Ch. 2: Motion in One Dimension

Part 1

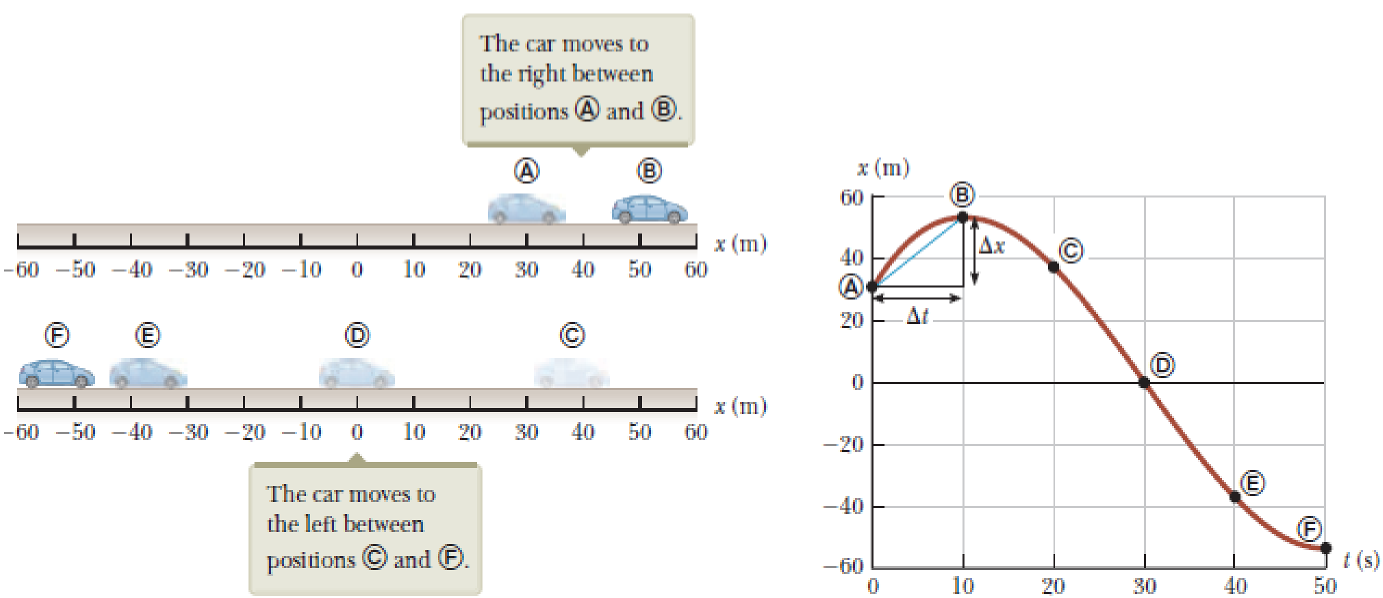
Kinematics: describes motion in terms of space and time, ignoring external agents that cause or modify that motion.

Motion: continuous change in position.

2.1 Position, Velocity, and Speed

position or location in space at any **time**, with respect to a **chosen reference point considered the origin of a coordinate system, (a reference point) O**

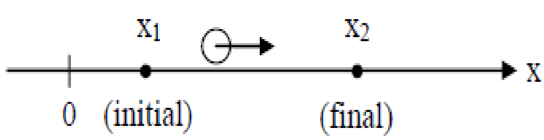
A car is moving back and forth along the *x* axis, Figure 2.1a. The car is 30 m to the right of the reference position $x = 0$. We record

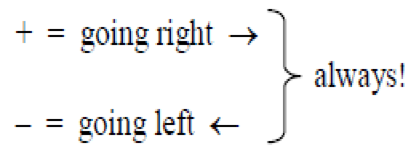


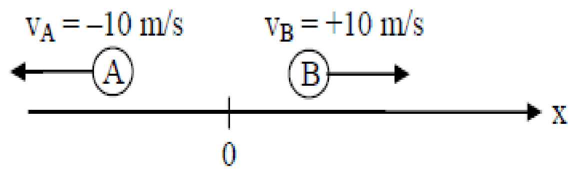
the car's position every 10 s, pictorially, graphically, and tabular.

The displacement: **Δx** change in the position in some time interval. Initial position x_i, final position x_f

$$\Delta x = x_f - x_i$$







Distance d is the **length of a path** followed by a particle.

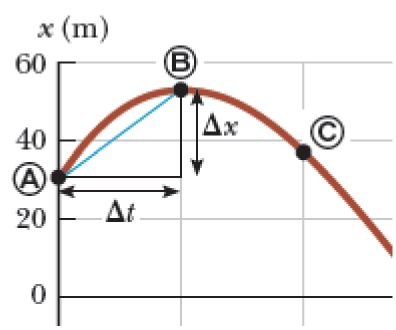
The displacement is a vector which has direction. **<u>Distance</u> d is scaler always positive.**

<u>The average velocity</u> $\bar{\upsilon}_x$ = displacement Δx divided by the time interval Δt of the displacement:

$$v_{x,avg} = \bar{v}_x = \frac{\Delta x}{\Delta t}$$ <u>Unit:</u> **meter/second (m/s)**

<u>For example</u>,

Between A and B in Figure 2.1b



$$v_{x,avg} = \bar{v}_x = \frac{\Delta x}{\Delta t} = \frac{52-30}{10-0} = 2.2\,m/s =$$

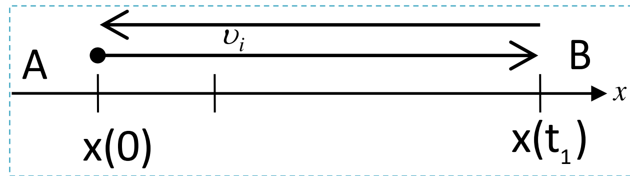
Slope of line between A and B

Average velocity = the slope of the line joining the two points

The **average speed of a particle**, is a scalar quantity,

Average speed $= \bar{v} = \dfrac{d}{\Delta t} = \dfrac{\text{total distance}}{\text{total time}}$ (scalar has no direction)

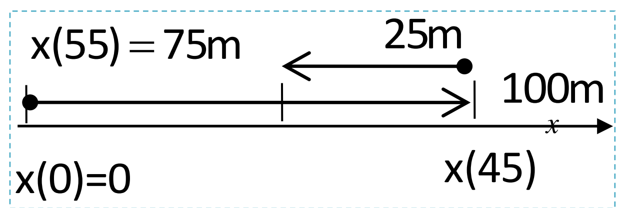
You travel from point ***A*** to point ***B*** and then back along the line from ***B*** to ***A***.
What is
(a) your total displacement?
(b) your distance traveled?



The **displacement is zero**: **Δx = x$_f$ - x$_i$ = 0**

The **distance is twice the length:** $d = 2\overline{AB}$

For example, suppose it takes **you 45.0 s to travel 100 m down** a long straight hallway toward your departure gate at an airport. At the 100 m mark, you realize you missed the rest room, and you **return back 25.0 m along the same hallway**, **taking 10.0 s to make the return trip**.



Average velocity $= v_{xavg} = \dfrac{\Delta x}{\Delta t} = \dfrac{+75.0}{55.0} = 1.36 m/s$

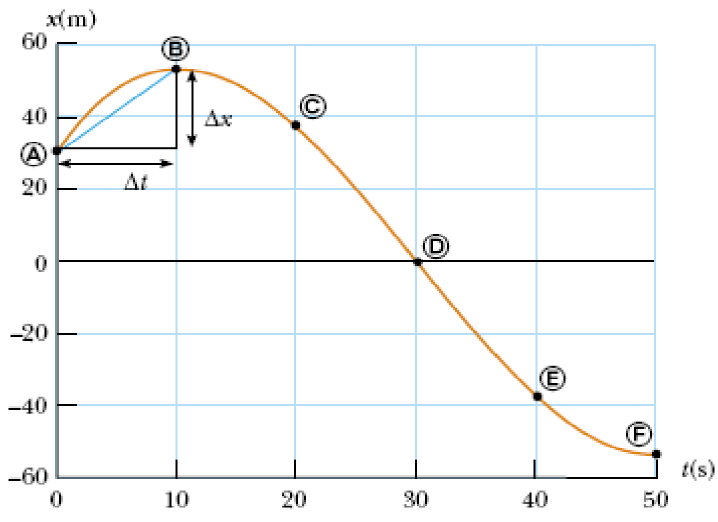
The **average speed** for your trip is

$$v_{avg} = \bar{v} = \frac{\text{total distance}}{\text{total time}} = \frac{d}{\Delta t} = \frac{+125}{55.0} = 2.27 m/s$$

Example 2.1 Calculating the Average Velocity and Speed

Find the displacement, average velocity, and average speed of the car in **Figure 2.1a** between positions A and F.

Solution From the position–time graph given in Figure 2.1b, note that $\boldsymbol{x_A = 30\ m}$ at $t_A = 0$ s and that $\boldsymbol{x_F = -53\ m}$ at $t_F = 50$ s



(b)

Fig.2.1

Table 2.1

Position of the Car at Various Times

Position	t(s)	x(m)
Ⓐ	0	30
Ⓑ	10	52
Ⓒ	20	38
Ⓓ	30	0
Ⓔ	40	−37
Ⓕ	50	−53

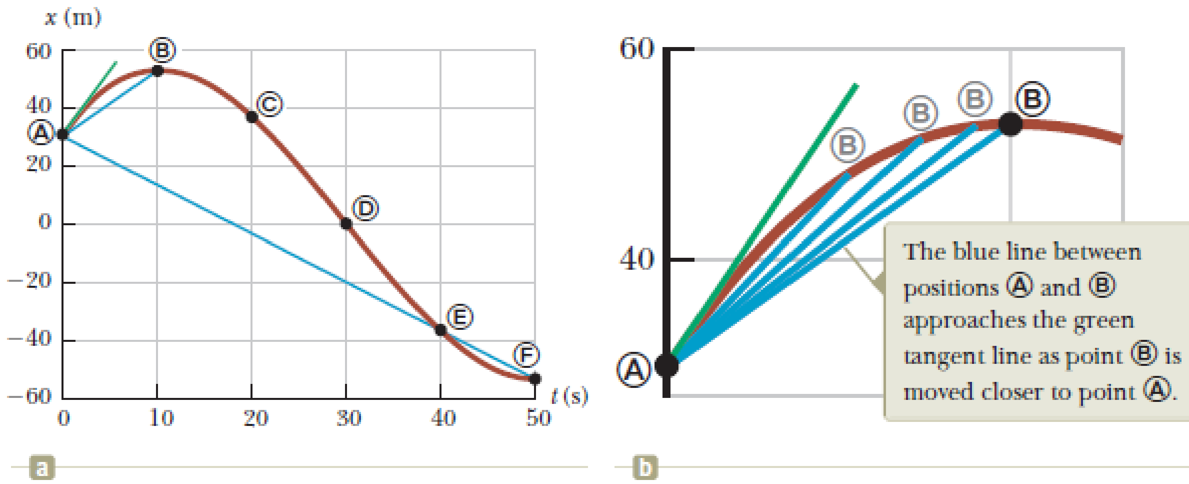
$$\bar{v}_x = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i} = \frac{x_F - x_A}{t_F - t_A} = \frac{-53-30}{50-0} = \frac{-83m}{50s} = -1.7\ \ m/s$$

From **A** to **B** $\boldsymbol{d^+ = 22\ m}$ + From **B** to **F** $\boldsymbol{d^- = 105\ m}$

$$\text{average speed} = \bar{v} = \frac{\text{total distance}}{\text{total time}} = \frac{d}{\Delta t} = \frac{+127}{50s} = 2.5\,m/s$$

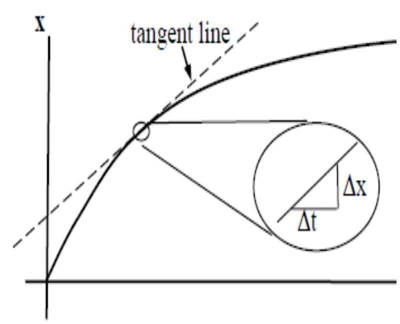
2.2 Instantaneous Velocity and Speed

Needed the velocity of a particle at some instant of time. If the point B moves toward point A. The line between the points becomes steeper and steeper, and as the two points become extremely close together, the line becomes a tangent line to the curve.



The ***instantaneous velocity* = the slope the tangent line at A = velocity of the car at the moment the car was at A**.

The instantaneous velocity $\mathbf{v_x}$ is equal to **the *derivative* of *x* with respect to *t***,



$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

The instantaneous speed = is defined as the **magnitude of its instantaneous velocity**. It is a scalar positive quantity.

If the instantaneous velocity is + 25 m/s along a given line

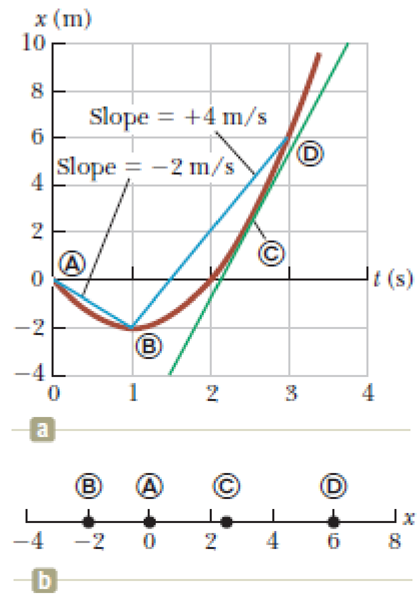
The speed or the instantaneous = **+ 25 m/s**

The **instantaneous velocity of - 25 m/s** along the same line,

The speed <u>or</u> the instantaneous speed = + 25 m/s

Example 2.3 Average and Instantaneous Velocity

A particle moves along the *x-axis*. Its position varies with time according to the expression $\boldsymbol{x(t) = -4t + 2t^2}$, where x is in meters and t is in seconds. The position–time graph for this motion is shown in Figure 2.4. Notice that the particle moves in the negative x direction for the first second of motion, is momentarily at rest at the moment $\boldsymbol{t = 1}$ **s**, and moves in the positive x direction at times $\boldsymbol{t > 1}$ **s**.



(A) Determine the **displacement** of the particle in the time intervals $t = 0$ to $t = 1$ s and $t = 1$ s to $t = 3$ s.

Solution

$$\Delta x(A \rightarrow B) = x_f - x_i = x(1) - x(0) = \left[-4(1) + 2(1)^2\right] - \left[-4(0) + 2(0)\right] = -2\text{m}$$

$$\Delta x_{BD} = x_f - x_i = x(3) - x(1) = \left[-4(3) + 2(3)^2\right] - \left[-4(1) + 2(1)\right] = +6 + 2 = +8\text{m}$$

Or simply from the graph one can obtain the same answers.

(B) Calculate the **average velocity** during these two time intervals.

Solution

$$\bar{v}_x(A \rightarrow B) = \frac{\Delta x}{\Delta t} = \frac{-2}{1} = -\ 2\text{m/s}$$

$$\bar{v}_x(B \rightarrow D) = \frac{\Delta x}{\Delta t} = \frac{+8}{3\text{-}1} = \ +4\text{m/s}$$

(C) Find the instantaneous velocity of the particle at $t = 2.5$ s.

By measuring the slope of tangent line at $t = 2.5$ s in Figure 2.4, we find that

From the graph, the slope $\approx \dfrac{10-(-4)}{3.8-1.5} = \dfrac{14}{2.3} \approx +6\,\text{m/s}$

From the equation

$$v_x(C) = \left.\frac{dx}{dt}\right|_{t=2.5} = \left[-4+4t\right]_{t=2.5} = \left[-4+4(2.5)\right] = +6\,\text{m/s}$$

2.3 Analysis Model: Particle under Constant Velocity

Any entity moving under constant velocity can be used as a model. The entity may be a car, a person, or a plane; which are modeled as a particle.

The particle velocity is constant, hence the **instantaneous velocity = the average velocity, and**

$$\bar{v}_x = v_x = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

Taking x_i at $t_i = 0$, $t_f = t$:

$$\frac{x_f - x_i}{t-0} = v_x \rightarrow x_f = x_i + v_x t \qquad \text{(for constant } v_x\text{)}$$

Example 2.4 Modeling a Runner as a Particle

A kinesiologist is studying the biomechanics of the human body. (*Kinesiology* is the study of the movement of the human body. Notice the connection to the word *kinematics*.) She determines the velocity of an experimental subject while he runs along a straight line at **a constant rate**. The kinesiologist starts the stopwatch at the moment the runner passes a given point and stops it after the runner has passed another point 20 m away. The time interval indicated on the stopwatch is 4.0 s.

(A) What is the runner's velocity?

Runner is moving at a constant rate or constant velocity.

$$v_x = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i} = \frac{20 - 0}{4.0} = 5.0 m/s$$

(B) If the runner continues his motion after the stopwatch is stopped, what is his position after **10 s** have passed?

Let the runner's position at **t = 10 s** be x_f

$$\Delta x = x_f - x_i = x_f - 0 = v\Delta t = 5.0 \times 10 = 50 m \quad \Rightarrow x_f = 50\, m$$

If the **Speed Constant: Speed = avg. Speed**

For example,

Consider a particle moving **at a constant speed of 5.0 m/s in a circular path** of **radius 10.0 m**. Calculate the time interval required to complete one trip around the circle.

$$\bar{v} = v = \frac{d}{\Delta t} \quad \rightarrow \quad \Delta t = \frac{d}{v} = \frac{2\pi r}{v} = \frac{2\pi(10.0 m)}{5.0} = \frac{62.87}{5.00} = 12.6\, s$$

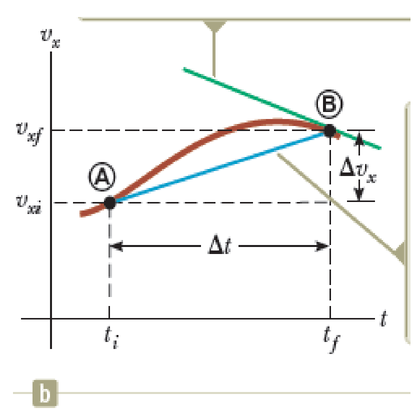
2.4 Acceleration

Average acceleration: $$\bar{a}_x = \frac{\Delta v_x}{\Delta t} = \frac{v_{xf} - v_{xi}}{t_f - t_i}$$

Units: SI system is m/s².

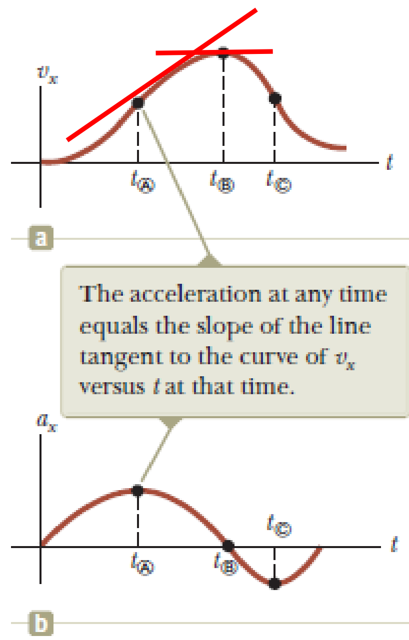
Instantaneous acceleration:

$$a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt}$$



Velocity and acceleration are in the same direction, the object **is speeding up**, when they are opposite, the object is **slowing down**.

Since: $v_x = \dfrac{dx}{dt} \Rightarrow a_x = \dfrac{dv_x}{dt} = \dfrac{d^2x}{dt^2}$



The instantaneous acceleration (Figure 2.6b) can be obtained from the velocity–time graph.

At any time, the acceleration equals the slope of the line tangent to the v_x versus t curve.

Positive values of acceleration correspond to those points in Figure 2.6a where the velocity is increasing in the positive *x* direction. The acceleration reaches a maximum at time t_A, when the slope of the velocity–time graph is a maximum. The acceleration then goes to zero at time t_B, when the velocity is a maximum.

The acceleration is negative when the velocity is decreasing in the positive *x* direction, and it reaches its most negative value at time t_C.

Conceptual Example 2.5

Graphical Relationships Between x, v_x, and a_x

The position of an object moving along the *x* axis varies with time as in Figure 2.7a. Graph the velocity versus time and the acceleration versus time for the object.

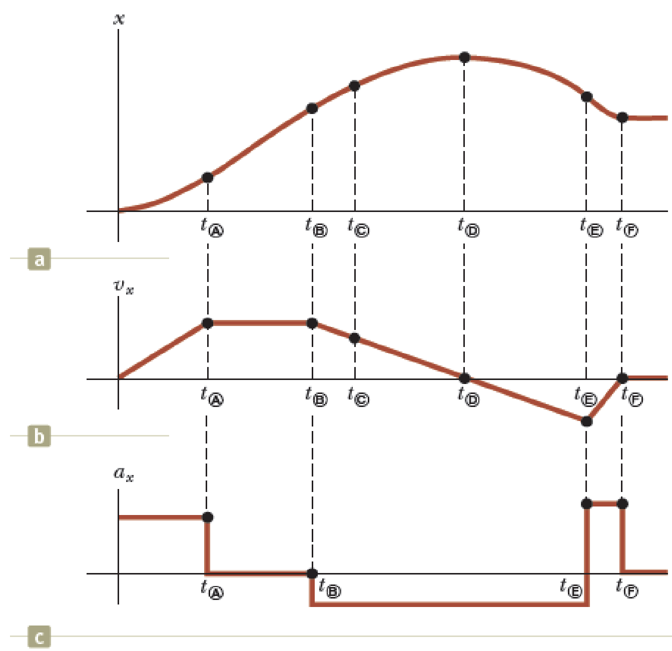


Figure 2.8 (Conceptual Example 2.5) (a) Position–time graph

Solution The velocity at any instant is the slope of the tangent to the *x* -*t* graph at that instant. Between $t = 0$ and $t = t_A$, the slope of the $x - t$ graph increases uniformly, and so the velocity increases linearly, as shown in Figure 2.7b.

Between t_A and t_B, the slope is constant, and so the velocity remains constant. At t_D, the slope of is zero, so the velocity is zero at that instant.

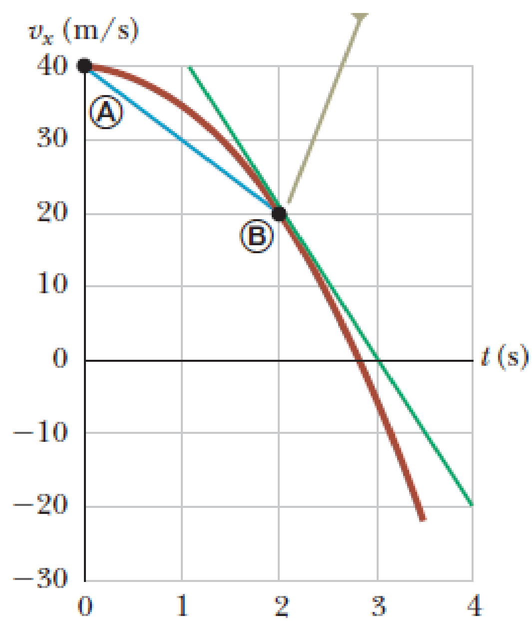
Between t_D and t_E, the slope and thus the velocity are negative and decrease uniformly in this interval. In the interval t_E to t_F, the slope is still negative, and at t_F it goes to zero.

Finally, after t_F, the slope of the x -t graph is zero, meaning that the object is at rest for $t > t_F$.

The acceleration is constant and positive between 0 and t_A, where the slope of the v_x -t graph is positive. It is zero between t_A and t_B and for $t > t_F$ because the slope of the $v_x - t$ graph is zero at these times. It is negative between t_B and t_E because the slope of the v_x -t graph is negative during this interval.

Example 2.6 Average and Instantaneous Acceleration

The velocity of a particle moving along the *x-axis* varies time according to the expression **$v_x = (40 - 5t^2)$ m/s**, where t is in seconds.



(A) Find the average acceleration in the time interval $t = 0$ to $t = 2.0$ s.

Solution

Because the slope of the entire v_x-t curve is negative, we expect the acceleration to be negative.

$$v_{xA} = \left[40 - 5t_A^{\ 2}\right] = \left[40 - 5(0)^2\right] = +40\,\text{m/s}$$

$$v_{xB} = \left[40 - 5t_B^{\ 2}\right] = \left[40 - 5(2)^2\right] = +20\,m/s$$

$$\bar{a}_x = \frac{\Delta v_x}{\Delta t} = \frac{v_{xf} - v_{xi}}{t_f - t_i} = \frac{20 - 40}{2 - 0} = -10 m/s^2$$

(B) Determine the acceleration at t = 2.0 s.

$a_x = \lim\limits_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dx}$, take $t_i = t = 2.0\ s$, $t_f = t + \Delta t$

$$v_{xf} = 40 - 5(t + \Delta t)^2 = 40 - 5t^2 - 10t - 10t\Delta t - 5(\Delta t)^2$$

$$v_{xi} = 40 - 5t^2$$

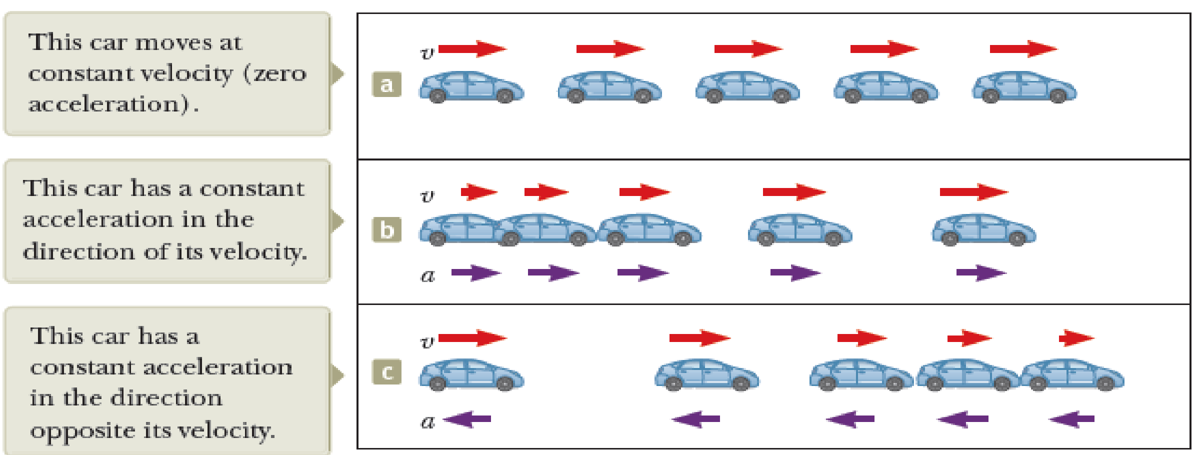
$$a_x = \lim_{\Delta t \to 0} \frac{v_{xf} - v_{xi}}{\Delta t} = \lim_{\Delta t \to 0} \frac{-10t\Delta t - 5(\Delta t)^2}{\Delta t} = \lim_{\Delta t \to 0}\left(-10t - 5\Delta t\right) = -10t\ \ m/s^2;$$

$$\Rightarrow a_x(2.0) = -20 m/s^2$$

Simply use derivative:

$$a_x = \frac{dv_x}{dt} = -10t = -10(2.0) = -20.0 m/s^2$$

2.5 Motion Diagrams



Motion diagrams describe the velocity and acceleration while an object is in motion.

In Figure **2.9a**, the car moves through the same displacement in each time interval. Therefore, the car is moving with ***constant***

positive velocity **and** ***zero acceleration***. We could model the car as a particle moving with constant velocity.

In Figure **2.9b**, **the velocity vector increases in time**, the car is moving with a *positive velocity* and a ***positive acceleration***. The velocity and acceleration are in the same direction. The car is speeding up. In Figure **2.9c**, we can tell that the **car slows** as it moves to the right because its displacement between adjacent images decreases with time. In this case, the car moves to the right with a constant **negative acceleration**. **The velocity vector decreases** in time and eventually reaches zero. From this diagram, we see that the acceleration and velocity vectors are *not* in the same direction.