Bayesian Decision Theory

1.1 Bayesian Detection Framework

2.4 Expectations only equations 12, 13, 14

2.5 Gaussian Distribution equations 16, 17

3. Bayesian Decision Theory

Bayes' theorem:

$$P(H_i/r) = \frac{P(r/H_i)\,P(H_i)}{P(r)} \quad (A)$$

Where, ① $i = 0, 1$, H: Hypothesis

② $P(H_i)$: Prior distribution.

③ $P(r/H_i)$: Conditional distribution of observation r given that $H = i$

④ Posterior distribution of Having $H = i$ given the observation $R = r$:

($P(H_i/r)$)

note: 5 and 6 From your slides Page 14

3.2 Bayesian Risk:

Expected cost for deciding H_0 given n observations:

$$r_1, \ldots, r_n = C_{00} P(H_0|r) + C_{10} P(H_0|r)$$

Expected cost for deciding H_1 given n observations:

$$r_1, \ldots, r_n = C_{11} P(H_1|r) + C_{10} P(H_0|r)$$

Now if we decide in favor of Hypothesis with Lower expected cost given n observations:

$$C_{00} P(H_0|r) + C_{01} P(H_1|r) \underset{H_0}{\overset{H_1}{\gtrless}} C_{10} P(H_0|r) + C_{11} P(H_1|r)$$

$$(C_{00} - C_{10}) P(H_0|r) \underset{H_0}{\overset{H_1}{\gtrless}} (C_{11} - C_{01}) P(H_1|r)$$

($P(H_0|r)$: from eq. A; $P(H_1|r)$: from eq. A)

$$(C_{00} - C_{10}) \frac{P(r|H_0) P(H_0)}{P(r)} \underset{H_0}{\overset{H_1}{\gtrless}} (C_{11} - C_{01}) \frac{P(r|H_1) P(H_1)}{P(r)}$$

Note: $P(r)$: total prob. $= P(r|H_1) P(H_1) + P(r|H_0) P(H_0)$

$$\frac{P(r|H_1)}{P(r|H_0)} \underset{H_0}{\overset{H_1}{\gtrless}} \frac{P(H_0)}{P(H_1)} \cdot \frac{C_{10} - C_{00}}{C_{01} - C_{11}}$$

Threshold Risk minimizing rule.

Therefore Bayesian Risk minimization can be written as:

$$\text{if } \{ C_{00} P(H_0|r) + C_{01} P(H_1|r), C_{10} P(H_0|r) + C_{11} P(H_1|r) \} =$$
$$C_{00} P(H_0|r) + C_{01} P(H_1|r) \Rightarrow \text{decide } H_0 \text{ otherwise decide } H_1 \quad (2)$$

For the online monitoring we need to use sequential detection;

check (examine) By:

Derive the sequential detection with conditionaly independent observation:

Final answer will be

$$J_m^{(n)}(\alpha_m) = \min\left[C_{00}[1-\alpha_m] + C_{10}\alpha_m,\ C_{10}[1-\alpha_m] + C_{11}\alpha_m,\ C + \phi\left(J_{m+1}^{(n)}(\alpha_{m+1})\right)\right]$$

Decide H_1 or H_0 or continue.

Example 3. + Example 4.

4. Numerical Example (MATLAB)