliptical end plate, and the location of the center of pressure, measured from the x axis.



SOLUTION

Fluid Pressure: The fluid pressure at an arbitrary point along y axis can be determined using Eq. 9–13, $p = \gamma(0.5 - y) = 900(9.81)(0.5 - y) = 8829(0.5 - y)$.



Resultant Force and its Location: Here, $x = \sqrt{1 - 4y^2}$. The volume of the differential element is $dV = dF_R = p(2x\,dy) = 8829(0.5 - y)[2\sqrt{1 - 4y^2}]\,dy$. Evaluating integrals using Simpson's rule, we have

$$F_R = \int_{FR} d\,F_R = 17658 \int_{-0.5\text{ m}}^{0.5\text{ m}} (0.5 - y)(\sqrt{1 - 4y^2})dy$$

$$= 6934.2 \text{ N} = 6.93 \text{ kN} \qquad \textbf{Ans.}$$

$$\int_{F_R} \tilde{y}d\,F_R = 17658 \int_{-0.5\text{ m}}^{0.5\text{ m}} y(0.5 - y)(\sqrt{1 - 4y^2})dy$$

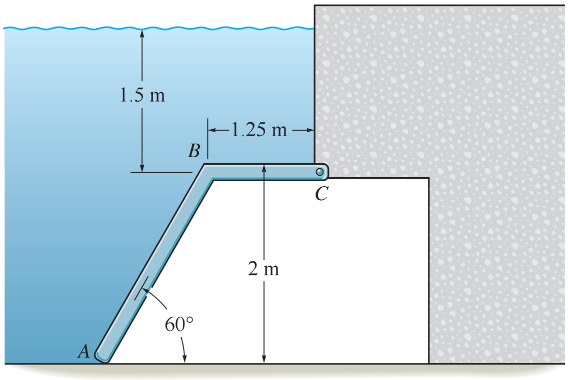
$$= -866.7 \text{ N} \cdot \text{m}$$

$$\bar{y} = \frac{\int_{F_R} \tilde{y}dF_R}{\int_{F_R} dF_R} = \frac{-866.7}{6934.2} = -0.125 \text{ m} \qquad \textbf{Ans.}$$

Ans:
$F_R = 6.93 \text{ kN}$
$\bar{y} = -0.125 \text{ m}$

9–129.

Determine the magnitude of the resultant force acting on the gate ABC due to hydrostatic pressure. The gate has a width of 1.5 m. $\rho_w = 1.0\ \text{Mg/m}^3$.



SOLUTION

$$w_1 = 1000(9.81)(1.5)(1.5) = 22.072\ \text{kN/m}$$

$$w_2 = 1000(9.81)(2)(1.5) = 29.43\ \text{kN/m}$$

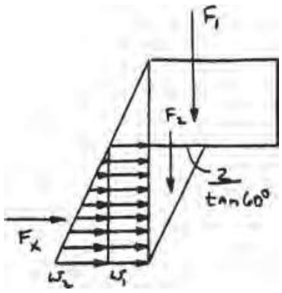
$$F_x = \frac{1}{2}(29.43)(2) + (22.0725)(2) = 73.58\ \text{kN}$$

$$F_1 = \left[(22.072)\left(1.25 + \frac{2}{\tan 60^\circ}\right)\right] = 53.078\ \text{kN}$$

$$F_2 = \frac{1}{2}(1.5)(2)\left(\frac{2}{\tan 60^\circ}\right)(1000)(9.81) = 16.99\ \text{kN}$$

$$F_y = F_1 + F_2 = 70.069\ \text{kN}$$

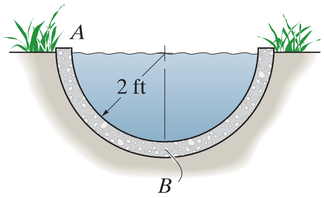
$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{(73.58)^2 + (70.069)^2} = 102\ \text{kN} \quad \textbf{Ans.}$$



Ans:
$F = 102\ \text{kN}$

9–130.

The semicircular drainage pipe is filled with water. Determine the resultant horizontal and vertical force components that the water exerts on the side AB of the pipe per foot of pipe length; $\gamma_w = 62.4\ \text{lb/ft}^3$.



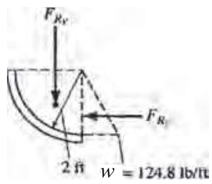
SOLUTION

Fluid Pressure: The fluid pressure at the bottom of the drain can be determined using Eq. 9–13, $p = \gamma z$.

$$p = 62.4(2) = 124.8\ \text{lb/ft}^2$$

Thus,

$$w = 124.8(1) = 124.8\ \text{lb/ft}$$



Resultant Forces: The area of the quarter circle is $A = \frac{1}{4}\pi r^2 = \frac{1}{4}\pi(2^2) = \pi\ \text{ft}^2$. Then, the vertical component of the resultant force is

$$F_{R_v} = \gamma V = 62.4[\pi(1)] = 196\ \text{lb} \qquad \textbf{Ans.}$$

and the horizontal component of the resultant force is

$$F_{R_h} = \frac{1}{2}(124.8)(2) = 125\ \text{lb} \qquad \textbf{Ans.}$$

Ans:
$F_{R_v} = 196\ \text{lb}$
$F_{R_h} = 125\ \text{lb}$