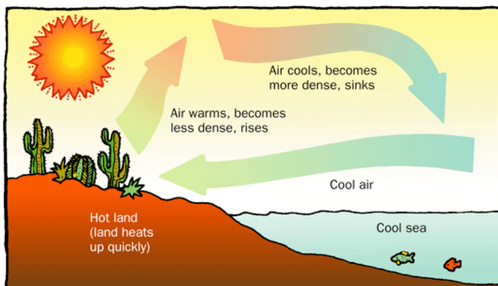
Renewable Energy Systems (12210588)

9.2 Calculations of the Wind Power

[a]Quaschning, V. (2005). Understanding Renewable Energy Systems. Earthscan, London, UK.



http://www.encyclopedia.com/topic/Coriolis_effect.aspx

Fathi Anayah, PhD

Lecture 9.2

Introduction[a]

1. The sun **heats up** air masses in the atmosphere. The spherical shape of the Earth, the **Earth's rotation** and seasonal and regional fluctuations of the solar irradiance cause spatial air pressure differentials.
2. These are the source of air movements. **Irradiation oversupply at the equator** is the source for compensating air streams between the equator and the poles.
3. Besides the spatial compensation streams, less extensive air currents exist due to the influence of local areas of **high** and **low pressure**.
4. There are two types of winds: **global** and **local** winds.

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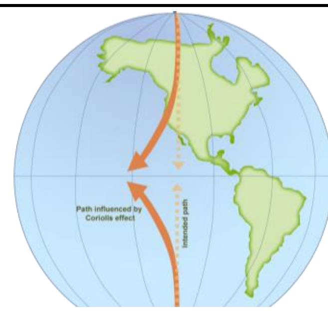
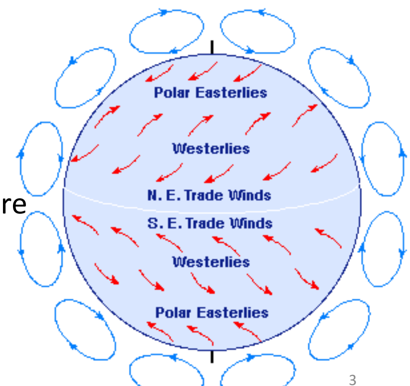
Global winds: Coriolis Effect[a]

- The Earth **rotates** on its axis, so circulating air is **deflected** or turns.
- the **Coriolis effect** is the apparent deflection of moving objects when the motion is described relative to a rotating reference frame.
- The **Coriolis force** diverts the compensating streams between high and low pressure areas.
- Due to the rotation of the Earth, the air masses in the **northern** hemisphere are diverted to the **right** and in the **southern** hemisphere to the **left**.
- Finally, the air masses rotate around the low-pressure areas.





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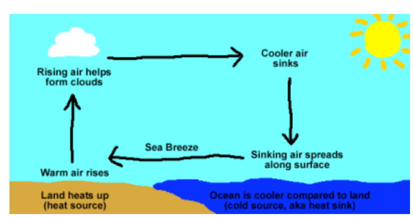
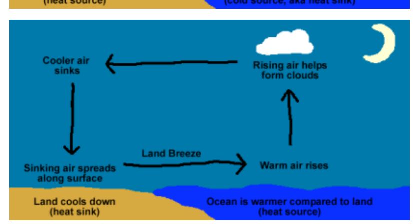
http://csep10.phys.utk.edu/astr161/lect/earth/coriolis.html

Local winds: Sea and land breezes[a]

- Wind resources are particularly high in **coastal** areas because **temperature differences** between water and land cause local **compensating streams**.

1. The **sunlight** heats the land more quickly than the water during the day. The results are compensating winds in the direction of the land. These winds can reach up to 50 km inland.
2. During the **night** the land cools much faster than the sea; this causes compensating winds in the opposite direction.





A **sea breeze** is a wind blowing from the water onto the land.
A **land breeze** is a breeze blowing from land out toward a body of water.

http://science.jrank.org/pages/3800/Land-Sea-Breezes.html

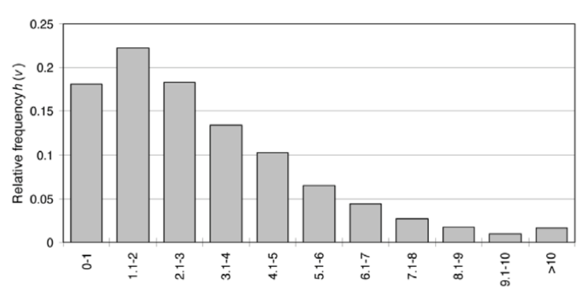
Table 5.1 *Wind Speed Classification of the Beaufort Wind Scale*[a]

Bf	*v in m/s*	*Description*	*Effects*
0	0–0.2	Calm	Smoke rises vertically
1	0.3–1.5	Light air	Smoke moves slightly and shows direction of wind
2	1.6–3.3	Light breeze	Wind can be felt. Leaves start to rustle
3	3.4–5.4	Gentle breeze	Small branches start to sway. Wind extends light flags
4	5.5–7.9	Moderate breeze	Larger branches sway. Loose dust on ground moves
5	8.0–10.7	Fresh breeze	Small trees sway
6	10.8–13.8	Strong breeze	Trees begin to bend, whistling in wires
7	13.9–17.1	Moderate gale	Large trees sway
8	17.2–20.7	Fresh gale	Twigs break from trees
9	20.8–24.4	Strong gale	Branches break from trees, minor damage to buildings
10	24.5–28.4	Full gale/storm	Trees are uprooted
11	28.5–32.6	Violent storm	Widespread damage
12	⩾ 32.7	Hurricane	Structural damage

Note: Bf, Beaufort force; v, wind speed in m/s (1 m/s = 3.6 km/h = 2.24 mph)

Wind speed distributions[a]

- The **mean wind speed** can only partly describe the **potential** of a site, because the wind distribution may be continuous wind or long calm periods interspersed with periods of very high wind speeds. The wind energy in these two cases can be totally different. Nevertheless, the **mean wind speed** is often used to give the **site quality**.
- A wind speed **frequency distribution** gives much **better** information about the wind conditions of a certain site than the **mean wind speed**.
- Most common statistical functions used for wind power calculations are the **Weibull** and the **Rayleigh** distributions



Power content of wind[a]

1. The kinetic energy E carried by a wind with speed v:

$$E = \tfrac{1}{2} \cdot m \cdot v^2$$

1. The density ρ and volume V determine the mass:

$$m = \rho \cdot V$$

1. The power P that the wind contains is calculated by differentiating the energy with respect to time:

$$P = \dot{E} = \tfrac{1}{2} \cdot \dot{m} \cdot v^2$$

1. The derivative with respect to time results in the air mass flow

$$\dot{m} = \rho \cdot \dot{V} = \rho \cdot A \cdot \dot{s} = \rho \cdot A \cdot v$$

2. Hence the power of the wind becomes:

$$P = \tfrac{1}{2} \cdot \rho \cdot A \cdot v^3$$

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Energy conversion of wind power[a]

1. Conservation of mass flow entails:

$$\dot{m} = \rho \cdot \dot{V} = \rho \cdot A_1 \cdot v_1 = \rho \cdot A \cdot v = \rho \cdot A_2 \cdot v_2 = const.$$

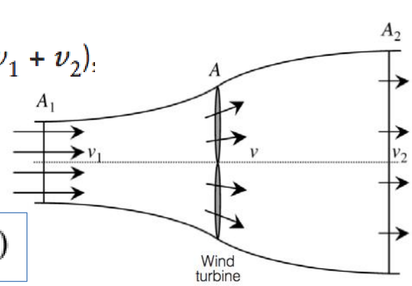
1. The wind velocity:

$$v = \tfrac{1}{2} \cdot (v_1 + v_2)$$

1. The power P_T taken from the wind can be calculated from the difference in wind speeds:

$$P_T = \tfrac{1}{2} \cdot \dot{m} \cdot (v_1^2 - v_2^2)$$

With $\dot{m} = \rho \cdot A \cdot v = \rho \cdot A \cdot \frac{1}{2} \cdot (v_1 + v_2)$:



$$P_T = \tfrac{1}{4} \cdot \rho \cdot A \cdot (v_1 + v_2) \cdot (v_1^2 - v_2^2)$$

The power coefficient (c_P)[a]

1. The power P_0 of the wind through the area A without the influence of the wind turbine

$$P_0 = \tfrac{1}{2} \cdot \rho \cdot A \cdot v_1^3$$

2. The ratio of the power used by the turbine P_T to the power content P_0 of the wind is called the power coefficient (c_P):

$$c_P = \frac{P_T}{P_0} = \frac{(v_1 + v_2)\cdot(v_1^2 - v_2^2)}{2 \cdot v_1^3} = \frac{1}{2} \cdot (1 + \frac{v_2}{v_1}) \cdot (1 - \frac{v_2^2}{v_1^2})$$

3. Betz has calculated the maximum power coefficient ($c_{P,Betz}$):

With $\zeta = \frac{v_2}{v_1}$ and $\frac{\mathrm{d}c_P}{\mathrm{d}\zeta} = \frac{\mathrm{d}\left(\tfrac{1}{2}\cdot(1+\zeta)\cdot(1-\zeta^2)\right)}{\mathrm{d}\zeta} = -\tfrac{3}{2}\cdot\zeta^2 - \zeta + \tfrac{1}{2} = 0$

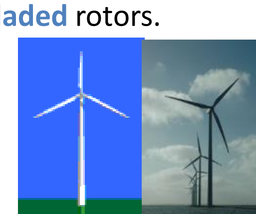
$$\zeta_{id} = \frac{v_2}{v_1} = \frac{1}{3} \quad \Rightarrow \quad c_{P,Betz} = \frac{16}{27} \approx 0.593$$

4. Power efficiency:

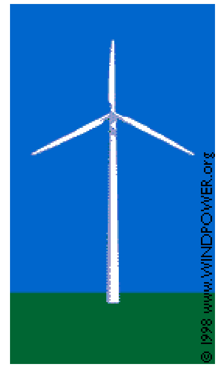
$$\eta = \frac{P_T}{P_{id}} = \frac{P_T}{P_0 \cdot c_{P,Betz}} = \frac{P_T}{\tfrac{1}{2} \cdot \rho \cdot A \cdot v_1^3 \cdot c_{P,Betz}} = \frac{c_P}{c_{P,Betz}}$$

Number of rotor blades[a]

- Modern horizontal axis wind generators can have **one**, **two** or **three** rotor blades. **More than three** blades are usually not used.
- The lower the number of rotor blades, the **less material** is needed during manufacturing.
- **Single-bladed** *rotors* must have a counterweight on the opposite side of the rotor. Single-bladed rotors do not have a smooth motion and therefore exhibit a **high material stress**.
- The optimal power coefficient of **three-bladed** rotors is slightly above that of two-bladed rotors.
- **Three-bladed** rotors have an optically smoother operation and hence visually integrate better into the landscape.
- The **mechanical strain** is also lower for **three-bladed** rotors.



Active Yaw and Pitch Control



- Active Yaw (all medium & large turbines produced today, & some small turbines from Europe)
  - Anemometer on nacelle tells controller which way to point rotor into the wind
  - Yaw drive turns gears to point rotor into wind
- Pitch Control
  - Blades rotate out of the wind when wind speed becomes too great

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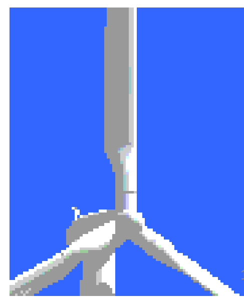

Example: Power Generated by Wind Turbine

How much power a wind turbine with 50 meters long blade can generate with a wind speed of 12 m/s? The site of the installation is about 1000 feet above sea level. Assume 40% efficiency (η).

Air density is lower at higher elevation. For 1000 feet above sea level, ρ is about 1.16 kg/m^3

Power = ½ $(\rho)(A)(V)^3$ (η)

$= 0.5(1.16)(\pi 50^2)(12)^3(0.4)$

$= 3.15 \times 10^6$ Watt

= 3.15 MW

where we assumed the turbine efficiency is 40%.

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