24

Gauss's Law

CHAPTER OUTLINE

24.1 Electric Flux
24.2 Gauss's Law
24.3 Application of Gauss's Law to Various Charge Distributions
24.4 Conductors in Electrostatic Equilibrium

ANSWERS TO QUESTIONS

Q24.1 The net flux through any gaussian surface is zero. We can argue it two ways. Any surface contains zero charge, so Gauss's law says the total flux is zero. The field is uniform, so the field lines entering one side of the closed surface come out the other side and the net flux is zero.

***Q24.2** (i) Equal amounts of flux pass through each of the six faces of the cube. Answer (e).
(ii) Move the charge to very close below the center of one face, through which the flux is then $q/2\epsilon_0$. Answer (c).
(iii) Move the charge onto one of the cube faces. Then the field has no component perpendicular to this face and the flux is zero. Answer (a).

Q24.3 The luminous flux on a given area is less when the sun is low in the sky, because the angle between the rays of the sun and the local area vector, $d\vec{\mathbf{A}}$, is greater than zero. The cosine of this angle is reduced. The decreased flux results, on the average, in colder weather.

Q24.4 The surface must enclose a positive total charge.

***Q24.5** (i) Both spheres create equal fields at exterior points, like particles at the centers of the spheres. Answer (c).
(ii) The field within the conductor is zero. The field within the insulator is 4/5 of its surface value. Answer (f).

Q24.6 Gauss's law cannot tell the different values of the electric field at different points on the surface. When E is an unknown number, then we can say $\int E\cos\theta dA = E\int\cos\theta dA$. When $E(x, y, z)$ is an unknown function, then there is no such simplification.

Q24.7 The electric flux through a sphere around a point charge is independent of the size of the sphere. A sphere of larger radius has a larger area, but a smaller field at its surface, so that the product of field strength and area is independent of radius. If the surface is not spherical, some parts are closer to the charge than others. In this case as well, smaller projected areas go with stronger fields, so that the net flux is unaffected.

***Q24.8** The outer wall of the conducting shell will become polarized to cancel out the external field. The interior field is the same as before. Answer (c).

***Q24.9** (a) Let q represent the charge of the insulating sphere. The field at A is $(4/5)^3 q/[4\pi(4\text{ cm})^2\epsilon_0]$. The field at B is $q/[4\pi(8\text{ cm})^2\epsilon_0]$. The field at C is zero. The field at D is $q/[4\pi(16\text{ cm})^2\epsilon_0]$. The ranking is A > B > D > C.

(b) The flux through the 4-cm sphere is $(4/5)^3 q/\epsilon_0$. The flux through the 8-cm sphere and through the 16-cm sphere is q/ϵ_0. The flux through the 12-cm sphere is 0. The ranking is B = D > A > C.

Q24.10 Inject some charge at arbitrary places within a conducting object. Every bit of the charge repels every other bit, so each bit runs away as far as it can, stopping only when it reaches the outer surface of the conductor.

Q24.11 If the person is uncharged, the electric field inside the sphere is zero. The interior wall of the shell carries no charge. The person is not harmed by touching this wall. If the person carries a (small) charge q, the electric field inside the sphere is no longer zero. Charge $-q$ is induced on the inner wall of the sphere. The person will get a (small) shock when touching the sphere, as all the charge on his body jumps to the metal.

***Q24.12** (i) The shell becomes polarized. Answer (e).
(ii) The net charge on the shell's inner and outer surfaces is zero. Answer (a).
(iii) Answer (c).
(iv) Answer (c).
(v) Answer (a).

SOLUTIONS TO PROBLEMS

Section 24.1 **Electric Flux**

P24.1 (a) $\Phi_E = \vec{\mathbf{E}}\cdot\vec{\mathbf{A}} = \left(a\hat{\mathbf{i}} + b\hat{\mathbf{j}}\right)\cdot A\hat{\mathbf{i}} = \boxed{aA}$

(b) $\Phi_E = \left(a\hat{\mathbf{i}} + b\hat{\mathbf{j}}\right)\cdot A\hat{\mathbf{j}} = \boxed{bA}$

(c) $\Phi_E = \left(a\hat{\mathbf{i}} + b\hat{\mathbf{j}}\right)\cdot A\hat{\mathbf{k}} = \boxed{0}$

P24.2 $\Phi_E = EA\cos\theta = \left(2.00\times10^4\text{ N/C}\right)\left(18.0\text{ m}^2\right)\cos 10.0° = \boxed{355\text{ kN}\cdot\text{m}^2/\text{C}}$

P24.3 $\Phi_E = EA\cos\theta \qquad A = \pi r^2 = \pi(0.200)^2 = 0.126\text{ m}^2$

$5.20\times10^5 = E(0.126)\cos 0° \qquad E = 4.14\times10^6\text{ N/C} = \boxed{4.14\text{ MN/C}}$

P24.4 (a) $A' = (10.0\text{ cm})(30.0\text{ cm})$

$A' = 300\text{ cm}^2 = 0.030\,0\text{ m}^2$

$\Phi_{E,A'} = EA'\cos\theta$

$\Phi_{E,A'} = \left(7.80\times10^4\right)\left(0.030\,0\right)\cos 180°$

$\Phi_{E,A'} = \boxed{-2.34\text{ kN}\cdot\text{m}^2/\text{C}}$

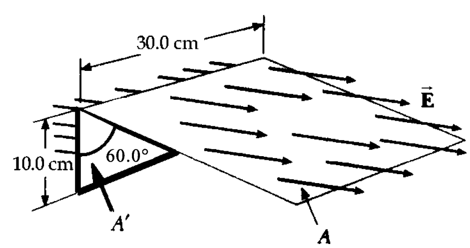

FIG. P24.4

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(b) $\Phi_{E,A} = EA\cos\theta = \left(7.80\times10^4\right)(A)\cos 60.0°$

$$A = (30.0\text{ cm})(w) = (30.0\text{ cm})\left(\frac{10.0\text{ cm}}{\cos 60.0°}\right) = 600\text{ cm}^2 = 0.060\,0\text{ m}^2$$

$$\Phi_{E,A} = \left(7.80\times10^4\right)(0.060\,0)\cos 60.0° = \boxed{+2.34\text{ kN}\cdot\text{m}^2/\text{C}}$$

(c) The bottom and the two triangular sides all lie *parallel* to $\vec{\mathbf{E}}$, so $\Phi_E = 0$ for each of these. Thus,

$$\Phi_{E,\text{total}} = -2.34\text{ kN}\cdot\text{m}^2/\text{C} + 2.34\text{ kN}\cdot\text{m}^2/\text{C} + 0 + 0 + 0 = \boxed{0}$$

P24.5 $\Phi_E = EA\cos\theta$ through the base

$$\Phi_E = (52.0)(36.0)\cos 180° = -1.87\text{ kN}\cdot\text{m}^2/\text{C}$$

Note that the same number of electric field lines go through the base as go through the pyramid's surface (not counting the base).

For the slanting surfaces, $\boxed{\Phi_E = +1.87\text{ kN}\cdot\text{m}^2/\text{C}}$.

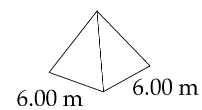

FIG. P24.5

Section 24.2 **Gauss's Law**

P24.6 (a) One-half of the total flux created by the charge q goes through the plane. Thus,

$$\Phi_{E,\text{plane}} = \frac{1}{2}\Phi_{E,\text{total}} = \frac{1}{2}\left(\frac{q}{\epsilon_0}\right) = \boxed{\frac{q}{2\epsilon_0}}$$

(b) The square looks like an infinite plane to a charge *very close* to the surface. Hence,

$$\Phi_{E,\text{square}} \approx \Phi_{E,\text{plane}} = \boxed{\frac{q}{2\epsilon_0}}$$

(c) $\boxed{\text{The plane and the square look the same to the charge.}}$

P24.7 (a) $\Phi_E = \frac{q_{\text{in}}}{\epsilon_0} = \frac{(+5.00\ \mu\text{C} - 9.00\ \mu\text{C} + 27.0\ \mu\text{C} - 84.0\ \mu\text{C})}{8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2} = -6.89\times10^6\text{ N}\cdot\text{m}^2/\text{C}^2$

$$\Phi_E = \boxed{-6.89\text{ MN}\cdot\text{m}^2/\text{C}}$$

(b) Since the net electric flux is negative, more lines enter than leave the surface.

P24.8 (a) $E = \frac{k_e Q}{r^2}$; $\qquad 8.90\times10^2 = \frac{\left(8.99\times10^9\right)Q}{(0.750)^2}$

But Q is negative since $\vec{\mathbf{E}}$ points inward. $\qquad Q = -5.57\times10^{-8}\text{ C} = \boxed{-55.7\text{ nC}}$

(b) The $\boxed{\text{negative}}$ charge has a $\boxed{\text{spherically symmetric}}$ charge distribution, concentric with the spherical shell.

P24.9 $\Phi_E = \dfrac{q_{\text{in}}}{\epsilon_0}$

Through S_1 $\quad \Phi_E = \dfrac{-2Q+Q}{\epsilon_0} = \boxed{-\dfrac{Q}{\epsilon_0}}$

Through S_2 $\quad \Phi_E = \dfrac{+Q-Q}{\epsilon_0} = \boxed{0}$

Through S_3 $\quad \Phi_E = \dfrac{-2Q+Q-Q}{\epsilon_0} = \boxed{-\dfrac{2Q}{\epsilon_0}}$

Through S_4 $\quad \Phi_E = \boxed{0}$

P24.10 (a) $\Phi_{E,\text{shell}} = \dfrac{q_{\text{in}}}{\epsilon_0} = \dfrac{12.0\times10^{-6}}{8.85\times10^{-12}} = 1.36\times10^{6}\ \text{N}\cdot\text{m}^2/\text{C} = \boxed{1.36\,\text{MN}\cdot\text{m}^2/\text{C}}$

(b) $\Phi_{E,\text{half shell}} = \dfrac{1}{2}\left(1.36\times10^{6}\ \text{N}\cdot\text{m}^2/\text{C}\right) = 6.78\times10^{5}\ \text{N}\cdot\text{m}^2/\text{C} = \boxed{678\,\text{kN}\cdot\text{m}^2/\text{C}}$

(c) $\boxed{\text{No}}$, the same number of field lines will pass through each surface, no matter how the radius changes.

P24.11 The total charge is $Q-6|q|$. The total outward flux from the cube is $\dfrac{Q-6|q|}{\epsilon_0}$, of which one-sixth goes through each face:

$$\left(\Phi_E\right)_{\text{one face}} = \frac{Q-6|q|}{6\epsilon_0}$$

$$\left(\Phi_E\right)_{\text{one face}} = \frac{Q-6|q|}{6\epsilon_0} = \frac{(5.00-6.00)\times10^{-6}\ \text{C}\cdot\text{N}\cdot\text{m}^2}{6\times8.85\times10^{-12}\ \text{C}^2} = \boxed{-18.8\ \text{kN}\cdot\text{m}^2/\text{C}}$$

.

P24.12 The total charge is $Q-6|q|$. The total outward flux from the cube is $\dfrac{Q-6|q|}{\epsilon_0}$, of which one-sixth goes through each face:

$$\left(\Phi_E\right)_{\text{one face}} = \boxed{\frac{Q-6|q|}{6\epsilon_0}}$$

P24.13 (a) With δ very small, all points on the hemisphere are nearly at a distance R from the charge, so the field everywhere on the curved surface is $\dfrac{k_e Q}{R^2}$ radially outward (normal to the surface). Therefore, the flux is this field strength times the area of half a sphere:

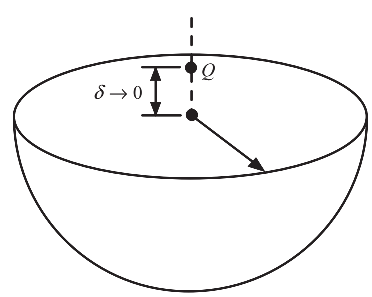

FIG. P24.13

$$\Phi_{\text{curved}} = \int \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = E_{\text{local}}A_{\text{hemisphere}}$$

$$\Phi_{\text{curved}} = \left(k_e\frac{Q}{R^2}\right)\left(\frac{1}{2}4\pi R^2\right) = \frac{1}{4\pi\epsilon_0}Q(2\pi) = \boxed{\frac{+Q}{2\epsilon_0}}$$

(b) The closed surface encloses zero charge so Gauss's law gives

$$\Phi_{\text{curved}} + \Phi_{\text{flat}} = 0 \qquad \text{or} \qquad \Phi_{\text{flat}} = -\Phi_{\text{curved}} = \boxed{\frac{-Q}{2\epsilon_0}}$$

P24.14 Consider as a gaussian surface a box with horizontal area A, lying between 500 and 600 m elevation.

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q}{\epsilon_0}: \quad (+120\,\text{N/C})A + (-100\,\text{N/C})A = \frac{\rho A(100\text{ m})}{\epsilon_0}$$

$$\rho = \frac{(20\,\text{N/C})(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)}{100\text{ m}} = \boxed{1.77\times10^{-12}\text{ C/m}^3}$$

The charge is $\boxed{\text{positive}}$, to produce the net outward flux of electric field.

P24.15 If $R \le d$, the sphere encloses no charge and $\Phi_E = \dfrac{q_{\text{in}}}{\epsilon_0} = \boxed{0}$.

If $R > d$, the length of line falling within the sphere is $2\sqrt{R^2 - d^2}$

so $\quad \Phi_E = \boxed{\dfrac{2\lambda\sqrt{R^2-d^2}}{\epsilon_0}}$

P24.16 $$\Phi_{E,\text{hole}} = \vec{\mathbf{E}}\cdot\vec{\mathbf{A}}_{\text{hole}} = \left(\frac{k_eQ}{R^2}\right)(\pi r^2) = \left(\frac{(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(10.0\times10^{-6}\text{ C})}{(0.100\text{ m})^2}\right)\pi(1.00\times10^{-3}\text{ m})^2$$

$$\Phi_{E,\text{hole}} = \boxed{28.2\text{ N}\cdot\text{m}^2/\text{C}}$$

P24.17 $$\Phi_E = \frac{q_{\text{in}}}{\epsilon_0} = \frac{170\times10^{-6}\text{ C}}{8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2} = 1.92\times10^7\text{ N}\cdot\text{m}^2/\text{C}$$

(a) $(\Phi_E)_{\text{one face}} = \dfrac{1}{6}\Phi_E = \dfrac{1.92\times10^7\text{ N}\cdot\text{m}^2/\text{C}}{6} \qquad (\Phi_E)_{\text{one face}} = \boxed{3.20\text{ MN}\cdot\text{m}^2/\text{C}}$

(b) $\Phi_E = \boxed{19.2\text{ MN}\cdot\text{m}^2/\text{C}}$

(c) The answer to (a) would change because the flux through each face of the cube would not be equal with an asymmetric charge distribution. The sides of the cube nearer the charge would have more flux and the ones further away would have less. The answer to (b) would remain the same, since the overall flux would remain the same.

Section 24.3 **Application of Gauss's Law to Various Charge Distributions**

P24.18 (a) $E = \dfrac{k_eQr}{a^3} = \boxed{0}$

(b) $E = \dfrac{k_eQr}{a^3} = \dfrac{(8.99\times10^9)(26.0\times10^{-6})(0.100)}{(0.400)^3} = \boxed{365\text{ kN/C}}$

(c) $E = \dfrac{k_eQ}{r^2} = \dfrac{(8.99\times10^9)(26.0\times10^{-6})}{(0.400)^2} = \boxed{1.46\text{ MN/C}}$

(d) $E = \dfrac{k_eQ}{r^2} = \dfrac{(8.99\times10^9)(26.0\times10^{-6})}{(0.600)^2} = \boxed{649\text{ kN/C}}$

The direction for each electric field is $\boxed{\text{radially outward}}$.

P24.19 The charge distributed through the nucleus creates a field at the surface equal to that of a point charge at its center: $E = \frac{k_e q}{r^2}$.

$$E = \frac{\left(8.99\times10^9\ \text{Nm}^2/\text{C}^2\right)\left(82\times1.60\times10^{-19}\ \text{C}\right)}{\left[(208)^{1/3}\,1.20\times10^{-15}\ \text{m}\right]^2}$$

$$E = \boxed{2.33\times10^{21}\ \text{N/C}} \quad \text{away from the nucleus}$$

P24.20 Note that the electric field in each case is directed radially inward, toward the filament.

(a) $$E = \frac{2k_e\lambda}{r} = \frac{2\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(90.0\times10^{-6}\ \text{C/m}\right)}{0.100\ \text{m}} = \boxed{16.2\ \text{MN/C}}$$

(b) $$E = \frac{2k_e\lambda}{r} = \frac{2\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(90.0\times10^{-6}\ \text{C/m}\right)}{0.200\ \text{m}} = \boxed{8.09\ \text{MN/C}}$$

(c) $$E = \frac{2k_e\lambda}{r} = \frac{2\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(90.0\times10^{-6}\ \text{C/m}\right)}{1.00\ \text{m}} = \boxed{1.62\ \text{MN/C}}$$

P24.21 $$E = \frac{\sigma}{2\epsilon_0} = \frac{9.00\times10^{-6}\ \text{C/m}^2}{2\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)} = \boxed{508\ \text{kN/C, upward}}$$

***P24.22** (a) A long cylindrical plastic rod 2.00 cm in radius carries charge uniformly distributed throughout its volume, with density 5.00 μC/m^3. Find the magnitude of the electric field it creates at a point *P*, 3.00 cm from its axis. As a gaussian surface choose a concentric cylinder with its curved surface passing through the point *P* and with length 8.00 cm.

(b) We solve for

$$E = \frac{(0.02\ \text{m})^2\,0.08\ \text{m}\ (5\times10^{-6}\ \text{C/m}^3)}{(8.85\times10^{-12}\text{C}^2/\text{N}\cdot\text{m}^2)2(0.03\ \text{m})\ 0.08\ \text{m}} = \boxed{3.77\ \text{kN/C}}$$

P24.23 If ρ is positive, the field must be radially outward. Choose as the gaussian surface a cylinder of length L and radius r, contained inside the charged rod. Its volume is $\pi r^2 L$ and it encloses charge $\rho\pi r^2 L$. Because the charge distribution is long, no electric flux passes through the circular end caps; $\vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = EdA\cos 90.0° = 0$. The curved surface has $\vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = EdA\cos 0°$, and E must be the same strength everywhere over the curved surface.

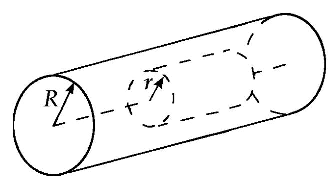

FIG. P24.23

Gauss's law, $\oint \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = \frac{q}{\epsilon_0}$, becomes $E\int_{\substack{\text{Curved}\\\text{Surface}}} dA = \frac{\rho\pi r^2 L}{\epsilon_0}$.

Now the lateral surface area of the cylinder is $2\pi rL$:

$$E(2\pi r)L = \frac{\rho\pi r^2 L}{\epsilon_0} \quad \text{Thus,} \quad \vec{\mathbf{E}} = \boxed{\frac{\rho r}{2\epsilon_0}\ \text{radially away from the cylinder axis}}.$$

***P24.24** $$\sigma = \left(8.60\times10^{-6}\ \text{C/cm}^2\right)\left(\frac{100\ \text{cm}}{\text{m}}\right)^2 = 8.60\times10^{-2}\ \text{C/m}^2$$

$$E = \frac{\sigma}{2\epsilon_0} = \frac{8.60\times10^{-2}}{2\left(8.85\times10^{-12}\right)} = \boxed{4.86\times10^9\ \text{N/C away from the wall}}$$

So long as the distance from the wall is small compared to the width and height of the wall, the distance does not affect the field.

P24.25 The volume of the spherical shell is

$$\frac{4}{3}\pi\left[(0.25\text{ m})^3-(0.20\text{ m})^3\right]=3.19\times10^{-2}\text{ m}^3$$

Its charge is

$$\rho V=\left(-1.33\times10^{-6}\text{ C/m}^3\right)\left(3.19\times10^{-2}\text{ m}^3\right)=-4.25\times10^{-8}\text{ C}$$

The net charge inside a sphere containing the proton's path as its equator is

$$-60\times10^{-9}\text{ C}-4.25\times10^{-8}\text{ C}=-1.02\times10^{-7}\text{ C}$$

The electric field is radially inward with magnitude

$$\frac{k_e|q|}{r^2}=\frac{|q|}{\epsilon_0\,4\pi r^2}=\frac{8.99\times10^9\text{ Nm}^2\left(1.02\times10^{-7}\text{ C}\right)}{\text{C}^2\left(0.25\text{m}\right)^2}=1.47\times10^4\text{ N/C}$$

For the proton

$$\sum F=ma \qquad eE=\frac{mv^2}{r}$$

$$v=\left(\frac{eEr}{m}\right)^{1/2}=\left(\frac{1.60\times10^{-19}\text{ C}\left(1.47\times10^4\text{ N/C}\right)0.25\text{ m}}{1.67\times10^{-27}\text{kg}}\right)^{1/2}=\boxed{5.94\times10^5\text{ m/s}}$$

P24.26 The distance between centers is $2\times5.90\times10^{-15}$ m. Each produces a field as if it were a point charge at its center, and each feels a force as if all its charge were a point at its center.

$$F=\frac{k_eq_1q_2}{r^2}=\left(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)\frac{(46)^2\left(1.60\times10^{-19}\text{ C}\right)^2}{\left(2\times5.90\times10^{-15}\text{ m}\right)^2}=3.50\times10^3\text{ N}=\boxed{3.50\text{ kN}}$$

P24.27 (a) $\vec{\mathbf{E}}=\boxed{0}$

(b) $E=\dfrac{k_eQ}{r^2}=\dfrac{\left(8.99\times10^9\right)\left(32.0\times10^{-6}\right)}{(0.200)^2}=7.19\text{ MN/C}\qquad\vec{\mathbf{E}}=\boxed{7.19\text{ MN/C radially outward}}$

P24.28 Consider two balloons of diameter 0.2 m, each with mass 1 g, hanging apart with a 0.05 m separation on the ends of strings making angles of 10° with the vertical.

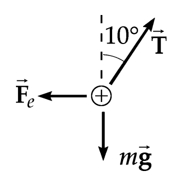

FIG. P24.28

(a) $\sum F_y=T\cos10°-mg=0\Rightarrow T=\dfrac{mg}{\cos10°}$

$\sum F_x=T\sin10°-F_e=0\Rightarrow F_e=T\sin10°$, so

$$F_e=\left(\frac{mg}{\cos10°}\right)\sin10°=mg\tan10°=(0.001\text{ kg})\left(9.8\text{ m/s}^2\right)\tan10°$$

$$F_e\approx2\times10^{-3}\text{ N}\ \boxed{\sim10^{-3}\text{ N or 1 mN}}$$

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(b) $F_e = \dfrac{k_e q^2}{r^2}$

$$2\times10^{-3}\ \text{N} \approx \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)q^2}{(0.25\ \text{m})^2}$$

$$q \approx 1.2\times10^{-7}\ \text{C} \boxed{\sim10^{-7}\ \text{C or } 100\ \text{nC}}$$

(c) $E = \dfrac{k_e q}{r^2} \approx \dfrac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.2\times10^{-7}\ \text{C}\right)}{(0.25\ \text{m})^2} \approx 1.7\times10^4\ \text{N/C} \boxed{\sim10\,\text{kN/C}}$

(d) $\Phi_E = \dfrac{q}{\epsilon_0} \approx \dfrac{1.2\times10^{-7}\ \text{C}}{8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2} = 1.4\times10^4\ \text{N}\cdot\text{m}^2/\text{C} \boxed{\sim10\,\text{kN}\cdot\text{m}^2/\text{C}}$

P24.29 (a) $E = \dfrac{2k_e\lambda}{r} = \dfrac{2\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left[\left(2.00\times10^{-6}\ \text{C}\right)/7.00\ \text{m}\right]}{0.100\ \text{m}}$

$$E = \boxed{51.4\ \text{kN/C, radially outward}}$$

(b) $\Phi_E = EA\cos\theta = E(2\pi r\ell)\cos0°$

$$\Phi_E = \left(5.14\times10^4\ \text{N/C}\right)2\pi(0.100\ \text{m})(0.020\,0\ \text{m})(1.00) = \boxed{646\ \text{N}\cdot\text{m}^2/\text{C}}$$

Section 24.4 Conductors in Electrostatic Equilibrium

P24.30 The fields are equal. The equation $E = \dfrac{\sigma_{\text{conductor}}}{\epsilon_0}$ suggested in the chapter for the field outside the aluminum looks different from the equation $E = \dfrac{\sigma_{\text{insulator}}}{2\epsilon_0}$ for the field around glass. But its charge will spread out to cover both sides of the aluminum plate, so the density is $\sigma_{\text{conductor}} = \dfrac{Q}{2A}$. The glass carries charge only on area A, with $\sigma_{\text{insulator}} = \dfrac{Q}{A}$. The two fields are $\dfrac{Q}{2A\epsilon_0}$, the same in magnitude, and both are perpendicular to the plates, vertically upward if Q is positive.

P24.31 $\oint EdA = E(2\pi rl) = \dfrac{q_{\text{in}}}{\epsilon_0}$ $\qquad E = \dfrac{q_{\text{in}}/l}{2\pi\epsilon_0 r} = \dfrac{\lambda}{2\pi\epsilon_0 r}$

(a) $r = 3.00$ cm $\qquad \vec{\mathbf{E}} = \boxed{0}$

(b) $r = 10.0$ cm $\qquad \vec{\mathbf{E}} = \dfrac{30.0\times10^{-9}}{2\pi\left(8.85\times10^{-12}\right)(0.100)} = \boxed{5\,400\ \text{N/C, outward}}$

(c) $r = 100$ cm $\qquad \vec{\mathbf{E}} = \dfrac{30.0\times10^{-9}}{2\pi\left(8.85\times10^{-12}\right)(1.00)} = \boxed{540\ \text{N/C, outward}}$

***P24.32** (a) All of the charge sits on the surface of the copper sphere at radius 15 cm. The field inside is $\boxed{\text{zero}}$.

(b) The charged sphere creates field at exterior points as if it were a point charge at the center:

$$\vec{\mathbf{E}} = \frac{k_e q}{r^2} \text{ away} = \frac{\left(8.99\times10^9 \text{ N}\cdot\text{m}^2\right)\left(40\times10^{-9} \text{ C}\right)}{\text{C}^2\,(0.17 \text{ m})^2} \text{ outward} = \boxed{1.24\times10^4 \text{ N/C outward}}$$

(c) $$\vec{\mathbf{E}} = \frac{\left(8.99\times10^9 \text{ N}\cdot\text{m}^2\right)\left(40\times10^{-9} \text{ C}\right)}{\text{C}^2\,(0.75 \text{ m})^2} \text{ outward} = \boxed{639 \text{ N/C outward}}$$

(d) All three answers would be the same. The solid copper sphere carries charge only on its outer surface.

P24.33 The charge divides equally between the identical spheres, with charge $\frac{Q}{2}$ on each. Then they repel like point charges at their centers:

$$F = \frac{k_e(Q/2)(Q/2)}{(L+R+R)^2} = \frac{k_e Q^2}{4(L+2R)^2} = \frac{8.99\times10^9 \text{ N}\cdot\text{m}^2\left(60.0\times10^{-6}\text{ C}\right)^2}{4\text{C}^2\,(2.01 \text{ m})^2} = \boxed{2.00 \text{ N}}$$

***P24.34** Let the flat box have face area A perpendicular to its thickness dx. The flux at $x = 0.3$ m is into the box $-EA = -(6\,000 \text{ N/C}\cdot\text{m}^2)(0.3 \text{ m})^2 A = -(540 \text{ N/C})\,A$.

The flux out of the box at $x = 0.3 \text{ m} + dx$

$+EA = -(6\,000 \text{ N/C}\cdot\text{m}^2)(0.3 \text{ m} + dx)^2 A = +(540 \text{ N/C})\,A + (3\,600 \text{ N/C}\cdot\text{m})\,dx\,A$

(The term in $(dx)^2$ is negligible.)

The charge in the box is $\rho A\,dx$ where ρ is the unknown. Gauss's law is

$-(540 \text{ N/C})\,A + (540 \text{ N/C})\,A + (3\,600 \text{ N/C}\cdot\text{m})\,dx\,A = \rho A\,dx/\epsilon_0$

Then $\rho = (3600 \text{ N/C}\cdot\text{m})\epsilon_0 = (3600 \text{ N/C}\cdot\text{m})(8.85\times10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2) = \boxed{31.9 \text{ nC/m}^3}$

P24.35 (a) Inside surface: consider a cylindrical surface within the metal. Since E inside the conducting shell is zero, the total charge inside the gaussian surface must be zero, so the inside charge/length $= -\lambda$.

$0 = \lambda\ell + q_{\text{in}}$ so $\frac{q_{\text{in}}}{\ell} = \boxed{-\lambda}$

Outside surface: The total charge on the metal cylinder is $2\lambda\ell = q_{\text{in}} + q_{\text{out}}$

$q_{\text{out}} = 2\lambda\ell + \lambda\ell$ so the outside charge/length is $\boxed{3\lambda}$

(b) $$E = \frac{2k_e(3\lambda)}{r} = \frac{6k_e\lambda}{r} = \boxed{\frac{3\lambda}{2\pi\epsilon_0 r} \text{ radially outward}}$$

***P24.36** The surface area is $A = 4\pi a^2$. The field is then

$$E = \frac{k_e Q}{a^2} = \frac{Q}{4\pi\epsilon_0 a^2} = \frac{Q}{A\epsilon_0} = \frac{\sigma}{\epsilon_0}$$

It is not equal to $\sigma/2\epsilon_0$. At a point just outside, the uniformly charged surface looks just like a uniform flat sheet of charge. The distance to the field point is negligible compared to the radius of curvature of the surface.

P24.37 (a) The charge density on each of the surfaces (upper and lower) of the plate is:

$$\sigma = \frac{1}{2}\left(\frac{q}{A}\right) = \frac{1}{2}\frac{\left(4.00\times10^{-8}\ \text{C}\right)}{(0.500\ \text{m})^2} = 8.00\times10^{-8}\ \text{C/m}^2 = \boxed{80.0\,\text{nC/m}^2}$$

(b) $$\vec{\mathbf{E}} = \left(\frac{\sigma}{\epsilon_0}\right)\hat{\mathbf{k}} = \left(\frac{8.00\times10^{-8}\ \text{C/m}^2}{8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2}\right)\hat{\mathbf{k}} = \boxed{(9.04\ \text{kN/C})\hat{\mathbf{k}}}$$

(c) $$\vec{\mathbf{E}} = \boxed{(-9.04\ \text{kN/C})\hat{\mathbf{k}}}$$

Additional Problems

P24.38 In general, $\vec{\mathbf{E}} = ay\hat{\mathbf{i}} + bz\hat{\mathbf{j}} + cx\hat{\mathbf{k}}$

In the xy plane, $z = 0$ and $\vec{\mathbf{E}} = ay\hat{\mathbf{i}} + cx\hat{\mathbf{k}}$

$$\Phi_E = \int \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = \int\left(ay\hat{\mathbf{i}} + cx\hat{\mathbf{k}}\right)\cdot\hat{\mathbf{k}}dA$$

$$\Phi_E = ch\int_{x=0}^{w} x\,dx = ch\left.\frac{x^2}{2}\right|_{x=0}^{w} = \boxed{\frac{chw^2}{2}}$$

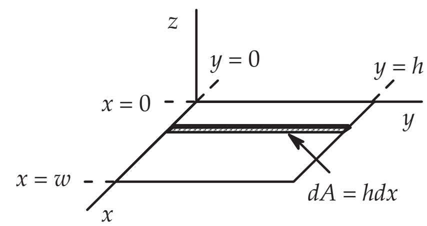

FIG. P24.38

P24.39 (a) Uniform $\vec{\mathbf{E}}$, pointing radially outward, so $\Phi_E = EA$. The arc length is $ds = Rd\theta$, and the circumference is $2\pi r = 2\pi R\sin\theta$.

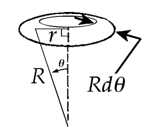

FIG. P24.39

$$A = \int 2\pi r\,ds = \int_0^\theta (2\pi R\sin\theta)Rd\theta = 2\pi R^2\int_0^\theta \sin\theta\,d\theta$$

$$= 2\pi R^2(-\cos\theta)\Big|_0^\theta = 2\pi R^2(1-\cos\theta)$$

$$\Phi_E = \frac{1}{4\pi\epsilon_0}\frac{Q}{R^2}\cdot 2\pi R^2(1-\cos\theta) = \boxed{\frac{Q}{2\epsilon_0}(1-\cos\theta)}$$ [independent of R!]

(b) For $\theta = 90.0°$ (hemisphere): $\Phi_E = \frac{Q}{2\epsilon_0}(1-\cos 90°) = \boxed{\frac{Q}{2\epsilon_0}}$.

(c) For $\theta = 180°$ (entire sphere): $\Phi_E = \frac{Q}{2\epsilon_0}(1-\cos 180°) = \boxed{\frac{Q}{\epsilon_0}}$ [Gauss's Law].

P24.40 The sphere with large charge creates a strong field to polarize the other sphere. That means it pushes the excess charge over to the far side, leaving charge of the opposite sign on the near side. This patch of opposite charge is smaller in amount but located in a stronger external field, so it can feel a force of attraction that is larger than the repelling force felt by the larger charge in the weaker field on the other side.

***P24.41** (a) The field is zero within the metal of the shell. The exterior electric field lines end at equally spaced points on the outer surface. The charge on the outer surface is distributed uniformly. Its amount is given by

$$EA = Q/\epsilon_0$$

$$Q = -(890\text{ N/C})\, 4\pi (0.75\text{ m})^2\, 8.85\times 10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2 = -55.7\text{ nC}$$

(b) and (c) For the net charge of the shell to be zero, the shell must carry +55.7 nC on its inner surface, induced there by −55.7 nC in the cavity within the shell. The charge in the cavity could have any distribution and give any corresponding distribution to the charge on the inner surface of the shell. For example, a large positive charge might be within the cavity close to its topmost point, and a slightly larger negative charge near its easternmost point. The inner surface of the shell would then have plenty of negative charge near the top and even more positive charge centered on the eastern side.

***P24.42** (a) $q_{\text{in}} = +3\ \mu\text{C} - 1\ \mu\text{C} = \boxed{+2.00\ \mu\text{C}}$

(b) The charge distribution is spherically symmetric and $q_{\text{in}} > 0$. Thus, the field is directed $\boxed{\text{radially outward}}$ or to the right at point D.

(c) $E = \dfrac{k_e q_{\text{in}}}{r^2} = \dfrac{8.99\times 10^9\ \ 2.00\times 10^{-6}\text{ N/C}}{(0.16)^2} = \boxed{702\text{ kN/C}}$

(d) Since all points within this region are located inside conducting material, $\boxed{E = 0}$.

(e) $\Phi_E = \int \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = 0 \quad\Rightarrow\quad q_{\text{in}} = \epsilon_0 \Phi_E = \boxed{0}$

(f) $q_{\text{in}} = \boxed{+3.00\ \mu\text{C}}$

(g) $E = \dfrac{k_e q_{\text{in}}}{r^2} = \dfrac{8.99\times 10^9\ 3.00\times 10^{-6}}{(0.08)^2} = \boxed{4.21\text{ MN/C to the right}}$ (radially outward).

(h) $q_{\text{in}} = \rho V = \left(\dfrac{+3\ \mu\text{C}}{\frac{4}{3}\pi 5^3}\right)\left(\dfrac{4}{3}\pi 4^3\right) = \boxed{+1.54\ \mu\text{C}}$

(i) $E = \dfrac{k_e q_{\text{in}}}{r^2} = \dfrac{8.99\times 10^9\ 1.54\times 10^{-6}}{(0.04)^2} = \boxed{8.63\text{ MN/C to the right}}$ (radially outward)

(j) As in part (d), $E = 0$ for $10\text{ cm} < r < 15\text{ cm}$. Thus, for a spherical gaussian surface with $10\text{ cm} < r < 15\text{ cm}$, $q_{\text{in}} = +3\ \mu\text{C} + q_{\text{inner}} = 0$ where q_{inner} is the charge on the inner surface of the conducting shell. This yields $q_{\text{inner}} = \boxed{-3.00\ \mu\text{C}}$.

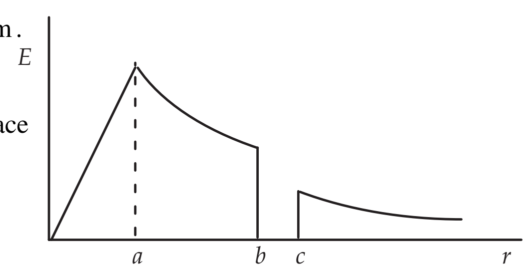

FIG. P24.42(l)

(k) Since the total charge on the conducting shell is $q_{\text{net}} = q_{\text{outer}} + q_{\text{inner}} = -1\ \mu\text{C}$, we have

$$q_{\text{outer}} = -1\ \mu\text{C} - q_{\text{inner}} = -1\ \mu\text{C} - (-3\ \mu\text{C}) = \boxed{+2.00\ \mu\text{C}}$$

(l) This is shown in the figure to the right.

P24.43 (a) $\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = E(4\pi r^2) = \dfrac{q_{\text{in}}}{\epsilon_0}$

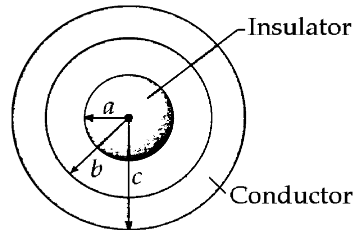

FIG. P24.43

For $r < a$, $q_{\text{in}} = \rho\left(\dfrac{4}{3}\pi r^3\right)$

so $E = \boxed{\dfrac{\rho r}{3\epsilon_0}}$

For $a < r < b$ and $c < r$, $q_{\text{in}} = Q$

So $E = \boxed{\dfrac{Q}{4\pi r^2 \epsilon_0}}$

For $b \le r \le c$, $E = 0$, since $\boxed{E = 0}$ inside a conductor.

(b) Let $q_1 =$ induced charge on the inner surface of the hollow sphere. Since $E = 0$ inside the conductor, the total charge enclosed by a spherical surface of radius $b \le r \le c$ must be zero.

Therefore, $q_1 + Q = 0$ and $\sigma_1 = \dfrac{q_1}{4\pi b^2} = \boxed{\dfrac{-Q}{4\pi b^2}}$

Let $q_2 =$ induced charge on the outside surface of the hollow sphere. Since the hollow sphere is uncharged, we require

$$q_1 + q_2 = 0 \quad \text{and} \quad \sigma_2 = \frac{q_1}{4\pi c^2} = \boxed{\frac{Q}{4\pi c^2}}$$

P24.44 First, consider the field at distance $r < R$ from the center of a uniform sphere of positive charge $(Q = +e)$ with radius R.

$$(4\pi r^2)E = \frac{q_{\text{in}}}{\epsilon_0} = \frac{\rho V}{\epsilon_0} = \left(\frac{+e}{\frac{4}{3}\pi R^3}\right)\frac{\frac{4}{3}\pi r^3}{\epsilon_0} \quad \text{so} \quad E = \left(\frac{e}{4\pi\epsilon_0 R^3}\right) r \text{ directed outward}$$

(a) The force exerted on a point charge $q = -e$ located at distance r from the center is then

$$F = qE = -e\left(\frac{e}{4\pi\epsilon_0 R^3}\right) r = -\left(\frac{e^2}{4\pi\epsilon_0 R^3}\right) r = \boxed{-Kr}$$

(b) $K = \dfrac{e^2}{4\pi\epsilon_0 R^3} = \boxed{\dfrac{k_e e^2}{R^3}}$

(c) $F_r = m_e a_r = -\left(\dfrac{k_e e^2}{R^3}\right) r$, so $a_r = -\left(\dfrac{k_e e^2}{m_e R^3}\right) r = -\omega^2 r$

Thus, the motion is simple harmonic with frequency $f = \dfrac{\omega}{2\pi} = \boxed{\dfrac{1}{2\pi}\sqrt{\dfrac{k_e e^2}{m_e R^3}}}$

(d) $f = 2.47 \times 10^{15}\ \text{Hz} = \dfrac{1}{2\pi}\sqrt{\dfrac{(8.99 \times 10^9\ \text{N} \cdot \text{m}^2/\text{C}^2)(1.60 \times 10^{-19}\ \text{C})^2}{(9.11 \times 10^{-31}\ \text{kg})R^3}}$

which yields $R^3 = 1.05 \times 10^{-30}\ \text{m}^3$, or $R = 1.02 \times 10^{-10}\ \text{m} = \boxed{102\ \text{pm}}$

P24.45 The vertical velocity component of the moving charge increases according to

$$m\frac{dv_y}{dt} = F_y \qquad m\frac{dv_y}{dx}\frac{dx}{dt} = qE_y$$

Now $\frac{dx}{dt} = v_x$ has the nearly constant value v. So

$$dv_y = \frac{q}{mv}E_y dx \qquad v_y = \int_0^{v_y} dv_y = \frac{q}{mv}\int_{-\infty}^{\infty} E_y dx$$

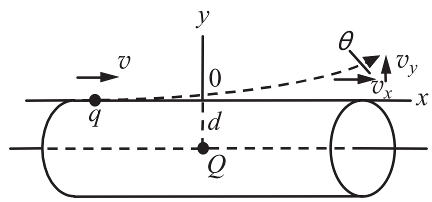

FIG. P24.45

The radially outward component of the electric field varies along the x axis, but is described by

$$\int_{-\infty}^{\infty} E_y dA = \int_{-\infty}^{\infty} E_y (2\pi d)\,dx = \frac{Q}{\epsilon_0}$$

So $\int_{-\infty}^{\infty} E_y dx = \frac{Q}{2\pi d\,\epsilon_0}$ and $v_y = \frac{qQ}{mv2\pi d\epsilon_0}$. The angle of deflection is described by

$$\tan\theta = \frac{v_y}{v} = \frac{qQ}{2\pi\epsilon_0\, dmv^2} \qquad \boxed{\theta = \tan^{-1}\frac{qQ}{2\pi\epsilon_0\, dmv^2}}$$

P24.46 Consider the field due to a single sheet and let E_+ and E_- represent the fields due to the positive and negative sheets. The field at any distance from each sheet has a magnitude given by the textbook equation

$$|E_+| = |E_-| = \frac{\sigma}{2\epsilon_0}$$

(a) To the left of the positive sheet, E_+ is directed toward the left and E_- toward the right and the net field over this region is $\vec{\mathbf{E}} = \boxed{0}$.

(b) In the region between the sheets, E_+ and E_- are both directed toward the right and the net field is

$$\vec{\mathbf{E}} = \boxed{\frac{\sigma}{\epsilon_0}\text{ to the right}}$$

(c) To the right of the negative sheet, E_+ and E_- are again oppositely directed and $\vec{\mathbf{E}} = \boxed{0}$.

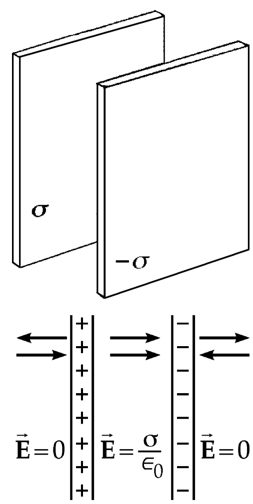

FIG. P24.46

P24.47 The magnitude of the field due to each sheet given by Equation 24.8 is

$$E = \frac{\sigma}{2\epsilon_0} \text{ directed perpendicular to the sheet}$$

(a) In the region to the left of the pair of sheets, both fields are directed toward the left and the net field is

$$\vec{\mathbf{E}} = \boxed{\frac{\sigma}{\epsilon_0}\text{ to the left}}$$

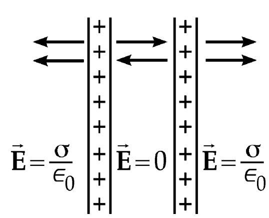

FIG. P24.47

continued on next page

(b) In the region between the sheets, the fields due to the individual sheets are oppositely directed and the net field is

$$\vec{\mathbf{E}} = \boxed{0}$$

(c) In the region to the right of the pair of sheets, both fields are directed toward the right and the net field is

$$\vec{\mathbf{E}} = \boxed{\frac{\sigma}{\epsilon_0} \text{ to the right}}$$

P24.48 The electric field throughout the region is directed along x; therefore, $\vec{\mathbf{E}}$ will be perpendicular to dA over the four faces of the surface which are perpendicular to the yz plane, and E will be parallel to dA over the two faces which are parallel to the yz plane. Therefore,

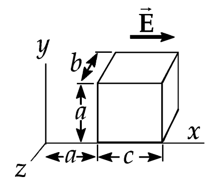

FIG. P24.48

$$\Phi_E = -\left(E_x\big|_{x=a}\right)A + \left(E_x\big|_{x=a+c}\right)A = -\left(3+2a^2\right)ab + \left(3+2(a+c)^2\right)ab$$
$$= 2abc(2a+c)$$

Substituting the given values for a, b, and c, we find $\Phi_E = \boxed{0.269 \text{ N}\cdot\text{m}^2/\text{C}}$.

$$Q = \epsilon_0 \Phi_E = 2.38\times10^{-12} \text{ C} = \boxed{2.38 \text{ pC}}$$

P24.49 $\oint \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = E\left(4\pi r^2\right) = \frac{q_{\text{in}}}{\epsilon_0}$

(a) For $r > R$, $q_{\text{in}} = \int_0^R Ar^2\left(4\pi r^2\right)dr = 4\pi\frac{AR^5}{5}$

and $E = \boxed{\frac{AR^5}{5\epsilon_0 r^2}}$

(b) For $r < R$, $q_{\text{in}} = \int_0^r Ar^2\left(4\pi r^2\right)dr = \frac{4\pi Ar^5}{5}$

and $E = \boxed{\frac{Ar^3}{5\epsilon_0}}$

P24.50 The resultant field within the cavity is the superposition of two fields, one $\vec{\mathbf{E}}_+$ due to a uniform sphere of positive charge of radius $2a$, and the other $\vec{\mathbf{E}}_-$ due to a sphere of negative charge of radius a centered within the cavity.

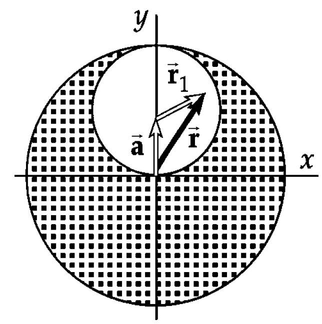

FIG. P24.50

$$\frac{4}{3}\left(\frac{\pi r^3\rho}{\epsilon_0}\right)=4\pi r^2E_+ \qquad \text{so} \qquad \vec{\mathbf{E}}_+=\frac{\rho r}{3\epsilon_0}\hat{\mathbf{r}}=\frac{\rho\vec{\mathbf{r}}}{3\epsilon_0}$$

$$-\frac{4}{3}\left(\frac{\pi r_1^3\rho}{\epsilon_0}\right)=4\pi r_1^2E_- \qquad \text{so} \qquad \vec{\mathbf{E}}_-=\frac{\rho r_1}{3\epsilon_0}\left(-\hat{\mathbf{r}}_1\right)=\frac{-\rho}{3\epsilon_0}\vec{\mathbf{r}}_1$$

Since $\vec{\mathbf{r}}=\vec{\mathbf{a}}+\vec{\mathbf{r}}_1$, $\qquad \vec{\mathbf{E}}_-=\dfrac{-\rho\left(\vec{\mathbf{r}}-\vec{\mathbf{a}}\right)}{3\epsilon_0}$

$$\vec{\mathbf{E}}=\vec{\mathbf{E}}_++\vec{\mathbf{E}}_-=\frac{\rho\vec{\mathbf{r}}}{3\epsilon_0}-\frac{\rho\vec{\mathbf{r}}}{3\epsilon_0}+\frac{\rho\vec{\mathbf{a}}}{3\epsilon_0}=\frac{\rho\vec{\mathbf{a}}}{3\epsilon_0}=0\hat{\mathbf{i}}+\frac{\rho a}{3\epsilon_0}\hat{\mathbf{j}}$$

Thus, $\boxed{E_x=0}$

and $\boxed{E_y=\dfrac{\rho a}{3\epsilon_0}\text{ at all points within the cavity}}$

P24.51 Consider the charge distribution to be an unbroken charged spherical shell with uniform charge density σ and a circular disk with charge per area $-\sigma$. The total field is that due to the whole sphere, $\dfrac{Q}{4\pi\epsilon_0R^2}=\dfrac{4\pi R^2\sigma}{4\pi\epsilon_0R^2}=\dfrac{\sigma}{\epsilon_0}$ outward plus the field of the disk $-\dfrac{\sigma}{2\epsilon_0}=\dfrac{\sigma}{2\epsilon_0}$ radially inward. The total field is $\dfrac{\sigma}{\epsilon_0}-\dfrac{\sigma}{2\epsilon_0}=\boxed{\dfrac{\sigma}{2\epsilon_0}\text{ outward}}$.

P24.52 In this case the charge density is *not uniform,* and Gauss's law is written as $\oint\vec{\mathbf{E}}\cdot d\vec{\mathbf{A}}=\dfrac{1}{\epsilon_0}\int\rho dV$. We use a gaussian surface which is a cylinder of radius r, length ℓ, and is coaxial with the charge distribution.

(a) When $r<R$, this becomes $E(2\pi r\ell)=\dfrac{\rho_0}{\epsilon_0}\displaystyle\int_0^r\left(a-\frac{r}{b}\right)dV$. The element of volume is a cylindrical shell of radius r, length ℓ, and thickness dr so that $dV=2\pi r\ell dr$.

$$E(2\pi r\ell)=\left(\frac{2\pi r^2\ell\rho_0}{\epsilon_0}\right)\left(\frac{a}{2}-\frac{r}{3b}\right) \quad \text{so inside the cylinder, } E=\boxed{\frac{\rho_0r}{2\epsilon_0}\left(a-\frac{2r}{3b}\right)}$$

(b) When $r>R$, Gauss's law becomes

$$E(2\pi r\ell)=\frac{\rho_0}{\epsilon_0}\int_0^R\left(a-\frac{r}{b}\right)(2\pi r\ell dr) \quad \text{or outside the cylinder, } E=\boxed{\frac{\rho_0R^2}{2\epsilon_0r}\left(a-\frac{2R}{3b}\right)}$$

P24.53 $\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \dfrac{q_{\text{in}}}{\epsilon_0} = \dfrac{1}{\epsilon_0}\int_0^r \dfrac{a}{r} 4\pi r^2 dr$

$$E 4\pi r^2 = \frac{4\pi a}{\epsilon_0}\int_0^r r dr = \frac{4\pi a}{\epsilon_0}\frac{r^2}{2}$$

$$\boxed{E = \frac{a}{2\epsilon_0}} = \text{constant magnitude}$$

(The direction is radially outward from center for positive a; radially inward for negative a.)

P24.54 The total flux through a surface enclosing the charge Q is $\dfrac{Q}{\epsilon_0}$. The flux through the disk is

$$\Phi_{\text{disk}} = \int \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}$$

where the integration covers the area of the disk. We must evaluate this integral and set it equal to $\dfrac{\frac{1}{4}Q}{\epsilon_0}$ to find how b and R are related. In the figure, take $d\vec{\mathbf{A}}$ to be the area of an annular ring of radius s and width ds. The flux through $d\vec{\mathbf{A}}$ is $\vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = EdA\cos\theta = E(2\pi s ds)\cos\theta$.

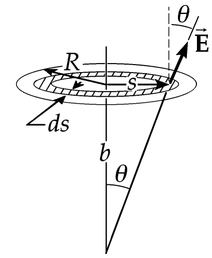

FIG. P24.54

The magnitude of the electric field has the same value at all points within the annular ring,

$$E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{Q}{s^2+b^2} \qquad \text{and} \qquad \cos\theta = \frac{b}{r} = \frac{b}{\left(s^2+b^2\right)^{1/2}}.$$

Integrate from $s = 0$ to $s = R$ to get the flux through the entire disk.

$$\Phi_{E,\text{ disk}} = \frac{Qb}{2\epsilon_0}\int_0^R \frac{sds}{\left(s^2+b^2\right)^{3/2}} = \frac{Qb}{2\epsilon_0}\left[-\left(s^2+b^2\right)^{1/2}\right]\Bigg|_0^R = \frac{Q}{2\epsilon_0}\left[1 - \frac{b}{\left(R^2+b^2\right)^{1/2}}\right]$$

The flux through the disk equals $\dfrac{Q}{4\epsilon_0}$ provided that $\dfrac{b}{\left(R^2+b^2\right)^{1/2}} = \dfrac{1}{2}$.

This is satisfied if $\boxed{R = \sqrt{3}b}$.

P24.55 (a) Consider a cylindrical shaped gaussian surface perpendicular to the yz plane with one end in the yz plane and the other end containing the point x:

Use Gauss's law: $\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \dfrac{q_{\text{in}}}{\epsilon_0}$

By symmetry, the electric field is zero in the yz plane and is perpendicular to $d\vec{\mathbf{A}}$ over the wall of the gaussian cylinder. Therefore, the only contribution to the integral is over the end cap containing the point x:

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{\text{in}}}{\epsilon_0} \quad \text{or} \quad EA = \frac{\rho(Ax)}{\epsilon_0}$$

so that at distance x from the mid-line of the slab, $\boxed{E = \dfrac{\rho x}{\epsilon_0}}$.

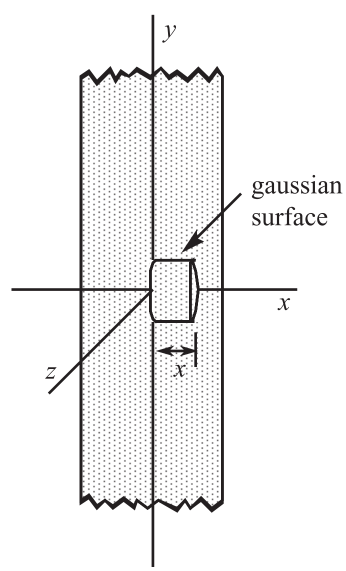

FIG. P24.55

continued on next page

(b) $a = \frac{F}{m_e} = \frac{(-e)E}{m_e} = -\left(\frac{\rho e}{m_e \in_0}\right)x$

The acceleration of the electron is of the form $a = -\omega^2 x$ with $\omega = \sqrt{\frac{\rho e}{m_e \in_0}}$

Thus, the motion is simple harmonic with frequency $f = \frac{\omega}{2\pi} = \boxed{\frac{1}{2\pi}\sqrt{\frac{\rho e}{m_e \in_0}}}$

P24.56 Consider the gaussian surface described in the solution to problem 59.

(a) For $x > \frac{d}{2}$, $dq = \rho\, dV = \rho A dx = CAx^2 dx$

$$\int \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{1}{\in_0}\int dq$$

$$EA = \frac{CA}{\in_0}\int_0^{d/2} x^2 dx = \frac{1}{3}\left(\frac{CA}{\in_0}\right)\left(\frac{d^3}{8}\right)$$

$E = \frac{Cd^3}{24\in_0}$ or $\boxed{\vec{\mathbf{E}} = \frac{Cd^3}{24\in_0}\hat{\mathbf{i}} \text{ for } x > \frac{d}{2}; \quad \vec{\mathbf{E}} = -\frac{Cd^3}{24\in_0}\hat{\mathbf{i}} \text{ for } x < -\frac{d}{2}}$

(b) For $-\frac{d}{2} < x < \frac{d}{2}$ $\int \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{1}{\in_0}\int dq = \frac{CA}{\in_0}\int_0^x x^2 dx = \frac{CAx^3}{3\in_0}$

$$\boxed{\vec{\mathbf{E}} = \frac{Cx^3}{3\in_0}\hat{\mathbf{i}} \text{ for } x > 0; \quad \vec{\mathbf{E}} = -\frac{Cx^3}{3\in_0}\hat{\mathbf{i}} \text{ for } x < 0}$$

P24.57 (a) A point mass m creates a gravitational acceleration $\vec{\mathbf{g}} = -\frac{Gm}{r^2}\hat{\mathbf{r}}$ at a distance r

The flux of this field through a sphere is $\oint \vec{\mathbf{g}} \cdot d\vec{\mathbf{A}} = -\frac{Gm}{r^2}(4\pi r^2) = -4\pi Gm$

Since the r has divided out, we can visualize the field as unbroken field lines. The same flux would go through any other closed surface around the mass. If there are several or no masses inside a closed surface, each creates field to make its own contribution to the net flux according to

$$\boxed{\oint \vec{\mathbf{g}} \cdot d\vec{\mathbf{A}} = -4\pi Gm_{\text{in}}}$$

(b) Take a spherical gaussian surface of radius r. The field is inward so

$$\oint \vec{\mathbf{g}} \cdot d\vec{\mathbf{A}} = g4\pi r^2 \cos 180° = -g4\pi r^2$$

and $-4\pi Gm_{\text{in}} = -4\pi G\frac{4}{3}\pi r^3 \rho$

Then, $-g4\pi r^2 = -4\pi G\frac{4}{3}\pi r^3 \rho$ and $g = \frac{4}{3}\pi r \rho G$

Or, since $\rho = \frac{M_E}{\frac{4}{3}\pi R_E^3}$, $g = \frac{M_E G r}{R_E^3}$ or $\boxed{\vec{\mathbf{g}} = \frac{M_E G r}{R_E^3} \text{ inward}}$

P24.58 The charge density is determined by $Q = \frac{4}{3}\pi a^3 \rho \qquad \rho = \frac{3Q}{4\pi a^3}$

(a) The flux is that created by the enclosed charge within radius *r*:

$$\Phi_E = \frac{q_{\text{in}}}{\epsilon_0} = \frac{4\pi r^3 \rho}{3\epsilon_0} = \frac{4\pi r^3 3Q}{3\epsilon_0\, 4\pi a^3} = \boxed{\frac{Qr^3}{\epsilon_0 a^3}}$$

(b) $\Phi_E = \boxed{\frac{Q}{\epsilon_0}}$. Note that the answers to parts (a) and (b) agree at $r = a$.

(c)

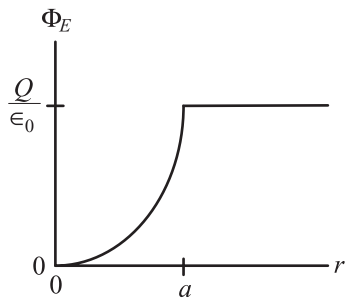

FIG. P24.58(c)

P24.59 $\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = E\left(4\pi r^2\right) = \frac{q_{\text{in}}}{\epsilon_0}$

(a) $\left(-3.60\times 10^3 \text{ N/C}\right) 4\pi \left(0.100 \text{ m}\right)^2 = \frac{Q}{8.85\times 10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2} \qquad (a < r < b)$

$Q = -4.00\times 10^{-9} \text{ C} = \boxed{-4.00 \text{ nC}}$

(b) We take Q' to be the net charge on the hollow sphere. Outside *c*,

$\left(+2.00\times 10^2 \text{ N/C}\right) 4\pi \left(0.500 \text{ m}\right)^2 = \frac{Q+Q'}{8.85\times 10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2} \qquad (r > c)$

$Q + Q' = +5.56\times 10^{-9} \text{ C}$, so $Q' = +9.56\times 10^{-9} \text{ C} = \boxed{+9.56 \text{ nC}}$

(c) For $b < r < c$: $E = 0$ and $q_{\text{in}} = Q + Q_1 = 0$ where Q_1 is the total charge on the inner surface of the hollow sphere. Thus, $Q_1 = -Q = \boxed{+4.00 \text{ nC}}$.

Then, if Q_2 is the total charge on the outer surface of the hollow sphere,

$Q_2 = Q' - Q_1 = 9.56 \text{ nC} - 4.0 \text{ nC} = \boxed{+5.56 \text{ nC}}$.

P24.60 The field direction is radially outward perpendicular to the axis. The field strength depends on r but not on the other cylindrical coordinates θ or z. Choose a gaussian cylinder of radius r and length L. If $r < a$,

$$\Phi_E = \frac{q_{\text{in}}}{\epsilon_0} \quad \text{and} \quad E(2\pi rL) = \frac{\lambda L}{\epsilon_0}$$

$$E = \frac{\lambda}{2\pi r\epsilon_0} \quad \text{or} \quad \boxed{\vec{\mathbf{E}} = \frac{\lambda}{2\pi r\epsilon_0}\hat{\mathbf{r}} \qquad (r < a)}$$

If $a < r < b$,

$$E(2\pi rL) = \frac{\lambda L + \rho\pi\left(r^2 - a^2\right)L}{\epsilon_0}$$

$$\boxed{\vec{\mathbf{E}} = \frac{\lambda + \rho\pi\left(r^2 - a^2\right)}{2\pi r\epsilon_0}\hat{\mathbf{r}} \qquad (a < r < b)}$$

If $r > b$,

$$E(2\pi rL) = \frac{\lambda L + \rho\pi\left(b^2 - a^2\right)L}{\epsilon_0}$$

$$\boxed{\vec{\mathbf{E}} = \frac{\lambda + \rho\pi\left(b^2 - a^2\right)}{2\pi r\epsilon_0}\hat{\mathbf{r}} \qquad (r > b)}$$

ANSWERS TO EVEN PROBLEMS

P24.2 $355\ \text{kN}\cdot\text{m}^2/\text{C}$

P24.4 (a) $-2.34\ \text{kN}\cdot\text{m}^2/\text{C}$ (b) $+2.34\ \text{kN}\cdot\text{m}^2/\text{C}$ (c) 0

P24.6 (a) $\dfrac{q}{2\epsilon_0}$ (b) $\dfrac{q}{2\epsilon_0}$ (c) Plane and square both subtend a solid angle of a hemisphere at the charge.

P24.8 (a) -55.7 nC (b) The negative charge has a spherically symmetric distribution concentric with the shell.

P24.10 (a) $1.36\ \text{MN}\cdot\text{m}^2/\text{C}$ (b) $678\ \text{kN}\cdot\text{m}^2/\text{C}$ (c) No; see the solution.

P24.12 $\dfrac{Q - 6|q|}{6\epsilon_0}$

P24.14 $1.77\ \text{pC}/\text{m}^3$ positive

P24.16 $28.2\ \text{N}\cdot\text{m}^2/\text{C}$

P24.18 (a) 0 (b) $365\ \text{kN/C}$ (c) $1.46\ \text{MN/C}$ (d) $649\ \text{kN/C}$

P24.20 (a) $16.2\ \text{MN/C}$ toward the filament (b) $8.09\ \text{MN/C}$ toward the filament (c) $1.62\ \text{MN/C}$ toward the filament

P24.22 (a) A long cylindrical plastic rod 2.00 cm in radius carries charge uniformly distributed throughout its volume, with density 5.00 $\mu\text{C/m}^3$. Find the magnitude of the electric field it creates at a point P, 3.00 cm from its axis. As a gaussian surface choose a concentric cylinder with its curved surface passing through the point P and with length 8.00 cm. (b) 3.77 kN/C

P24.24 4.86 GN/C away from the wall. It is constant close to the wall.

P24.26 3.50 kN

P24.28 (a) ~1 mN (b) ~100 nC (c) ~10 kN/C (d) ~10 kN · m²/C

P24.30 $\vec{\mathbf{E}} = Q/2\epsilon_0 A$ vertically upward in each case if $Q > 0$

P24.32 (a) 0 (b) 12.4 kN/C radially outward (c) 639 N/C radially outward (d) No answer changes. The solid copper sphere carries charge only on its outer surface.

P24.34 31.9 nC/m³

P24.36 The electric field just outside the surface is given by σ/ϵ_0. At this point the uniformly charged surface of the sphere looks just like a uniform flat sheet of charge.

P24.38 $\dfrac{chw^2}{2}$

P24.40 See the solution.

P24.42 (a) 2.00 μC (b) to the right (c) 702 kN/C (d) 0 (e) 0 (f) 3.00 μC (g) 4.21 MN/C radially outward (h) 1.54 μC (i) 8.63 MN/C radially outward (j) −3.00 μC (k) 2.00 μC (l) See the solution.

P24.44 (a, b) See the solution. (c) $\dfrac{1}{2\pi}\sqrt{\dfrac{k_e e^2}{m_e R^3}}$ (d) 102 pm

P24.46 (a) 0 (b) $\dfrac{\sigma}{\epsilon_0}$ to the right (c) 0

P24.48 0.269 $\text{N}\cdot\text{m}^2/\text{C}$; 2.38 pC

P24.50 See the solution.

P24.52 (a) $\dfrac{\rho_0 r}{2\epsilon_0}\left(a - \dfrac{2r}{3b}\right)$ (b) $\dfrac{\rho_0 R^2}{2\epsilon_0\, r}\left(a - \dfrac{2R}{3b}\right)$

P24.54 See the solution.

P24.56 (a) $\vec{\mathbf{E}} = \dfrac{Cd^3}{24\epsilon_0}\hat{\mathbf{i}}$ for $x > \dfrac{d}{2}$; $\vec{\mathbf{E}} = -\dfrac{Cd^3}{24\epsilon_0}\hat{\mathbf{i}}$ for $x < -\dfrac{d}{2}$ (b) $\vec{\mathbf{E}} = \dfrac{Cx^3}{3\epsilon_0}\hat{\mathbf{i}}$ for $x > 0$;

$\vec{\mathbf{E}} = -\dfrac{Cx^3}{3\epsilon_0}\hat{\mathbf{i}}$ for $x < 0$

P24.58 (a) $\dfrac{Qr^3}{\epsilon_0\, a^3}$ (b) $\dfrac{Q}{\epsilon_0}$ (c) See the solution

P24.60 For $r < a$, $\vec{\mathbf{E}} = \lambda/2\pi\epsilon_0 r$ radially outward.

For $a < r < b$, $\vec{\mathbf{E}} = [\lambda + \rho\pi(r^2 - a^2)]/2\pi\epsilon_0 r$ radially outward.

For $r > b$, $\vec{\mathbf{E}} = [\lambda + \rho\pi(b^2 - a^2)]/2\pi\epsilon_0 r$ radially outward.