Renewable Energy Systems (12210588)
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10. Wind Distribution and Tutorial 2

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Lecture 10

Wind speed distributions

- Wind speed distributions are commonly used to indicate the annual available wind energy
- These distributions are estimated using measurements, wind maps or computer analysis
- Tables or statistical functions can give the distribution
- Good wind maps that show the mean wind speed exist for most countries

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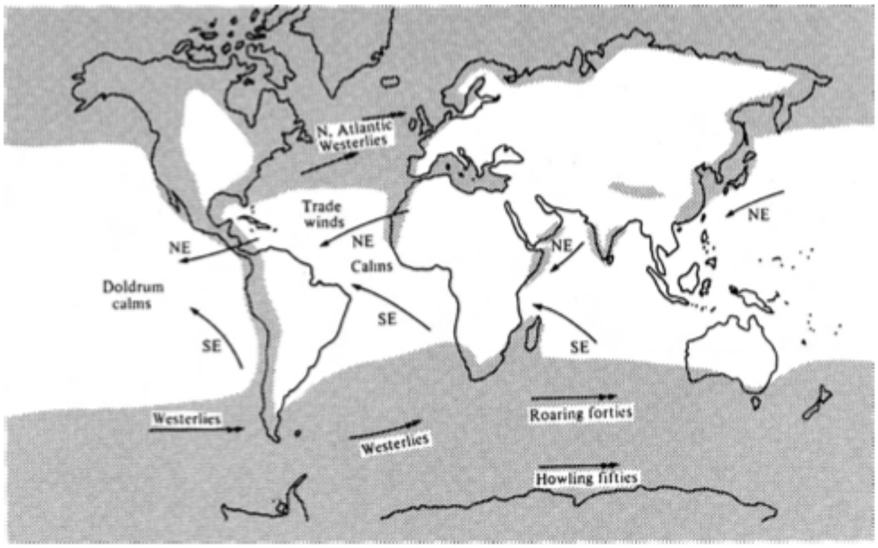

Figure 9.2 Prevailing strong winds. The shaded areas indicate regions of wind attractive for wind power development, with average wind speed $>5\,m\,s^{-1}$ and average generation ~33% of rated power. Note the importance of marine situations and upland sites.

Wind speed distributions

- In coastal areas, mean wind speeds of 6 m/s or more can be reached; in inland areas it can be below 3 m/s
- Mountainous regions also offer good wind conditions
- A wind speed frequency distribution gives much better information about the wind conditions of a certain site than the mean wind speed
- The most common statistical functions that are used for wind power calculations are the Weibull and the Rayleigh distributions

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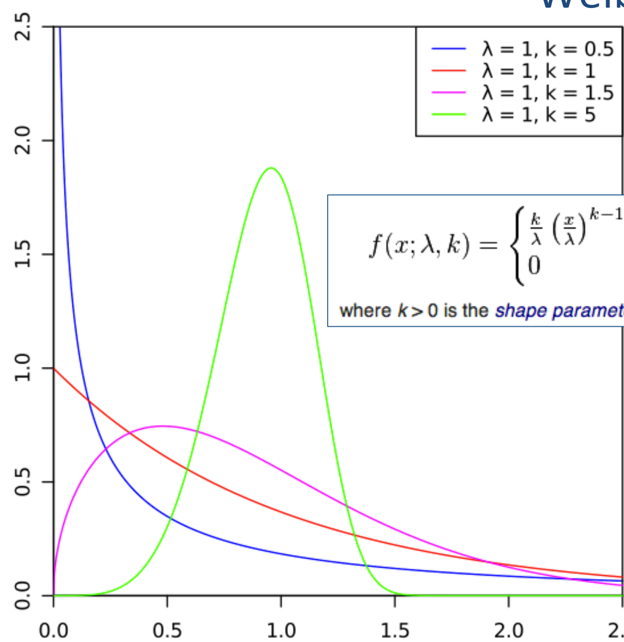
Weibull distribution



$$f(x;\lambda,k) = \begin{cases} \frac{k}{\lambda}\left(\frac{x}{\lambda}\right)^{k-1} e^{-(x/\lambda)^k} & x \geq 0, \\ 0 & x < 0, \end{cases}$$

where $k > 0$ is the *shape parameter* and $\lambda > 0$ is the *scale parameter*

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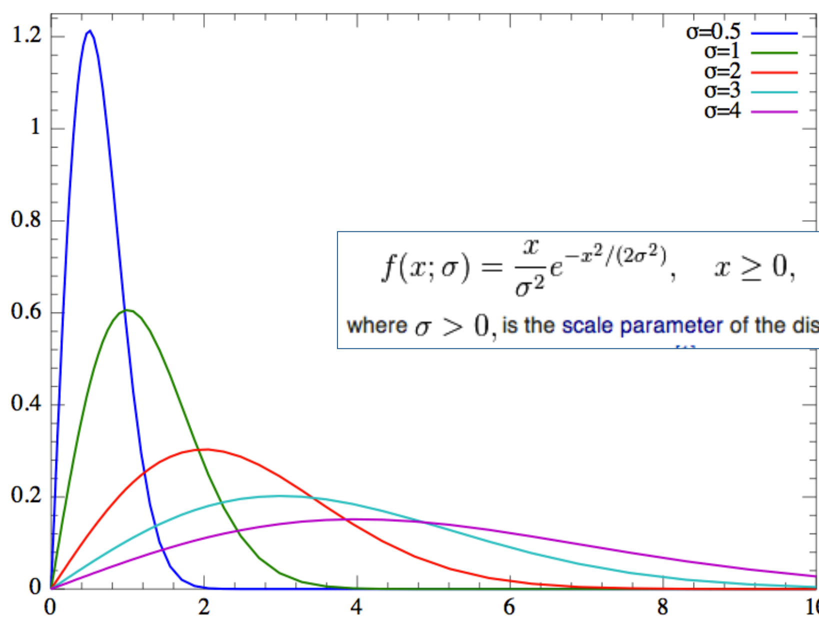
Rayleigh distribution



$$f(x;\sigma) = \frac{x}{\sigma^2} e^{-x^2/(2\sigma^2)}, \quad x \geq 0,$$

where $\sigma > 0$, is the scale parameter of the distribution.

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The Rayleigh distribution needs only the average wind speed as a parameter

Note:
For the Rayleigh PDF, the cumulative distribution function is:

$$prob(u \le U) = F(U) = 1 - \exp\left[-\frac{\pi}{4}\left(\frac{U}{u_{av}}\right)^2\right]$$

$$prob(u > U) = F(U) = \exp\left[-\frac{\pi}{4}\left(\frac{U}{u_{av}}\right)^2\right]$$

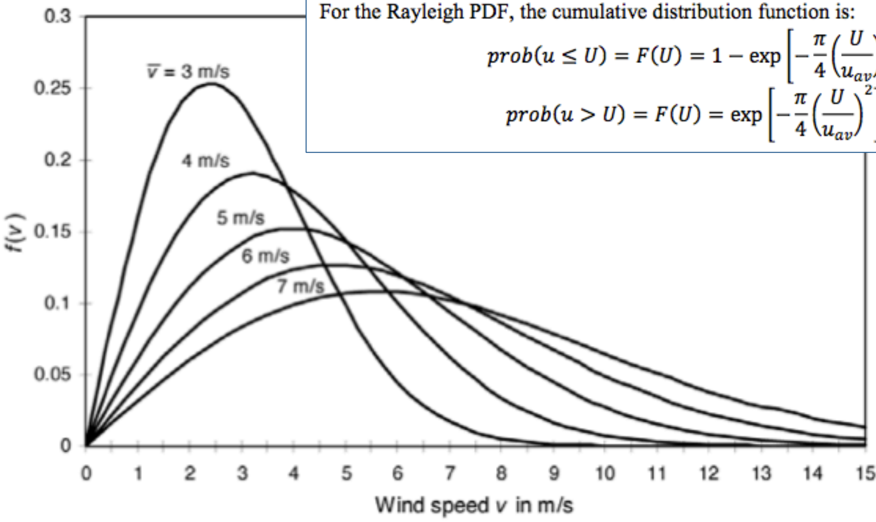
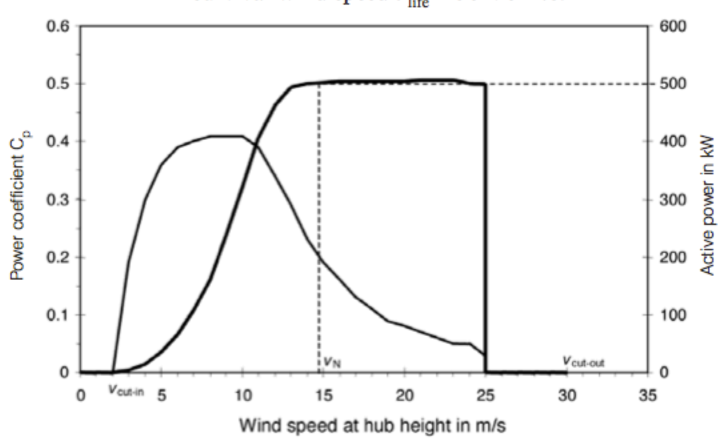


Figure 5.2 *Rayleigh Distributions for Different Mean Wind Speeds* $\bar{v}$

- cut-in wind speed $v_{\text{cut-in}}$ = 2.5–4.5 m/s
- design wind speed v_D = 6–10 m/s
- nominal wind speed v_N = 10–16 m/s
- cut-out wind speed $v_{\text{cut-out}}$ = 20–30 m/s
- survival wind speed v_{life} = 50–70 m/s.



Source: data from Enercon, 1997

Figure 5.13 *Generator Active Power and Power Coefficient against Wind Speed for the 500-kW Enercon E-40 Wind Generator*

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Output Power of Wind I

$$C_p = \frac{\text{Turbine power}}{\text{Power content of wind}} = \frac{P_T}{P_o}$$

When wind speed has a Rayleigh distribution

$$C_p = \frac{P_{output}}{P_{rated}} = \frac{P_{out}}{P_r}$$

$$C_p = 0.087 \times V_{average} - \frac{P_r}{1000D^2}$$

$$P_{out} = 0.087 \times V_{average} \times P_r - \frac{P_r^2}{1000D^2}$$

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Output Power of Wind II

$$\eta = \frac{P_T}{P_{ideal}} = \frac{P_T}{\frac{1}{2}\rho A V_{average}^3 C_{P.Betz}} = \frac{C_P}{P_{P.Betz}}$$

Since wind speed has a Rayleigh distribution

$$\eta = \frac{P_{out}}{P_{wind}} = \frac{P_{out}}{P_w} \rightarrow P_w = \frac{1}{2}\rho A V_{average}^3 \times \frac{6}{\pi}$$

$$\eta = \frac{0.087 \times V_{average} \times P_r - \frac{P_r^2}{1000D^2}}{\frac{1}{2}\rho A V_{average}^3 \times \frac{6}{\pi}}$$

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Tutorial 2

Question 1

Suppose an anemometer mounted at a height of 10 m on a level field with tall grass (α=0.15) shows an average wind speed of 6 m/s.

a) Assuming Rayleigh statistics and an air density of 1.225 kg/m3 estimate the average wind power (W/m2) at a height of 80 m.
b) Suppose a 1300 kW wind turbine with 60 m rotor diameter has its hub located at a height of 80 m in this site. Estimate the annual energy delivered (kWh/yr) if the turbine has an overall efficiency of 30%.
c) What would the turbine's power coefficient (capacity factor) be?

Question 1 - Solution

a) Assuming Rayleigh statistics and an air density of 1.225 kg/m^3, estimate the average wind power (W/m^2) at a height of 80 m.

$$\frac{V_{80}}{V_{10}} = \left(\frac{h_{80}}{h_{10}}\right)^{\alpha} \rightarrow V_{10} = 6m/s, h_{10} = 10m, \& \alpha = 0.15$$

$$\frac{V_{80}}{6} = \left(\frac{80}{10}\right)^{0.15} \rightarrow V_{80} = 6 \times (8)^{0.15} = 8.2m/s$$

$$P_w = \frac{1}{2}\rho A V_{average}^3 \times \frac{6}{\pi} = \frac{1}{2} \times (1) \times 1.225 \times (8.2)^3 \times \frac{6}{\pi}$$

$$P_w = 644.7 Watt/m^2$$

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Question 1 - Solution

b) Suppose a 1300 kW wind turbine with 60 m rotor diameter has its hub located at a height of 80 m in this site. Estimate the annual energy delivered (kWh/yr) if the turbine has an overall efficiency of 30%.

$$E_{annual} = P_{out} \times 8760$$

$$\eta = \frac{P_{out}}{P_w} = 0.3 \rightarrow P_{out} = 0.3 \times 644.7 = 193.4W/m^2$$

$$E_{annual} = 193.4W/m^2 \times 8760 hr/yr$$

$$E_{annual} = 1.69 MWhr/yr/m^2$$

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Question 1 - Solution

c) What would the turbine's power coefficient (capacity factor) be?

$$C_p = 0.087 \times V_{average} - \frac{P_r}{1000 D^2} \rightarrow P_r = 1300kW$$

$$C_p = 0.087 \times 8.2 - \frac{1300 \times 10^3}{1000 \times 60^2} = 0.352$$

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Question 2

The wind speeds at a certain location can be represented by Rayleigh statistics. Using the approximate relation for the capacity factor of a wind turbine, derive an expression for the wind speed that yields to the highest efficiency for the wind turbine as a function of its rated power and blade diameter. What is the optimum wind speed for:

a) The NEG/Micon1000/60 wind turbine
b) The NEG/Micon 1000/54 wind turbine

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Question 2 - Solution

Since wind speed has a Rayleigh distribution

$$\eta = \frac{P_{out}}{P_{wind}} = \frac{P_{out}}{P_w}$$

$$\rightarrow P_w = \frac{1}{2}\rho A V_{average}^3 \times \frac{6}{\pi} \;\&\; P_{out} = C_p P_r$$

$$\eta = \frac{0.087 \times V_{average} \times P_r - \frac{P_r^2}{1000 D^2}}{\frac{1}{2}\rho A V_{average}^3 \times \frac{6}{\pi}}$$

$$\frac{d\eta}{dV_{average}} = 0 \rightarrow V_{average} = 0.017 \times \frac{P_r}{D^2} \rightarrow \eta_{\max imum}$$

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Question 2 - Solution

a) The NEG/Micon 1000/60 wind turbine

$$V_{average} = 0.017 \times \frac{P_r}{D^2} = 0.017 \times \frac{1000}{60^2} = 4.77 m/s$$

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b) The NEG/Micon 1000/54 wind turbine

$$V_{average} = 0.017 \times \frac{P_r}{D^2} = 0.017 \times \frac{1000}{54^2} = 5.89 m/s$$

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Question 3

A wind turbine is used to deliver 90000 kWh in a 30-day month using a 350 kW three phase synchronous generator. The air density is 1.225 kg/m^3.

a) What power coefficient (capacity factor) would be needed for the machine?
b) If the average wind speed is 5.25 m/s, and Rayleigh statistics apply, what should the rotor diameter be? (Note: use the approximate relation for the capacity factor)
c) If at a certain moment the turbine is producing its rated power (350 kW), what would the wind speed be if the machine is 35% efficient at this operating point?
d) If the tip-speed ratio is assumed to be 4, what gear ratio would be needed to match the speed of the blades to the speed of the rotor of the generator if the generator needs to spin at 1500 rpm to deliver its rated power?

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Question 3 - Solution

a) What capacity factor would be needed for the machine?

$$C_p = \frac{\text{Energy delivered per month}}{\text{P}_r \times \text{ No. of hours per month}}$$

$$C_p = \frac{90000 \times 10^3}{(350 \times 10^3) \times (30 \times 24)} = 0.357$$

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b) If the average wind speed is 5.25 m/s, and Rayleigh statistics apply, what should the rotor diameter be? (Note: use the approximate relation for the capacity factor)

$$C_p = 0.087 \times V_{average} - \frac{P_r}{1000D^2}$$

$$0.357 = 0.087 \times 5.25 - \frac{350 \times 10^3}{1000 \times D^2} \rightarrow D = 59.23m$$

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Question 3 - Solution

c) If at a certain moment the turbine is producing its rated power (350 kW), what would the wind speed be if the machine is 35% efficient at this operating point?

$$\eta = \frac{P_{out}}{P_{wind}} = \frac{P_{out}}{\frac{1}{2}\rho A V_{average}^3 \times \frac{6}{\pi}}$$

$$0.35 = \frac{350 \times 10^3}{\frac{1}{2}(1.225)(\frac{\pi}{4}59.23^2) \times V^3 \times \frac{6}{\pi}}$$

$$V = 8.4 m/s$$

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Question 3 - Solution

d) If the tip-speed ratio is assumed to be 4, what gear ratio would be needed to match the speed of the blades to the speed of the rotor of the generator if the generator needs to spin at 1500 rpm to deliver its rated power?

$$TSR = \frac{\pi D \times N}{60 \times V} \rightarrow 4 = \frac{\pi(59.23) \times N}{60 \times 8.4}$$

$$N = 10.8 rpm$$

$$\text{Gear Ratio} = \frac{1500}{10.8} = 138.44 rpm/rpm$$

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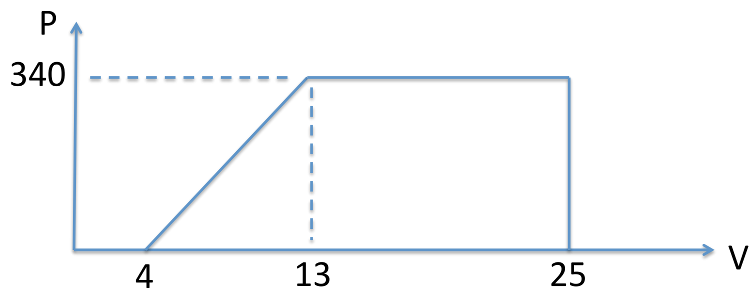
Question 4

A 31 m, 340 kW wind turbine is installed at a site that a wind speed statistics represented by Rayleigh PDF with an average wind speed of 8 m/s. The ideal power curve of the turbine has a cut-in speed of 4 m/s, a rated speed of 13 m/s and cut-out speed of 25 m/s.

a) Sketch the ideal wind power curve for this turbine.



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Question 4 - Solution

b) Find the number of hours per year at which the wind speed is less than or equal 13 m/s.

c) Find the number of hours per year at which the wind speed is more than 25m/s.

Note:
For the Rayleigh PDF, the cumulative distribution function is:

$$prob(u \leq U) = F(U) = 1 - \exp\left[-\frac{\pi}{4}\left(\frac{U}{u_{av}}\right)^2\right]$$

$$prob(u > U) = F(U) = \exp\left[-\frac{\pi}{4}\left(\frac{U}{u_{av}}\right)^2\right]$$

b) Prob(V<=13) = 0.87

→ # of hours = 0.87*8760 = 7621 hours

c) Prob(V>25) = 0.000467

→ # of hours = 0.000467*8760 = 4 hours

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Question 4 - Solution

d) Calculate the annual energy (kWh/yr) that the wind turbine will deliver when the wind speed is between its rated value the cut-out speed.

Prob(13 <= V <= 25) = 1 – Prob(V<=13) – Prob(V>25)
= 1- 0.87 - 0.000467 = 0.125

→ # of hours = 0.125 * 8760 = 1095 hr/year

E(13 <= V < 25) = P_r * 1095 = 340 kW * 1095 hr/year
= 372.3 MW.hr/year

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Question 4 - Solution

e) Using the approximate relation for the capacity factor, estimate the fraction of the annual energy delivered with winds that are above the rated wind speed.

$$C_p = 0.087 \times V_{average} - \frac{P_r}{1000D^2} = 0.342$$

$$0.342 = \frac{E_{annual}}{P_r \times 8760} \rightarrow E_{annual} = 1018.6 MWh/yr$$

$$\% = \frac{372.3}{1018.6} \times 100\% = 36.5\%$$

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f) Find the annual energy in the wind if efficiency of the wind turbine rotor is 46%, the gear box is 60% & the generator efficiency is 93%.

$$\eta = \eta_1 \times \eta_2 \times \eta_3 = 0.46 \times 0.60 \times 0.93 = 0.26$$

$$E_w = E_{annual} / \eta = 1018.6 / 0.26$$

$$E_w = 3917.7 MWhr / yr$$

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Homework 2 – due date is April 30th

A 40 m diameter, 700 kW wind turbine has its hub installed at a height of 50 m in a site with wind speeds that can be represented by Rayleigh wind statistics having an average wind speed of 8 m/s. The cut-in speed of the wind turbine is 3 m/s, the rated speed is 14 m/s and the cut-out speed is 23 m/s. The gear ratio of the gear box connecting the shaft of the turbine to the shaft of the 4 pole three phase synchronous generator of the turbine is 1:45. If the overall efficiency of the wind turbine is assumed to be constant at all wind speeds and the air density at the hub of the turbine is 1.225 kg/m^3, find:

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Homework 2 – due date is Lecture 12

a) The rotational speed of the rotor at rated wind speed if the tip speed ratio is 5.0.
b) The speed of the rotor of the generator.
c) Find the number of hours per year at which the wind speed is less than or equal 13 m/s.
d) The output power of the turbine at a wind speed of 7 m/s.
e) The output power of the turbine at a wind speed of 25 m/s.
f) The rated output power of the turbine if the hub height is 65m, given that the friction coefficient is assumed to be unchanged.
g) The annual output energy that can be generated from the wind turbine.

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