## Ch. 2: Motion in One Dimension

*<u>Kinematics</u>*: describes motion in terms of space and time, ignoring external agents that cause or modify that motion.

Motion: continuous change in position.

**2.1** Position, Velocity, and Speed

**<u>position</u>** or <u>location</u> in space at any time, with respect to a chosen reference point considered the origin of a coordinate system, (a reference point) O

A car is moving back and forth along the x axis, Figure 2.1a. The car is 30 m to the right of the reference position x = 0. We record



the car's position every 10 s, pictorially, graphically, and tabular.

**The displacement:**  $\Delta x$  change in the position in some time interval. Initial position  $x_i$ , final position  $x_f$ 



**Distance d** is the **length of a path** followed by a particle.

The displacement is a vector which has direction. <u>Distance</u> d is scaler always positive.

<u>The average velocity</u>  $\overline{u_x}$  = displacement  $\Delta x$  divided by the time interval  $\Delta t$  of the displacement:

$$v_{x,avg} = \overline{v}_x = \frac{\Delta x}{\Delta t}$$
 Unit: meter/second (m/s)

For example,

Between A and B in Figure 2.1b

$$v_{_{x,avg}} = \overline{v}_{_x} = \frac{\Delta x}{\Delta t} = \frac{52 - 30}{10 - 0} = 2.2 \text{ m/s} =$$

Slope of line between A and B

**Average velocity** = the slope of the line joining the two points

The average speed of a particle, is a scalar quantity,



Average speed  $= \overline{v} = \frac{d}{\Delta t} = \frac{\text{total distance}}{\text{total time}}$  (scalar has no direction)

You travel from point *A* to point *B* and then back along the line from *B* to *A*. What is
(a) your total displacement?
(b) your distance traveled?

 $A \xrightarrow{v_i} B$   $x(0) x(t_1)$ 

The displacement is zero:  $\Delta x = x_f - x_i = 0$ 

The distance is twice the length:  $d = 2\overline{AB}$ 

**For example**, suppose it takes **you 45.0 s to travel 100 m down** a long straight hallway toward your departure gate at an airport. At

the 100 m mark, you realize you missed the rest room, and you return back 25.0 m along the same hallway, taking 10.0 s to make the return trip.



Average velocity = 
$$v_{xavg} = \frac{\Delta x}{\Delta t} = \frac{+75.0}{55.0} = 1.36 \text{ m/s}$$

The average speed for your trip is

$$v_{avg} = \overline{v} = \frac{\text{total distance}}{\text{total time}} = \frac{d}{\Delta t} = \frac{+125}{55.0} = 2.27 \text{m/s}$$

#### **Example 2.1** Calculating the Average Velocity and Speed

Find the displacement, average velocity, and average speed of the car in **Figure 2.1a** between positions A and F.

Solution From the position-time graph given in Figure 2.1b, note that  $x_A = 30$  m at  $t_A = 0$  s and that  $x_F = -53$  m at  $t_F = 50$  s



Position of the Car at Various Times		
Position	$t(\mathbf{s})$	<i>x</i> ( <b>m</b> )
۲	0	30
®	10	52
©	20	38
O	30	0
Ē	40	- 37
Ē	50	-53

$$\frac{1}{v_{x}} = \frac{\Delta x}{\Delta t} = \frac{x_{f} - x_{i}}{t_{f} - t_{i}} = \frac{x_{F} - x_{A}}{t_{F} - t_{A}} = \frac{-53 - 30}{50 - 0} = \frac{-83m}{50s} = -1.7 \text{ m/s}$$

From A to B  $d^+ = 22 m + From B$  to F d=105 m

average speed =  $\overline{v} = \frac{\text{total distance}}{\text{total time}} = \frac{d}{\Delta t} = \frac{+127}{50s} = 2.5 \text{ m/s}$ 

#### **2.2** Instantaneous Velocity and Speed

Needed the velocity of a particle at some instant of time. If the point B moves toward point A. The line between the points becomes steeper and steeper, and as the two points become extremely close together, the line becomes a tangent line to the curve.



The *instantaneous velocity* = the slope the tangent line at A = velocity of the car at the moment the car was at A. The instantaneous velocity  $v_x$  is equal to the *derivative* of x with respect to t,

$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$



**The instantaneous speed** = is defined as the **magnitude of its instantaneous velocity**. It is a scalar positive quantity.

If the instantaneous velocity is + 25 m/s along a given line

The speed or the instantaneous = +25 m/s

The instantaneous velocity of - 25 m/s along the same line,

The speed <u>or</u> the instantaneous speed = +25 m/s

#### **Example 2.3** Average and Instantaneous Velocity

### A particle moves along the *x*-axis. Its position varies with time 1

according to the expression  $x(t) = -4t + 2t^2$ , where x is in meters and t is in seconds. The position-time graph for this motion is shown in Figure 2.4. Notice that the particle moves in the negative x direction for the first second of motion, is momentarily at rest at the moment t = 1 s, and moves in the positive x direction at times t > 1 s.



(A) Determine the **displacement** of the particle in the time intervals t = 0 to t = 1 s and t = 1 s to t = 3 s.

#### Solution

 $\Delta x(A \to B) = x_{f} - x_{i} = x(1) - x(0) = \left[-4(1) + 2(1)^{2}\right] - \left[-4(0) + 2(0)\right] = -2m$   $\Delta x_{BD} = x_{f} - x_{i} = x(3) - x(1) = \left[-4(3) + 2(3)^{2}\right] - \left[-4(1) + 2(1)\right] = +6 + 2 = +8m$ Or simply from the graph one can obtain the same answers.

(B) Calculate the average velocity during these two time intervals.

#### Solution

$$\overline{v}_{x}(A \rightarrow B) = \frac{\Delta x}{\Delta t} = \frac{-2}{1} = -2 \text{ m/s}$$
$$\overline{v}_{x}(B \rightarrow D) = \frac{\Delta x}{\Delta t} = \frac{+8}{3 - 1} = +4 \text{ m/s}$$

(C) Find the instantaneous velocity of the particle at t = 2.5 s. By measuring the slope of tangent line at t = 2.5 s in Figure 2.4, we find that

## From the graph, the slope $\approx \frac{10 - (-4)}{3.8 - 1.5} = \frac{14}{2.3} \approx +6 \text{m/s}$

From the equation

$$v_{x}(C) = \frac{dx}{dt}\Big|_{t=2.5} = [-4 + 4t]_{t=2.5} = [-4 + 4(2.5)] = +6m/s$$

#### **2.3** Analysis Model: Particle under Constant Velocity

Any entity moving under constant velocity can be used as a model. The entity may be a car, a person, or a plane; which are modeled as a particle.

The particle velocity is constant, hence the **instantaneous velocity** 

#### = the average velocity, and

$$\overline{v}_{x} = v_{x} = \frac{\Delta x}{\Delta t} = \frac{x_{f} - x_{i}}{t_{f} - t_{i}}$$

Taking  $x_i$  at  $t_i = 0$ ,  $t_f = t$ :

$$\frac{\mathbf{x}_{f} - \mathbf{x}_{i}}{t - 0} = \mathbf{v}_{x} \rightarrow \mathbf{x}_{f} = \mathbf{x}_{i} + \mathbf{v}_{x} \mathbf{t} \qquad \text{(for constant } \mathbf{v}_{x}\text{)}$$

#### **Example 2.4** Modeling a Runner as a Particle

A kinesiologist is studying the biomechanics of the human body. (*Kinesiology* is the study of the movement of the human body. Notice the connection to the word *kinematics*.) She determines the velocity of an experimental subject while he runs along a straight line at **a constant rate**. The kinesiologist starts the stopwatch at the moment the runner passes a given point and stops it after the runner has passed another point 20 m away. The time interval indicated on the stopwatch is 4.0 s.

(A) What is the runner's velocity?

Runner is moving at a constant rate or constant velocity.

$$v_x = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i} = \frac{20 - 0}{4.0} = 5.0 \text{ m/s}$$

(B) If the runner continues his motion after the stopwatch is stopped, what is his position after 10 s have passed? Let the runner's position at t = 10 s be  $x_f$ 

$$\Delta x = x_f - x_i = x_f - 0 = v\Delta t = 5.0 \times 10 = 50 m \implies x_f = 50 m$$

#### If the Speed Constant: Speed = avg. Speed

#### For example,

Consider a particle moving **at a constant speed of 5.0 m/s in a circular path** of **radius 10.0 m**. Calculate the time interval required to complete one trip around the circle.

$$\overline{v} = v = \frac{d}{\Delta t} \rightarrow \Delta t = \frac{d}{v} = \frac{2\pi r}{v} = \frac{2\pi (10.0m)}{5.0} = \frac{62.87}{5.00} = 12.6 s$$

2.4 Acceleration

Average acceleration: 
$$\bar{a}_x = \frac{\Delta v_x}{\Delta t} = \frac{v_{xf} - v_{xi}}{t_c - t_c}$$

#### Units: SI system is m/s<sup>2</sup>.

Instantaneous acceleration:

$$a_{x} = \lim_{\Delta t \to 0} \frac{\Delta v_{x}}{\Delta t} = \frac{dv_{x}}{dt}$$

Velocity and acceleration are in the same direction, the object **is speeding up**, when they are opposite, the object is **slowing down**.



Since: 
$$v_x = \frac{dx}{dt} \Longrightarrow a_x = \frac{dv_x}{dt} = \frac{d^2x}{dt^2}$$

The instantaneous acceleration (Figure 2.6b) can be obtained from the velocity–time graph.

# At any time, the acceleration equals the slope of the line tangent to the $v_x$ versus t curve.

Positive values of acceleration correspond to those points in Figure 2.6a where the velocity is increasing in the positive x direction. The

acceleration reaches a maximum at time  $t_A$ , when the slope of the velocity–time graph is a maximum. The acceleration then goes to zero at time  $t_B$ , when the velocity is a maximum.

The acceleration is negative when the velocity is decreasing in the positive x direction, and it reaches its most negative value at time  $t_{\rm C}$ .

#### Conceptual Example 2.5 Graphical Relationships Between x, v<sub>x</sub>, and a<sub>x</sub>

The position of an object moving along the *x* axis varies with time as in Figure 2.7a. Graph the velocity versus time and the acceleration versus time for the object.

Solution The velocity at any instant is the slope of the





tangent to the *x* -*t* graph at that instant. Between t = 0 and  $t=t_A$ , the



slope of the x - t graph increases uniformly, and so the velocity increases linearly, as shown in Figure 2.7b.

Between  $t_A$  and  $t_B$ , the slope is constant, and so the velocity remains constant. At  $t_D$ , the slope of is zero, so the velocity is zero at that instant.

Between  $t_D$  and  $t_E$ , the slope and thus the velocity are negative and decrease uniformly in this interval. In the interval  $t_E$  to  $t_F$ , the slope is still negative, and at  $t_F$  it goes to zero.

Finally, after  $t_F$ , the slope of the *x* -*t* graph is zero, meaning that the object is at rest for  $t > t_F$ .

The acceleration is constant and positive between 0 and  $t_A$ , where the slope of the  $v_x$  -t graph is positive. It is zero between  $t_A$ and  $t_B$  and for  $t > t_F$  because the slope of the  $v_x$  -t graph is zero at these times. It is negative between  $t_B$  and  $t_E$  because the slope of the  $v_x$  -t graph is negative during this interval.

#### Example 2.6 Average and Instantaneous Acceleration

The velocity of a particle moving along the *x*-axis varies time according to the expression  $v_x = (40 - 5t^2) \text{ m/s}$ , where t is in seconds.

(A) Find the average acceleration in the time interval t = 0 to t = 2.0 s.

#### Solution

Because the slope of the entire  $v_x$ -t

curve is negative, we expect the acceleration to be negative.

$$v_{xA} = [40 - 5t_{A}^{2}] = [40 - 5(0)^{2}] = +40 \text{ m/s}$$



$$v_{xB} = \left[ 40 - 5t_{B}^{2} \right] = \left[ 40 - 5(2)^{2} \right] = +20 \text{ m/s}$$

$$\bar{a}_{x} = \frac{\Delta v_{x}}{\Delta t} = \frac{v_{xf} - v_{xi}}{t_{f} - t_{i}} = \frac{20 - 40}{2 - 0} = -10 \text{ m/s}^{2}$$
(B) Determine the acceleration at  $t = 2.0 \text{ s}$ .
$$a_{x} = \lim_{\Delta t \to 0} \frac{\Delta v_{x}}{\Delta t} = \frac{dv_{x}}{dx}, \text{ take } t_{i} = t = 2.0 \text{ s}, t_{f} = t + \Delta t$$

$$v_{xf} = 40 - 5(t + \Delta t)^{2} = 40 - 5t^{2} - 10t - 10t\Delta t - 5(\Delta t)^{2}$$

$$v_{xi} = 40 - 5t^{2}$$

$$a_{x} = \lim_{\Delta t \to 0} \frac{v_{xf} - v_{xi}}{\Delta t} = \lim_{\Delta t \to 0} \frac{-10t\Delta t - 5(\Delta t)^{2}}{\Delta t} = \lim_{\Delta t \to 0} (-10t - 5\Delta t) = -10t \text{ m/s}^{2};$$

$$\Rightarrow a_{x}(2.0) = -20 \text{ m/s}^{2}$$

Simply use derivative:

$$a_x = \frac{dv_x}{dt} = -10t = -10(2.0) = -20.0 \text{ m/s}^2$$

**2.5** Motion Diagrams



Motion diagrams describe the velocity and acceleration while an object is in motion.

In Figure **2.9a**, the car moves through the same displacement in each time interval. Therefore, the car is moving with *constant* 

*positive velocity* and *zero acceleration*. We could model the car as a particle moving with constant velocity.

In Figure 2.9b, the velocity vector increases in time, the car is moving with a *positive velocity* and a *positive acceleration*. The velocity and acceleration are in the same direction. The car is speeding up. In Figure 2.9c, we can tell that the **car slows** as it moves to the right because its displacement between adjacent images decreases with time. In this case, the car moves to the right with a constant **negative acceleration**. The velocity vector **decreases** in time and eventually reaches zero. From this diagram, we see that the acceleration and velocity vectors are *not* in the same direction.

## **2.6** Analysis Model: Particle Under Constant Acceleration

For one-dimensional motion with constant acceleration $\bar{a}_x = a_x$ , take  $t_i = 0$ , and  $t_f = t$  for any later time, we have

$$\overline{a}_{x} = a_{x} = \frac{v_{xf} - v_{xi}}{t - 0} \quad \text{or,}$$

$$\mathbf{v}_{xf} = \mathbf{v}_{xi} + \mathbf{a}_{x} \mathbf{t} \implies \mathbf{t} = \frac{\mathbf{v}_{xf} - \mathbf{v}_{xi}}{\mathbf{a}_{x}}$$
 (1)

This expression enables us to determine an object's velocity at *any* time *t* if we know the object's initial velocity  $v_{xi}$  and its (constant) acceleration  $a_x$ . A velocity–time graph for this constant-acceleration motion is shown in



Figure 2.10b. The graph is a straight line, the (constant) slope of which is the acceleration  $a_x$ ; when the acceleration is constant, the graph of acceleration versus time (Fig.2.10c) is a straight line having a slope of zero.

Because velocity at **constant acceleration** varies linearly in time according to Equation 2.9, we can express the average velocity in any time interval as the arithmetic mean of the initial velocity  $v_{xi}$  and the final velocity  $v_{xf}$ :

$$\overline{v}_{x} = \frac{v_{xf} + v_{xi}}{2}$$

$$\Delta \mathbf{x} = \mathbf{x}_{f} - \mathbf{x}_{i} = \mathbf{v}_{x} \mathbf{t} = \frac{\mathbf{v}_{xi} + \mathbf{v}_{xf}}{2} \cdot \mathbf{t} \Longrightarrow \mathbf{x}_{f} - \mathbf{x}_{i} = \left(\frac{\mathbf{v}_{xi} + \mathbf{v}_{xf}}{2}\right) \left(\frac{\mathbf{v}_{xf} - \mathbf{v}_{xi}}{a_{x}}\right)$$
$$\mathbf{v}_{xf}^{2} = \mathbf{v}_{xi}^{2} + 2 \cdot \mathbf{a}_{x} \left(\mathbf{x}_{f} - \mathbf{x}_{i}\right)$$
(2)

One can also write

$$x_{f} - x_{i} = \frac{v_{xi} + v_{xf}}{2} \cdot t \Longrightarrow x_{f} - x_{i} = \frac{v_{xf} + v_{xi}}{2} t = \frac{v_{xi} + v_{xi} + a_{x}t}{2} \cdot t$$

$$\Delta x = x_{f} - x_{i} = v_{xi} \cdot t + \frac{1}{2}a_{x} \cdot t^{2}$$
(3)

The three derived equation are the kinematic equations that may be used to solve any problem involving one-dimensional motion at constant acceleration.

**Quick Quiz 2.6** In Figure 2.12, match each  $v_x$ -*t* graph on the top with the  $a_x$ -*t* graph on the bottom that best describes the motion.



Figure 2 - 12 Solution: (a)  $\rightarrow$  (e), (b)  $\rightarrow$  (d), (c)  $\rightarrow$  (f).

#### **Example 2.7 Carrier Landing**

A jet lands on an aircraft carrier at a speed of 140 mi/h (= 63 m/s).
(A) What is its acceleration (assumed constant) if it stops in 2.0 s due to an arresting cable that snags the jet and brings it to a stop?

**Solution** The **final speed is zero**, and the acceleration of the jet is assumed to be constant.

$$a_x = \frac{v_{xf} - v_{xi}}{t_f - t_i} = \frac{0.0 - 63}{2} \approx -32 \text{ m/s}^2$$
,

$$v_{xf} = v_{xi} + a_x t \Longrightarrow 0 = 63 + a_x(2) \Longrightarrow a_x = -63/2 \approx -32m/s$$

(Jet is slowing down)

(B) If the jet touches down at position  $x_i = 50$ , what is its final position?

$$x_{f} - x_{i} = \frac{v_{xi} + v_{xf}}{2}t \implies x_{f} - 50 = \frac{1}{2}(63 + 0) \times 2.0 = 63 \text{ m}$$
$$x_{f} = 63 + 50 = 113 \text{ m}$$

One may also use:  $x_f - x_i = v_{xi}t + \frac{1}{2}a_xt^2$ 

#### Example 2.8 Watch Out for the Speed Limit!

A car traveling at a <u>constant speed of 45.0</u> **m/s** passes a trooper hidden behind a billboard. **One second after** the speeding car passes the billboard;



the **trooper sets out from** the billboard to catch it, accelerating at a constant rate of **3.00 m/s<sup>2</sup>**. How long does it take her to overtake the car?

**Solution** Fig. 2.12 helps clarify the sequence of events.

Choose the billboard as the origin and t = 0 to be the time the trooper begins moving. At t=0 the car is at  $x_B = 45.0$  m because it has traveled at a constant speed of  $v_x = 45.0$  m/s for 1 s. The car's position at any time *t*:

 $x_{car} = x_B + v_{x car}t = 45.0 + 45.0 t$ 

The trooper starts from rest at  $t_B = 0$  and accelerates at 3.00 m/s<sup>2</sup> away from the origin. Hence, her position at any time t is given by

$$x_{f^{-}}x_{i} = v_{xi}t + \frac{1}{2}a_{x}t^{2} = 0 + 0 + \frac{1}{2}a_{x}t^{2} = \frac{1}{2}(3.0)t^{2}$$

The trooper overtakes the car when

$$x_{trooper} = x_{car} \implies 1.5t^2 - 45 - 45t = 0;$$
  
⇒  $t = \frac{45 \pm \sqrt{(45)^2 - 4(1.5)(-45)}}{2 \times 1.5}; \implies t = 31.0s$ 

#### **2.7** Freely Falling Objects

All objects dropped toward the Earth near its surface and neglecting air resistance fall with the same constant acceleration under the influence of the Earth's gravity. Such motion near the surface of earth when air-resistance is absent is referred to a free-fall.

Objects thrown upward or downward and those released from rest are all falling freely once they are released. Freely falling object experiences acceleration directed downward, regardless of its initial motion. The magnitude of the *free-fall acceleration is denoted* by the **symbol** *g*.

Since the "up" is taken as the + y direction, then for free falling objects  $a_y = g = -9.8 \text{ m/s}^2$ .

# Example 2.10 Not a Bad Throw for a Rookie!

A stone thrown from the top of a building is given an **initial velocity of 20.0 m/s straight upward**. The building **is 50.0 m high**, and the stone just misses the edge of the roof on its way down, as shown in Figure 2.14.

(A) Using  $t_A = 0$  as the time the stone leaves the thrower's hand at position A, determine the time at which the stone reaches its maximum height.

#### Solution

As the stone reaches maximum height at B,  $v_{yB} = 0$ 



$$v_{yB} = v_{yA} - gt = 0$$
  
$$\Rightarrow v_{yA} = gt, \quad t = \frac{v_{yA}}{g} = \frac{20}{9.8} = 2.04s$$

(B) Find the maximum height of the stone.

#### Solution

The maximum height as measured from the position of the thrower A, where we set  $y_A = 0$ :

$$v_{yB}^{2} = v_{yA}^{2} - 2g(y_{B} - y_{A})$$
  
 $\Rightarrow 0 = 400 - 2 \times 9.8(y_{B} - 0),$ 
 $y_{B} = \frac{400}{2 \times 9.8} = 20.4 \text{m}$ 

On the other hand, you can also use:

$$y_{B} = y_{A} + v_{yA}t - \frac{1}{2}gt^{2}$$
  
 $\Rightarrow y_{B} = 0 + (20)(2.04) - \frac{1}{2}(9.8)(2.04)^{2} = 20.4 \text{ m}$ 

(C) Determine the velocity of the stone when it returns to the height from which it was thrown.

#### Solution

When the stone is back at the height from which it was thrown position A), the *y* coordinate is again zero.

$$v_{yc}^2 = v_{yA}^2 - 2g(y_c - y_A) \Rightarrow v_{yc}^2 = 400 - 2 \times 9.8(0 - 0), v_{yc} = -20 \text{ m/s}$$
  
(E) Find the velocity and position of the stone at *t*=5.00 s.  
Solution For t = 5 s;

$$v_{yD} = v_{yA} - gt = +20 - 9.8 \times 5 = -29.0 \text{ m/s}$$
  
 $y_{D} = y_{A} + v_{yA}t - \frac{1}{2}gt^{2} \Longrightarrow y_{D} = 0 + (20)(5) - \frac{1}{2}(9.8)(5)^{2} = -22.5 \text{ m}$ 

**Example:** A stone is fired from the ground vertically **upwards with initial velocity** +10 m/s.

(1) What is t when the **maximum height** reached by the stone measured from the point of release?



Choose upward direction relative to ground (y=0) to be positive

$$y_0 = 0$$
,  $a = -g$ ,  $v_0 = +10 \text{ m/s}$   
 $v_y = v_{0y} - gt \implies 0 = +10 - 10t \implies t = 1 \text{ sec}$ 

(2) plot the graph of v(t) vs. t



At y=0 v = +10, v=0 at  $y_{max}$  for some value of t (t=1s), v = -10 just as it touches the ground

(3) What is y<sub>max.</sub>

$$y_{max} = y_o + v_{y0}t - \frac{1}{2}gt^2$$
  
 $\Rightarrow y_{max} = 0 + (+10)(1) - \frac{1}{2}(10)(1)^2 = 5m$ 

#### **2.8** Kinematic Equations **Derived from Calculus**

Figure : $(v_x-t)$  for a particle moving along the x axis.

The area of the shaded rectangle is equal to the **displacement**  $\Delta \mathbf{x}$  in the **time** 



interval  $\Delta t_n$ , while the total area under the curve is the total displacement of the particle and is given by

$$\Delta \mathbf{x} = \sum_{n} \mathbf{v}_{x} \cdot \Delta \mathbf{t}_{n}$$

 $n \rightarrow \infty$ ,  $\Delta t_n \rightarrow 0$ , average velocity = instantaneous velocity,

**Displacement:**  $\Delta x = \int v_x(t) dt$ 

#### **Displacement** = area under $V_x$ -t graph.

Kinematic Equations for acceleration

$$a_x = \frac{dv_x}{dt} \implies dv_x = a_x dt$$
 Integrating (antiderivative) gives  
 $v_{xf} - v_{xi} = \int_0^t a_x dt$ 

For constant acceleration: ٧

$$v_{xf} - v_{xi} = a_x \int_{0}^{t} dt = a_x t$$

Equation for velocity

$$v_{x} = \frac{dx}{dt} \implies dx = v_{x}dt, \text{ And by integration}$$

$$x_{f} - x_{i} = \int_{0}^{t} v_{x}dt = \int_{0}^{t} (v_{xi} + a_{x}t)dt \quad (\text{Since} \quad v_{x} = v_{xf} = v_{xi} + a_{x}t)$$

$$x_{f} - x_{i} = v_{xi}t + \frac{1}{2}a_{x}t^{2}$$

#### Example:



$$\Delta x = Area ( )$$



$$\Delta x = \frac{1}{2}(2)(6+2) + \frac{1}{2}(3+2)(6) + \frac{1}{2}(2)(-4) = 19m$$

(b) Find the position x at t=7 sec.

$$x(7) - x(0) = x(7) - 4 = 19 m \implies x(7) = 23m$$

(c) Draw the acceleration curve.

