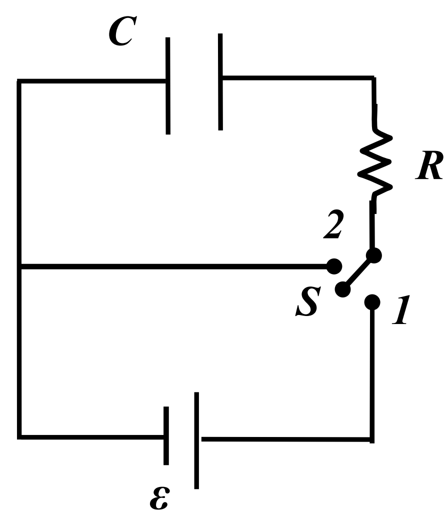
Experiment No. 10: RC-Circuits

Charging and discharging a capacitor

Purpose:

(1) To study the charging and discharging mechanism and behavior of a capacitor experientially.
(2) To determine the time constant for an RC circuit.



Fg.1: RC-Circuit

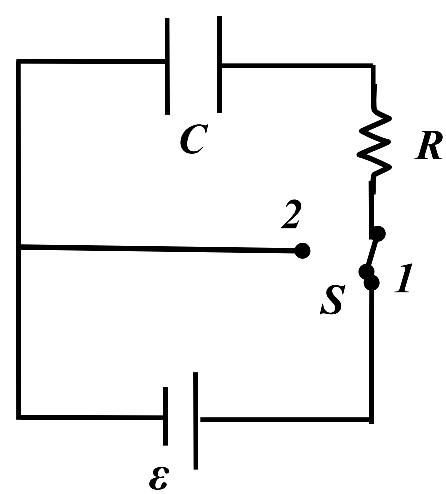
Equipment:

Dc power supply **Ɛ** (5 V), carbon resistor **R=1MΩ**, Capacitor **C=100 µF**, dc power supply multi-meter, a calculator, and a few connecting wires with alligator clips

Theory:

Consider a capacitor in series with a resistor, switch, and battery, as shown in the Fig.1.

Charging a Capacitor



Fg.2: Charging the Capacitor

When a circuit with a *resistor* (R) and a *capacitor* (C) in series is closed as in Fig.2, (S is closed to 1) the capacitor is initially uncharged and so **the voltage across it is equal to 0**. The capacitor starts charging and its voltage will increase but opposes the voltage across the batteries or power supply. This means that **the current in the circuit begins with its maximum** value and then **decrease to zero** when the capacitor becomes **fully charges**. Eventually, the capacitor will be very nearly fully charged and the current will effectively go to zero when the voltage across the capacitor becomes nearly equivalent to the voltage of the power source.
For time t >>> 0 the capacitor is fully charged up to **maximum charge Q_f** and the current in the **circuit is zero (steady state)**

$$Q_f = Q_{max} = CV_0 = C\varepsilon \qquad V_0 \approx \varepsilon, \quad I_f = 0$$

Apply Kirchhoff's loop rule to the circuit after the switch S is thrown to 1. Traversing the loop clockwise gives

ε - $q/C - IR = 0$ ($q/C = V_C(t)$ = potential drop of the capacitor)

$$Q(t) = Q_{max}(1 - e^{-t/RC}), \quad Q_{max} = CV_0 = C\varepsilon$$

$$I(t) = \frac{dq}{dt} = \frac{Q}{RC} e^{-t/RC} = \frac{\varepsilon}{R} e^{-t/RC}, \qquad \frac{\varepsilon}{R} = I_{max}$$

$$V(t) = V_0(1 - e^{-t/RC}), \qquad V_0 \approx \varepsilon$$

Where ***e*** is the base of the natural logarithm and we have made the substitution ($Q_{max} = C\varepsilon$) for the maximum charge at $t >> 0$.

The quantity ***RC*** is called the **time constant τ** of the circuit:

τ =RC

It has the dimensions of time **[τ] = [R][C] = [V/I][Q/V]= [Q/I]=$C \times \frac{s}{C}$ = s = sec**

Plot of capacitor voltage V versus time t is shown in **Fig.3**.

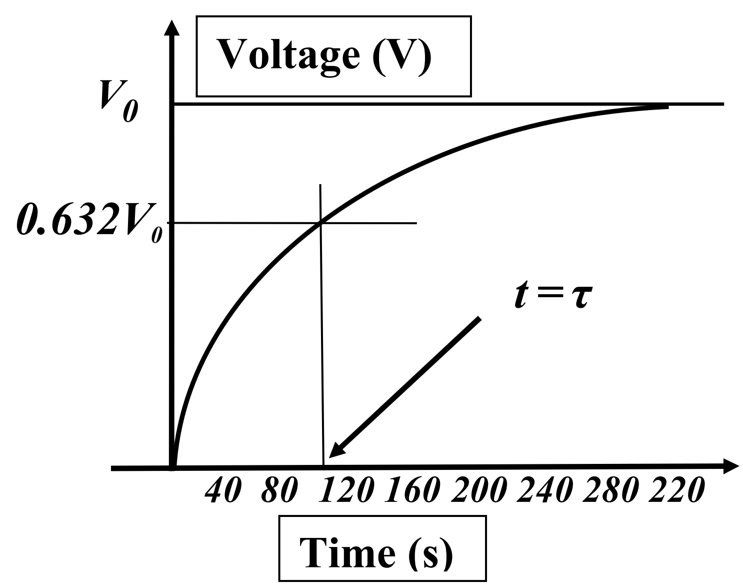


Fig.3 : Voltage vs. Time for cahrging the capacitor

The voltage of the capacitor V increases from 0 until the time

$t = \tau = RC$,

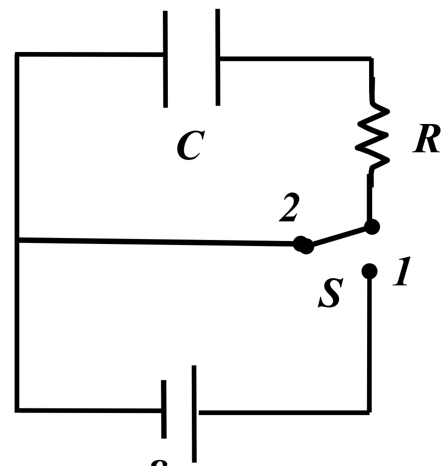
The voltage has the value

$$V(\tau)=V_0(1-e^{-\tau/\tau}) \quad = V_0(1-e^{-1})$$

$$= V_0(1-0.368) \quad = 0.632V_0 \qquad V_0 \approx \varepsilon$$

Discharging the Capacitor

The initial potential difference a cross the capacitor is V_0 and the initial charge has maximum value of $Q_0 = CV_0$ and there is zero potential difference across the resistor because I=0. When the switch is thrown down to **2** at t = 0 as shown in Fig.4 besides, the capacitor begins to discharge through R.



Fg.4: Discharging the Capacitor

At some time *t* during the discharge, the current in the circuit is *I* and the charge on the capacitor is *q,* we have

$$at \quad t>0 \qquad \frac{-q}{C} = IR = \frac{dq}{dt}R$$

$$\Rightarrow \frac{dq}{-q} = \frac{dt}{RC}$$

to obtain assuming q(t=0) = Q_o

$$ln\left(\frac{q}{Q_0}\right) = \frac{-t}{RC} \quad \Rightarrow Q(t) = Q_0 e^{-t/RC}, \qquad I(t) = -I_o e^{-t/RC}$$

The voltage of C at any time is

$$V(t) = V_0 e^{-t/\tau}, \quad \tau\text{=RC}$$

C is discharging

The potential will decrease with time according to the above relation, V_0 represents the voltage at time $t = 0$, and **τ** represents the "time constant" or time that it

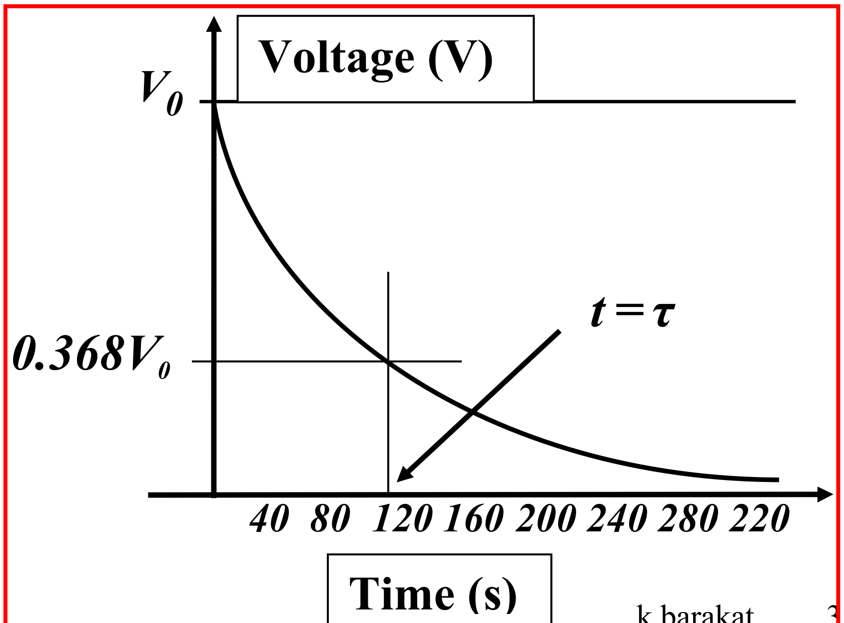


Fig.5 : V(t) vs. t for discharging the capacitor

takes for the voltage to decrease by a factor of $\frac{1}{e}V_0$. **At t = τ**

$$V(\tau) = V_0 \frac{1}{e} = 0.368\ V_0$$

<u>One can plot **lnV**</u> against **time** to obtain a curve like Fig.6.

$$V(t) = V_0 e^{-t/\tau} \qquad \Rightarrow lnV = lnV_0 - \frac{t}{\tau}$$

$$slope = \frac{-1}{\tau} \quad \Rightarrow \tau = \frac{-1}{S} \qquad S = \frac{lnV - lnV_0}{t-0} = \frac{0 - lnV_0}{t-0}$$

Procedure:

(1) Charging: Connect the RC circuit as shown in Fig. 2. Close the circuit so that the current flows and the capacitor will begin to charge.

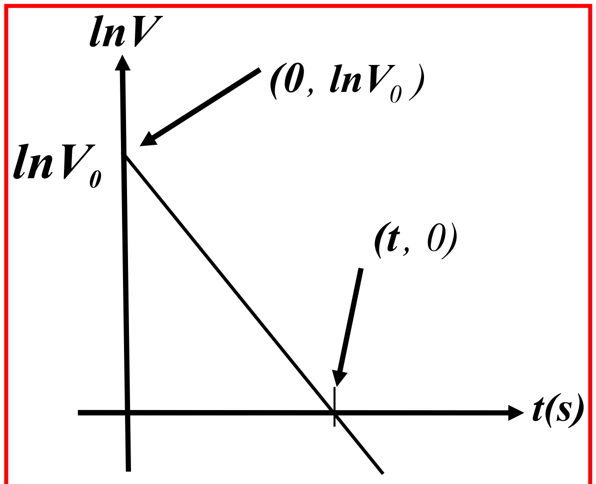


Fig.6 : lnV(t) vs. t

Measure the voltage across the capacitor. At time t = 0, the voltage across the capacitor will be zero because the capacitor has not charged up yet. Record the voltage every 20 seconds until the voltage across the capacitor is close to the voltage across the battery or power supply when you stop taking data. Tabulate your results in **table 1**.

(2) Discharging: At t=0 and when the capacitor was fully charged with maximum voltage $V=V_0$. Connect the circuit as shown in fig.4 by closing S to 2. The capacitor begins to discharge through R. Its voltage drops to $(1/e)V_0=0.368$ at t=τ, and decays exponentially to zero with time. Measure V(t) for every 20 seconds and record your data in **table 2**.

Experiment No. 10: RC-Circuits

Charging and discharging a capacitor

Name: …………………………………………….... Day and Date: …..…………………

Student's No.: …………………………..…….….…….. Sec: ……………………….

Partners Names: ………………………………………………………………………......

Data: Ɛ=5V, R=1MΩ, C = 100μF, RC= 100 sec.

(1) Charging

t (sec)	**0**																	
V(t) (V)																		
I μA																		

Q1. Make a graph of the voltage across the capacitor versus time. (it looks like fig.3)

Q2. What is the initial values of Q, I, V at t=0.

Q3. What is the maximum value of voltage (V_0) and final charge Q.

Q4. Locate on the V-axis the value 0.632 $\times V_0$.

Q5. From $V=0.632V_0$ find the value of τ on the time axis.

(2) Discharging

t(sec)																		
V(t)(V)																		
ln V																		

Q1. What is the value of V at t=0.

Q2. Calculate the time constant **τ=RC** for **R=1MΩ** and **C = 100µF**.

Q3. Plot **Voltage** vs. **time t.**

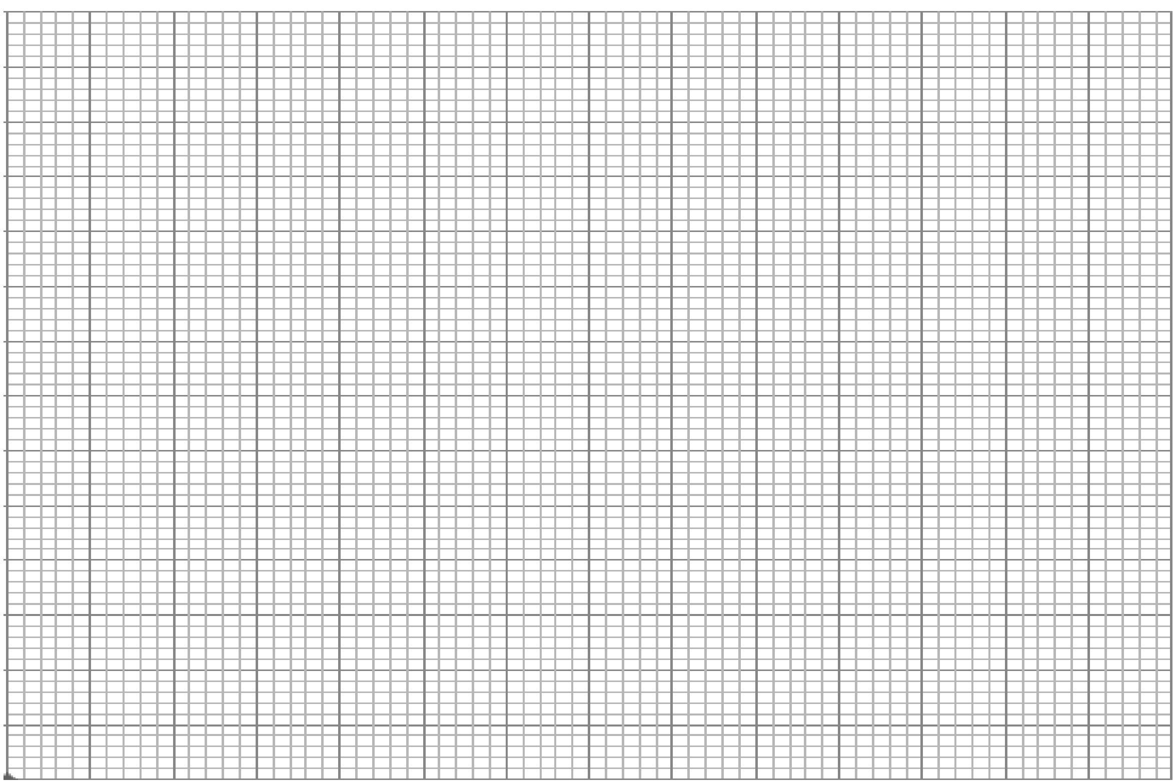
Q4. Locate on V-axis the value **$0.368V_0$**.

Q5. From this value find τ on the time axis.

Q6. Plot **lnV** vs. **t .**

Q7. From the **(lnV, t)** curve, determine:

The vertical intercept =**$\ln V_0$** = (   )   the V_0 =



$$S = \frac{\ln V - \ln V_0}{t - 0} \quad , \text{ take } \ln V = 0 \text{ at some time } t, \Rightarrow \tau = -\frac{1}{slope}$$