

## Experiment No. 10

### Pendulum

#### Objectives:

- 1-To determine a spring's constant
- 2-To determine the dependence of the period on the length of a pendulum
- 3-To determine the acceleration of gravity.

#### Theory:

An oscillating system that undergoes uniform periodic rectilinear motions, is called "Simple Harmonic Motion, (SHM)". In general, any motion that repeats itself at regular intervals is SHM, when the object is subjected to a restoring force proportional to the displacement and acts in the opposite direction. Examples, are an oscillating block hanging vertically from a spring, and a simple pendulum.

A body that executes simple harmonic motion in the y-direction, the location in the y-axis at any given time (t) from the start is given according to the equation

$$y = A \sin(\omega t)$$

Where y is the displacement of the body from its equilibrium position at  $y = 0$ , t is the lapse of time and is measured such that at  $t = 0$ ,  $y = 0$ ; A, the amplitude of the motion, is the maximum displacement of the body from its equilibrium position, and is **the angular velocity** of the motion. The velocity at any time

$$v = \frac{dy}{dt} = A\omega \cos(\omega t)$$

$$\Rightarrow a = \frac{d^2y}{dt^2} = -\omega^2 A \sin(\omega t) = -\omega^2 y \quad \Rightarrow \quad \omega^2 = \frac{-a}{y}$$

The angular velocity  $\omega = 2\pi f$  is related to the period of the oscillation by the equation:

$$\omega = \frac{2\pi}{T}$$

The frequency f is in the number of vibrations per unit time. The period T of the vibration (or oscillation) is the reciprocal of frequency.

In this experiment, you examine two types of oscillatory motion:

(I) spring system: (fig. 9.1) mass attached to spring displaced a distance (-y) from the equilibrium point ( $y=0$ ), for small displacements the restoring force in the spring is given by Hook's law ( $F =$

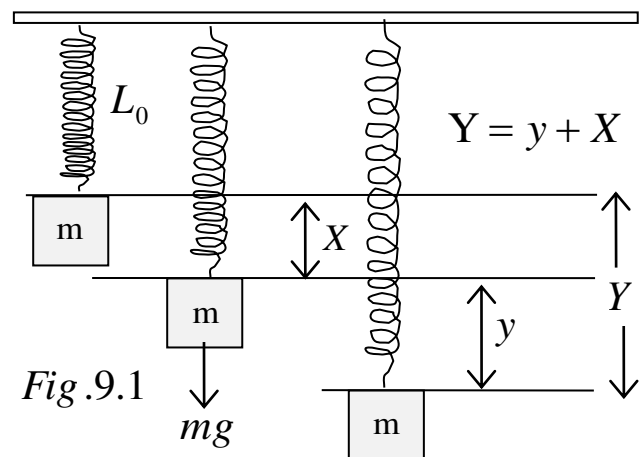


Fig. 9.1

- ky) where k is the spring constant. Using Newton's second law and Hook's law for the displacement  $Y = y + X$ :

$$ma = -k(y + X) + mg$$

But since

$$-kX = mg \quad \Rightarrow \quad ma = -ky$$

Where  $X$  is the elongation made when the mass  $m$  attached to the spring and held stationary (**no oscillations**).

Note: When the spring stretches a distance  $X$ , and the attached mass is stationary, you can find  $k$  from  $m$  and the elongation  $X$  from equilibrium position ( $x=0$ ), the spring is with original length  $L_0$ .

$$k = \frac{mg}{X}$$

When the mass sets into oscillation, then

$$ma = -ky$$

Therefore: 
$$\omega^2 = \frac{-a}{y} = \frac{k}{m}$$

$$T = 2\pi\sqrt{\frac{m}{k}} \quad \text{and} \quad T^2 = 4\pi^2 \frac{m}{k}$$

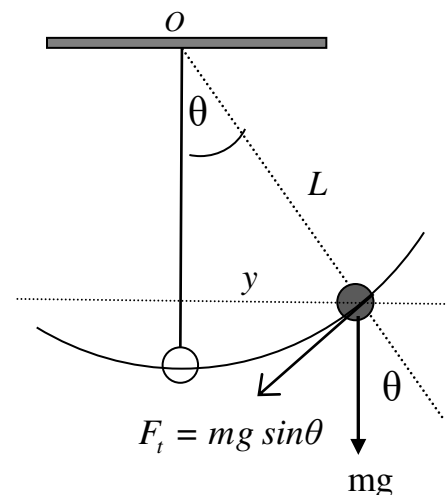
(II) Simple Pendulum: the second example you examined for SHM is the simple pendulum, which consists of a mass ( $m$ ) called the bob, attached to the string of length  $L$ . If the bob is moved from the rest at the equilibrium position at the bottom through a small angle of displacement  $\theta$  as shown in Fig.9.2, the bob will experience a restoring force due to the component of gravity tangent to the path, which acts to bring the bob back to its equilibrium position at the bottom.

When  $m$  is displaced for small angles  $\theta$ , applying Newton's law for the motion tangent to the path of motion:

$$F_t = ma = -mg \sin \theta, \quad \sin \theta = \frac{y}{L}$$

$$a = -m \sin \theta = -m \frac{y}{L}, \quad \omega^2 = \frac{a}{y} = -\frac{g}{L}$$

$$T = 2\pi\sqrt{\frac{L}{g}} \quad \text{and} \quad T^2 = 4\pi^2 \frac{L}{g}$$



**Procedure:****(I) mass on a spring:**

- (1) Suspend the spring vertically from a rigid support.
- (2) Use the slotted weights to elongate the **spring from  $y=0$** , and record the displacements  $b$  from the equilibrium position when the block becomes **stationary**.
- (3) Tabulate your data in Table 9.1
- (4) Plot  **$F_g = mg$**  versus  **$X$**  and from the graph **find  $k$**  of the spring.
- (5) **Attach a mass to the spring and pull it downward a distance (A) from the equilibrium position.**
- (6) Find **the period of a vertical oscillation**, use the stopwatch to determine the time needed for 10 oscillations, **do this 2 times, and find the average period T.**
- (7) Tabulate your results in Table 9.2.
- (8) **Plot  $T^2$  versus  $m$ . and find  $K$  from the graph.**

**(II) Simple Pendulum:**

- (1) Displace the bob **to one side for a small angle** of displacement.
- (2) Release the bob **letting it oscillate**.
- (3) Determine the time **needed for 10 oscillations, do this step 2 times** and take the average.
- (4) Change  $L$  and find the average period  $T$  each time.
- (4) Tabulate your results in Table 9.3.
- (5) **Plot  $T^2$  vs  $L$  and from the slope determine  $g$ .**

## Exp. No. 10 The Pendulum

Name: .....Grade: .....

Student's No.: ..... Day and Date: .....

Partners Names: ..... Sec: .....

**Part 1:** Determine the spring constant.

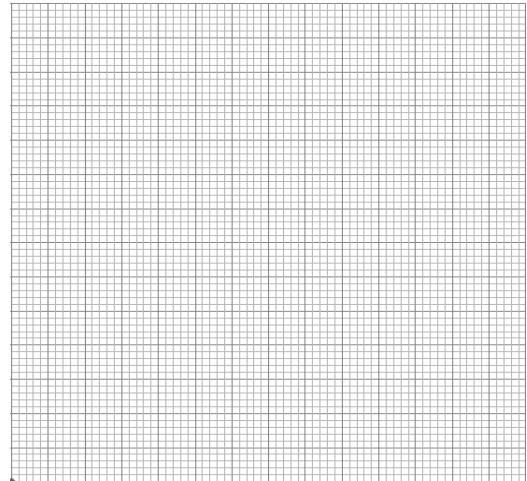
Trial	Total mass	Spring elongation	Force of Gravity(dyne)	Spring constant (dyne/cm)
	M (gm)	X (cm)	F= mg (dyne)	K = mg/X (dyne/cm)
1				
2				
3				
4				
5				

**Find the value of k:**

(1)  $k_{avg} = (k_1+k_2+k_3+k_4+k_5)/5 =$   
.....

(2) Plot **mg vs. x.**

Determine k from the slope = S = k =  
.....

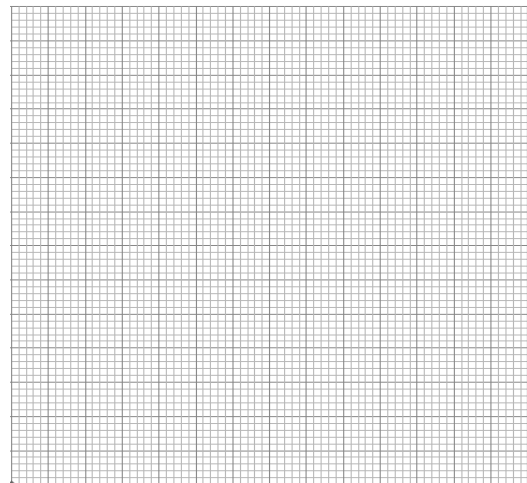


**Part 2:** Spring Harmonic Motion.

m(g)	t <sub>1</sub> (for 10 Oscillations)	t <sub>2</sub> (for 10 Oscillations)	Avg. Period T = (t <sub>1</sub> +t <sub>2</sub> )/20	T <sup>2</sup>

Plot **T<sup>2</sup>** versus **m** and determine **k from slope S**  
(from the table below)

Slope = S=  $4 \pi^2/k$  →  $k = 4 \pi^2/S =$



**Part 3:** Pendulum.

L(length) (cm)	t <sub>1</sub> (for 10 Oscillations)	t <sub>2</sub> (for 10 Oscillations)	T = (t <sub>1</sub> +t <sub>2</sub> )/20	T <sup>2</sup>

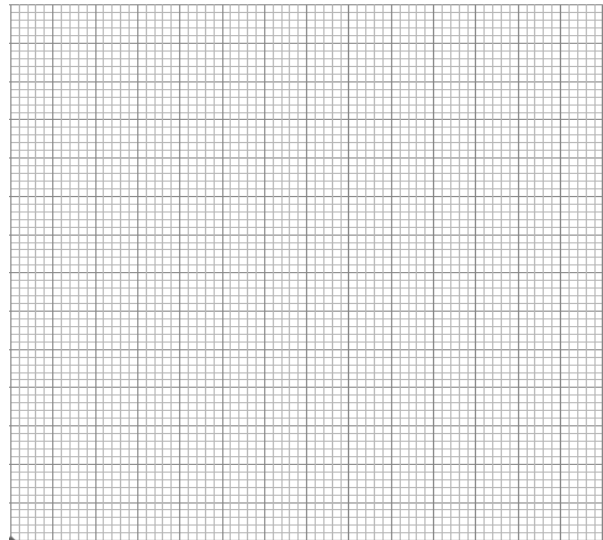
Plot  $T^2$  versus L and determine  $g$  from slope  $S$

$$\text{Slope} = S = 4\pi^2/g$$

$$g = 4\pi^2/S =$$

**Questions:**

1- Does the period of the simple pendulum, in general, depend on the amplitude?



2-What is the relation between  $k$ 's and  $k$ 'eq for parallel and series combinations?

3- If the length of the pendulum clock depends on temperature, in summer will the clock gain or lose time? Explain your answer.