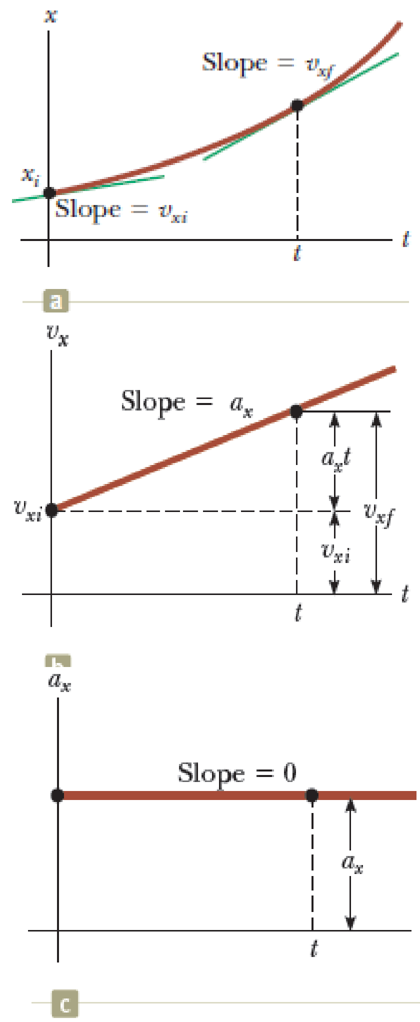
Ch. 2: Motion in One Dimension

Part 2

2.6 Analysis Model: Particle Under Constant Acceleration



For one-dimensional motion with constant acceleration $\bar{a}_x = a_x$, take $t_i = 0$, and $t_f = t$ for any later time, we have

$$\bar{a}_x = a_x = \frac{v_{xf} - v_{xi}}{t - 0} \qquad \text{or,}$$

$$v_{xf} = v_{xi} + a_x t \Rightarrow t = \frac{v_{xf} - v_{xi}}{a_x} \qquad (1)$$

This expression enables us to determine an object's velocity at *any* time *t* if we know the object's initial velocity v_{xi} and its (constant) acceleration a_x. A velocity–time graph for this constant-acceleration motion is shown in Figure 2.10b. The graph is a straight line, the (constant) slope of which is the acceleration a_x; when the acceleration is constant, the graph of acceleration versus time (Fig.2.10c) is a straight line having a slope of zero.

Because velocity at **constant acceleration** varies linearly in time according to Equation2.9, we can express the average velocity in any time interval as the arithmetic mean of the initial velocity v_{xi} and the final velocity v_{xf} :

$$\bar{v}_x = \frac{v_{xf} + v_{xi}}{2}$$

$$\Delta x = x_f - x_i = \bar{v}_x t = \frac{v_{xi} + v_{xf}}{2} \cdot t \Rightarrow x_f - x_i = \left(\frac{v_{xi} + v_{xf}}{2}\right)\left(\frac{v_{xf} - v_{xi}}{a_x}\right)$$

$$v_{xf}^2 = v_{xi}^2 + 2 \cdot a_x \left(x_f - x_i\right) \qquad (2)$$

One can also write

$$x_f - x_i = \frac{v_{xi} + v_{xf}}{2} \cdot t \Rightarrow x_f - x_i = \frac{v_{xf} + v_{xi}}{2} t = \frac{v_{xi} + v_{xi} + a_x t}{2} \cdot t$$

$$\Delta x = x_f - x_i = v_{xi} \cdot t + \frac{1}{2} a_x \cdot t^2 \qquad (3)$$

The three derived equation are the kinematic equations that may be used to solve any problem involving one-dimensional motion at constant acceleration.

Quick Quiz 2.6 In Figure 2.12, match each v_x–t graph on the top with the a_x–t graph on the bottom that best describes the motion.

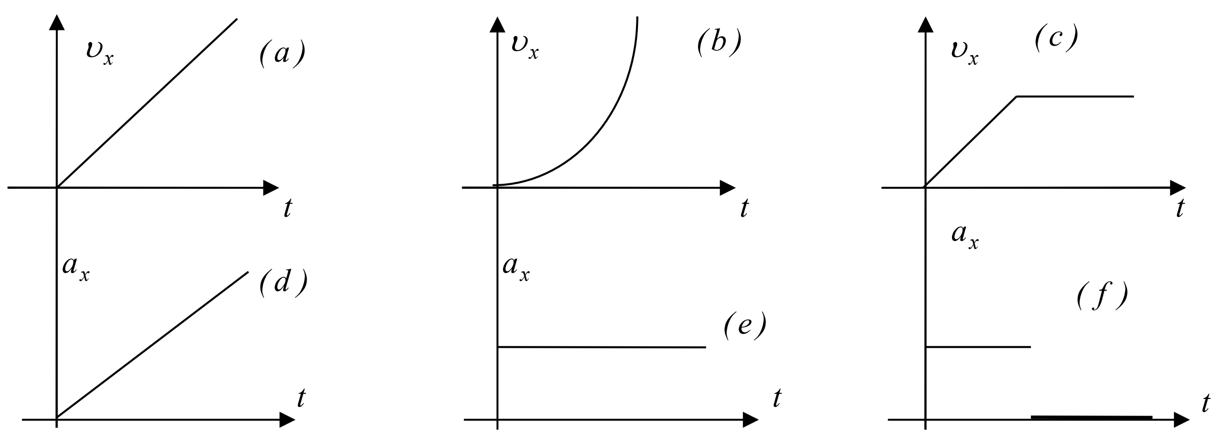
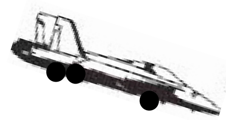


Figure 2 - 12 **Solution:** (a)→(e), (b) →(d), (c) → (f).

Example 2.7 Carrier Landing



A jet lands on an aircraft carrier at a speed of 140 mi/h (= 63 m/s).
(A) What is its acceleration (assumed constant) if it stops in 2.0 s due to an arresting cable that snags the jet and brings it **to a stop**?

Solution The **final speed is zero**, and the acceleration of the jet is assumed to be constant.

$$a_x = \frac{v_{xf} - v_{xi}}{t_f - t_i} = \frac{0.0 - 63}{2} \approx -32\,m/s^2,$$

$$v_{xf} = v_{xi} + a_x t \Rightarrow 0 = 63 + a_x(2) \Rightarrow a_x = -63/2 \approx -32 m/s$$

(Jet is slowing down)

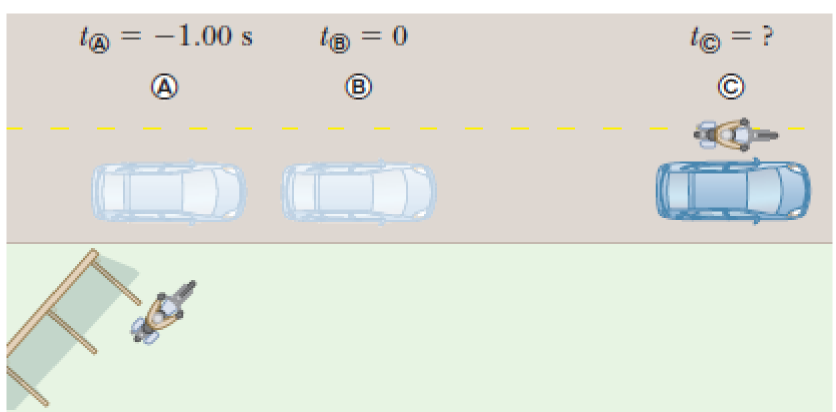
(B) If the jet touches down at position x_i= 50, what is its final position?

$$x_f - x_i = \frac{v_{xi} + v_{xf}}{2} t \quad \Rightarrow x_f - 50 = \frac{1}{2}(63 + 0) \times 2.0 = 63\,m$$

$$x_f = 63 + 50 = 113m$$

One may also use: $x_f - x_i = v_{xi} t + \frac{1}{2} a_x t^2$

Example 2.8 Watch Out for the Speed Limit!



A car traveling at a <u>constant speed of **45.0 m/s**</u> passes a trooper hidden behind a billboard. **One second after** the speeding car passes the billboard;

the **trooper sets out from** the billboard to catch it, accelerating at a constant rate of **3.00 m/s²**. **How long does it take her to overtake the car?**

Solution Fig. 2.12 helps clarify the sequence of events.
Choose the billboard as the origin and $t = 0$ to be the time the trooper begins moving. **At t=0 the car is at x_B = 45.0 m** because it has traveled at a constant speed of **v_x = 45.0 m/s for 1 s**. The car's position at any time t:

$$x_{car} = x_B + v_{x\,car} t = 45.0 + 45.0\,t$$

The trooper starts from rest at t_B = 0 and **accelerates at 3.00 m/s²** away from the origin. Hence, her **position at any time t** is given by

$$x_f - x_i = v_{xi}t + \frac{1}{2}a_x t^2 = 0 + 0 + \frac{1}{2}a_x t^2 = \frac{1}{2}(3.0)t^2$$

The trooper overtakes the car when

$$x_{trooper} = x_{car} \Rightarrow 1.5t^2 - 45 - 45t = 0;$$

$$\Rightarrow t = \frac{45 \pm \sqrt{(45)^2 - 4(1.5)(-45)}}{2 \times 1.5}; \quad \Rightarrow t = 31.0\text{s}$$

2.7 Freely Falling Objects

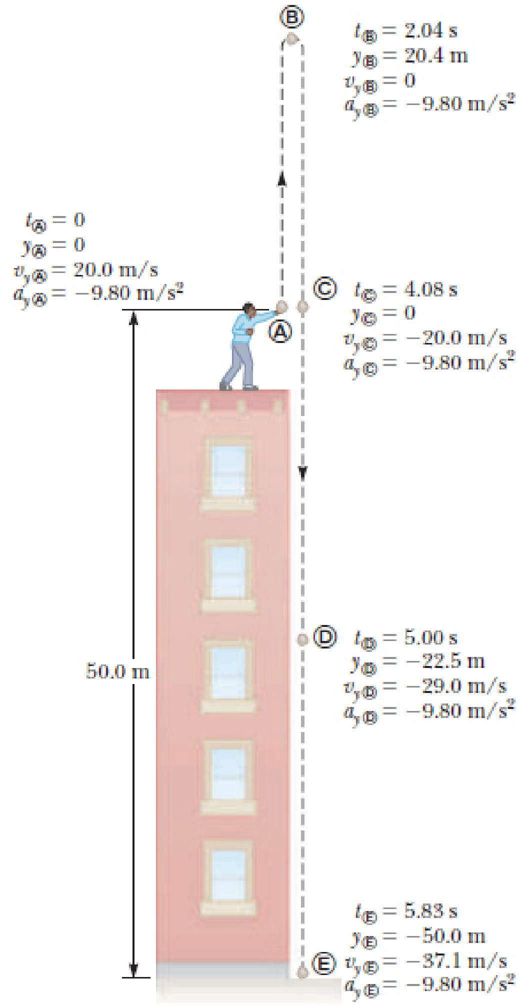
All objects dropped toward the Earth near its surface and neglecting air resistance fall with the same constant acceleration under the influence of the Earth's gravity. Such motion near the surface of earth when air-resistance is absent is referred to a free-fall.

Objects thrown upward or downward and those released from rest are all falling freely once they are released. Freely falling object experiences acceleration directed downward, regardless of its initial

motion. The magnitude of the ***free-fall acceleration*** *is denoted* by the **symbol *g***.
Since the "**up**" **is taken as the +*y* direction**, then for free falling objects **$a_y = g = -9.8\ m/s^2$.**

Example 2.10 Not a Bad Throw for a Rookie!

A stone thrown from the top of a building is given an **initial velocity of 20.0 m/s straight upward**. The building **is 50.0 m high**, and the stone just misses the edge of the roof on its way down, as shown in Figure 2.14.



(A) Using $t_A = 0$ as the time the stone leaves the thrower's hand **at position A,** determine **the time** at which the stone **reaches its maximum height**.

Solution

As the stone reaches maximum height at B, $v_{yB} = 0$

$$v_{yB} = v_{yA} - gt = 0$$

$$\Rightarrow v_{yA} = gt, \quad t = \frac{v_{yA}}{g} = \frac{20}{9.8} = 2.04\,s$$

(B) Find the maximum height of the stone.

Solution

The maximum height as measured from the position of the thrower A, where we set $y_A = 0$:

$$v_{yB}^2 = v_{yA}^2 - 2g(y_B - y_A)$$
$$\Rightarrow 0 = 400 - 2\times 9.8(y_B - 0), \qquad y_B = \frac{400}{2\times 9.8} = 20.4\text{m}$$

On the other hand, you can also use:

$$y_B = y_A + v_{yA}t - \frac{1}{2}gt^2$$
$$\Rightarrow y_B = 0 + (20)(2.04) - \frac{1}{2}(9.8)(2.04)^2 = 20.4 \ \text{m}$$

(C) Determine the velocity of the stone **when it returns to the height from** which it was thrown.

Solution

When the stone is back at the height from which it was thrown position A), **the *y* coordinate is again zero.**

$$v_{yC}^2 = v_{yA}^2 - 2g(y_C - y_A) \Rightarrow v_{yC}^2 = 400 - 2\times 9.8(0-0),\ v_{yC} = -20 \ \text{m/s}$$

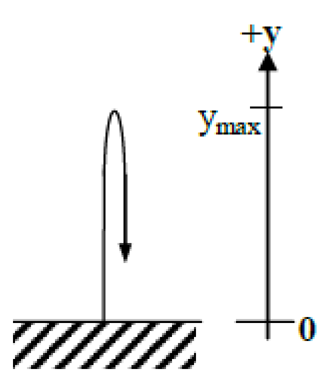
(E) Find the velocity and position of the stone at ***t*=5.00 s**.

Solution For t = 5 s;

$$v_{yD} = v_{yA} - gt = +20 - 9.8\times 5 = -29.0\text{m/s}$$

$$y_D = y_A + v_{yA}t - \frac{1}{2}gt^2 \Rightarrow y_D = 0 + (20)(5) - \frac{1}{2}(9.8)(5)^2 = -22.5\,\text{m}$$

Example: A stone is fired from the ground vertically **upwards with initial velocity +10 m/s.**

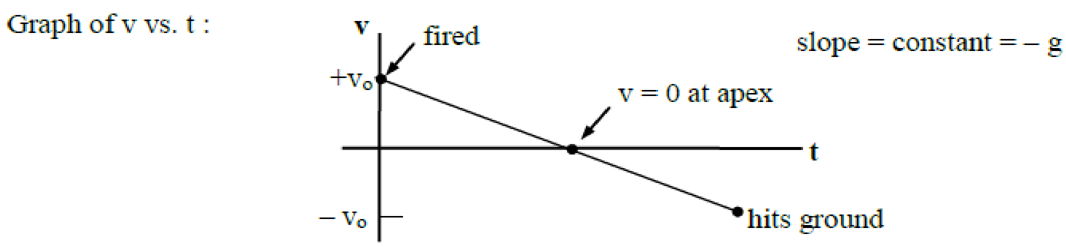


(1) What is t when the **maximum height** reached by the stone measured from the point of release?

Choose upward direction relative to ground (y=0) to be positive

$$y_0 = 0, \quad a = -g, \quad v_o = +10 \text{ m/s}$$
$$v_y = v_{oy} - gt \quad \Rightarrow 0 = +10\text{-}10t \quad \Rightarrow t = 1 \text{ sec}$$

(2) plot the graph of v(t) vs. t



At y=0 v = +10, v=0 at y_{max} for some value of t (t=1s), v = -10 just as it touches the ground

(3) What is y_{max}.

$$y_{max} = y_o + v_{y0}t - \frac{1}{2}gt^2$$

$$\Rightarrow y_{max} = 0 + (+10)(1) - \frac{1}{2}(10)(1)^2 = 5\text{m}$$

2.8 Kinematic Equations Derived from Calculus

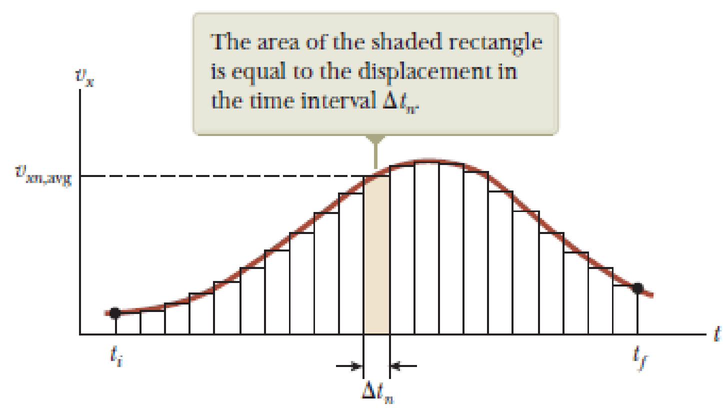


Figure :(v_x-t) for a particle moving along the *x* axis.

The area of the shaded rectangle **is equal to the displacement** Δx in the **time interval** Δt_n, while the total area under the curve is the total displacement of the particle and is given by

$$\Delta x = \sum_n \overline{v}_x \cdot \Delta t_n$$

$n \to \infty$, $\Delta t_n \to 0$, average velocity = instantaneous velocity,

Displacement: $\Delta x = \int v_x(t)dt$

Displacement = area under V_x–t graph.

Kinematic Equations for acceleration

$$a_x = \frac{dv_x}{dt} \Rightarrow dv_x = a_x dt$$ Integrating (antiderivative) gives

$$v_{xf} - v_{xi} = \int_0^t a_x dt$$

For constant acceleration: $$v_{xf} - v_{xi} = a_x \int_0^t dt = a_x t$$

Equation for velocity

$$v_x = \frac{dx}{dt} \Rightarrow dx = v_x dt,$$ And by integration

$$x_f - x_i = \int_0^t v_x dt = \int_0^t (v_{xi} + a_x t)dt \qquad (\text{Since} \quad v_x = v_{xf} = v_{xi} + a_x t)$$

$$x_f - x_i = v_{xi} t + \frac{1}{2} a_x t^2$$

Example:

Fig shows **v_x vs. t** in sec. the object starts at t = 0 from x = 4.0m.

(a) Find displacement of the object for the entire trip (i.e. from t=0 to t =7s.)

Total **displacement Δx = area under the (*v*, *t*)** curve from ***t*=0 to 7 s**.

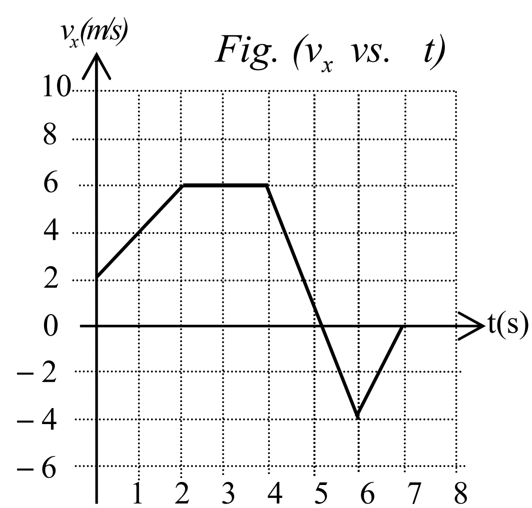
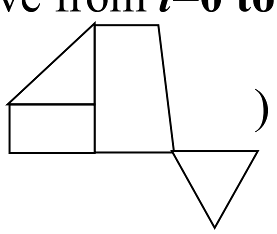


Fig. (v_x vs. t)

Δx= **Area (** )

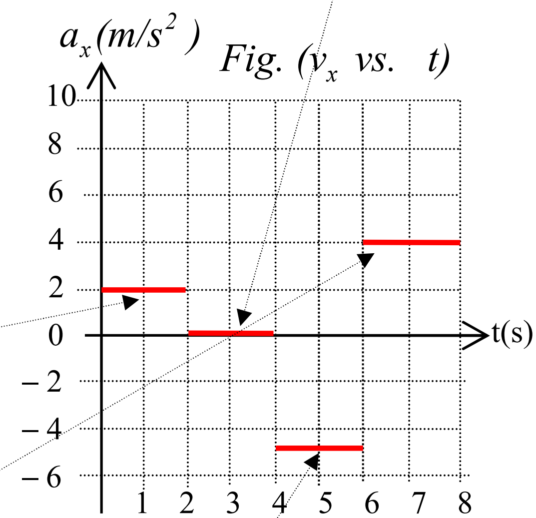
$$\Delta x = \frac{1}{2}(2)(6+2) + \frac{1}{2}(3+2)(6) + \frac{1}{2}(2)(-4) = 19m$$

(b) Find the position x at t= 7 sec.

$$x(7) - x(0) = x(7) - 4 = 19\,m \quad \Rightarrow \quad x(7) = 23m$$

(c) Draw the acceleration curve.

$$a_x = 0$$



$$a_x = \underbrace{\frac{dx}{dt} = \frac{\Delta x}{\Delta t}}_{\text{slope of tangent=S of line}} = \frac{x(2) - x(0)}{2 - 0} = \frac{6-2}{2-0} = 2m/s^2$$

$$a_x = \frac{0-(-4)}{1} = 4m/s^2$$

$$a_x = \frac{-4-6}{2} = -5m/s^2$$