Chapter 3 Problems

Ch3-Ed.9 page 72	**Problems ch3: 4, 12, 21, 23, 37, 51**

Prob. 3-4. Two points in a plane have **polar coordinates** (2.50 m, 30.0°) and (3.80 m, 120.0°). Determine (a) the Cartesian coordinates of these points and (b) the distance between them.

$$x_1 = r_1 \cos\theta \Rightarrow x_1 = 2.5\cos 30 = 2.17 \text{ m}$$

$$y_1 = r_1 \sin 30 = 2.5\sin 30 = 2.5\times\frac{1}{2} = 1.25 \text{ m}$$

$$(x_1, y_1) = (2.17, 1.25) \text{ m}$$

$$x_2 = r_2 \cos\theta \Rightarrow x_2 = 3.8\cos 120 = -1.90 \text{ m}$$

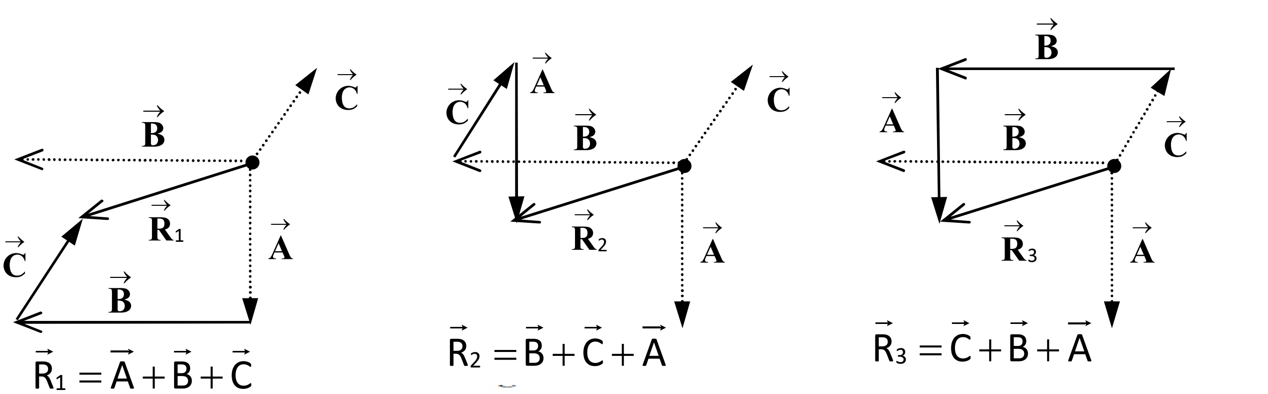
$$y_2 = 3.8\sin 120 = 3.29 \text{ m}, \qquad (x_2, y_2) = (-1.90, 3.29) \text{ m}$$

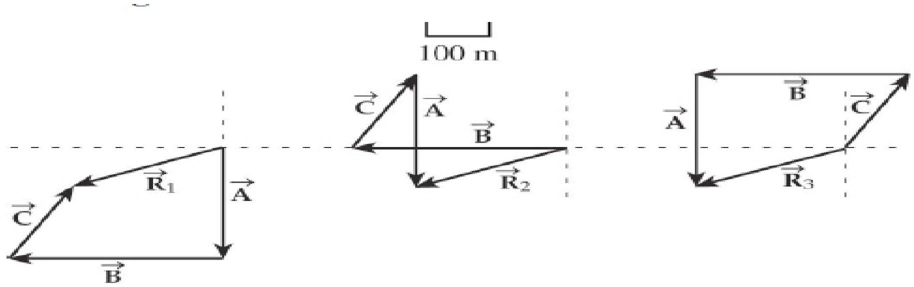
$$d = \sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{4.07^2 + 2.04^2} = 4.55 \text{ m}$$

Prob. 3-12. Three displacements are $\vec{A} = 200$ m **due south**, $\vec{B} = 250$ m **due west**, and $\vec{C} = 150$ m at 30° **east of north**

(a) Construct a separate diagram for each of the following possible ways of adding these vectors: $\vec{R}_1 = \vec{A} + \vec{B} + \vec{C}$; $\vec{R}_2 = \vec{B} + \vec{C} + \vec{A}$; $\vec{R}_3 = \vec{C} + \vec{B} + \vec{A}$

(b) Explain what you can conclude from comparing the diagrams.





(b) $\vec{R} = \vec{R}_1 = \vec{R}_2 = \vec{R}_3$

Prob. 3-21. While exploring a cave (استكشاف كهف), a spelunker (مغاوري) starts at the entrance (مدخل) and moves the following distances in a horizontal plane. She goes **75.0 m north**, **250 m east**, **125 m** at an angle **θ = 30.0° north of east**, and **150 m south**. Find her **resultant displacement** from the cave entrance. Figure P3.21 suggests the situation but is not drawn to scale.

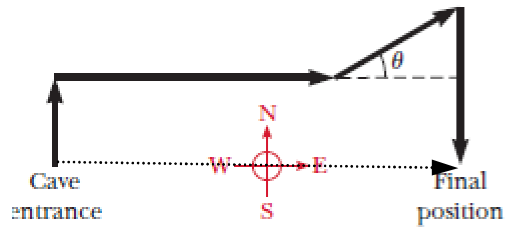


Figure P3.21

$$\vec{d}_1 = 75\hat{j}\ \text{m}, \qquad \vec{d}_2 = 250\hat{i}\ \text{m},$$

$$\vec{d}_3 = (125\cos 30^\circ\,\hat{i} + 125\sin 30^\circ\,\hat{j})\ \text{m},$$

$$\vec{d}_4 = (-150\,\hat{j})\ \text{m}$$

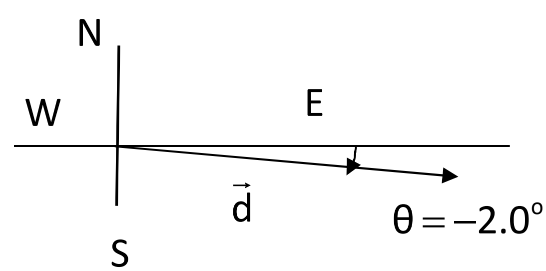
$$\vec{d} = (385\hat{i} - 12.5\hat{j})\ \text{m}$$

$$d_x = 250 + 125\cos 30 = 358\ \text{m},$$

$$d_y = 75 + 125\sin 30 - 150 = -12.5\ \text{m},$$

$$R = \sqrt{358^2 + (-12.5)^2} = 358.2\,\text{m},$$

$$\tan\theta = \frac{R_y}{R_x} = \frac{-12.5}{358} = -0.0349, \quad \theta = -2.0^\circ$$



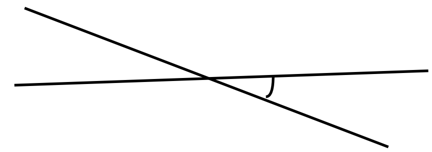
Prob. 3-23 Consider the two vectors $\vec{A} = (3\hat{i} - 2\hat{j})$ and $\vec{B} = -\hat{i} - 4\hat{j}$. Calculate **(a)** $\vec{A} + \vec{B}$, **(b)** $\vec{A} - \vec{B}$, **(c)** $|\vec{A} + \vec{B}|$, **(d)** $|\vec{A} - \vec{B}|$, and **(e)** the direction of $\vec{A} + \vec{B}$ and $\vec{A} - \vec{B}$.

(a) $\vec{A} + \vec{B} = (3\hat{i} - 2\hat{j}) + (-\hat{i} - 4\hat{j}) = 2\hat{i} - 6\hat{j}$

(b) $\vec{A} + (-\vec{B}) = (3\hat{i} - 2\hat{j}) - (-\hat{i} - 4\hat{j}) = 4\hat{i} + 2\hat{j}$

(c) $|\vec{A} + \vec{B}| = \sqrt{2^2 + 6^2} = 2\sqrt{10} = 6.32$

(d) $|\vec{\mathbf{A}} - \vec{\mathbf{B}}| = \sqrt{4^2 + 2^2} = \sqrt{20} = 4.47$



(e) $\theta_{|A+B|} = \tan^{-1}(-6/2) = \tan^{-1}(-3) = -71.6^0 = 287.4^0$

$\theta_{|A-B|} = \tan^{-1}(2/4) = \tan^{-1}(0.5) = 26.6^0$

Prob. 3-37. (a) Taking $\vec{A} = (6.0\hat{i} - 8.0\hat{j})$ and $\vec{B} = -8.0\hat{i} + 3.0\hat{j}$ units, and $\vec{C} = 26.0\hat{i} + 19.0\hat{j}$ units, determine ***a*** and ***b*** such that $a\vec{A} + b\vec{B} + \vec{C} = 0$.
(b) A student has learned that a single equation cannot be solved to determine values for more than one unknown in it. How would you explain to him that both *a* and *b* can be determined from the single equation used in part (a)?

$a\vec{A} - b\vec{B} + \vec{C} = 0$

$a\vec{A} = 6a\hat{i} - 8a\hat{j},$

$b\vec{B} = -8b\hat{i} + 3b\hat{j}$

$\vec{C} = 26.0\hat{i} + 19.0\hat{j}$ Add them

$a\vec{A} + b\vec{B} + \vec{C} = 0 \quad \Rightarrow \quad 0 = (6a - 8b + 26)\hat{i} + (-8a + 3b + 19.0)\hat{j}$

$6a - 8b + 26 = 0, \quad \Rightarrow \quad a = 1.33\,b - 4.33$

$-8a + 3b + 19 = 0 \quad \Rightarrow \quad -8(1.33b - 4.33) + 3b + 19 = 0$

$7.67b = 53.67 \quad \rightarrow b = 7.00$

$a = 1.33(7.00) - 4.33 = 5.00.$

Prob. 3-51. A person going for a walk follows the path shown in Fig. P3.51. the total trip consists of four straight-line paths. At the end of the walk, what is the person's resultant displacement measured from the starting point?

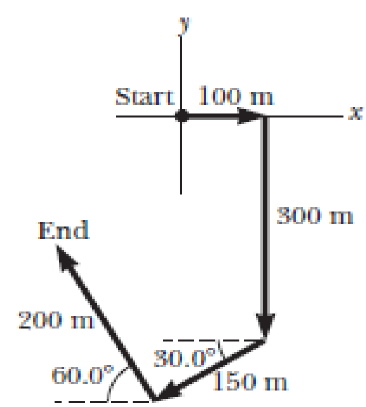


Figure P3.51

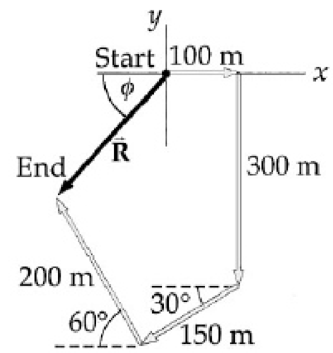
$\vec{\mathbf{d}}_1 = 100\,\hat{\mathbf{i}}$ m,

$\vec{\mathbf{d}}_2 = (-300\,\hat{\mathbf{j}})$ m

$$\vec{\mathbf{d}}_3 = (-150\cos30^\circ\,\hat{\mathbf{i}} - 150\sin30^\circ\,\hat{\mathbf{j}}) = (-130\,\hat{\mathbf{i}} - 75\,\hat{\mathbf{j}})\ \text{m}$$

$$\vec{\mathbf{d}}_4 = (-200\cos60^\circ\,\hat{\mathbf{i}} + 200\sin60^\circ\,\hat{\mathbf{j}}) = (-100\,\hat{\mathbf{i}} + 173\,\hat{\mathbf{j}})\ \text{m}$$

$$\vec{\mathbf{d}} = \vec{\mathbf{d}}_1 + \vec{\mathbf{d}}_2 + \vec{\mathbf{d}}_3 + \vec{\mathbf{d}}_4 = (-130\,\hat{\mathbf{i}} - 202\,\hat{\mathbf{j}})\ \text{m}$$



$$d = \sqrt{(-130)^2 + (-202)^2} = 240\ \text{m}$$

$$\theta = \tan^{-1}\left(\frac{-202}{-130}\right) = 57.2^\circ + 180 = 237.2^\circ$$

The resultant points into **the third quadrant** instead of the first quadrant. The angle counterclockwise from the +*x* axis is ($\theta^0 = 180 + 57.2 = 237.2^\circ$)

Prob. 3-24. A map suggests that **Atlanta** is **730 miles** in a direction **of 5.00° north of east** from **Dallas**. The same map shows that **Chicago is 560 miles** in a direction of **21.0° west of north** from **Atlanta**. Figure P3.22 shows the locations of these three cities. Modeling the Earth as flat, use this information to find the **displacement from Dallas to Chicago**.

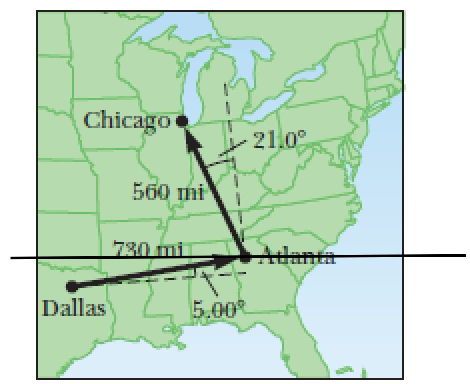


Figure P3.24

$$\mathbf{d}_x = 730\cos5^\circ - 560\sin21^\circ = 527\ \text{miles}$$

$$\mathbf{d}_y = 730\sin5^\circ + 560\cos21^\circ = 586\ \text{miles}$$

$$\vec{\mathbf{d}} = (527\,\hat{\mathbf{i}} + 586\,\hat{\mathbf{j}})\ \text{m}$$

$$|\mathbf{d}| = \sqrt{527^2 + 586^2} = 788\ \text{mi}$$

at $\theta = \tan^{-1}\frac{586}{527} = \tan^{-1}(1.11) = 48^\circ$